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The Modeling of Anisotropic Fuselage Lining Material

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THE MODELING OF ANISOTROPIC FUSELAGE LINING MATERIAL

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INTRODUCTION

- **Objective:**
  - — development of a theoretical model that can account for the effect of lining anisotropy on sound transmission through fuselage structures
- **Present Work:**
  - — measurements of physical properties of polyimide foam samples
  - — measurements of random incidence transmission loss
  - — numerical simulations of random incidence transmission loss with isotropic and anisotropic models

Physical Properties to be Measured

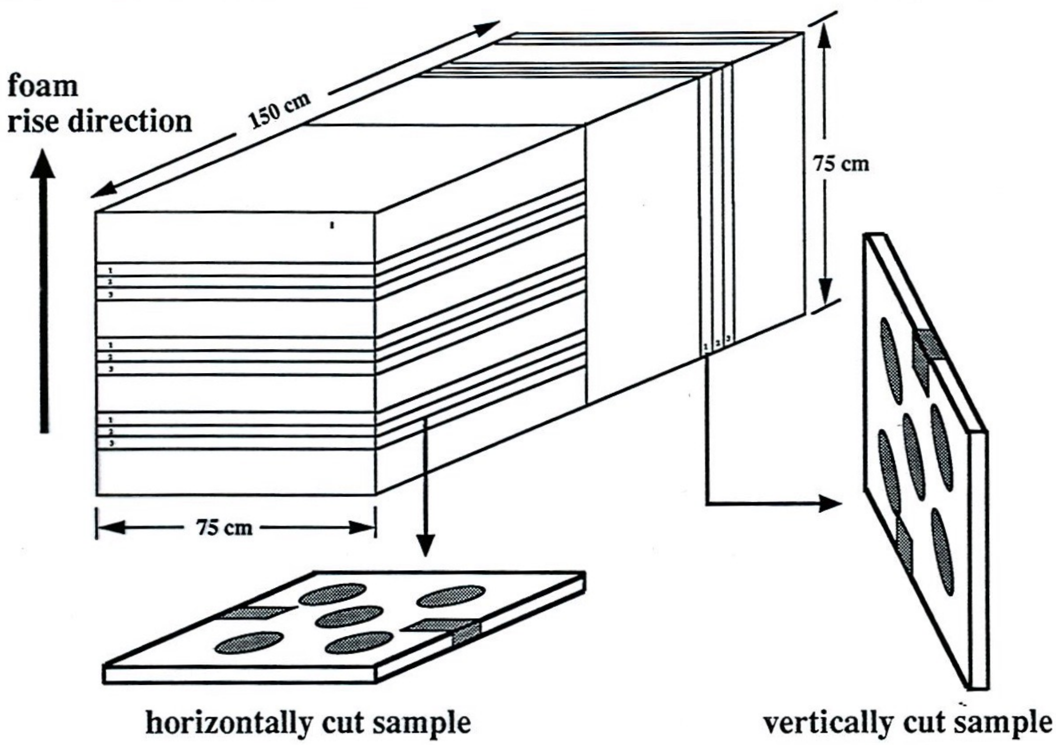
- Bulk Density
- Flow Resistivity*
- Bulk Modulus* & Loss Factor
- Bulk Shear Modulus*
- Porosity
- Tortuosity*

(Bulk Density, Flow Resistivity, Bulk Modulus & Loss Factor, Bulk Shear Modulus — measured)

