Optimization of vibratory energy harvesters with stochastic parametric uncertainty: a new perspective

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ABSTRACT

Vibration energy harvesting has been shown as a promising power source for many small-scale applications mainly because of the considerable reduction in the energy consumption of the electronics and scalability issues of the conventional batteries. However, energy harvesters may not be as robust as the conventional batteries and their performance could drastically deteriorate in the presence of uncertainty in their parameters. Hence, study of uncertainty propagation and optimization under uncertainty is essential for proper and robust performance of harvesters in practice. While all studies have focused on expectation optimization, we propose a new and more practical optimization perspective; optimization for the worst-case (minimum) power. We formulate the problem in a generic fashion and as a simple example apply it to a linear piezoelectric energy harvester. We study the effect of parametric uncertainty in its natural frequency, load resistance, and electromechanical coupling coefficient on its worst-case power and then optimize for it under different confidence levels. The results show that there is a significant improvement in the worst-case power of thus designed harvester compared to that of a naively-optimized (deterministically-optimized) harvester.

Keywords: energy harvesting, vibration, optimization, uncertainty, piezoelectric

1. INTRODUCTION

Maintenance, replacement, and recharging costs, environmental or health-related complexities, and above all scalability issue of the conventional batteries have convinced researchers to look for other powering sources or mechanisms for electronic devices. Reduction in power consumption of electronics has made harvesting energy from ambient vibration, a universal and abundant source of energy, a viable alternative to the bulky and costly conventional batteries.[1] However, vibratory energy harvesters (VEH) are less robust in the presence of parametric uncertainties, that is their effectiveness could drop drastically when their parameters deviate from the nominal design values.

Optimizing and fine tuning mechanical and electrical parameters of a harvester are essential for an effective harvesting. Finding exact or approximate optimal deterministic parameters for electromagnetic[2] and piezoelectric[3–7] energy harvesters has been comprehensively studied in the literature for the linear harvesters. For the nonlinear energy harvesters, researchers have mainly studied the effects of mechanical potential shape[8–12] or the harvesting circuitry.[13–15] All these studies have assumed deterministic system parameters to optimize the harvested power; however, manufacturing tolerances, wear and tear, material degradation, humidity, temperature, and other environmental conditions result in parametric uncertainty in the system and hence in a less-effective detuned harvester. Therefore, uncertainty in the system usually necessitate two types of analysis: sensitivity (uncertainty propagation) analysis and optimization under uncertainty for a robust design.

Although being well explored in other fields like controls, finance, and production planning, uncertainty propagation and optimization under uncertainty have not received much attention in the field of energy harvesting. Ng and Liao,[16] and Godoy and Trindade[17] studied parametric sensitivity of linear VEHs while Mann et al.[18] and Madankan et al.[19] studied the uncertainty quantification in nonlinear harvesters.

There are even fewer studies who in addition to uncertainty propagation have explored optimization under uncertainty. Ali et al.[20] studied using Monte Carlo (MC) simulations, the effect of uncertainty in harmonic

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excitation frequency, mechanical damping and electromechanical coupling on the mean (ensemble expectation) harvested power of a linear PEH, and then they optimized deterministic dimensionless time constant and electromechanical coupling coefficient as a function of standard deviation in the excitation frequency. Franco and Varoto[21] studied the geometric and electrical parametric uncertainty in a cantilever piezoelectric energy harvester (PEH). They numerically quantified system sensitivity to parametric uncertainty with the help of MC simulations and they used stochastic optimization to optimize the parameters for the ensemble expectation of the harvested power.

Approaches to optimization under uncertainty have followed a variety of modeling philosophies, including expectation minimization, minimization of deviations from goals, minimization of maximum costs, and optimization over soft constraints.[22] The two optimization studies mentioned here are of the expectation-minimization type (minimization of the negative of average harvested power).

Maximizing expected power is an appropriate approach when a large number of harvesters are to be used together (uncertainty in harvester parameters) to power up a device or when one harvester is to be used in an uncertain environment. Imagine 100 harvesters are to be used to power up a device or charge a battery, then maximizing the expected power over parametric uncertainties makes perfect sense as the expected power of the ensemble is a good measure of the total delivered power. Now consider a case where a single harvester powers up a device which requires a minimum power to operate properly. This would be a common setup for self-powered medical implants, wireless sensors and many other applications of energy harvesters. For instance, suppose a hospital decides to purchase medical devices say pacemakers, powered by energy harvesters. In this case it is crucial that for each single device, its harvester delivers a minimum power; otherwise, it will cause serious health-related complexities. In this case, the customer i.e. the hospital will be interested in a batch of devices with the maximum number of devices fulfilling the minimum power requirement or alternatively, in a batch of devices with the largest minimum power for a given percentage of the total number of devices. It is obvious that the expected power of the batch will be of minimal interest in this case; hence, optimizing the harvesters for the maximum expected power is not practically helpful. This type of demands and problems requires another optimization philosophy: optimization of minimum power (worst-case scenario) and *not* expectation optimization. This paper addresses this type of optimization which is of great importance in the field of energy harvesting and has not yet been addressed.

