

Calibration of Muon Reconstruction Algorithms Using an External Muon Tracking System at the Sudbury Neutrino Observatory

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Abstract

To help constrain the algorithms used in reconstructing high-energy muon events incident on the Sudbury Neutrino Observatory (SNO), a muon tracking system was installed. The system consisted of four planes of wire chambers, which were triggered by scintillator panels. The system was integrated with SNO's main data acquisition system and took data for a total of 95 live days. Using cosmic-ray events reconstructed in both the wire chambers and in SNO's water Cherenkov detector, the external muon tracking system was able to constrain the uncertainty on the muon direction to better than 0.6° .

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1. Introduction

The Sudbury Neutrino Observatory (SNO) was a large water Cherenkov detector optimized for detecting solar neutrinos created from the ${}^8\text{B}$ reaction in the main pp fusion chain. In addition to solar neutrinos, the Sudbury Neutrino Observatory was also sensitive to high-energy muons that traverse the volume of the detector. A small fraction of these events are neutrino-induced muons from atmospheric neutrinos, while the large remaining fraction come from cosmic rays created in the upper atmosphere. It is possible to discriminate between these muon sources by looking at the angular distribution of incoming muons. The combination of large depth and the relatively flat topography in the vicinity of the detector attenuates almost all cosmic ray muons entering the detector at zenith angle $\cos(\theta_z) > 0.4$. The study of muon events in the SNO detector provides measurements of the absolute flux of atmospheric neutrinos and constraints on the atmospheric neutrino mixing parameters Δm_{23}^2 and θ_{23} [1]. While the latter measurement is more strongly constrained by other experiments [2, 3], the former is unique to the SNO experiment.

To facilitate a clean measurement of the zenith distribution of muons entering the SNO fiducial volume, an accurate understanding of the muon reconstruction algorithm is necessary. This includes both the angular and spatial resolution of high-energy muons which enter the detector. Determining the accuracy of the muon tracking reconstruction algorithm, however, relies almost entirely on Monte Carlo simulations. Although the detector response to muons was benchmarked against selected cosmic-ray data, there is not an external calibration source that can provide a consistency-check to the accuracy of the reconstruction algorithm. This is in sharp contrast to the case for SNO's response to neutrons and low energy electrons, which was calibrated with multiple sources to a precision of $\sim 1\%$ [4].

We present in this paper a means by which the SNO experiment was able to calibrate its muon tracking algorithm via the use of an external muon tracking system. The External Muon System (EMuS) allowed SNO to simultaneously reconstruct selected cosmic-ray events in two independent systems, thereby providing a cross-check on the tracking algorithm. The EMuS experiment ran for a total of 94.6 live days during the last phase of the SNO experiment.

This paper is divided as follows: Section 2 describes the main SNO experiment, Section 3 describes the SNO muon reconstruction algorithm, Section 4 describes the characteristics of the EMuS apparatus, Section 5 describes the criterion for accepting events, and finally Section 6 discusses the analysis used to calibrate the SNO tracking algorithm against data taken with the EMuS system.

2. The Sudbury Neutrino Observatory

The SNO detector consisted of a 12-meter-diameter acrylic sphere filled with 1 kiloton of D_2O . The 5.5-cm-thick acrylic vessel was surrounded by 7.4 kilotons of ultra-pure H_2O encased within a barrel-shaped cavity, 34 m in height and 22 m in diameter. A 17.8-meter-diameter geodesic structure surrounded the acrylic vessel and supported 9456 20-cm-diameter photomultiplier tubes (PMTs) pointed toward the center of the detector. A non-imaging light concentrator was mounted on each PMT to increase the effective photocathode coverage to 54% [5]. The detector is described in detail elsewhere [6].

SNO was located in the Vale Creighton mine in Ontario, Canada at a depth of 2.092 km (5890 ± 94 meters water equivalent) with a flat overburden. At this depth, the muon rate incident over the geodesic sphere and integrated over the seasonal variation is $62.9 \pm 0.2 \mu/\text{day}$ across an impact area of 216 m^2 [1]. Muons entering the detector produce Cherenkov light at an angle

45 of 42° with respect to the propagation direction of the muon. Cherenkov light and light from
 46 delta rays produced by the muon illuminate an average of 5500 PMTs, whose charge and timing
 47 information are recorded. The amplitude and timing response of the PMTs were calibrated *in*
 48 *situ* using a light diffusing sphere illuminated by a laser at six distinct wavelengths [4]. This laser
 49 ball calibration was of particular relevance to the muon fitter because it provides a timing and
 50 charge calibration for multiple photon hits on a single PMT. Other calibration sources used in
 51 SNO are described elsewhere [6, 7].

52 Data taking in the SNO experiment was subdivided into three distinct phases for measure-
 53 ment of the solar neutrino flux. In the first phase, the experiment ran with pure D₂O only. The
 54 solar neutral current reaction was observed by detecting the 6.25 MeV γ -ray following the cap-
 55 ture of the neutron by a deuteron. For the second phase of data taking, approximately 0.2% by
 56 weight of purified NaCl was added to the D₂O volume to enhance the sensitivity to neutrons
 57 via their capture on ³⁵Cl. In the third and final phase of the experiment, 40 discrete ³He and
 58 ⁴He proportional tubes were inserted within the fiducial volume of the detector. This enhanced
 59 the neutron capture cross-section to make an independent measurement of the neutron flux, by
 60 observing neutron capture on ³He in the proportional counters. Results from the measurements
 61 of the solar neutrino flux for these phases have been reported elsewhere [8, 9, 10, 11, 12, 13, 14].

