

## Rating and pricing: state of the art for the proposal of new methodologies

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Paraphrasing a sentence from the Nobel Prize Robert Merton, without the Risk we would have not need of Finance; Economy would be sufficient to describe and manage financial transactions across agents and Countries. The ability of transferring the risk expresses the main aim of Finance; a correct evaluation of the counterparty risk is one of the main goal of credit risk theory. Financial crisis remark this necessity: if a financial contract such as a Credit Default Swap, which, by definition, is a tool to protect against the default risk of a counterpart, is traded between banks and insurance companies that can not exactly evaluate the financial position of customers, the effects on global Equilibrium and the loss of value can be severe. The question of pricing methodologies is strictly linked to the previous topic: is rating sufficient to catch all price fluctuations? What are the advantages and disadvantages of nowadays pricing methodologies? In which directions can them be switched or improved? Mathematics can give an answer to us? How information underlying market risk can be allowed into a Mathematical model of pricing in order not to affect a correct credit risk evaluation?

These are the questions this article will answer.

**Key words:** pricing, counterparty credit risk, Basel Committee, Credit Rating Agencies

**JEL-classification:** G12, G18, G32, G33

### Introduction

At the web-page: <http://www.bis.org/publ/bcbs279.htm> it can be read:

“The Basel Committee's final standard on the standardised approach for measuring counterparty credit risk exposures includes a comprehensive, non-modelled approach for measuring counterparty credit risk associated with OTC derivatives, exchange-traded derivatives, and long settlement transactions. [...] The SA-CCR will take effect on 1 January 2017. [...] The SA-CCR limits the need for discretion by national authorities, minimizes the use of banks' internal estimates, and avoids undue complexity by drawing upon prudential approaches already available in the capital framework. [...] The Exposure At Default is multiplied by the risk weight of a given counterparty in accordance with either the Standardised or Internal Ratings-Based approaches for credit risk to calculate the corresponding capital requirement”.

So, the recent regulation, in the European Union framework, about counterparty credit risk evaluation, is quite contradictory in itself. Indeed, the paper of March 2014, of which is cited above both the web-page link and a part of the introduction, shows two basic policy features: on one hand, it processes the intention to make less rely on policy instruments which could allow for the rise of unjustified discretions. On the other hand, it fairly explains the presence of evaluation parameters, such as the Exposure At Default, which actually do depend on third parties (the Credit Rating Agencies) or on internal-bank models; this amplifies the possibility of errors or misleading evaluations of some financial instruments.

Why is the European Union still stuck in such a regulatory framework? In which sense could still the mathematical tools underlying credit derivatives pricing be improved or why they are not? The paper is structured as follows: after a brief literature review, section 1 outlines an introductory framework about the current situation concerning credit risk and the demand for loans, in order to stress out the importance of a correct credit evaluation. Section 2 introduces the underlying mathematical framework required to understand the idea that Merton had; idea which completely revolutioned the financial instruments scenario and their pricing methodology, opening the so called ‘structural-form’ models. In

Section 3 the Basel framework is presented, while Section 4 deals with the problem of the Credit Rating Agencies. Last Section presents two basic kind of approach which have given birth to models alternative to structural-form ones.

### **Literature review**

For how much it concerns the world of financial derivatives, a good introduction is provided by a white paper by the Deutsche Börse Group, titled “The global derivatives market: an introduction” (Deutsche Börse AG, April 2008). A complete explanation of pricing models, detailed both in mathematics and in banking managements, can be found in the book by Andrea Resti, Andrea Sironi “Rischio e valore nelle banche: misura, regolamentazione, gestione” (Resti, 2008) or (B. Schmid, 2002) which includes references to ratings.

For which may concern structural-form models and reduced-form models, a practical mathematical introduction is provided by (D. Filipovič, 2002). For deeper mathematics insight good references are the books (Bielecky-Rutkowski, 2014) or (Duffie-Singleton, 2003). Apart from the article (Duffie, 2000), several mathematical models have been introduced to overcome limitations of both structural-form models and reduced-form models; examples are (Guo-Jarrow-Zeng, 2009), (Yu, 2005) (U. Cetin, 2004, Vol. 14, No. 3). How to deal with credit derivatives (J.P.Morgan, December 2013) or (Pallavicini, 2014). About credit ratings: empirical tests on Credit Default Swaps is in (Chen-Cheng-Wu) or (Benmelech, 2009); about rating methodology (Standard & Poor's Rating Services, 2012). For further ideas on systemic risk implications of Basel framework: (Bonollo, Crimaldi, Flori, Pammolli Riccaboni, 2014).

