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Generating fuzzy rules by learning from olive tree transpiration measurement – An algorithm to automatize *Granier* sap flow data analysis



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ABSTRACT

The present study aims at developing an intelligent system of automating data analysis and prediction embedded in a fuzzy logic algorithm (FAUSY) to capture the relationship between environmental variables and sap flow measurements (Granier method). Environmental thermal gradients often interfere with Granier sap flow measurements since this method uses heat as a tracer, thus introducing a bias in transpiration flux calculation. The FAUSY algorithm is applied to solve measurement problems and provides an approximate and yet effective way of finding the relationship between the environmental variables and the natural temperature gradient (NTG), which is too complex or too ill-defined for precise mathematical analysis. In the process, FAUSY extracts the relationships from a set of input-output environmental observations, thus general directions for algorithm-based machine learning in fuzzy systems are outlined. Through an iterative procedure, the algorithm plays with the learning or forecasting via a simulated model. After a series of error control iterations, the outcome of the algorithm may become highly refined and be able to evolve into a more formal structure of rules, facilitating the automation of Granier sap flow data analysis. The system presented herein simulates the occurrence of NTG with reasonable accuracy, with an average residual error of 2.53% for sap flux rate, when compared to data processing performed in the usual way. For practical applications, this is an acceptable margin of error given that FAUSY could correct NTG errors up to an average of 76% of the normal manual correction process. In this sense, FAUSY provides a powerful and flexible way of establishing the relationships between the environment and NTG occurrences.

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1. Introduction

The need for improved irrigation management and better understanding of plant water relations led to the development of several methods for estimating water flow in plant stems. The whole plant transpiration can be estimated by sap flow measurement methods. The heat dissipation method is one of those methods that performs a direct measurement in a trunk section using simple physical principles. This method is often identified as the *Granier* method (Granier, 1985; Lu et al., 2004; González-Altozano et al., 2008).

The *Granier* method bases upon the heat flux provided by means of an adjustable source of electrical energy and constant power through a probe inserted in the trunk, which is correlated

to the heat removed by sap flow. For this purpose, two probes are used to determine the temperature difference (ΔT , [°C]) between the upper heating point and an unheated probe placed in a lower point. The ΔT value is introduced into the equation:

$$k = (\Delta T_{max} - \Delta T)/\Delta T \tag{1}$$

where ΔT_{max} is the maximum value of ΔT and k is the flux index. The flux is calculated from k with a relationship admitted to be species independent (Granier, 1985; Granier et al., 1990; Valancogne and Granier, 1997) based on empirical calibrations:

$$u = 118.99 \times 10^{-6} k^{1.231} \tag{2}$$

where u is the sap flux density $[m^3 m^{-2} s^{-1}]$. On a daily course, the maximum temperature difference ΔT_{max} between both probes in a sensor indicates the minimum or null sap flux rate for that specific day. The sap flux $F[m^3 s^{-1}]$ can be calculated as:

$$F = u \cdot A \tag{3}$$

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Nomenclature area of the conducting xylem section (m²) RE_f residual error of sap flux predicted (m³ s⁻¹) a_i , b_i and c_i parameters of a triangular membership function for RE_{na} residual error of sap flux not adjusted (m³ s⁻¹) relative humidity (%) the i_{th} FS RH solar radiation (W m⁻²) DOY day of year R_s event time variable. t fraction of day (1/48,12/48,...,48/48)) variable sap flux density (m^3 m^{-2} s^{-1}) ET_{o} reference evapotranspiration Time sap flux $(m^3 s^{-1})$ F и adjusted sap flux density $(m^3 m^{-2} s^{-1})$ adjusted sap flux (m³ s⁻¹) F_{adi} u_{ad} fraction of ground covered by vegetation UD universe of discourse f_c F_{na} not adjusted sap flux density $(m^3 m^{-2} s^{-1})$ not adjusted sap flux $(m^3 s^{-1})$ u_{na} predicted sap flux density $(m^3 m^{-2} s^{-1})$ predicted sap flux (m³ s⁻¹) F_{pred} u_{pred} fuzzy set the output relative to each event e FS_i Vd FV explicative fuzzy variables actual value of a variable x fraction of soil surface wetted by irrigation maximum normalized value of a variable f_w x_{max} triangular membership function minimum normalized value of a variable $f_{\Delta j}$ χ_{min} GN reference sensor χ_n normalized value of a variable not adjusted temperature difference at t time (°C) k flux index (Granier coefficient) $\Delta T_{na(t)}$ k_{adj} temperature difference (°C) adjusted flux index (Granier coefficient) ΔT k_{na} not adjusted flux index (Granier coefficient) $\Delta T_{(t)}$ temperature difference at t time (°C) k_{pred} predicted flux index (Granier coefficient) $\Delta T_{adi(t)}$ adjusted temperature difference at t time (°C) maximum temperature difference (°C) MSE mean square error ΔT_{max} ΔT_{max_adj} maximum adjusted temperature difference (°C) number of fuzzy regions ΔT_{max_na} maximum not adjusted temperature difference (°C) $NTG_{pred(t)}$ NTG predicted via algorithm at t time (°C) NTG natural temperature gradient (°C) $\Delta T_{pred(t)}$ predicted temperature difference at t time (°C) $NTG_{obs(t)}$ observed temperature difference of GN at t time (°C) **PFC** the percentage of FAUSY contribution (%) RF residual error

where A is the area $[m^2]$ of the conducting xylem section. This method has been successfully used in numerous studies, including in Portugal (Ferreira et al., 2004; Silva et al., 2008; Paço et al., 2009, 2012).

