

**CRACK ANALYSIS USING EXTENDED FINITE  
ELEMENT METHOD WITH VIRTUAL CRACK  
CLOSURE TECHNIQUE**

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**2016**

**CRACK ANALYSIS USING EXTENDED FINITE ELEMENT METHOD  
WITH VIRTUAL CRACK CLOSURE TECHNIQUE**

**by**

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**Thesis submitted in fulfilment of the**

**requirements for the degree of**

**Master of Science**

**MARCH 2016**

## **DECLARATION**

I hereby declare that the work reported in this thesis is the result of my own investigation and that no part of the thesis has been plagiarized from external sources. Materials taken from the sources are duly acknowledged by giving explicit references.

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## ACKNOWLEDGEMENT

First and foremost, I would like to express the highest appreciation to my main supervisor, Dr.-Ing Muhammad Razi Abdul Rahman for his extremely supportive and ever accessible throughout this research work. His expertise, understanding, tolerance and patience, added considerably to my post-graduate experience and provided a good basis for the project and present thesis.

Secondly, I would also like to express my gratitude to my co-supervisor, Prof. Dr. Ishak Hj. Abdul Azid from Universiti Kuala Lumpur (UniKL) for his encouragement and invaluable guidance in helping me to cope in every stages of this project. Besides, I gratefully acknowledge the financial support of the ERGS, RUI grant scheme and MyMaster scholarship for me to complete the research work.

Furthermore, I also wish to thank my friends from School of Mechanical Engineering, Mr Lim Chong Hui, Mr Woo Wing Hon, Mr Leong Thye Jien and Miss Lee Tze Pin for their useful advice in this project. Moreover, I also thank all the staff of Institute of Postgraduate Studies, Universiti Sains Malaysia (USM) who have helped me during the research.

Last but not least, I would like to thank my family members especially my mother for their moral support, encouragement and tolerance, which have enabled me to complete this research work.

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## LIST OF ABBREVIATION

SIF	Stress intensity factor
SERR	Strain energy release rate
VCCT	Virtual crack closure technique
FE	Finite element
FEM	Finite element method
XFEM	Extended finite element method
XFEM-VCCT	Extended finite element method based virtual crack closure technique
FEA	Finite element analysis
DCB	Double cantilever beam
LEFM	Linear elastic fracture mechanics
Al	Aluminum
Cu	Copper
VCE	Virtual crack extension
MTS	Maximum tangential stress
SED	Strain energy density
FMM	Fast marching method
PUM	Partition of unity method
LSM	Level set method
EP	Electronic packaging
DOF	Degree of freedom
PCB	Printed circuit board

## LIST OF SYMBOL

SYMBOL	Description	UNIT
$\sigma_{UTS}$	Material Tensile Strength	Pa
$\sigma$	Normal Stress	$N/m^2$
$K$	Stress Intensity Factor	MPa
$G$	Strain Energy Release Rate	N/mm
$G_{IC}$	Critical Strain Energy Release Rate	N/mm
$\Gamma$	Line Integral	-
$K_I$	Stress Intensity Factor in Mode I	MPa
$K_{II}$	Stress Intensity Factor in Mode II	MPa
$\nu$	Poisson's Ratio	-
$\kappa$	Constant in Plane Strain/Stress	-
$m$	Total Number of Element	-
$\eta$	Error percentage	-
$\varepsilon$	Strain	-
$h_e$	Old element	-
$p$	Interpolation shape function order	-
$r$	Distance of force	m
$E$	Young's Modulus	GPa
$J$	Crack driving force, $J$ -integral	$J/m^2$
$W_s$	Strain energy density	$J/m^3$
$T$	Traction vector	$N/m^2$
$u$	Displacement	m

$X_i$	Shear force in VCCT	N
$Z_i$	Opening Force in VCCT	N
$\theta$	Angle ( ° )	-
$\tau$	Shear stress	$N/m^2$
$a$	Initial crack length	mm
$Y$	Finite-geometry correction factor	-
$\beta$	Crack angle ( ° )	-
$\gamma_s$	Elastic surface energy	N/m
$\gamma_p$	Plastic deformation energy	N/m
$I$	Second moment of area	$mm^4$
$C$	Elastic compliance	-
$B$	Width	m
$T$	Thickness	m
$\Delta$	Deflection	m
$u_I$	Nodal displacement vector	m
$a_j$	Nodal enriched degree of freedom of jump discontinuity	-
$N_\Gamma$	Nodes belonging to elements cut by the crack	-
$b_K^\alpha$	Nodal degree of freedom of the crack-tip enrichment	-
$N_\Lambda$	Nodes belonging to elements containing crack-tip	-
$t_s$	Time	s
$\phi$	Signed distance function	-
$\psi$	Tangetial level set function	-
$F$	Applied load	N