#### **1. Correct credit evaluation request**

##### **Credit loans statistics at the end of 2013**

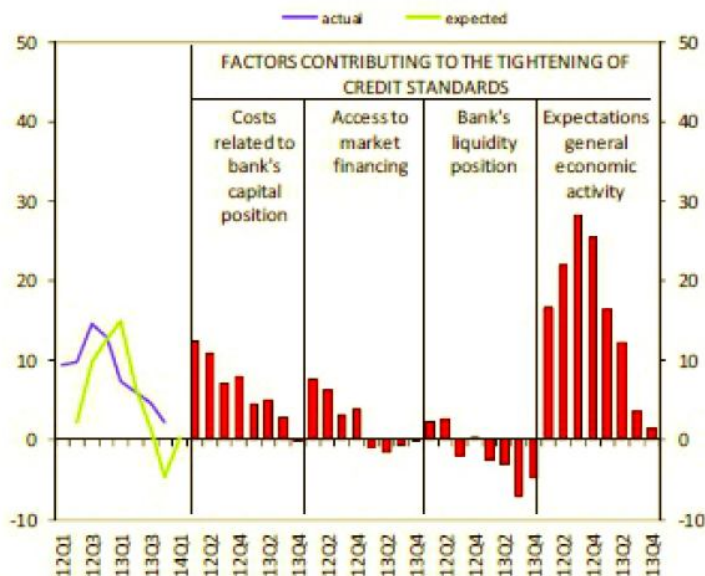
Thanks to chart number 1 it's possible to notice the situation concerning credit standards for the approval of loans to enterprises. The net percentage of banks that tightened credit standards for loans to households for house purchase became slightly negative (-1%, from 3%). For consumer credit, the net tightening remained broadly unchanged (at 2%). For the first quarter of 2014, euro area banks expect a further reduction in the net tightening on loans to nonfinancial corporations – to reach nil – and a more intense net easing for loans to households. In the last quarter of 2013, the demand for credit remained weak across all loan categories. However, the net decline in demand for loans to non-financial corporations slowed further (-10%, from -12% in the previous quarter), thus approaching its historical average. This reflected the marked decline in the contractive impact of financing needs related to fixed investments (- 9%, from -22%), while the contribution to demand of other financing needs, including those related to inventories and working capital, faded away (1%, from 6%).

That can be read: financing institutions want to overcome the period of market decline after which it could be notices very low demand for loans; this can be achieved by relaxing credit standards for the approval of loans to enterprises.

Anyway, a market recovery seems still far away; at least, requests for credit are quite less than the expected values, even if with increasing trend, as it is shown by chart number 3.

**Chart 1 Changes in credit standards applied to the approval of loans or credit lines to enterprises**

(net percentages of banks contributing to tightening credit standards)

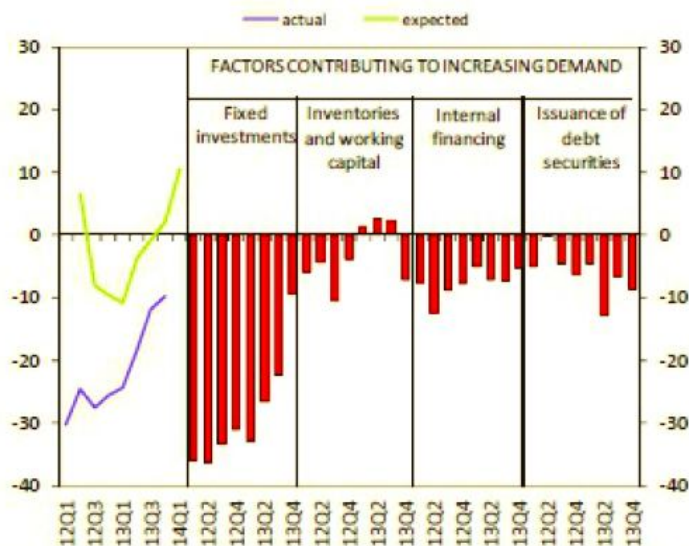


Notes: "Actual" values are changes that have occurred, while "expected" values are changes anticipated by banks. Net percentages are defined as the difference between the sum of the percentages of banks responding "tightened considerably" and "tightened somewhat" and the sum of the percentages of banks responding "eased somewhat" and "eased considerably". The net percentages for responses to questions related to the factors are defined as the difference between the percentage of banks reporting that the given factor contributed to a tightening and the percentage reporting that it contributed to an easing.

