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A Comparison of Two Equivalent Source Methods for Noise Source Visualization

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RAY W. HERRICK LABORATORIES

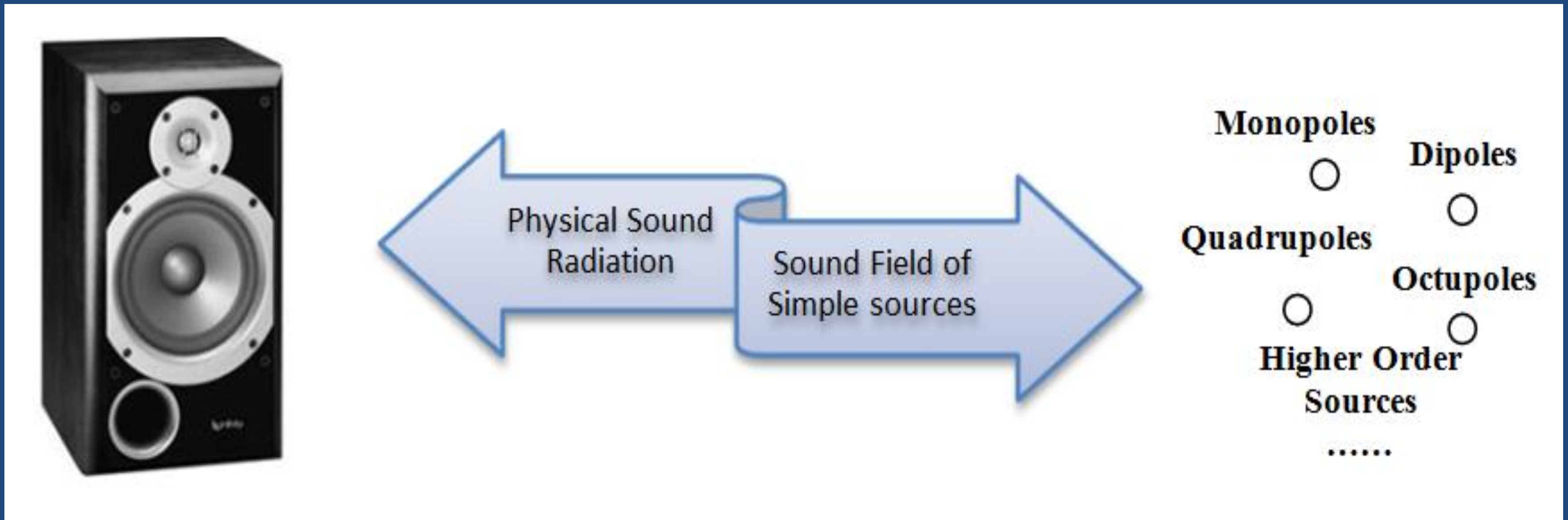
Cummins

Acoustics Holography

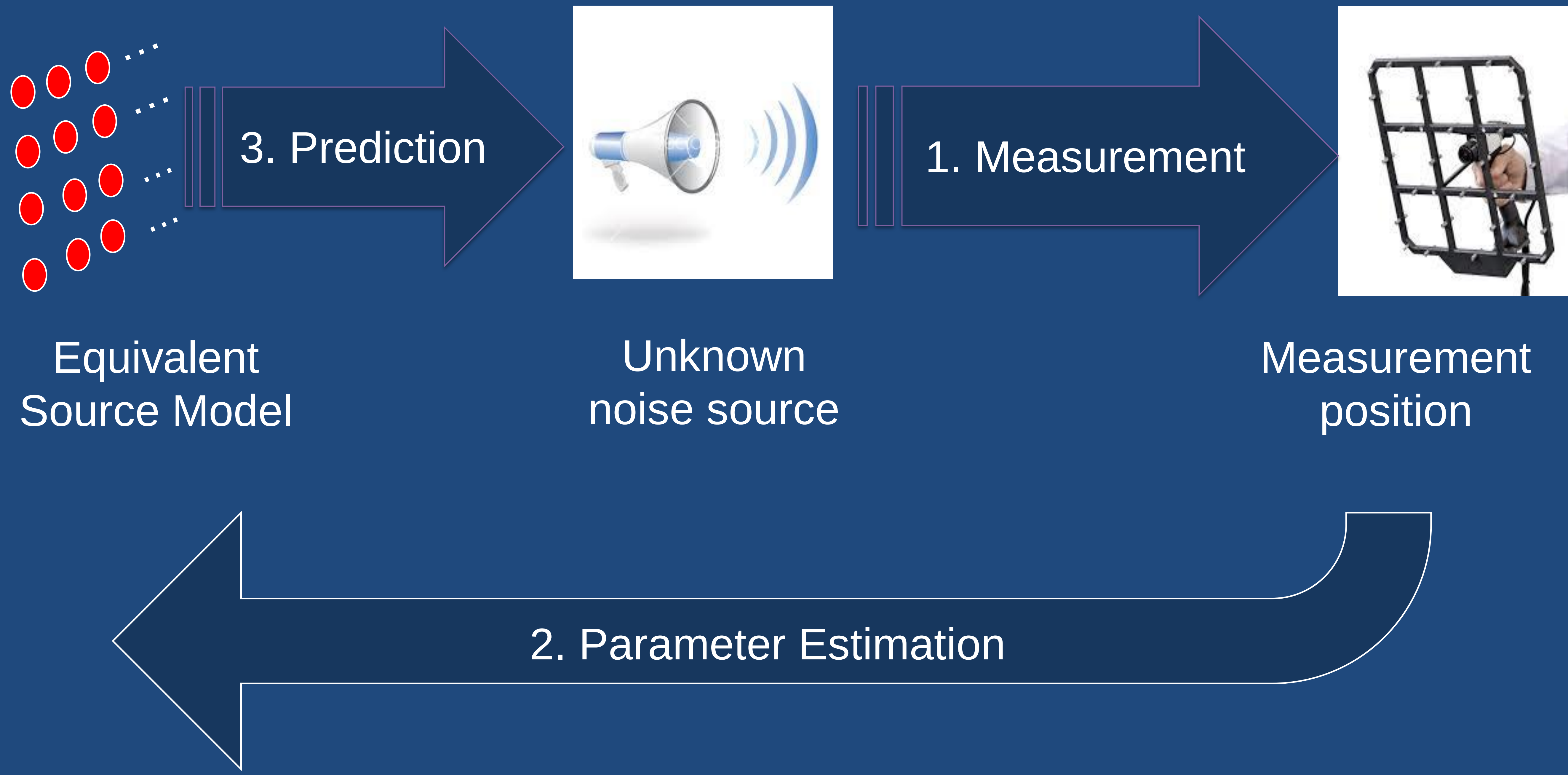
- Inverse Fourier Method
  - Statistically Optimised Near Acoustic Holography (SONAH)
- Beamforming
- Inverse Boundary Element Method (IBEM)
- Equivalent Source Method (ESM)
- Inverse Radiation Mode
  - Jiawei Liu, “Noise source Identification based on an Inverse Radiation Mode Procedure”, Noise-Con 16, Providence, Rhode Island.

