Modeling of Microbial Approach in Wastewater Treatment Process: A Case Study of mPHO in Taman Timor Oxidation Pond, Johor, Malaysia

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Abstract. In this study, we consider the application of biological based product mPHO that contains Phototrophic bacteria (PSB) for the degradation of bacteria Coliform (pollutant) in Taman Timor Oxidation Pond, Johor, Malaysia. A mathematical model is developed to describe the reaction between microorganism and pollutant. The model facilitates the determination of mPHO optimum amount for achieving the maximum pollutant decontamination in the oxidation pond. A partial differential equation model with coupled equation is developed, and the parameters of the model are estimated using the real data collected from the oxidation pond under study. The numerical simulations are also presented to illustrate the performance of proposed model.

Keywords: Mathematical Model; Phototrophic Bacteria; Oxidation Pond.
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INTRODUCTION

It is well known that over two thirds of the region in our planet is covered by water, while the rest is taken up by land. As the human population continues to grow, people are putting pressure on the planet's water resources. The early history of human civilization has shown how important this resource in almost all major civilizations such as the Greeks that began in the river basin. However, due to the rapid development that took place in the area of inland, water resources such as rivers, lakes and sea are about to become fully utilized. Unfortunately, the effects of what are considered as unbalanced development have been polluting the main source of natural environments on our planet, known as water. Recognizing the important interaction between water resources and human life, an attempt should be made to ensure that natural resources are being managed as good as possible and provide benefits to us. Water pollution to some extent will affects and disrupts the activities of human life in obtaining water supply and also for other domestic purposes.

An original source can be considered as contaminated if there are changes regarding its quality. Water pollution is caused by a combination of different types of environmental components such as organisms, organic materials, and inorganic materials. The major part of it is the result of human activities that contributed the largest part in pollution compared to other sources. This phenomenon can be described by the outbreak of an epidemic that swept our world today. Many studies have been conducted to ensure that wastewater treatment system could produce a good quality of water. Oxidation pond treatment has proved its effectiveness to meet the needs of medium-sized communities [1]. This treatment does not involve expensive construction and maintenance costs as compared to other treatment systems that are being widely used today. The procedure is primarily to decompose organic and inorganic contaminants in either anaerobic or aerobic reaction.

This pilot scale study was carried out in Taman Timor oxidation pond, Tampoi, Johor, estimated about 1,909 square meters and about 1.5 meters in depth, 54 meters length and 2,864.13 cubic meters of total volume of water [2]. To ensure that maximum levels of decontamination can be achieved in the oxidation pond technique, a biological-based product mPHO has been applied regularly within three months period of study using an original schedule provided by J-Biotech Company [2]. As a result, the effects of mPHO are observed through two sampling locations, which are CP1 (inlet and the application of mPHO) and CP2 (outlet) [2]. Comparisons of data at both sampling points have shown that mPHO has a good effect in enhancing the degradation process of organic and inorganic matter. In this study, the relationship between mPHO and pollutant is highlighted through developing and simulating a mathematical model.

LITERATURE REVIEW

Almost all problems involving real-life applications can be modeled using mathematical modeling. In solving the real problems, the main issue should be taken seriously so that the nature of problems itself will not be affected. Therefore, we shall explore both sides of the problem, which are the physical phenomenon and its mathematical structure. Later, we will be able to build a good relationship between both side's point of view. There are several issues that are being discussed globally such as water pollution in ponds, rivers and sea. When discussing issues related to the quality of water, we have actually brought together a study that has been developed since long time ago by Streeter and Phelps around 1925 [3] to see the relationship between biological oxygen demand (BOD) and dissolved oxygen (DO) in the River Cam, England. This model explains how the nature of BOD and DO can vary with the time of observation. Many mathematical models have been developed to help the conservation and preservation in a given locality. For instance, a study carried out on Tha Chin river stream in Thailand, which considered the effect of substances contaminated with dissolved oxygen [4]. This study proposed the model of two-dimensional coupled advection-dispersion equations for both state variables, respectively. In this model, contamination and oxygen concentration are permitted to fluctuate along the length of stream and these were dealt as homogeneous over the cross-segment of river, subjected to Dobbin's criterion [5].