Source: The Euro area bank lending survey, January 2014

**Chart 3 Changes in demand for loans or credit lines to enterprises**

(net percentages of banks reporting a positive contribution to demand)



Notes: "Actual" values are changes that have occurred, while "expected" values are changes anticipated by banks. Net percentages for the questions on demand for loans are defined as the difference between the sum of the percentages of banks responding "increased considerably" and "increased somewhat" and the sum of the percentages of banks responding "decreased somewhat" and "decreased considerably". The net percentages for responses to questions related to each factor are defined as the difference between the percentage of banks reporting that the given factor contributed to increasing demand and the percentage reporting that it contributed to decreasing demand.

Source: The Euro area bank lending survey, January 2014

## Credit risks overview at the end of 2013

Finally, it's possible to understand the systemic implications' scenario for credit risk by looking at the red ball in the following table: risk intensity is very high at systemic level; markets are correlated both 'geographically' and at different levels of financial instruments, so it emerges an extremely fragile framework to deal with.

Main risks: Sources		Main risks: Categories			
Risk	Change since 3Q13	Risk category	Change since 3Q13	Outlook for 1Q14	Systemic risk
Risks in EU sovereign debt markets	➔	Liquidity risk	➔	➔	●
Market clustering	➔	Market risk	➔	➔	●
Funding risk	➔	Contagion risk	➔	➔	●
Valuation risk	➔	Credit risk	➔	➔	●
Market functioning	➔				

Note: Assessment of main risk sources for markets under ESMA remit, change since the last assessment. Upward arrows indicate an increase in the contribution to risks, downward arrows a decrease.

Note: Assessment of main risk categories for markets under ESMA remit since past quarter, and outlook for current quarter. Systemic risk assessment based on categorisation of the ESA Joint Committee. Colours indicate current risk intensity. Coding: green=low, yellow=moderate, orange=high, red=very high. Upward arrows indicate a risk increase, downward arrows a risk decrease.

Source: Trends Risks Vulnerabilities, ESMA, January 2014

Summing up the overview: the context of credit risk is extremely fragile in the last times, due to a very high contagion possibility and correlations between different financial markets. This situation, in which it appears as if every market's move is a possible bullet shot at systemic level, is worsened by the low level of requests for loans, which are in turn generated by slack underlying economic activities.

Therefore, in such an alerting time, it's not possible to override an extremely accurate risk evaluation: it does appear anymore merely as a risk management issue, since it affects entire economic systems.

## 2. Structural-form models

### Merton and the contingent claim analysis

A contingent claim is a claim that can be made only if one or more specified outcomes occur<sup>1</sup>; mathematically it can be defined as an asset  $V$  whose value depends on a stochastic process  $X$  and on time  $t$ .

Merton, in 1974, was the first to apply this simple idea to model the evolution of a firm; consequently he could give a formula for firms' insolvency risk. He exploited the principles of option pricing discovered one year before by Black and Scholes, by assuming that the company's debt is entirely represented by a zero-coupon bond. With this assumption, the default process of a company can be seen as driven by the value of the company's asset: default occurs when the market value of the firm (the value of its assets) is lower than that of its liabilities. Indeed, by the principle of limited responsibility, the stakeholder who can't generate cash flows sufficient to repay the debt he got towards a financial institution, cannot go default on his own means but just as a firm. Therefore, the payment to the debtholders at the maturity of the debt is the smaller between the face value of the debt or the market value of the firm's assets  $V_T$ , just as the payoff of a put option: if the entire company

1 <http://www.nasdaq.com/investing/glossary/c/contingent-claim>.

debt is represented by a zero coupon bond (that is the simplest financial title of debt, with just the repayment at the ending time of the contract, called “maturity”<sup>2</sup>), then, in this analogy, the payment to debtholders corresponds to the payoff at maturity to the bondholder, i.e. the bank; the final time for the due debt corresponds to the maturity of the bond, and technically the due debt corresponds face value of the bond minus a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond.