# **ANALISIS RETAKAN DENGAN MENGGUNAKAN KAEDAH LANJUTAN UNSUR TERHINGGA BERSEMPENA TEKNIK MAYA PENUTUPAN RETAK**

## **ABSTRAK**

Kegagalan retakan adalah masalah umum dalam struktur kejuruteraan di mana retak kecil berpotensi untuk menjejaskan keseluruhan struktur. Walaupun terdapat teori yang sedia ada dan penemuan secara berterusan dalam bidang mekanik patah, kajian analisis perambatan retakan masih dalam penyelidikan. Faktor Kamatan tegasan (SIF) dan Kadar Tenaga Keterikan (SERR) adalah parameter retakan penting yang digunakan untuk menganggar kelakuan struktur yang mengandungi retakan dan sekitar hujung retakan.

Dalam kajian ini, XFEM bersempena dengan VCCT diperkenalkan untuk mengira parameter SERR dan SIF. Dalam kombinasi tersebut, VCCT dinyatakan sebagai kriteria kegagalan untuk mengira SERR di hujung retakan. Selain itu, analisis tiada perambatan retakan dibentangkan di plat retakan pinggir dan plat retakan condong. Sementara itu, analisis perambatan retakan ditunjukkan oleh rasuk dua julur (DCB). Keputusan simulasi dibandingkan dengan keputusan teori di mana ia sebagai rujukan. Dalam analisis penumpuan, analisis jejaring berstruktur menggunakan XFEM-VCCT telah terbukti secara stabil dalam analisis apabila retakan terletak di muka elemen dan juga di pinggir elemen. Dalam analisis retakan pinggir, purata perbezaan diperolehi oleh XFEM-VCCT adalah kira-kira 0.5% manakala ralat purata dikira dengan FEM konvensional adalah lebih daripada 1% berbanding dengan penyelesaian teori. Selain itu, dalam analisis retakan condong yang berbeza sudut, purata perbezaan yang diperolehi XFEM-VCCT adalah 1.06%



dan 1.45% bagi  $K_I$  dan  $K_{II}$ . Di samping itu, purata perbezaan dalam analisis perubahan panjang retakan adalah 2.58% bagi  $K_I$  dan 1.62% bagi  $K_{II}$ . Untuk analisis perambatan retakan, XFEM-VCCT menunjukkan beban maksimum yang dicapai adalah 1246 N dengan ralat 1.11% berbanding dengan 1260 N yang dikira secara teori dan panjang retakan maksimum ialah 840mm, manakala pengiraan teori adalah 880m. Daripada keputusan yang diperolehi, bilangan elemen yang digunakan oleh XFEM-VCCT dalam semua model adalah jauh kurang daripada FEM konvensional. Secara keseluruhannya, kaedah yang kini dicadangkan telah menunjukkan keputusan yang baik melalui perbandingan. Ia banyak mengurangkan masa pengiraan komputer dan meningkatkan penyelesaian dalam analisis retakan untuk industri kejuruteraan.

# CRACK ANALYSIS USING EXTENDED FINITE ELEMENT METHOD WITH VIRTUAL CRACK CLOSURE TECHNIQUE

## ABSTRACT

Fracture and failure are general problems in engineering structures where a small crack can potentially compromise the structural integrity. Although there are significant theories and findings in fracture mechanics field, the study of crack propagation analysis is still actively pursued. Stress Intensity Factor (SIF) and Strain Energy Release Rate (SERR) are the important fracture parameters used to estimate the structures behavior containing crack and surrounding of crack-tip.

In this study, XFEM in conjunction with VCCT was utilized to calculate the parameters of SERR and SIF. In the combination, the VCCT was specified as the fracture criterion to calculate the SERR at the crack tip. As case studies, non-crack propagation analysis was presented in edge-crack plate and inclined-crack plate. For crack propagation analysis, the double cantilever beam (DCB) problem was used as case study. The simulation results were compared to the theoretical results as the reference point. In the convergence analysis, the structured mesh analysis using XFEM-VCCT was proven to be stable for analysis when the crack lied on the face of element and also on the edge of element. In the edge-crack analysis, the average error obtained by XFEM-VCCT was approximately 0.5% whereas the average error computed by conventional FEM in conjunction with  $J$ -integral was more than 1% compared to theoretical results. Furthermore, in the different inclined-crack angle analysis, the average error produced by XFEM-VCCT was 1.06% and 1.45% for  $K_I$  and  $K_{II}$ , respectively. Besides, the average error for different crack length analysis was 2.58% in  $K_I$  and 1.62%  $K_{II}$ . For crack propagation analysis, XFEM-VCCT showed the maximum load achieved was 1246 N with 1.11% error compared to 1260

N determined theoretically and the maximum crack length was 840mm, while the theoretical result was 880m. From the results, the number of elements used by XFEM-VCCT applied to all the models was significantly lower than the conventional FEM. On the whole, the proposed method showed good agreement through the results comparison. It greatly reduced the computational time and enhanced solution to fracture analysis in the engineering industry.