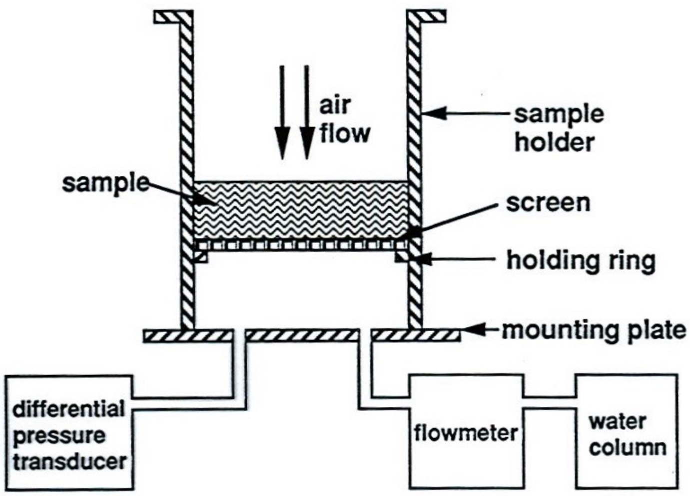
(Porosity, Tortuosity — estimated)

* anisotropic in polyimide foam

HORIZONTALLY CUT *VS.* VERTICALLY CUT SAMPLES

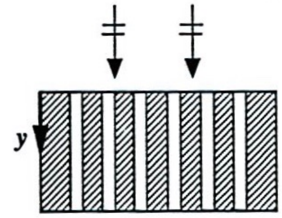


Flow Resistivity Measurement



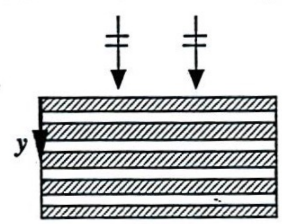
Horizontally cut sample

5.81×10^4 mks Rayls/m

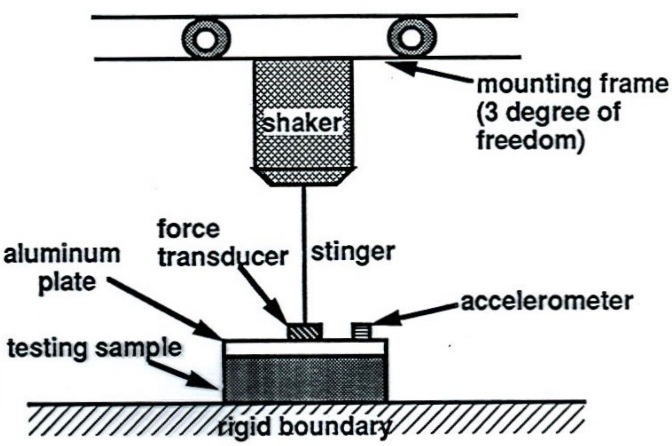


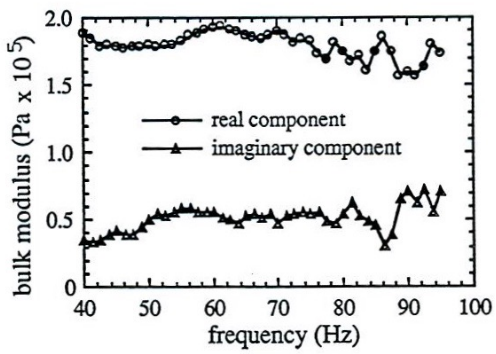
Vertically cut sample

9.64×10^4 mks Rayls/m



Bulk Modulus Measurement





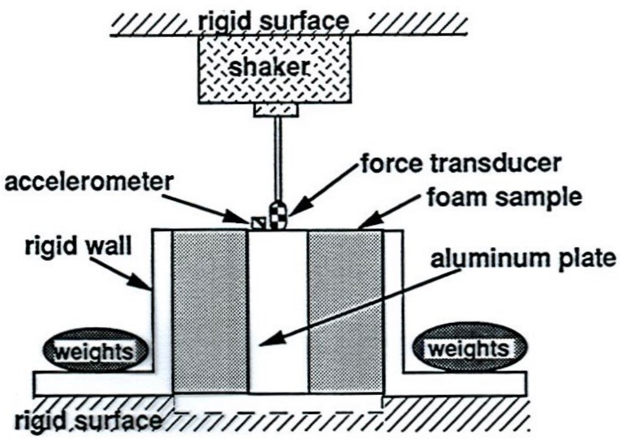
Horizontally cut sample

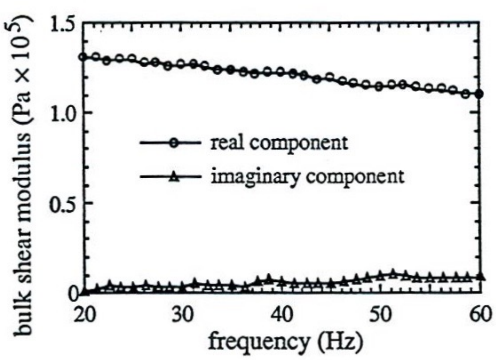
$$1.5 \times 10^5 (1+j0.33) \text{ Pa}$$

Vertically cut sample

$$8.7 \times 10^4 (1+j0.28) \text{ Pa}$$