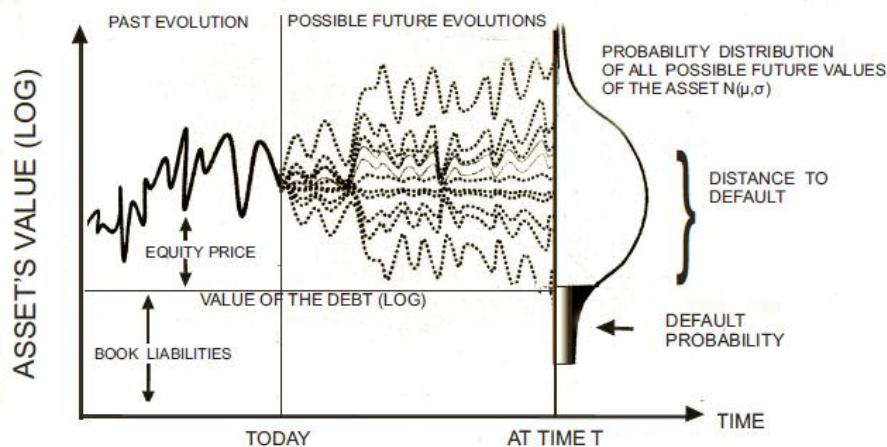
In the following are represented in details the two mechanism of the evolution of a firm’s asset and the payoff profile of a bank and in which way they reconcile, giving as a result the formulas we’re looking for, that are the value of the debt and the price of the bond.

Since the evolution of a firm’s asset<sup>3</sup> is driven by the value of the company's asset, the risk of a firm's default is therefore explicitly linked to the variability of the firm's asset value. In this meaning, bonds, which are one of the most ‘secure’ financial contracts, become risky, since it’s not sure their repayment will take place at maturity. However, they are linked to structural observable data for quoted firms, that are the share prices: from this data it’s possible to gain some useful parameters, such as the assets’ instantaneous return and the volatility of return of firms’ assets. The firms’ asset percentage return is given by the brownian motion formula:

$$\frac{dV}{V} = \mu dt + \sigma_V (\varepsilon \sqrt{dt})$$

with  $V_t$  the market value of the firm, dependent on time;  $\mu$  a parameter which catches the assets’ instantaneous return;  $\sigma_V$  the variability of a random noise, that increases over time; more precisely, it’s the volatility of return of firms’ assets, measured by the standard deviation of assets’ return.

Picture 1. The possible evolutions  $dV_t / V_t$



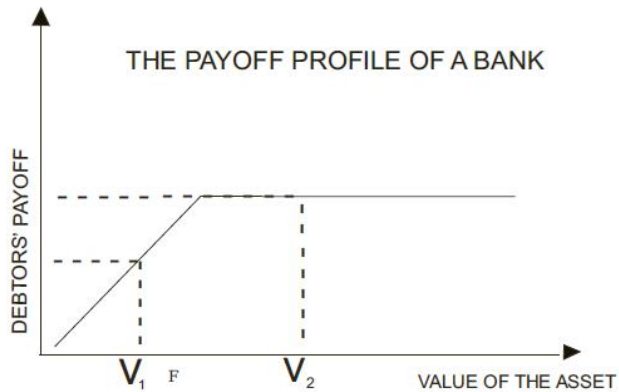
The credit risk is represented by the possibility that, at maturity T of the debt, the firm’s asset value  $V_T$  is less than the value of the loan repayment.

The power of this idea lies in the fact that in one graph it’s possible to capture both the business risk, that is the degree of firm’s risk from being in the market, which is captured by the parameter of the Gaussian volatility, and the financial risk, that is the horizontal line ‘value of the debt’, which represents the ratio between assets/liabilities, i.e. the financial leverage of the firm  $L = \frac{Fe^{-iT}}{V}$ .

<sup>2</sup> A simple example could be found at: <http://www.investinganswers.com/financial-dictionary/bonds/zero-coupon-bond-859>.

