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Digging Deeper: Observing Primordial Gravitational Waves below the Binary-Black-Hole-Produced Stochastic Background

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The merger rate of black hole binaries inferred from the detections in the first Advanced LIGO science run implies that a stochastic background produced by a cosmological population of mergers will likely mask the primordial gravitational wave background. Here we demonstrate that the next generation of ground-based detectors, such as the Einstein Telescope and Cosmic Explorer, will be able to observe binary black hole mergers throughout the Universe with sufficient efficiency that the confusion background can potentially be subtracted to observe the primordial background at the level of $\Omega_{\mathrm{GW}} \simeq 10^{-13}$ after 5 years of observation.

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Introduction.—According to various cosmological scenarios, we are bathed in a stochastic primordial gravitational wave background (PGWB) produced in the very early stages of the Universe. Proposed theoretical models include the amplification of vacuum fluctuations during inflation [1–3], pre-big bang models [4–6], cosmic (super) strings [7–10], or phase transitions [11–13]. The detection of a primordial background would have a profound impact on our understanding of the evolution of the Universe, as it represents a unique window on its first instants, up to the limits of the Planck era, and on the physical laws that apply at the highest energy scales.

In addition to the PGWB, an astrophysical background is expected to result from the superposition of a large number of unresolved sources since the beginning of stellar activity (see Ref. [14] for a review of different sources that could produce an astrophysical background). The astrophysical background potentially contains a wealth of information about the history and evolution of a population of point sources, but it is a confusion noise background that is detrimental to the observation of the PGWB. In this Letter we show that at the sensitivity levels envisaged for third generation detectors such as the Einstein Telescope (ET) [15] and Cosmic Explorer (CE) [16], it will be possible to detect most of the sources, giving hope that the confusion background can be subtracted from the data, enabling the study of a PGWB. This problem is similar to the one investigated in Refs. [17,18] in the context of the Big Bang Observer.

On September 14, 2015, Advanced LIGO [19–21] directly detected gravitational waves (GWs) from the collision of two stellar-mass black holes at a redshift of $z \sim 0.1$ (GW150914) [22,23]. The inferred component masses of $m_1 = 36\ M_\odot$ and $m_2 = 29\ M_\odot$ are larger than those of candidate black holes in x-ray binaries inferred from reliable dynamical measurements [24]. This first detection suggests the existence of a population of black holes with relatively large masses, that might have formed in low-metallicity stellar environments [24], either through the evolution of an isolated massive binary in a galaxy [25] or through mass segregation and dynamical interactions in a dense globular system [26].

LIGO discoveries during the first observing run included a high-confidence ($> 5\sigma$) detection of a second merger event GW151226 and a marginal event of lower significance ($< 2\sigma$) LVT151012, both believed to be binary black hole (BBH) mergers. GW151226 resulted from the merger of black holes of mass $m_1 = 14.2M_\odot$ and $m_2 = 7.5M_\odot$ [27], and LVT151012 is believed to have resulted from the merger of black holes of mass $m_1 = 23M_\odot$ and $m_2 = 13M_\odot$ [27]. These observations indicate that many more detections will occur in the future and have provided the tightest constraints on the rate of such events [28].

Besides the loudest and closest events that can be detected individually by the Advanced LIGO–Advanced Virgo network, the population of undetected sources at larger redshift is expected to create a significant astrophysical background [29]. The background from the population of binary neutron stars (BNSs) and BBHs has been investigated by many authors in the past (see Refs. [14,30–35] for the most recent papers), who suggested that Advanced LIGO and Advanced Virgo had a realistic chance of detecting this background after a few years of operation with the standard cross-correlation method, even if this background is nor continuous (no overlap of the sources) or Gaussian [36].

In Ref. [29] the LIGO and Virgo Collaborations calculated the contribution to the stochastic background from BBHs with the same masses as GW150914. Taking into account the statistical uncertainty in the rate, they found that the stochastic signal could be detected, in the most optimistic case, even before the design sensitivity of the

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instruments is reached, but more likely after a few years of their operation at design sensitivity. It was also shown that lower mass systems that are too faint to be detected individually could add a significant contribution to the background. Following this first Letter, other authors have investigated the implication of GW150914 for the confusion background, including models of metallicity evolution with redshift and mass distributions [37,38], and arrived at the same conclusion: the background from BBHs is likely to be higher than previously expected and may dominate over the primordial background.