Apart from that, there are also studies that were developed to study the important phenomena that occur in the water bodies such as eutrophication process [6]. It is the enrichment of an ecosystem with chemical nutrients, typically compounds containing nitrogen, phosphorus, or both. When household and industrial wastes are discharged into water, organic matters and nutrients presented in them are uptaken by bacteria and other biological species such as algae using dissolved oxygen in the interacting processes [7-10]. As these processes becoming the parts of a food chain in the water bodies, the level of DO was decreased due to various interactive biochemical and biodegradation processes [11-13]. This encourages the growth of algae (algal bloom) and other aquatic plants. Following this are the occurrences of overcrowding and plants compete for sunlight, space and oxygen. Eutrophication can be a natural process occurring in lakes, ponds or rivers as they age through time. Considering the biodegradation process involving the relationship between biological species and its nutrient, the Michaelis-Manten equation is the suitable model that we really need to apply. The model describes the rate of enzymatic reactions by relating the reaction rate v with substrate S by the formula $v = V_{\max}[S]/(K_m + [S])$, where $V_{\max}$ is the maximum rate attained by the system that can be called as saturating substrate concentrations and K_m is the substrate concentration that takes the half value of $V_{\max}$.

Lastly, a dynamic mathematical model has also been developed to predict the effluent quality of facultative wastewater stabilization ponds. A two-dimensional hydraulic model was employed considering dispersed flow and diffusion in horizontal and vertical directions, respectively. Resulting partial differential equation system was solved using finite difference methods and matrix manipulation techniques. The model has been calibrated and evaluated on the basis of collected data from a full-scale facultative stabilization pond in Izmir, Turkey [13].

Although many studies have been presented that are related to wastewater and its treatment by oxidation pond, there are still less results that demonstrate the control problem of mathematical models. This might happened because of the difficulty in having a reliable data to be used in the simulation procedure. This is the purpose of our study, which is to use the experimental data as the basis, and constructing the control experiment of mathematical model.

MATHEMATICAL MODEL

We modeled the wastewater treatment process using a system of partial differential equations (PDE), which is the first order PDE with coupled-equation:

The variables and parameters used in this mathematical model are as follows:

$M(t)$ is the concentration of PSB in the pond (mg/liter) where t varies from initial time up to 84 days.
$P(t)$ is the concentration of pollutant in the pond (mg/liter).
m_0 is the concentration of PSB in one liter of mPHO (mg/liter).
$U(t)$ is the amount of mPHO applied to the pond according to the JBMI schedule per liter in 84 days.
p_0 is the concentration of pollutant in one liter of sewage at CP1 (mg/liter).
v_s is the average amount of sewage coming in (liter/day).
v_p is the volume of the pond in liter.
υ is the velocity of wastewater in the pond (m/day).
c_1 to c_9 are constants determined by parameter estimations based on the experimental data at CP2.

$$\frac{\partial P(x,t)}{\partial t} + \upsilon \frac{\partial P(x,t)}{\partial x} = (c_1 - c_2 P(t) - c_3 M(t))P(t) + c_4 \frac{v_s\, p_0}{v_p} - c_5 \frac{(U(t)+v_s)\, P(x,t)}{v_p}, \tag{1}$$

$$\frac{\partial M(x,t)}{\partial t} + \upsilon \frac{\partial M(x,t)}{\partial x} = (c_6 - c_7 M(t) - c_8 P(t))M(t) + c_9 \frac{m_0\, U(t)}{v_p} - c_5 \frac{(U(t)+v_s)\, M(x,t)}{v_p}, \tag{2}$$

where $0 \le t \le 84$ day, $0 \le x \le 54$ meter, and $\upsilon = 7.74144$ m/day.