Bulk Shear Modulus Measurement





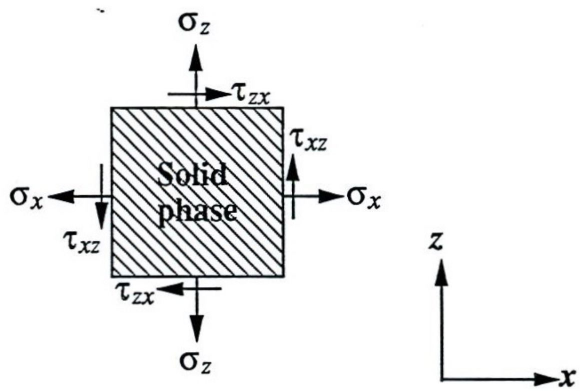
Horizontally cut sample

$1.1 \times 10^5 (1+j0.06)$ **Pa**

Vertically cut sample

$6.0 \times 10^4 (1+j0.11)$ **Pa**

STRESS-STRAIN RELATIONS



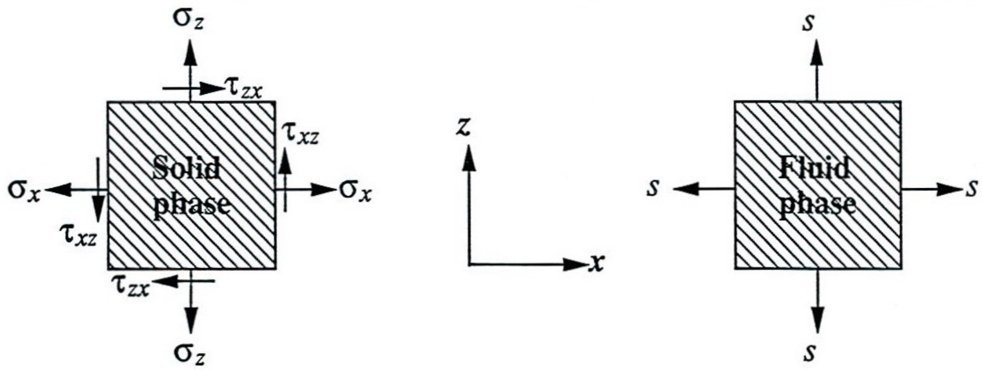
$$\sigma_x = (2N + A)e_x + Fe_z + M\varepsilon$$

$$s = Me_x + Qe_z + R\varepsilon$$

$$\sigma_z = Fe_x + Ce_z + Q\varepsilon$$

$$\tau_{zx} = \tau_{xz} = G\gamma_{zx}$$

EQUATIONS OF MOTION



$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = \rho_1 \frac{\partial^2 u_x}{\partial t^2} + \rho_2 (q_1^2 - 1) \frac{\partial^2}{\partial t^2}(u_x - U_x) + b_1 \frac{\partial}{\partial t}(u_x - U_x)$$