62 3. Muon Reconstruction with the SNO Detector

63 The SNO muon reconstruction algorithm fits for a through-going muon track based on the
 64 charge, timing, and spatial distribution of triggered PMTs. Using a maximum likelihood method,
 65 the fitter is able to determine a variety of muon tracking parameters, including the muon's propa-
 66 gation direction, impact parameter with respect to the center of SNO, the total deposited energy,
 67 and a timing offset. The likelihood is defined as:

$$L = \prod_i^{PMTs} \left[\sum_{n=1}^{\infty} P_N(n|\lambda_i) P_Q(Q_i|n) P_T(t_i|n) \right] \quad (1)$$

68 where n is the number of detected photons, $P_N(n|\lambda_i)$ is the probability of n photoelectrons being
 69 detected for λ_i expected number of detected photoelectrons, $P_Q(Q_i|n)$ is the probability of seeing
 70 charge Q_i given n photon hits, and $P_T(t_i|n)$ is the probability of observing a PMT trigger at time
 71 t given n photon hits.

72 The heart of the fitter lies in the first probability term, which is calculated based on Monte
 73 Carlo simulations. Muons were simulated at discrete impact parameter values with random di-
 74 rections through the detector. These simulations were used to create lookup tables for how many
 75 photoelectrons are expected to be detected by a PMT at a given position with respect to a muon
 76 track with a given impact parameter.

77 The second term further refines the fit by including the charge information from the PMTs,
 78 and allows an estimate of the total energy deposited by the muon, correcting for offline PMTs
 79 and the neck of the detector. This probability was calculated by simulating multiple photon hits
 80 on all of the PMTs in SNO. For a given number of photon hits, the resulting charge distribution is
 81 modeled as an asymmetric Gaussian with the widths extracted from simulations. This fit model
 82 agrees well with the simulations for many photon hits, and acceptably for few photon hits (see
 83 Figure 1).

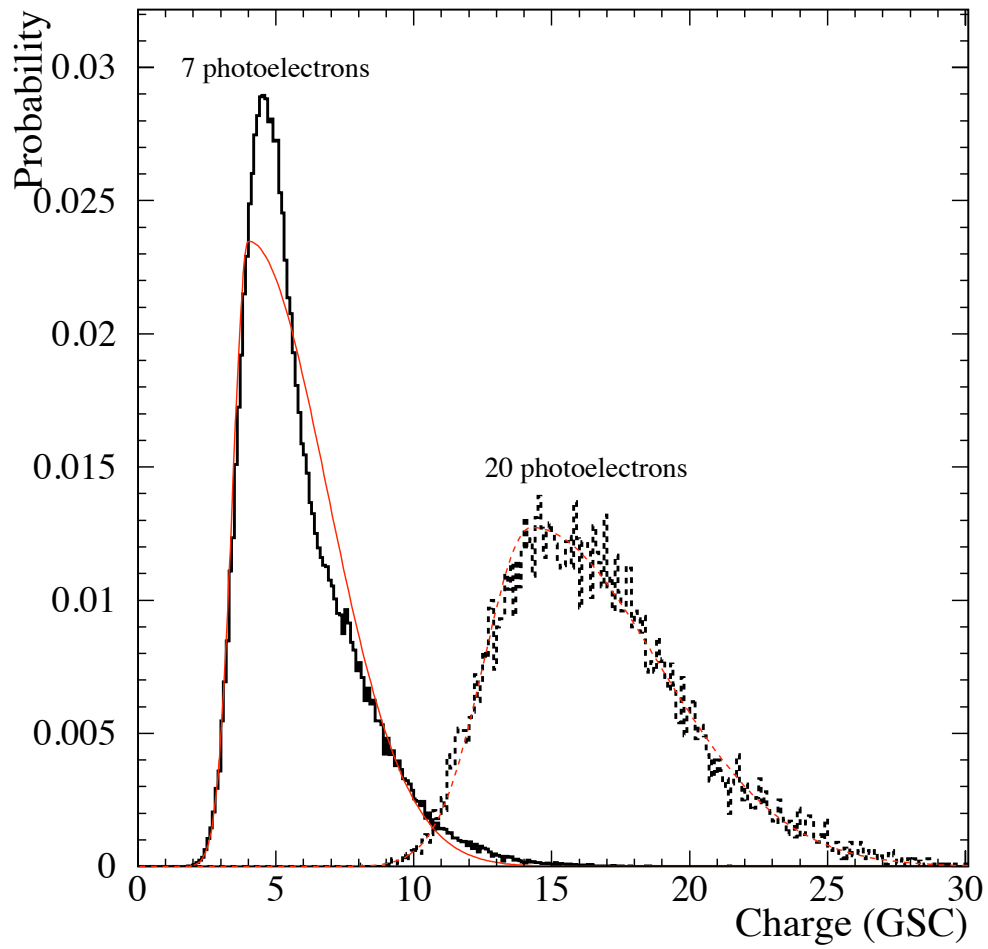


Figure 1: The normalized PMT charge distribution measured in (scaled) pedestal-subtracted ADC charge for the case of 7 and 20 photoelectrons striking a single PMT. The smooth curve (red) indicates the prediction from the charge parameterization model used in the reconstruction.

	μ	σ	$1 - f$	$m\sigma$
Angular Difference	0° (fixed)	0.4°	0.01	1.6°
Impact Parameter Difference	-0.08 cm	3.0 cm	0.012	21 cm

Table 1: Accuracy of the muon fitter based on Monte Carlo simulations. Fit parameters for mean (μ), widths (σ and $m\sigma$), and relative weight ($1 - f$) are given in Equations 4 and 5.

84 The third term in the likelihood refines the fit by including the PMT timing. For each PMT,
85 the time residual can be calculated as:

$$t_{res} = t_{PMT,i} - t_0 - \frac{d_1}{c} - \frac{d_2}{c_D} \quad (2)$$

86 where $t_{PMT,i}$ is the recorded time on a given PMT, t_0 is the time offset term in the likelihood fit,
87 d_1 is the distance the muon travels within the detector before emitting the Cherenkov photon, c
88 is the speed of light in vacuum, d_2 is the distance the Cherenkov photon traveled, and c_D is the
89 average speed of light in D₂O/H₂O medium (21.8 cm/ns). The Cherenkov photon is assumed
90 to have an angle of 42° with respect to the muon track, making d_1 and d_2 well-defined. The
91 probability of the time residual is modeled as a Gaussian centered at zero with corrections to
92 include estimates of prepulsing and late light as a function of the number of photon hits.