<sup>3</sup> In this framework, we’re not directly interested in firms’ liabilities: we assume the assets represent the whole information necessary to understand the financial performance of a firm.

On its side, the bank can fully cover its position by buying a put option, since the bank payoff profile is the same of a short put: for values of  $V_T$  greater to the nominal value of loan reimbursement, the asset's value is such that it is possible to repay the bank (or the creditors) of the entire principal plus accrued interest, and an eventual residual value goes to stakeholders; viceversa, for values of  $V_T$  smaller than the debt, the firm is insolvent and the bank receives just a portion of the due payment. So, in order to cover its credit risk, the bank can buy a put option on the value  $V_t$  of the firm's asset, with maturity equal to the maturity of the loan and strike price equal to the value of debt reimbursement. In this way, the combination of loan and purchase of put option gives as result a guaranteed payoff equal to the amount of the loan  $F$ .



### Some results

If, to the bank, the syntetical position given by the sum of the price  $P_0$  of the put option at time  $t = 0$  and the value of the loan  $B_0$  is at equilibrium, that means without any possible insolvency risk, then it must value as the payoff of a risk-free title which at maturity pays the amount of the loan  $F$ :  $P_0 + B_0 = Fe^{-iT}$ . Thanks to the option pricing model from Black and Scholes, it's possible to get the price of the put option  $P_0$  of a title that at maturity  $T$  repays  $F$ , with a discount factor  $i$ :

$$P_0 = Fe^{-iT} N(-d_2) - N(-d_1)V_0$$

where  $V_0$  is the value of the firm assets today (time  $t = 0$ ),  $N$  is the standard normal distribution function, which is tabulated, and

$$d_2 = \frac{\frac{1}{2}\sigma_v^2 T + \ln L}{\sigma_v \sqrt{T}} = d_1 - \sigma_v \sqrt{T}$$

Consequently one obtains the value  $B_0$  of the loan, that increases with lower maturity of the loan  $T$  and lower financial leverage  $L$  already defined:

$$B_0 = Fe^{-iT} \left[ N(d_2) + \frac{1}{L} N(-d_1) \right]$$

Furthermore, the insolvency or default probability  $PD$ , which simply corresponds, for the bank, to exercise the put option in order not to lose its money:

$$PD = N(-d_2) = 1 - N(d_2).$$

### 3. The Standardised Approach for measuring Counterparty Credit Risk exposures

#### Counterparty Credit Risk measures

From (J.P.Morgan, December 2013)<sup>4</sup> “Derivative contract are dynamic in nature and can therefore give rise to either an asset or a liability for a counterparty, depending on market movements”. So, the question is how to deal with the risk those contracts convey. We have at least to consider four disposable metrics conventionally used in the practice to monitor and measure counterparty exposure:

- the notional of the contracts, which provides information around the total size of a product with a counterparty; it's a metric to be careful with, because unlike bonds and loans, the notional of a derivative does not reflect the actual risk;
- the current mark-to-market, that is a snapshot of the current exposure to a counterparty typically adjusted to reflect any netting (eg ISDA agreements) and collateral arrangements. This provides more information than the notional amount of derivatives in question. However, it is still limited in its information, particularly when the forward mark-to-market is expected to change, for example depending on the shape of the interest rate yield curve; in Basel framework, this metric is enhanced by incorporating a sense of the potential future exposure using a specific percentage of the notional of each transaction ('add-on factor') based on a grid for each underlying asset class and maturity.
- the expected exposure, which quantifies how much the bank expects eventually to lose in case of an insolvency event; This represents the expected positive mark-to-market profile of a swap or portfolio of transactions reflecting any netting and collateral arrangements at different points in the future the potential future exposure add-on factor, when dealing with a bunch of derivatives, is based on the underlying asset classes and on the remaining maturity of the contract.
- the stressed potential exposure, sometimes referred to as a peak exposure measure. represents amplified exogenous conditions due to possible contingent instabilities.

Those measures synthetically enter the risk evaluation by the short formula:

Counterparty credit risk = (Current net exposure + Potential future exposure) – collateral.