$$\frac{\partial s}{\partial x} = \rho_2 \frac{\partial^2 U_x}{\partial t^2} + \rho_2 (q_1^2 - 1) \frac{\partial^2}{\partial t^2}(U_x - u_x) + b_1 \frac{\partial}{\partial t}(u_x - U_x)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = \rho_1 \frac{\partial^2 u_z}{\partial t^2} + \rho_2 (q_2^2 - 1) \frac{\partial^2}{\partial t^2}(u_z - U_z) + b_2 \frac{\partial}{\partial t}(u_z - U_z)$$

$$\frac{\partial s}{\partial z} = \rho_2 \frac{\partial^2 U_z}{\partial t^2} + \rho_2 (q_2^2 - 1) \frac{\partial^2}{\partial t^2}(U_z - u_z) + b_2 \frac{\partial}{\partial t}(U_z - u_z)$$

System of Governing Differential Equations in 2-Dim.

From Dynamic Relations and Stress-Strain Relations,

Solid Phase:
(in x-direction)

$$\left[N\frac{\partial^2 u_x}{\partial x^2}+G\frac{\partial^2 u_x}{\partial z^2}\right]+\frac{\partial}{\partial x}\left[N\frac{\partial u_x}{\partial x}+(G+F-A)\frac{\partial u_z}{\partial z}\right]+\frac{\partial}{\partial x}(Ae_s+M\varepsilon)$$
$$=-\omega^2(\rho^*_{111}u_x+\rho^*_{121}U_x)$$

Fluid Phase:
(in x-direction)

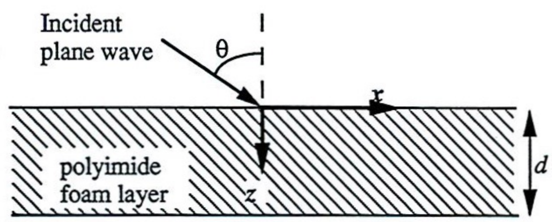
$$\frac{\partial}{\partial x}(Me_s+R\varepsilon)+(Q-M)\frac{\partial^2 u_z}{\partial z\partial x}=-\omega^2(\rho^*_{121}u_x+\rho^*_{221}U_x)$$

where

u_x, u_z, U_x, U_z	solid and fluid displacements
$e_s=\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}$	volumetric strain of solid phase
$\varepsilon=\frac{\partial U_x}{\partial x}+\frac{\partial U_z}{\partial z}$	volumetric strain of fluid phase
A, C, F, G, M, N, Q	elastic coefficients

SOLUTION PROCEDURE

STEP 1 **Substitute assumed solutions for displacement fields into the system of governing equations**



Incident sound field: $\Phi_i = e^{-j(k_x x + k_z z)}$

Assumed solutions:

$$u_x = a_i e^{-j(k_x x + k_{iz} z)}$$

$$u_z = b_i e^{-j(k_x x + k_{iz} z)}$$

$$U_x = c_i e^{-j(k_x x + k_{iz} z)}$$

$$U_z = d_i e^{-j(k_x x + k_{iz} z)}$$

SOLUTION PROCEDURE

STEP 2 **Solve the characteristic equations for wavenumbers of three types of waves within polyimide foam layer**

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda_{44} \end{bmatrix} \begin{Bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{Bmatrix} = 0$$

$$A_1 k_{iz}^6 + A_2 k_{iz}^4 + A_3 k_{iz}^2 + A_4 = 0$$

SOLUTION PROCEDURE

STEP 3 **Rewrite assumed solutions for displacement fields in terms of 6 unknown coefficients using calculated wavenumbers**

- **Solid Phase Displacements:**

$$u_x = e^{-jk_x x}\left(\sum_{i=1}^{4}\alpha_i C_i e^{-jk_{iz}z} + \sum_{i=5}^{6} C_i e^{-jk_{iz}z}\right)$$

$$u_z = e^{-jk_x x}\left(\sum_{i=1}^{4} C_i e^{-jk_{iz}z} + \sum_{i=5}^{6}\alpha_i C_i e^{-jk_{iz}z}\right)$$