In this paper we formulate two problems in a generic form: (*i*) propagation of parametric uncertainty in terms of the worst-case (minimum) power and (*ii*) optimization of the worst-case power in presence of parametric uncertainty. The later is cast as a min-max optimization. Parametric uncertainties are modelled as Gaussian random variables which are assumed to be controllable in a mean-value sense. These two problems are then applied to a simple model of a PEH and the results of uncertainty propagation and min-max optimization are presented.

2. MATHEMATICAL MODELING

Cantilever piezoelectric energy harvester as the most common PEH is often adequately modelled as a single-degree-of-freedom (SDOF) oscillator coupled with an electrical circuit as shown in Fig.1. In its simplest form when the piezoelectric patches are connected to an electrical resistance, and when the vibratory system is base excited, the governing dynamic equations of the system could be written as[23, 24]

$$m\ddot{x} + c\dot{x} + kx + \theta v = -m\ddot{x}_b$$
$$C_p\dot{v} + \frac{v}{R} = \theta\dot{x}, \tag{1}$$

where, m, k, and c are the oscillator's mass, linear stiffness and damping coefficient, respectively. C_p, θ and R are the inherent capacitance of the piezoelectric layer, electromechanical coupling coefficient, and the load resistance, respectively. x, x_b and v are the oscillator's displacement relative to its base, base displacement, and the voltage across the load resistance, respectively.

Average power is the measure of the performance of the harvester. Since the system is linear a closed-form solution for the power could be easily found by applying the Fourier transform to Eq.(1). Power is conventionally
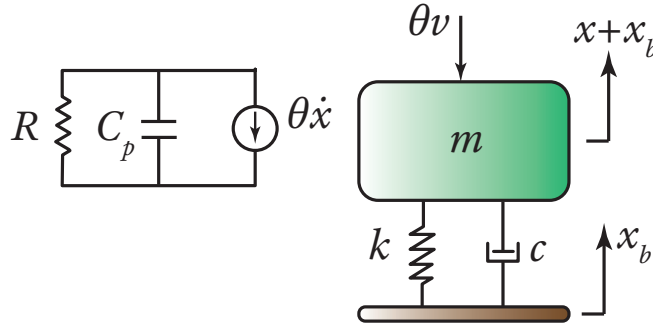
Figure 1. A base-excited PEH modeled as a SDOF oscillator coupled with an electric circuit modeling a load resistance and the inherent capacitance of the piezoelectric layer.

normalized by the square of input acceleration for the harmonic excitation. The normalized peak power* could then be written as

$$
\left| \frac{P(\omega)}{(X_b\omega^2)^2} \right| = \frac{1}{R}\left| \frac{V}{X_b\omega^2} \right|^2 = \frac{R\theta^2\omega^2}{\left((RC_p\omega_n^2 + 2\zeta\omega_n + \frac{R\theta^2}{m})\omega - RC_p\omega^3 \right)^2 + \left(-\omega_n^2 + (1 + 2\zeta RC_p\omega_n\omega^2)\omega^2 \right)^2}, \tag{2}
$$

where, $X_b(\omega)$ and $V(\omega)$ are the Fourier transforms of the base displacement and load voltage, respectively, and ω is the excitation frequency. Also, by convention natural frequency ω_n and damping ratio ζ are introduced which are defined as $\omega_n = \sqrt{k/m}$, and $\zeta = c/2\sqrt{km}$.