#### The Exposure at Default in Basel III

In the document (Bank for International Settlements, 2014) the exposures under Standardised Approach for measuring Counterparty Credit Risk exposures, or SA-CCR<sup>5</sup>, consist of two components, the replacement cost (RC) and the potential future exposure (PFE):

$$EAD = 1,4 * (RC + PFE)$$

Without entering technical definitions, let's just give some formulas that should remember something familiar. The formula for the replacement cost for unmargined transactions<sup>6</sup> is given by<sup>7</sup>:

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<sup>4</sup> Available at:

[https://www.jpmorgan.com/cm/cs?pagename=JPM/DirectDoc&urlname=is\\_napfms2013\\_pensions.pdf&track=no](https://www.jpmorgan.com/cm/cs?pagename=JPM/DirectDoc&urlname=is_napfms2013_pensions.pdf&track=no), National Association of Pension Funds Limited 2013.

<sup>5</sup> The SA-CCR will apply to over-the-counter derivatives, exchange-traded derivatives and long settlement transactions.

<sup>6</sup> Unmargined transactions are those in which the variation margin is not exchanged, but collateral other than variation margin may be present. The variation margin consists in additional funds net to the initial margin; see more at: <http://www.nasdaqomx.com/transactions/markets/commodities/clearing/risk-default-management/margin-requirement>

$$RC = \max(V - C; 0)$$

precisely the payoff of a call option; and the one for margined transactions is similar<sup>8</sup>:

$$RC = \max(V - C; TH + MTA - NICA; 0).$$

Furthermore, here there are the supervisory delta adjustments<sup>9</sup>: they exactly recall the parameters  $d_1$  and  $d_2$  of the Merton formulas.

$\delta_i$	Bought	Sold
Call Options <sup>13</sup>	$+\Phi\left(\frac{\ln(P_i / K_i) + 0.5 * \sigma_i^2 * T_i}{\sigma_i * \sqrt{T_i}}\right)$	$-\Phi\left(\frac{\ln(P_i / K_i) + 0.5 * \sigma_i^2 * T_i}{\sigma_i * \sqrt{T_i}}\right)$
Put Options <sup>7</sup>	$-\Phi\left(\frac{\ln(P_i / K_i) + 0.5 * \sigma_i^2 * T_i}{\sigma_i * \sqrt{T_i}}\right)$	$+\Phi\left(\frac{\ln(P_i / K_i) + 0.5 * \sigma_i^2 * T_i}{\sigma_i * \sqrt{T_i}}\right)$
With the following parameters that banks must determine appropriately: $P_i$ : Underlying price (spot, forward, average, etc) $K_i$ : Strike price $T_i$ : Latest contractual exercise date of the option The supervisory volatility $\sigma_i$ of an option is specified on the basis of supervisory factor applicable to the trade (see Table 2 in paragraph 183).		

Source: Basel Committee on Banking Supervision, March 2014

## Where do ratings set in

According to Basel regulation, for each asset class and subclass, there exist some supervisory factors, correlations and supervisory option volatility add-ons. For single-name credit asset class, subclasses differentiate for the credit rating, giving as a result different supervisory factor parameters.

<sup>7</sup> V is the value of the derivative transaction in the netting set; C is the haircut value of net collateral held. A collateral is something pledged as security for repayment of a loan, to be forfeited in the event of a default.

<sup>8</sup> The additional parameters in the last formula are related to the mark-to-market, i.e. the value of the portfolio at its current worth; their definition is omitted here for simplicity.

<sup>9</sup> Bank supervisors may also require more complex trades to be allocated to more than one asset class, resulting in the same position being included in multiple classes. In that case, for each asset class to which the position is allocated, banks must determine appropriately the sign and delta adjustment of the relevant risk driver. A supervisory delta adjustment is made to the trade-level adjusted notional amount based on the position (long or short) and whether the trade is an option, CDO tranche or neither, resulting in an effective notional amount. For also an EAD calculation process scheme see more at: <http://blog.usbasel3.com/basel-committee-standardized-approach-for-calculating-counterparty-credit-risk-exposure-nimm-sa-ccr/>



