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DYNAMIC OPTIMIZATION IN SUPPLY CHAINS

ABSTRACT

Production and stock optimization in supply chains represents extremely complex problem, because it determines the optimum quantity production in time. Due to their multileveled nature, those problems are mostly solved by application of different methods and models of dynamic programming. The problem addressed in this scientific debate refers to determination of optimum quantity of production and stocks within the supply chain in a certain period of time, as well as in each sub-interval of the said period, but with the condition that the production and stock expenditure remains minimal and that the predetermined demand in every sub-interval and throughout the entire observation period remains satisfied.

KEY WORDS

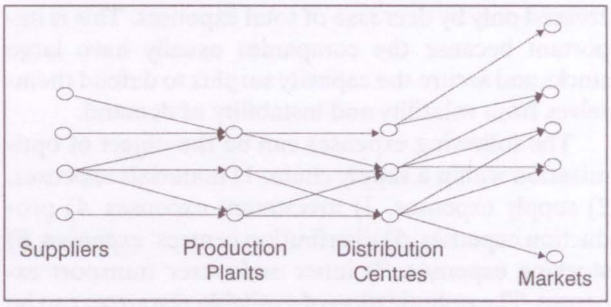
supply chain, optimization, dynamic programming

1. INTRODUCTION

The modern business arena, swamped with numerous challenges and ever-stronger global competition, is characterised by creation of many supply chains in an attempt to optimally use the existing resources and the potential of the chains already in place. A swift response to market demands requires effective solutions in all elements of a chain: supply, production, warehousing, transport and distribution. Real processes in competitive networks encompass large number of mutually dependant and changing variables, that with the help of advanced mathematical and computer techniques, need optimization. As majority of the problems in respect to supply chain management can be represented in the form of multileveled processes, the subject of this scientific debate is their optimization through application of dynamic programming and information modelling methods.

2. RELEVANT CHARACTERISTICS OF SUPPLY CHAINS

Supply chain involves all the participants and processes that are directly or period involved in meeting the buyers' demands. Besides producers and suppliers, supply chain includes also transport, warehousing, wholesale, retail sales and buyers. They represent the integral part of the supply chain. The activities of a supply chain start with buyer's order, and finish when the satisfied buyer pays his receipt for goods delivered. The supply chain represents a dynamic system where information, products and money continue to flow among the participants. A typical supply chain represents a network (Scheme 1), and it seems more appropriate to be speaking of supply networks than supply chains.



Scheme 1 - Supply chain network

Supply chain network as shown in Scheme 1 is made of four object levels. The production process is situated downstream from the supplier up to production plants, from production plants to distribution centres and from distribution centres to markets. Supply chain can be made of any number of object levels. Furthermore, sometimes supply flows take place downstream when semi-products or product parts are being returned to production plants for further improvement or when the products that are not intended for further use are being returned from retail points of sale to distribution centres for recycling.

Modern supply chains represent dynamic, flexible and responsive networks, which operate on the "anticipate and execute" principle, as opposed to traditional approach "produce, then sell". The fast response to changes requires effective solutions on all levels of supply chain: production, supply, warehousing, transport and distribution.

Generated supply chain value represents the difference between the values it has for the buyer

$$D = q \times \phi\,(q) \tag{1}$$

where q – production (demand) volume and ϕ (q) is an inverted form of the demand function, and the expenses made within the supply chain in order to meet the buyers' requirements

$$C = f(q) \tag{2}$$

Supply chain profitability is calculated from the difference of the total profit function divided among all the participants within the supply chain and total expenses, i. e.

$$P = D - C \tag{3}$$

or

$$P = q \times \phi\,(q) - F(q) \tag{4}$$

where P stands for total profit function in the form of production volume function for the named product. In this way, the supply chains profitability debate, as opposed to analysis based solely on supply chains optimal production based on total and average expenses includes also the market conditions expressed through appropriate demand functions.