93 The SNO muon fitter maximizes the likelihood function for the impact parameter, direction,
94 deposited energy, and timing offset using the method of simulated annealing with downhill sim-
95 plex [16]. After determining the parameters that maximize the likelihood, a set of data quality
96 measurements are used for background rejection.

97 The muon fitter is found to have good reconstruction accuracy for simulated muons. Fig-
98 ure 2 shows the angle (θ_{mr}) between the Monte Carlo generated muon direction (\vec{u}_g) and the
99 reconstructed muon direction (\vec{u}_r):

$$\theta_{mr} = \cos^{-1}(\vec{u}_g \cdot \vec{u}_r) \quad (3)$$

100 This is fit to a weighted double Gaussian function:

$$p(\theta) = A\theta \left[f e^{-\frac{\theta^2}{2\sigma^2}} + (1 - f) e^{-\frac{\theta^2}{2(m\sigma)^2}} \right] \quad (4)$$

101 The additional θ -dependence is introduced in order to account for the phase space available.

102 The fit parameters are summarized in Table 1. Although the tails are non-Gaussian, this fit
103 gives a reasonable estimate for the uncertainty in the angular resolution. Figure 3 shows the
104 impact parameter reconstruction accuracy. The distribution is fit to the sum of two Gaussians:

$$p(x) = A \left[f e^{-\frac{(x-\mu)^2}{2\sigma^2}} + (1 - f) e^{-\frac{(x-\mu)^2}{2(m\sigma)^2}} \right] \quad (5)$$

105 with the fit parameters also summarized in Table 1. Monte Carlo studies show that the recon-
106 struction accuracy of the muon direction and impact parameter are uncorrelated.

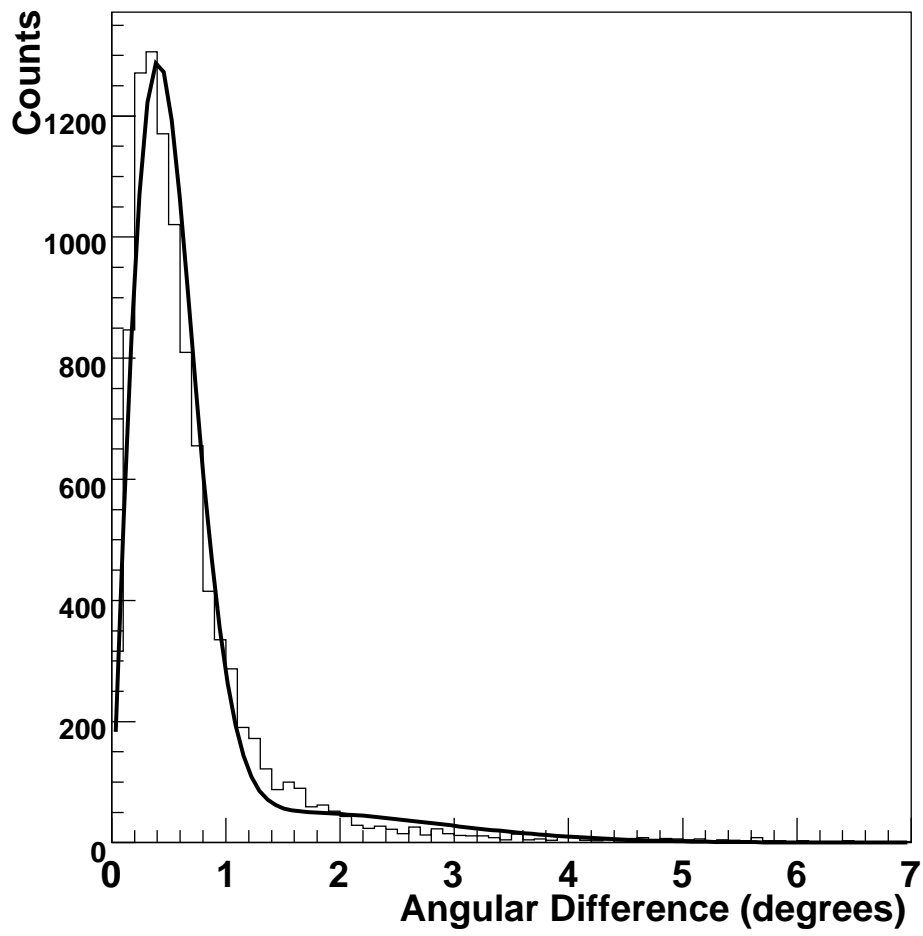


Figure 2: The angular difference (as defined in Eq. 3) of Monte-Carlo muon tracks through the SNO detector (solid histogram). The angular distribution is fit to the function outlined in Eq. 4 (solid line). The results from the fit are given in Table 1.