Equivalent Source Method

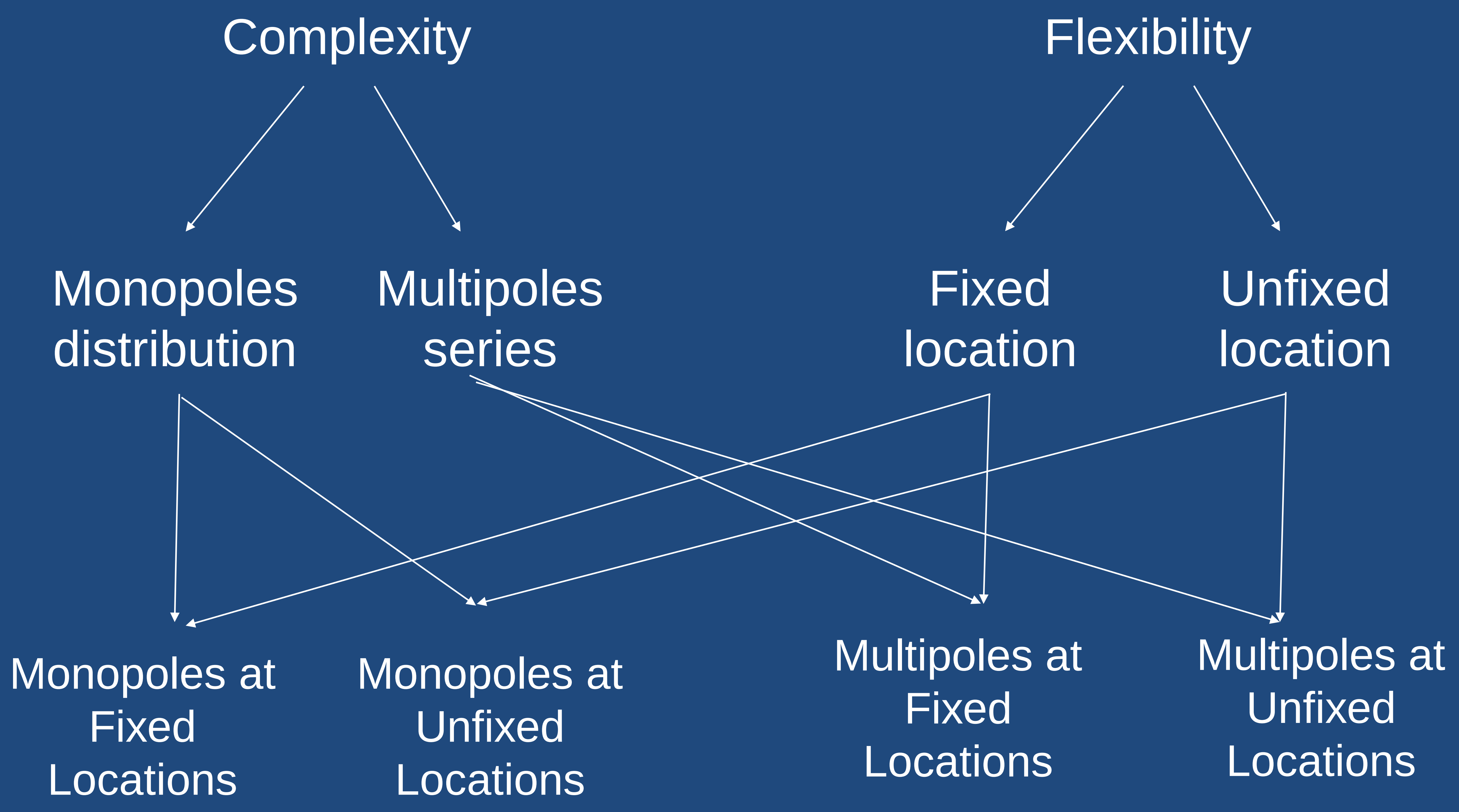
- Equivalent model having strongly associated physical meaning.
  - monopole: volume contraction and expansion, dipole: an oscillating force, lateral quadrupole: rotational torque or a vortex.
  - More similar the acoustic field of a single component is to the physical sound radiation, the fewer number of parameters we need.
- Mathematically straightforward.
- Equivalent source model don't have the limitation on frequency or measurement distance.



