Proceeding of 2nd International Science Postgraduate Conference 2014 (ISPC2014)

MHD FREE CONVECTION FLOW OVER AN INCLINED PLATE THAT APPLIES ARBITRARY SHEAR STRESS TO THE FLUID

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Abstract. An exact analysis of heat transfer past an infinite inclined plate that applies arbitrary shear stress to the fluid with Newtonian heating is presented. The fluid is considered electrically conducting and passing through a porous medium. The influence of thermal radiation in the energy equations is also considered. General solutions of the problem are obtained in closed form using the Laplace transform technique. They satisfy the governing equations, initial and boundary conditions and can set up a huge number of exact solutions correlatives to various fluid motions. The effects of various parameters on velocity and temperature profiles are shown graphically and discussed in details.
Keywords Free convection, Mass diffusion, Newtonian heating, MHD, Shear stress, Laplace transform.

1.0 INTRODUCTION

The concept of free convection flow of a viscous incompressible electrically conducting fluid in the presence of a transverse magnetic field in porous media is of much importance due to the important role of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. Furthermore, such type of flows has many applications in plasma studies, cooling of nuclear reactors, thermal insulation and heat transfer from pipes and transmission lines. Free convection flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical plate under the action of a magnetic field has been studied by Raptis and Kafousias [1]. Magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium was investigated by Aldoss et al. [2]. Many findings which consider continuous and well-defined thermal boundary conditions are found in [3–5]. MHD free convection from a vertical plate embedded in a thermally stratified porous medium are investigated by Chamkha [6]. The effects of magnetic field on steady free convection flow through a porous medium bounded by an infinite vertical plate was analyzed by [7].

However, recently the attention has been diverted mostly towards the conjugate boundary condition also knows as the Newtonian heating condition [8]. Soon after the pioneering work of Merkin [8], the Newtonian heating condition has been used by several researchers including Pop et al. [9], Lesnic et al. [10], Salleh et al. [11–13], Mohamed et al. [14], Uddin et al. $[15]$, Merkin et al. $[16]$, and Ramazan et al. $[17]$ considering different flow situations. Most of these researchers investigated the Newtonian heating problems using numerical techniques. However, very few studies on the Newtonian heating are available in the literature for which the exact solutions are obtained some of them are $[18-20]$.

Natural convection over an inclined plate was first studied experimentally by Rich $[21]$. A solution for the boundary layer on a horizontal plate showing that if the plate is heated and faces downwards or is cooled and faces upwards was presented by Stewartson $[22]$. Free convection heat transfer from an isothermal plate with arbitrary inclination was investigated by $[23]$. Chen et al. $[24]$ have obtained a numerical solution for the problem of natural convection over an inclined plate with variable surface temperature.

From the above discussion we conclude that the free convection flows over a vertical and inclined plate with different initial and boundary conditions has been investigated by different authors. Most of these studies deal with the problems where the velocity field is given on the boundary and the exact solutions are obtained. There are only few cases in which the shear stress is prescribed at the boundary $[25-27]$. However, such studies are only limited to the problems of momentum transfer. Indeed the solutions of such problems for free convection flows are scarce. On the other hand, it is important to bear in mind that the no slip boundary condition may not be necessarily applicable to flows of polymeric fluids that can slip or slide on the boundary. Having in mind such motivation, Fetecau et al. $[28]$, for the first time investigated free convection flow near a vertical plate that applies arbitrary shear stress to the fluid when the thermal radiation and porosity effects are taken into consideration. However, so far no study has been reported in the literature which focuses on the free convection flow with Newtonian heating past a inclined plate that applies arbitrary shear stress to the fluid. Even such studies are not available for viscous fluids. Therefore, in the present investigation, we study this problem for viscous fluid. However, for future research this problem can be also extended to other non-Newtonian fluids.

In fact the main purpose of this paper is to investigate the effects of Newtonian heating and mass diffusion on MHD free convection flow over a inclined plate that applies arbitrary shear stress to the fluid passing through a porous medium. General solutions of the problem are obtained using the Laplace transform technique. The results for velocity and temperature profiles are plotted graphically and discussed for the embedded flow parameters.