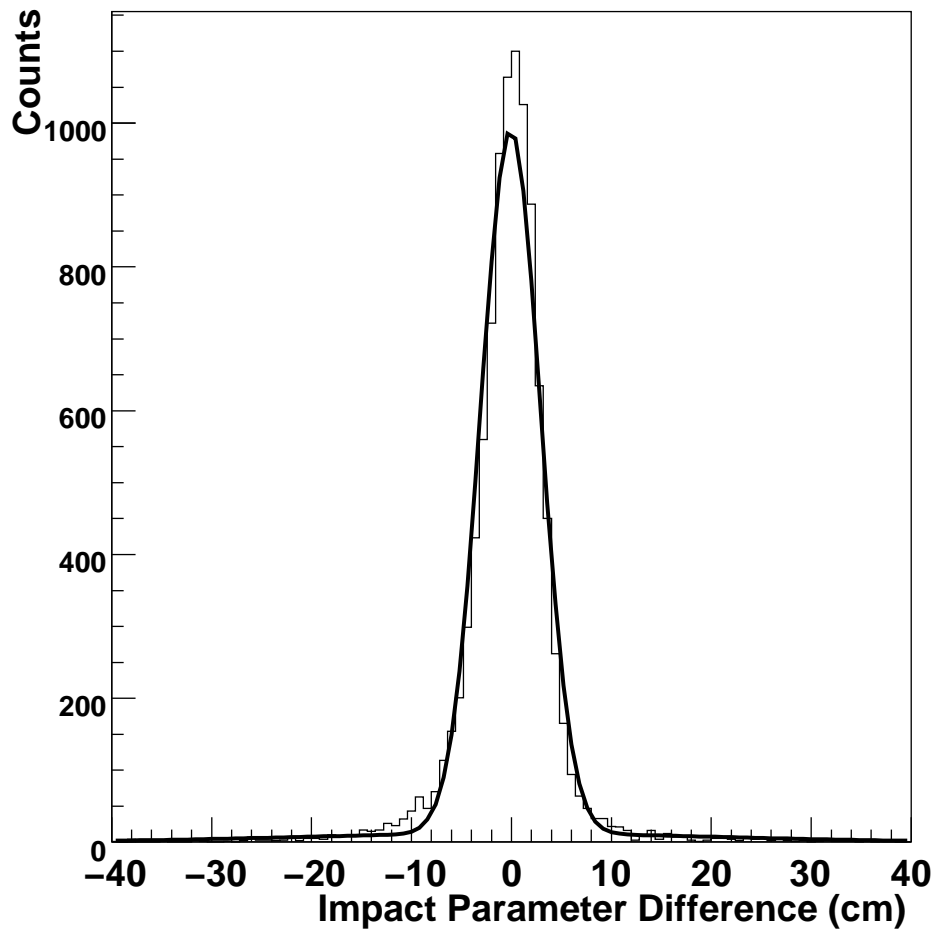


Figure 3: The impact parameter difference of Monte-Carlo muon tracks through the SNO detector (solid histogram). The distribution is fit to the function outlined in Eq. 5 (solid line). The results from the fit are given in Table 1.

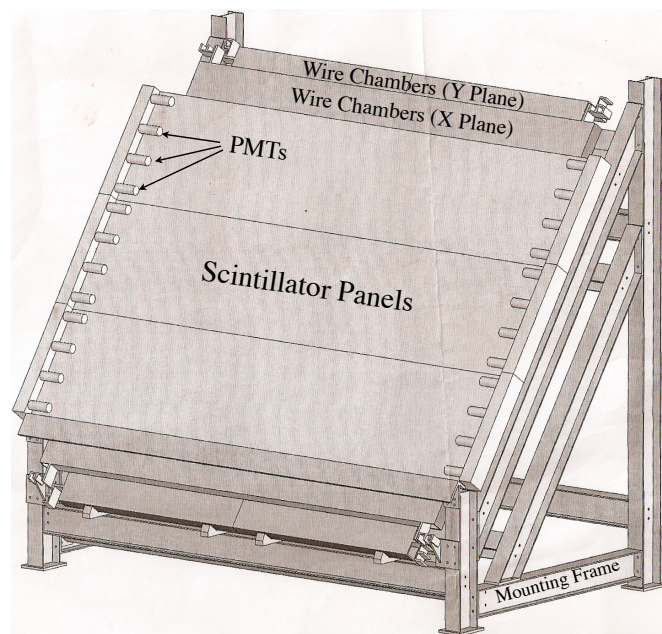


Figure 4: Diagram of the EMuS detector. See the text for more details.

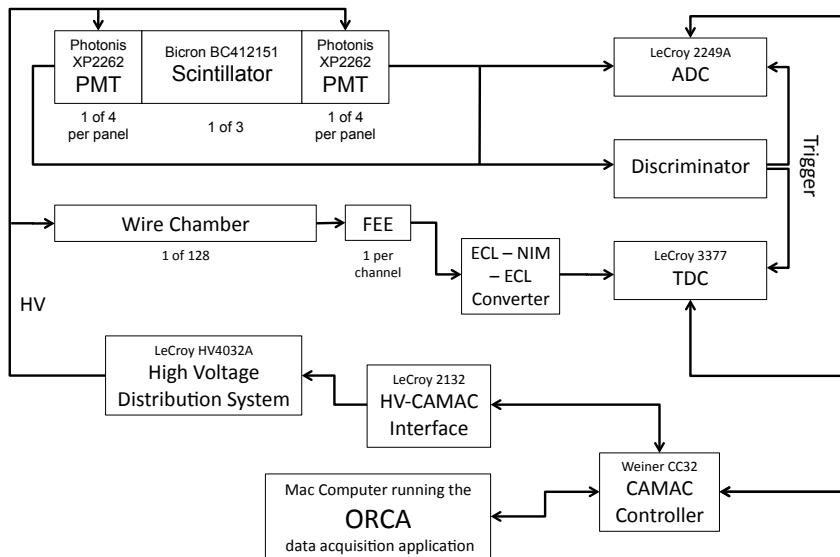


Figure 5: Diagram of the EMuS electronics system. See the text for more details.

107 4. The External Muon System

108 The External Muon System consists of a series of 128 single-wire chambers arranged into
109 four planes and triggered by three large scintillator panels (see Figure 4). The wire chamber cells
110 and electronics were provided by Indiana University. Each cell is 7.5 cm wide and has a square
111 cross-section with the corners trimmed into a near-octagonal shape. The cells are 2.564 m in
112 length and possess a single 50 μm diameter tungsten wire running through the center. The wire
113 is held at a positive potential of 2500 V (2700 V) while running on the surface (underground)
114 for electron drift and collection. A gas mixture of 90%Ar-10%CO₂ was used in order to achieve
115 high efficiency and stability, and to meet safety regulations for underground operations.

