

# Assessment of Resilience in Desalination Infrastructure Using Semi-Markov Models

Abdulaziz Khiyami, Andrew Owens, Abdelkrim Doufene, Adnan Alsaati, and Olivier de Weck

**Abstract** As the supply of desalinated water becomes significant in many countries, the reliable long-term operation of desalination infrastructure becomes paramount. As it is not realistic to build desalination systems with components that never fail, instead the system should be designed with more resilience. To answer the question how resilient the system should be, we present in this paper a quantitative approach to measure system resilience using semi-Markov models. This approach allows to probabilistically represent the resilience of a desalination system, considering the functional or failed states of its components, as well as the probability of failure and repair rates. As the desalination plants are connected with the end-user through water transportation and distribution networks, this approach also enables an evaluation of various network configurations and resilience strategies. A case study addressing a segment of the water system in Saudi Arabia is given with the results, benefits, and limitations of the technique discussed.

**Keywords:** *System Resilience, Water System, Semi-Markov Process.*

## List of Acronyms

CDF: Cumulative Distribution Function  
MRP: Markov Renewal Process  
MTBF: Mean Time Between Failures  
MTTR: Mean Time to Repair  
PDF: Probability Distribution Function  
SDR: Standard Deviation in Repair Time  
SMP: Semi-Markov Processes  
SWCC: Saline Water Conversion Corporation

## 1 Introduction

Water is a prerequisite for life and its provision in modern society is contingent on numerous interacting components that include the water source; physical infrastructure; the services it provides; the organizations that govern its use; and the people and industry that consume it, and produce waste water. As the interdependence between these components is strong, and in order to make water use more efficient, together these components may collectively be aggregated in one system, that we call the ‘water system’ in this paper.

Given water’s criticality, water system planners must continuously assess and manage a host of challenges to ensure the satisfactory performance of their

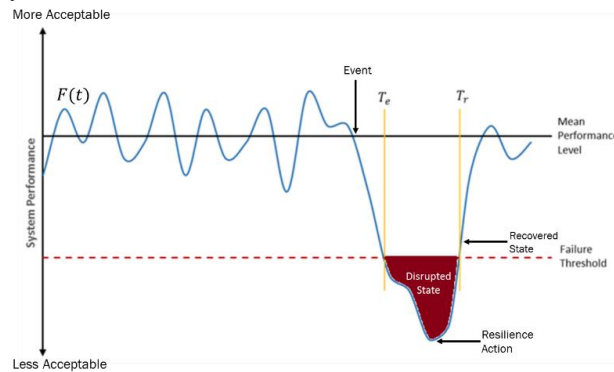
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systems. These challenges include the ever-present need to balance costs and impacts to the environments as well as the preparation for a variety of potential hazards such as natural disasters, and terrorist attacks, etc. This undertaking requires a continuous cycle of evaluation and planning activities following adverse events to upgrade and adapt the water system based on lessons learned. In an effort to aid and quantify this process, numerous attributes and objectives with which to assess the performance of water systems have been proposed. These include but are not limited to: cost, sustainability, reliability, robustness, preparedness, responsiveness, vulnerability, etc.

Key among these many overlapping and oftentimes conflicting objectives has been the concept of water system ‘resilience.’ Resilient systems have been described in the literature as those with “the ability to reduce the magnitude and/or the duration of disruptive events” (NIAC, 2009) or “the ability to minimize the costs of a disaster, return to the status quo, and to do so in the shortest feasible time” (McAllister, 2013). Fiksel et al., 2014 define resilience as “the capacity for a system to survive, adapt, and flourish in the face of turbulent change and uncertainty.” Hashimoto (1982) describes resilience as one of three key special risk-related system performance criteria in the widely utilized Reliability, Resiliency, and Vulnerability (RRV) framework and defines it as “how quickly a system is likely to recover or bounce back from failure once failure has occurred.”



**Figure 1** Graph of Resilience. Adapted from (Hashimoto, 1982)

Figure 1 graphically illustrates these definitions of system resilience. The function  $F(t)$  may represent any system performance measure provided that higher values correlate to higher performance. At a time  $T_e$ , the systems performance has fallen below a prescribed failure threshold entering a Disrupted State. Following a resilience action to repair the system, performance reaches above the failure threshold at time  $T_r$ . The difference  $T_r - T_e$  is the time spent in a failed (disrupted) state. The design of a resilient system should seek to minimize this time period, crafting systems that are both unlikely to fall below the prescribed failure threshold and quickly recover from failure should a failure occur.

A review of the literature on resilience reveals that many of its aspects bear similarity to the concepts of risk, reliability, preparedness, vulnerability assessment, disaster management and risk management. The question thus becomes: how does resilience differ from these concepts; and is it a distinct concept or just a different word for the same activities?