2.0 MATHEMATICAL FORMULATION

Let us consider the unsteady MHD free convection flow of an incompressible viscous fluid over an infinite inclined plate. The $x-$ axis is taken along the vertical plate and the $y-$ axis is taken normal to the plate. Initially, both the plate and fluid are at stationary condition with the constant temperature T_∞ . After time $t \geq 0$, the plate applies a time dependent shear stress $f(t)$ to the fluid along the $x-$ axis. Meanwhile, the temperature of the plate is raised to T_w . The radiation term is considered in the energy equation. However, the radiative heat flux is considered to be negligible in the $x-$ direction in comparison to the $y-$ direction. We assume that the flow is laminar and the fluid is grey absorbing-emitting radiation but no scattering medium. Under the usual Boussinesq's approximation and neglecting the viscous dissipation, the unsteady free convection flow is governed by the following equations of momentum and energy:

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta_T\left(T-T_\infty\right)\cos\left(\alpha\right)-\frac{\nu}{K}u-\frac{\sigma B_0^2}{\rho}u, \tag{1}$$

$$\rho C_p\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}\quad y,\ t>0, \tag{2}$$

where $u,\ T,\ \nu,\ \rho,\ g,\ \beta_T,\ K,\ \sigma,\ B_0,\ C_p,\ k,$ and q_r are the velocity of the fluid in $x-$ direction, its temperature, the kinematic viscosity, the constant density, the gravitational acceleration, the heat transfer coefficient, the permeability of the porous medium, the electric conductivity of the fluid, the applied magnetic field, the heat capacity at the constant pressure, the thermal conductivity and the radiative heat flux.

The corresponding initial and boundary conditions are

$$\begin{gathered} u\left(y,0\right)=0,\ T\left(y,0\right)=T_\infty, \\ \frac{\partial u\left(0,t\right)}{\partial y}=\frac{f\left(t\right)}{\mu},\ \frac{\partial T\left(0,t\right)}{\partial y}=-hsT, \\ u\left(\infty,t\right)=0,\ T\left(\infty,t\right)=T_\infty;\quad t>0, \end{gathered} \tag{3}$$

where μ, hs are the coefficient of viscosity, the heat transfer parameter for Newtonian heating and the function $f(t)$ satisfies the condition $f(0)=0.$ The radiation heat flux under Rosseland approximation $[29]$ is given by

$$q_r=-\frac{4\sigma^*}{3k_R}\frac{\partial T^4}{\partial y}, \tag{4}$$

where σ^* and k_R are the Stefan-Boltzmann constant and the mean spectral absorption coefficient, respectively. Here we limit our analysis to optically thick fluids while using Rosseland approximation. It is supposed that the temperature difference within the flow are sufficiently small, then Eq. (4) can be linearized by expanding T^4 into Taylor series about T_∞, and neglecting higher order terms, we find that

$$T^4\approx 4T_\infty^3T-3T_\infty^4. \tag{5}$$

Introducing Eq. (5) into Eq. (4) and putting the obtained result in Eq. (2), we get

$$\Pr\frac{\partial T}{\partial t}=\nu\left(1+Nr\right)\frac{\partial^2 T}{\partial y^2};\quad y,t>0, \tag{6}$$

where $\Pr$, ν and Nr are defined by

$$\Pr=\frac{\mu C_p}{k},\ \nu=\frac{\mu}{\rho},\ N_r=\frac{16\sigma T_\infty^3}{3kk_R}. \tag{7}$$

In order to reduce Eqs. $(1),(3)$ and (6) into their non-dimentional forms, we introduce the following dimensionless variables

$$\begin{gathered}u^*=\frac{u}{U},T^*=\frac{T-T_\infty}{T_\infty},\ y^*=\frac{U}{\nu}y,\\ t^*=\frac{U^2}{\nu}t,\ f^*\left(t^*\right)=\frac{1}{\rho U^2}f\left(\frac{\nu}{U^2}t^*\right),\end{gathered} \tag{8}$$

into Eqs. (1) and (6) and dropping out the " $*$ " notation, it yields

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+GrT\cos\alpha-K_p u-Mu, \tag{9}$$

$$\Pr_{eff}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial y^2}, \tag{10}$$

where $\Pr_{eff}=\frac{\Pr}{1+Nr}$ is the effective Prandtl number $[29,\ (10)]$ and

$$\begin{gathered}Gr=\frac{g\beta_T\nu T_\infty}{U^3},\\ M=\frac{\sigma B_0^2\nu}{\rho U^2},\ K_p=\frac{\nu^2}{U^2}\frac{1}{K},\end{gathered} \tag{11}$$

are the Grashof number, modified Grashof number, magnetic parameter, Schmidt number and the inverse permeability parameter for the porous medium respectively.