In theory, the Granier method is functional and measurements are easily automated. However, it requires heavy data processing and analysis for its systematic use while these tasks are difficult to automate (Do and Rocheteau, 2002a; Bush et al., 2010) since they require frequent user intervention. Beyond, it is assumed that the temperature measurements are not changed by the occurrence of natural temperature gradients (NTG, [°C]) in the trunk. Nonetheless, a NTG occurrence is easily verified by turning off the probe heating. The NTG are most likely originated by factors such as the azimuth of the probes inserted into the trunk, the distance between probes and the effect of radiation, soil and air temperature in the vicinity of the trunk. The variations of the ΔT circadian curve induced by the NTG contribute to errors in ΔT_{max} and consequently in sap flux calculation since the error is propagated to k. These errors are difficult to be detected and corrected in automatic calculus (Lu et al., 2004; Allen et al., 2011); thus, the process frequently needs a human decision, which constitutes an obstacle to automation. This is particularly pertinent for long temporal series.

Correction techniques for *Granier* sap flow measurements have been applied in order to eliminate or reduce inaccuracy introduced by NTG (Lundblad et al., 2001; Regalado and Ritter, 2007). Two general methodologies are referred in literature: the shutdown of the heating probe during specific periods (Lundblad et al., 2001; Do and Rocheteau, 2002a; Silva et al., 2008) and the use of unheated probes specifically for this purpose (Köstner et al., 1998). Cabibel and Do (1991) proposed a more elaborated correction method, which involves correlations between natural gradients for each probe and micrometeorological measurements. A temperature measurement close to the heated and unheated points, but

apart enough to avoid any interference with the heat source, can also be performed as a proxy to infer NTG (Cermák and Kucera, 1981). However, this is not exact information since the measurement points do not coincide. The correction methods mentioned above are usually adequate to specific applications, but do not provide a universal adjustment. The NTG is spatial and temporally variable, not only changing among different trees but also when different sensor installations are used in the same tree. The alternative of combining switched on and switched off sensors to remove NTG bias is limited because it requires that the NTG variability around the trunk is negligible, which is occasional in practice, and that the heating sensor does not affect the NTG occurrences, meaning that the method is restricted to large trees only (Lu et al., 2004). Do and Rocheteau (2002b) tested a new measurement approach employing cyclic heating just in particular situawhere significant natural temperature gradients (NTG > 0.2 °C) are expected and the residual error is possibly diluted. Beyond that, this solution delivers a lower time resolution, needs more expensive equipment to provide for cyclic heating and makes the system far more complex. Additionally, Lubczynski et al. (2012) concluded that the cyclic method in its current state of implementation is not yet optimized with respect to species and variability of environmental conditions, and that data processing is also not fully automated.

Recently, Vandegehuchte and Steppe (2013) reported the *Granier* method as being the most cited concerning NTG occurrence, contrarily to other methods where the influence of NTG on measurements has not been reported yet (e.g., heat field deformation method, Nadezhdina et al., 1998, 2012), although it is expected to be smaller, given the shorter distances to the heat source. Corrections have also been suggested for the tissue heat balance (THB) method (Cermák et al., 1973, Kucera et al., 1977) as reported by Köstner et al. (1998) but Tatarinov et al. (2005) have further

concluded that the small heat field generated by the *Granier* method turns it more sensitive than the THB technique to NTG, among other factors. Köstner et al. (1998) also report that heat pulse velocity methods are, in their turn, less prone to thermal imbalances than heat balance methods.

An improved solution to overcome the difficulties arising from the existence of NTG and a generalization of the process would be useful. Furthermore, it would be beneficial to find relationships between NTG and environmental variables to help increase the accuracy of NTG determination and its potential automation. Concerning the relationship between NTG and environmental conditions, Do and Rocheteau (2002a) observed that the magnitude and variability of NTG were likely exacerbated by the combination of open stand, high solar radiation and low sap flow rate. Gutiérrez et al. (1994) have also observed that large temperature differentials in the environment and conditions leading to a high radiation load on an exposed stem may adversely affect the accuracy of the *Granier* technique, particularly if the gauge is not appropriately insulated and shielded. Those authors minimized the rate of change in stem temperature by additional stem insulation.

