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DOUBLE DIFFUSION ON UNSTEADY MAGNETOHYDRODYNAMIC FREE CONVECTION FLOW IN A ROTATING MEDIUM

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Abstract. The heat and mass transfer on the unsteady radiative magnetohydrodynamic (MHD) free convection flow over an impulsively started vertical plate in a rotating medium is studied. The fluid is considered gray, absorbing-emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in this analysis. The Laplace transform technique is used to obtain the exact solutions of the governing equations. The numerical results of fluid velocity, fluid temperature, fluid concentration, and skin friction, as well as the rate of heat transfer and the rate of mass transfer at the plate are displayed through graphs and tables in order to see the effects of the various parameters involved. The present results obtained here may be used to verify the validation of obtained numerical solutions for more complicate fluid flow problems.

Keywords **:** MHD Free Convection Flow, Rotating Medium,

1.0 INTRODUCTION

Free convection is common in nature and has numerous applications and occurrences in industry. It is a major cause for atmospheric and oceanic recirculation and plays an increasingly important role in the passive emergency cooling systems of advanced nuclear reactors [1]. The study of MHD free convection flow in a rotating medium has drawn attention of many researchers due to its wide range of applications in cosmological and geophysical fluid dynamics, astrophysics, meteorology, petroleum, designing thermo syphon tubes, in cooling turbine blades and in hydrology to study the movement of underground water. In the rotating medium, there exist an external force, which called the Coriolis force, acts on the fluid. The force is responsible for the deflection in a clockwise sense to a moving objects on the surface of the Earth in the

Northern Hemisphere and in a counter-clockwise sense in the Southern Hemisphere. Chandrasekhar[2] pointed out the significant roles of the Coriolis force on problems of thermal instability and on stability of viscous flow has made significant contributions to the theory of hydrodynamic flow phenomena in numerous situations. Then, several others studies are carried out by various investigators disclosed that the Coriolis force is significant compared to viscous and inertia occurring in the basic equations of the problems involving unsteady free-convection flow past an infinite vertical plate in a rotating fluid. Singh [3] studied analytically the hydromagnetic free convection flow past an impulsively started vertical plate in a rotating fluid by using the Laplace transform technique and pointed out that with increasing rotation parameter, the primary velocity decreases. Later, Bestman and Adjepong[4] studied the unsteady MHD free convection flow near a moving infinite flat plate in a rotating medium in the presence of radiation heat transfer using perturbation method. Continuously, Singh *et al*.[5] studied the MHD free convection flow past an impulsively started vertical plate with the magnetic field fixed on the plate. Lahurikar[6] investigated the flow past an impulsively started vertical isothermal plate in a rotating fluid. The exact solutions of the problem were obtained by using the Laplace transform technique. Subsequently, Vijayalakshmi[7] extended the study of Lahurikar[6] by taking into account the radiation effect. Recently, Vijayalakshmi and Kamalan[8] extended his work [7] for the constant accelerated plate motion. Futhermore, Jana *et al*.[9] investigated the unsteady flow of viscous fluid through a bounded porous plate in a rotating system. Recently, Farhad*et al*.[10] extended the work of Jana *et al*.[9] to the electrically conducting viscous fluid with the consideration of Hall current and slip condition at the boundary. Sahoo*et al*.[11]and Seth *et al*.[12] also consider the MHD viscous fluid flow in a rotating system.

It has been noticed that the effect of mass diffusion is not reported in the previous investigation, whereas in many real world problems the flow, the heat transfer and the mass diffusion are always coupled. When the heat and mass diffusion occur simultaneously, it generate a complex fluid motion named double diffusive convection. In a wide range of scientific fields the double diffusive convection appear in the field of biology, oceanology, astrophysics, geology, chemical process and crystal growth techniques. Extensive reviews have been reported on the subject by Viskan*et al*.[13]. As early as 1988, Tokis[14] evaluated the free convection and mass transfer effects on the MHD flows near a moving plate in a rotating fluid. Recently, Ahmed and Sarmah[15] investigated the effect of mass and heat transfer analytically on a MHD flow past an impulsively fixed vertical plate. More study on the double diffusion convection can be found in the work done by others researchers [16-18].