In this Letter, we use Monte Carlo simulations to calculate the confusion background from BBHs observed by networks of ground-based detectors. We study the potential reduction in the level of this background as more BBH signals are detected, and can be subtracted from the data, because of the improved sensitivity of ET [15] and CE [16] compared to advanced detectors. We show that the confusion background of astrophysically produced GWs can be significantly reduced, paving the way to observe the primordial background. We do not investigate subtraction techniques in detail, nor the residual resulting from the subtraction, but assume that the signals can be removed with high enough accuracy to search for an underlying stochastic gravitational wave background of a different origin.

Simulation of a population.—In order to calculate the total contribution of BBHs to the confusion background, we consider the fiducial model of Ref. [29] and generate an extra-galactic population of BBHs using the Monte Carlo procedure described in Refs. [36,39,40] and summarized below.

(1) The intrinsic masses m_1, m_2 (in the source frame) are selected from one of the two astrophysical distributions considered in Ref. [27]: (i) model A, power-law distribution of the primary (i.e., larger mass) companion $p(m_1) \propto m_1^{-2.35}$, and uniform distribution of the secondary, and (ii) model B, uniform distribution in the logarithm of the component masses $p(m_1, m_2) \propto m_1^{-1} m_2^{-1}$. In addition, we require that the component masses take values in the range 5–$100M_\odot$ with $m_1 + m_2 < 100M_\odot$.

(2) The redshift is drawn from a probability distribution $p(z)$,

$$p(z) = \frac{R_z(z)}{\int_0^{20} R_z(z)dz}, \tag{1}$$

obtained by normalizing the merger rate (in the observer frame) per interval of redshift, over the range $z \in 0$–20, and

$$R_z(z) = \int \frac{R_m(z)}{1+z} \frac{dV}{dz}(z)dz. \tag{2}$$

Here dV/dz is the comoving volume element and R_m (in the source frame) is the rate per volume, given by

$$R_m(z) = \int_{t_{\min}}^{t_{\max}} R_f(z_f) P(t_d) dt_d, \tag{3}$$

where $R_f(z)$ is the massive binary formation rate, $P(t_d)$ is the distribution of the time delay t_d between the formation of the massive progenitors and their merger, z_f is the redshift at the formation time $t_f = t(z) - t_d$, and $t(z)$ is the age of the Universe at merger. The value of R_m at $z = 0$ corresponds to the local rate estimated from the first LIGO observation run [27], which is 99^{+138}_{-70} Gpc^{-3} yr^{-1} for model A and 30^{+43}_{-21} Gpc^{-3} yr^{-1} for model B.

We assume that $R_f(z)$ follows the cosmic star formation rate and we use the recent model of Ref. [41], based on the gamma-ray burst rate of Ref. [42] and on the normalization described in Refs. [43,44]. We also assume that black holes of $30M_\odot$ or larger can only be formed below the metallicity threshold $Z_c = Z_\odot/2$ [24,29]. The metallicity is drawn from a $\log_{10}$-normal distribution with a standard deviation of 0.5 around the mean at each redshift [45] calculated from the mean metallicity-redshift relation of Ref. [46], rescaled upwards by a factor of 3 to account for local observations [41,47]. We further assume that the time delay distribution follows $P(t_d) \propto t_d^\alpha$, with $\alpha = -1$ for $t_d > t_{\min}$ [48–55], where $t_{\min} = 50 \times 10^6$ years is the minimum delay time for a massive binary to evolve until coalescence (see, e.g., Ref. [56]), and a maximum time delay $t_{\max}$ equal to the Hubble time.

(3) The location in the sky $\hat{\Omega}$, the cosine of the orientation ι, the polarization ψ, and the phase of the signal at coalescence ϕ_0 were drawn from uniform distributions.

(iv) For each BBH, we determine if its resultant GW emission is detectable in a given detector network. The signal-to-noise ratio (SNR) ρ_A detected by matched filtering with an optimum filter in the ideal case of Gaussian noise, in a detector labeled A, is

$$\rho_A^2 = 4 \int_0^\infty \frac{|F_{+,A}\tilde{h}_+ + F_{\times,A}\tilde{h}_\times|^2}{S_{n,A}} df, \tag{4}$$

where f is the GW frequency in the observer frame, $\tilde{h}_+$ and $\tilde{h}_\times$ are the Fourier transforms of the GW strain amplitudes of + and × polarizations that includes inspiral, merger, and ringdown phases of the signal [57], $F_{+,A}$ and $F_{\times,A}$ are the antenna response functions to the GW + and × polarizations, and $S_{n,A}(f)$ is the one-sided noise power spectral density of detector A. The coherent SNR for a network, assuming uncorrelated noises in the detectors, is simply given by the quadrature sum of the individual SNRs, $\rho_T^2 = \sum \rho_A^2$. We assume that sources with $\rho_T > 12$ can be removed with enough accuracy from the data and that only sources with $\rho_T < 12$ contribute to the confusion background. We are currently investigating this assumption using mock data challenges.