Automatic and universal correction methods accounting for NTG require specific algorithms capable of (a) processing high quantity of data, (b) picking up relationships with environmental parameters, and (c) being smart enough to mimetize human decision capabilities, requiring appropriate treatment of data in order to obtain useful information and processing techniques. Such an algorithm could be a relational fuzzy algorithm (Delgado et al., 2009) developed by means of fuzzy rules, understood as an alternative method to treat models with approximate or inexact reasoning, generalizing and making data processing faster and accurate, then becoming an important tool for approximate analysis of systems and decision processes, that are much too complex for the application of conventional mathematical techniques (Zadeh, 1965, 1968, 1973, 2009). The fuzzy algorithm based on automatic fuzzy inference must stand computational simplicity and flexibility, aiming at being embedded within the measurement device to support the model parameterization (Sigueira and Rocio,

The concepts of fuzzy logic can be used to translate into mathematical language the approximate information contained in sentences expressed in natural language inserted into the system. This is done by simplifying, selecting or rejecting simulations and generating a restricted set of parameter solutions or linearizing a mathematical model. Fuzzy logic has been widely applied in agricultural studies (e.g., Passam et al., 2003; Mustafa, 2009; Huang et al., 2010). Also, Tzimopoulos et al. (2008) estimated potential evapotranspiration with a fuzzy logic approach; the model was driven by fuzzy reasoning with the system through learning from observed temperature and evapotranspiration, with results compared with those of the Blaney-Criddle method. Odhiambo and Yoder (2001) showed that fuzzy evapotranspiration (ET) models can achieve accurate estimation of daily reference ET, comparable to the FAO Penman-Monteith equation, with advantages over other methods. Keskin et al. (2004) developed fuzzy models to estimate daily pan evaporation based on meteorological data for Lake Eğirdir and compared with the Penman method. Casper et al. (2007) introduced a system based on a fuzzy rules approach and used soil moisture and precipitation as input variables to predict flow rates at the outlet of a watershed. Similarly, a rule-based fuzzy logic crop water stress index was developed using growth chamber data and was successfully tested on tall fescue grown in a greenhouse (Al-Faraj et al., 2001). De La Torre et al. (2005) used a computational tool Predictive Fuzzy Rules Generator (PreFuRGe) based on fuzzy logic and data mining to investigate the optimal strawberry production in relation to nitrate concentration, in soil water at different depths, and determined the optimal fertilizer dose that allows equilibrium between production and minimized environmental damage. Kim et al. (2008) developed a fuzzy logic control algorithm to compensate for illumination variation and to achieve a consistent image from a sensor used for in-field plant sensing. The referred studies show the scientific relevance of fuzzy logic when applied to agricultural and environmental issues.

This paper suggests a fuzzy algorithm to linearize the biased data automatically and in real time, thus allowing the correction of NTG in plant transpiration measurements. In addition, the algorithm is expected to smooth the input of resulting transpiration values into the SIMDualKc model (Rosa et al., 2012) used for water balance and ET calculations. The integration of time series and optimization techniques, using means of learning from the environment in present and past time, is expected to obtain an innovative automatic data processing for the *Granier* method applied to olive trees.

2. Materials and methods

2.1. Data acquisition

The present study has used a dataset acquired by sap flow sensors inserted in tree trunks of a commercial olive grove located in Alentejo, Southern Portugal (38°24'N, 7°43'W, 143 m a.s.l.), during 61 days in 2012 (day of year (DOY) 100-160). Climate is dry subhumid, of Mediterranean type, with a dry and hot summer, an average annual rainfall of 600-800 mm, most of it falling in autumn and winter, and average monthly temperatures varying from 9 °C in January to 24 °C in July. The production system in the site, an olive grove of the cultivar Arbequina explored by the commercial farm "Olivais do Sul", uses a "super high density" management technique, adopting a high density of tree planting (1.35 m \times 3.75 m). The olive grove was irrigated nearly daily during spring and summer, with a drip system having emitters with discharge of 2.3 L h⁻¹ and 0.75 m spacing along the row. This resulted in average irrigation daily amounts close to 3 mm d⁻¹ during the irrigation season. The fraction of soil surface wetted by irrigation was $f_w = 0.21$ and the fraction of ground covered by vegetation was f_c = 0.35. The topography is undulated. The experimental plot is part of a large olive grove of approximately 78 ha. An automatic weather station (Pessl Instruments, model iMETOS Avr, Austria), located at about 12 km distance to the north-northwest direction, was used to collect meteorological data. Meteoroalso used to compute reference data was evapotranspiration (ET_0) following Allen et al. (1998). From these data, air temperature $(T, [^{\circ}C])$ relative humidity $(RH, [^{\otimes}])$, solar radiation (R_s , [W m⁻²]) and reference evapotranspiration (ET_o) were preliminary selected as explicative variables in the relationship with NTG, since these variables are supposedly those that most influence the occurrence of natural thermal gradients. The

Table 1 Installation details of the sap flow sensors.

Sensor	Line	Tree	$\mathcal{O}^{a}\left(m\right)$	Trunk cross sectional area (m ²)	Azimuth
G1	16	34	0.075	0.004	NE
G2	16	36	0.054	0.002	W
G3	16	38	0.080	0.005	E
G5	16	43	0.072	0.004	NW
G6	16	44	0.074	0.004	NE
G7	13	38	0.071	0.004	E
G8	13	39	0.078	0.005	N
G10	13	42	0.079	0.005	S
GN^b	13	41	0.076	0.005	E

^a Trunk diameter at 30 cm height.

^b Heater switched off.

relation between R_s , T, RH and the NTG was studied through linear regression models. However, the model NTG = 0.32–0.60 R_s –0.37 RH (r^2 = 0.72) presented the most adequate relation to estimate NTG, meaning that 72% of events were explained by the model at the 5% significance level. The remaining models were not significant. The criteria for the selection of the explicative variables were based on the analysis above mentioned, thus the explicative variables solar radiation and relative humidity have been selected.

