Reasoning Skills among Students: a Meta-Analysis

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Abstract

Many countries have undergone a paradigm shift in their approach of teaching, learning and assessment where emphasis is heavily placed on thinking skills. This skill is indeed a vital requirement in developing twenty-first century skills among students as there is a need for high levels of thinking, reasoning and collaborating in today's era. In the Malaysian context, there is a current need in elevating reasoning skills among high achievers as the Ministry of Education has realized that high achievers have not been performing excellently in global assessments, specifically. Therefore, this conceptual paper aims at identifying the components of reasoning skills that high achievers face difficulties in when learning Mathematics. From the meta-analysis conducted, two major components of reasoning skills were identified; knowledge reasoning and systemic thinking. Based on these findings from literature, these two components can be used as a learning framework to elevate students reasoning skills.

Keywords: Reasoning Skills; Knowledge Reasoning; Systemic Thinking; High Achievers

1.0 Introduction

One of the fundamental aspects in mathematics is reasoning. This is because reasoning enables one to understand mathematics in an effective manner as well as to create meaningful learning (Bahagian Pembangunan Kurikulum, 2013; Brodie, 2010). This skill has been identified to be one of the critical requirements among students to enable them to survive in the twenty-first century (The Partnership for 21st Century Learning, 2015; Kementerian Pendidikan Malaysia, 2013; Singapore Ministry of Education, 2013; Griffin, McGaw, & Care, 2012; Lankinen, 2010).

The development of reasoning skills is closely associated not only with intellectual development but with communication as well (Bahagian Pembangunan Kurikulum, 2013). To achieve this, it is not sufficient to just consider the content of the curriculum but also on how a teacher teaches in the mathematics classroom (Qualifications and Curriculum Authority, 2004). Numerous reports have shown that students, specifically high achievers are facing difficulty is displaying reasoning (Johnny, 2015; Rosmawati binti Ismail, 2014; Kementerian Pendidikan Malaysia, 2013; Mohini Mohamed & Johnny, 2013; Cheah, 2010; Maria Salih, 2010; Mohd Ariff Ahmad Tarmizi, Rohani Ahmad Tarmizi & Mohd Zin Bin Mokhtar, 2010).

Therefore this article aims to identify the components of reasoning skills that high achievers face difficulties in when learning mathematics. Besides that, certain strategies in honing reasoning skills that have been practiced and proven effective are also discussed.

2.0 Reasoning Skills

According to Qualifications and Curriculum Authority (2004), reasoning skill is defined as recognising patterns and opportunities to generalise; explaining why an answer must be correct; constructing chains of deductions; understanding the difference between a practical demonstration and proof; appreciating assumptions and constraints and knowing how varying these would affect results. This can be achieved through the process called 'specialising' (Mason, Burton, & Stacey, 2010). As one selects and tries different examples, he will eventually discover a pattern or rule developing within himself. The key is to notice it and try to bring it to the surface; transforming it into words or numbers. To articulate it will cause the individual to check and perhaps modify the schema constructed while one systematically prepares the ground for generalisation. This process of checking will lead to students identifying errors made, hence, leading to better conjectures. Mason *et al*. (2010) describes mathematical reasoning as a process that involves trying to convince yourself first, then a friend who asks pointed but friendly questions, and lastly, a sceptic who refuses to take anything at face value.

Griffin *et al*. (2012) outlined that reasoning skills should be emphasised in the three main domains of learning; knowledge, skill and attitude. In the knowledge domain, students should learn to reason effectively, use systemic thinking, evaluate evidence, solve problem as well as to articulate the result of an inquiry. In the skill domain, students should be able to use various types of reasoning and systemic thinking in the execution of procedures. Reasoning attitudes comprise of making reasoned judgments and decisions, solve problems as well as attitudinal disposition throughout the whole process of Ways of Thinking.

These three domains of learning can be processed through three main levels; self-system, metacognitive system and cognitive system (Marzano and Kendall, 2008). The levels of processing a mathematical knowledge should encompass two sources; general understanding and procedural knowledge. Procedural knowledge begins from information when you begin working on a problem and identifying what you need to establish (Mason, Burton, & Stacey, 2010; Marzano and Kendall, 2008; Schoenfeld, 1994). These eventually develop skills such as tactics, algorithms, single rules etc. Understanding of the knowledge becomes critical when pupils move from simple problems to complex problems or unfamiliar problems (Qualifications and Curriculum Authority, 2004; Schoenfeld, 1994).