Summary table of supervisory parameters					Table 2
Asset Class	Subclass		Supervisory factor	Correlation	Supervisory option volatility
Interest rate			0.50%	N/A	50%
Foreign exchange			4.0%	N/A	15%
Credit, Single Name	AAA		0.38%	50%	100%
	AA		0.38%	50%	100%
	A		0.42%	50%	100%
	BBB		0.54%	50%	100%
	BB		1.06%	50%	100%
	B		1.6%	50%	100%
	CCC		6.0%	50%	100%
Credit, Index	IG		0.38%	80%	80%
	SG		1.06%	80%	80%
Equity, Single Name			32%	50%	120%
Equity, Index			20%	80%	75%
Commodity	Electricity		40%	40%	150%
	Oil/Gas		18%	40%	70%
	Metals		18%	40%	70%
	Agricultural		18%	40%	70%
	Other		18%	40%	70%

Source: Basel Committee on Banking Supervision, March 2014

#### 4. The Credit Rating Agencies problem

##### Rating evaluation

The Rating evaluation is a method used to evaluate firms and financial instruments on the basis of their financial risk. The assessments of the rating are issued by the so-called credit rating agencies; The valuation it is expressed through a vote in letters, according to which the market establishes a risk premium to require the company to accept that particular investment, or the regulatory framework establishes calibrating parameters: down-going into rating scales increases the risk premium required and then the issuer must pay a spread greater than the risk-free rate. It has been talked of a “dictatorship of the analysts”, for their market power to influence the stock market, recognized them from the fact that the market in that part does not take into account the sometimes conflicting interests exists, and because on the other hand market is relatively interested in a truthful rating at a fair price of the securities. Even if in Basel regulatory framework a clear source for fair rating evaluations it’s not provided <sup>10</sup>, it appears just as a kind of third externality, yet necessary, in the Standardized Approach valuation process.

##### Timeline to reduce reliance on Credit Rating Agency ratings

In November 2012, the Financial Stability Board (FSB) published a roadmap<sup>11</sup> to reduce reliance on Credit Rating Agency (CRA) ratings. The FSB roadmap set out a timeline with concrete actions for

<sup>10</sup> A different kind of critique to the SA-CCR, regarding to choice of percentage parameters fro Credit single name asset class, is provided in this blog: <http://www.bobsguide.com/guide/news/2014/May/27/sa-ccr-ebleu-basel-finalises-new-standardised-approach-for-measuring-counterparty-credit-risk-exposures.html>

<sup>11</sup> FSB roadmap, available at: [https://www.financialstabilityboard.org/publications/r\\_121105b.pdf](https://www.financialstabilityboard.org/publications/r_121105b.pdf)

jurisdictions with a view to implementing the FSB Principles<sup>12</sup>. In May 2014, there is the European Union response<sup>13</sup> to FSB request for actions plans on reducing reliance on CRA ratings. This joint response provides an overview and summarizes all the existing and ongoing policy actions following adopted EU Regulations, Directives and implementing legislation and binding technical standards. It also includes guidelines adopted by the European Supervisory Authorities (European Securities and Markets Authority (ESMA), the European Banking Authority (EBA) and the European Insurance and Occupational Pensions Authority (EIOPA)). This response is complemented by any specific actions adopted at national level and which are not included in this response. For specific actions undertaken at national level, we refer to individual actions plans submitted by EU FSB Members. For central banking operations in the Euro area, we refer to the individual response by the European Central Bank (ECB).

## **European Union response**

In the EU response, it can be read that “The Capital Requirements Directive (CRD)/Capital Requirements Regulation (CRR) require credit institutions to have their own sound credit granting criteria and credit decision processes in place. This applies irrespective of whether institutions grant loans to customers or whether they incur securitization exposures. [...] The CRD/CRR entered into force on 1st of January 2014, reflecting the approach taken to reduce reliance on external credit ratings.”

But also that: “For the specific purposes of calculating regulatory bank capital requirements, rating agency assessments are, in certain instances, applied as a basis for differentiating capital requirements according to risks, and not for determining the minimum required quantum of capital itself. The CRD framework as a whole provides banks with an incentive to use internal rather than external credit ratings even for purposes of calculating regulatory capital requirements. In the specific case of securitisation exposures, due to a lack of sufficiently objective internal methodologies within banks, most of them would still be expected to calculate their regulatory capital requirements by reference to external ratings”.

Summing up, something is changing towards a more independent and self-contained regulatory framework, but the road ahead is still long.