Detected sources.—In this section we investigate the evolution of the number of detections as the detector sensitivity increases from second to third generation and the number of detectors in the network increases from three to five. The advanced version of the two LIGO detectors at

Hanford (H) and Livingston (L) [20,21] started collecting data in September 2015 and are expected to reach design sensitivity in 2019, followed by Advanced Virgo (V) a few months later [58]. Two other detectors will join the network over the next 8 years: the Japanese detector KAGRA (K) [59] and a new detector in India (I) [60], whose sensitivity will be similar to the two LIGO detectors. Third generation detectors are currently under design study, such as the Einstein Telescope [15] and the Cosmic Explorer [16]. Between the second and the third generation, we expect to reach intermediate sensitivities, referred to as A+ and Voyager. Figure 1 plots the strain sensitivity of the various detectors considered in this Letter.

The total number of BBHs that coalesce in the observable Universe, as derived from the actual constraints on the local rate [27], is in the range $[1, 40] \times 10^4$ a year; therefore, the average waiting time between two consecutive events in the detector frame is between 100 and 2000 s. With advanced detectors, only a small fraction of the sources will be detected (less than 3% assuming the maximal rate) and the chance that two detections overlap in time is very small (less than 0.05%) given that the average duration of the signal is only 0.7 s (model A) or 0.2 s (model B) with the low frequency limit of the detectors of 10 Hz. With a network of third generation detectors, on the other hand, most of the sources will be above the detection threshold (more than 99.9%) and the signal will last much longer since the low frequency limit will be pushed to about 5 Hz. For the average rates, we expect 27% of sources to have some overlap (model A with an average duration of 83 s) or 3.5% (model B with an average duration of 26 s) and up to 48% (model A) or 11% (model B) for the maximal rates. However, when there is an overlap in the time domain the sources can still be resolved individually. In fact, the low frequency part of the signal contributes little to the SNR. In the frequency band starting at 20 Hz, we have more than a 99% chance of detection while decreasing the chance of overlap to 0.8% (model A with an average duration of 2 s) or 0.25% (model B with an average duration of 0.65 s) for the average rate, and about 2.3% (model A) and 1% (model B) for the maximal rate.

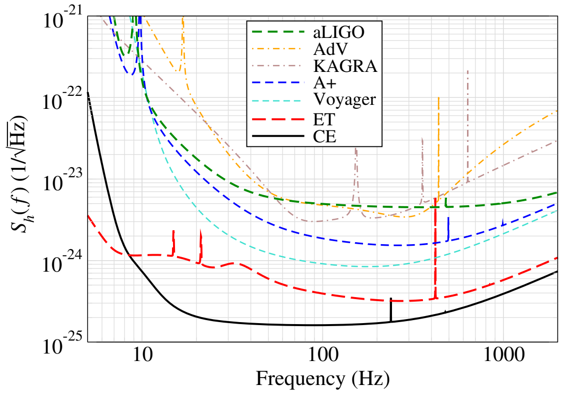

FIG. 1. Design power spectral density of second generation detectors, Advanced LIGO (aLIGO), Advanced Virgo (AdV), and KAGRA, and proposed sensitivity of third generation detectors Einstein Telescope (ET) and Cosmic Explorer (CE). Expected intermediate sensitivities such as Advanced LIGO Plus (A+) and Voyager are also shown.

Binary background.—The superposition of the gravitational waves from sources at all redshifts and integrated over all directions of the sky creates a stochastic background, whose energy-density spectrum in GWs is described by the dimensionless quantity [61]

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\mathrm{GW}}}{df}, \tag{5}$$

where $d\rho_{\mathrm{GW}}$ is the energy density in the frequency interval f to $f + df$, $\rho_c = 3H_0^2 c^2 / 8\pi G$ is the closure energy density of the Universe, and $H_0 = 67.8 \pm 0.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant [62].