Construct Equivalent Source Model



Equivalent Source Model



Equivalent Source Model

Complexity

Flexibility

Monopoles distribution

Multipoles series

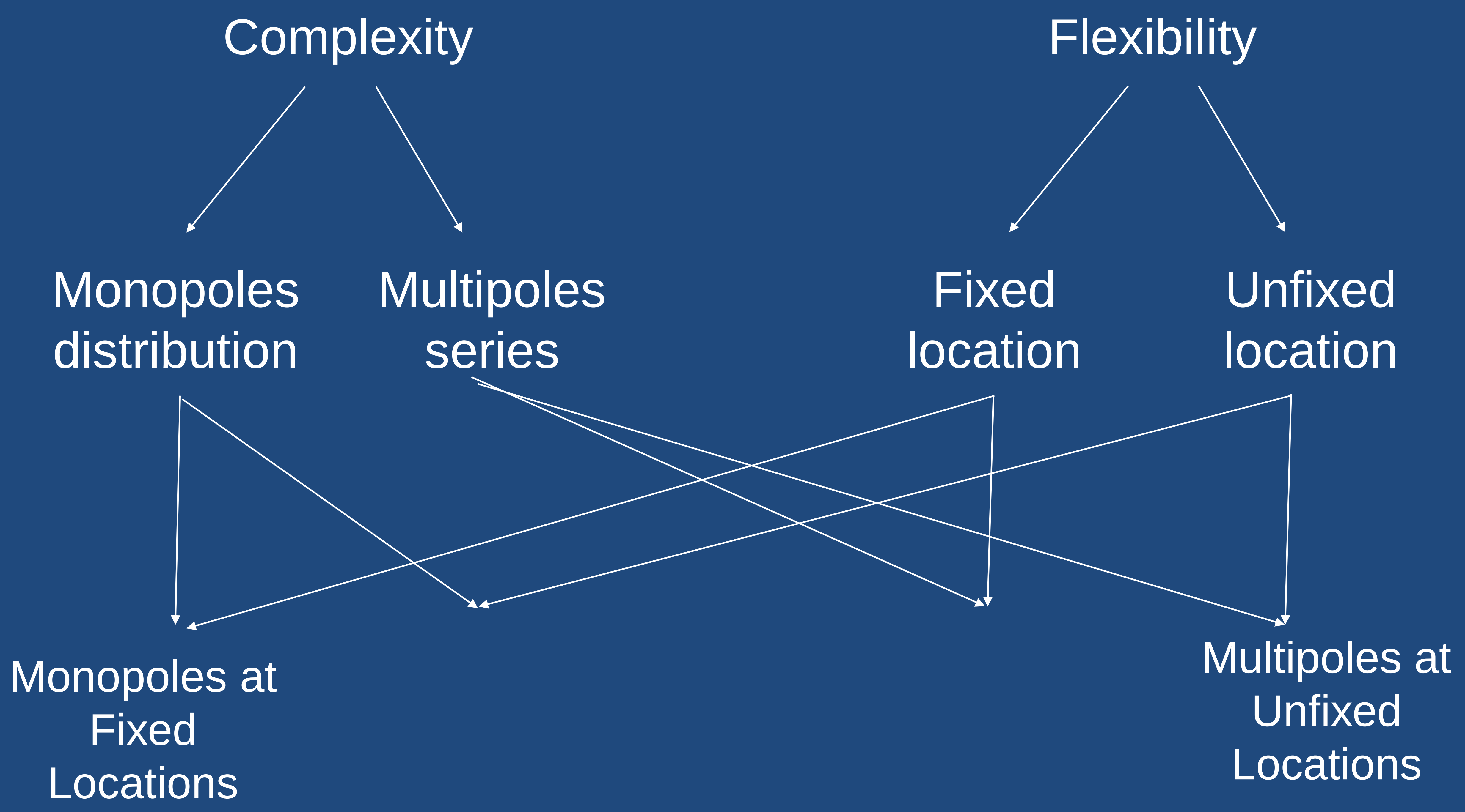
Fixed location

Unfixed location

Multipoles at Unfixed Locations

Yangfan Liu, J. Sturat Bolton, “The Use of Non-Collocated Higher Order Sources in the Equivalent Source Method”, Internoise 2012 New York City, New York

Equivalent Source Model



Monopoles at Fixed Locations

- The equation of the model generated acoustic field at all locations can be derived in a matrix form:

$$\overrightarrow{\hat{P}_m} = A(\vec{X}_S)\vec{S} \;\rightarrow\; \vec{S} = \overrightarrow{\hat{P}_m} A(\vec{X}_S)^{-1},$$

Where

$$\overrightarrow{\hat{P}_m} = [\hat{P}_m(\vec{\xi}_1|\vec{X}_S,\omega), \ldots \hat{P}_m(\vec{\xi}_W|\vec{X}_S,\omega)]^T, \quad P\;(\vec{X}|\overrightarrow{X_0},\omega) = \frac{e^{-jk\|\vec{X}-\overrightarrow{X_0}\|}}{4\pi\|\vec{X}-\overrightarrow{X_0}\|},$$

$$A(\vec{X}_s,\omega) = \begin{bmatrix} \vec{P}(\vec{\xi}_1|\vec{X}_S,\omega)^T \\ \vec{P}(\vec{\xi}_2|\vec{X}_S,\omega)^T \\ \ldots \\ \vec{P}(\vec{\xi}_W|\vec{X}_S,\omega)^T \end{bmatrix},$$

- Therefore the general parameter estimation problem is formulated as

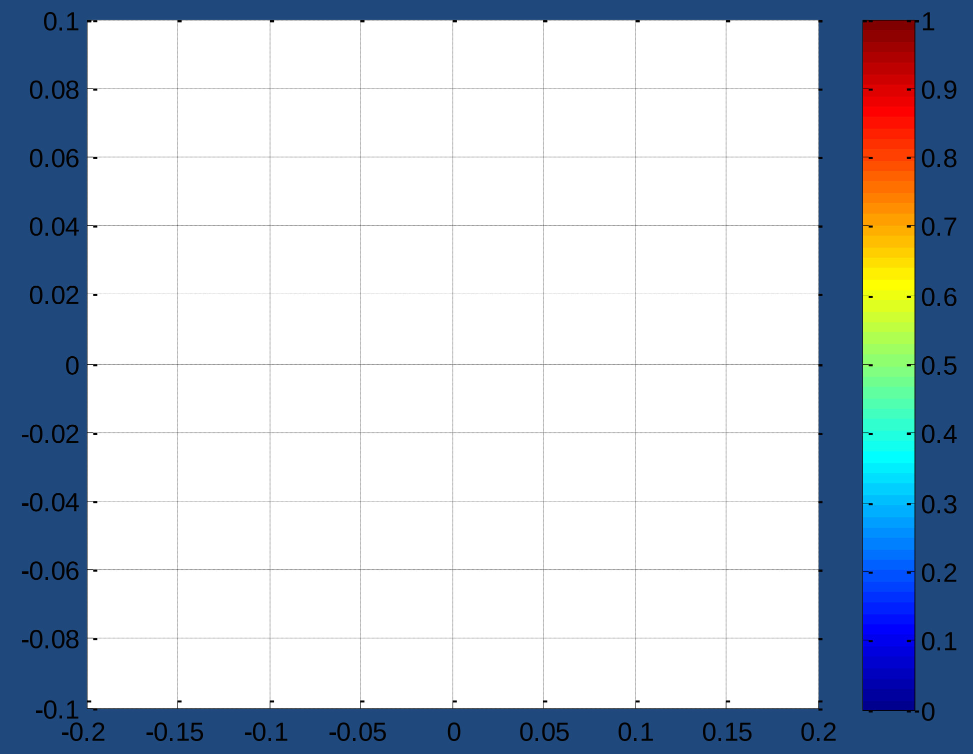
$$min\left(\left\|\vec{P}_m - \vec{\hat{P}}_m\right\|^2\right)$$

Wideband Holography

- Jorgen Hald. Wideband Holography. *Inter-noise Melbourne Australia,* November 2014.

Source strength vector q_k

take source strength as zero at first iteration



Wideband Holography

- Jorgen Hald. Wideband Holography. *Inter-noise Melbourne Australia,* November 2014.