The values of ΔT were obtained by 9 sensors (G1, G2, G3, G5, G6, G7, G8, G10 and GN) installed in olive trees just below the bark, with a distance of 10 cm between heated and unheated gauges (Table 1) (1 cm probes from UP GmbH, Germany). The reference sensor (GN) has traced the temperature variability (NTG) into the trunk when the heater was switched off. Mean sap flow data was stored every 30 min in a datalogger (Model CR1000, Campbell Scientific, Inc., Logan, UT, USA).

Three ΔT approaches were used in function of the variable time (t, [s]):

$$\Delta T_{adj(t)} = \Delta T_{(t)} - NTG_{obs(t)} \tag{4}$$

$$\Delta T_{na(t)} = \Delta T_{(t)} \tag{5}$$

$$\Delta T_{pred(t)} = \Delta T_{(t)} - NTG_{pred(t)}$$
(6)

where $\Delta T_{(t)}$ is the temperature difference acquired by the sensor when switched ON at t time, $\Delta T_{adj(t)}$ [°C] is the ΔT adjusted by the natural temperature gradient observed at t time, which NTG_{obs(t)} [°C] is acquired by the reference sensor GN when switched OFF, $\Delta T_{na(t)}$ [°C] is the ΔT not adjusted, $\Delta T_{pred(t)}$ [°C] is the ΔT adjusted by the NTG predicted via algorithm at t time (NTG_{pred(t)}).

To accurately calculate the sap flux, the basic term ΔT_{max} must be determined separately for each sensor (Granier, 1987). Two estimates of ΔT_{max} , *i.e.*, ΔT_{max_adj} and ΔT_{max_na} were calculated for each sensor by searching the maximum temperature difference obtained with a sensor switched ON (heated) over the initial 10 days.

Accordingly, by Eqs. ((1)–(3)) and for each sensor, the values of k, sap flux density (u [m³ m⁻² s⁻¹]) and sap flux rate (F [m³ s⁻¹]) were obtained by range of temporal sequence (Fig. 1), namely k, u and F adjusted (k_{adj} , u_{ad} and F_{adj}), not adjusted (k_{na} , u_{na} and F_{na}) and predicted (k_{pred} , u_{pred} and F_{pred}).

The temporal sequence of data collected from sensors in the interval DOY (day of year) [100,160] was divided into learning time and forecasting time. The learning time operation was defined as the time used for capturing inferences by generated fuzzy rules.

Table 2 Maximum not adjusted temperature difference ($\Delta Tmax_na$ [°C]) and Maximum adjusted temperature difference ($\Delta Tmax_adj$ [°C]) obtained from measurements in the 10 initial days (DOY[100,110]).

Sensor	ΔT_{max_na}	ΔT_{max_adj}	$ \Delta T_{max_na} - \Delta T_{max_adj} $
G1	11.740	11.408	0.332
G2	14.880	14.551	0.329
G3	13.510	13.254	0.256
G5	13.260	12.984	0.276
G6	11.990	11.705	0.285
G7	11.660	11.314	0.346
G8	14.140	13.942	0.198
G10	10.790	10.397	0.393

Alternatively, the forecasting time operation was defined as the time used in estimating NTG occurrences when environmental variables and fuzzy rules were generated. The learning time and the forecasting time are illustrated in Fig. 1.

The $\Delta T_{max_na(t)}$ and ΔT_{max_adj} were extracted from the sensor's measurements in the 10 initial days (Table 2) and recorded for later computations. Lastly, FAUSY started the forecasting time process by estimating k through $\Delta T_{pred(t)}$ and ΔT_{max_adj} , for all sensors.

2.2. Development of the fuzzy algorithm

2.2.1. Knowledge base and data base

The Fuzzy Algorithm Automation System (FAUSY) is structured in two modules: (a) knowledge base and database, and (b) inferential machine (Fig. 2). The SWI-PROLOG computational environment was used for practical implementation of the FAUSY algorithm given its easy installation and use because it does not require an ample knowledge in programming and is freely available (Wielemaker et al., 2012).

The knowledge base was adequately represented by a fuzzy logic approach, which provides a computational framework for fuzzy inference in an environment of approximate information. It was used to define and qualify parameters that describe relationships in each event. The knowledge base was prepared aiming to supply information about the discretization and fuzzification processes.

The database is set as a structured collection of data. Data, *i.e.*, input/output events, are registered and recorded into a database by the algorithm. These records were treated as unsupervised learning. The term unsupervised learning is used as referring to automatically mapping of an input-output event's vector into

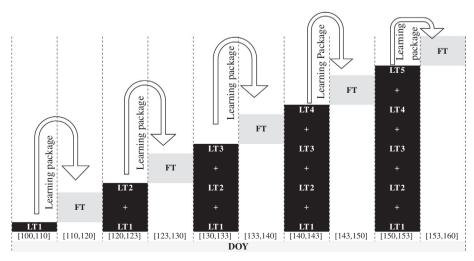


Fig. 1. Learning time (LT) and forecasting time (FT) temporal sequence of data collected from sensors in interval Day of Year (DOY) [100, 160].