The GW spectrum from the population of BBHs is given by the expression

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c c} f F(f), \tag{6}$$

where $F(f)$ is the total flux and f is the observed frequency. The total flux (in erg Hz^{-1}) is the sum of the individual contributions:

$$F(f) = T^{-1} \frac{\pi c^3}{2G} f^2 \sum_{k=1}^{N} [\tilde{h}_{+,k}^2(f) + \tilde{h}_{\times,k}^2(f)], \tag{7}$$

where N is the number of undetected sources in the Monte Carlo sample (in order to obtain a smooth average of the spectrum, we set $N = 10^5$ for the sources with $\rho_T < 12$). The normalization factor T^{-1} assures that the flux has the correct dimension, T being the length of the data sample.

Our waveform model includes inspiral, merger, and ringdown phases of the signal. In the inspiral regime, before the black holes reach the last stable orbit, the slope of the spectrum has the well-known $f^{2/3}$ behavior:

$$\Omega_{\mathrm{GW}}^{\mathrm{insp}}(f) = \frac{5\pi^{2/3} G^{5/3} c^{5/3}}{18 c^3 H_0^2} T^{-1} f^{2/3} \sum_{k=1}^{N} \frac{(1+z_k)^{5/3} (\mathcal{M}_k)^{5/3}}{D_L(z_k)^2} \times \left(\frac{(1+\cos^2 \iota_k)^2}{4} + \cos^2 \iota_k \right), \tag{8}$$

where $M = m_1 + m_2$ is the total mass, $\mathcal{M} = (m_1 m_2)^{3/5} M^{-1/5}$ the chirp mass, and $D_L(z)$ is the luminosity distance at redshift z. We shall see below that we retrieve this behavior over the relevant range of frequencies.

Figure 2 shows the energy density Ω_{GW} in GWs from undetected BBHs ($\rho_T < 12$) within advanced (top plot), A+ (middle plot), and third generation (bottom plot) detectors. Solid (green) curves are the total backgrounds

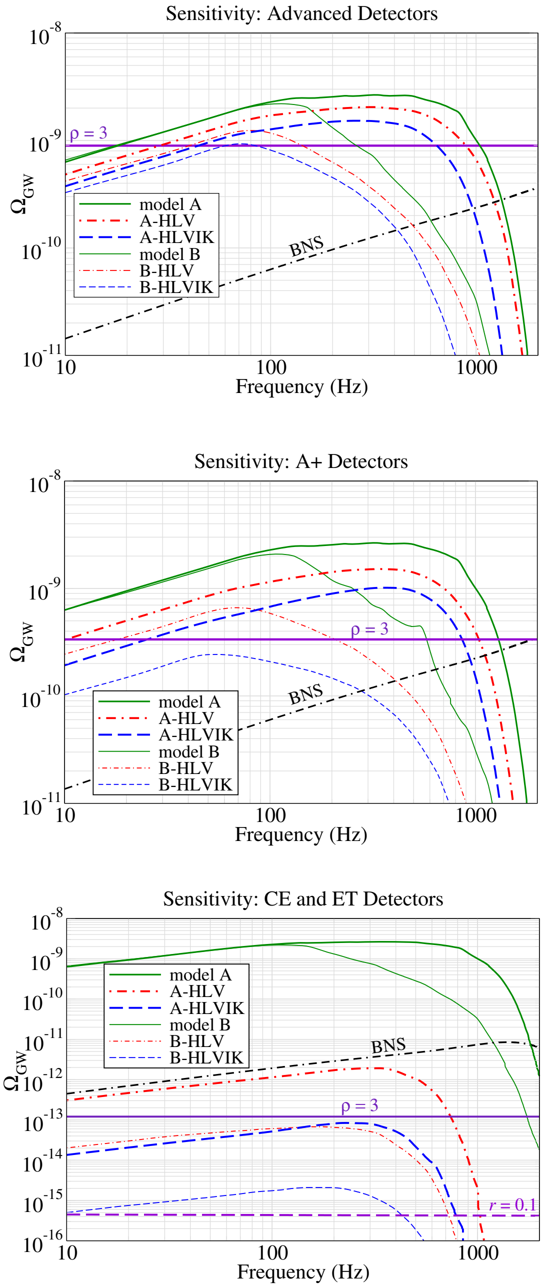

FIG. 2. Energy density spectrum Ω_{GW} in GWs from undetected BBHs ($\rho_T < 12$) within advanced (top plot), A+ (middle plot), and third generation (bottom plot) detectors. Solid (green) curves are the total backgrounds for models A (thick lines) and B (thin lines), respectively, when detected BBH signals are not removed from the data, so they are the same in each plot. We see that in the tens of Hz region one obtains the characteristic $f^{2/3}$ slope. The cosmological background from inflation assuming a tensor-to-scalar ratio of $r = 0.1$ is shown for comparison, and confusion background from unresolved binary neutron stars, assuming an average local rate of 60 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ [63]. The horizontal solid line is the minimal flat spectrum that can be detected with $\rho = 3$ with a five-detector network after 5 years.

