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The Casimir force at high temperature

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Abstract. – The standard expression of the high-temperature Casimir force between perfect conductors is obtained by imposing macroscopic boundary conditions on the electromagnetic field at metallic interfaces. This force is twice larger than that computed in microscopic classical models allowing for charge fluctuations inside the conductors. We present a direct computation of the force between two quantum plasma slabs in the framework of non relativistic quantum electrodynamics including quantum and thermal fluctuations of both matter and field. In the semi-classical regime, the asymptotic force at large slab separation is identical to that found in the above purely classical models, which is therefore the right result. We conclude that when calculating the Casimir force at non-zero temperature, fluctuations inside the conductors can not be ignored.

Casimir showed in 1948 [1] that the zero-point energy of the quantum electromagnetic field generates an attractive force between two perfectly conducting metallic plates at distance d and zero temperature. In his calculation, the microscopic structure of the conductors is not taken into account. The latter are merely treated as macroscopic boundary conditions for the electromagnetic fields requiring the vanishing of the tangential electric field. This geometrical constraint modifies the field eigenmodes depending on d. The d-dependence of the modified zero-point energy is the source of the well known Casimir force

$$f^{\mathrm{vac}}(d) = -\frac{\pi^2 \hbar c}{240\, d^4} \tag{1}$$

($\hbar$ denotes Planck's constant, c the speed of light).

The generalisation of Casimir's calculation to thermalized fields was given some years later in [2, 3], see [4] for a recent account. When the temperature T is different from zero, one can form the dimensionless parameter $\alpha = \beta\pi\hbar c/d$ (the ratio of the thermal wave length of the photon to the conductors separation; β is the inverse temperature). A large value of α (low temperature, short separation) characterizes the quantum regime whereas a small value of α (high temperature, large separation) yields a purely classical asymptotic result (independent of $\hbar$ and c)

$$f = -\frac{\zeta(3)}{4\pi\beta d^3} + \mathrm{O}(\mathrm{e}^{-b/\alpha}), \quad \alpha \to 0, \quad b > 0 \tag{2}$$

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where $\zeta(s)$ is the Riemann zeta function. Each field mode is a thermalized quantum mechanical oscillator with frequencies obtained from the previously described macroscopic boundary conditions. All fluctuations inside the conductors are ignored. We note that in fact, on purely dimensional grounds, a term $\propto d^{-3}$ must also be proportional to $k_B T$, the only issue being the numerical value of the proportionality constant. This issue is the subject of this letter.

In recent times, a number of works have adressed the question of the incidence of the microscopic charge and field fluctuations inside the conductors on the Casimir force [5], [6], [7]. The considered models are classical : the conductors are represented by slabs (or surfaces) containing mobile charges in thermal equilibrium and interacting through the sole Coulomb potential. These models all yield the same universal result for the mean electrostatic force between the slabs at fixed temperature and large distance

$$\langle f \rangle = -\frac{\zeta(3)}{8\pi\beta d^3} + \mathrm{o}(d^{-3}), \quad d \to \infty \tag{3}$$

Universality means that the asymptotic force does not involve any parameter characterizing the material constitution of the conductors: particle charges and masses, densities ρ and slab thicknesses.([1]) In [5], the authors study a statistical mechanical system of charges confined to a plane at distance d of a macroscopic (non fluctuating) planar conductor. In [6], they show that replacing the above macroscopic conductor by fluctuating charges does not alter the result (3). We provide in [7] a general derivation of (3) showing that universality is guarantied by perfect screening sum rules [8].

If one compares the result (3) with (2), one sees that the extrapolation of Casimir's calculation to the classical regime is larger by a factor 2 than that obtained in the classical microscopic models. The two approaches are based on different premises : (2) was derived from the frequency spectrum of the full electromagnetic field but treating the metals as macroscopic bodies without internal structure. One the contrary, the force in (3) is purely electrostatic (longitudinal field) and it originates from the particle fluctuations inside the conductors.