Resilience is indeed heavily intertwined with these concepts, however there appears to be a consensus that its key lies in the anticipation of unexpected events (EPA, 2015).

In this vein we propose a probabilistic framework devised using semi-Markov models to quantitatively model and assess the expected resilience of a water system. Each component in the system is defined by its status (functional/failed) and transition probability distributions defined by failure rates, repair rates, and

the time that the system can maintain its performance after component failures. This technique enables the calculation of all likely potential system states, and the probability of system failure within a chosen study period, thereby anticipating conceivable system failures.

To exemplify this approach we analyze a case study from the Kingdom of Saudi Arabia, an arid country that has turned to desalination for much of its municipal water supply. With a heavy reliance upon desalination and an extensive network of plants and pipelines, the Kingdoms water system performance is especially beholden to plant outages, pipe breaks, and pump failures. These failure conditions are easily anticipated but occur unexpectedly. Our approach provides a framework for these events to be anticipated and planned for so that they are less disruptive to the overall system performance, thereby increasing resilience.

The paper is organized as follows: in the Background section we present the context of Saudi Arabia, in the Methodology section we discuss the theoretical and mathematical procedures of the resilience framework; and in the Application section we utilize the developed methodology for the Saudi context. Future work and conclusions are presented in the final section.

## 2 Background.

The Kingdom of Saudi Arabia (KSA) is the largest country in the world with no permanent natural rivers or lakes, an arid land with seldom rainfall. As such the vast majority of water consumed in the Kingdom comes from non-renewable ground water resources (SSDN 2015, SIPS 2015).

To compensate for its lack of natural freshwater the Kingdom has increasingly turned to desalination to satisfy its water needs. Today Saudi Arabia is the world's largest market for desalinated water with a capacity of 5.72 million m<sup>3</sup>/day accounting for as much as 60% of the total urban water supply (SWCC, 2014).

Perhaps no city can better demonstrate Saudi Arabia's extreme reliance upon desalination better than its capital Riyadh. Initially a small oasis town of no more than 10,000 inhabitants (Al-Naim, 2008) at the start of the 20<sup>th</sup> century the capital is now a bustling metropolis with a population close to 7 million. Having long ago outgrown its local water resources, Riyadh now meets nearly half its municipal demand from desalinated water that is produced at giant facilities on the East Coast and then pumped via pipelines over hundreds of kilometers.

Thus, the optimal operation of the desalination system depends not only upon the stand-alone plants but the network as a whole. The evaluation of a desalination system as network of production nodes (desalination plants) and consumption nodes (cities) connected by edges of water pipelines is therefore informative for enhancing the design of the system in its entirety.

Ishimatsu, et al., 2015 presented such a deterministic network model that allowed for a desalination network's optimization in space, that is, where geographically a new infrastructure component should be located at a given time. This procedure utilized a graph theoretic framework with a multi-objective optimization to design the network for cost and/or sustainability.

However, the stated mission of the Saline Water Conversion Corporation (SWCC), the main institution tasked with the supply of desalinated water, is *the secure and maintained provision of water to the nation*.

Therefore to truly optimize KSA's desalination network, a model that considers failure and resilience is necessary. An optimization that only considers nominal operating conditions is not realistic indeed as it will overestimate the systems capabilities and underestimate its operating costs.

### 3 Methodology.

In this paper, we utilize Semi-Markov Processes (SMPs) to examine the resilience of water pipeline networks for a given operating duration, looking in particular at the amount of downtime, the amount of unmet demand, and the number of repair actions that will be required. All of these metrics are stochastic, not deterministic, since the underlying processes behind them –failures and repairs– are inherently stochastic. As such, the outputs of the model are not single point values, but rather distributions. These can then be used by decision-makers to make risk-informed decisions regarding local storage capacity, resource allocation for maintenance actions, and operating cost projections.

#### 3.1 Semi-Markov Processes

SMPs are probabilistic, state-based models of system behavior that are an extension of Markov chains. Like Markov chains, SMPs represent system behavior in a directed graph of states and transitions, where states (nodes) represent a given configuration of the system and transitions (edges) are events that cause the system configuration to change from one state to another. Each transition has an associated probability distribution which describes the amount of time until that transition occurs once the state it leaves is entered. An important requirement on SMPs is that, similar to Markov chains, the states must be “memoryless,” meaning that the future evolution of the system is dependent only on the current state and not on the pathway taken to reach that state. However, whereas in Markov chains these distributions must be exponential, SMPs allow the use of any distribution (Warr and Collins, 2012; Nunn and Desiderio 1977; Lisnianski and Levitin, 2003). An excellent overview of SMPs and techniques for solving them is presented by Warr and Collins (Warr and Collins, 2012).