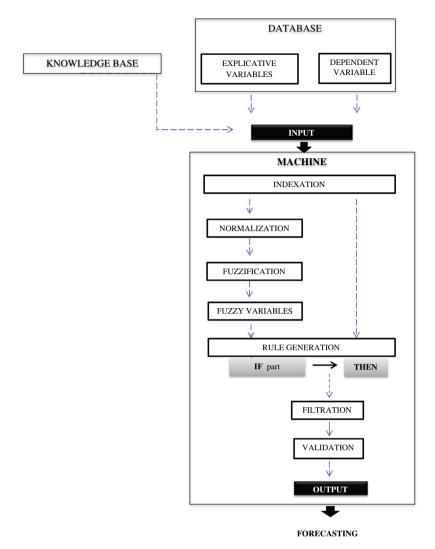


Fig. 2. Schematic flow chart of the FAUSY algorithm. For this study, the input, i.e. dependent variable, was NTG (natural temperature gradient) and explicative variables were solar radiation, relative humidity and reference evapotranspiration. The outputs were learned and predicted temperature difference values.

classes and building systematic learning, i.e., generating rules mechanically.

2.2.2. Inferential machine

The inferential machine comprises the procedural steps in the learning time and the generation of output, in the forecasting time. Five steps are operated in learning time.

2.2.2.1. Step 1: data processing. The database is composed by two categories of variables, explicative and dependent, and supplied by inputs per Day of Year (DOY). All input values – explicative variables (V) and dependent variables (Vd) are indexed. The input dataset indexed should guarantee correct matching between inputs in the time line and avoid computational effort in the fuzzification stage.

The dependent variable NTG has arisen from sensor reference measurements. The explicative variables solar radiation (R_s) and relative humidity (RH) are selected from meteorological observations. The explicative variable time is discrete within the interval [1,48] which means one unit each ½ hour, in one day. Otherwise, the continuous explicative variables should be normalized. The normalization of the database causes a fine-tuning in the fuzzy rules generation (El-Shafie et al., 2011) since assuming a relative magnitude for variables clarifies the comparison process. The

explicative variable Time (fraction of day: 1/48,12/48,...,48/48) was selected and associated to each NTG of the switched off reference sensor GN. This means that the input values of continuous variables are normalized into an appropriate scale defined by the maximum and minimum values by,

$$x_n = (x - x_{min})/(x_{max} - x_{min}) \tag{7}$$

where x_n is the normalized value, x is the actual value, and x_{min} and x_{max} are the minimum and maximum values of x in the actual series. In case information about x_{min} and x_{max} is lacking, they should be estimated from the empirical knowledge on the range of these variables. Thus, the R_S and RH attributes were normalized into a real number in the unit interval [0,1].

2.2.2.2. Step 2: Fuzzification. Automatic generation of fuzzy rules is based on the fuzzification procedure, in which the input values, supplied in a digital form, are converted to labelled values associated at degree of membership, namely, fuzzy variables or linguistic variables. These linguistic variables are combined to generate fuzzy rules in accordance with the objective of the system, that are casually delivered to other system devices.

The fuzzification procedure consists of the universe of discourse (UD) of continuous explicative variables being divided into fuzzy regions. The UD is divided into a number of overlapping (fuzzy)

regions and, to each region, a triangular membership function $(f_{\Delta j})$ is defined. The number of fuzzy regions (n_{fr}) , which are overlapped sets of variables, are quantified by $n_{fr} = 2N + 1$, where N is an adequate numerical choice that ensures an odd number for n_{fr} , with a central region. The accuracy of the algorithm can be regulated through an adequate choice of N, since larger values generally imply higher accuracy levels.

The simple triangular membership function $(f_{\Delta j})$ is easily computed as,

$$F_{\Delta j}(x_i, a_j, b_j, c_j) = FS_j(x_i) = \begin{cases} 0 & x_i < a_j \\ (x_i - a_j)/(b_j - a_j) & a_j \leqslant x_i \leqslant b_j \\ (c_j - x_i)/(c_j - b_j) & b_j \leqslant x_i \leqslant c_j \\ 0 & x_i > c_i \end{cases}$$
(8)

where a_j and c_j are defined by a lower limit of membership with $a_j < c_j$, b_j is defined by an upper limit of membership, and x_i is the input value to fuzzification. The parameters a_j , b_j and c_j of a triangular membership function for the jth fuzzy set (FS $_j$) can be calculated by:

$$b_1 = 0, b_2 = b_1 + 1/(n_{\textit{fr}} - 1), \ldots, bn_{\textit{fr}} = bn_{\textit{fr}-1} + 1/(n_{\textit{fr}} - 1) \eqno(9)$$

$$a_i = b_i - 1 \tag{10}$$

$$c_i = b_i + 1 \tag{11}$$

with the domain interval of FS_j [0; 1] divided into $(n_{fr} - 1)$ equal regions. The information on parameters a_j , b_j and c_j of the function (8) should be included in the knowledge base.