Although we do not consider random excitation in this study, for the sake of completeness we present how the time-average power could be calculated if the system is excited by stationary random excitation. In this case we use Parseval's identity which relates the average energy in a signal to its finite Fourier transform as

$$
\overline{P}(t) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\mathrm{d}t = \int_0^\infty \frac{S_v(\omega)}{R}\mathrm{d}\omega, \tag{3}
$$

where, $S_v(\omega)$ is the power spectral density of the voltage across the load and is related to the input acceleration power spectral density $S_{\ddot{x}_b}(\omega)$ by the relation[25–28]

$$
S_v(\omega) = \left| H_{\ddot{x}_b}^v(\omega) \right|^2 S_{\ddot{x}_b}(\omega). \tag{4}
$$

In Eq.(4) $S_{\ddot{x}_b}(\omega)$ is one-sided power spectral density of input acceleration and $H_{\ddot{x}_b}^v(\omega)$ is the transfer function from input base acceleration $\ddot{x}_b$ to the load voltage v and could be derived based on the governing dynamic equations in Eq.1 as

$$
H_{\ddot{x}_b}^v(\omega) = \frac{R\theta\omega}{\left((RC_p\omega_n^2 + 2\zeta\omega_n + \frac{R\theta^2}{m})\omega - RC_p\omega^3 \right) + \left(-\omega_n^2 + (1 + 2\zeta RC_p\omega_n\omega^2)\omega^2 \right) j}, \tag{5}
$$

where, $j = \sqrt{-1}$.

For a deterministic harvester Eqs.(2,3) could be used to study the effect of different parameters on the harvested power and to optimize them. Here we assume that some of the parameters are random. This uncertainty

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*For a harmonic excitation the average power is simply half the peak power; hence, we simply use the peak power as a performance measure.

in parameters could be a result of manufacturing tolerances or defects, material degradation, or environmental effects such as temperature or humidity. Random parameters ξ_i are modelled as Gaussian variables with mean value of ξ_{mi} and standard deviation of σ_{ξ_i}.

3. UNCERTAINTY PROPAGATION AND OPTIMIZATION UNDER UNCERTAINTY

In this study we investigate the effect of uncertainty on the minimum harvested power i.e. the worst-case performance, and then optimize the mean uncertain parameters to maximize the minimum power i.e. optimization for the best worst-case performance. The random parameters are modeled as Gaussian with a mean and a standard deviation. Here we assume the mean value of the parameters (ξ_{mi}) are controllable. Hence we write the ith random parameter as $\xi_i = \xi_{mi} + \delta\xi_i$ where $\delta\xi_i$ is the variation from the mean value. We know that for random variables with Gaussian distribution this variation extends from $-\infty$ to $+\infty$; however, the closer it gets to the tails the smaller gets the probability of the parameter in that range. Therefore, to make the optimization tractable and non-trivial we have to limit the variation $\delta\xi_i$ for a desired confidence level. For example for a 99.7% confidence level, $-3\sigma_{\xi_i} < \delta\xi_i < +3\sigma_{\xi_i}$, and for a 95.5% confidence level we should limit $\delta\xi_i$ as $-2\sigma_{\xi_i} < \delta\xi_i < +2\sigma_{\xi_i}$.

Suppose a manufacturer mass produces a batch of harvesters with parametric uncertainties. It is important for the customer to know that a certain percentage of the harvesters i.e. the confidence level, say 95.5% of the harvesters, deliver a minimum required power. To answer this question, the manufacturer should be able to quantify the effect of uncertainties on the worst-case performance (minimum power) for a given confidence level. Moreover, it is clear that the larger the confidence level fulfilling a minimum power requirement or the larger the minimum power for a given confidence level, the better the quality of that batch. Assuming that the mean value of the uncertain parameters are controllable in the manufacturing process, then the manufacturer should optimize the mean values of the parameters to maximize the worst-case power for a given confidence level or to maximize the confidence level for a given worst-case power. For the optimization in this study we do the former i.e. maximizing the worst-case power for a given confidence level.

As discussed we have two types of problems here:

(**P1**): **uncertainty propagation:** *Given the confidence level, find the worst-case (minimum) power as a function of parametric uncertainties (standard deviations σ_{ξ_i}), deterministic parameters, and mean values of uncertain parameters ξ_{mi}:*

$$\overline{P}^{\text{wc}}(\xi_{mi}, \sigma_{\xi_i}, \xi_j^{\text{det}}) = \min_{\xi_i}\{\overline{P}(t;\xi_i,\xi_j^{\text{det}}) : \ \xi_i \in (\xi_{mi} - \max(\delta\xi_i), \xi_{mi} + \max(\delta\xi_i))\}. \tag{6}$$