An SMP is fully characterized by the kernel matrix  $\mathbf{Q}(\mathbf{t})$  and the unconditional waiting time density matrix  $\mathbf{H}(\mathbf{t})$ , each of which have entries that are calculated as follows (Warr and Collins, 2012):

$$Q_{ij}(\mathbf{t}) = f_{ij}(\mathbf{t}) \prod_{k \neq j} \left( 1 - \int_0^{\mathbf{t}} f_{ik}(\xi) d\xi \right)$$

$$H_{ii}(\mathbf{t}) = \sum_j Q_{ij}(\mathbf{t})$$

where  $f_{ij}(\mathbf{t})$  is the Probability Distribution Function (PDF) describing the amount of time  $t$  that passes after entry into state  $i$  before a transition from state  $i$  to state  $j$  occurs, given that a transition to state  $j$  does occur (as opposed to some other state). Each entry  $Q_{ij}(\mathbf{t})$  of the kernel matrix is a PDF describing the amount of time  $t$  that passes after entry into state  $i$  before a transition from state  $i$  to state  $j$  occurs, assuming no transition to any other state occurs in the interim. This can be seen from the fact that it is a product of the PDF of the time until transition from state  $i$  to state  $j$  and the complements of the Cumulative Distribution Functions (CDFs) of all other transitions. The unconditional waiting time density matrix is a diagonal matrix with entries  $H_{ii}(\mathbf{t})$  that give the PDFs describing the amount of time  $t$  that passes after entry into state  $i$  until a transition out of state  $i$  occurs, regardless of the destination state. Given  $\mathbf{Q}(\mathbf{t})$  and  $\mathbf{H}(\mathbf{t})$ , several key metrics describing the behavior of the system modeled by the SMP can be solved for. These metrics are listed in Table 1 (Warr and Collins, 2012). The process of calculating these metrics from  $\mathbf{Q}(\mathbf{t})$  and  $\mathbf{H}(\mathbf{t})$  using the Laplace domain is described in greater detail below.

**Table 1:** Symbols, names, and descriptions of key SMP metrics. All metrics assume that the system starts in state  $i$  at time 0 (Warr and Collins, 2012).

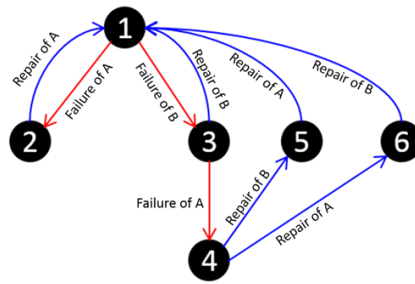
Symbol	Name	Description
$\phi_{ij}(t)$	Time-dependent state probability	Probability that the system will be in state $j$ at time $t$
$E_{ij}(t)$	Expected time in state	Expected amount of time that the system will have spent in state $j$ up to time $t$
$g_{ij}(t)$	PDF of first passage time	PDF describing the time $t$ taken to reach state $j$ the first time
$G_{ij}(t)$	CDF of first passage time	CDF giving the probability that the system has reached state $j$ by time $t$
$V_{ij}(k, t)$	MRP probability	CDF giving the probability that the system has reached state $j$ a total of $k$ or fewer times by time $t$

### 3.2 Application to Resilience Modeling

SMPs have previously been used to examine the resilience and maintenance logistics requirements of space systems (Owens, 2014; Owens and de Weck 2014; Owens et al. 2015; Owens et al. 2015; Do et al. 2015; Owens and de Weck 2015; Do et al. 2016), and we use a similar approach here. In this formulation, each state in the SMP is characterized in terms of the status – functional or failed – of each element – pipeline or desalination plant– within the system.

As is suggested by the state formulation, the transitions between states represent failure and repair events. (In the case where degraded states are included, these would include degradation and partial repair events.) The PDF used depends on the transition being represented. Failures are characterized by exponential distributions – a common first-order model of random component failures known as the constant failure rate model (Ebeling, 2000). The rate parameter of this distribution is equal to the inverse of the Mean Time Between Failures (MTBF) for each particular element. Repairs are modeled using a lognormal distribution, which provides a good estimate of the time required for corrective repair (Kline, 1984; Jones, 2010). In this case, the distribution is formed to have a mean and standard deviation equal to the Mean Time to Repair (MTTR) and Standard Deviation in Repair Time (SDR) for each particular repair activity.