The aim of the present study is to investigate the double diffusive free convection MHD flow past an impulsively started infinite vertical plate in a rotating fluid. More exactly, this research is the extension of Ahmed and Sarmah[15]V by considering the rotating medium. Exact solutions of the governing partial differential equations are obtained using Laplace transform method. The influence of the embedded parameters on the flow, thermal fields and concentration fields are graphically presented and analyzed.

2.0 MATHEMATICAL ANALYSIS

An unsteady double diffusion free convection MHD flow in a viscous incompressible fluid over an impulsively started vertical plate in a rotating medium is considered. We assume that the fluid is electrically conducted under the assumption of small magnetic Reynolds number and the plate is thermally conducted but electrically insulated. Both the plate and the fluid are in a state of rigid body rotation with uniform angular velocity Ω about z-axis. A uniform magnetic field of strength B_o is acting perpendicular to the flow i.e. along z-axis in outward direction. Initially ($t = 0$), the plate and the fluid are at rest such that the free stream temperature T_∞ and concentration C_∞ are the same everywhere. At time $t > 0$, the plate starts moving in its own plane with a constant velocity U_0 along the x-axis. At the same time, the temperature of the plate and concentration are raised or lowered to T_w and C_w respectively, which are hereafter maintained constant. Since the plate occupying the plane z=0 is of infinite extent, all the physical quantities depend only on z and t. Moreover, the induced magnetic field is assumed to be negligible and the plate is taken electrically non-conducting. The effects of thermal radiation in the energy equation and thermal diffusion in the concentration equation are also taken into consideration. Under the foregoing assumptions and Boussinesq approximation, the equations governing the MHD free convection flow with combined heat and mass transfer are as follows:

$$\frac{\partial u'}{\partial t'} - 2\Omega v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} + g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty) - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho} v' \quad (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} \quad (4)$$

where υ is the kinematic viscosity, t is the time, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of mass expansion, σ is the electrical conductivity, B_0 is the applied magnetic field, ρ is the fluid density, C_p is the specific heat of the fluid at constant pressure, k is the thermal conductivity of the fluid and q_r is the radiative flux in the z-direction, D is the mass diffusivity, K_T is the thermal-diffusion ratio and T_m is the mean fluid temperature. Moreover, the term $(\partial q_r / \partial z')$ in equation (3) represents the change in the radioactive heat flux with distance normal to the plate. The initial and boundary conditions corresponding to the problem are

$$\begin{array}{llllll} t' \leq 0 & u' = 0, & T' = T_\infty, & C' = C_\infty & \text{for all } z' \\ t' > 0 & \begin{cases} u' = U_0, & T' = T_w, \quad C' = C_w \quad \text{at } z' = 0 \\ u' = 0, & T' \to T_\infty, \quad C' \to C_\infty \quad \text{as } z' \to \infty. \end{cases} \end{array} \tag{5}$$

By Rosseland approximation, the radiative heat flux of an optically thin grey gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^* \sigma^* (T_\infty^4 - T'^4) \tag{6}$$

wherea^* is the absorption coefficient and σ^* is the Stefan-Boltzmann constant. It is assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T_∞ and neglecting higher order terms we finally get

$$T'^4 \cong 4T_\infty^3 T' - 3T_\infty^4. \tag{7}$$

Substituting equations (6) and (7) into equation (3), we have

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} + 16 a^* \sigma^* T_\infty^3 (T_\infty - T'). \tag{8}$$

Following Tokis[14] and Vijayalakshmi[8], the dimensionless variables are introduced as

$$\begin{array}{lll} u = \dfrac{u'}{U_0}, & v = \dfrac{v'}{U_0}, & z = \dfrac{z' U_0}{\upsilon}, \\ t = \dfrac{t' U_0^2}{\upsilon}, & \theta = \dfrac{T' - T_\infty}{T_w - T_\infty}, & C = \dfrac{C' - C_\infty}{C_w - C_\infty}. \end{array} \tag{9}$$

