A MICROMECHANICAL DAMAGE MODEL FOR INITIALLY ANISOTROPIC MATERIALS

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Summary We propose a new model of brittle damage for initially orthotropic materials. The proposed strain energy-based formulation allows to account for the interaction between initial and induced anisotropies and to address the very challenging issue of opening-closure effects (unilateral behaviour). In order to derive the complete model including the damage growth, we take advantage of micromechanical developments suitably combined with the thermodynamics framework of the standard generalized materials. The model has been implemented within the finite-element code ABAQUS and various numerical simulations have been carried out to illustrate its predictive capabilities. In particular, emphasis is put on the evolution of the material symmetry and the influence of microcracks opening-closure states on the damage process.

CONTEXT AND ISSUES

Several experimental studies have shown that microcracking governs the mechanical behavior of various engineering materials. Some of the main macroscopic consequences induced by these microcracks are degradation elastic properties, load-induced anisotropy and microcracks opening-closure effects (also called unilateral effects). For many industrial applications, an appropriate modelling of such degradation process is required to derive a reliable evaluation of mechanical systems. Continuum Damage Mechanics offers an appropriate theoretical framework to account for the inelastic response associated to these features. Accounting for these phenomena in the context of initially anisotropic materials such as composites or sedimentary rocks constitutes a very challenging issue. The interaction between primary anisotropy and damage-induced one is itself a difficult task [1,2]. Addressing the additional representation of damage activation–deactivation effects is then most often restricted to damage in some given specific directions [3]. The present study intends to present a new model for brittle damage for initially anisotropic materials based on results obtained in Goidescu et al. [4]. The resulting analytical 2D micromechanical model (see [5]) avoids inconsistencies often associated to the description of the unilateral behavior [6,7], while exhibiting a physical motivation of the deactivation representation.

CONSTITUTIVE LAW

Small strain, rate independent and isothermal conditions are considered. Dilute concentration of frictionless microcracks is assumed. Following micromechanical arguments [4], a discrete approach, consisting in a finite number of microcracks systems, is considered for the damage description based on the set of scalar microcracks density parameters $\mathbf{d} = (d_i)_{i=1,N}$, with d_i the microcrack density of the i$^{\text{th}}$ microcrack system. Orientations $\mathbf{n}_i$ (unit normal) of microcracks are regularly distributed and not restricted to initial structural axes.

In the virgin state (without damage), the material is assumed to exhibit an orthotropic elastic linear behaviour defined by the orthonormal basis $(\mathbf{e}_1, \mathbf{e}_2)$ ($W_0(\varepsilon, \mathbf{A})$ denotes the virgin free energy where second tensor $\mathbf{A} = \mathbf{e}_1 \otimes \mathbf{e}_1$ accounts for initial anisotropy). The macroscopic free energy corresponding to such a material weakened by arbitrarily oriented microcracks systems is essentially derived from standard micromechanical analysis, with the help of Hill lemma in the context of cracked bodies and the use of crack displacements expressions in the context of anisotropic elasticity [4]. The thermodynamic potential comes thus to the following form:

$$W(\varepsilon, \mathbf{d}) = W_0(\varepsilon, \mathbf{A}) + \sum_{i=1}^{N} d_i \begin{cases} h_{open}(\varepsilon, \mathbf{A}) + h'_{open}(\varepsilon, \mathbf{n}_i) + h''_{open}(\varepsilon, \mathbf{A}, \mathbf{n}_i), & if\ g(\varepsilon, \mathbf{A}, \mathbf{n}_i) > 0 \\ h_{clos}(\varepsilon, \mathbf{A}) + h'_{clos}(\varepsilon, \mathbf{n}_i) + h''_{clos}(\varepsilon, \mathbf{A}, \mathbf{n}_i), & if\ g(\varepsilon, \mathbf{A}, \mathbf{n}_i) \le 0 \end{cases}$$

Such formulation accounts for the different behaviour of microcracks according to their status (open or closed) defined by the opening-closure criterion *g*. Moreover, for both situations, elementary free energy contributions may represent various interaction modes between initial and induced anisotropies: *h* addresses isotropic-like damage effect that keeps the initial symmetry of the material, *h'* introduces weak-coupling with a directional dependence similar to the one derived in the isotropic context, whereas *h''* accounts for strong interaction between initial and damage induced anisotropies that may lead to a complex resulting elasticity. It is to note that appropriate mathematical conditions are satisfied, especially the objectivity and the continuity of class C^1 of *W* (that ensures the existence and continuity of the stress-strain response and of the thermodynamic forces) and the piecewise continuity of class C^∞ of the tangent operator. Moreover, micromechanics provides a physical justification to the representation of activation-deactivation effects, regarding both the complex recovery

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phenomenon occurring at the closure of defects and the opening-closure criterion that derives from the cancellation of the normal jump displacement of microcracks.