116 When a muon passes through the system, it deposits energy in the scintillator and ionizes
117 atoms in each of the wire chambers it passes through. The scintillator converts the energy into
118 light that is then detected by PMTs in a fast process ($\sim\text{ns}$). In the wire chambers, the high voltage
119 draws the ionization electrons to the wire in a slow drift process ($\sim\mu\text{s}$). The drift time is propor-
120 tional to the closest distance between the muon track and the wire, allowing track reconstruction
121 using timing and position. The measured drift time for each wire is the time difference between
122 when the scintillator fired and when the drift electrons reached the wire.

123 The scintillator consists of three large rectangular panels ($350 \times 70 \times 5 \text{ cm}^3$) which cover
124 the active region of the EMuS detector. The panels were acquired from the KARMEN neutrino
125 experiment [17], and consisted of Bicron BC412 scintillator read out at each end by four Photo-
126 nis XP2262 PMTs. The signals from the PMTs were sent to a LeCroy 2249A Analog to Digital
127 Converter (ADC) and a discriminator. If both ends of a panel fire in coincidence, a start signal
128 was sent to the wire readout modules, and the ADC modules recorded the pulse-height of each
129 PMT.

130 Each wire chamber was monitored by an individual Front-End Electronics (FEE) card which
131 output an ECL signal if a pulse is detected on the wire. The ECL signal was sent to a LeCroy 3377
132 Time to Digital Converter (TDC) with a readout window of 4.1 μs . In order to mitigate high levels
133 of electronic noise in the pre-amplifiers, the readout cables were sent through an additional ECL-
134 NIM-ECL converter (see Figure 5).

135 The EMuS system was deployed on the deck of the SNO experiment, 12 m above and 3 m
136 west of the center of the detector. Due to space and solid-angle considerations, the planes were
137 inclined at a 55° from horizontal. A survey was performed to determine the position of each of the
138 wires with respect to the SNO detector. The dominant sources of uncertainty associated with the
139 wire positions relative to the SNO detector are summarized in Table 2. The largest uncertainty
140 stems from determining X-Y coordinates of the EMuS detector. By comparing survey results
141 with other known location markers at the detector, the X-Y coordinate was determined to better
142 than $\pm 0.53 \text{ cm}$. The reference point used for the Z-coordinate of the detector was only known
143 to $\pm 0.32 \text{ cm}$, and thus added as an uncertainty to the EMuS location. Other uncertainties on the
144 locations of the wires included uncertainties on the floor level, the placement of the wires within
145 the modules, the spacing between wires, and the gaps between the modules. These additional
146 uncertainties do not apply equally to all wires, and have a maximum combined value of $\pm 0.30 \text{ cm}$.
147 The final uncertainty on the SNO-EMuS coordinate translation based on this survey was ± 0.68
148 cm.

149 4.1. Time to Radius Conversion

150 Well-determined models of electron drift and diffusion in a gas [18] predict that the timing of
151 a wire chamber hit with respect to the scintillator trigger can be used to measure the distance of

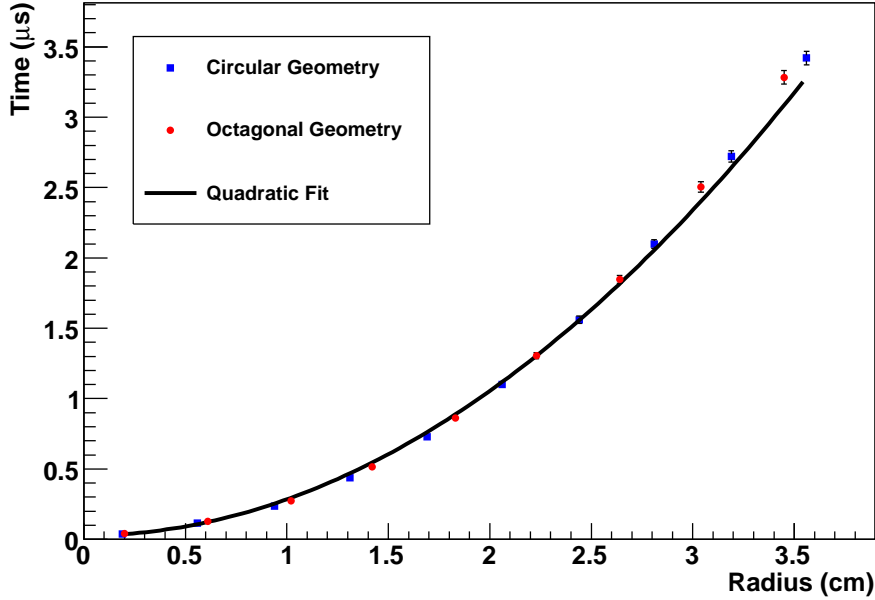


Figure 6: The drift time for simulated electrons inside the EMuS wire chambers plotted as a function of starting radius. The plot shows drift times for both circular (boxes) and octagonal (circles) cross-sectional geometries. The quadratic fit (solid line) is accurate to within 5% at the maximum simulated radius.

152 closest approach of the muon. This time-to-radius conversion function, $r(t)$, has been simulated
 153 and measured for the EMuS system.

154 The Garfield gas simulation [19] was used to generate expected $r(t)$ curves as a function of
 155 gas pressure and applied voltage. The code was not able to perfectly model the shape of the wire
 156 chambers so two similar geometries were used to check the effects of this imperfect modeling:
 157 a circle with radius 3.75 cm, and a regular octagon with a longest radius of 4.06 cm. Simulated
 158 electrons were generated at 10 points along the longest radius, and the mean drift time for each
 159 point was calculated. Figure 6 shows that the two $r(t)$ curves agree to within 2%. A parabolic fit
 160 to this data is accurate to 5%.