This calls for a more complete model that incorporates the dynamical part of the field (transverse field) in addition to the internal degrees of freedom of the conductors. A preliminary remark is in order: it is well known that classical matter in thermal equilibrium always decouples from the transverse field because of the Bohr–van Leeuwen theorem [9]. It is therefore necessary to treat the conductors' charges quantum mechanically. The complete model is formulated as follows. One considers two parallel slabs A and B of surface L^2, thickness a and at a distance d apart. They contain non relativistic quantum charges (electrons, ions, nuclei) with appropriate statistics. The total charge in each slab is taken equal to zero. The slabs are immersed in a quantum electromagnetic field, which is itself enclosed into a larger box K with sides of length R, $R \gg L, a$. The Hamiltonian of the total finite volume system reads in Gaussian units ([2])

$$H = \sum_i \frac{\left(\boldsymbol{p}_i - \frac{e_{\gamma_i}}{c}\boldsymbol{A}(\boldsymbol{r}_i)\right)^2}{2m_{\gamma_i}} + \sum_{i<j} \frac{e_{\gamma_i} e_{\gamma_j}}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} + \sum_i V^{\mathrm{walls}}(\gamma_i, \boldsymbol{r}_i) + H_0^{\mathrm{rad}} \tag{4}$$

The sums run on all particles with position $\boldsymbol{r}_i$ and species index γ_i; $V^{\mathrm{walls}}(\gamma_i, \boldsymbol{r}_i)$ is a steep external potential that confines the particles in the slabs . It can eventually be taken infinitely steep at walls' position implying Dirichlet boundary conditions for the particle wave functions.

([1])In microscopic conductor models, there is a new energy parameter $e^2/\rho^{-1/3}$, the mean potential energy, so that universality does not follow from a simple dimensional analysis.
([2])The Pauli coupling terms between spins and magnetic field are not taken into account here.

The electromagnetic field is written in the Coulomb (or transverse) gauge so that the vector potential $\boldsymbol{A}(\boldsymbol{r})$ is divergence free and $H_0^{\rm rad}$ is the Hamiltonian of the free radiation field. For it we impose periodic boundary conditions on the faces of the large box K. Hence expanding $\boldsymbol{A}(\boldsymbol{r})$ in the plane waves modes $\boldsymbol{k} = (\frac{2\pi n_x}{R}, \frac{2\pi n_y}{R}, \frac{2\pi n_z}{R})$ gives the usual formulae

$$\boldsymbol{A}(\boldsymbol{r}) = \left(\frac{4\pi\hbar c^2}{R^3}\right)^{1/2} \sum_{\boldsymbol{k},\lambda} g(\boldsymbol{k}) \frac{\boldsymbol{e}_{\boldsymbol{k}}(\lambda)}{\sqrt{2\omega_{\boldsymbol{k}}}} (a^*_{\boldsymbol{k},\lambda} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + a_{\boldsymbol{k},\lambda} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}) \tag{5}$$

$$H_0^{\rm rad} = \sum_{\boldsymbol{k},\lambda} \hbar\omega_{\boldsymbol{k}}\, a^*_{\boldsymbol{k},\lambda} a_{\boldsymbol{k},\lambda}, \qquad \omega_{\boldsymbol{k}} = c|\boldsymbol{k}| \tag{6}$$

In (5), $\boldsymbol{e}_{\boldsymbol{k}}(\lambda)$, $\lambda = 1, 2$, are the polarization vectors and $g(\boldsymbol{k})$, $g(0) = 1$, is a form factor that takes care of ultra-violet divergences.

We suppose that the matter in the slabs is in thermal equilibrium with the radiation field and therefore introduce the finite volume free energy of the full system at temperature T

$$\Phi_{R,L,d} = -k_B T \ln \mathrm{Tr} e^{-\beta H} \tag{7}$$

where the trace $\mathrm{Tr} \equiv \mathrm{Tr}_{\rm mat}\mathrm{Tr}_{\rm rad}$ is carried over particles' and field's degrees of freedom. The force between the slabs by unit surface is now defined by

$$f(d) = \lim_{L\to\infty} \lim_{R\to\infty} f_{R,L}(d) \quad \text{with} \quad f_{R,L}(d) = -\frac{1}{L^2}\frac{\partial}{\partial d}\Phi_{R,L,d} \tag{8}$$