Introducing equation (9) into equations (1)-(5), and equation (8), we obtain the following non-dimensional equations:

$$\frac{\partial q}{\partial t}+2Eqi=\frac{\partial^2 q}{\partial z^2}+Gr\theta+Gr^*C-mq, \tag{10}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial z^2}-\frac{R}{\Pr}\theta, \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} \tag{12}$$

and the non-dimensional initial and boundary conditions (5) reduced to:

$$\begin{array}{llllll} t\le 0 & q=0, & \theta=0, & C=0 & \text{for all } z \\ t>0 & \begin{cases} q=1, \\ q=0, \end{cases} & \begin{array}{l}\theta=1,\\ \theta\to 0,\end{array} & \begin{array}{l}C=1\\ C\to 0\end{array} & \begin{array}{l}\text{at } z=0\\ \text{as } z\to\infty,\end{array} \end{array} \tag{13}$$

where

$$q=u+iv, \qquad i=\sqrt{-1}, \qquad E=\frac{\Omega\upsilon}{U_0^2}, \qquad \text{Re}=\frac{U_0 z}{\upsilon},$$

$$m=\frac{\upsilon\sigma B_0^2}{\rho U_0^2}, \quad Gr=\frac{z^3 g\beta(T_w-T_\infty)}{\upsilon^2\,\text{Re}^3}, \quad Gr^*=\frac{z^3 g\beta^*(C_w-C_\infty)}{\upsilon^2\,\text{Re}^3}, \qquad \Pr=\frac{\rho\upsilon C_p}{k},$$

$$R=\frac{16a^*\upsilon^2\sigma^* T_\infty^3}{kU_0^2}, \qquad Sc=\frac{\upsilon}{D}.$$

and Equation (10) is obtained by combining Equation (1) and (2) by introducing complex term q. Here E is the rotation parameter, Re is the local Reynolds number, Gr is the Grashof number, Gr^* is the modified Grashof number, m is the magnetic parameter, Pr is the Prandtl number, R is the radiation parameter and Sc is the Schmidt number. The Laplace transform technique is applied to obtain the solutions of equations (10)-(12) subjected to the initial and boundary conditions (13).Therefore, we have

$$\theta(z,t)=\varphi(0,\text{R}/\Pr,z\sqrt{\Pr}). \tag{14}$$

$$C(z,t)=\varphi(0,0,z\sqrt{Sc}). \tag{15}$$

$$
\begin{aligned}
q(z,t) &= (1+A_1-A_2)\varphi(0,a,z) - A_1\varphi(-\frac{R-a}{\Pr-1},a,z) + A_2\varphi(\frac{a}{Sc-1},a,z) - \\
&\quad A_1\varphi(0,\frac{R}{\Pr},z\sqrt{\Pr}) + A_1\varphi(-\frac{R-a}{\Pr-1},\frac{R}{\Pr},z\sqrt{\Pr}) + A_2\varphi(0,0,z\sqrt{Sc}) - \\
&\quad A_2\varphi(\frac{a}{Sc-1},0,z\sqrt{Sc}) \qquad \text{for } \Pr \neq 1 \,\&\, Sc \neq 1, \\
q(z,t) &= (1+A_1-A_2)\varphi(0,a,z) + A_2\varphi(\frac{a}{Sc-1},a,z) - A_1\varphi(0,R,z) + A_2\varphi(0,0,z\sqrt{Sc}) - \\
&\quad A_2\varphi(\frac{a}{Sc-1},0,z\sqrt{Sc}) \qquad \text{for } \Pr=1 \,\&\, Sc \neq 1, \\
q(z,t) &= (1+A_1-A_2)\varphi(0,a,z)(\frac{1}{2}) - A_1\varphi(\frac{R-a}{1-\Pr},a,z) - A_1\varphi(0,\frac{R}{\Pr},z\sqrt{\Pr}) + \\
&\quad A_1\varphi(\frac{R-a}{1-\Pr},\frac{R}{\Pr},z\sqrt{\Pr}) + A_2\varphi(0,0,z) \qquad \text{for } \Pr \neq 1 \,\&\, Sc = 1, \\
q(z,t) &= (1+A_1-A_2)\varphi(0,a,z) - A_1\varphi(0,R,z) + A_2\varphi(0,0,z). \qquad \text{for } \Pr = Sc = 1.
\end{aligned}
\tag{16}
$$

where,

$$
\varphi(x,y,z) = \frac{e^{xt}}{2}[e^{-z\sqrt{x+y}} erfc\{\frac{z}{2\sqrt{t}} - \sqrt{(x+y)t}\} + e^{z\sqrt{x+y}} erfc\{\frac{z}{2\sqrt{t}} + \sqrt{(x+y)t}\}],
$$