2.2.2.3. Step 3: Fuzzy rule generation. After the fuzzification procedure, the fuzzy rule generating algorithm can be executed. The absolute target is developing a fuzzy model that is capable of learning and afterwards forecasting the dependent variables of a given input set in real time by the fuzzy inferential mapping (Bisht et al., 2009) as

The fuzzy-rules generation algorithm is computed at the learning time and is based on multi-input/single-output events in the rule matrix.

$$\textit{rule matrix} = \begin{vmatrix} IF & \cdots & IF & THEN \\ \vdots & & \vdots & \vdots \\ [FV]_{l}^{e} & \cdots & [FV]_{l}^{e} & [Vd]_{e \times l+1}^{e} \end{vmatrix}$$
 (13)

In the rule matrix, each column IF represents the inputs of the *i* explicative fuzzy variables (FV) and the single column THEN is the output Vd relative to each event *e*. Each row characterizes a generated rule, which is recorded to be later filtered as referred below. Occasionally, it is possible that some situations do not occur in the learning time but arise only in the forecasting time. This may result from learning directly from environmental variables that behave as non-continuous functions. In this case, the algorithm FAUSY will do the learning considering directly the explicative variable Time to fill in the gap of the non-continuous function and not the function itself.

2.2.2.4. Step 4: Filtering. Conflicting situations can arise when rules with the same antecedents, i.e. the same IF parts, but different consequents (the THEN parts) are generated. In case of a conflict situation, Wang and Mendel (1992) suggested that a degree of membership should be assigned to each rule and that the conflicting rules with the highest degree should be retained and the other ones rejected. However, because all rules generated are information, rejects are not possible. Vojtáš (2001) referred that it is

important to have sound and complete semantics because it empowers to compare representative results of computation with real world data. It should be assured that all occurrences must be computed to describe the *status quo*. Thus, if there are conflicting rules, the model should look for the best solution based in the fuzzy logic theory. In FAUSY the conflicting rules are overcome through calculation of the average of the consequent part. For example, considering two rules:

$$FV_1 \dots FV_n \to Vd_x$$
 (14)

$$FV_1 \dots FV_n \to Vd_y \tag{15}$$

where the aforementioned consequent part is $Vd_x \neq Vd_y$, the average of the rule pair is calculated as

$$Vd_{x,y} = (Vd_x + Vd_y)/2 \tag{16}$$

2.2.2.5. Step 5: Performance assessment. The FAUSY algorithm should identify the explicative variables, selected for fuzzy rules set generation, better related to NTG. The performance assessment is done by analysing the residual error (RE) between the predicted and observed values of the dependent variable and through the mean square error (MSE). RE refers to the discrepancy between the observed and predicted outputs (i.e., $NTG_{obs(t)}$ and $NTG_{pred(t)}$):

$$RE = |Y_i - Y_i| \tag{17}$$

where Y_i is $NTG_{obs(t)}$ and \widehat{Y}_i is $NTG_{pred(t)}$ at the *i*th event. The mean squared error (MSE) is evaluated using the equation:

$$MSE = 1/n \sum_{i=1}^{n} (\widehat{Y}_i - Y_i)^2$$
 (18)

where n is the total number of events in the specific time interval. The selection of the best explicative variables set will be defined by the minimum MSE.

If a large residual error occurs, then the first set of generated rules in the learning time is not sufficient to explain the reasoning of the system, or a weak relation was established between the explicative variables and the NTG observed. Therefore, the learning time should be larger, or the explicative variables should be replaced by new generated fuzzy rules. The decision on how to proceed should be taken by the expert; thus the expert's information should be added into the knowledge base. In addition, if the magnitude of the residual error is very small, then it is adequate to initiate the forecasting time implementation, which works following the last event in *t* learning time when learned with the explicative variables.

2.3. Methodology for performance evaluation of FAUSY

The performance evaluation of FAUSY algorithm is achieved using the following residual error equations

$$RE_f = |F_{pred} - F_{adj}| \tag{19}$$

$$RE_{na} = |F_{na} - F_{adj}| \tag{20}$$

$$PFC = (RE_f - RE_{na})100/RE_{na} \tag{21}$$

where RE_f and RE_{na} are the residual errors obtained when comparing the results of the sap flux rate predicted respectively by the FAUSY algorithm (F_{pred}) and the sap flux rate not adjusted (F_{na}) with the sap flux rate manually adjusted F_{adj} . PFC is the percentage of FAUSY contribution to the improvement of sap flux rate estimation without requiring manual computations.

The performance evaluation through the RE_f, RE_n and PFC calculations infer if the FAUSY algorithm contributes to the ΔT adjust-

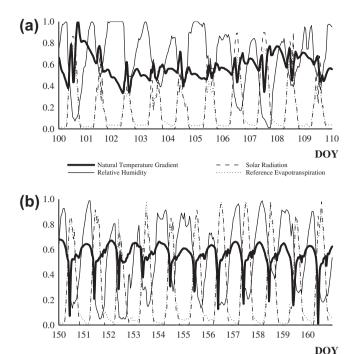


Fig. 3. Visual analysis of daily behaviour of natural temperature gradients and the accessible explicative variables (normalized data) in equal time line, namely, reference evapotranspiration (ET_o), solar radiation (R_s) and relative humidity (RH) over (a) [100,110] and (b) [150,160].

Reference Evapotranspiration

Natural Temperature Gradient

ment and if it can supplant the non-automatic data processing when the sensor is switched on. The non-automatic data processing is less advantageous since it is a procedure in which the k value is computed after the complete data sets (relative to each sensor and for the periods between data collection events) have been obtained.