Adding and substracting the free energy of the free photon field in (7) leads to

$$\Phi_{R,L,d} = -k_B T \ln\left(\frac{\mathrm{Tr} e^{-\beta H}}{Z_0^{\rm rad}}\right) - k_B T \ln Z_0^{\rm rad} \tag{9}$$

where $Z_0^{\rm rad}$ is the partition function of the free photon field in the volume K. Since the last term of (9) is independent of d, it does not contribute to the force (8). Therefore

$$f(d) = k_B T \lim_{L\to\infty}\lim_{R\to\infty} \frac{1}{L^2}\frac{\partial}{\partial d} \ln\left(\frac{\mathrm{Tr} e^{-\beta H}}{Z_0^{\rm rad}}\right) \tag{10}$$

In principle $f(d)$ yields the Casimir force taking into account quantum and thermal fluctuations of both matter and field.

The main result presented in this letter is that, in the semi-classical regime, the dominant term of the large distance behaviour of the force (10) is still given by the universal classical behaviour (3). This regime is obtained when the particle thermal wave lengths $\lambda_\gamma = \hbar(\beta/m_\gamma)^{1/2}$ are much smaller than the slabs' thickness and separation ($\lambda_\gamma \ll a,\ d$).

More precisely, the force is of the form

$$f(d) = -\frac{\zeta(3)}{8\pi\beta d^3} + \mathcal{R}(\beta,\hbar,d), \qquad \text{where} \quad \mathcal{R}(\beta,\hbar,d) = \mathrm{O}(d^{-4}) \tag{11}$$

namely, the quantum corrections included in the remainder $\mathcal{R}(\beta,\hbar,d)$ only occur at the subdominant order d^{-4}.

The formalism adapted to the investigation of the high temperature (or semi-classical) regime is the Feynman-Kac-Itô path integral representation of the Gibbs weight. In this formalism a quantum point particle of species γ is represented by a closed Brownian path

$\boldsymbol{r}+\lambda_\gamma\boldsymbol{\xi}(s),\ 0\le s<1,\ \boldsymbol{\xi}(0)=\boldsymbol{\xi}(1)=0$, starting at $\boldsymbol{r}$ and of extension λ_γ: it can be viewed as a charged random wire at $\boldsymbol{r}$. Thus the ensemble of wires can be treated as a classical-like system with phase space points $(\boldsymbol{r}_i,\boldsymbol{\xi}_i)$. The wire shape $\lambda_\gamma\boldsymbol{\xi}(s)$ (the quantum fluctuation) plays the role of an internal degree of freedom; see [10], section IV, for more details on this formalism. Here, for simplicity, we use Maxwell-Boltzmann statistics for the particles. We also treat the field classically on the ground that the spacing between the dimensionless energy levels $\beta\hbar\omega_{\boldsymbol{k}}$ of the $\boldsymbol{k}$ field mode become vanishingly small in the high temperature and large distance asymptotics ($\alpha\ll 1$). A complete presentation will be found in [11], [12].

The Gibbs weight associated to n wires is

$$\exp\left(-\beta\sum_{i<j}^{n}e_{\gamma_i}e_{\gamma_j}V(\boldsymbol{r}_i,\boldsymbol{\xi}_i,\boldsymbol{r}_j,\boldsymbol{\xi}_j)+i\sum_{j=1}^{n}\sqrt{\frac{\beta e_{\gamma_j}^2}{m_{\gamma_j}c^2}}\int_0^1 d\boldsymbol{\xi}_j(s)\cdot\boldsymbol{A}(\boldsymbol{r}_j+\lambda_{\gamma_j}\boldsymbol{\xi}_j(s))\right)\tag{12}$$

where

$$V(\boldsymbol{r}_i,\boldsymbol{\xi}_i,\boldsymbol{r}_j,\boldsymbol{\xi}_j)=\int_0^1 ds\frac{1}{|\boldsymbol{r}_i+\lambda_{\gamma_i}\boldsymbol{\xi}_i(s)-\boldsymbol{r}_j-\lambda_{\gamma_j}\boldsymbol{\xi}_j(s)|}\tag{13}$$