161 In order to directly measure the $r(t)$ curve, the EMuS system was run on the surface at the

SNO X-Y Coordinate	0.53 cm
SNO Z Coordinate	0.32 cm
Floor Level*	0.17 cm
Wire Placement	0.08 cm
Wire Spacing*	0.18 cm
Gaps Between Modules*	0.14 cm
Time to Radius Conversion	0.28 cm
Overall	0.74 cm

Table 2: Uncertainties associated with wire positioning. Uncertainties marked by an * do not apply to all wires.

162 MIT-Bates Linear Accelerator Center in Middleton, MA. Candidate muon tracks are selected if
 163 they pass through two adjacent chambers on two parallel planes. A series of data cleaning cuts
 164 are applied to remove hit pairs created by noise and accidental triggers. Since the positions of
 165 the wire chambers that fire are known, an estimate of the angle of the muon trajectory (θ) can be
 166 calculated. Once the angle is known, the radii of closest approach are related as:

$$R_1 + R_2 = D \cos \theta \quad (6)$$

167 where D is the distance between each wire. A trial $r(t)$ function ($\rho(t) = at^2 + b$) is used to estimate
 168 R_2 as a function of the time from the other chamber:

$$R'_2 = D \cos \theta - \rho(t_1) \quad (7)$$

169 A least-squared parameter B is constructed

$$B = (\rho(t_2) - R'_2)^2 \quad (8)$$

170 and then minimized. The resulting $r(t)$ curve is shown in Figure 7. Slices in time show a Gaussian
 171 shape, where the maximum width of these slices is 0.24 cm, which is taken as the uncertainty on
 172 the time-to-radius conversion. The fit also extracts a negative time offset of 70 ns, which is caused
 173 by delays introduced by the electronic signal chain. This time offset slightly decreases the effi-
 174 ciency for reconstructing events, but does not significantly change the reconstruction accuracy.
 175 Running conditions varied slightly between Bates lab and underground at SNO (mainly due to
 176 ambient pressure and operating voltage) and simulations were used to correct for these changes.
 177 The extrapolation provides an additional uncertainty of ± 0.14 cm, yielding a total uncertainty of
 178 ± 0.28 cm on the time-to-radius conversion model.

179 5. Data Selection

180 A number of data quality checks were made to find candidate muons that went through both
 181 SNO and the EMuS system. Six of the EMuS wires were removed from the analysis because of
 182 their abnormally low or high trigger rates. A small number of channels had multiple recorded
 183 hits in a single event. For such events, only the first hit in time was considered part of the muon
 184 track reconstruction algorithm.

185 EMuS event level cuts were defined to select muon events throughout the run of the experi-
 186 ment. A minimum of three wire planes had to fire in order to ensure proper reconstruction. The
 187 event also had to have fewer than 30 wires fired so as to reduce contamination from electrical
 188 pickup. Finally, runs with increased human activity above the detector, due to calibrations or
 189 source manipulation runs, were removed from the data analysis. A total of 62 EMuS events
 190 passed all run selection criteria.

191 To correlate these candidate events with the SNO detector, all of the relevant SNO runs were
 192 examined with an event viewer. Of the 62 EMuS events, 32 corresponded to a muon track passing
 193 within the volume of the detector confined by the PSUP structure, while 16 corresponded to an
 194 event where a muon passed external to SNO's PMT support structure and was therefore seen
 195 only by the outward looking PMT tubes. The remaining 14 EMuS events did not traverse the
 196 cavity. Of the 32 muon tracks within the SNO detector volume, 30 were properly reconstructed
 197 by SNO's muon fitter. The EMuS system ran for 94.6 days of livetime, giving a rate of 0.32
 198 reconstructed coincident events per day.

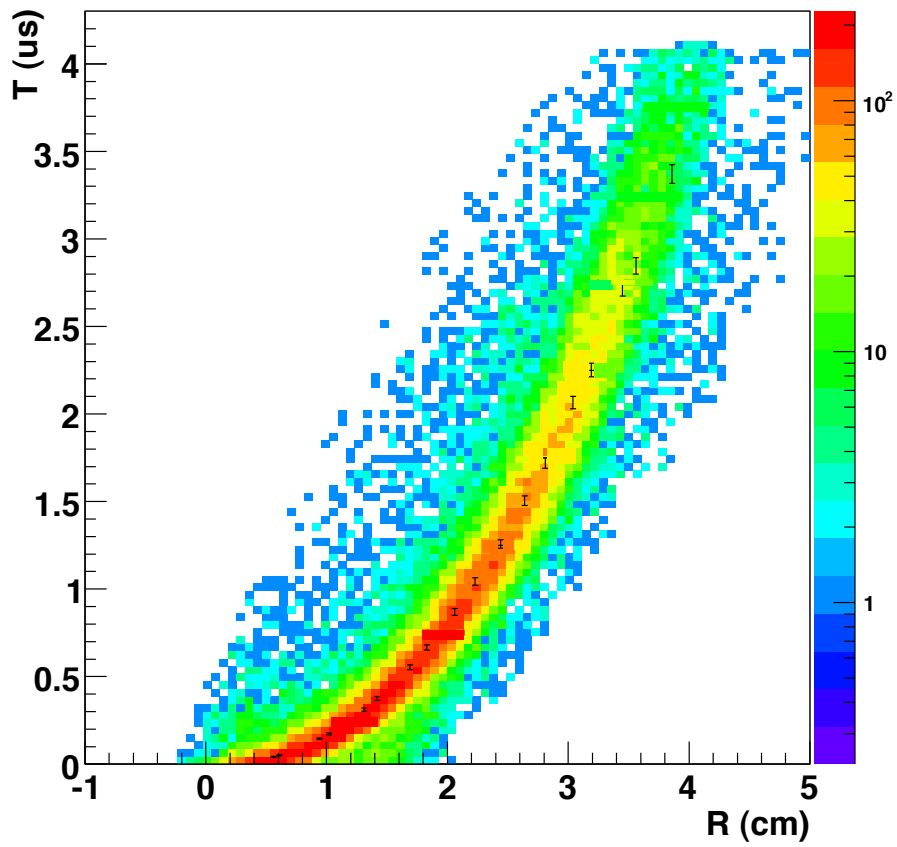


Figure 7: Drift time as a function of radius for data taken at Bates Laboratory (surface measurement). The color axis indicates the number of events that reconstruct with the given radius and time. The vertical error bars are Garfield simulations of the drift time.

