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The Use of Wideband Acoustical Holography for Noise Source Visualization

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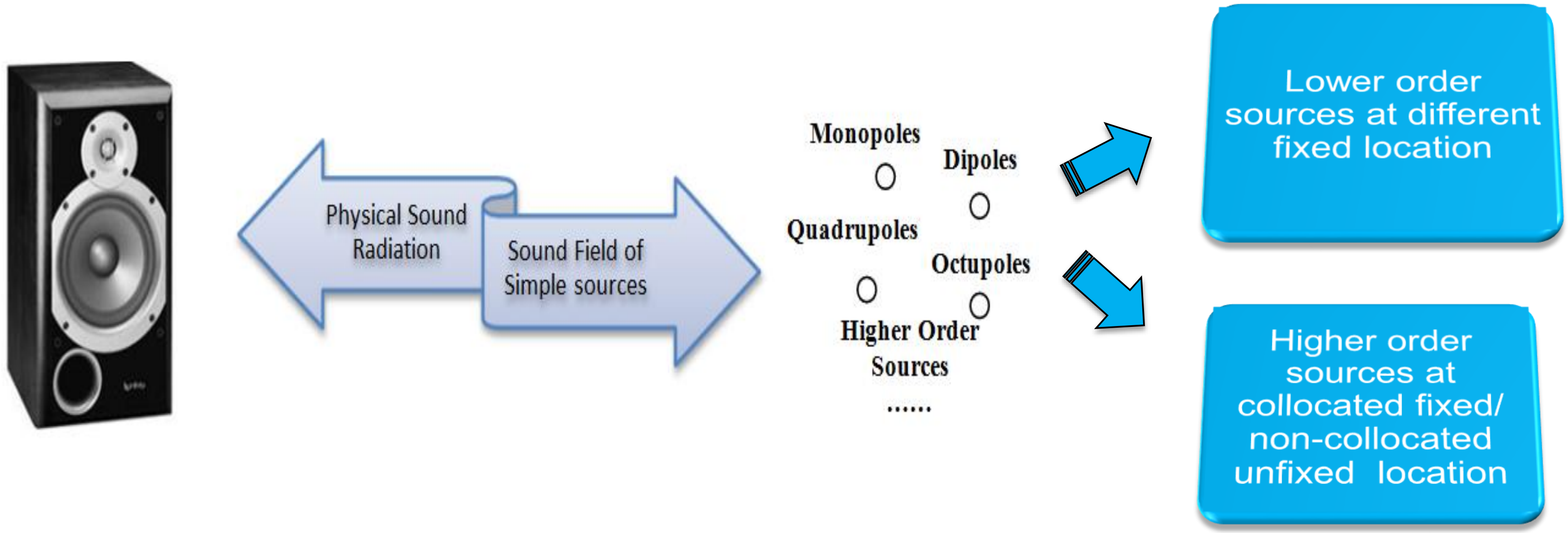
Sponsor: Cummins

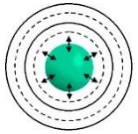
Introduction

- Equivalent Source Method (ESM)

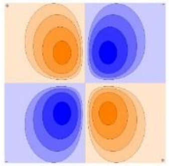
  - Basic idea: From measurements, reconstruct the sound field in free space by using a number of equivalent sources located in a certain region with appropriate source strengths, which will generate approximately the same sound field as the actual physical source.



- Monopole



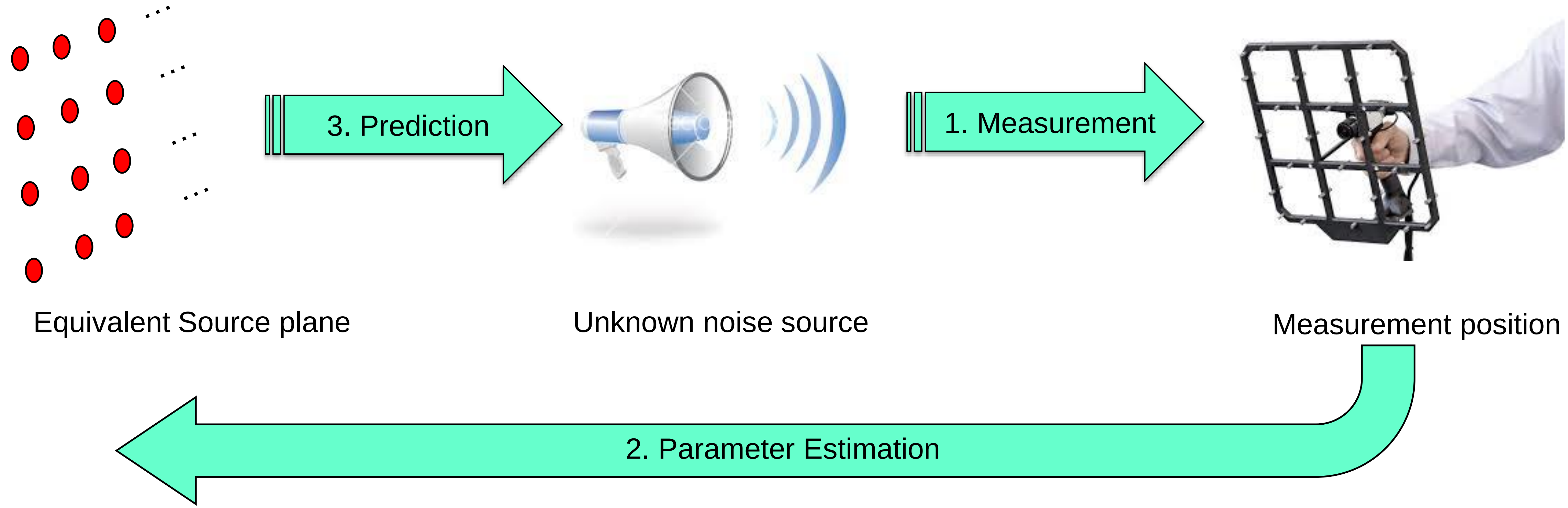
- Multipole



- Spherical Harmonic

Introduction to Lower Order Equivalent Source Model

- Idea of lower order sources at different fixed location



● : monopoles are used as lower order equivalent source in the present work

Analytical Acoustic Field Expression of Simple Source

- Expression of a monopole with source strength S

$$P_{S0}\left(\vec{X}\middle|\overrightarrow{X_0},\omega\right) = S * P_0\left(\vec{X}\middle|\overrightarrow{X_0},\omega\right) = \frac{Se^{-jk\left\|\vec{X}-\overrightarrow{X_0}\right\|}}{4\pi\left\|\vec{X}-\overrightarrow{X_0}\right\|},$$

Where ||.|| denotes the Euclidian norm of a vector, and the wave number *k=ω/c*, with *c* is the speed of sound. S is a complex number containing the information of both amplitude and phase needs to be estimated.

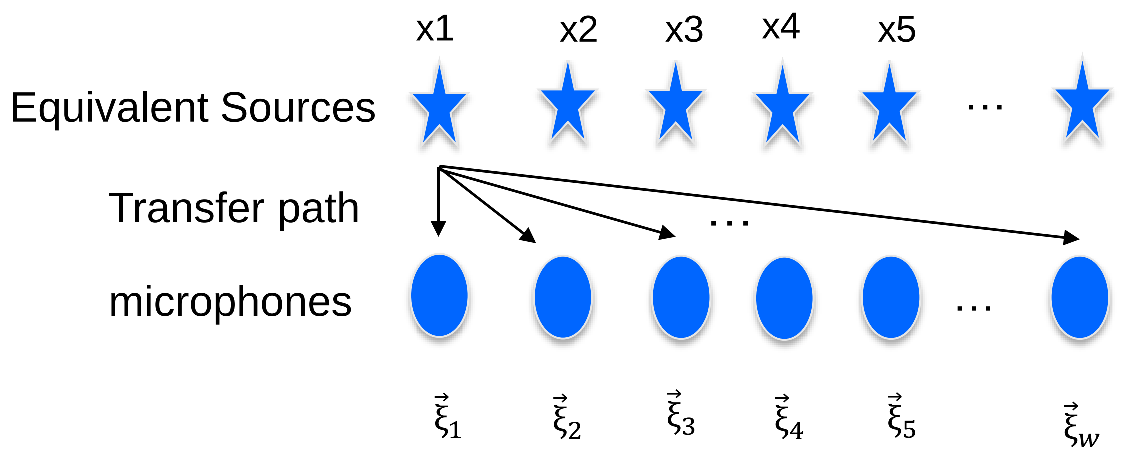
- The equation of the model-generated acoustic field at all locations can be derived in a matrix form:

$$\overrightarrow{\hat{P}_m} = A\left(\vec{X}_S\right)\vec{S},$$

Where

$$\overrightarrow{\hat{P}_m} = \left[\hat{P}_m\left(\vec{\xi}_1\middle|\vec{X}_S,\omega\right),\ldots\hat{P}_m\left(\vec{\xi}_W\middle|\vec{X}_S,\omega\right)\right]^T,$$

$$A\left(\vec{X}_S,\omega\right) = \begin{bmatrix} \vec{P}\left(\vec{\xi}_1\middle|\vec{X}_S,\omega\right)^T \\ \vec{P}\left(\vec{\xi}_2\middle|\vec{X}_S,\omega\right)^T \\ \ldots \\ \vec{P}\left(\vec{\xi}_W\middle|\vec{X}_S,\omega\right)^T \end{bmatrix},$$

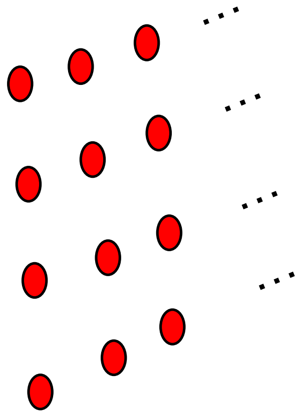


Parameter Estimation

- Therefore the general parameter estimation problem is formulated as

$$min\left(\left\|\vec{P}_m - \hat{\vec{P}}_m\right\|^2\right)$$

  - For this model structure, the location of simple sources is fixed and known, and the measurement locations are known. So the vector $\vec{P}$ is turned into a vector of numbers rather than functions.
  - So based on parameter estimation equation, *A* becomes a matrix of fixed numbers. So only strengths of the simple sources $\vec{S}$ are regarded as model parameters. So the parameter estimation problem is a linear process.