for models A (thick lines) and B (thin lines), respectively, when detected BBH signals are not removed from the data, so they are the same in each plot. For each generation of sensitivity, we consider two different networks: a network of three detectors (HLV) located at the sites of LIGO-Hanford, LIGO-Livingston, and Virgo and a network of five detectors (HLVIK) that includes LIGO India and KAGRA, in addition to HLV. In the top plot, the detectors are assumed to have projected sensitivity levels of advanced detectors shown in Fig. 1. In the middle plot, we assume that all the detectors have the same intermediate sensitivity (A+). In the bottom plot, for the third generation we assume the sensitivity of ET in a triangle detector configuration at the location of Virgo and CE for all other detectors.

Results.—The total background from BBHs when detected BBH signals are not removed from the data is expected to dominate over all other sources of stochastic background, up to a few hundred Hz with an average energy density of $\Omega_{GW}(10\text{ Hz}) = 6 \times 10^{-10}$. With advanced detectors, the BBH confusion background is more than 50% of the total background, and still above $\Omega_{GW}(10\text{ Hz}) = 10^{-10}$ with the A + sensitivity. With third generation detectors, on the other hand, the level of the confusion background is decreased by orders of magnitude, reaching $\Omega_{GW}(10\text{ Hz}) = 10^{-14}$–$10^{-13}$ with a network of three detectors and $\Omega_{GW}(10\text{ Hz}) = 10^{-16}$–$10^{-14}$ with five detectors.

With a network of five third generation detectors we are able to decrease the confusion background below the minimal detectable flat energy spectrum (5 years of integration) of $\Omega_{GW}(10\text{ Hz}) = 10^{-13}$. The detectable value is derived requiring a SNR of $\rho = 3$, where [29]

$$\rho = \frac{3H_0^2}{10\pi^2}\sqrt{2T}\left(\int_0^\infty df \sum_{i=1}^{n} \sum_{j>i} \frac{\gamma_{ij}^2(f)\Omega_{GW}^2(f)}{f^6 S_{n,i}(f) S_{n,j}(f)}\right)^{1/2}, \tag{9}$$

for a network of detectors $i = 1, 2, \ldots, n$. In this expression γ_{ij} is the overlap reduction function characterizing the reduction of sensitivity due to the separation and the relative orientation of the detectors. The contribution to the SNR comes mostly from the closest pair of detectors, namely, the LIGO Hanford–LIGO Livingston detector pairs over a frequency interval of 50 Hz. The entire network of detectors is needed to identify the signals, estimate their parameters [64], and then remove their presence from the data. This minimal detectable value is above the current upper limit for the standard inflation model assuming a tensor-to-scalar ratio $r = 0.1$, meaning that the detectors' sensitivity should be improved by at least another factor of about 10 to reach a level of $\Omega_{\min} \sim 10^{-15}$.

An improvement by a factor of 10 in sensitivity past ET and CE would also allow for the removal of the extra confusion background from BNSs that could remain in the data at the level of $\Omega_{GW}(10\mathrm{Hz}) = 4.5 \times 10^{-13}$, as shown in Fig. 2. The level of this confusion background is uncertain, but future detections will provide constraints on the rate of such events and allow for more accurate predictions.

Conclusions.—In this study we have demonstrated that third generation GW detectors will have sensitivities sufficient to directly observe almost every coalescing BBH system in the Universe. However, a more detailed analysis is needed to assess how well one can subtract BBH signals from the data, for example, using methods similar to those developed for the Big Bang Observer [17,18] or LISA [65]; this will be addressed in an ongoing mock data challenge. With the binary black hole coalescences removed, these detectors would be sensitive to a PGWB at the level of $\Omega_{\rm GW} \simeq 10^{-13}$, after 5 years of observation, comparable to the sensitivity of LISA [66]. A potential limitation to this sensitivity comes from other astrophysically produced GWs, such as those from the coalescence of binary neutron stars, but there is still much uncertainty on the magnitude of this background. Observations of compact binary coalescence events in the coming years will provide the necessary information on their merger rate. The removal of BBH confusion background with third generation detectors opens up the possibility to observe the PGWB.

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