Knowledge reasoning should be about how one tackles questions while reflecting on experiences (Mason *et al*., 2010; Schoenfeld, 1994). When one works on difficult problems, reasoning becomes the fundamental part of the discovery process, Familiar problems lead to familiar routine while, unfamiliar situations leads to reconsideration during the thinking and executing processes. Hence, thinking takes place to reason and understand (Schoenfeld, 2011). This progression is important as teachers can lead students from knowing a fact, procedure or concept to using these prerequisites to solve more complex mathematical problems. This progression develops systemic reasoning (Mullis et al., 2003) or systemic thinking (Barros-Castro, Midgley, & Pinzón, 2013; Griffin *et al*., 2012).

3.0 Components of reasoning found difficult by high achievers

High achieving students are found to face difficulties in reasoning in the aspect of tackling non routine problems (Johnny, 2015; Rosmawati binti Ismail, 2014; Mohini Mohamed & Johnny, 2013). When faced with problems such as this, they tend to adopt the 'trial and error' method and rush to obtain a solution by merely using the information given in the problem instead of defining the nature and belief of the problem posed (Heng & Sudarshan, 2013; Yeo, 2009). High achievers are able to recall rules associated with the problem posed. However, they face difficulty in applying these rules into non routine problems (Mohini Mohamed & Johnny, 2013). This can be described as the inability to match prerequisites and experiences with new situations.

The activity of analysing information and procedures is another hurdle faced by high achievers. This is evident when students pay more attention towards keywords in the problems posed to identify the procedure that has to be executed rather than understanding the concept itself (Heng & Sudarshan, 2013). Research have even found that students generally parrot routine task based on what they have learned previously in solving given mathematical problems rather than analysing them as they are hardly asked to engage in the activity of analysing during lessons (Mohd Ariff Ahmad Tarmizi, Rohani Ahmad Tarmizi & Mohd Zin Bin Mokhtar, 2010; Dindyal, 2006). This can be held true as reports from the Ministry of Education has also revealed that 82% of lessons observed did not engage students efficiently in the process of analysing and interpreting data but focusing more on surface level content understanding (Kementerian Pendidikan Malaysia, 2013).

The same is also evident in the process of synthesising where on 15% of lessons observed engaged students effectively in synthesising information (Kementerian Pendidikan Malaysia, 2013). In a research to study how high achievers form analogical reasoning through the activity of synthesising and creating new matches, findings showed that they did not have a clear conceptualisation of how to perform it but were only able to identify the mismatches from the information in the problem given (Maria Salih, 2010).

High achievers were also found to face difficulty in making reasoned judgments or evaluating the logic of the solutions they obtained from mathematical problems solved (Heng & Sudarshan, 2013; Mohini Mohamed & Johnny, 2013; Mohini Mohamed & Johnny, 2010). Even Kementerian Pendidikan Malaysia (2013) has reported that there is a need to guide these students to employ sound judgments. Mohd Ariff Ahmad Tarmizi, Rohani Ahmad Tarmizi & Mohd Zin Bin Mokhtar (2010) stressed that this can be achieved if they are able to consider and evaluate major alternative points of view, reflect critically on learning experiences and processes and incorporate reflections into decision-making. Students need to play the role of constructors of knowledge rather than be receivers of knowledge (Cheah, 2010).

There are a few reasons identified as to why high achievers are facing difficulty in reasoning in the mathematics classroom. One reason is the type of teacher questioning. Cheah (2010) reported that teacher questioning in the mathematics classroom are generally ad hoc and close ended or as Mason (2010) names it cloze technique. This is usually done by giving a pause and expecting students to fill in the missing word verbally. For example, "*This shape is called a …*" or "*the next step will be to multiply 7 with …*" Cloze techniques questioning generally directs attention to key details in problem solving hence, reinforcing memory of students which is useful but not to be used extremely during lessons. This is because students may end up rehearsing patterns or parroting procedures to solve similar problems to what they have learned in the classroom (Cheah, 2010; Mohd Ariff Ahmad Tarmizi et al., 2010).

Another typical mathematics classroom activity that perhaps may have led high achievers to face difficulty in reasoning is teachers showing or explaining step by step to solve problems (Mohd Ariff Ahmad Tarmizi et al., 2010; Lau, Singh, & Hwa, 2009). This leads to rote learning of mathematical rules and procedures (Shafia Abdul Rahman, 2010) that place emphasis on memorisation and recall of learnt algorithms. This way of learning does not encourage students to use their reasoning skill (Abdul Halim Abdullah & Effandi Zakaria, 2013; Abdul Halim Abdullah & Mohini Mohamed, 2008).

Many teachers use examples to illustrate definitions and exemplify specific rules or theorems (Shafia Abdul Rahman, 2010; Lau *et al*., 2009). However, this can lead to students developing restricted thinking that only those types of situations fit the specific rule hence, overlooking the actual sense of the concept which the teacher originally intended them to grasp. Therefore, high achievers should be encouraged to discern generality of a concept with an awareness of particular details that form the example given.