If the demand for products of a certain supply chain remains unaltered, then the profit can be increased only by decrease of total expenses. This is important because the companies usually have large stocks and secure the capacity surplus to defend themselves from volatility and instability of demand.

The following expenses can be the object of optimization within a supply chain: 1) materials expenses, 2) supply expenses, 3) investment expenses, 4) production expenses, 5) distribution centres' expenses, 6) stocking expenses, 7) inner and outer transport expenses. The optimization of available resources can be achieved through appropriate planning, management and decision-making. According to the traditional approach, the supply chains are linear systems where the raw material is - input and the finished product in buyer's hands is - output. Supply chain participants act as closed, independent subjects, with little or no direct information from other participants. Such approach represents a great risk with potentially unfavourable consequences, and is redefined primarily due to the following factors: 1) consumers and industrial buyers pressure to adjust the products as per their specifications, 2) shorter lifecycle of products, 3) reduction of time necessary for a product to reach the market and 4) demand for improvement of services and support for buyers.

The supply chains represent complex systems with large number of subjects, numerous periods, uncertainty and causality of phenomena and change within time. Supply chains management represents one of the greatest challenges for modern management practitioners and theoreticians. It is an area of operational research that favours the concept of optimal management.

3. DEFINING THE PRODUCTION OPTIMIZATION PROBLEM IN A SUPPLY CHAIN AS PER PRODUCTION AND WAREHOUSING EXPENSES

Optimization represents one of the most significant tools in implementing and planning efficient operations and the increase of competitive advantage. The companies must make intelligent decisions to be able to achieve optimal usage of all the available resources, such as workforce, equipment, raw materials and capital. As many companies keep large stock and secure the surplus of capacity in order to defend themselves from demand volatility and instability, we will further on discuss the problem of production planning in time, as per production expenses and warehousing expenses.

The initial assumption is that it is necessary to determine the production quantity of a product per time unit t = 1, 2, ..., T, of a given T period, while keeping the production and warehousing expenses at the minimum and at the same time satisfying the known demand d t = 1, 2, ..., T per every time unit of a planned period T. Production expenses per product unit, c_t, t = 1, 2, ..., T include cost of raw material, equipment, workforce and other expenses and are known for every time unit.

Stock expenses will surface in case the production increases over demand, i. e. when

$x_t > d_t$, t = 1, 2, ..., T,

Where x_t, t = 1, 2, ..., T denotes the production range in "t" time period to be determined, and dt denotes known demand. Warehousing costs can be demonstrated as follows

$F(x_t - d_t)$, t = 1, 2, ..., T

with the assumption that they are known at any time unit.

Furthermore, if K_t, t = 1, 2, ..., T represents maximum possible production quantity, a k_t, t = 1, 2, ..., T obligatory or minimum production quantity per every time unit of the planned period T, then one can set the following limitation

$k_t \leq xt \leq K_T$, $t = 1, 2, ..., T$,

or limitations

$x_t \geq k_t$ and $x_t \leq K_t$, $t = 1, 2, ..., T$.

This problem can be formulated in the following manner as well: it is necessary to determine the optimum production quantity of a certain product in every time period, keeping the production and warehousing expenses to the minimum and satisfying the predetermined limitations, including the demand.

Such problem should be solved applying the dynamic programming method, i. e. using the appropriate recursive relations. Accordingly, one should start with the last time period and determine the costs for it, which means that the goal function will be minimised

$$f_T = \min \{F(S - d_T + x_T) + c_T x_T\}, \qquad (5)$$

with limitations:

$x_T \geq d_T - S$,

$k_T \leq x_T \leq K_T$,

where S signifies product quantity produced in the previous time period, and stocked.

After calculating the minimum cost for all the S values in T period, we pass on to the calculation of minimum cost for the period before last one (T – 1). This means that minimum functions should be found:

$$f_{T-1}(S) = \min \{F(S - d_T + x_{T-1}) + c_{T-1}x_{T-1} + f_T(S + x_{T-1} - d_{T-1})\} \qquad (6)$$

with limitations:

$x_{T-1} \geq d_{T-1} - S$,

$k_{T-1} \leq x_{T-1} \leq K_{T-1}$.