The corresponding dimensionless initial and boundary conditions are

$$
\begin{aligned}
& u(y,0)=0,\ T(y,0)=0,\ y\geq 0, \\
& \frac{\partial u}{\partial y}|_{y=0}=f(t),\ \frac{\partial T}{\partial y}|_{y=0}=-\gamma(1+T), \\
& T(\infty,t)=0,\ u(\infty,t)=0,
\end{aligned} \tag{12}
$$

where $\gamma=\frac{\nu}{U}hs$ is the Newtonian heating parameter. We note that Eq. (12) gives $T=0$ when $\gamma=0$, corresponding to having $hs=0$ and therefore there is no heating from the plate.

3.0 EXACT SOLUTION

Applying Laplace transform to Eqs. (9) and (10) and using the initial conditions from Eq. (12), we reduce to the following differential equations

$$
\begin{aligned}
q\bar{u}(y,q)=&\frac{\partial^2\bar{u}(y,q)}{\partial y^2}+Gr\bar{T}(y,q)\cos\alpha \\
&-K_p\bar{u}(y,q)-M\bar{u}(y,q),
\end{aligned} \tag{13}
$$

$$
\bar{T}(y,q)=\frac{1}{\Pr_{eff}q}\frac{\partial^2\bar{T}(y,q)}{\partial y^2}, \tag{14}
$$

The corresponding transformed boundary conditions are

$$
\begin{aligned}
& \bar{T}(\infty,q)=0, \\
& \bar{u}(\infty,q)=0,\ \frac{\partial\bar{u}(y,q)}{\partial y}|_{y=0}=F(q), \\
& \frac{\partial\bar{T}(y,q)}{\partial y}|_{y=0}=-\gamma\left(\frac{1}{q}+\bar{T}(0,q)\right).
\end{aligned} \tag{15}
$$

Solution of Eq. (14) is obtained as

$$
\bar{T}(y,q)=\frac{-\gamma}{q\left(\gamma-\sqrt{q\Pr_{eff}}\right)}e^{-y\sqrt{q\Pr_{eff}}}. \tag{16}
$$

By taking the inverse Laplace transform of Eq. (16), we find

$$T(y,t)=e^{b_1^2 t-b_1 y\sqrt{\Pr_{eff}}}\operatorname{erf}c\left(\frac{y\sqrt{\Pr_{eff}}}{2\sqrt{t}}-b_1\sqrt{t}\right)$$
$$-\operatorname{erf}c\left(\frac{y\sqrt{\Pr_{eff}}}{2\sqrt{t}}\right) \tag{17}$$

and

$$\frac{\partial T(y,t)}{\partial y}|_{y=0}=-b_1 e^{b_1^2 t}\left(1+\operatorname{erf}\left(b_1\sqrt{t}\right)\right), \tag{18}$$

is the corresponding heat transfer rate also known as Nusselt number. Here $\operatorname{erf}(.)$ and $\operatorname{erf}c(.)$ denote the error function and complementary error function of Gauss $[28]$.

Solution of Eq. (13) using boundary conditions from Eq. (15) yieldswhich upon inverse Laplace transform gives

$$\bar{u}(y,q)=\frac{-\frac{-Grb_1\cos\alpha}{\Pr_{eff}-1}b_5}{\sqrt{q}\left(\sqrt{q}-b_1\right)\left(q-b_2\right)\sqrt{q+b_6}}e^{-y\sqrt{q+b_6}}$$
$$-\frac{F(q)}{\sqrt{q+b_6}}e^{-y\sqrt{q+b_6}}$$
$$+\frac{-Gr\gamma\cos\alpha}{q\left(\gamma-\sqrt{q\Pr_{eff}}\right)\left(\left(\Pr_{eff}-1\right)q-b_6\right)}e^{-y\sqrt{q\Pr_{eff}}} \tag{19}$$