is the Coulomb potential between two wires and the vector potential part a stochastic line integral that represents the flux of the magnetic field across the wire. The vector potential is itself a random field distributed by the normalized Gaussian thermal weight $\mathrm{e}^{-\beta H_0^{\mathrm{rad}}}/Z_0^{\mathrm{rad}}$. Then the partial trace $\langle\ \cdots\ \rangle_{\mathrm{rad}}=\frac{1}{Z_0^{\mathrm{rad}}}\mathrm{Tr}_{\mathrm{rad}}(\mathrm{e}^{-\beta H_0^{\mathrm{rad}}}\cdots)$ over the transverse field degrees of freedom in (10) is easily performed

$$\left\langle\exp\left(i\sum_{j=1}^{n}\sqrt{\frac{\beta e_{\gamma_j}^2}{m_{\gamma_j}c^2}}\int_0^1 d\boldsymbol{\xi}_j(s)\cdot\boldsymbol{A}(\boldsymbol{r}_j+\lambda_{\gamma_j}\boldsymbol{\xi}_j(s))\right)\right\rangle_{\mathrm{rad}}$$
$$=\left(\prod_{i=1}^{n}e^{-\beta e_{\gamma_i}^2W_m(\boldsymbol{0},\boldsymbol{\xi}_i,\boldsymbol{0},\boldsymbol{\xi}_i)}\right)e^{-\beta\sum_{i<j}^{n}e_{\gamma_i}e_{\gamma_j}W_m(\boldsymbol{r}_i,\boldsymbol{\xi}_i,\boldsymbol{r}_j,\boldsymbol{\xi}_j)}\tag{14}$$

In (14) W_m is a double stochastic integral

$$e_{\gamma_i}e_{\gamma_j}W_m(\boldsymbol{r}_i,\boldsymbol{\xi}_i,\boldsymbol{r}_j,\boldsymbol{\xi}_j)=\frac{1}{\beta\sqrt{m_{\gamma_i}m_{\gamma_j}}c^2}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\sum_{\mu,\nu=1}^{3}j_\mu^*(\boldsymbol{k},i)G^{\mu\nu}(\boldsymbol{k})j_\nu(\boldsymbol{k},j)\tag{15}$$

where

$$G^{\mu\nu}(\boldsymbol{k})=\frac{4\pi|g(\boldsymbol{k})|^2}{|\boldsymbol{k}|^2}\delta_{tr}^{\mu\nu}(\boldsymbol{k}),\quad \delta_{tr}^{\mu\nu}(\boldsymbol{k})=\delta^{\mu\nu}-\frac{k^\mu k^\nu}{|\boldsymbol{k}|^2}\tag{16}$$

is the free field covariance and $\delta_{tr}^{\mu\nu}(\boldsymbol{k})$ the transverse Kronecker symbol. ([3]) In (15), $\boldsymbol{j}(\boldsymbol{k},i)$ is the Fourier transform of the line current $\boldsymbol{j}(\boldsymbol{x},i)=e_{\gamma_i}\int_0^1 d\boldsymbol{\xi}_i(s)\delta(\boldsymbol{x}-\boldsymbol{r}_i-\lambda_{\gamma_i}\boldsymbol{\xi}_i(s))$ associated to the wire $\boldsymbol{\xi}$. One sees that the transverse part of the field gives rise to an effective pairwise magnetic interaction W_m that has (up to a factor) the same form as the classical energy of a pair of current wires. Its ratio to the Coulomb energy (13) is of the order of k_BT divided by the rest mass energy of the particles. It accounts for orbital diamagnetic effects, which

([3])The product in (14) contains the magnetic self energies of the wires.

are small in normal conductors. Performing a small $\boldsymbol{k}$ expansion in the integrand of (15) and noting that $\int_0^1 d\boldsymbol{\xi}(s) = 0$ one sees that the large distance behaviour of W_m is dipolar

$$e_{\gamma_i} e_{\gamma_j} W_m(\boldsymbol{r}_i, \boldsymbol{\xi}_i, \boldsymbol{r}_j, \boldsymbol{\xi}_j) \sim \frac{1}{\beta\sqrt{m_{\gamma_i} m_{\gamma_j}} c^2} \int_0^1 d\boldsymbol{\xi}_i(s_1) \cdot \int_0^1 d\boldsymbol{\xi}_j(s_2) \times \left(e_{\gamma_i} \lambda_{\gamma_j} \boldsymbol{\xi}_i(s_1) \cdot \nabla_{\boldsymbol{r}_i}\right) \left(e_{\gamma_i} \lambda_{\gamma_j} \boldsymbol{\xi}_j(s_2) \cdot \nabla_{\boldsymbol{r}_j}\right) \frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} \tag{17}$$