3. Results and discussion

3.1. Setting parameters

The search space of the learning parameters is potentially very large, as a countless number of associations can be performed

between environmental parameters and different learning time steps. Thus it was essential to set parameters in accordance with the knowledge base and observations. In view of this, it was observed (Fig. 3A and B) that potential relationships, could be established over the DOY intervals [100, 110] and [150, 160] between the natural temperature gradients (NTG) and the explicative variables available, namely reference evapotranspiration (ET_0) , solar radiation (R_s) and relative humidity (RH). These time intervals were selected from several intervals established for learning time to show the unpredictability of the process. However, the relationship was better between the NTG and the variables ETo, Rs, RH at time interval [150,160], than at time interval [100,110]. This corroborates the need of a phased learning time procedure (Fig. 1), aiming to attenuate the unpredictability over time. Following this first assessment, the fuzzy rules were generated considering the relationship between the variables ET_o , R_s and RH and NTG, for the time interval [100,110], being the next step the search for the best related variable.

The deviations of environmental variables, namely R_s and RH, when related to NTG (Fig. 3) showed a minor residual error when compared with the ET_o variable (Fig. 4). The reason arises probably from the fact that although ET_o integrates the same meteorological information, it also integrates biophysical plant parameters, however relative to grass, the reference crop considered, different from the parameters of the crop under study. The NTG development, in its turn, is influenced by the effect of tree shadowing and row orientation of the crop, with shadow changing over the day, thus dynamically modifying the NTG value near the probes. This supports the observed fact that NTG forecasting related to ET_o is undefined and that the fuzzy rules associated with the explicative variables (R_s plus RH), by themselves, show the best fitting.

In time interval [100,110], FAUSY searched for relationships between the available explicative variables and the dependent variable NTG. After comparing the residual error values, it was verified that the rules generated with R_s and RH led to smaller absolute average residual errors than the rules generated with ET_o (Fig. 4), respectively 0.0551 and 0.0996. Therefore, R_s and RH have been adopted to predict the NTG values.

In the next step, three categories of t time variable combinations, *i.e.*, combinations between explicative variables R_s and RH were set: (t-4), (t-2) and (t) steps backward over time (SBT). SBT means the collected data in different time, *i.e.*, the algorithm searches for data four, two or zero steps backward from t time. For example, if the algorithm picks the values for R_s and RH, for time t and time t-4, this variable combination is described as $R_{s(t-4)} \times R_{s(t)} \times RH_{(t-4)} \times RH_{(t)}$. During [100,110], an increasing

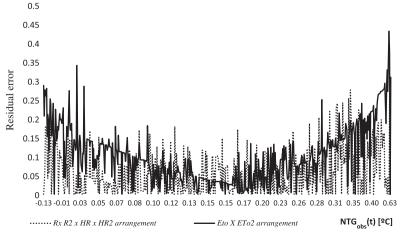


Fig. 4. Comparing the residual error obtained for the $R_{s(t-2)} \times R_{s(t)} \times RH_{(t-2)}$ $RH_{(t)}$ and the $ET_{o(t-2)} \times ET_{o(t)}$ variable combinations.

Table 3Mean squared error of the four variable combinations during the period DOY 100, 110 (learning time).

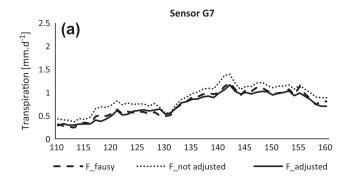
Variable combinations	MSE
$R_{s(t-4)} \times R_{s(t)} \times RH_{(t-4)} \times RH_{(t)}$	0.0021
$R_{s(t-2)} \times R_{s(t)} \times RH_{(t-2)} \times RH_{(t)}$	0.0030
$R_{s(t)} \times RH_{(t)}$	0.0060
$ET_{o(t-2)} \times ET_{o(t)}$	0.0099

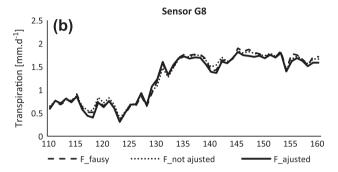
MSE was observed for the variable combinations: $R_{s(t-4)} \times R_{s(t)} \times RH_{(t-4)} \times RH_{(t)}$; $R_{s(t-2)} \times R_{s(t)} \times RH_{(t-2)} \times RH_{(t)}$; $R_{s(t)} \times RH_{(t)}$ and $ET_{o(t-2)} \times ET_{o(t)}$ (Table 3). The variable combination $R_{s(t-4)} \times R_{s(t)} \times RH_{(t-4)} \times RH_{(t)}$ produced the lowest MSE (about 0.0021) for [100,110]; hence this variable combination was chosen for learning time, as soon as the sensor was switched off (without heating).

3.2. Comparison between manual and fuzzy logic data treatments

FAUSY results were compared with those obtained with manually processed data. In the time interval [110,160], the performance of FAUSY algorithm was analysed by residual error Eqs. ((19)–(21)). The average values of residual errors, for FAUSY treatment or without any correction for NTG (REf, REna, respectively) are shown in Table 4, as well as the improvement obtained with FAUSY (PFC). The residual error observed in all sensors showed lower residual error (RE_f), when compared to the not adjusted procedure (RE_{na}). The FAUSY algorithm simulated the sap flux rate of sensors with a mean residual error of 0.028 mm d^{-1} , lower than the mean residual error of sap flux rate obtained without NTG corrections (using $\Delta T_{na(t)}$), which was about $0.12 \text{ mm} \text{ d}^{-1}$. PFC indicated an acceptable improvement when using the FAUSY algorithm comparatively to the situation of no NTG correction (76.14%, where 100% represents the manual NTG correction), meaning that the algorithm is potentially advantageous for daily sap flow rate estimations.