Applying Laplace inverse transform to Eq. (19) , we get

$$u(y,t)=u_c(y,t)+u_m(y,t), \tag{20}$$

where

$$u(y,t) = +\frac{b_{12}b_{13}}{b_{15}\sqrt{\pi}}\int_0^t\left[e^{b_9^2(t-s)} - b_9 e^{b_9^2(t-s)}\,\mathrm{erf}\left(b_9\sqrt{t-s}\right)\right]\frac{e^{-\frac{y^2}{4s}-b_6 s}}{\sqrt{s}}ds$$

$$+\frac{b_{12}b_{13}}{b_{15}\sqrt{\pi b_{10}}}\int_0^t\left[\sqrt{b_{10}}b_{14}e^{b_{10}(t-s)} + b_9 e^{b_{10}(t-s)}\,\mathrm{erf}\left(\sqrt{b_{10}}\sqrt{t-s}\right)\right]\frac{e^{-\frac{y^2}{4s}-b_6 s}}{\sqrt{s}}ds$$

$$-\int_0^t f(t-s)\frac{e^{-\frac{y^2}{4s}-b_6 s}}{\sqrt{\pi s}}ds$$

$$+\frac{1}{2b_9}\int_0^t e^{b_{10}s+y\sqrt{\Pr}\sqrt{b_{10}}}\left[-1+e^{b_9^2(t-s)}\left(1+\mathrm{erf}\left(b_9\sqrt{t-s}\right)\right)\right]\mathrm{erf}\,c\left(\frac{y\sqrt{\Pr}}{2\sqrt{s}}+\sqrt{b_{10}s}\right)ds$$

$$+\frac{1}{2b_9}\int_0^t e^{b_{10}s-y\sqrt{\Pr}\sqrt{b_{10}}}\left[-1+e^{b_9^2(t-s)}\left(1+\mathrm{erf}\left(b_9\sqrt{t-s}\right)\right)\right]\mathrm{erf}\,c\left(\frac{y\sqrt{\Pr}}{2\sqrt{s}}-\sqrt{b_{10}s}\right)ds$$

$$+\frac{2b_{12}b_{13}}{\sqrt{\pi}b_{10}}\int_0^t\frac{e^{b_{10}(t-s)-\frac{y^2}{4s}-b_6 s}}{\sqrt{s}}ds \tag{21}$$

and

$$u_m(y,t) = -\frac{1}{\sqrt{\pi}}\int_0^t f(t-s)\frac{e^{-\frac{y^2}{4s}-b_6 s}}{\sqrt{s}}ds, \tag{22}$$

correspond to the convective and mechanical parts of velocity.

4.0 LIMITING CASE

In this section we intend to discuss one of limiting case of our general solutions.

Solution in the absence of thermal radiation ($Nr \to 0$)

In the absence of thermal radiation, the corresponding solutions can directly be obtained from the general solutions by taking $Nr \to 0$ into Eqs. (16) and (19) , and replace $\Pr_{eff}$ by $\Pr$, we find that

$$u(y,t)=+\frac{b_{12}b_{13}}{b_{15}\sqrt{\pi}}\int_0^t\left[e^{b_9^2(t-s)}-b_9e^{b_9^2(t-s)}\operatorname{erf}\left(b_9\sqrt{t-s}\right)\right]\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$+\frac{b_{12}b_{13}}{b_{15}\sqrt{\pi b_{10}}}\int_0^t\left[\sqrt{b_{10}}b_{14}e^{b_{10}(t-s)}+b_9e^{b_{10}(t-s)}\operatorname{erf}\left(\sqrt{b_{10}}\sqrt{t-s}\right)\right]\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$-\int_0^t f(t-s)\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{\pi s}}ds$$
$$+\frac{1}{2b_9}\int_0^t e^{b_{10}s+y\sqrt{\Pr}\sqrt{b_{10}}}\left[-1+e^{b_9^2(t-s)}\left(1+\operatorname{erf}\left(b_9\sqrt{t-s}\right)\right)\right]\operatorname{erf}c\left(\frac{y\sqrt{\Pr}}{2\sqrt{s}}+\sqrt{b_{10}s}\right)ds$$
$$+\frac{1}{2b_9}\int_0^t e^{b_{10}s-y\sqrt{\Pr}\sqrt{b_{10}}}\left[-1+e^{b_9^2(t-s)}\left(1+\operatorname{erf}\left(b_9\sqrt{t-s}\right)\right)\right]\operatorname{erf}c\left(\frac{y\sqrt{\Pr}}{2\sqrt{s}}-\sqrt{b_{10}s}\right)ds$$
$$+\frac{2b_{12}b_{13}}{\sqrt{\pi}b_{10}}\int_0^t\frac{e^{b_{10}(t-s)-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds \tag{23}$$