Having now identified the basic effective pair interactions between the random wires, namely the Coulomb potential $V(i,j)$ (13) and the magnetic potential $W_m(i,j)$ (15), it is possible to proceed exactly as in the treatment of classical charged fluids [13]. One sees that $V(i,j)$ differs from the genuine classical electrostatic interaction between two charged wires

$$V_{\rm elec}(i,j) = \int_0^1 ds_1 \int_0^1 ds_2 \frac{1}{|\boldsymbol{r}_i + \lambda_{\gamma_i}\boldsymbol{\xi}_i(s_1) - \boldsymbol{r}_j - \lambda_{\gamma_j}\boldsymbol{\xi}_j(s_2)|} \tag{18}$$

by the quantum-mechanical "equal-time constraint" imposed by the Feynman-Kac formula. It is therefore useful to split $V(i,j) = V_{\rm elec}(i,j) + W_c(i,j)$, where

$$W_c(i,j) = \int_0^1 ds_1 \int_0^1 ds_2 (\delta(s_1 - s_2) - 1) \frac{1}{|\boldsymbol{r}_i + \lambda_{\gamma_i}\boldsymbol{\xi}_i(s_1) - \boldsymbol{r}_j - \lambda_{\gamma_j}\boldsymbol{\xi}_j(s_2)|} \tag{19}$$

is the part of $V(i,j)$ due to intrinsic quantum fluctuations ($W_c(i,j)$ vanishes if $\hbar$ is set equal to zero). Its large distance behaviour originates from the term bilinear in $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ in the multipolar expansion of the Coulomb potential in (19). It is dipolar and formally similar to that of two electrical dipoles of sizes $e_1\lambda_1\boldsymbol{\xi}_1$ and $e_2\lambda_2\boldsymbol{\xi}_2$.

$$e_{\gamma_i} e_{\gamma_j} W_c(\boldsymbol{r}_i, \boldsymbol{\xi}_i, \boldsymbol{r}_j, \boldsymbol{\xi}_j) \sim \int_0^1 ds_1 \int_0^1 ds_2 \left(\delta(s_1 - s_2)) - 1\right) \left(e_{\gamma_i} \lambda_{\gamma_i} \boldsymbol{\xi}_i(s_1) \cdot \nabla_{\boldsymbol{r}_i}\right) \left(e_{\gamma_j} \lambda_{\gamma_j} \boldsymbol{\xi}_j(s_2) \cdot \nabla_{\boldsymbol{r}_j}\right) \frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} \tag{20}$$

Introducing the diagrammatic representation of the correlation functions by Mayer graphs, we perform the usual resummations of $V_{\rm elec}$-chains to sum the Coulomb divergences. This provides a short range screened potential $\Phi_{\rm elec}(i,j)$, as in the classical Debye-Hückel mean-field theory. Mayer graphs are reorganized in integrable prototype graphs with bonds

$$F(i,j) = -\beta e_{\gamma_i} e_{\gamma_j} \Phi_{\rm elec}(i,j) \tag{21}$$

$$F^{\rm R}(i,j) = \exp[-\beta e_{\gamma_i} e_{\gamma_j} (\Phi_{\rm elec}(i,j) + W_c(i,j) + W_m(i,j))] - 1 + \beta e_{\gamma_i} e_{\gamma_j} \Phi_{\rm elec}(i,j) \tag{22}$$

with the constraint of excluded convolution rule between $F(i,j)$ bonds, namely chains of F bonds are forbidden to avoid double counting of the original Mayer graphs.