Strongly under-determined system

- ill-posed
- ghost sources
- …

Number of estimated parameters >> Number of measurements

Wideband Holography (WBH)

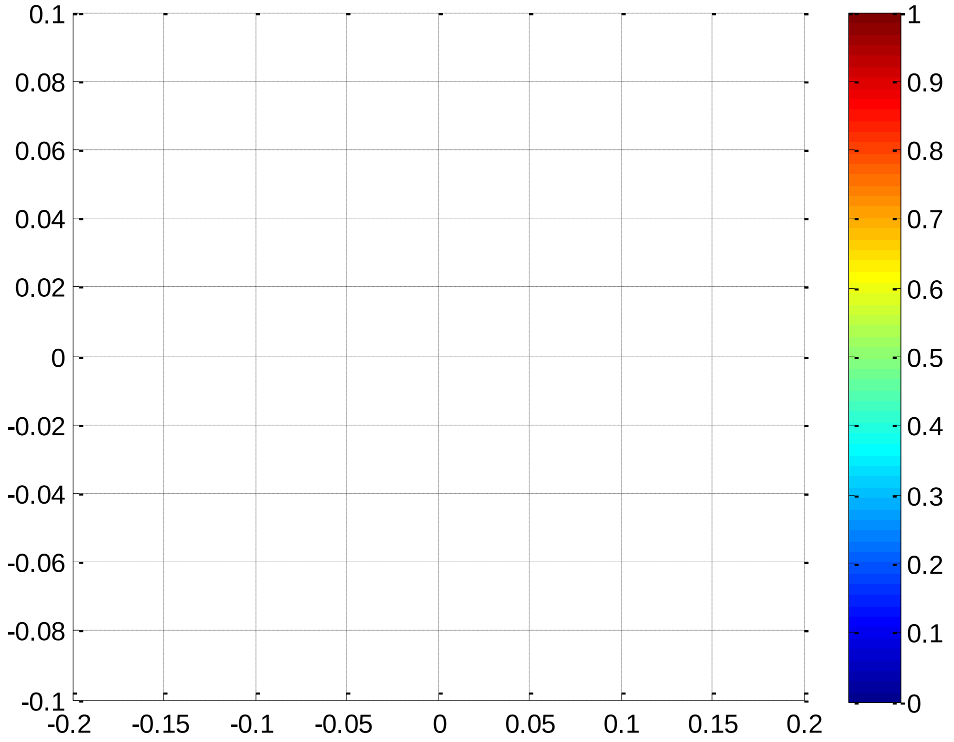
- Regularization
  - Jorgen Hald. Wideband Holography. *Inter-Noise, Melbourne, Australia,* November 2014.

Source strength vector q_k

Residual vector:
$r(\vec{q}) \equiv \left\|\vec{P}_m - A\vec{q}\right\|^2$

Calculate residual vector

Calculate the step Δq to minimize the residual vector



Wideband Holography (WBH)

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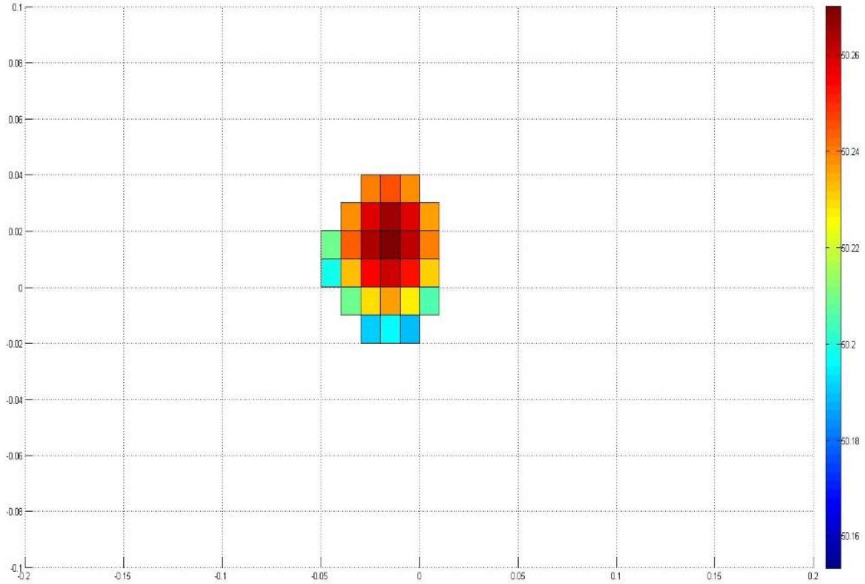

Wideband Holography (WBH)

➢ Regularization

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Residual vector:

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Calculate residual vector

Calculate the step Δq to minimize the residual vector

$\Delta q_k = s_k \vec{w}_k$, where $\vec{w}_k$ is the negative gradient vector, $\vec{w}_k \equiv A^H r(q_k) = A^H(p - Aq_k)$, s_k is the step length to minimum along that direction $s_k \equiv \frac{g_k^H r(q_k)}{g_k^H g_k}$, the vector g_k defined as $g_k \equiv A\vec{w}_k$

$q_{k+1} = q_k + \alpha \Delta q$

Get the next source strength candidate

Turn off the sources below threshold

$$q_{k+1,i} = \begin{cases} \tilde{q}_{k+1,i} & if\ \left|\tilde{q}_{k+1,i}\right| \geq T_k \\ 0 & otherwise \end{cases},$$

where $T_k \equiv 10^{\frac{-D_k}{20}} \left|\tilde{q}_{k+1,max}\right|$

Check the threshold condition

Wideband Holography (WBH)

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Residual vector: $r(\vec{q}) \equiv \left\| \vec{P}_m - A\vec{q} \right\|^2$

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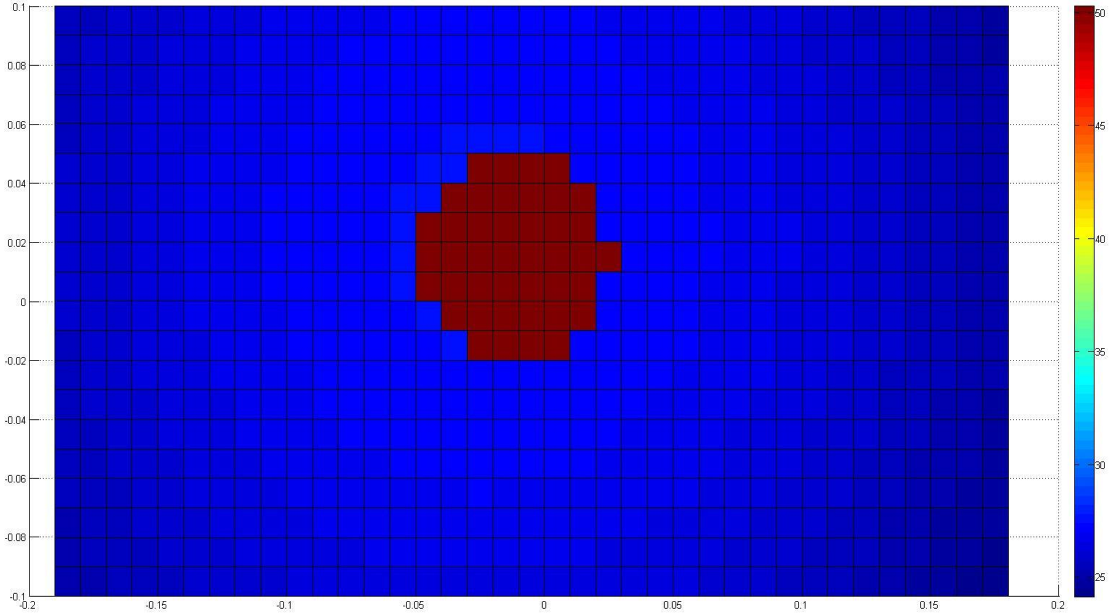

Wideband Holography (WBH)