Source strength vector q_k

take source strength as zero at first iteration

Calculate residual vector

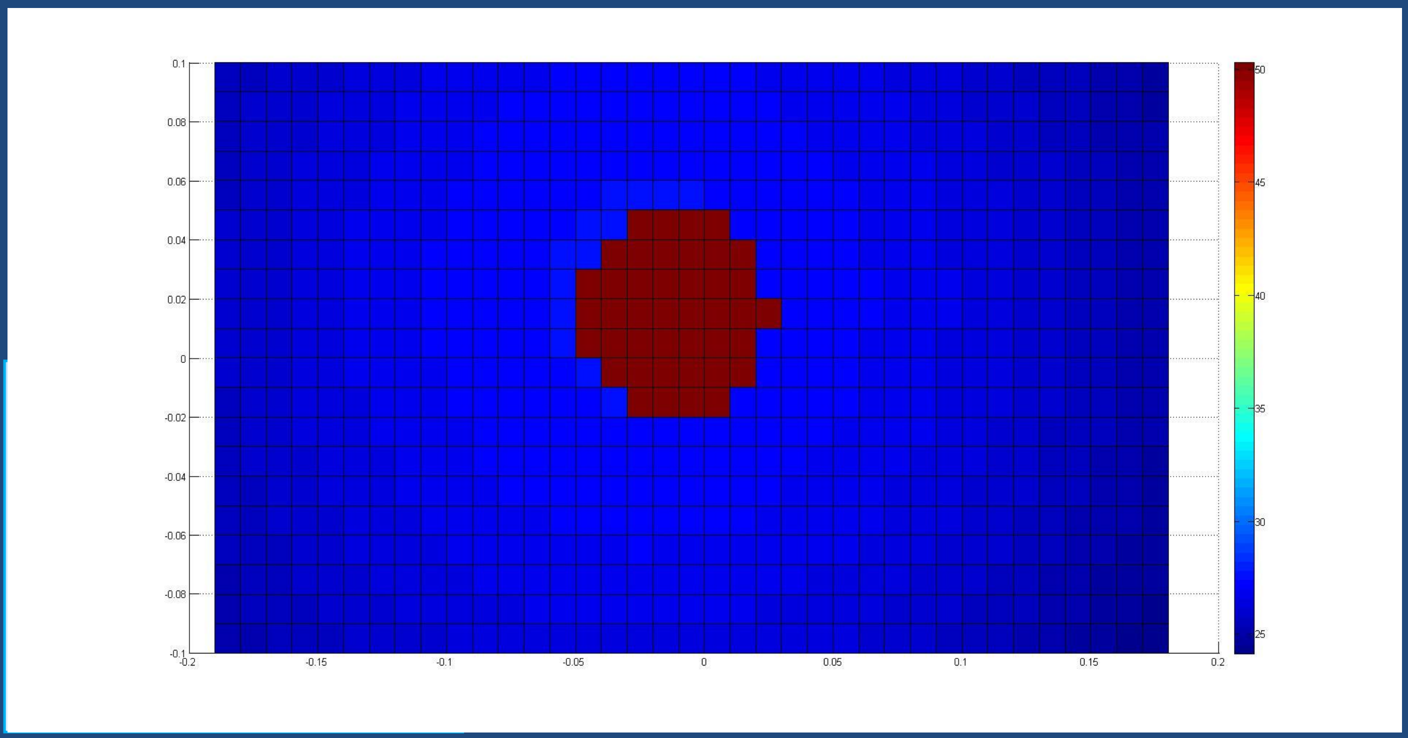
Residual vector: $r(\vec{q}) \equiv \left\|\vec{P}_m - A\vec{q}\right\|^2$

Calculate the step Δq to minimize the residual vector

$\Delta q_k = s_k \vec{w}_k$, where $\vec{w}_k$ is the negative gradient vector, $\vec{w}_k \equiv A^H r(q_k) = A^H(p - Aq_k)$, s_k is the step length to minimum along that direction $s_k \equiv \frac{g_k^H r(q_k)}{g_k^H g_k}$, the vector g_k defined as $g_k \equiv A\vec{w}_k$

$q_{k+1} = q_k + \alpha \Delta q$

Get the next source strength candidate



Wideband Holography

- Jorgen Hald. Wideband Holography. *Inter-noise Melbourne Australia,* November 2014.

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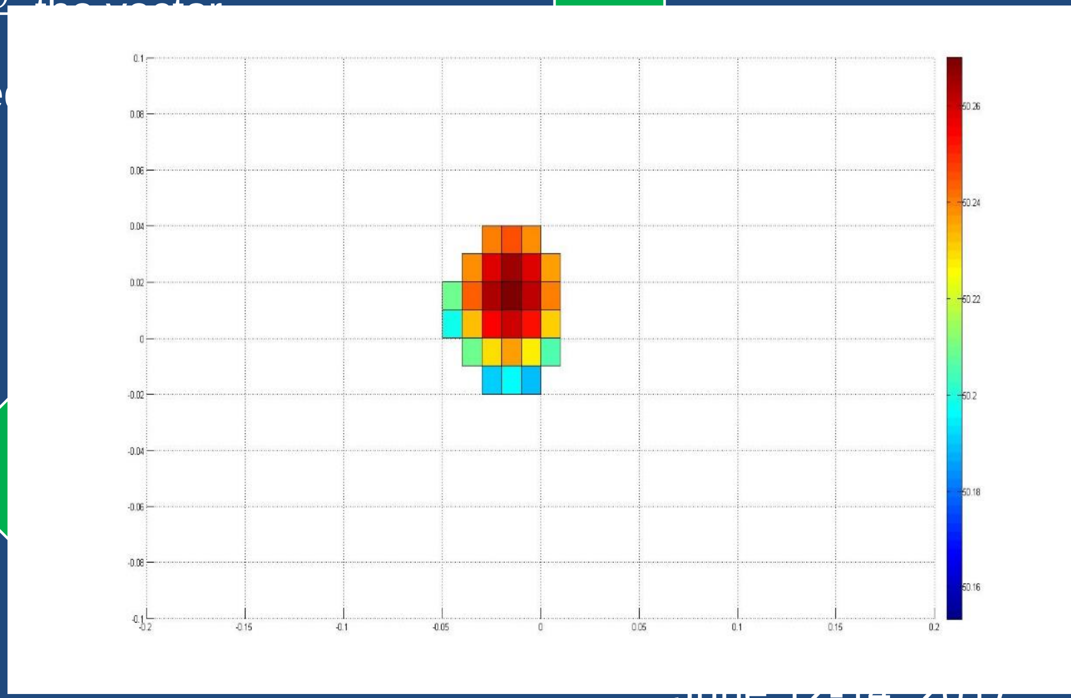
$q_{k+1} = q_k + \alpha \Delta q$



Turn off the sources below threshold

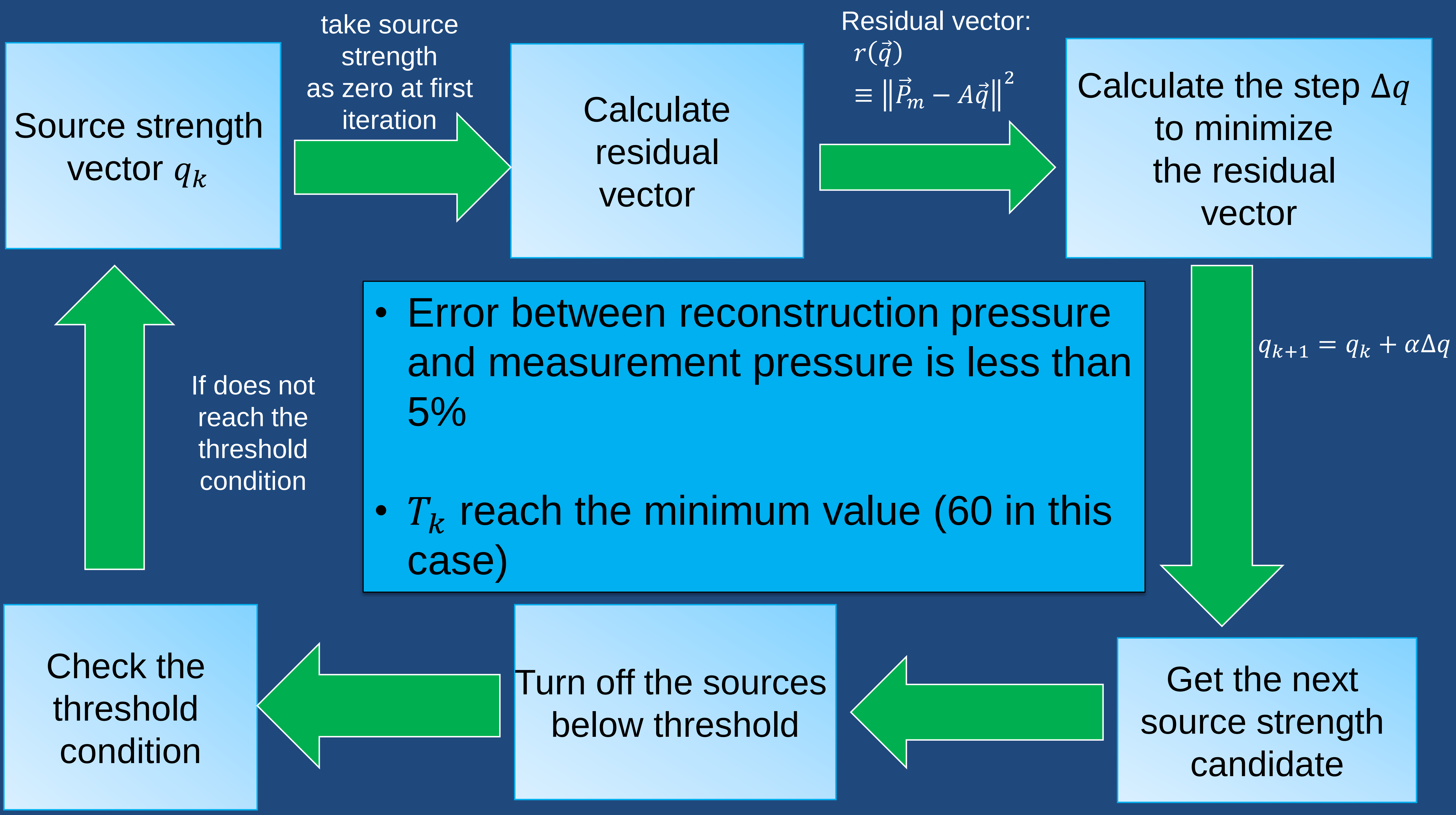
$$q_{k+1,i} = \begin{cases} \tilde{q}_{k+1,i} & if\ |\tilde{q}_{k+1,i}| \geq T_k \\ 0 & otherwise \end{cases},$$