## **5. Reduced-form and partial information models**

### **Mathematical underlying assumptions**

So far, the structural-forms models have the following underlying assumption: every agent in the market can get the same level of disposable information. The mathematical device to represent information is called “filtration”. More precisely, a filtration  $\mathfrak{F}$  is a family of sets  $\mathfrak{F}_t$  on a probability space  $\Omega$ . This family is increasing over time, i.e. each set is contained in the next one in time. The sets  $\mathfrak{F}_t$ ,  $0 \leq t \leq T^*$ , represent the level of disposable information at each point in time; so, the assumption of increasing sets means that agents don’t lose information as time goes by. Every structural-form model assumes complete knowledge, that means whole filtration underlying.

### **Reduced-form models**

If everything is known or predictable, there is no space for the chance: unpredictable events may still happen in this framework, and may cause serious economic effects. The reduced-form models have

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<sup>12</sup> FSB Principles on Reducing Reliance on CRA ratings, available at: [http://www.financialstabilityboard.org/publications/r\\_101027.pdf](http://www.financialstabilityboard.org/publications/r_101027.pdf)

<sup>13</sup> Staff Working Paper EU Response to the Financial Stability Board (FSB)-EU Action Plan to reduce reliance on Credit Rating Agencies, May 2014.

been introduced because, by exogeneity, not every default can be predicted. The mathematical framework beneath those models, sometimes called intensity based method, is the following. On a probability space  $(\Omega, \mathcal{F}, P)$ , consider two filtrations:  $F$  and  $G$ , defined over the same time interval  $[0, T^*]$ , one contained into the other:  $G = (G_t)_{[0, T^*]} \subseteq F = (F_t)_{[0, T^*]}$ . The default arrival is modeled by an aleatory time  $\tau_d : \Omega \rightarrow [0, +\infty]$ . The hazard rate (default indicator function)  $H = I_{\{\tau_d \leq t\}}$  is a  $(F - G)$ -adapted stochastic process; this means that a market participant with access to partial market information  $G_t$  cannot observe whether default has occurred by time  $t$  ( $\tau_d \leq t$ ) or not ( $\tau_d > t$ ). Intuitively speaking, events in  $F_t$  are  $G_t$ -observable only in the second case, when default didn't happen until time  $t$ . At time  $t$ , the probability of a default before maturity  $T$  is given by  $PD(t) = E[H(T) | G_t]$ .

### **A partial information model**

Even if very versatile, intensity-based models are inconvenient to use for who knows many things about a firm; that could be stakeholders or well-informed private companies. The model provided in (Duffie, 2001) is based on the assumption that not everyone has the same information regarding a firm's asset; so the idea is once again to use different filtrations on the same probability space. It can be called a 'hybrid model', because of the mixture of the two kind of model seen so far. It allow for the possibility of controlling the firm: the asset's value is represented by a stochastic process  $V$ . The "structural part" of the model consists in choosing an optimal liquidation policy until the condition "asset less than liabilities" hold, that is done by solving a Hamilton-Jacobi-Bellman equation. At the same time, it accounts for an unpredictable default arrival, which is "controlled" by the partial filtration<sup>14</sup>.

### **Conclusions**

One should ask why modeling implementations are not often well reflected into the regulatory frameworks. In the financial field, more than in sociology, rules come after the mathematical or economical findings, but, more often, rules come after financial practices. The importance of credit ratings should give speed and motivation to private or public Credit Rating Agencies, in order to improve themselves, towards more efficient and accurate models of evaluation. The challenges for financial mathematicians are fair and always open. But what about a different risk management approach? Where does the border between risk appetite and resilience of financial systems lie? New findings are also desirable in this framework.

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<sup>14</sup> According to (U. Cetin, 2004, Vol. 14, No. 3) "Duffie and Lando obtain a reduced form model by constructing an economy where the market sees the manager's information set plus noise. The noise makes default a surprise to the market. In contrast, we obtain a reduced form model by constructing an economy where the market sees a reduction of the manager's information set. The reduced information makes default a surprise to the market. [...]From a manager's perspective, default is an accessible stopping time (predictable). Usually, in the structural approach default occurs when the firm's value, a continuous sample path process, hits a barrier. This formulation is consistent with the manager's perspective but inconsistent with reduced form models."

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