- Regularization

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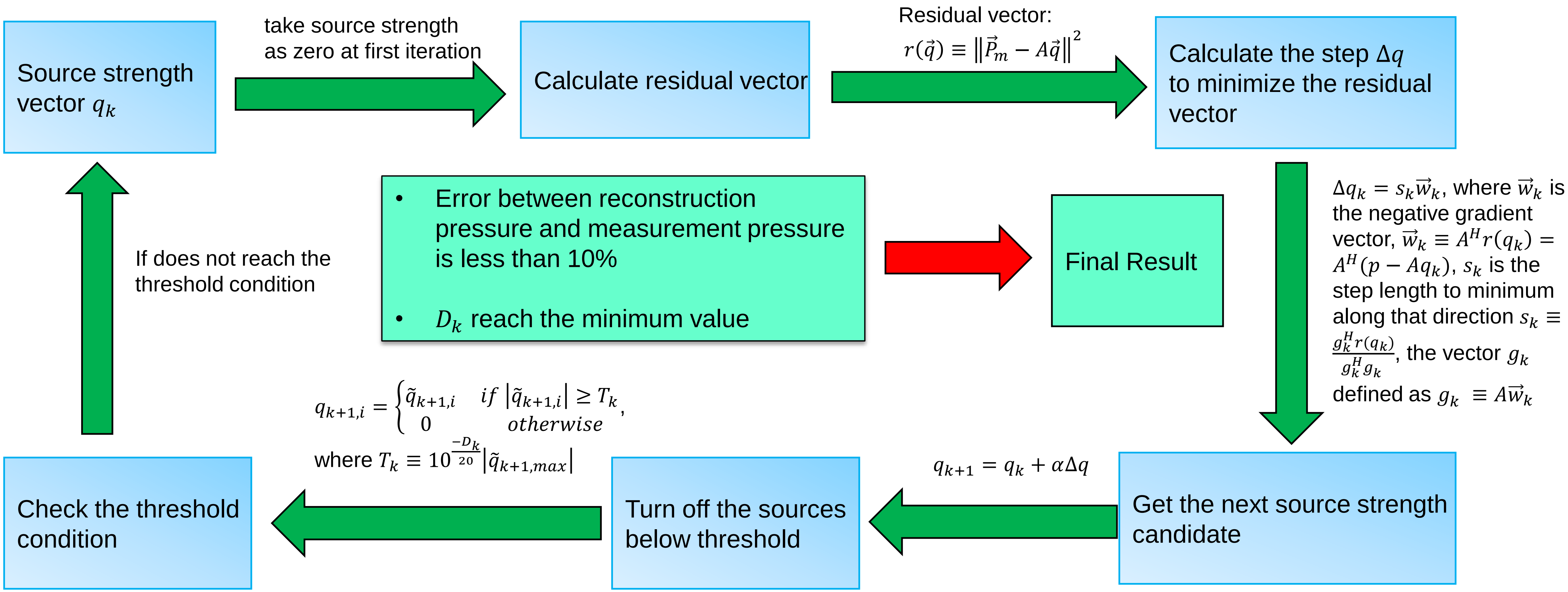
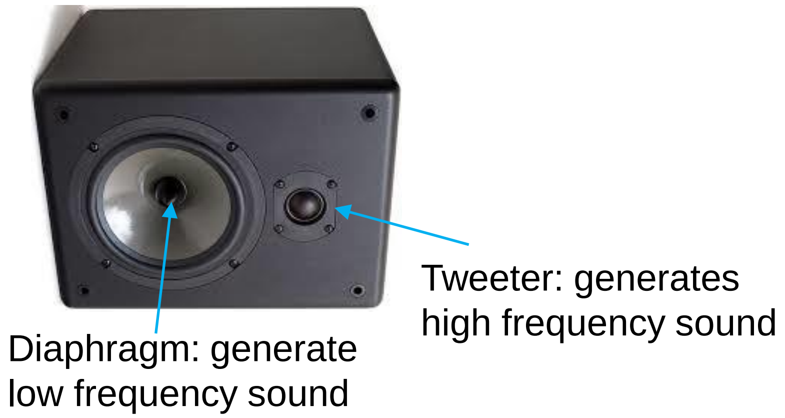
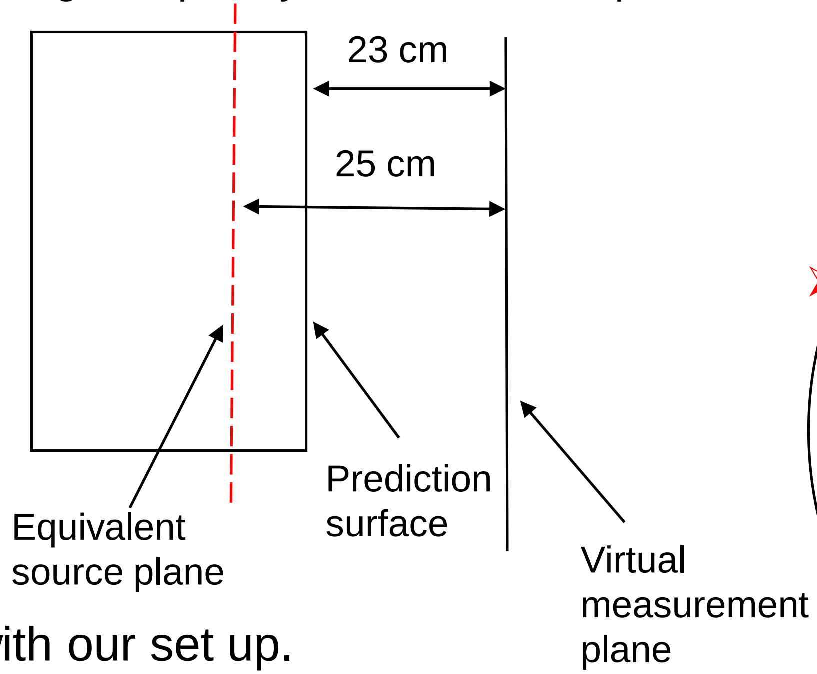
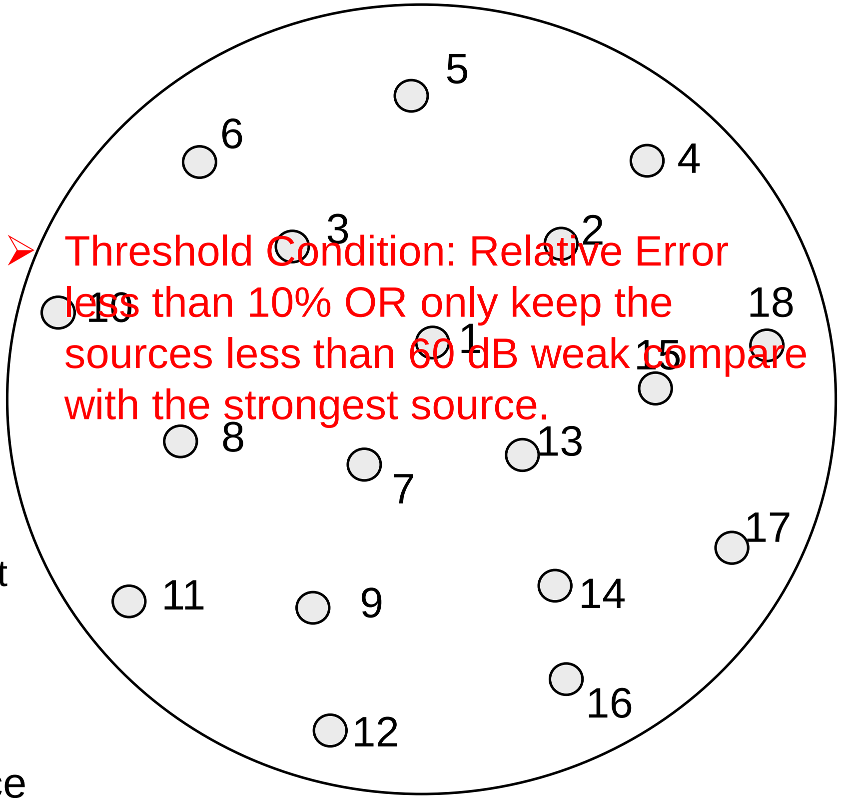


Simulation of Artificial Noise Source

- Simulate a loudspeaker on the reconstruction plane
  - Put a group of nine attached monopoles at different positions depending on frequency: at diaphragm position at low frequency; at tweeter position at high frequency. 18 virtual microphones to “measure” data.