$$T(y,t)=e^{\frac{\gamma^2}{\Pr}t-\frac{\gamma}{\Pr}y\sqrt{\Pr}}\operatorname{erf}c\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\frac{\gamma}{\Pr}\sqrt{t}\right)-\operatorname{erf}c\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right). \tag{24}$$

The corresponding expression for the heat transfer rate reduces to

$$\frac{\partial T(y,t)}{\partial y}\Big|_{y=0}=-\frac{\gamma}{\Pr}e^{\frac{\gamma^2}{\Pr}t}\left[1+\operatorname{erf}\left(\frac{\gamma}{\Pr}\sqrt{t}\right)\right]. \tag{25}$$

All the arbitrary constants appearing in this paper are given by

$$b_1=\frac{\gamma}{\sqrt{\Pr_{eff}}},\ b_2=\frac{b_6}{b_3}, b_3=\Pr_{eff}-1,\ b_4=\frac{-Grb_1\cos\alpha}{\Pr_{eff}-1},$$

$$b_5=\sqrt{\Pr_{eff}},\ b_6=K_p+M,\ b_7=1+\frac{b_1\sqrt{b_2}}{b_2}-\frac{b_1}{\sqrt{b_2}},\ b_8=b_1^3-b_1b_2,$$

$$b_9=\frac{\gamma}{\sqrt{\Pr}},\ b_{10}=\frac{b_6}{b_{11}}, b_{11}=\Pr-1,\ b_{12}=\frac{-Grb_9\cos\alpha}{b_{11}},$$

$$b_{13}=\sqrt{\Pr},\ b_{14}=1+\frac{b_9\sqrt{b_{10}}}{b_{10}}-\frac{b_9}{\sqrt{b_{10}}},\ b_{15}=b_9^3-b_9b_{10},$$

$$a_3=\frac{K_p}{b_3},$$

$$a_5=1+\frac{b_1\sqrt{a_3}}{a_3}-\frac{b_1}{\sqrt{a_3}},\ a_6=b_1^3-b_1a_5, a_7=\frac{M}{b_3},\ a_8=\frac{M}{a_2},$$

$$a_9=1+\frac{b_1\sqrt{a_7}}{a_7}-\frac{b_1}{\sqrt{a_7}},\ a_{10}=b_1^3-b_1a_9.$$

5.0 RESULTS AND DISCUSSION

Exact analysis of heat and mass transfer past an infinite inclined plate that applies arbitrary shear stress to the fluid with Newtonian heating is presented. More general solutions of the problem are obtained using the Laplace transform technique. The graphical results of velocity profiles for various flow parameters such as porosity parameter K_p and Newtonian heating parameter γ are discussed. All these graphs correlate to the case when the plate bring to bear a constant shear stress to the fluid. The effects of inverse permeability parameter K_p on the velocity is presented in Fig. 1. The decrease in the permeability is the cause of increase in resistance of porous medium. As a consequence, the fluid velocity decreases with respect to K_p. The fluid temperature decreases from maximum at the boundary to a minimum value as far from the plate. The effects of the Newtonian heating parameter on velocity and temperature are studied in Figs. 2 and 3. It is observed that an increase in the Newtonian heating parameter increase the momentum and thermal boundary layers, increase the fluid velocity and surface temperature of the plate decreases.