At initial time and length namely t=0, and x=0, the value of pollutants is 0.32 mg/L, while the value of PSB is $1.5\text{x}10^{-6}$ mg/L. The initial conditions are presented by quadratic interpolation of the data at CP1 and CP2 when t=0,

$$P(x,0) = \frac{1}{54}\left(\frac{1.0\times10^3}{10^8} - \frac{7.08\times10^6}{10^8}\right)0.01851\,x^2 + \frac{7.08\times10^6}{10^8}, \tag{3}$$

$$M(x,0) = \frac{1}{54}\left(\frac{0.5\times10^2}{10^8} - \frac{0.36\times10^2}{10^8}\right)1.2751\,x^2 + \frac{0.36\times10^2}{10^8}. \tag{4}$$

The boundary conditions $P(0,t)$ and $M(0,t)$ can be calculated through linear interpolation of given data at CP1.

Parameter Estimation

The parameters of the proposed model can be estimated using a set of data collected through sampling from the pond in 84 days. Based on the given data, we want to determine the unknown parameters in equations (1-4) by the solution of parameter estimation problem. Then, a derivative-free optimization algorithm is employed to estimate the optimum value of the parameters $c_1,\ c_2, \ldots, c_9$. A random value for each parameter is initially generated, where the cost function of this problem can be formulated as follows

$$f(c_1,...,c_9) = \sum_{i=1}^{12} \left| P(t_i) - P^*(t_i) \right| + \sum_{i=1}^{12} \left| M(t_i) - M^*(t_i) \right|. \qquad (5)$$

Here $P^*(t_i)$ and $M^*(t_i)$ is the amount of pollutant and PSB measured at CP2 at time t_i.

This cost function has to be minimized subject to the mathematical model, which has been described in (5). The current schedule of mPHO gave us the following parameters for the problem. These procedures were iteratively repeated until some acceptable values for the parameters are obtained. After performing the aforementioned optimization process, the values for the parameters are obtained as shown in Table 1.

TABLE 1. The Parameters Determined by Parameter Estimation.

Parameters	Value
c_1	0.32168
c_2	0.00237307
c_3	11.5246
c_4	0.0500592
c_5	0.0500258
c_6	0.0052575
c_7	0.677609
c_8	0.0831078
c_9	993.064

Using the above parameters, the mathematical model has been simulated and the numerical results are shown in Figure 1 and Figure 2. Figure 1 shows that although initially we do not have pollutant at that much, the dynamics of the system have caused the amount of pollutant to be slowly increases until it reaches some peak level because of the input of pollutant that enters the system from CP1. Eventually, the amount of pollutant can be reduced until it reaches a safe level by the end of treatment. While the variation of PSB along the pond has fluctuated with some pattern, it still cannot be reduced until the end of treatment as depicted in Figure 2. However, since PSB is not harmful to the environment, it is acceptable to release some unnecessary amount of PSB at discharged area.

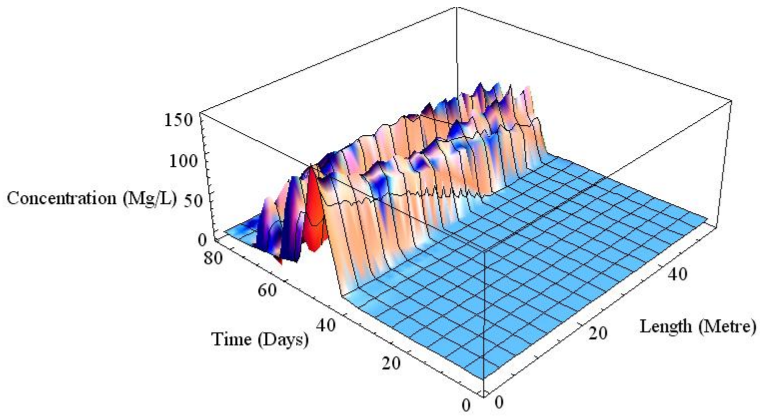

FIGURE 1. 3D Graph of Pollutant Based on Mathematical Model Using Schedule of J-Biotech.