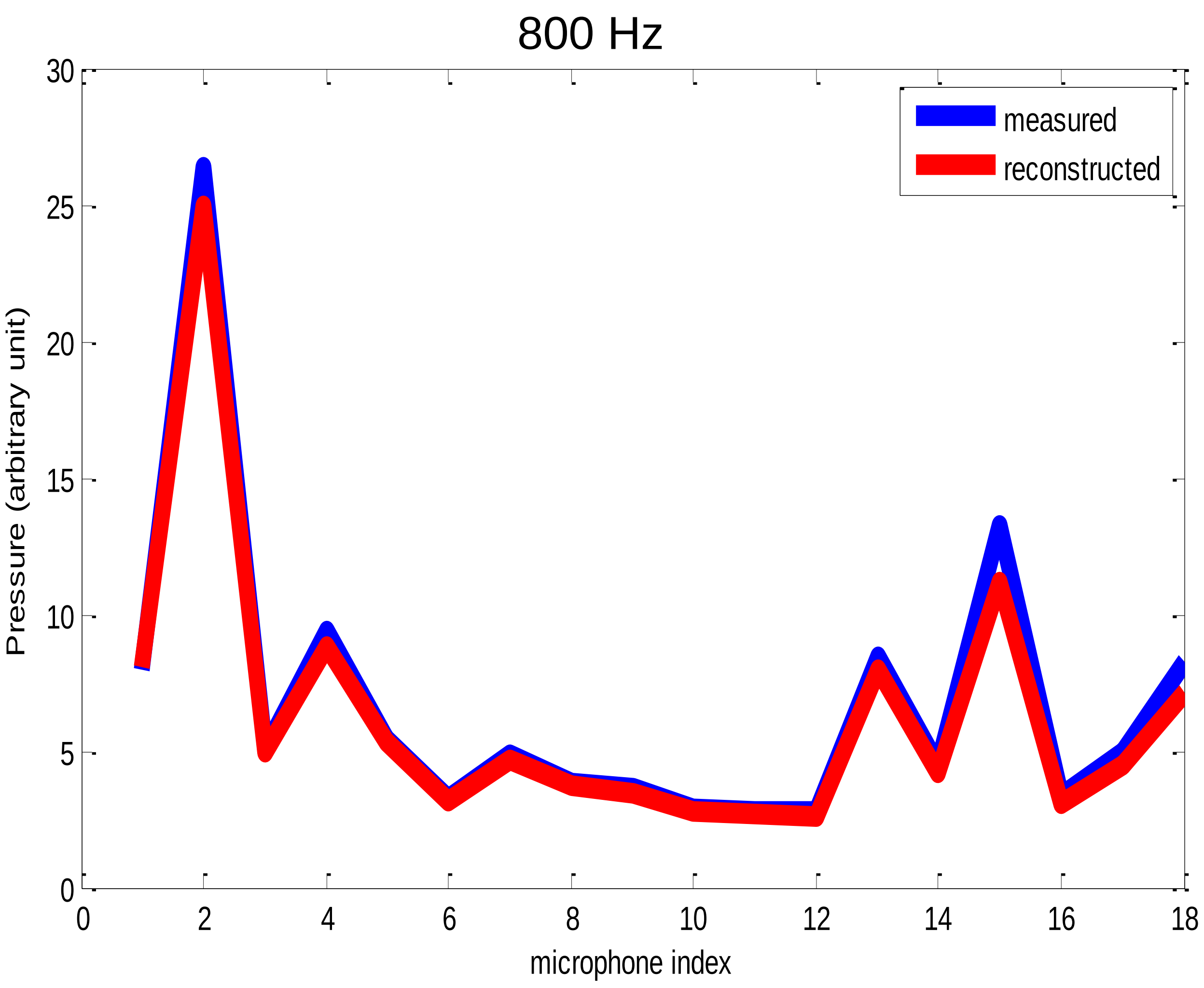


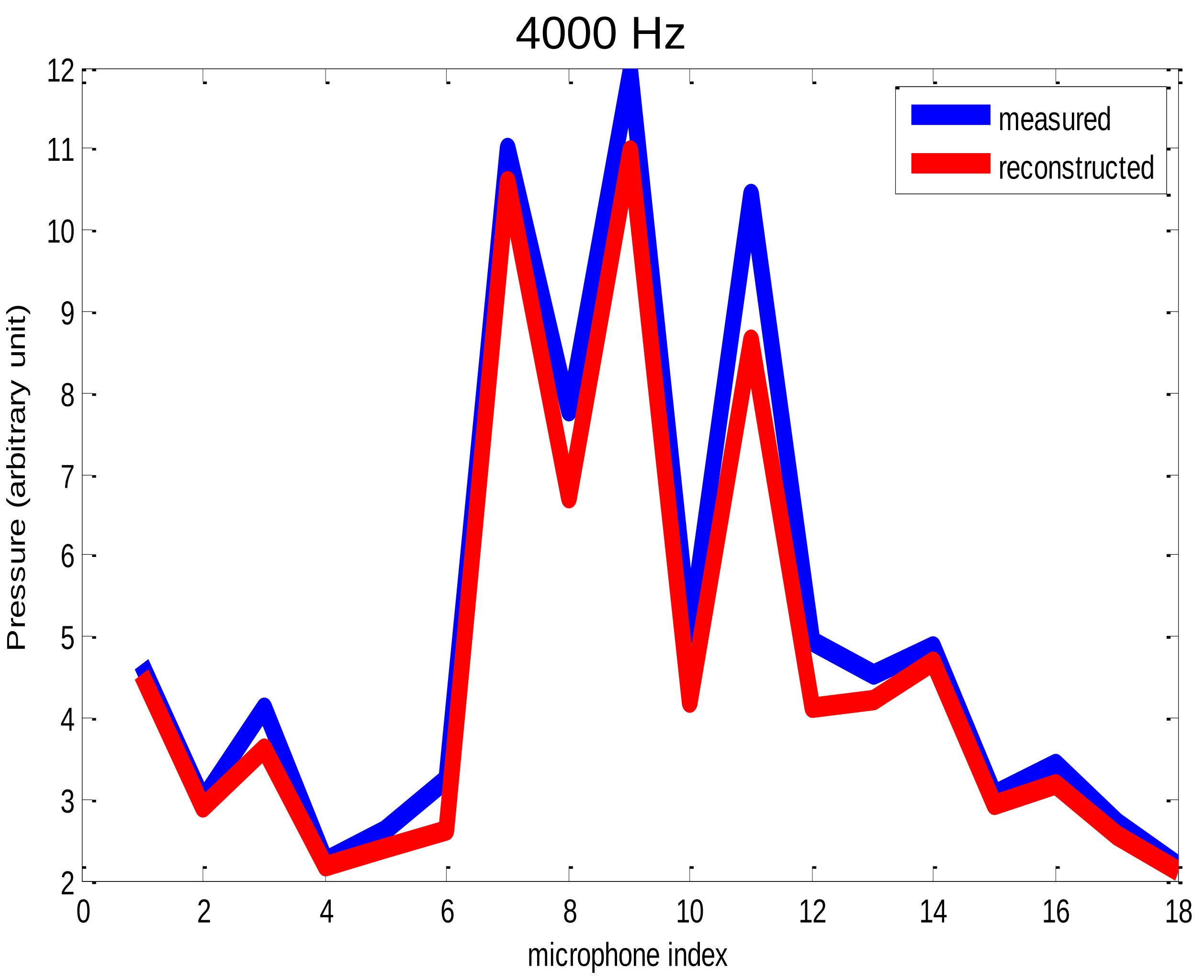


- Threshold Condition: Relative Error less than 10% OR only keep the sources less than 60 dB weak compare with the strongest source.

- Compare the reconstruction result with our set up.
  - Pressure at the virtual microphone position
  - Source strength, pressure and velocity distribution at the loudspeaker front face

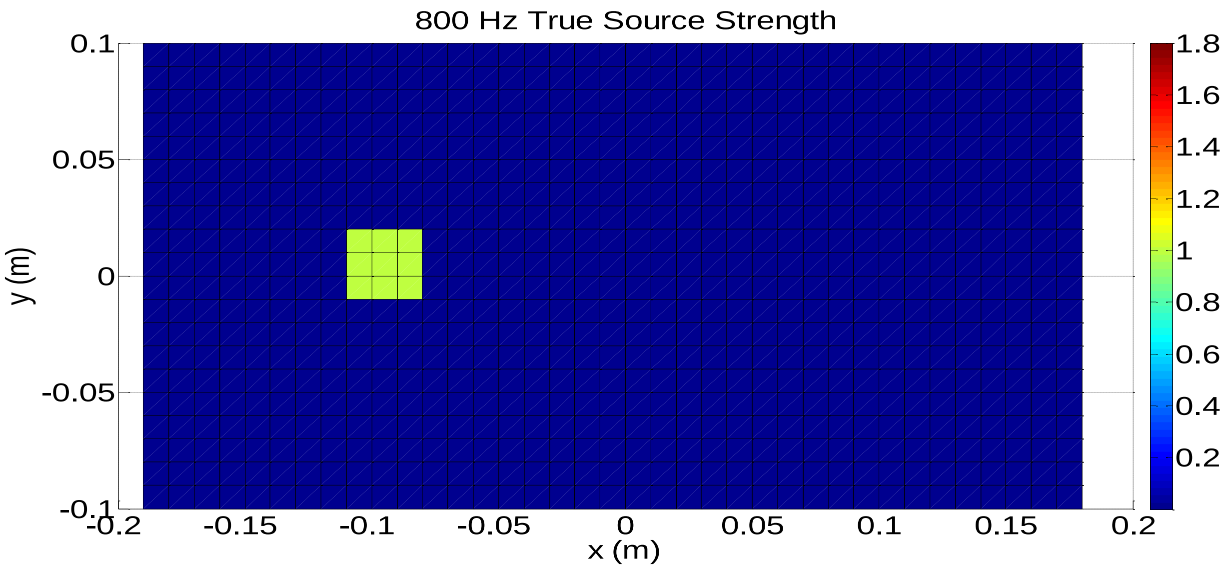
Comparison between measured and predicted pressures at virtual measurement positions