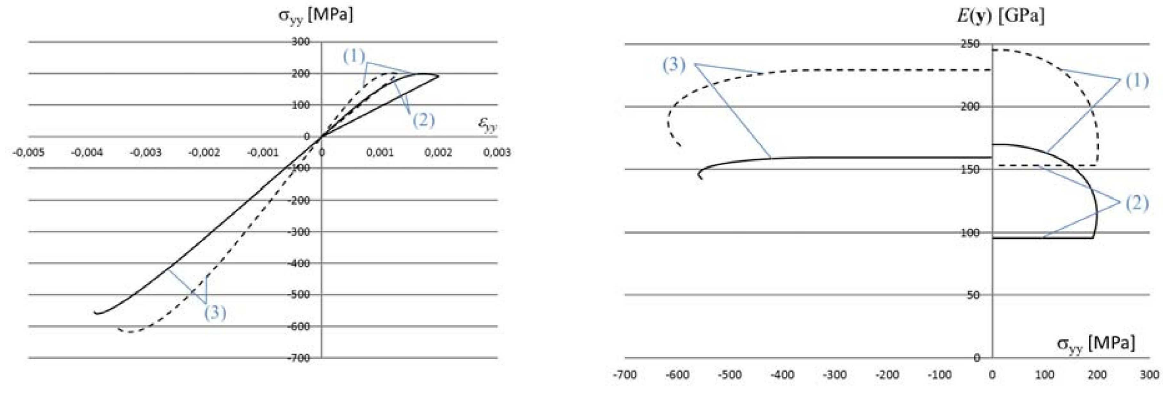
The framework of generalized standard materials is chosen to easy the satisfaction of the thermodynamical admissibility of the formulation. Following classical form, damage evolution is formulated for each system independently (see [8] for instance):

$$f(F^{d_i}, d_i) = F^{d_i} - k_0\,(1+\eta\, d_i), \quad \textit{with} \quad F^{d_i} = -\partial W / \partial d_i$$

Dealing with a reduced number of coefficients (4 elastic coefficients of the virgin medium and 2 parameters related to the damage evolution, k_0 and η), such formulation may take into account very complex damage growth. Indeed, since the damage thermodynamic force F^{d_i} is considered as the driving force of the damage process, the damage evolution depends also on the opening and closure of microcracks and on the interaction between anisotropies.

APPLICATIONS TO A CERAMIC COMPOSITE

The mechanical response of a ceramic composite (SiC-SiC) under uniaxial tension load followed by compression is studied. Axis and off-axis loads have been investigated. Considering a sufficiently large number N of systems provides a relevant representation of microcracks density distribution obtained in such a case. Fig. 1 shows first that the model is able to describe the specific asymmetric behavior between tensile and compressive loads of brittle materials. The different opening-closure state between these two situations explains the differences in the yield stress and in the damage evolution: all microcracks are open during tensile load and some of them get closed during compression. Such a test highlights also the influence of unilateral effects on elastic properties. As shown by the evolution of Young modulus E(**y**) in the axial direction **y**, elastic properties recover to some extent their initial value (partial damage deactivation) when compressive load induces the closure of some microcracks. Finite element simulations on structure case studies (notched samples) also illustrate these features. Note finally, that such model provides a suitable basis for the consideration of a supplementary dissipative mechanism such as the friction on the closed microcracks lips.



Axial stress-strain response

Axial Young modulus E(**y**) according to the axial stress

Fig. 1. Uniaxial tension load (1) followed by unloading (2) and then compression (3) along unit axis **y** on a ceramic composite (full line corresponds to axis load along a structural axis of the material, dashed line denotes an off-axis loading case).

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