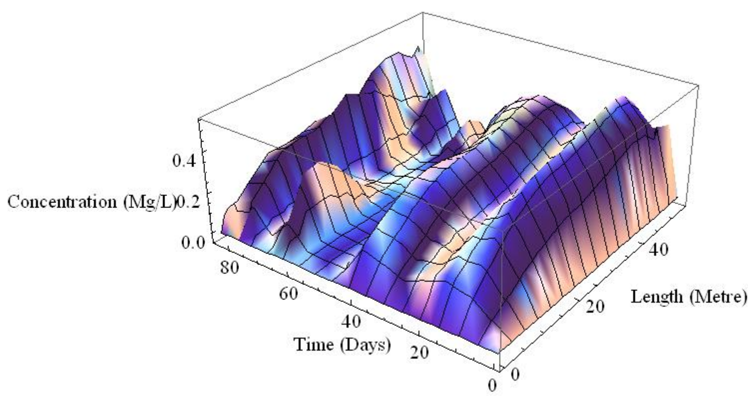

FIGURE 2. 3D Graph of Phototrophic Bacteria Based on Mathematical Model Using Schedule of J-Biotech.

Optimal Control of mPHO

Now, using the parameters obtained in the previous section, an optimal control problem needs to be solved to obtain the optimum schedule for the application of mPHO in the pond in such a way that it could reduce both the cost of application of mPHO itself as well as the environmental cost.

The cost function of optimal control is given by

$$min\int_0^{84}[50\ U(t)^2+\int_0^{54}\omega_p\ P(x,t)^2\,dx+\int_0^{54}\omega_m\ M(x,t)^2\,dx]\ dt, \tag{6}$$

where $U(t)$ is the schedule of mPHO application and some suitable weights of ω_P and ω_M need to be carefully determined to balance the environmental cost of each parameters. In this study, we take the values of these weights all equal to 1. The solution of the above problem through control parameterization method gave us the optimal value for the application of mPHO in the pond as presented in Figure 3 and Figure 4. Based on the new schedule for mPHO application, we simulate the mathematical model again to see the effects of schedule on the value of pollutant and PSB. Figure 5 and Figure 6 show the application of a new schedule that significantly changes the amount of pollutant while it reduces the amount of mPHO.

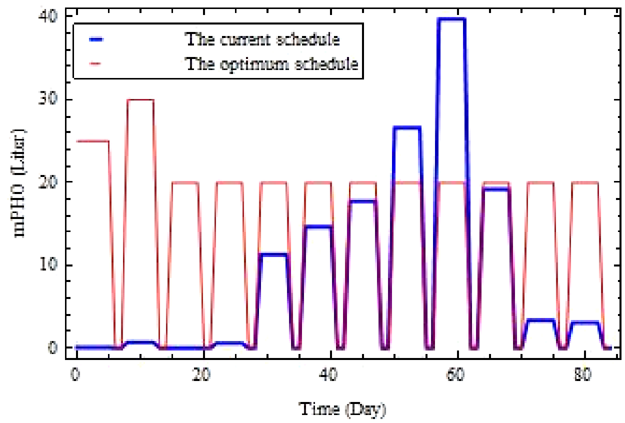

FIGURE 3. Optimum Schedule of mPHO Against the Current Schedule of J-Biotech.