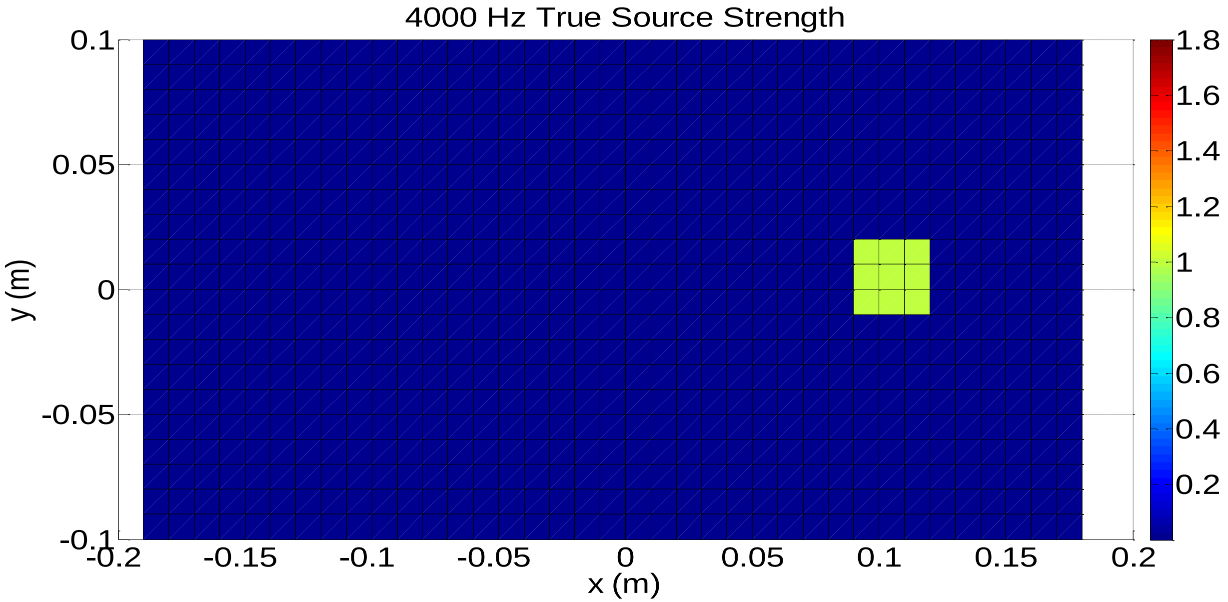




- Good agreement even at relatively high frequency

Artificial Sources Reconstruction Result

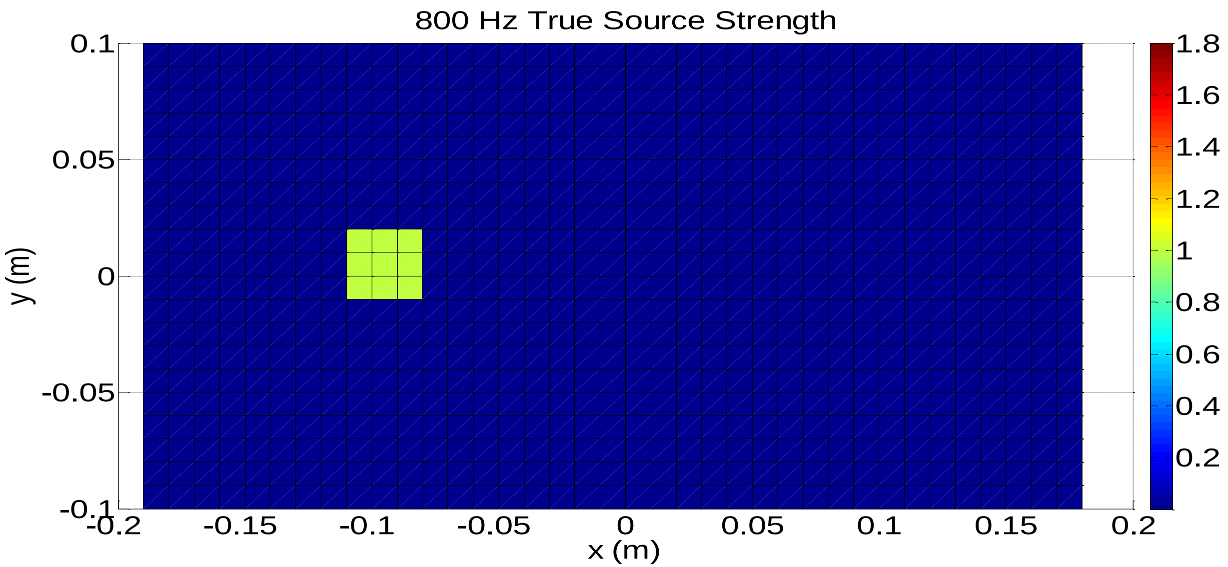


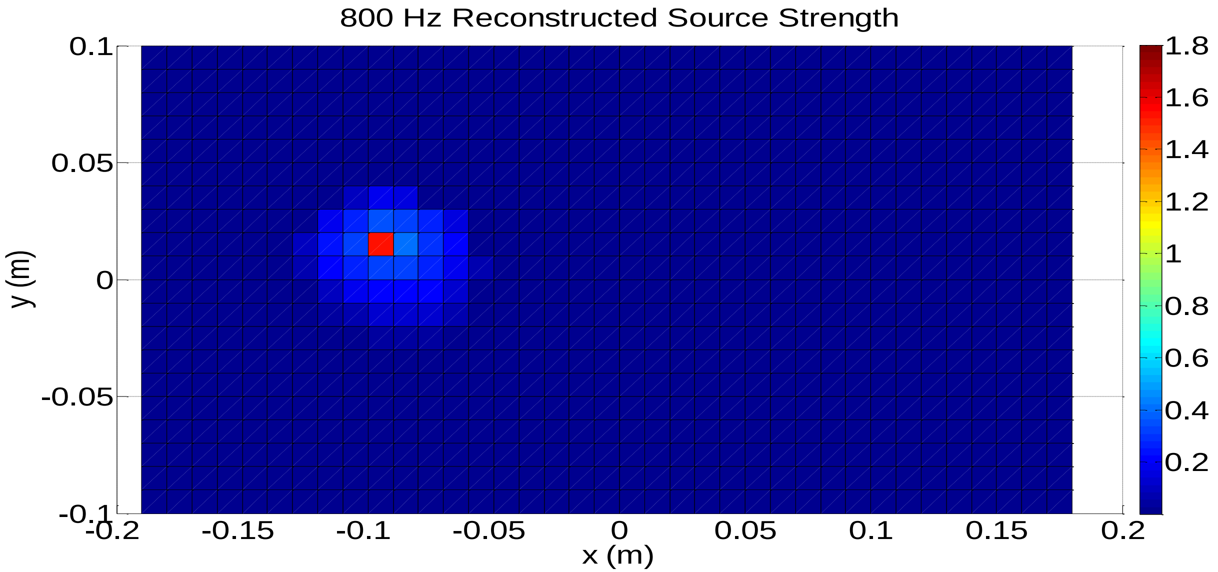


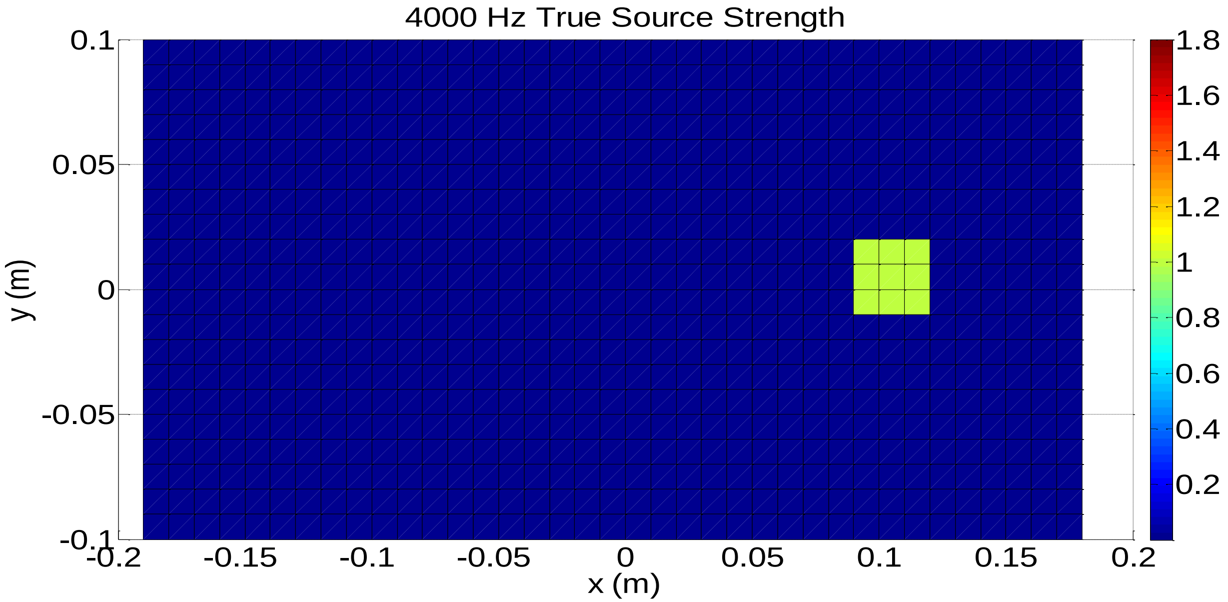
- Equivalent source plane set up

- -0.19 m – 0.18 m in x-direction
- -0.1 m – 0.1m in y-direction
- 0.01m spacing in both x and y directions, 798 monopoles in total