where $T_k \equiv 10^{\frac{-D_k}{20}} |\tilde{q}_{k+1,max}|$

Check the threshold condition

Wideband Holography

- Jorgen Hald. Wideband Holography. *Inter-noise Melbourne Australia,* November 2014.



Multipole at Unfixed Locations

- Sound field expression for multipole sources:

Sound field of order $n+1$ source and order n source can be related by directional derivative:

$$P_{Sn+1}(\vec{X}|\vec{X_0},\omega) = d < \nabla P_{Sn}(\vec{X}|\vec{X_0},\omega), \vec{u}_{n+1} >$$

$$= d < \left[\frac{\partial P_{Sn}(\vec{X}|\vec{X_0},\omega)}{\partial x_0}, \frac{\partial P_{Sn}(\vec{X}|\vec{X_0},\omega)}{\partial x_0}, \frac{\partial P_{Sn}(\vec{X}|\vec{X_0},\omega)}{\partial x_0}\right]^T, \vec{u}_{n+1} >$$

Dipole: $P_{S1} = Sd_1 < \nabla P_0, \vec{u}_1 >,$ → $[\frac{\partial P_0}{\partial x}, \frac{\partial P_0}{\partial y}, \frac{\partial P_0}{\partial z}]^T$

Dipole strength: $S\, d_1$

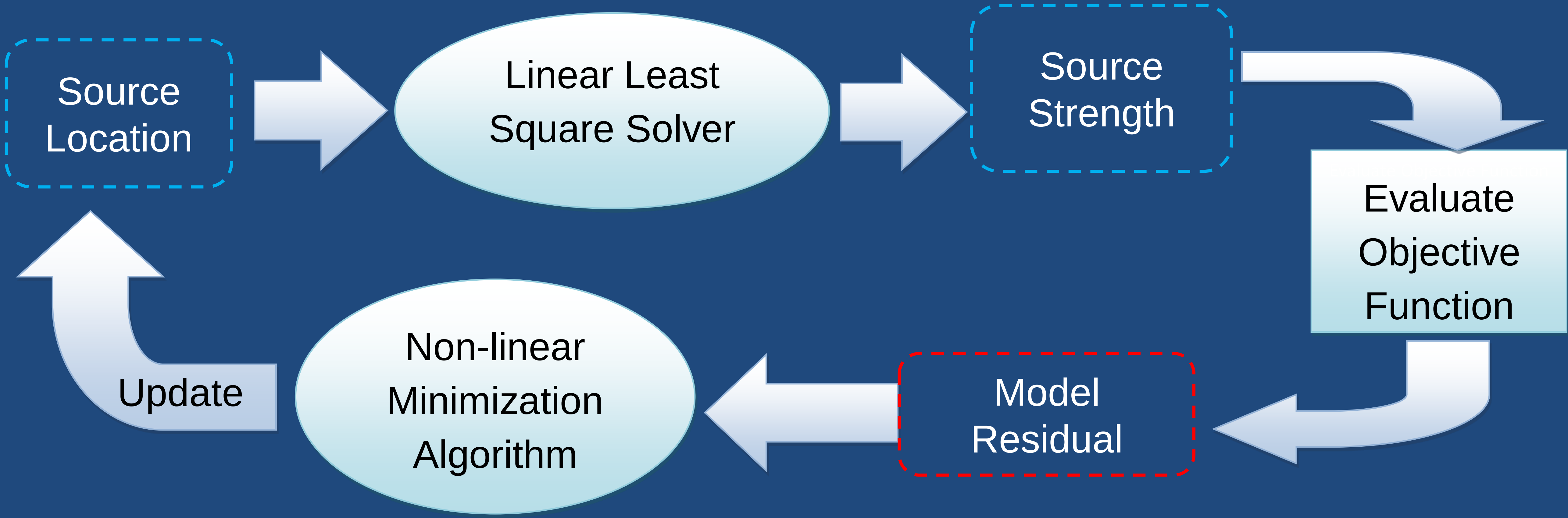
Quadrupole: $P_{S2} = Sd_1 d_2 \vec{u}_2^T R_2 \vec{u}_1,$ →

$$R_2 = \begin{bmatrix} \frac{\partial^2 P_0}{\partial x^2} & \frac{\partial^2 P_0}{\partial x \partial y} & \frac{\partial^2 P_0}{\partial x \partial z} \\ \frac{\partial^2 P_0}{\partial y \partial x} & \frac{\partial^2 P_0}{\partial y^2} & \frac{\partial^2 P_0}{\partial y \partial z} \\ \frac{\partial^2 P_0}{\partial z \partial x} & \frac{\partial^2 P_0}{\partial z \partial y} & \frac{\partial^2 P_0}{\partial z^2} \end{bmatrix}$$

Quadrupole $Sd_1 d_2$

Multipole at Unfixed Locations

➢ Process of estimating source strength and locations:



➢ Algorithms in two parts of the above problem

Linear Part:
1. Standard Least-square Solution
2. Regularization (ill-posed)

Non-linear Part: Trust Region Reflective algorithm

T.F. Coleman and Y. Li, "An Interior Trust Region Approach for Nonlinear Minimization Subject to Bounds", *SIAM J. Optimization*, **6**(2), (1996), pp. 418-445.