The structure of the network of states and transitions representing the SMP is specifically constructed to link the generic SMP metrics described in Table 1 to system metrics. In particular, the structure of the SMP links the Markov Renewal Process (MRP) probabilities – which give the distribution of the number of times a given state will be visited in a given period of time – to the number of failures experienced by a particular element by ensuring that each state is linked to the failure of a particular component. This is done by ensuring that every state is entered by one and only one failure transition. Therefore, the number of times that a given state is visited corresponds to the number of times that that failure occurs. An example of this network structure is given in Figure 2. When multiple states are entered by failure of the same element, the MRP distributions for these states are convolved together to determine the total number of failures experienced by that element. Additional details on the connection between state structure and system metrics, as well as restrictions on SMP structure, are discussed by Owens (Owens, 2014).



**Figure 2:** Example SMP state/transition network for a system with two elements, A and B. Each transition is labeled with the event it represents. Red transitions indicate failure events, and blue transitions indicate repairs.

The impacts of failures are captured via the state definitions. Since each state is characterized by the status of each element within the system, a model that can characterize system performance as a function of element status can then produce key metrics for each state, such as the rate of unmet demand in a given city. This information can be combined with SMP metrics relating to states, including distributions for the number of times a state is visited and the amount of time spent in that state, in order to develop distributions for these key metrics (Owens and de Weck, 2015).

### 3.3 Automated SMP Generation

A key limitation for the application of SMPs to systems analysis of this type is that the number of states that a given system could be extremely large. As a result, the generation of the SMP model itself can be a very time-consuming process unless some form of automation can be utilized. While some previous applications of SMPs have used manually-generated state network models that limit state-space with simplifying assumptions (Owens, 2014; Owens et al. 2015), we implement an automated SMP generation algorithm based on one presented previously for space systems by Owens and de Weck (Owens and de Weck, 2015).

The algorithm consists of a systematic enumeration of new states based on existing ones, starting from the nominal state (i.e. all systems operational). New states – called “children” of the current state – are produced by examining all possible transitions away from the current state. In general, elements that are currently functional can fail, and elements that are currently failed can be repaired. For example, the nominal state has a set of transitions away from it representing the failure of each element in the system, each of which ends at a new state representing the configuration of the system in which that element is failed. Additional failures and repairs produce additional new states, unless the configuration of the resulting state is equivalent to the nominal state (all systems operational), in which case the transition returns to the nominal state rather than creating a new state (Owens and de Weck, 2015).

This iterative generation of new states grows the SMP network, and a pruning algorithm is used to remove states that have a probability of occurrence below a given threshold. This is done by calculating the first passage probability  $G_{ij}(t)$  for each new state to determine the probability that it is visited at least once within the time horizon of the analysis; if this probability is below a given threshold, and if the state was entered by a failure event and not a repair event, the state is removed from the network. States entered by repair events are not removed from the network since they are a part of the pathway back to the nominal state, forming the

loops that enable the use of MRP probabilities to examine spares requirements (Owens and de Weck, 2015).

The main difference between the algorithm used here and the one described by Owens and de Weck (Owens and de Weck, 2015) is that in this case new states are produced in generations, rather than one at a time, before pruning is applied. Generation 0 is the nominal state, generation 1 consists of all of the children of the nominal state, generation 2 consists of all the children of the children of the nominal state, and so on. Pruning of states in generations rather than individually significantly decreases the amount of computational time required to generate the SMP network.

### 3.4 Model Solution

Once an SMP model of the system is produced, it can be solved for the key metrics of interest. This process consists of two steps. First, the SMP is solved for the metrics described in Table 1, or whatever subset of them is desired for a particular problem. In this case, we are particularly interested in the MRP probabilities  $V_{ij}(t)$ , which are partially based on the first passage time PDFs  $g_{ij}(t)$ . These metrics can be solved for quickly using matrix multiplication in the Laplace domain followed by numerical Laplace transform inversion (Warr and Collins, 2012). For convenience, following the convention of Warr and Collins (Warr and Collins, 2012), we abbreviate the symbol for the Laplace transform as a tilde ( $\sim$ ) over the relevant matrix. The equations for first passage time and MRP probabilities in the Laplace domain are:

$$\begin{aligned}\tilde{g}(s) &= \tilde{Q}(s)(I - \tilde{Q}(s))^{-1} (I \circ (I - \tilde{Q}(s))^{-1})^{-1} \\ \tilde{V}(k, s) &= \frac{1}{s} (\mathbf{1} - \tilde{g}(s) \circ (\mathbf{1}(I \circ \tilde{g}(s))^k))\end{aligned}$$

Where  $I$  is the identity matrix,  $\circ$  is the Hadamard product of two matrices (elementwise multiplication), and  $\mathbf{1}$  is a matrix of ones (Warr and Collins, 2012). Once the Laplace transform of the MRP probabilities is obtained using the equations above, the EULER numerical Laplace transform inversion technique developed by Abate and Whitt (Abate and Whitt, 1995) is utilized to obtain the time-domain MRP probabilities. Owens (Owens, 2014) presents a brief overview and explanation of the numerical Laplace and inverse Laplace transform algorithms used here in Appendix A of his thesis, and more detail, including derivations and background, is presented by Warr and Collins (Warr and Collins, 2012) and Abate and Whitt (Abate and Whitt, 1995).