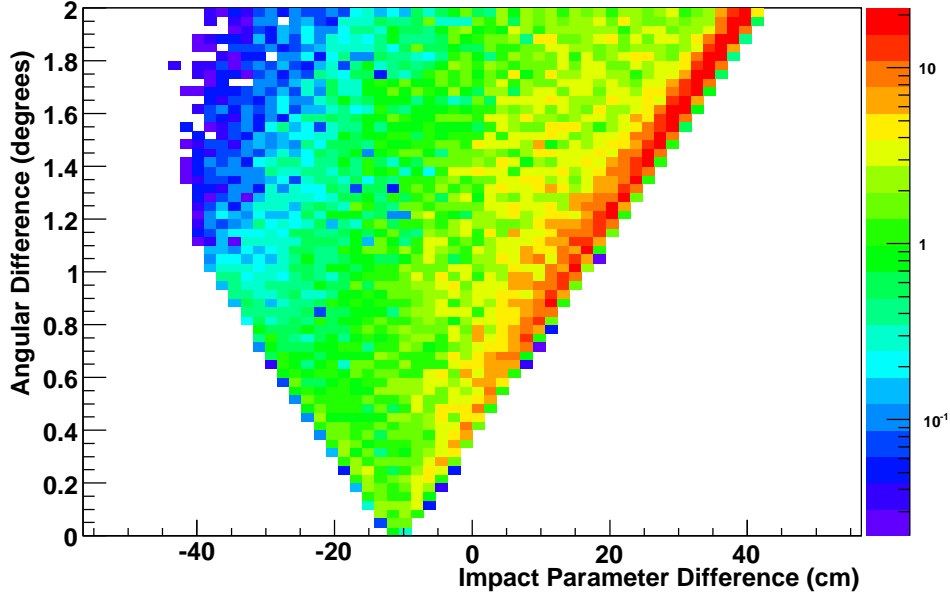


Figure 8: Angular difference vs impact parameter difference between SNO's muon fitter and the EMuS system for one event. The color scale indicates the density of possible tracks weighted by their likelihood.

199 6. EMUs Reconstruction

200 By utilizing tracks that reconstruct in both SNO and the EMuS system, one can determine
 201 the final muon track reconstruction accuracy. A Monte Carlo-based method is used to determine
 202 such reconstruction characteristics. For each real data event that is reconstructed in both the SNO
 203 and EMuS detector, a series of random test tracks are generated. These Monte Carlo generated
 204 random tracks use the muon track as reconstructed by the SNO detector alone as a seed track, but
 205 its vertex and direction are allowed to vary; with up to $\delta\theta \leq 10^\circ$ variations in reconstruction angle
 206 and up to $\delta b_\mu \leq 100$ cm variations in impact parameter. Subsequently, these generated Monte
 207 Carlo tracks are then compared to the hit pattern as recorded in the EMuS tracking chamber. The
 208 negative log likelihood value (hereafter referred to as the likelihood) for each generated track is
 209 calculated to determine the overall compatibility of the SNO muon reconstruction algorithm with
 210 tracks reconstructed in the EMuS system. The likelihood is given by the following functional
 211 form:

$$\mathcal{L} = \sum_{\text{wires } i} \frac{[b_i - \rho(t_i)]^2}{\sigma_i^2} \quad (9)$$

212 where b_i is the impact parameter between the simulated track and the i^{th} wire, $\rho(t_i)$ is the expected
 213 radius given the TDC time recorded for the wire and σ_i is the wire position uncertainty. Wire
 214 hits that reconstruct at greater than 5σ from the main track are essentially removed to avoid
 215 reconstruction bias.

216 Figure 8 shows the most likely tracks for a single event based on this method. The distribution
 217 is the projection of a cone, and indicates that there is a degeneracy between the angle and track