- **Fluid Phase Displacements:**

$$U_x = e^{-jk_x x}\sum_{i=1}^{6}\beta_i C_i e^{-jk_{iz}z}$$

$$U_z = e^{-jk_x x}\sum_{i=1}^{6}\gamma_i C_i e^{-jk_{iz}z}$$

SOLUTION PROCEDURE

STEP 4 **Express the solid and fluid stresses of the foam in terms of displacement field solutions**

- **Solid Phase Stress:**

$$\sigma_z = -je^{-jk_x x}\left[\sum_{i=1}^{4}\{k_x(F\alpha_i + Q\beta_i) + k_{iz}(C + Q\gamma_i)\}C_i e^{-jk_{iz}z}\right.$$
$$\left. + \sum_{i=5}^{6}\{k_x(F + Q\beta_i) + k_{iz}(C\alpha_i + Q\gamma_i)\}C_i e^{-jk_{iz}z}\right]$$

$$\tau_{xz} = -je^{-jk_x x}G\left[\sum_{i=1}^{4}(k_x + k_{iz}\alpha_i)C_i e^{-jk_{iz}z} + \sum_{i=5}^{6}(k_x\alpha_i + k_{iz})C_i e^{-jk_{iz}z}\right]$$

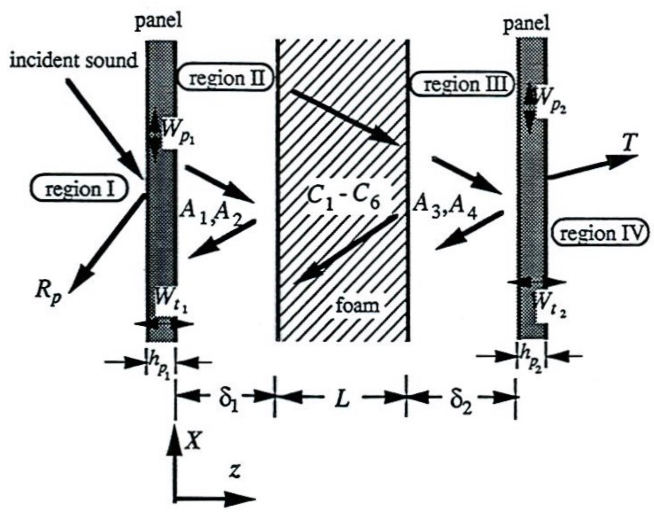
- **Fluid Phase Stress:**

$$s = -je^{-jk_x x}\left[\sum_{i=1}^{4}\{k_x(M\alpha_i + R\beta_i) + k_{iz}(Q + R\gamma_i)\}C_i e^{-jk_{iz}z}\right.$$
$$\left. + \sum_{i=5}^{6}\{k_x(M + R\beta_i) + k_{iz}(Q\alpha_i + R\gamma_i)\}C_i e^{-jk_{iz}z}\right]$$

SOLUTION PROCEDURE

STEP 5

Apply the appropriate boundary conditions and solve for reflection and/or transmission coefficients along with 6 unknown coefficients