Artificial Sources Reconstruction Result



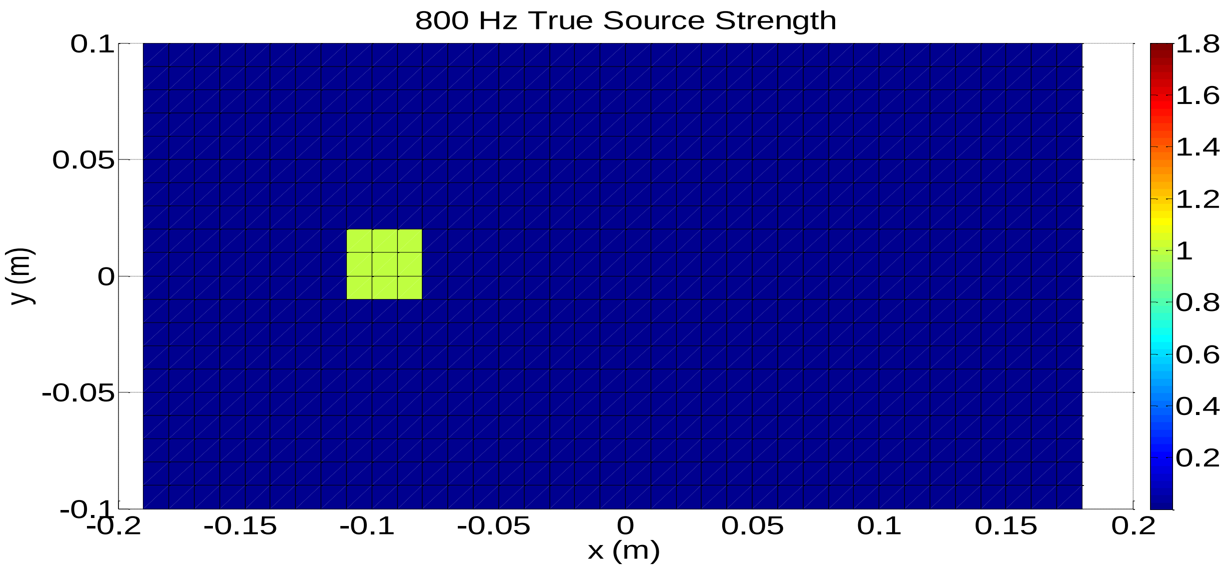




- Calculation Result

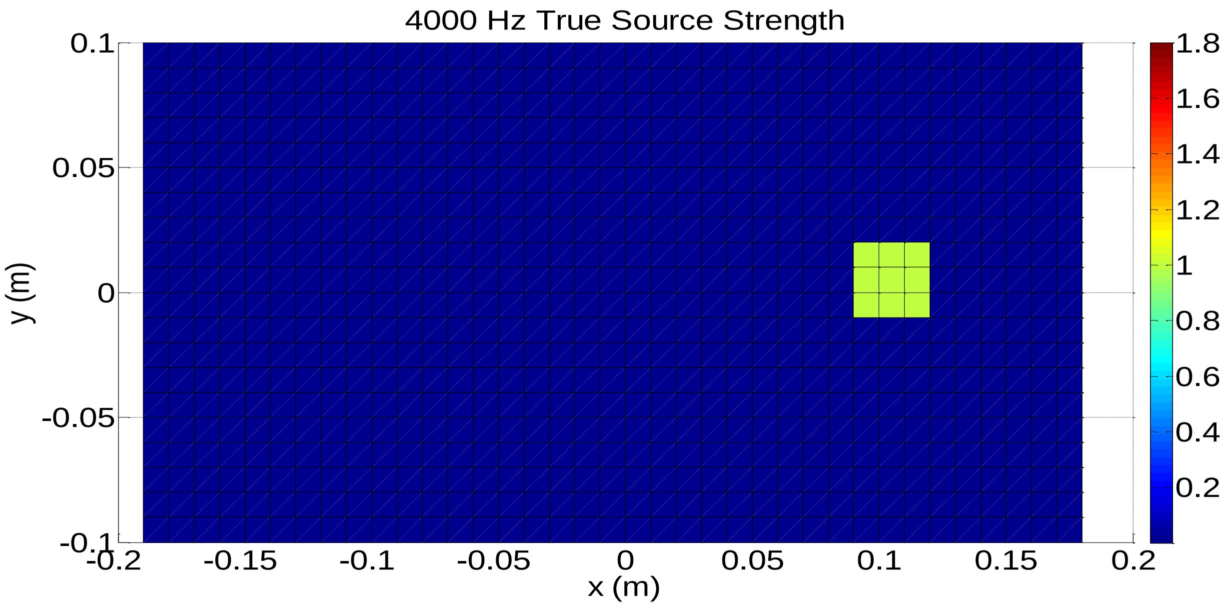
- At 800 Hz when D = 56.1dB, the relative error is less than 10%, which stops the iteration
- 48 reconstructed virtual sources, with total source strength 8.551
- Relative error 5%

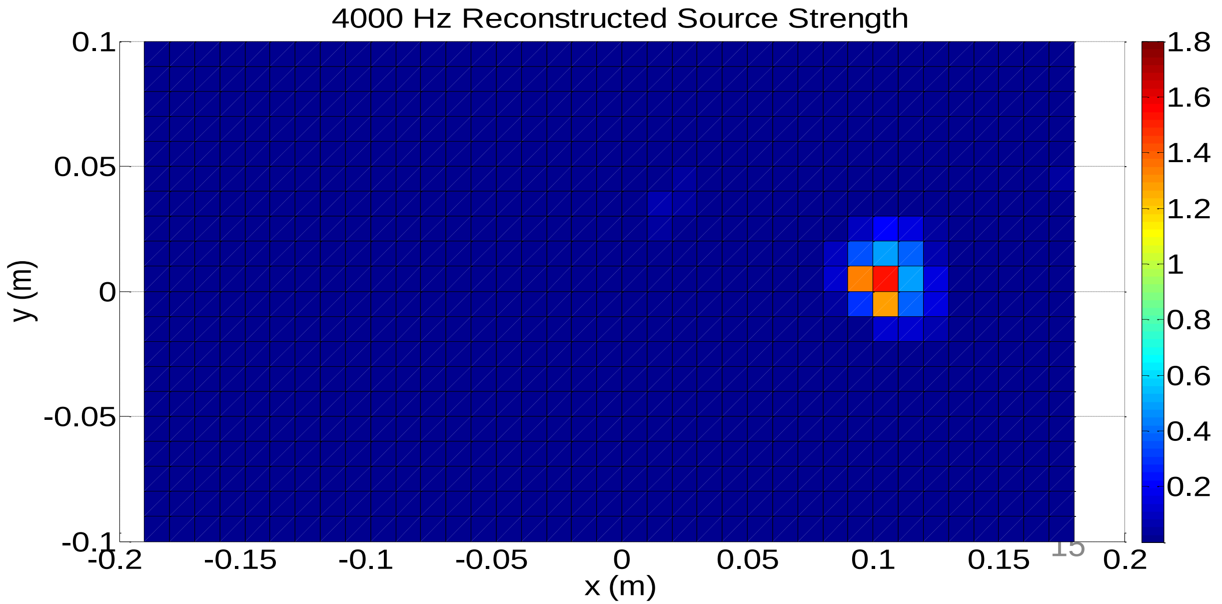
Artificial Sources Reconstruction Result



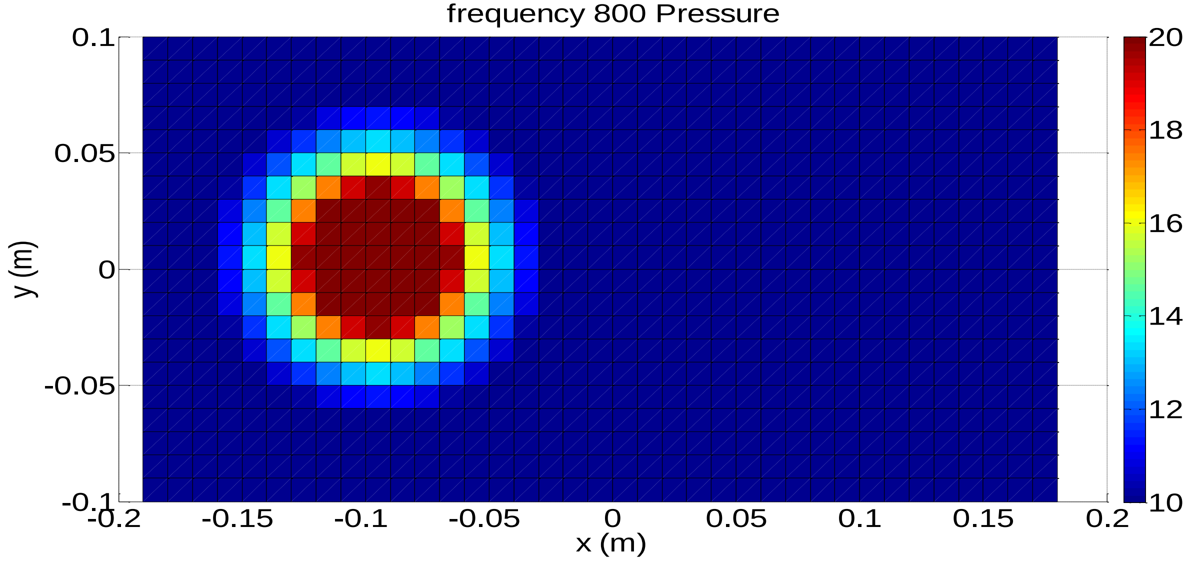
- Calculation Result

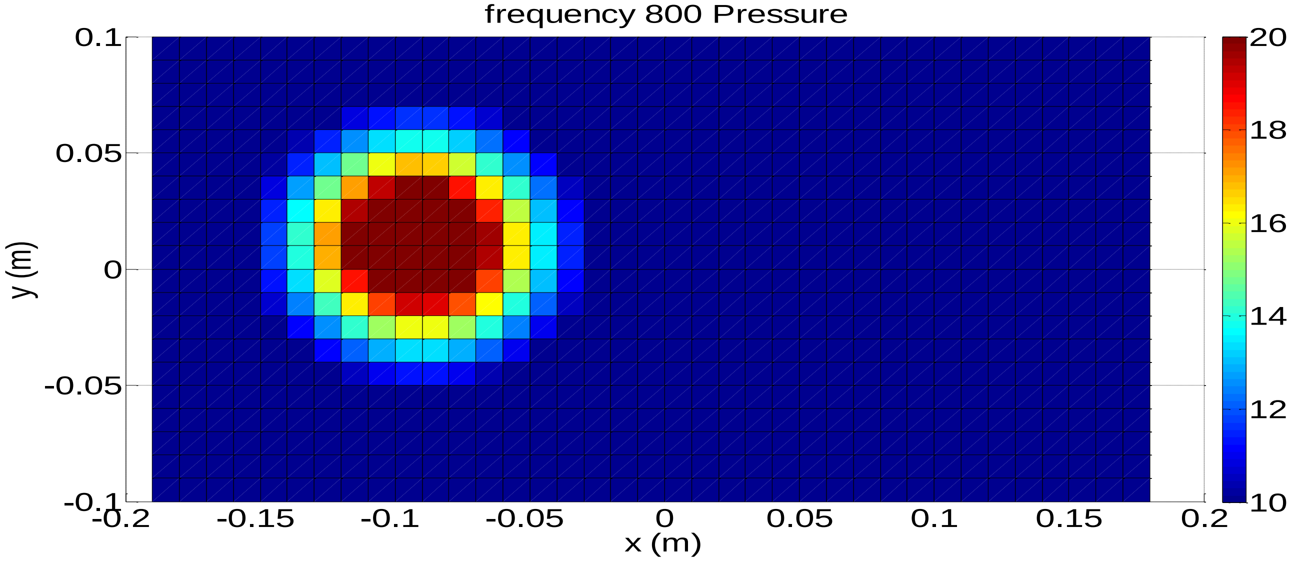
- At 4000 Hz when D = 60.1dB, the relative error is less than 10%, which stops the iteration
- 54 reconstructed virtual sources, with total source strength 8.522
- Relative error 5%