The result of the above procedure is the distribution of the number of times each state in the SMP is visited. This result can be used directly to determine the distribution of the number of failures that each element in the system will experience, as described above. When combined with the unconditional waiting time density  $H_{jj}(t)$  for each state  $j$ , the distribution of the number of visits to state  $j$  (assuming a start in state 0, the nominal state)  $V_{0j}(k, t)$  can also be used to generate  $T_j(t)$ , the distribution of the total amount of time that will be spent in state  $j$  for the time period examined.

$$T_j(t) = V_{0j}(0, t) \delta(0) + \sum_{k=1}^{\infty} (V_{0j}(k, t) - V_{0j}(k-1, t)) (\text{Conv}_k(H_{jj}(t)))$$

Here  $\delta(0)$  is the Dirac delta function and  $\text{Conv}_k(f(t))$  is a function representing the convolution of  $k$  instances of a function  $f(t)$  – that is,  $\text{Conv}_1(f(t)) = f(t)$ ,  $\text{Conv}_2(f(t)) = f(t) * f(t)$ , and so on. When applied to the unconditional waiting time density for a particular state, this convolution produces the distribution of the total amount of time spent in that state given that the state is visited exactly  $k$

times. This distribution is then conditioned by the probability that the state is visited exactly  $k$  times, and the sum of these conditioned distributions (representing the possible cases for the number of times the state will be visited) gives the distribution of the total amount of time spent in that state. In practice, the summation in the equation above is only carried out as far as there is a non-negligible probability of  $k$  visits to the state rather than continuing to infinity.

As described above, each state in the SMP is characterized by the status of each element within it. For this case study, this means the status of each pipeline and desalination plant as either functional or failed. For high-level decision-making, however, a more relevant metric of interest may be the impact of these failures on water delivery to consumers (in this case, cities). Therefore, each state is characterized in terms of the rate of unmet demand at each city by solving an optimization problem to determine the flow configuration in the network that minimizes the total rate of unmet demand across all cities. In the nominal state, each pipeline and desalination plant has a maximum capacity indicating the amount of water it can transport or produce. States in which a failure has occurred in one or more elements have the capacities of that element set to zero in order to simulate the impacts of that failure. This reduction in network capability results in reduced ability to meet consumer demands, which in turn results in some rate of unmet demand at some (or all) of the cities in the network. The optimization problem for a system with  $n$  cities and  $m$  elements (pipelines and desalination plants) is formulated as follows:

$$\begin{aligned} & \text{minimize} \quad \sum_{i=1}^n u_i \\ \text{subject to:} \quad & u_i + \sum_{j \in \text{IN}_i} x_j - \sum_{j \in \text{OUT}_i} x_j = d_i \quad \forall i \in \{1, \dots, n\} \\ & 0 \leq u_i \leq d_i \quad \forall i \in \{1, \dots, n\} \\ & -c_j \leq x_j \leq c_j \quad \forall j \in \{1, \dots, m\} \end{aligned}$$

where  $u_i$  is the rate of unmet demand at city  $i$ ,  $d_i$  is the rate of demand at city  $i$ ,  $c_j$  is the flow capacity for element  $j$ ,  $x_j$  is the flow rate in element  $j$ , and  $\text{IN}_i$  and  $\text{OUT}_i$  are the sets of elements flowing into and out of city  $i$ , respectively. Note that self-loops, which represent desalination plants, appear only in the set of elements flowing into their city, and not the set flowing out. This linear optimization problem is quickly and easily solved using MATLAB's built-in `linprog()` function in order to determine the rate of unmet demand at each city in each state of the SMP.

It is possible that some states in the SMP are identical in terms of their system-level characteristics. Therefore, once the amount of time spent in each state and the rate of unmet demand for each city in each state are determined, the distributions for the amount of time spent in states with identical unmet demand profiles are convolved together to determine the total amount of time the system spends in that condition. Alternatively, these distributions could be convolved together based on the unmet demand rate for a particular city. Once the distribution of the total amount of time spent at a given rate of unmet demand is obtained, it can be used with the specific rate of unmet demand to determine the distribution of the total amount of unmet demand in the time period being examined, which can then be used to inform storage capacity decisions.