Experiment Setup

- Experiment at Ray W. Herrick Labs, Purdue.
  - Test with loudspeaker (Infinity Primus P163) as a noise source
  - Brule and Kjaer 18 channel irregular array
  - Measurements were taken 0.3 m from the loudspeaker.
  - Monopole model only need 18 channels measurement in front of the loudspeaker
  - Multipole model need 18 channels measurement around the loudspeaker, 108 measurements in total
  - Compare the holography result with real noise source location

Partial Field Decomposition

➢Partial field decomposition in equivalent source method

- Concentrate on major noise source
- Moohyung Lee and J. Stuart Bolton, “Scan-based near-field acoustical holography and partial field decomposition in the presence of noise and source level variation,” *J. Acoust. Soc. Am*.(2005)

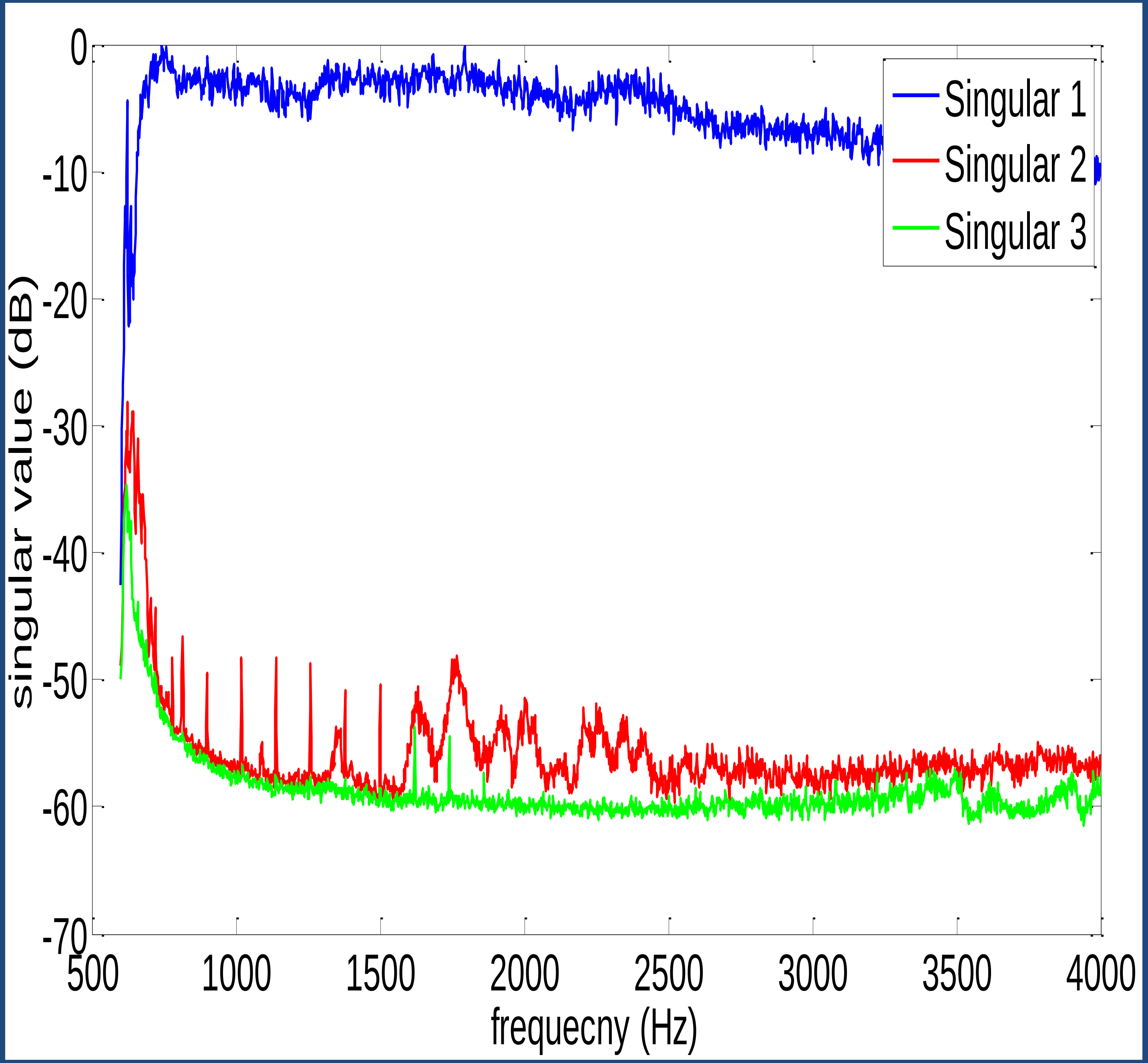


Figure 1: *Singular value decomposition result.*

Pressure, Velocity, Intensity Calculation

- Reconstruct Pressure

$$\mathbf{p} = \mathbf{A}\mathbf{q}\,,$$

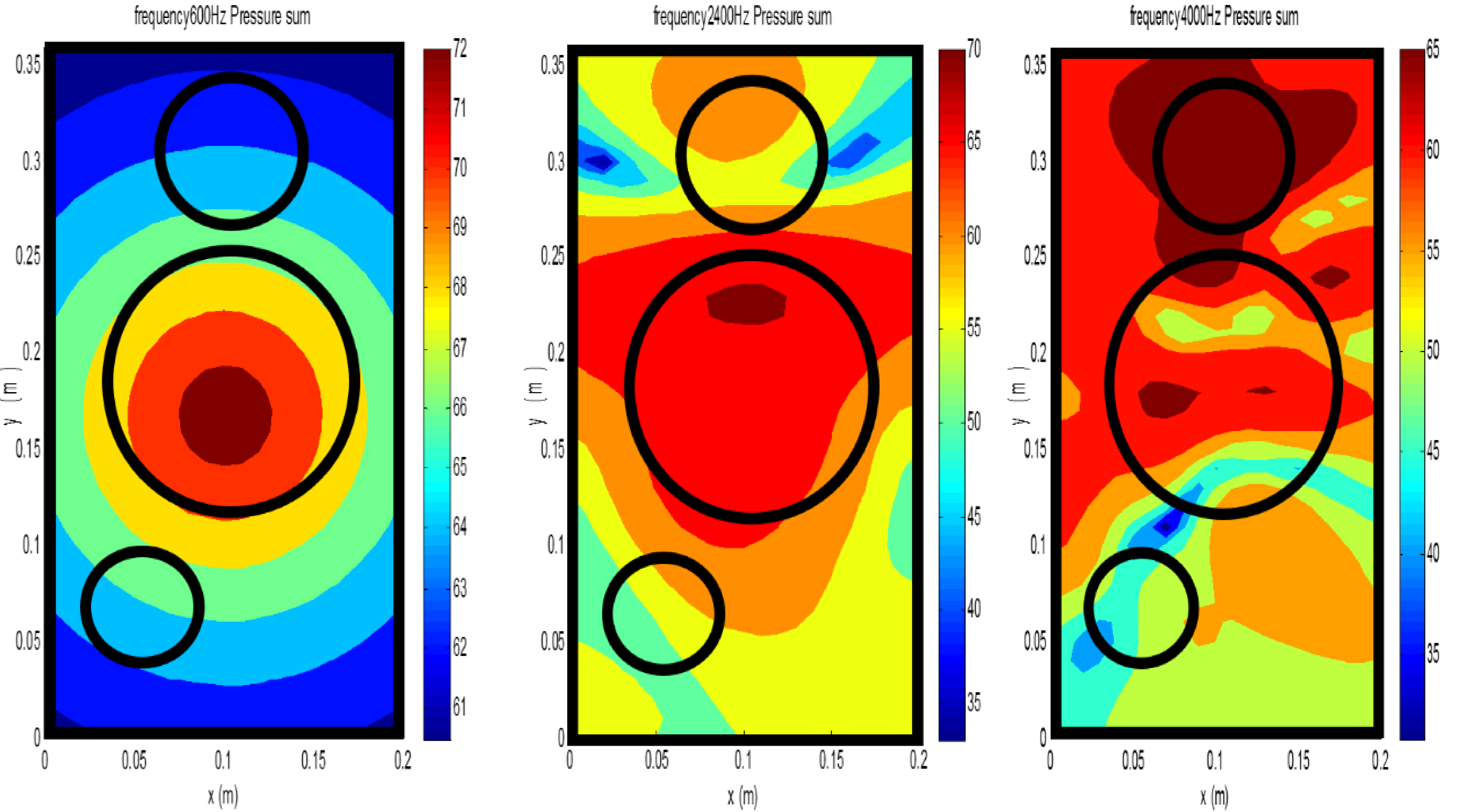
- Reconstruct Velocity

$$V = \frac{P}{\rho c}\left(1 + \frac{1}{jk\left\|\vec{X} - \overrightarrow{X_0}\right\|}\right)$$