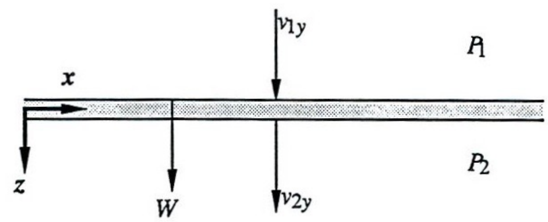
BOUNDARY CONDITIONS

PANEL

(i) $v_{1z} = j\omega W_t$

(ii) $v_{2z} = j\omega W_t$

(iii) $P_1 - P_2 = (Dk_x^4 - \omega^2 m_s)W_t$



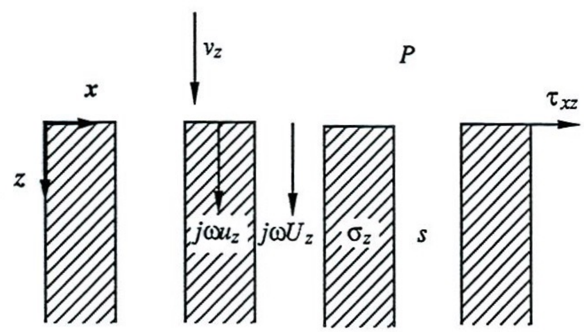
OPEN SURFACE

(i) $-hP = s$

(ii) $-(1-h) = \sigma_z$

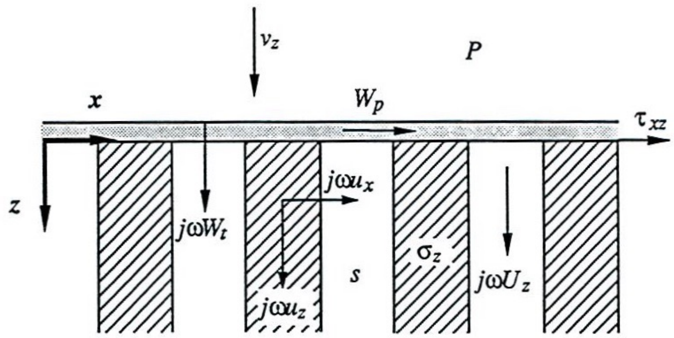
(iii) $v_z = j\omega(1-h)u_z + j\omega h U_z$

(iv) $\tau_{xz} = 0$



BOUNDARY CONDITIONS

SEALED SURFACE



(i) $v_z = j\omega W_t$

(ii) $u_z = W_t$

(iii) $U_z = W_t$

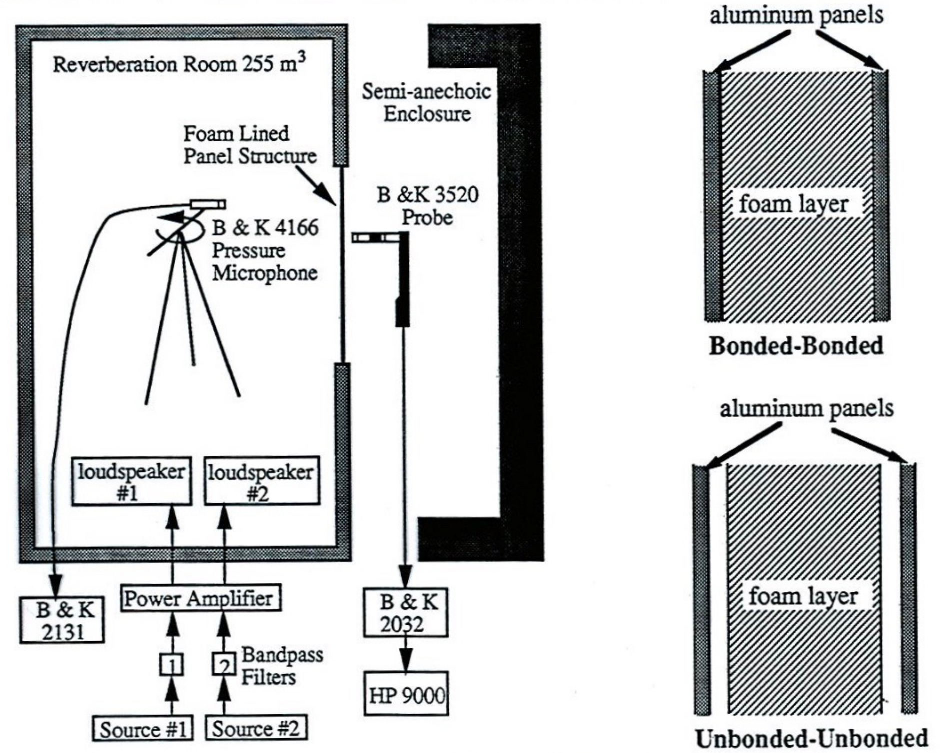
(iv) $u_x = W_p(-/+)\frac{h_p}{2}\frac{dW_t}{dx}$