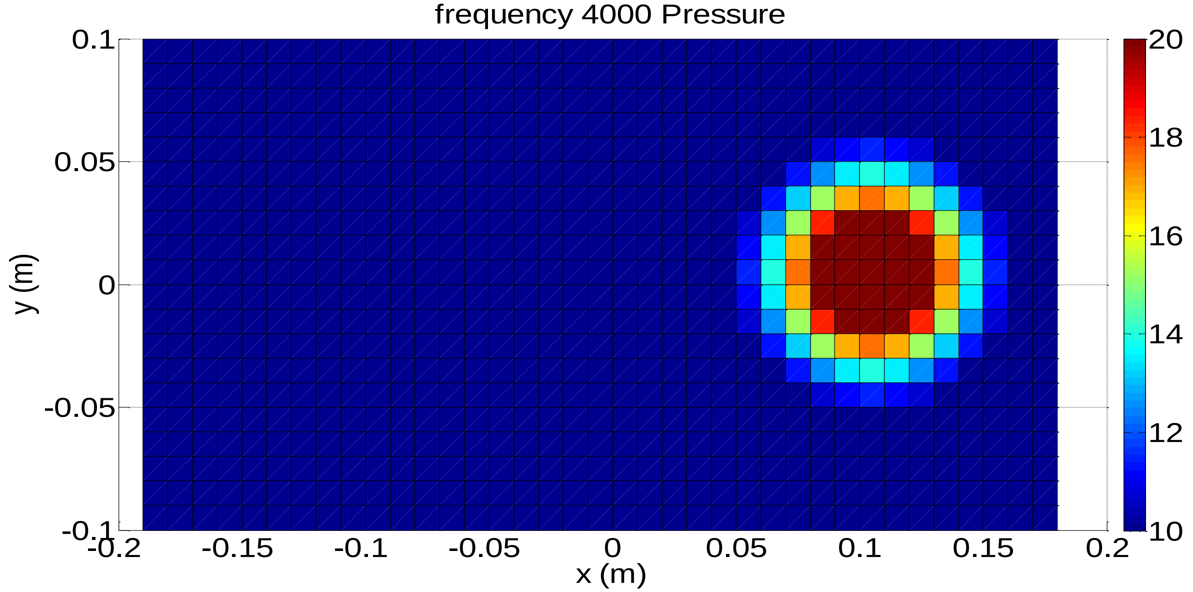


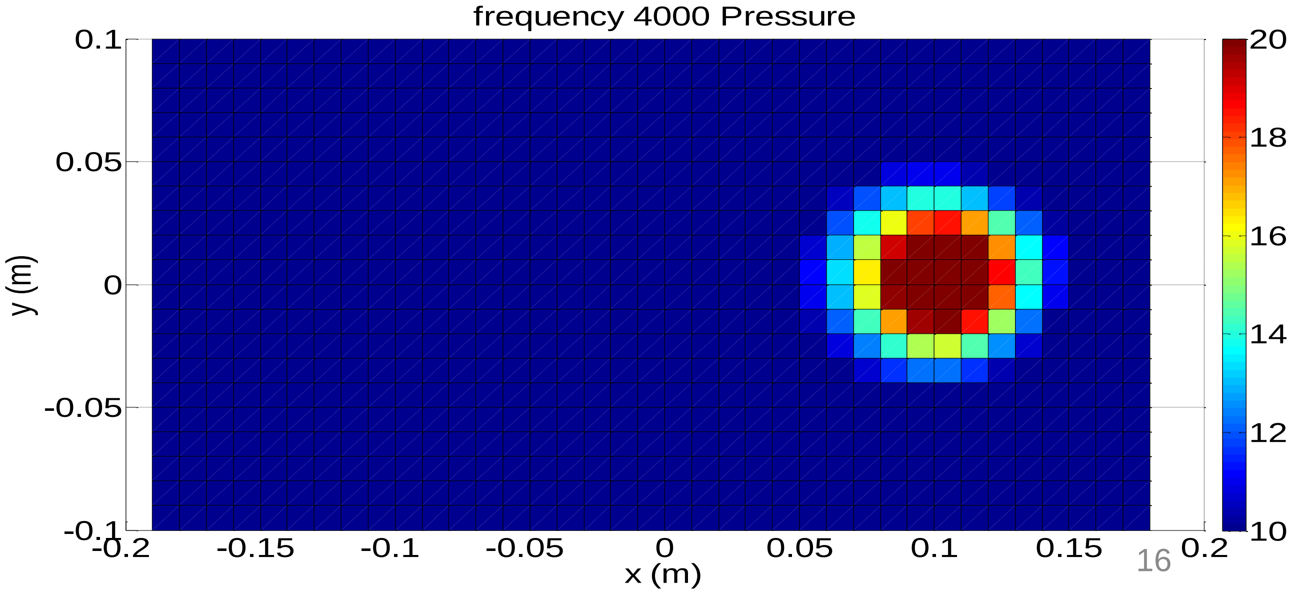


Pressure Prediction Result

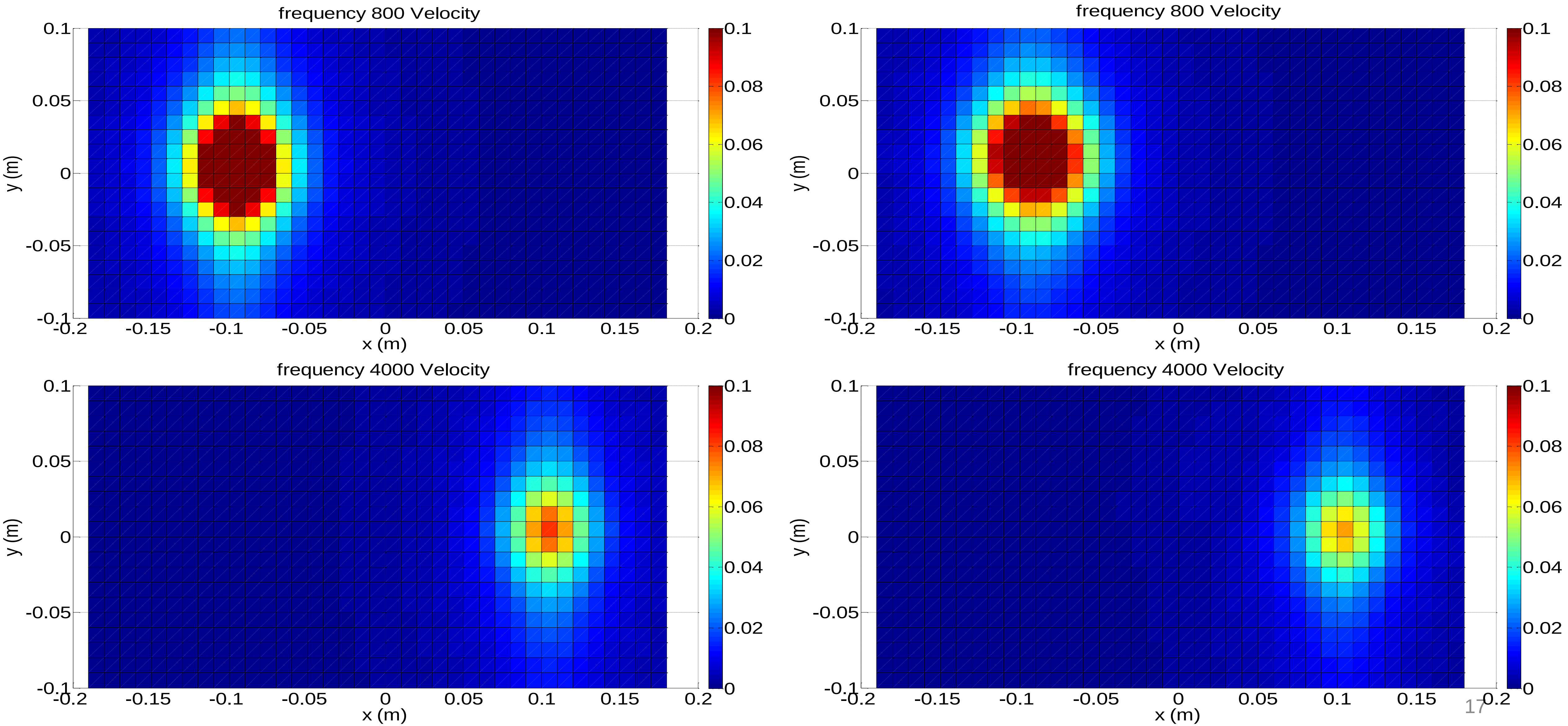








Normal Velocity Prediction Result



Experiments

- Experiment at Ray W. Herrick Labs, Purdue.
  - Test with loudspeaker (Infinity Primus P163) as a noise source
  - Brule and Kjaer 18 channel irregular array
  - Only one measurement of loudspeaker front face was conducted, 0.3 m from the loudspeaker front face, 0.35 m from the equivalent source plane.
  - Compare the holography result with real noise source location
- Partial field decomposition in equivalent source method
  - Concentrate on major noise source
  - Moohyung Lee and J. Stuart Bolton, “Scan-based near-field acoustical holography and partial field decomposition in the presence of noise and source level variation,” *J. Acoust. Soc. Am.*(2005)
- Threshold Condition: Relative Error less than 20% OR only keep the sources less than 50 dB weaker compared with the strongest source.