4.0 Strategies to hone reasoning in the mathematics classroom

Good process of learning and instruction has been regularly conceptualized in the West as a transition from a teacher-centered to a student-centered instructional approach where mathematics should be treated conceptually (Hemmi & Ryve, 2014). Reasoning can happen through conceptual change or scientific understanding in a classroom (Lai & Murray, 2015; Zeeuw, Craig, & Hye, 2013; Zimmerman, 2005). Therefore, reasoning skills are supported through formation and modification of concepts and theories. This section attempts to discuss effective processes of teaching, learning and assessment that have helped promote reasoning skills in the mathematics classrooms.

Incorporating open-ended problems in formative assessments have shown to promote reasoning in mathematics (Leong & Tan, 2014; Griffin *et al*., 2012; Pehkonen, 2008). This is conducted through coursework, research projects and written examinations where the tasks posed are embedded in the curriculum. Pehkonen (2008) stresses that open-ended problems works as a tool that enhances understanding and creativity in mathematics.

Another strategy identified is to incorporate reflective thinking or review segments in classroom activities where the whole class are actively engaged (Savola, 2008; Yoong, 2002). One technique is to guide students through teacher reflecting and questioning aloud. This is to encourage students on how to start reflecting without worrying on being right or wrong. This eventually helps students on how to monitor their thinking. Gradually, students will be able to practice the skills on their own with minimal teacher guidance. Another technique is teachers to lead in these review activities with the whole class being actively engaged in the process using the Review-Lesson-Practice approach (Savola, 2008). Among the favoured activities in conducting review segments are class discussions and student presentations. Incorporating review segments in lessons is aligned with the theory of Mathematical Thinking by Mason *et al*. (2010) where Review is an important phase for honing reasoning skills.

Besides that, placing emphasis on questioning in the mathematics classroom has proven effective (Ida Ah Chee Mok, Berinderjeet Kaur, Zhu, & Yau, 2013; Wong, 2012). There are two types of questioning analysed; teacher questioning techniques and student questioning. The former focuses on teacher asking a combination of closed and higher order questions to the students using the IRF standard; Teacher Initiates, Student Responds, Teacher Feedback (Wong, 2012). However, there is a critical need to move beyond this standard as encouraging students' questioning in the classroom directly empowers their learning in the aspect of building understanding and justifying situations posed (Ida Ah Chee Mok et al., 2013; Wong, 2012). Wong (2012) and Mason (2010) stresses that active learners are demonstrated by their skill of posing questions in the classroom. Therefore, hesitant students need to be initiated to make it a habit of asking questions in the classroom. Merely asking a lot of questions is not effective to hone reasoning skills but the quality of the questions asked. Wong (2012) described a guide used in the classroom to encourage students to ask questions which cover four categories; meaning, method, reasoning and application. Thus, enabling students to learn to ask "what if" and "what if not" questions through changing of values, generalizing or specializing situations, representations and operations.

5.0 Conclusion

From the discussion above, it is evident that high achievers are not given much opportunity to independently work on mathematical problems or even to conjecture their mathematical ideas in the classroom. Classroom instruction with these students were also found to be inadequately engaged in synthesising and analysing but being passive receivers as most teachers use lecture style of instruction (Kementerian Pendidikan Malaysia, 2013; Dindyal, 2006). Even practitioners who teach high achievers admit that their students are very good listeners but face difficulty in expressing themselves or even asking questions in the classroom. This is further supported by Kementerian Pendidikan Malaysia (2013) where 50% of the observed lessons delivered by teachers were reported to be at an unsatisfactory level in the aspect of incorporating higher order thinking skills.

Clearly, the process of learning and instruction needs a transformation in terms of instructional practice. Effective knowledge reasoning and systemic thinking can be honed through effective instructional strategy in the classroom. The current practice of students being passive receivers and teachers using lecture style of instruction in teaching mathematics in the classroom will not enable students to hone reasoning skills. It can only be achieved if students are encouraged to be active learners and play the role of knowledge constructors (Cheah, 2010; Mason et al., 2010; Mason, 2010). In the field of mathematics, specifically algebra and geometry, play an important role in developing reasoning skills (Areti Panaoura, 2014; Mason et al., 2010; Chin & Liu, 2009; Qualifications and Curriculum Authority ;2004). Therefore, these topics can be used as platforms to hone knowledge reasoning and systemic thinking among high achievers in the Malaysian mathematics classroom setting with emphasis on non routine problems.

Hence, a suitable and feasible instructional strategy needs to be incorporated parallel with the current transition of the mathematics curriculum that focuses on higher order thinking skills. This is to enable high achievers to think mathematically, specifically reason in the context of knowledge reasoning and systemic thinking in an interactive way in the classroom.

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