$$
a = m + 2Ei, \quad A_1 = \frac{Gr}{R-a}, \quad A_2 = \frac{Gr^*}{a}.
$$

and "$erfc\{x\}$" represents the complementary error function. Moreover, the present solutions (14)-(16) satisfy the imposed initial and boundary conditions (13). The physical quantities of interest are the local skin friction C_f, local Nusselt number Nu and local Sherwood number Sh, which are defined as

$$
C_f = \left.\frac{\partial q}{\partial z}\right|_{z=0}, \qquad Nu\,\mathrm{Re}^{-1} = -\left.\frac{\partial \theta}{\partial z}\right|_{z=0}, \qquad Sh\,\mathrm{Re}^{-1} = -\left.\frac{\partial C}{\partial z}\right|_{z=0}. \tag{17}
$$

The present solutions are more general and some of the existing solutions in the literature can be obtained as limiting cases, as follow

(i) when the radiation parameter $R=0$, equation (16) is reduced to the equation (21) obtained by Tokis[14].

(ii) when the radiation parameter $R=0$, Schmidt number $Sc=0$ and modified Grashof number $Gr^*=0$, equation (16) is reduced to the equation (9) obtained by Singh [3].

(iii) when the magnetic number m=0, Schimidt number Sc=0 and modified Grashof number $Gr^*=0$, equation (16) is reduced to (12) and (13) in Vijayalakshimi[7].

3.0 Discussion of the Results

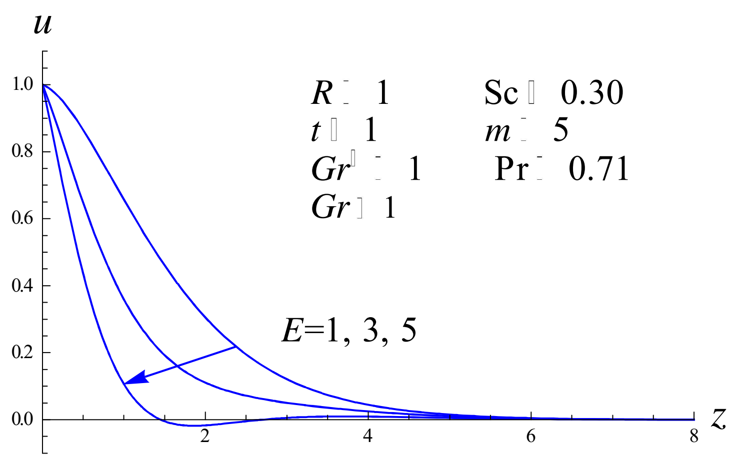
Initially, we compared our results with Ahmed and Sarmah[15], in order to verify the correctness of the present obtained results. The results are found to be excellent agreement and some of the comparison are shown in Table (1) and Table (2). Present outcome will be illustrated now to reveal the effects of various parameters on the temperature field, concentration field, velocity field, Nusselt number, Sherwood number and shear stress coefficient. To be realistic, the values of Pranthl number (Pr) are taken for air (Pr=0.71), water (Pr=7) and mercury (Pr=0.015). The values of Schmidt number (Sc) are chosen for Helium (Sc=0.30), Oxygen (Sc=0.66) and Benzene (Sc=2.62). Fig 1 demonstrates the velocity profiles at Pr=0.71, Sc=0.30 and $Gr=Gr^{*}=R=t=m=1$for variety of rotating parameter E =1, 3 and 5. It is worth to note here that $Gr > 0$ and $Gr^{*} > 0$ imply the thermal buoyancy and concentration buoyancy assisting flow respectively and negative for the opposing flow ($Gr < 0$ and $Gr^{*} < 0$).In Fig 1, $Gr = Gr^{*} = 1$ are considered and that illustrate both the thermal and concentration buoyancy force act on assisting the flow. The effect of E on the velocity field is such that the velocity boundary layer thickness decreases sharply when$E$ is increased as shown in Fig 1. This trend is consistent withVijayalakshmi[7]. For example, in Fig 1 (a) the boundary layer thickness for velocity u is approximated to 5.2677 when$E$=1whereas when$E$=5 the thickness is reduced to 1.4277 (approximately). It is interesting to notice that initially the magnitude of v at z=0 decrease to some values and then increase to some values until reach the far end boundary condition, v=0 as $z \to \infty$ .Thus, the velocity profile v exhibits the reverse flow region and this do not happen in the velocity profile u. Accordingly, the skin friction coefficient $C_f$displays significant variation for a change in the rotation parameter as shown in Table (3). For example, the values of Re[C_f] and Im[C_f] decrease to -1.14489 and -1.81513 respectively when$E$=1 increase to$E$=5. In Table (4) shows that the rate of mass transfer is enhanced when the Schmidt number Sc is increased. Further, in the Fig 2, it is noticed that the magnitude of the concentration decreases with increasing on the value of Sc. Physically, the molecular diffusivity (D) is decreasing as the value of Sc increase and the concentration profile shows significant variation for different values of Sc. Consistently, this find out show similar trend as illustrated in Ahmed and Sarmah[15].The variation of temperature profile for different values of Pr and R arepresented in Fig 3. The thermal boundary layer thickness decrease sharply with an increase in Pr and R and hence induced an increase in the surface temperature gradient. This result is consistent with the outcome present in Ahmed and Sarmah[15]. The rate of heat transfer is increased when the magnitudes of Prand$R$are increased as revealed in Table (5).