(**P2**): **optimization for the worst-case scenario under parametric uncertainty:** *Given the confidence level, find the optimum mean value of the uncertain parameters ξ_{mi}, and the deterministic parameters ξ_j^{det} to maximize the worst-case (minimum) power:*

$$\overline{P}^{\text{wc}}_{\max}(\sigma_{\xi_i}) = \max_{\xi_{mi},\ \xi_j^{\text{det}}} \quad \min_{\xi_i}\{\overline{P}(t;\xi_i,\xi_j^{\text{det}}) : \ \xi_i \in (\xi_{mi} - \max(\delta\xi_i), \xi_{mi} + \max(\delta\xi_i))\}, \tag{7}$$

where ξ_j^{det} is the jth deterministic parameter. **P2** is also known as min-max optimization problem. To study **P1**, for given mean values of the uncertain parameters ξ_i i.e. ξ_{mi}, a search over a grid of $\xi_{mi} - \max(\delta\xi_i) < \xi_i < \xi_{mi} + \max(\delta\xi_i)$ is conducted to find the minimum power. Depending on the confidence level $\max(\delta\xi_i)$ can adopt different values in terms of the standard deviation σ_{ξ_i}. For instance, for a confidence level of 95.5%, $\max(\delta\xi_i) = 2\sigma_{\xi_i}$. In addition, depending on the number of simultaneous uncertain parameters being studied ($i = 1, 2, ..., n$) the search grid will be on a line, surface, or in an n-dimensional hypercube in general. Also, optimum parameters for a deterministic harvester are used as the corresponding mean values for the uncertain parameters (ξ_{mi}) and deterministic parameters (ξ_j^{det}) in **P1**[†]. This is what we would refer to as *naive* optimization i.e. the optimization of the parameters without considering uncertainties. When ξ_{mi} and ξ_j^{det} are chosen this way, **P1** shows how uncertainty in parameters affects the worst-case power of a naively-optimized harvester.

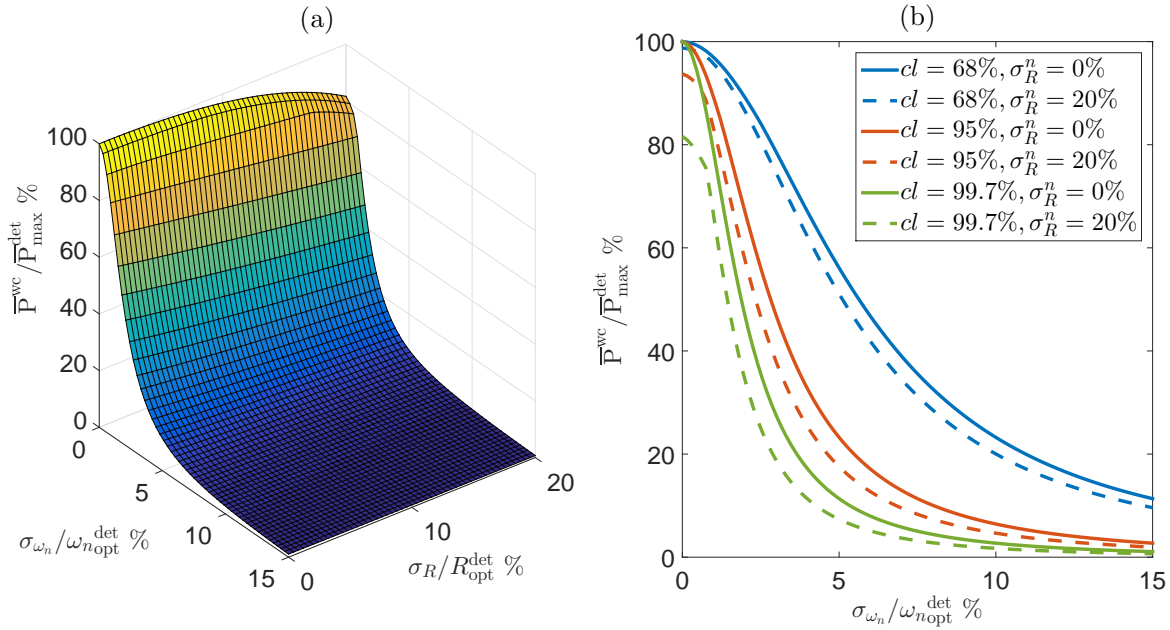[†]If there is no optimum value, a practically reasonable value is selected.

Figure 2. Dependence of normalized worst-case power on normalized uncertainty in natural frequency and load resistance under harmonic excitation: (a) dependence as surface plot for confidence level of 99.7%, (b) dependence on uncertainty in natural frequency for two different normalized uncertainty values in load resistance ($\sigma_R^n = \sigma_R/R_{\mathrm{opt}}^{\mathrm{det}}\%$), and for three confidence levels of 68%, 95.5%, and 99.7%.