(v) $(+/-)\tau_{zx} = (D_p k_x^2 - \omega^2 m_s)W_p$

(vi) $(+/-)P(-/+)q_p - jk_x\frac{h_p}{2}\tau_{xz}$

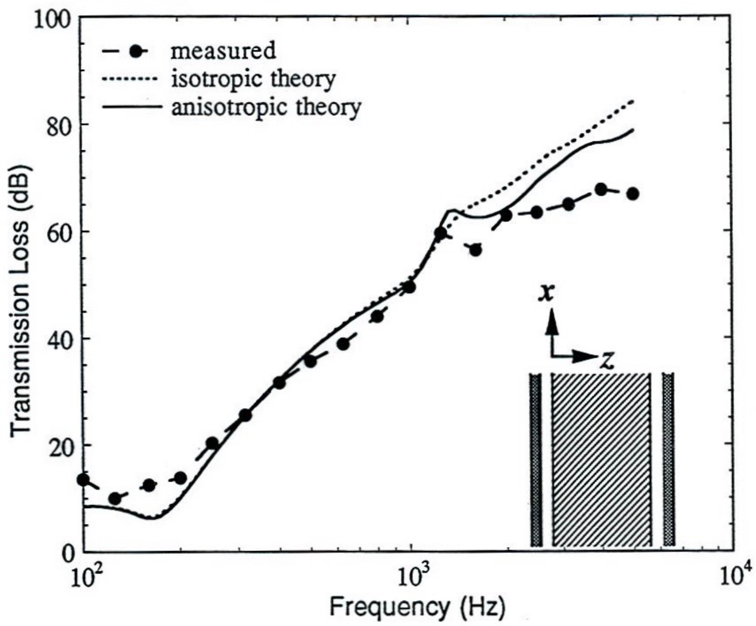
$= (Dk_x^4 - \omega^2 m_s)W_t$

Sound Transmission Loss Measurement



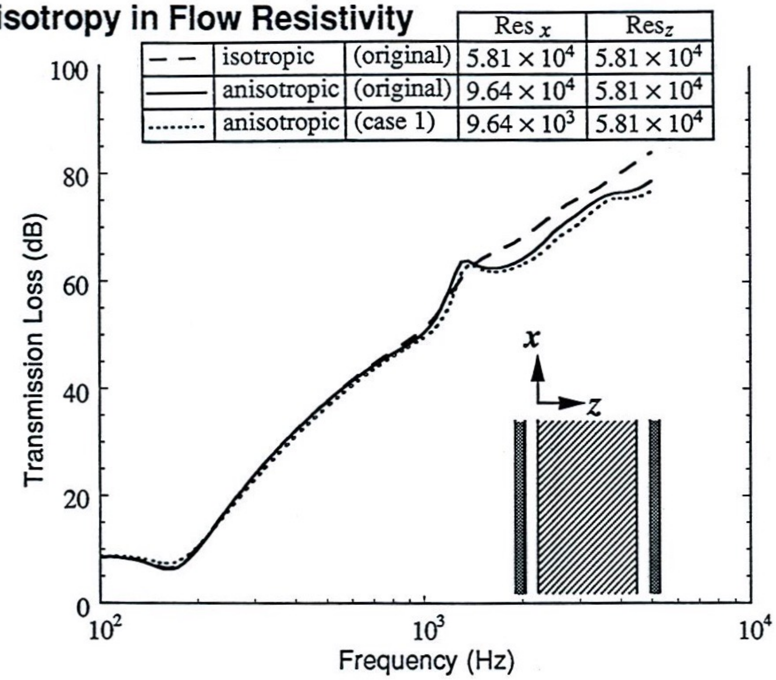
Transmission Loss for Horizontally Cut Layer