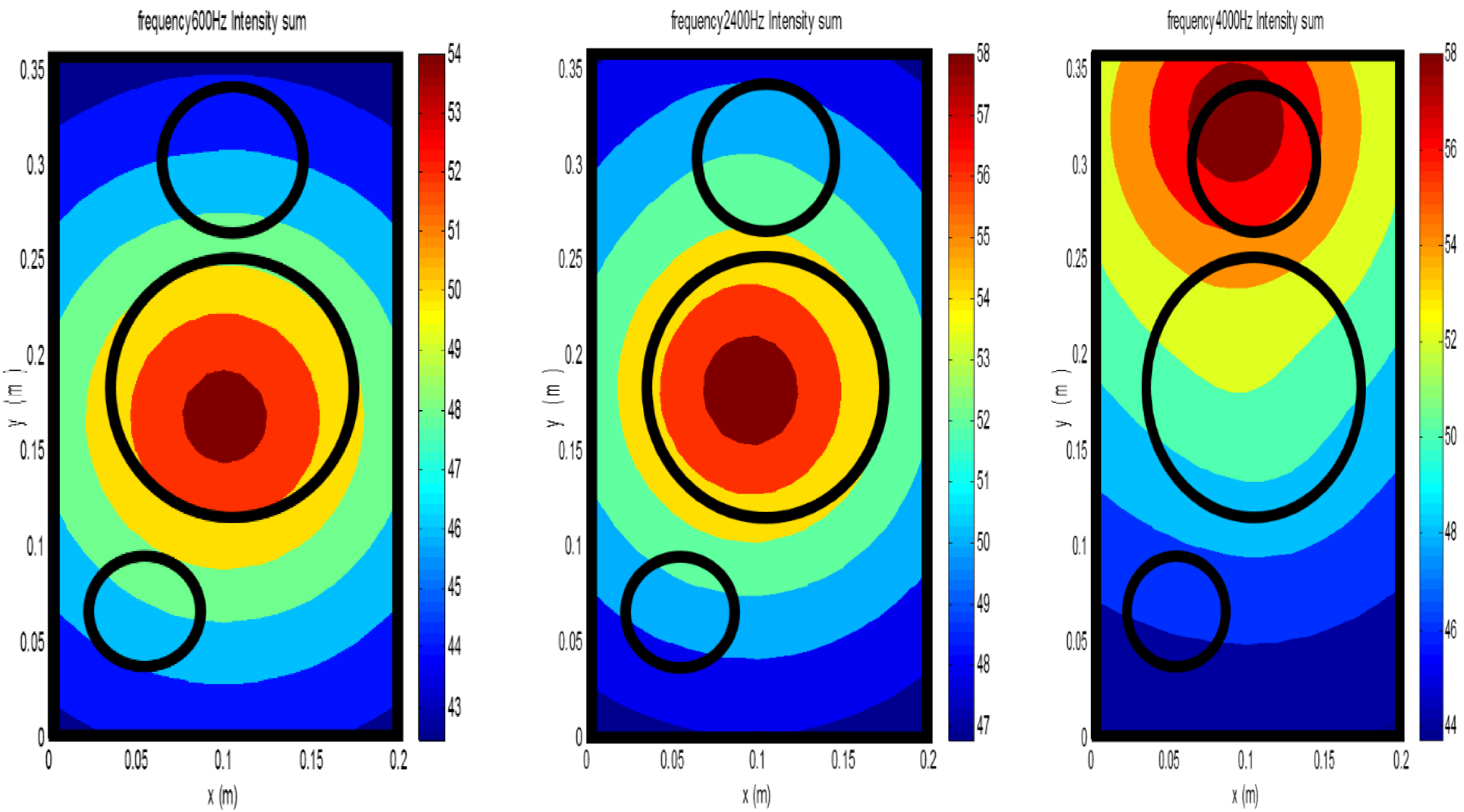
- Reconstruct Intensity

$$I = \frac{1}{2}\mathrm{Re}(PV^*)$$

Monopoles Distribution at Fixed Location
Pressure Reconstruction Result

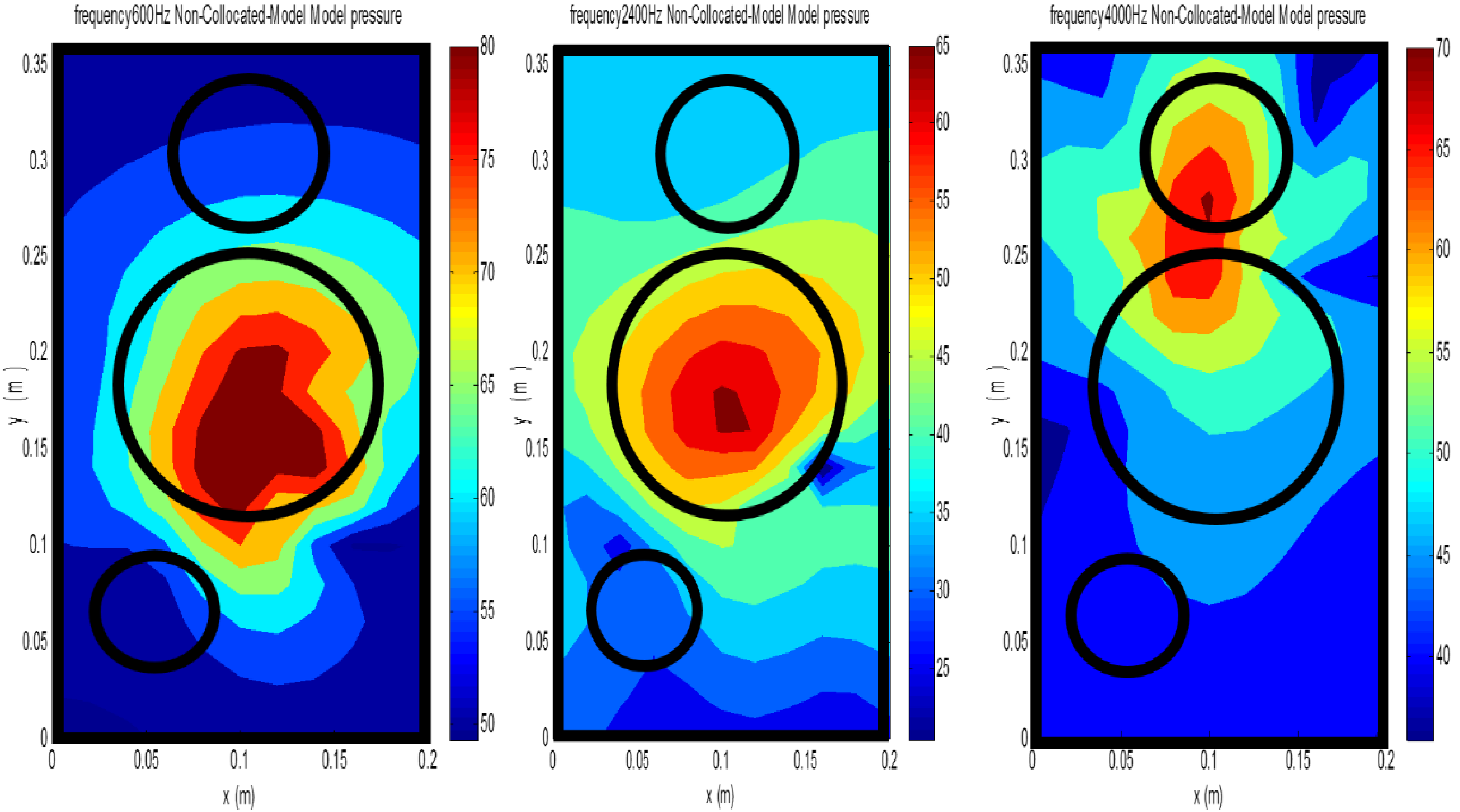


Monopoles Distribution at Fixed Location Intensity Reconstruction Result

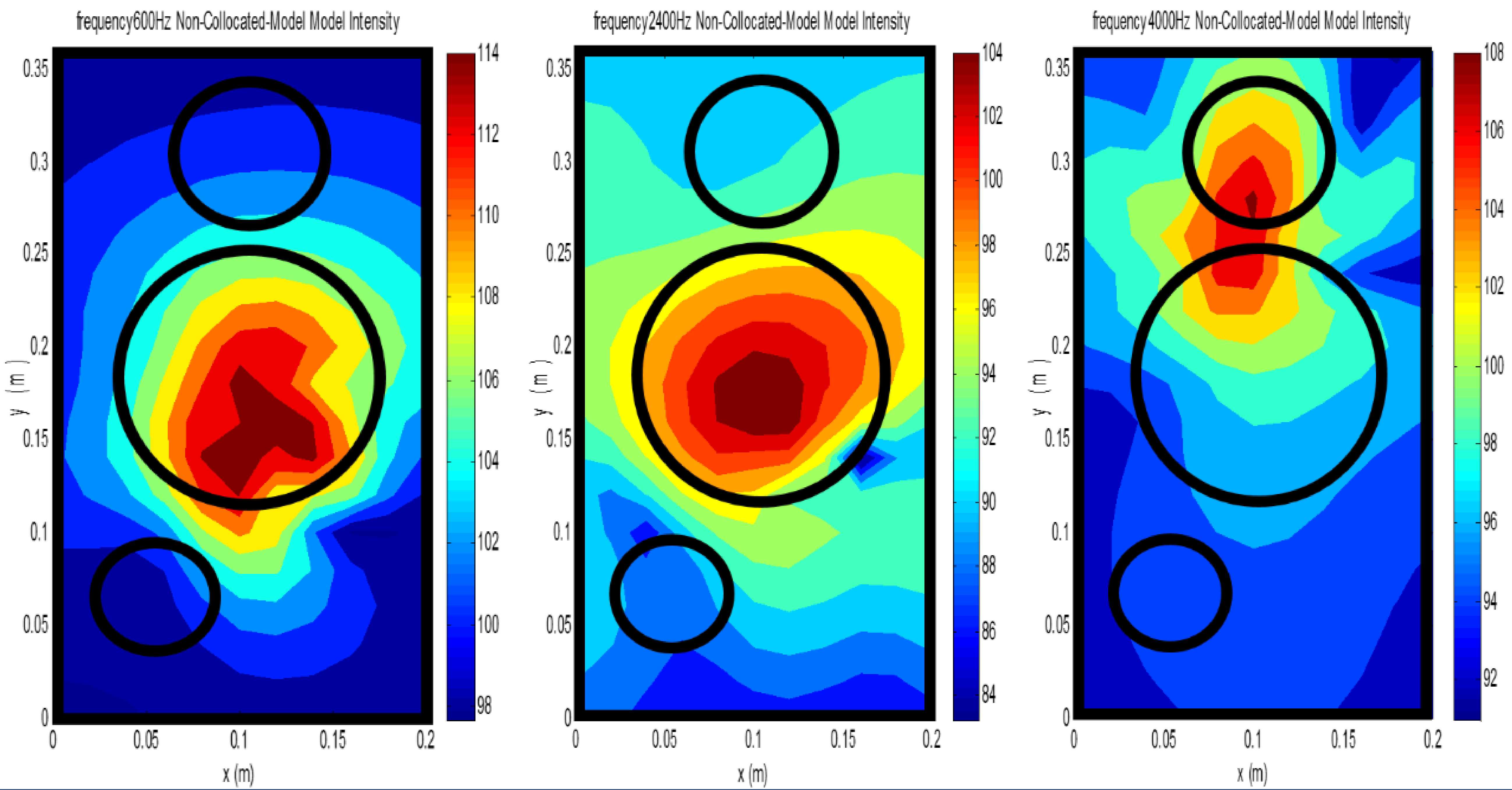


- 33 monopoles left

Multipole Series at Unfixed Location
Pressure Reconstruction Result



Multipole Series at Unfixed Location Intensity Reconstruction Result



- 16 parameters estimated

Conclusion

- Advantage of monopoles distribution at fixed location model

  - Easier to construct model
  - Mathematically straight forward
  - Easy to conduct the experiment if only one side information is needed

- Advantage of multipole series at unfixed location model

  - Better model for a complex noise source
  - Prediction position can be any surface in desired space
  - Less parameter needed to estimated in multipole model

Acknowledgement

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