To study **P2**, the same procedure as described above for **P1** is carried out over feasible deterministic parameters ξ_j^{det} and the mean values of the uncertain parameters ξ_{mi} to find the optimum mean values maximizing the worst-case power. In the next section numerical results are presented and discussed.

4. NUMERICAL RESULTS AND DISCUSSION

We explore the effects of uncertainty in three parameters namely, natural frequency ω_n, load resistance R and electromechanical coupling coefficient θ on the worst-case harvested power for different confidence levels (**P1**). Then considering these uncertainties, we optimize the deterministic or mean value of the uncertain parameters to maximize the worst-case power (**P2**). To be able to visualize the effects we consider two uncertain parameters at a time and optimize over the mean values of those two parameters.

Figure 2 shows the normalized worst-case power as a function of normalized uncertainty in natural frequency and load resistance when subjected to harmonic base excitation. Uncertainties are applied to the harvester optimized for deterministic parameters (naive optimization). Worst-case power is normalized by the maximum power of a deterministic harvester and the uncertainties in parameters are normalized by their deterministic optimum values. In all the simulations $m = 0.001$ kg, $\zeta = 0.02$, and $C_p = 100$ nF. Also, $\omega = 70$ rad/s for harmonic excitation. According to Fig.2(a), the worst-case power is very sensitive to the natural frequency but not much to the load resistance. Sharp resonance peak and wide peak for the optimum load resistance in linear harvesters explain it. Figure 2(b) depicts this dependence on uncertainty in natural frequency for two different uncertainty levels in the load resistance i.e. zero and 20% uncertainty, and for different confidence levels of 68%, 95.5%, and 99.7%. According to the figure the larger the confidence level the smaller the worst-case power. This is because the larger confidence level simply means the larger deviation in the parameter from its optimum value.

Figure 3 shows dependence of the normalized worst-case power on uncertainties in natural frequency and electromechanical coupling. According to the figure the sensitivity of the worst-case power to the electromechanical coupling coefficient is considerable and larger than that of the load resistance. Based on Figs. 2 and 3, we see that uncertainty in naively-optimized parameters of a harvester could drastically decrease its worst-case power.
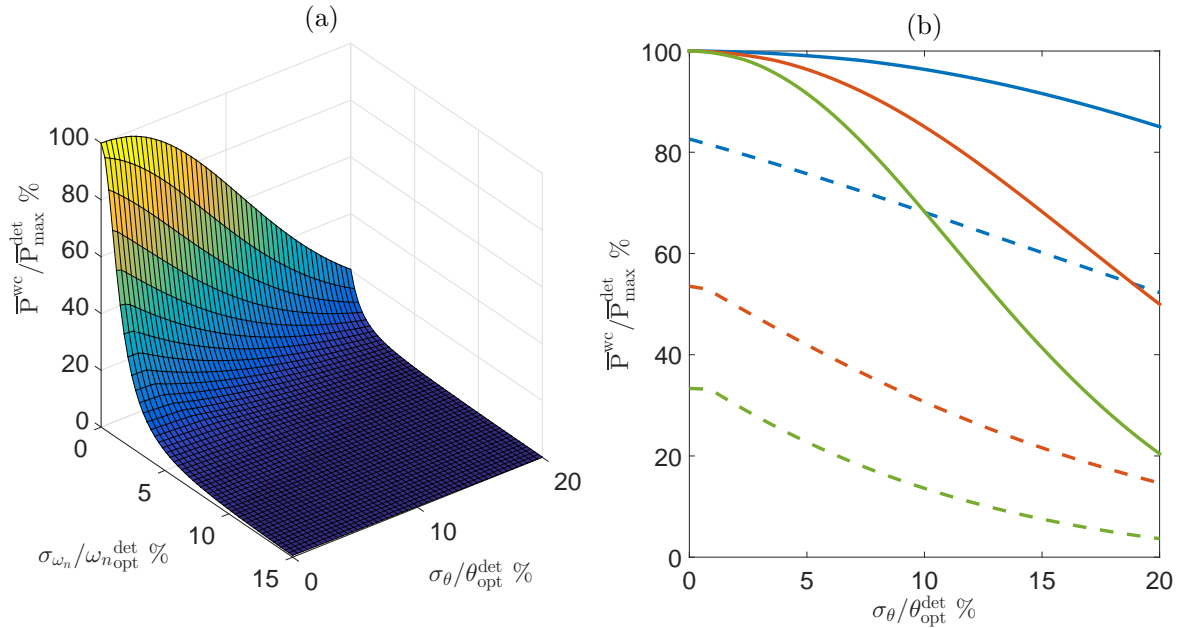
Figure 3. Dependence of normalized worst-case power on normalized uncertainty in natural frequency and electrome-chanical coupling coefficient under harmonic excitation: (a) dependence as surface plot for confidence level of 99.7%, (b) dependence on uncertainty in electromechanical coupling coefficient for two different normalized uncertainty values in natural frequency of 0% (solid line) and 20% (dashed line), and for three confidence levels of 68% (blue), 95.5% (red), and 99.7% (green).