With this procedure we can determine total minimum cost for every t = T, T – 1, ... 2. 1, periods. So, a certain "t" period will be:

$$f_t(S) = \min \{F(S - d_t + x_t) + c_t x_t + f_{t+1}(S - d_t + x_t)\} \qquad (7)$$

with limitations:

$x_t \geq d_t - S$,

$k_t \leq x_t \leq K_T$.

4. CALCULATION TABLE IN DYNAMIC OPTIMIZATION OF SUPPLY CHAINS

It is assumed that demand dt for a certain product during five periods t = 1, 2, 3, 4, 5 is 7, 5, 6, 7, 5 units of a product, and that the production costs per unit ct, t = 1, 2, 3, 4, 5 as per time units are 6, 10, 5, 9, 4 monetary units.

If the production is larger than demand, warehousing costs will arise, and they amount to 3 (xt – dt) of monetary units for every t = 1, 2, 3, 4, 5.

In every period at least two units of the product should be produced, and maximum production per period is 10 units of the product. It is necessary to determine how many units of the product should be produced per period in order to retain minimum costs.

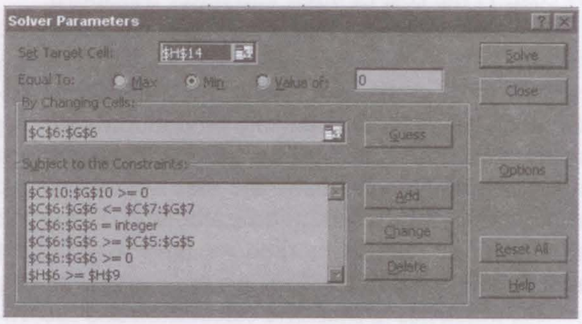
The calculation table (Table 1) gives the model for solving the said problem

Table 1 - Supply chain dynamic optimization model

	A	B	C	D	E	F	G	H
1								
2		Unit Costs	Jan	Feb	Mar	Apr.	May	
3		Prod.	6,00 kn	10,00 kn	5,00 kn	9,00 kn	4,00 kn	
4		Warehousing	3,00 kn	3,00 kn	3,00 kn	3,00 kn	3,00 kn	
5		Obligatory prod.	2	2	2	2	2	
6		**Opt. prod.**	**0**	**0**	**0**	**0**	**0**	0
7		Prod. Limits	10	10	10	10	10	
8		Init. Inv.	0	-	-	-	-	
9		Del. Reqmts	7	5	6	7	5	30
10		Ending Inv.	-	-	-	-	-	
11								
12		Prod. Cost	- kn	- kn	- kn	- kn	- kn	- kn
13		Warehousing Cost	-	-	-	-	-	- kn
14		Total Cost	-	-	-	-	-	- kn
15								

Primarily, production and warehousing costs per unit are being period into the address areas C3: G3 and C4: G4. Then, information on obligatory production is period into address area C5: G5. Address area C6: G5 contains decisive variables, and address field C7: G7 the information on production limits for every period. The initial stocks for the first month are known and are period in the address field B7, while the stocks at the end of the month are determined by the formula = C8+C6–C9. This formula is being copied into the address area C10: G10. The final stock at the end of the first month represents the initial stock for the second month and so on. In address field B12 there is a formula =C6xC3, which is being copied into the period address area C12: G12. This formula calculates monthly production costs. Address field C13 contains the formula =C4xC10 which calculates monthly warehousing costs. The total production and warehousing costs are calculated separately for each month in address area C14: G14, while total production and warehousing costs for all five months in address field H14, which contains the formula =SUM(C14: G14).

After formulating the dynamic optimization model in such a manner, we choose the programme Solver from the Tools menu of the calculation sheet and begin data input in Solver Parameters window, as demonstrated in Scheme 2.