TABLE (1): Comparison of Sherwood number with Ahmed and Sarmah[15]for Pr=0.7 and *t*=1.

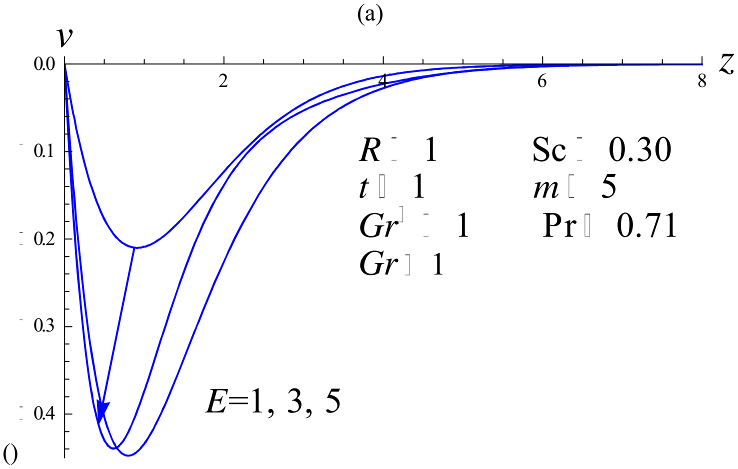
Sc	Present	Ahmed and Sarmah[15]
0.24	0.552790	0.552791
0.60	0.874038	0.874039
0.78	0.996558	0.996557

TABLE (2): Comparison of Nusselt number with Ahmed and Sarmah[15]for Pr=0.7 and *t*=1.

R	Present	Ahmed and Sarmah[15]
0.5	1.54834	1.54828
1.0	2.04430	2.04431
5.0	4.47218	4.47218



(a)



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(b)

FIGURE 1. Velocity profile (a) *u* and (b) *v*for various value of *E*.