Next we would like to see if optimization of the deterministic parameters and/or mean value of the uncertain parameters with the knowledge of uncertainties in the system will help to decrease the effect of uncertainty on the worst-case power. This could be done by numerically solving the min-max optimization problem in **P2**.

Optimization procedure formulated in **P2** is applied to the harvester under harmonic excitation. Figures 4 and 5 illustrate how optimization under parametric uncertainty improves worst-case power compared to the naively-optimized system i.e. the system optimized for deterministic parameters. Figure 4 shows the normalized maximum worst-case power as a function of normalized uncertainty in natural frequency and load resistance. Optimization is done over mean values of the said uncertain parameters. For comparison, worst-case power of the naively-optimized harvester is also plotted. Figure 5 shows the optimized worst-case power as a function of normalized uncertainty in natural frequency and electromechanical coupling over mean values of which the optimization is applied. As could be seen in Figs. 4 and 5, optimization under uncertainty greatly improves the worst-case power over the naively-optimized harvester.

The main reason and idea behind optimization in **P2** reflected in Figs. 4 and 5, are the trend and shape of dependence of the harvested power on the system parameters. For instance, an asymmetric concave dependence on a random parameter can have different optimum parameters for worst-case and naive optimization. In this case, an optimum value on the less-steep side of the curve rather than on the very peak could result in a larger worst-case power. This is the case for example when there is randomness only in the electromechanical coupling coefficient. It is shown in Fig.6 (a) that a choice of mean value of θ to the right of the peak point would result in larger-value 3-σ tails (larger worst-case power) which are designated by starts on the curves. The naive and worst-case optimum values for the mean electromechanical coupling coefficients are marked with red and blue hexagrams respectively.

Another way that **P2** could improve the worst-case power for a specific set of random parameters is via the curve-flattening effect that some parameters have on the harvested power. Despite having a smaller peak value, the flattened curve could have higher 3-σ tails than those of the naively-optimized harvester. Figure 6(b) exemplifies this mechanism. This figure compares the naively-optimized power curve with optimum mean values
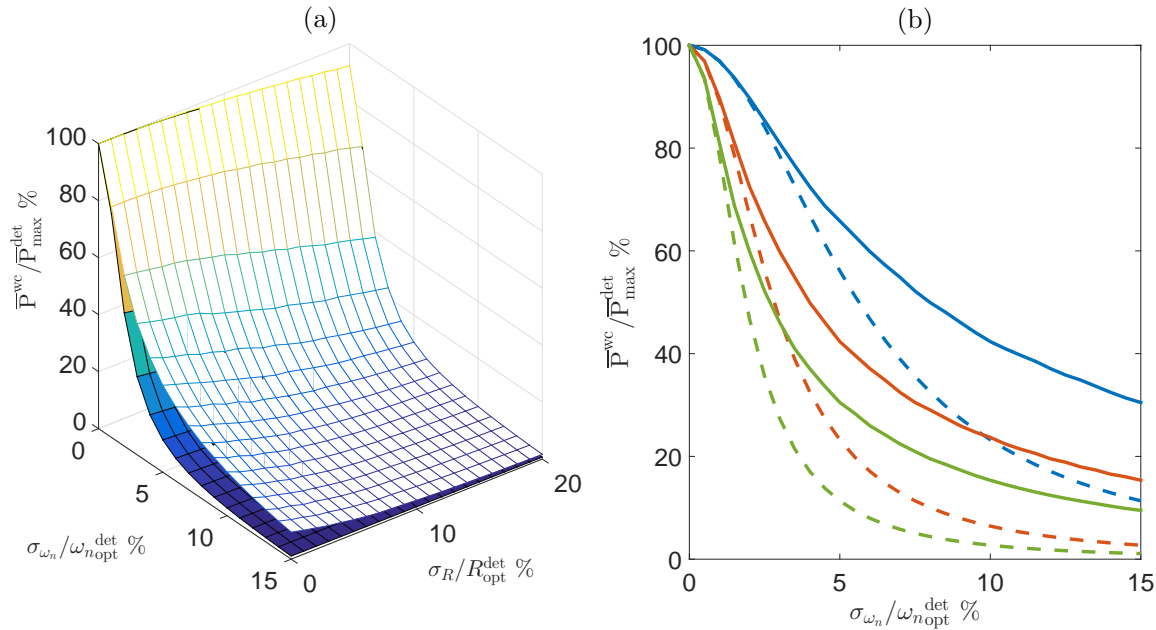
Figure 4. Maximized worst-case power as a function of uncertainty in natural frequency and load resistance under harmonic excitation. (a) the maximum worst-case power (wireframe mesh) compared to the worst-case power of the naively-optimized harvester (solid surface) for confidence level of 99.7% (b) maximum worst-case power (solid line) as a function of uncertainty in the natural frequency (no uncertainty in load resistance) compared to the naively-optimized harvester (dashed line) for confidence levels of 68% (blue), 95.5% (red), and 99.7% (green).