6.0 CONCLUSIONS

An analytical study is performed to examine the effects of Newtonian heating an unsteady free convection flow over an inclined plate that applies arbitrary shear stress to the fluid and is passing through a porous medium. The Laplace transform method is used to obtain the exact solutions. These solutions are presented in simple forms in terms of exponential functions and complementary error function of Gauss. It is found that they satisfy both the governing equations and all imposed initial and boundary conditions. The effects of various parameters on velocity and temperature are graphically studied. The fluid velocity is presented as a sum of mechanical and convection components. We found that all the results of velocity are new and its mechanical component reduces to known results from the literature when some suitable parameters are equated to zero. In addition one of the important limiting case is obtained from general solutions. We noted that both the temperature of the fluid do not depend on porosity and magnetic parameters together with shear stress on the boundary.

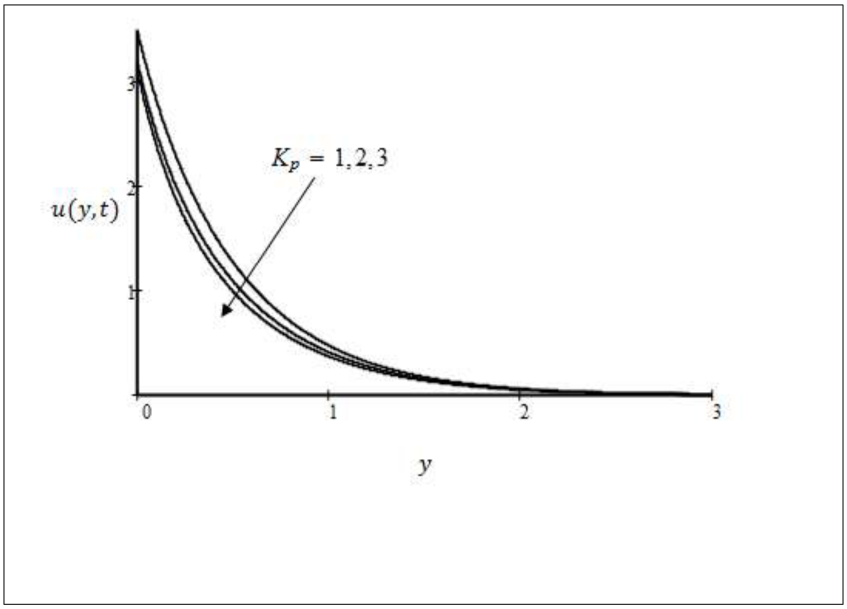

Figure 1 Velocity profiles for for different values of K_p when the plate applies a constant shear stress $f = 0.2$.

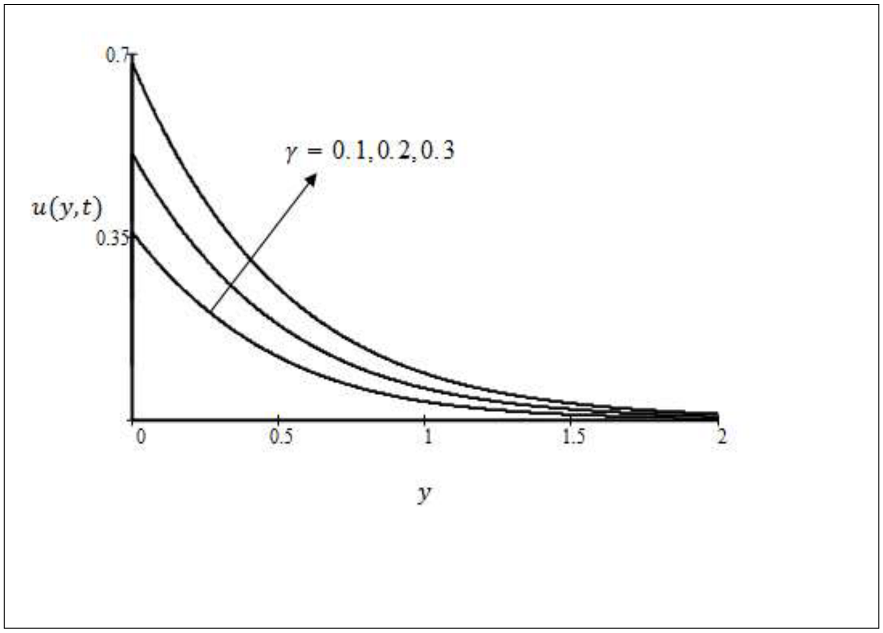

Figure 2 Velocity profiles for different values of γ when the plate applies a constant shear stress $f = 0.2$.

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