## 4 Application

The proposed methodology is applied to a subsection of Saudi Arabia's easterly desalination network. Figure 3 (left) shows the system containing the capital city



of Riyadh and associated desalination plants and cities on the Arabian Gulf. Though in reality the network extends beyond Riyadh, and also branches out onto other Eastern cities, for this case study the analysis is focused upon the largest and most significant population centers of the region. The simplified network representation considered in the case study is shown in Figure 3 (right).

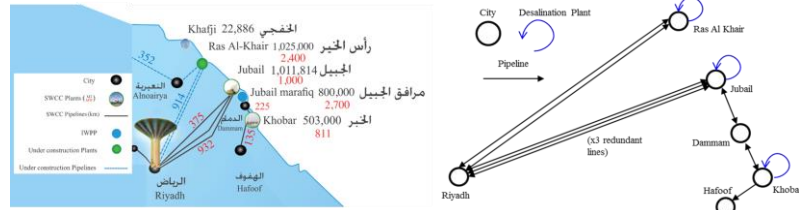


Figure 3 Eastern Desalination Network (SWCC, 2013) & Case Study Representation

### 4.1 Network Case Study Parameters

The parameters of the desalination network are recorded in Table 2 and Table 3 with the chosen analysis units of cubic meters and days. Daily city desalinated water demands were calculated using the population, per capita daily water consumption, and percentage contribution of desalination in a manner similar to the methodology previously utilized by Ishimatsu, et al., 2015. Desalination plant capacities and pipeline throughputs were found as specified in designs by SWCC and associated contractors (SWCC, 2016 and Lasser & Heinz, 2011).

Indications regarding plant failures were received from plant failure logs of SWCC. These logs included the duration and specific reason for outages e.g. steam line leaks, boiler maintenance; as well as the calculated MTBF, MTTR, and SDR for a desalination plant in 2015. Exact information regarding failure and repair rates was not made available for the specific desalination plants considered in the case study, and so the provided plants MTBF and MTTR were used as representative.

Information on failure and repair rates of pipelines was not forthcoming and was therefore estimated from news reports (Khan, 2011), technical reports (Malik, Andijani, Mobin, & Al-Hajri, 2005), and the recommendations of SWCC staff. On average, desalination pipelines were found to break less often than desalination plants, but require longer to repair.

The data therefore used in this case study is merely notional and intended to only demonstrate the proposed methodology, not to provide concrete results or recommendations.

Table 2 Node Parameters

Node ID	Node Name	Demands [1000 m <sup>3</sup> /day]
1	Riyadh	701
2	Ras Al Khair	0
3	Jubail	42
4	Dammam	113
5	Khobar	572
6	Hafoof	83

**Table 3** Edge Parameters

ID From	ID To	Name	MTBF [days]	MTTR [days]	SDR [days]	Edge Capacities [1000 m <sup>3</sup> /day]
2	2	Ras Al Khair Desalination Plant	60	4	3	1025
3	3	Jubail Desalination Plant	60	4	3	1782
5	5	Khobar Desalination Plant	60	4	3	547
1	2	Riyadh - Ras Al Khair D	110	14	7	474
1	2	Riyadh - Ras Al Khair E	110	14	7	474
1	3	Riyadh - Jubail A	110	14	6	415
1	3	Riyadh - Jubail B	100	14	6	415
1	3	Riyadh - Jubail C	100	14	6	380
3	4	Jubail - Dammam	90	5	1	305
4	5	Dammam - Khobar	75	4	1	305
5	6	Khobar- Hafoof	80	5	1	266

## 4.2 Case Study Execution and Results

The model was formulated in MATLAB and executed for a time horizon of 10 years with a state probability threshold of 0.25%. Computationally this required about 15 minutes of running time on a single machine using an Intel® Xeon® CPU E5-2650 v3 with 32 GB of installed RAM.

The CDF of unmet demand for each city was calculated and this is plotted in Figure 4. The analysis reveals for example that Riyadh, with its numerous feeder pipelines is relatively safe to the risk of unmet demand with nearly an 80% probability that unmet demand will not exceed 3 million m<sup>3</sup> throughout the 10 years considered. Strategic reserves of only 1 million m<sup>3</sup> are necessary to ensure that the city has a near zero chance of any unmet demand.

By contrast the Eastern Region cities of Dammam, Khobar, and Hafoof are far more vulnerable with Khobar, the largest of the three, being most at risk. Throughout the same 10 year period, Khobar has an 80% probability of experiencing nearly 50 million m<sup>3</sup> of unmet demand and would require reserves of 75 million m<sup>3</sup> to ensure against failure. This is intuitive, Khobar approaches Riyadh in its daily desalination demand but does not have the benefit of a direct connection to the Ras Al Khair facility or anywhere near as many redundant feeder pipelines.