of $\omega_n = 67.65$ rad/s and $\theta = 2.1287 \times 10^{-4}$N/V to the power curve optimized for the worst-case (minimum) power with optimum parameters of $\omega_n = 59.84$ rad/s and $\theta = 4.2575 \times 10^{-4}$N/V. Also, the only random parameter for the problem of Fig.6(b) is natural frequency of the system with standard deviation equal to 3% of the optimum natural frequency of the deterministic harvester. In this case, the electromechanical coupling (θ) has the flattening effect.

In general, there are other geometrical ways that **P2** improves the worst-case power and this becomes more complicated to visualize when there are two or more random variables; however, the main idea still lies in the multi-dimensional geometry of the dependence of the harvested power on the system parameters. Last but not least, it's worth mentioning that although harvesters designed for the worst-case have better worst-case power, they do not necessarily have better ensemble expected power than naively-optimized harvesters or obviously than harvesters optimized for ensemble expected power.

5. CONCLUSIONS

In this study, we proposed a new modeling philosophy for optimization of energy harvesters under parametric uncertainty. Instead of optimizing for ensemble expectation of average harvested power, we optimize for the worst-case (minimum) power based on some confidence level over the deterministic parameters and mean values of the random parameters. The proposed optimization perspective is practically very useful when there is a minimum requirement on the harvested power such as those in medical implants and wireless sensors. We also introduced a different notion of uncertainty propagation i.e. propagation in the worst-case power instead of the ensemble expected power. Based on this new modeling philosophy, we presented a very generic and architecture-independent formulation for uncertainty propagation (**P1**) and optimization under uncertainty (**P2**).