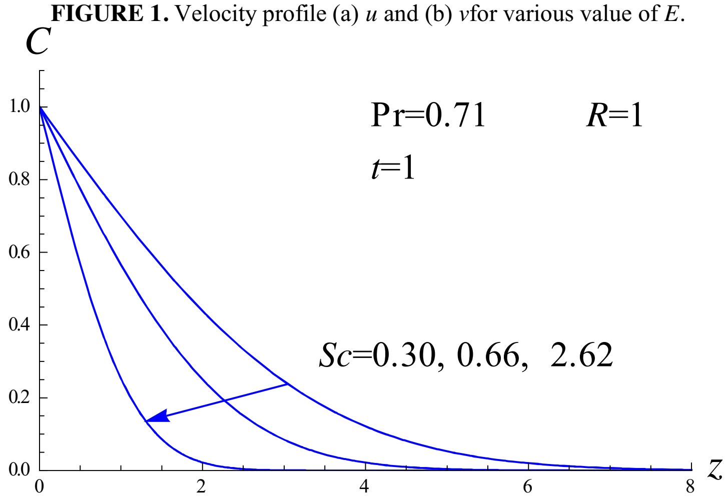
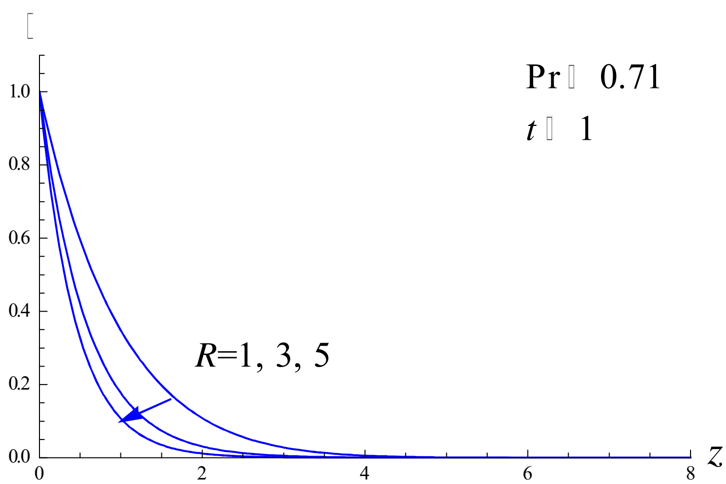
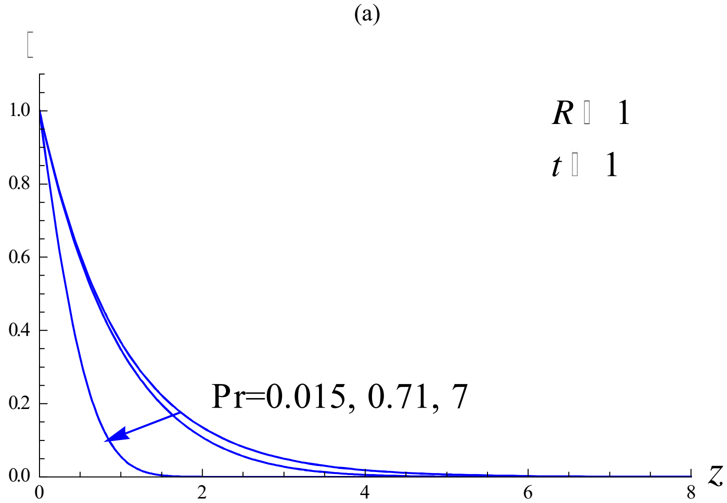


FIGURE 2. Variation of concentration profile *C* for various values of *Sc*.



(a)



(b)

FIGURE 3. Variation of temperature profile θ for various values of (a) *R*and (b) Pr.

TABLE (3): Skin friction coefficient $C_f$atPr=0.71, *Sc*=0.30 and $R=m=Gr=Gr^*=t=1$for various *E*.

E	*Re*[C_f]	*Im*[C_f]
1.	-0.09364	-0.56794
3	-0.60771	-1.41800
5	-1.14489	-1.81513

TABLE (4): Local Sherwood number $ShRe^{-1}$at Pr=0.71 for various *Sc*..

Sc	$ShRe^{-1}$
0.30	0.30902
0.66	0.45835
2.62	0.91322

TABLE (5): Local Nusselt number $NuRe^{-1}$ for various Prand*R*.

R	Pr	$NuRe^{-1}$
1	0.015	1.00000
	0.71	1.02297
	7	1.70100
3	0.71	1.73268
5	0.71	2.23609

4.0 Conclusion

Theoretically, the double diffusion free convection on the unsteady MHD flow in a rotating medium past a suddenly started infinite vertical plate has been studied. Closed form exact solutions for the velocity, concentration and temperature fields are obtained by Laplace transform technique. These enable the skin friction, the rate of heat transfer and the rate of concentration can be efficient determined. Some interesting finding of the study can be concluded as follows:

1. The velocity gradient is increased when the values of the rotating parameter *E* is increased. Thus, the momentum boundary layer thickness is enhanced by reducing the rotation of the medium.

2 The skin friction coefficient is reduced when the rotation parameter *E* is increased. In fact, when *E*=3 increase to *E*=5, the percentage of reduction on skin friction coefficient are almost 88.39% for real part and 28.00% for the imaginary part.
3 Higher values of Prandtl number Pr or radiation parameter *R* caused significant reduction in the thermal boundary layer thickness. However, increasing of the value of Pr or*R* has enhanced the rate of heat transfer in the fluid.
4 The concentration boundary layer thickness is thinner for higher values of Schmidt number *Sc*. Further, the Sherwood number is reduced when increasing the magnitudes of*Sc*.

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