Scheme 2 - Solver while resolving the dynamic programming problem

Once all the data have been period, clicking on the Solve button in the Solver parameters window activates the programme that calculates the value of decisive variables in address sequence C6: G6. Decisive variables that are being calculated in address sequence C6: G6 define the optimal solution. Table 2 demonstrates the optimal solution of the problem by using the calculation sheet MS Excel.

Table 2 - Optimal solution to the problem of supply chain dynamic optimization

	A	B	C	D	E	F	G	H
1								
2		Unit Costs	Jan	Feb	Mar	Apr.	May	
3		Prod.	6,00 kn	10,00 kn	5,00 kn	9,00 kn	4,00 kn	
4		Warehousing	3,00 kn	3,00 kn	3,00 kn	3,00 kn	3,00 kn	
5		Obligatory prod.	2	2	2	2	2	
6		**Opt. prod.**	**10**	**2**	**10**	**3**	**5**	30
7		Prod. Limits	10	10	10	10	10	
8		Init. Inv.	0	3	-	4	-	
9		Del. Reqmts	7	5	6	7	5	30
10		Ending Inv.	3	-	4	-	-	
11								
12		Prod. Cost	60,00 kn	20,00 kn	50,00 kn	27,00 kn	20,00 kn	177,00 kn
13		Warehousing Cost	9,00 kn	- kn	12,00 kn	- kn	- kn	21,00 kn
14		Total Cost	69,00 kn	20,00 kn	62,00 kn	27,00 kn	20,00 kn	**198,00 kn**
15								

As per information contained in Table 2 we can observe the production allocation, stock, production costs and stock cost per periods. The total minimum costs for all five months are 198 monetary units. The solution obtained in this manner is more than 2.5 times more favourable than the most unfavourable solution achieved through solution of the aimed function as per maximum values.

5. CONCLUSION

Supply chains represent complex systems with large number of subjects, numerous periods, uncertainties and causality and volatility in time. Managing supply chains represents one of the greatest challenges for modern practitioners and management theoreticians. It is the area of operational research, favouring the concept of optimal management. Usually, the object of optimization within a supply chain is: 1) cost of material, 2) supply cost, 3) investment cost, 4) production cost, 5) distribution centres cost, 6) stocking cost, 7) the cost of inner and outer transport.

Since many companies keep large stock and secure the surplus of capacities in order to defend themselves from volatility and uncertainty of demand, this scientific debate has deliberated on production planning programme over time and as related to production and warehousing cost, using the method of dynamic programming and information modelling. Using the information modelling method we have avoided the main deficit in the majority of dynamic programming models, which lies in excessive calculation and accordingly, application possibility to problems of smaller dimension. The importance of supply chains optimization by the said methods is emphasised by the fact that in case one should take no account of production planning and stocking as per market requirements, the production and warehousing costs would be as far as two and a half times greater than the optimal solutions, which would directly and permanently damage the competitiveness of a supply chain.

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SAŽETAK

DINAMIČKA OPTIMALIZACIJA U OPSKRBNIM LANCIMA

Optimalizacija proizvodnje i zaliha u opskrbnim lancima predstavlja iznimno složen problem, jer se radi o određivanju optimalne količine proizvodnje u vremenu. Takvi problemi se zbog svoje više-etapnosti najčešće rješavaju primjenom raznih metoda i modela dinamičkog programiranja. Problem koji se obrađuje o ovoj znanstvenoj raspravi, odnosi se na određivanje optimalne količine proizvodnje i zaliha unutar opskrbnog lanca u određenom razdoblju, kao i u svakom pod-intervalu toga razdoblja, pod uvjetom da troškovi proizvodnje i zaliha, budu minimalni i da se zadovolji unaprijed zadana potražnja u svakom pod-intervalu, a tako i u cijelom promatranom razdoblju.

KLJUČNE RIJEČI

opskrbni lanci, optimalizacija, dinamičko programiranje

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