Unbonded-Unbonded



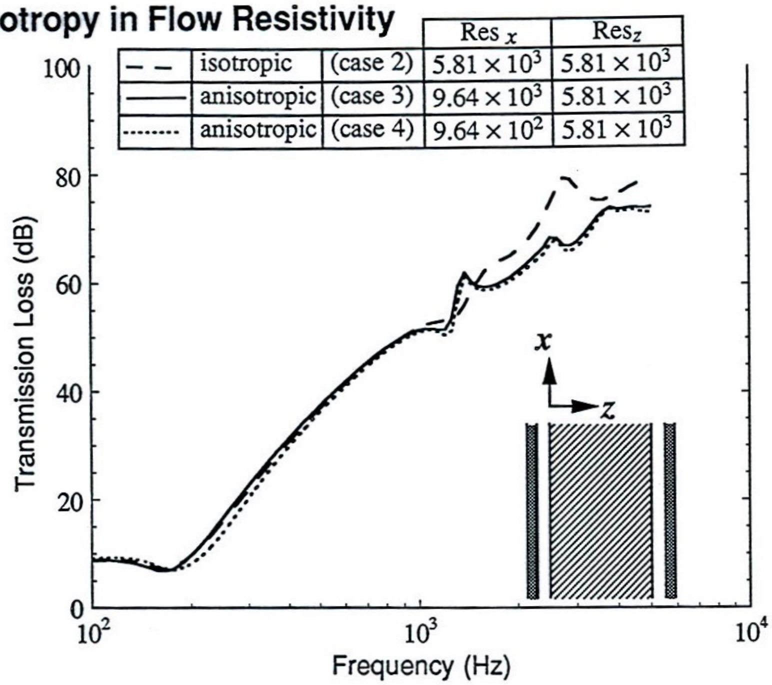
Effect of Parameter Change on the Transmission Loss

Anisotropy in Flow Resistivity

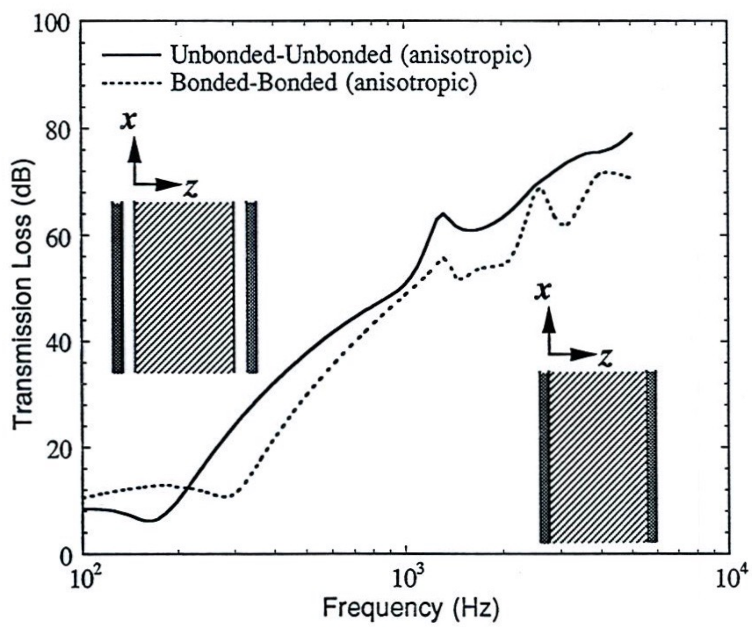


Effect of Parameter Change on the Transmission Loss

Anisotropy in Flow Resistivity



Effect of Boundary Conditions on Transmission Loss



CONCLUSIONS

- Development of a theory to model anisotropic fuse lining material.
- Anisotropic theory can give closer agreement to measurement than isotropic theory.
- In the anisotropic case:
  - — Layer resonances may exist at higher frequencies.
  - — The magnitude of flow resistivity normal to the layer is more important parameter than the anisotropy in flow resistivity.