To design for system resilience various strategies can now be explored using the proposed approach. For example adding a new desalination plant at Dammam, or connecting Ras Al Khair to Jubail with a new pipeline. Increasing plant/pipeline reliability through upgrades and more vigilant maintenance of the network elements can be investigated via variance of the failure and repair rates.

It was discovered that among the most effective ways to reduce the risk of unmet demand was by improving the reliability of the Khobar desalination plant. Doubling the MTBF from once every 60 days to once every 120 days reduces the expected unmet demand at probability of 80% by nearly half as shown in Figure 5. Further increasing the reliability of the Khobar desalination plant found further reductions in expected unmet demand but at diminishing returns as shown in Figure 6.

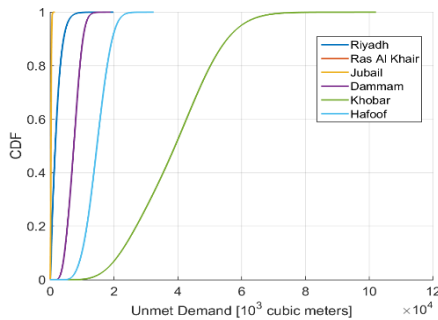


Figure 4 CDF of Total Unmet Demand

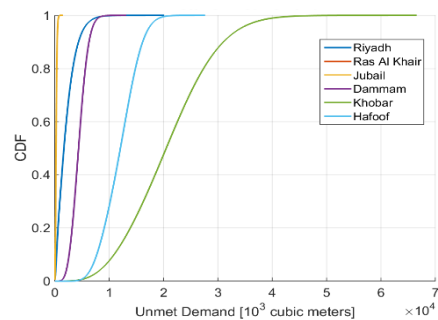


Figure 5 CDF of Total Unmet Demand for Khobar MTBF 120

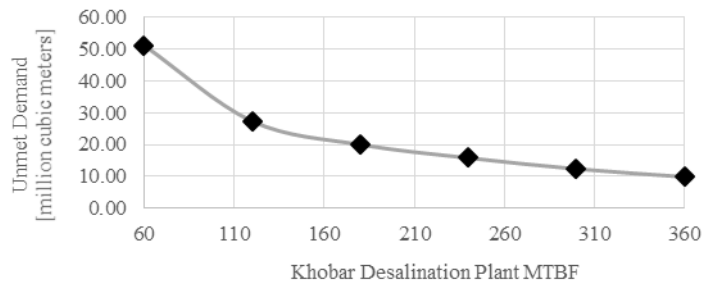


Figure 6 Khobar 80<sup>th</sup> Percentile Unmet Demand

## 5 Conclusions and Further Work.

This paper introduces an approach to quantitatively evaluate the resilience of water systems. The modelling procedure was illustrated via a notional case study of a portion of Saudi Arabia’s desalination network.

The current approach provides a starting framework upon which to improve for an advanced assessment of resilience in water systems. For starters, the current approach employs a binary fail/repair status for each network element; further work should explore the representation of partially degraded states to more fully represent the operation of the system. The current application utilizes static network demands to evaluate resilience well into the future. A model that incorporates dynamically changing demand and future growth scenarios will contribute to the understanding of how efficiency and end-user programs may affect the system resilience. Additionally the characterization of specific outages and failures needs to be introduced to the framework. For example, if an extreme event could cause all desalination plants to be shut-down simultaneously, the

likelihood and consequences of such an event is not currently considered in the model. Finally, the methodology should be enhanced by the implementation of a resilience optimization that will automatically find the best combination of network upgrades and expansions to maximize resilience. Future work should also more holistically evaluate the water system, considering agricultural demands and groundwater reserves, as well as waste water treatment, rather than just the desalination system in isolation to assess the resilience of the water system in its entirety.