# CHAPTER 1

## INTRODUCTION

### 1.1 Background of research

Fracture mechanics is a specific field which deals with fracture and failure process in engineering of materials and constructions. Failures occur in many manufacturing defects, including uncertainties in the mechanical or thermal loading, environment, initial cracks, deficiency of materials, and inadequacies in design. Fracture and failure are general problems in engineering structures where a small crack can potentially compromise the structural integrity. The stresses and strains field analysis near the crack-tip are the basis for understanding the behaviour of cracks. Although a plastic and damaged zone is present at the crack-tip, the linear elastic analysis provides an accurate mapping of reality for materials such as steel. For high ductility materials or extreme loads, elastic-plastic behaviour laws are taken into account (Recho, 2012).

Common three modes of fracture are shown in Figure1.1. Mode I fracture called Opening mode where a tensile stress applied in normal direction to the plane of the crack (a traction mode). Mode II fracture is Sliding mode where a shear stress acting parallel to the plane of the crack and perpendicular to the crack front (a shear mode). Mode III fracture is Tearing mode where a shear stress acting parallel to the plane of the crack and parallel to the crack front (a torsion mode). A fracture body can be loaded in one or combination of two or three modes.

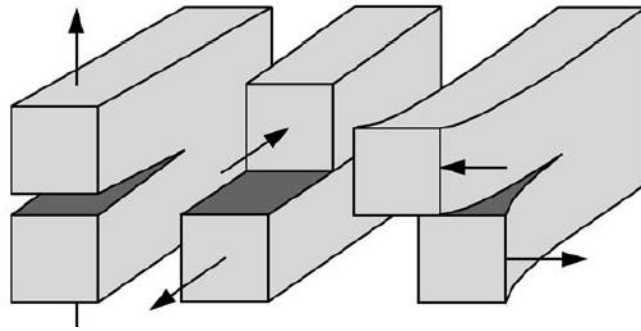


Figure 1.1 Loading Modes I, II, and III.

The object of fracture mechanics is to deliver quantitative solutions to identify the problems concerning cracks in structures. As illustrated below, by considering a structure contains pre-existing flaws or initial crack, the cracks may grow with time due to various causes such as fatigue, stress and creep, Fig 1.2(a). The residual strength is the failure strength of the structure as a function of crack size, decreases with increasing crack size as shown in Fig 1.2(b). The structure may fail after a time the residual strength becomes lower (Janssen et al., 2002).

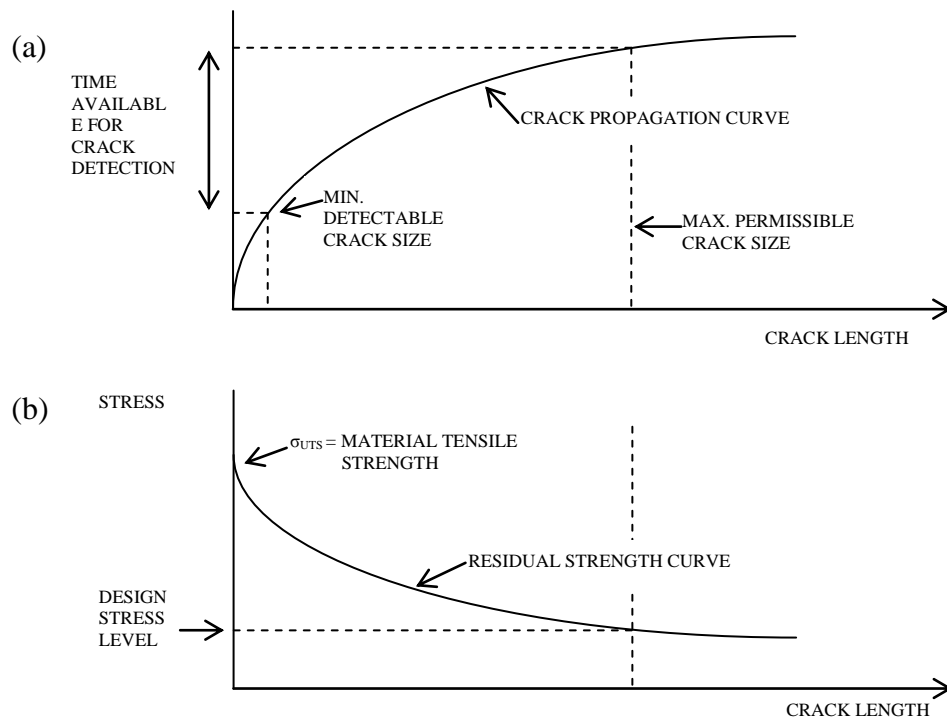


Figure 1.2(a) and (b) The Engineering Problem of a Crack in a Structure (Janssen et al., 2002).