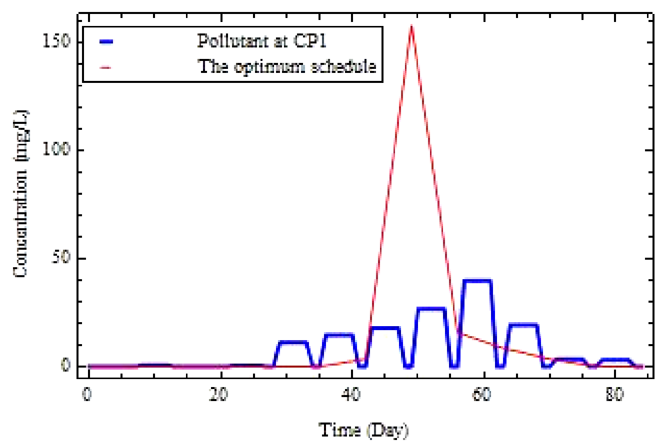

FIGURE 4. Optimum Schedule of mPHO Against the Input of Pollutant from CP1.

Figure 3 suggests that, rather than applying constant amount of mPHO for 10 remaining weeks of treatment, we should vary its application according to the quality of wastewater entering the pond. By using the optimum schedule, we have already saved about 590 liters of mPHO and it will cost us RM 29, 500 since each liter of mPHO cost RM 50. Figure 4 shows how the input of pollutant and optimal schedule vary with time.

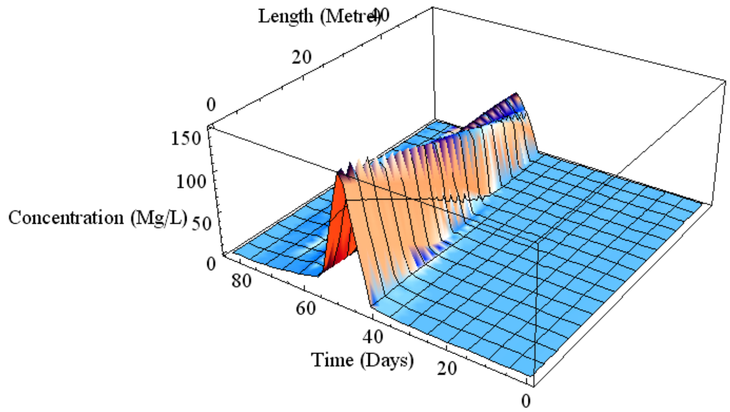

FIGURE 5. 3D Graph of Pollutant Based on the Optimum Schedule.

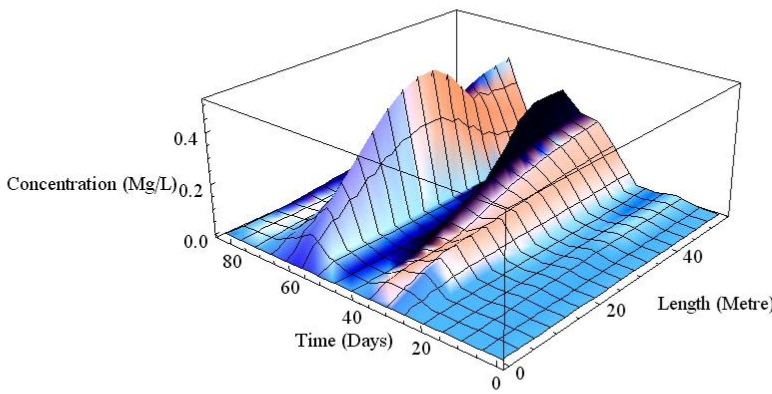

FIGURE 6. 3D Graph of PSB Based on the Optimum Schedule.

Figure 5 shows that the optimum schedule could control a huge amount of pollutant much quicker than current schedule. By using the control parameter, we could sooner control the amount of pollutant while taking care of the uncontrollable amount of PSB, which is not really necessary to the environment as shown in Figure 6.

CONCLUSION

In this study, a mathematical model for wastewater treatment process of an oxidation pond was developed and using real data, a set of optimum parameters were obtained for the model. Using the optimum parameters, we then constructed the optimal control problem for the schedule of mPHO. Again, simulating the models using the optimum schedule where the numerical results show that the models could produce an optimum schedule of application of mPHO. The mathematical model has shown the effectiveness of mPHO in improving water quality of oxidation pond.

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