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## References

1. Abate, J., and Whitt, W., "Numerical inversion of Laplace transforms of probability distributions," *ORSA Journal on computing*, vol. 7, 1995.
2. Al-Naim, M., 2008. Riyadh: A City of 'Institutional' Architecture. In: Y. Elsheshtawy, ed. *The Evolving Arab City: Tradition, Modernity and Urban Development*. Abingdon: Routledge, pp. 118-152.
3. Do, S., Owens, A. C., and de Weck, O. L., "HabNet - An Integrated Habitation and Supportability Architecting and Analysis Tool," *45th International Conference on Environmental Systems*, ICES-2015-289, Bellevue, WA: 2015.
4. Do, S., Owens, A., Ho, K., Schreiner, S., and de Weck, O., "An independent assessment of the technical feasibility of the Mars One mission plan – Updated analysis," *Acta Astronautica*, vol. 120, Mar. 2016, pp. 192–228.
5. Ebeling, C. E., *An Introduction to Reliability and Maintainability Engineering*, New Delhi: Tata McGraw-Hill Publishing Co. Ltd., 2000.
6. EPA, 2015. *Systems Measures of Water Distribution System Resilience*, Cincinnati: United States Environmental Protection Agency.
7. Fiksel, J., Goodman, I. & Hecht, A., 2014. Resilience: Navigating a Sustainable Future. *Solutions for a Sustainable and Desirable Future*, 5(5), pp. 38-47.
8. Hashimoto, T., 1982. Reliability, Resiliency, and Vulnerability Criteria For Water Resource System Performance Evaluation. *Water Resources Research*, 18(1), pp. 14-20.
9. Ishimatsu, T., Doufene, A. & de Weck, O., 2015. *Desalination Network Model Driven Decision Support System: A Case Study of*. San Diego, International Desalination Association World Congress 2015.
10. Jones, H., "Life Support Dependability for Distant Space Missions," *40th International Conference on Environmental Systems*, AIAA-2010-6287, Barcelona: American Institute of Aeronautics and Astronautics, 2010.
11. Khan, S. A., 2011. Riyadh residents breathe sigh of relief as water supply restored. *Saudi Gazette*, 16 09.
12. Kline, M. B., "Suitability of the Lognormal Distribution for Corrective Maintenance Repair Times," *Reliability Engineering*, vol. 9, 1984, pp. 65–80.
13. Lisnianski, A., and Levitin, G., *Multi-State System Reliability*, Singapore: World Scientific, 2003.

14. Lasser, B. & Heinz, A., 2011. *Innovations and Technological Developments in Long Distance Water Transmission Systems: Case Study of Riyadh, Saudi Arabia*, Essen: 3R International Technical Journal for Piping System Integrity and Efficiency.
15. Malik, A. U., Andijani, I., Mobin, M. & Al-Hajri, M., 2005. *Investigations on the Leakage in RWTS, Line – A at 24 km (A/B)1*, Al Jubail: SWCC.
16. McAllister, T., 2013. *Developing Guidelines and Standards for Disaster Resilience of the Built Environment: A Research Needs Assessment*, Gaithersburg: NIST.
17. MoWE, 2014. *2013 Annual Report*, Riyadh: Ministry of Water and Electricity.
18. NIAC, 2009. *Critical Infrastructure Resilience Final Report and Recommendations*, s.l.: National Infrastructure Advisory Council.
19. Nunn, W. R., and Desiderio, A. M., *Semi-markov processes: an introduction*, Arlington, Virginia: 1977.
20. Owens, A. C., “Quantitative Probabilistic Modeling of Environmental Control and Life Support System Resilience for Long-Duration Human Spaceflight,” Massachusetts Institute of Technology, 2014.
21. Owens, A. C., and de Weck, O. L., “Automated Risk and Supportability Model Generation for Repairable Systems,” *66th International Astronautical Congress*, IAC-15-D1.3.10, Jerusalem: International Astronautical Federation, 2015, pp. 1–9.
22. Owens, A. C., and de Weck, O. L., “Use of Semi-Markov Models for Quantitative ECLSS Reliability Analysis: Spares and Buffer Sizing,” *44th International Conference on Environmental Systems*, ICES-2014-116, Tucson, AZ: 2014.
23. Owens, A., de Weck, O., Mattfeld, B., Stromgren, C., and Cirillo, W., “Comparison of Spares Logistics Analysis Techniques for Long Duration Human Spaceflight,” *45th International Conference on Environmental Systems*, ICES-2015-288, Bellevue, WA: International Conference on Environmental Systems, 2015.
24. Owens, A., Do, S., Kurtz, A., and de Weck, O., “Benefits of Additive Manufacturing for Human Exploration of Mars,” *45th International Conference on Environmental Systems*, ICES-2015-287, Bellevue, WA: International Conference on Environmental Systems, 2015.
25. SIPS - Sustainable Infrastructure Planning System Project. Final Report, Center for Complex Engineering Systems at KACST and MIT, <http://www.cces-kacst-mit.org>, July 2015.
26. SSDN – Strategic Sustainable Desalination Network. Final Report, Center for Complex Engineering Systems at KACST and MIT, <http://www.cces-kacst-mit.org>, July 2015.
27. SWCC, 2013. *Annual Report*, Riyadh: Saline Water Conversion Corporation.
28. SWCC, 2014. *Annual Report*, Riyadh: Saline Water Conversion Corporation.
29. SWCC, 2016. *SWCC Projects Pipeline*. [Online] Available at: <http://www.swcc.gov.sa/english/Projects/Pipelines/Pages/PipeLine.aspx> [Accessed 11 April 2016].
31. Warr, R. L., and Collins, D. H., “An Introduction to Solving for Quantities of Interest in Finite-State Semi-Markov Processes,” 2012.