Stress Intensity Factor (SIF),  $K$  and Strain Energy Release Rate (SERR),  $G$  are the important fracture parameters to estimate the performance of structures containing cracks and singular stress field. SIF shows the stress field intensity measurement in the crack-tip region and the possibility to analyse crack growth or the failure when a load is applied to the structure. Generally, SIF can be calculated by analytical method and also numerical method (explained in Chapter 3 and 4). Since SIF and SERR are the crack driving forces to open a crack, a relationship between  $K$  and  $G$  must exist. The derivation of such relationship will be discussed in the further chapter. Through the stress and strain analysis or the energy release rate during the crack growth, SIF can be determined by several approaches. By noticing that crack will propagate when SERR exceeds the material property critical strain energy release rate,  $G_C$ , i.e., when  $G > G_C$ . Virtual Crack Closure Technique (VCCT) is an energy release rate based method where the crack grows with an infinitesimal increment. The calculation of SERR using VCCT is depending on the energy variation when an extension of crack length is imposed. This technique was proposed by Rybicki and Kanninen in 1977 (Rybicki and Kanninen, 1977b). Analytical solution can be directly applied due to physical effects through the model problem. Compared to analytical method, numerical method can be used to perform an analysis which takes into account all the boundary condition effects.

Numerical calculation is one of the efficient and universal methods for solving partial differential equations of applied mechanics for fracture analysis. The Finite Element Method (FEM) has been established in engineering industry as a powerful and effective tool of modern engineering design and stress field analysis. The evaluation of stress intensity factors by FEM is a technique widely used in recent years. However, some mechanical phenomena are difficult to model with

FEM in the study of cracks which demands significant mesh refinement along the entire crack front, especially crack propagation (McNary, 2009). FEM remains substantially advantages merely for continuous field problems, but when in discontinuous field problems, a new mesh to ensure that the element edges align with the discontinuity and considerable high mesh refinement around the discontinuity is required.

Through the improvements of numerical method, the limitation of FEM in discontinuous field can be eliminated by Extended Finite Element Method (XFEM) introduced by Belytschko and Black (Belytschko and Black, 1999). XFEM emerged as a solution to solve the problem of conventional FEM by applying enrichment functions at the position of the material interface or topology discontinuity instead of re-meshing the entire structure for crack growth simulation (Villanueva and Yu, 2013). XFEM allows a crack represented by finite elements with no requirement of mesh modification to follow the crack propagation compared to conventional crack modelling techniques. In XFEM, the crack is geometrically independent of the mesh (Levn and Rickert, 2012).

Although there are significant theories and findings in fracture mechanics field, the study of crack propagation analysis is actively pursued. In this study, a technique of combination XFEM and Virtual Crack Closure Technique (VCCT) is introduced to calculate the parameters of SERR and SIF. The results of SIF obtained by XFEM-VCCT will be compared based on the theoretical and numerical solution

## **1.2 Problem Statement**

Finite element method (FEM) is a numerical technique to solve partial differential equations as well as integral equations. It has undoubtedly become the most popular analytical tool for solving a wide range of engineering and physical

problem. Despite all achievement, the basis of FEM remains a disadvantage for discontinuous fracture mechanics (Mohammadi, 2008). Modelling a stationary crack with FEM requires a mesh that conforms to the geometric discontinuity. The singular stress field at crack-tip needs mesh refinement in crack propagation analysis. In addition, to model crack propagation is even more cumbersome because re-meshing process must be executed continuously to match the geometry of the discontinuity as the crack progresses (Cadge et al., 2011). Difficulties arise when using the traditional FEM for analysing in fracture mechanics such as changing the shape of the material structure requires a new mesh to ensure that the element edges align with the discontinuity (Abdelaziz and Hamouine, 2008). The discontinuous field problem is well known to be computationally expensive to obtain accurate solutions with polynomial approximations (Keswani et al., 2012).

The extended finite element method (XFEM) is a technique that extends FEM approach for the solutions to differential equations with discontinuous functions. The XFEM technique greatly simplifies the mesh generation requirements for models with pre-existing cracks. It was developed to reduce the difficulties in solving problems with localized features that are not efficiently resolved by mesh refinement. One of the significant advantage of XFEM is the finite element mesh does not need to be updated or it does not require to change the mesh during crack propagation (Levn and Rickert, 2012). The XFEM emerged as a solution to the shortcoming of the FEM by applying enrichment functions at the position of the material interface or topology discontinuity instead of re-meshing the entire structure.

In this study, considering the fact of rare research of effectively using the combination of XFEM and VCCT to compute SERR and SIF, this technique is applied in a two-dimensional benchmark edge crack finite plate, inclined crack plate,