The best results for daily transpiration [mm d $^{-1}$], when comparing manual with FAUSY data treatment, were found for sensor G7, with a PFC equal to 86.67% (Fig. 5a). The sensor G8 showed the worst adjustment, with a PFC of 18.18% (Fig. 5b). Results for sensor G5 have shown an intermediary behaviour, with PFC = 66.29% (Fig. 5c). These dissimilarities are explicated by Table 2, which presents the difference between ΔT_{max_adj} and ΔT_{max_na} showing that sensor G8 had the smallest difference and sensor G7 the largest. It corroborates the fact that the biased ΔT_{max} values are propagated in the k calculations. Thus, the magnitude of adjustment of ΔT_{max} is proportional to the magnitude of ΔT adjustment and, consequently, to the k calculations. Furthermore, ΔT_{max_adj} has been used to calculate k adjusted predicted (k_{pred}) and therefore, FAUSY uses information in the initial learning time (initial 10 days) to learn





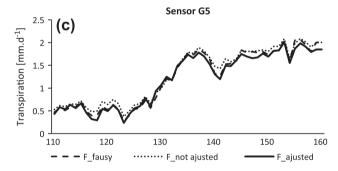


Fig. 5. Comparison of sap flux rate estimates with the FAUSY algorithm, using the not adjusted procedure and the adjusted procedure for sensors (a) G7, (b) G8 and (c) G5

with explicative variables and obtain an estimate of the ΔT_{max} value observed. The differences observed in FAUSY contribution for the different sensors corroborate the fact that NTG are spatially and temporally variable. For that reason, for a more accurate approach, each sensor should use itself as reference sensor, thus switched off in specific periods. This is possible if there is an algorithm (e.g. FAUSY algorithm) capable of replacing the periods when the heater is on. Operating in this mode or after calibration, and in relation to other NTG correction systems, it discards the need for additional equipment (e.g., that needed to shut down the heating

Table 4Performance of FAUSY algorithm computed by residual error Eqs. ((19)–(21)) computed from mean transpiration [mm d⁻¹].

Sensor	Mean transpi	Mean transpiration (mm d^{-1})			Mean residual error (mm d^{-1})		Fausy contribution	
	$\overline{F_{na}}$	F_{adj}	F_{pred}	RE _{na}	RE_f	RE _f (%)	PFC (%)	
G1	1.831	1.672	1.718	0.159	0.046	2.75	71.07	
G2	0.594	0.500	0.514	0.094	0.014	2.80	85.11	
G3	1.215	1.139	1.155	0.076	0.016	1.40	78.95	
G5	1.259	1.170	1.200	0.089	0.030	2.56	66.29	
G6	1.120	1.022	1.054	0.098	0.032	3.13	67.35	
G7	0.861	0.711	0.731	0.150	0.020	2.81	86.67	
G8	1.246	1.213	1.240	0.033	0.027	2.23	18.18	
G10	1.703	1.459	1.499	0.244	0.040	2.74	83.61	
Mean	1.23	1.11	1.14	0.118	0.028	2.53	76.14	

system periodically or additional unheated probes), relying only on the use of meteorological variables, commonly available on experimental sites.

In a global view of the process, FAUSY contribution to automation could be clearly demonstrated for prompt searching over available explicative variables, returning the lowest residual error possible. This fact corroborates the rapidness of the automation process, since the simple residual error calculation may be accepted as an efficient statistical tool. Additionally, in the manually adjusted procedure, the k calculation was computed after the complete data set has been collected; consequently, it was not possible to compute it in real time. Alternatively, FAUSY routine computed k at the same input data time, after the learning time, which allowed automation and availability in real time. As FAUSY relies only on measureable environmental conditions, it can theoretically be applied to other woody species and different conditions, although it must be more extensively tested, since this is the first application performed and factors influencing NTG (e.g., distance of the sensors from soil, presence/absence of irrigation) can change. The algorithm is also potentially applicable to other sap flow measurement methodologies in order to perform possible adjustments and further automate measurements.

4. Conclusions

In the present study, research by fuzzy rules generation was applied to *Granier* sap flow data treatment. In this context, the FAUSY algorithm shows to be an effective approach for system optimization, contributing positively to the estimation of adjusted ΔT values corrected for natural temperature gradients, therefore allowing to automate the data analysis process. The use of FAUSY allowed for a less time-consuming process, yet not discarding the human decision capacity, since it mimetizes the process. In addition, it provides the opportunity for earlier reaction to data because it allows for a real-time treatment possibility.

This algorithm consists of an innovative approach for data treatment when using the *Granier* method for sap flow measurement and allows to overcome the major restrictions of extensive data processing, especially when practical applications in irrigation management are intended. FAUSY algorithm presents a simpler solution to correct NTG in relation to other systems, since it relies only on the use of meteorological variables, commonly available on experimental sites.

In the future, FAUSY algorithm is potentially applicable to other sap flow measurement methodologies and other woody species, although further testing is needed.

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