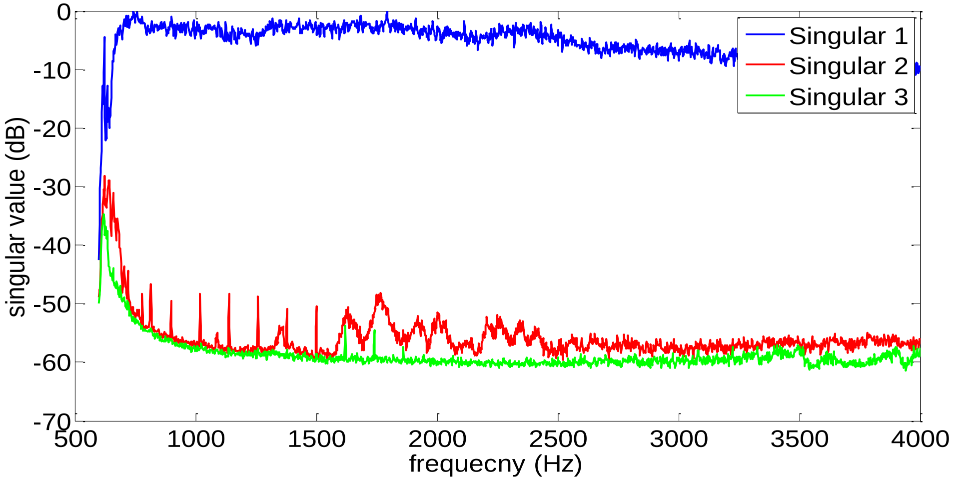
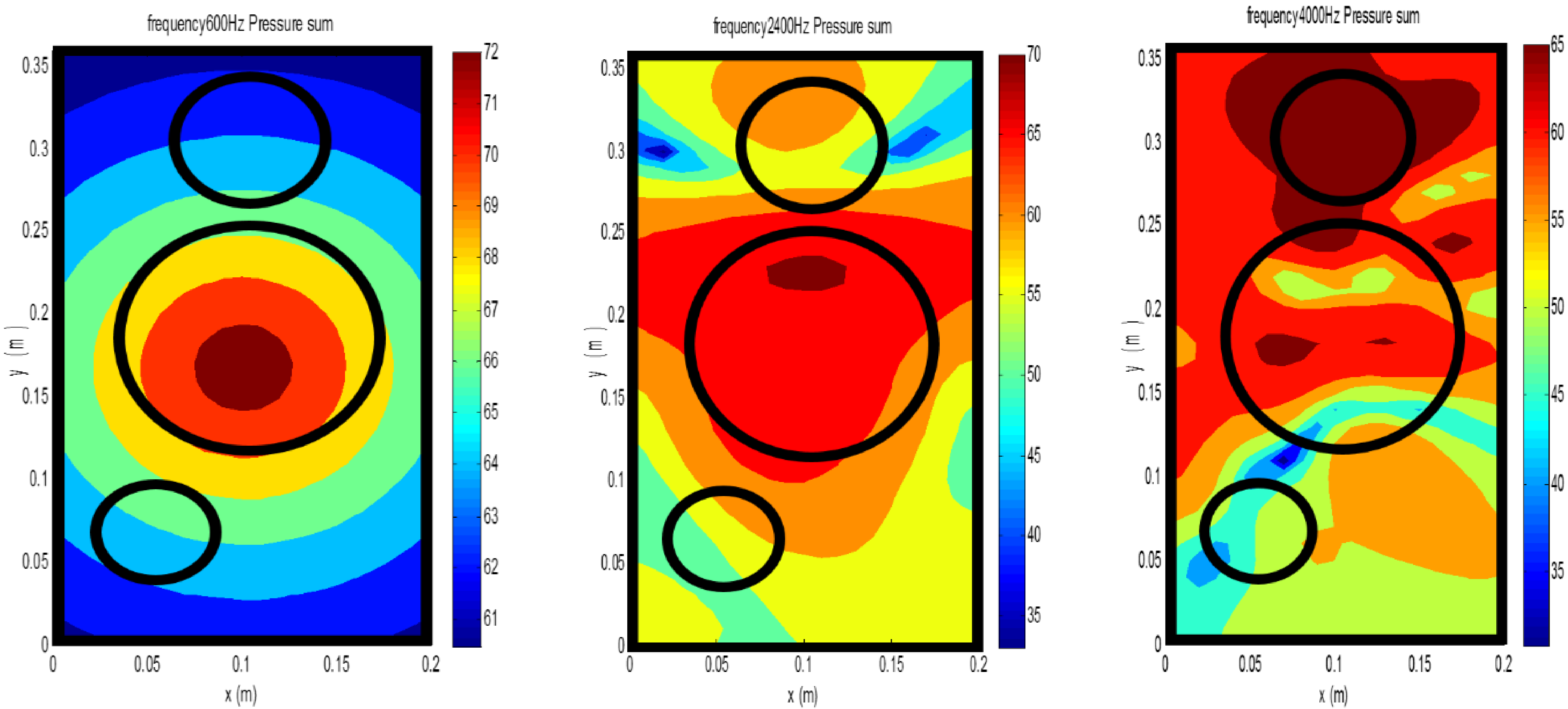
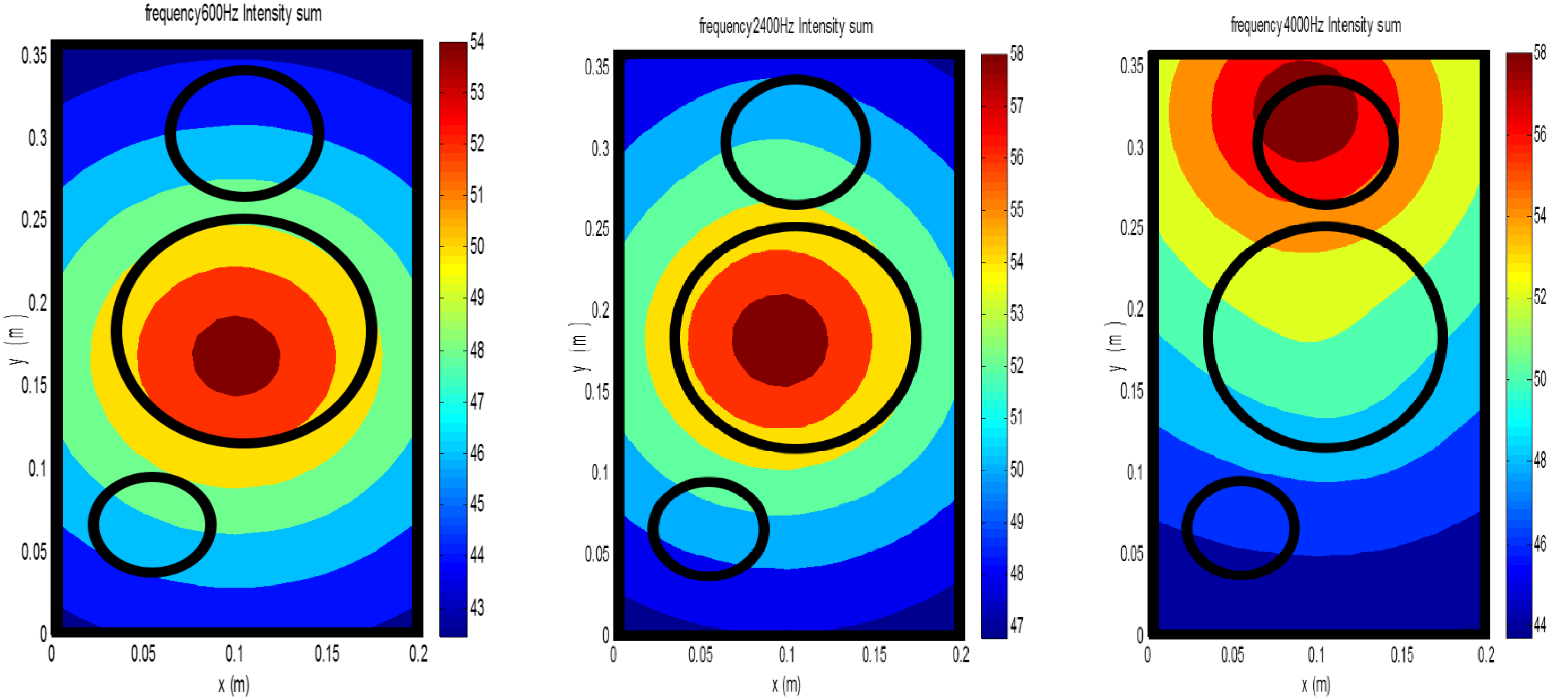


***Figure 5:** Singular value decomposition result.*

Pressure Prediction Result



Intensity Prediction Result



- Intensity gives very clear indication of sources locations
- Good results with remarkably small of microphones

Conclusions

- Based on Hald’s Wideband Holography (WBH) method, an equivalent source model composed of a monopole distribution at fixed location was created.
- The model was validated in both simulation experiment and measurement with a loudspeaker.
- Minimum source strength included in model. If the threshold value is too large will cause the ghost source problem, if the threshold value is too small the information of sound field will lost. This condition will decide that enough information of the true noise source will be calculated.
- The model accuracy is another threshold condition. If the model is accurate enough then no more sources need to be included in the model which can avoid the ghost source problem and save the calculation time.
- The measurement process to create the model is simple.
- Experiments show good results with relatively small number of microphones.

Acknowledgement

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