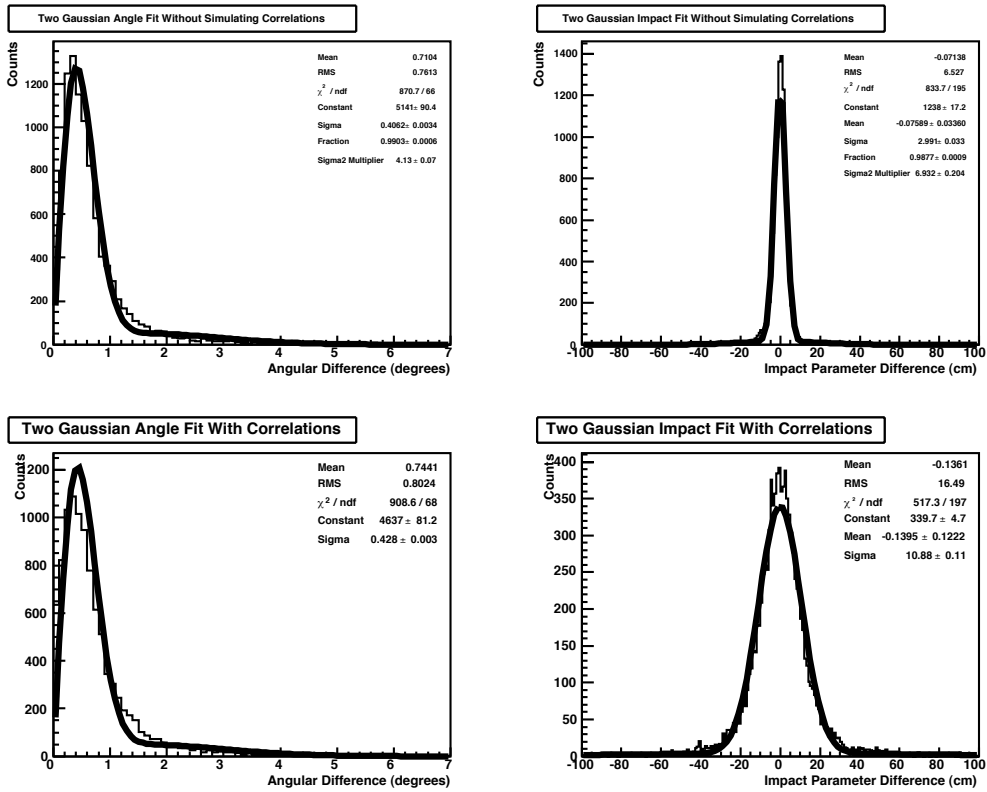


Figure 9: Results from fitting the angular (left) and impact parameter (right) distributions of the ensemble of the generated simulated data sets according to Equations 4 and 5, respectively. The top plots show the results of fitting the distributions directly from SNOMAN Monte Carlo simulation package without taking into account correlations between angle and impact parameter reconstruction in the EMuS data. The bottom plots show the results with the inclusion of these correlations.

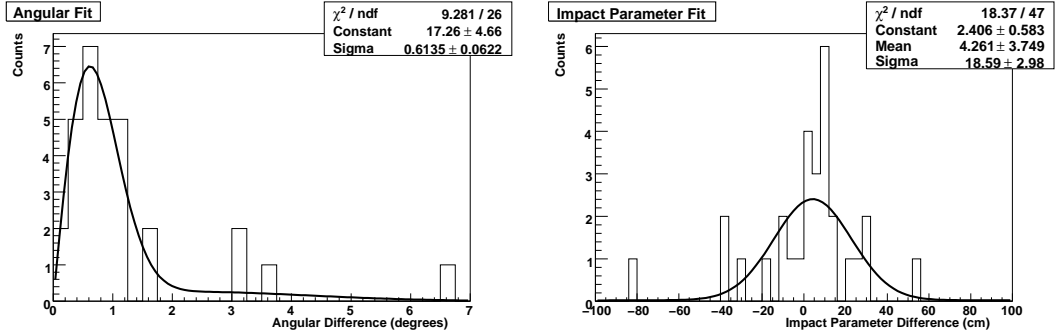


Figure 10: Gaussian fit to the data jointly reconstructed by the EMuS-SNO systems. Figure shows both angular (left) and impact parameter (right) difference.

reconstructed by the EMuS system. This is expected because if the track direction is changed (raising the angular difference) the placement of the track can be changed (raising the impact parameter difference) without significantly altering the hit pattern recorded by the EMuS system.

Since this ambiguity exists only in the EMuS system and not in SNO's muon tracking algorithm, we can compare tracks reconstructed in the two systems by assuming either (a) the impact parameter is fixed or (b) the reconstructed track direction is fixed. To test the validity of these assumptions, an ensemble of fake data sets is generated both with and without accounting for track correlations in the EMuS system. The results from these Monte Carlo tests are shown in Figure 9. Correlations have no effect on the angular mis-reconstruction or the means of the distributions, but they do broaden the impact parameter mis-reconstruction by as much as 10 cm. We conclude that the EMuS-SNO tracks are sensitive enough to constrain the angular reconstruction and impact parameter bias of the SNO muon fitting algorithm, but not the resolution of the impact parameter reconstruction.

Figure 10 shows the results of applying the two assumptions to the 30 reconstructed EMuS-SNO events. The data are fitted to the functional forms of Equations 4 and 5. Due to the small number of events, the weights and relative widths of the secondary gaussians are fixed to their values from the earlier simulations. We find that the angular width is $0.61^\circ \pm 0.06^\circ$. The impact parameter bias is 4.2 ± 3.7 cm, while fit impact parameter width is 18 ± 11 cm.

7. Conclusions

The combined data from the SNO detector and the External Muon System have demonstrated that the SNO muon reconstruction algorithm is accurate to the level needed by the neutrino-induced atmospheric flux analysis. The EMuS analysis places a constraint on the angular reconstruction to better than $0.61^\circ \pm 0.06^\circ$ and on the impact parameter bias to better than 4.2 ± 3.7 cm. The latter constraint is in good agreement with other methods using cosmic-ray data in SNO [1]. We believe the method employed here is a unique, low-cost way to explicitly verify the validity of muon track reconstruction for deep underground experiments.

244 8. Acknowledgements

245 This research was supported by: Canada: Natural Sciences and Engineering Research Council,
246 cil, Industry Canada, National Research Council, Northern Ontario Heritage Fund, Atomic Energy
247 of Canada, Ltd., Ontario Power Generation, High Performance Computing Virtual Laboratory,
248 Canada Foundation for Innovation; US: Dept. of Energy, National Energy Research Scientific
249 Computing Center; UK: Science and Technology Facilities Council; Portugal: Fundação
250 para a Ciência e a Tecnologia. We would like to thank Indiana University, Los Alamos National
251 Laboratory, and K. Eitel for loan of equipment to make the measurement possible. We would
252 also like to thank the SNO technical staff for their strong contributions and Vale (formerly Inco)
253 for hosting this project.

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