We now sketch the final steps. To obtain the force, one needs to find the asymptotic form of the correlation between a wire in A and a wire in B. Set $F(i,j) = F_{AB}$ (F_{AA}) when particle i belongs to slab A and particle j belongs to slab B (A), and likewise for $F^{\rm R}(i,j)$. Following the methods of [7], one shows that the bond F_{AB} is responsible for the universal term $-\zeta(3)/(8\pi\beta d^3)$ of (11). Some care has to be exercised with the bond $F^{\rm R}{}_{AB}$ that embodies the effect of field and particle quantum fluctuations through W_m and W_c. It has a dipolar long distance behaviour $F^{\rm R}(i,j) \sim -\beta e_{\gamma_i} e_{\gamma_j} (W_c(i,j) + W_m(i,j)) \sim |\boldsymbol{r}_i - \boldsymbol{r}_j|^{-3}$ that might contribute to the force. In forming the complete correlation function of the two slab system, the bonds F_{AB} and ${F^{\rm R}}_{AB}$ have to be dressed at their extremities by appropriate internal correlations of the individual slabs in conformity with the diagrammatic rules. Thus, the complete expressions that enters in the force formula at large separation are of the form $G_{AA} \star F_{AB} \star G_{BB}$ and $H_{AA} \star {F^{\rm R}}_{AB} \star H_{BB}$. The formation of the slabs' internal correlations G_{AA} and H_{AA} in these terms is not the same because of the excluded convolution rule that applies to F_{AB} but not to ${F^{\rm R}}_{AB}$. Working out the explicit expressions, one sees that perfect screening sum rules in the system of wires applied to $G_{AA} \star F_{AB} \star G_{BB}$ imply the universality of the d^{-3} term in (11), but the term $H_{AA} \star {F^{\rm R}}_{AB} \star H_{BB}$ yields no contribution at order d^{-3} because of the same sum rules.

Even without going through the detailed calculations, it is clear from the asymptotic forms (17), (20) that the corrections to the electrostatic result (3) due to the quantum nature of the charges and the radiation field are controlled by the thermal wave lengths $\lambda_\gamma = \hbar\sqrt{\beta/m_\gamma}$, thus small at high-temperature. Because of the Bohr-van Leeuwen theorem, the free energy (7) of the complete model continuously approaches that of the corresponding pure electrostatic classical system as the λ_γ vanish. The force cannot jump by a factor 2 in this limit.

One must conclude from this analysis that the discrepancy between (2) and (3) is not due to the omission of the transverse part of the electromagnetic interaction in the classical Coulombic models of refs. [5–7] but should be attribuated to the very fact that fluctuations inside the conductors are ignored in the calculation leading to (2). Hence (3) is the correct asymptotic form of the high-temperature Casimir force. In other words, the description of conductors by mere macroscopic boundary conditions is physically inappropriate whenever the effect of thermal fluctuations on the force are considered.

One the other hand, recent experiments validate the zero temperature formula (1). In [14] the authors find an experimental agreement with the value of Casimir force's strength $\pi^2\hbar c/240$ to a 15% precision level. This indicates that fluctuations in conductors are drastically reduced as the temperature tends to zero and possibly have no more effect on the force at $T = 0$. A full understanding of the cross over from the high temperature regime (11) to the zero temperature case together with the role plaid by matter and field fluctuations in the conductors is an open problem.

Finally we like to comment on the Lifshitz versus Schwinger method to take the metallic limit in their theories of forces between macroscopic dielectric bodies. In [15] Lifshitz obtained the high-temperature large-distance ($\alpha \ll 1$) force between two dielectric slabs having a static dielectric constant ϵ as

$$f(d) \sim -\frac{1}{16\pi\beta d^3}\int_0^\infty ds \frac{s^2}{\Delta^2 e^s - 1}, \quad \Delta = \frac{\epsilon+1}{\epsilon-1} \tag{23}$$

which is easily seen to reduce to (3) in the perfect conductor limit of electrostatics $\epsilon \to \infty$. In [16], Schwinger *et al.* have proposed to take the limits in the reverse order, *i.e.* the perfect conductor limit is taken first and the high-temperature large-distance asymptotics afterwards, resulting in the value (2). In the light of the preceding considerations, the Lifshitz procedure is the right one to recover the high temperature regime for conductors.

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