Next, we applied analysis methodologies **P1** and **P2** to a simple model of a piezoelectric energy harvester. We have considered parametric uncertainty in natural frequency, load resistance, and electromechanical coupling
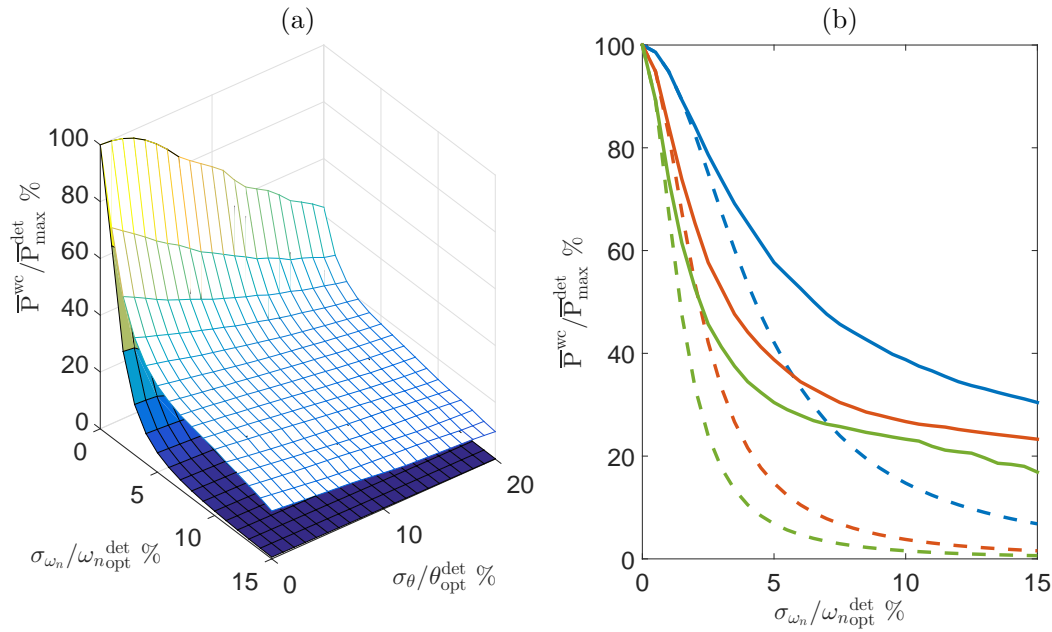
Figure 5. Maximized worst-case power as a function of uncertainty in natural frequency and electromechanical coupling coefficient under harmonic excitation. (a) the maximum worst-case power (wireframe mesh) compared to the worst-case power of the naively-optimized harvester (solid surface) for confidence level of 99.7% (b) maximum worst-case power (solid line) as a function of uncertainty in the natural frequency (no uncertainty in electromechanical coupling coefficient) compared to the naively-optimized harvester (dashed line) for confidence levels of 68% (blue), 95.5% (red), and 99.7% (green).
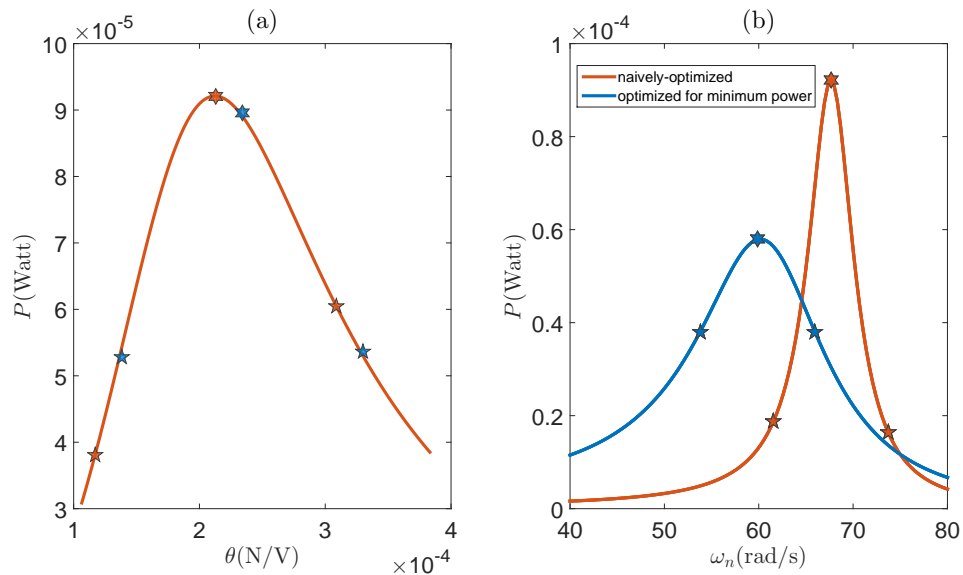

Figure 6. Two examples on how **P2** improves the worst-case power. (a) the power curve as a function of electromechanical coupling coefficient with a standard deviation of 15% of its optimum deterministic value and with optimum natural frequency of $\omega_n = 67.65$ rad/s (b) the power curve as a function of natural frequency with a standard deviation of 3% of its optimum deterministic value and with optimum coupling coefficients $\theta = 2.1287 \times 10^{-4}$N/V and $\theta = 4.2575 \times 10^{-4}$N/V for naively-optimized and worst-case-optimized harvesters, respectively. Optimum mean value of random parameters and their corresponding 3-σ tails are marked with hexagrams and stars, respectively. They are also color-coded as red and blue for naive and **P2** optimizations, respectively.

coefficient of the harvester under harmonic excitation. Direct application of **P1** showed that for harmonically-excited PEH, the worst-case power of the harvester is highly sensitive to its natural frequency and then to its electromechanical coupling but not very sensitive to the load resistance.

Then optimization **P2** was done over mean values of the natural frequency and load resistance or natural frequency and electromechanical coupling. It was shown that the optimized system taking into account the parametric uncertainties is much more robust to parametric uncertainties in terms of its worst-case power compared to the naively-optimized (deterministically-optimized) harvester.

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