ECONOMICS OF CONSUMER CITIES

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DECLARATION

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

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28 July 2016

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Thesis Summary

In past decades, we have witnessed an increasing economic status of the service sector in the US economy. American consumers spend more of their income on services, and the share of service employments in the economy also rises rapidly. As urban economists, what implications can we draw from this dramatic transformation of economic structure? In this thesis, we try to provide a few insights on this issue.

We define a consumer city where residents cluster to share the agglomeration benefits of non-traded goods. We abstract from the agglomeration economics in the traded sector and assume there are increasing returns in non-traded service sector. The increasing returns can arise from preference for diversity of non-traded services or indivisibilities in production process. We try to provide a consumption-based theory of skill sorting across cities to explore how skill sorting interacts with welfare inequality. Also, we try to understand the internal structure of city of a consumer city and shed some lights on the efficiency issues.

This thesis consists of two studies. The first study explores the emergence of "superstar cities". An important insight in Gyourko, Mayer and Sinai (*American Economic Journal: Economic Policy* 5(4), 2013), that rising aggregate demand, rather than diverging local productivity, accounts for the widening house price dispersion across US cities after WWII, rests on the assumption of idiosyncratic

location preferences and asymmetric housing supply elasticity across cities. Under such assumptions, cities with inelastic housing supply are “superstar” cities—they get more expensive, hence more exclusive to high-income households, as aggregate demand increases. We sharpen and extend this insight by presenting a model where “superstar” cities emerge from the interaction between increasing returns to local demand for differentiated non-traded services and non-homothetic preferences, instead of idiosyncratic location preferences and asymmetric housing supply elasticity. We consider an economy with heterogeneous workers differentiated by skill level, who earn income from employment either in the traded-good sector, where worker productivity depends on skill but not location, or in the non-traded-service sector, where worker’s wage depends on local demand but not skill. A fixed cost is required for each variety of local service, giving rise to increasing return to local demand, which is income elastic. In equilibrium, high-skill workers share the location with a greater variety of local services and higher land rent, middle-skill workers prefer the location with less variety of local services and lower land rent, low-skill workers, who specialize in non-traded sector, are indifferent between locations. The model can also account for skill dispersion within cities, rising non-traded sector employment share, and a U-shaped welfare change across skill spectrum, as a result of increased skill disparity in the economy.

The second study explores the internal structure and efficiency of consumer cities. In past decades, economic status of the non-traded service sector increased in the US. We build a model to study a consumer city, where consumers cluster to share the positive externalities and fixed cost of non-traded services. We assume that the non-traded services are produced and consumed in a single central location, referred to as Central Commercial District, or CCD. This assumption captures positive externalities in non-traded service consumption, which arise from access to diversity by consumers when the services are concentrated in one location. Heterogeneous workers are differentiated by skill level, earn income from employment either in the traded-good sector or the non-traded service sector. Worker productivity in the former sector depends on skill but not location, whereas in the latter sector it depends on aggregate local demand but not skill. A fixed cost is required for the production of non-traded service, which rises with service quality, giving rise to increasing return to local demand. The preferences are non-homothetic such that the demand for non-traded services is income elastic. We assume a single CCD landlord, who tenders the space out to a commercial service operator capable of paying the highest land rent. Service workers travel to work in the CCD and pay a commuting cost that linearly increases with distance to the CCD. Consumers visit the CCD to purchase non-traded services. On each trip, a consumer purchases one unit of service goods and pays a travel cost that linearly increases with distance to the CCD. In equilibrium, low-skill workers choose non-traded service occupation according to comparative advantage. High-skill traded-sector workers live in the central city to share the location with better access to non-traded services and higher housing price, low-skill service workers live adjacent to the central city, while middle-skill traded-sector workers live in the suburban region with the worst access to non-traded services and lowest housing price. The model can also account for rising service quality, non-traded sector employment share, non-traded service expenditure share, housing price premium at the city center and a downward-sloping welfare change across skill spectrum, as a result of increased skill disparity in the city.

One of the main objects of this study is to examine the efficiency of the consumer city. We find that maximizing commercial land rent in the CCD implies that the non-traded services are priced above the marginal production costs, hence, leading to a deadweight loss. Also, maximizing commercial land rent will cause inefficiency in resource allocation--the share of employments in the non-traded service sector is below social optimal and dispersed city structure generates more urban frictions. We argue that government should adopt the marginal-cost pricing regime by regulating the price of services and subsidizing the fixed costs changed on the service producer.

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1 Introduction

During past decades, big cities are gaining importance in offering consumption amenities. Recent literature documents the rise of "consumer cities", where the rich workers live in the central city, even though they work in suburbs. Demand for urban lifestyle has been increasing as a luxury good (Brueckner, Thisse and Zenou, 1999; Lee, 2010; Fu and Liao, 2012). When rich families cluster, it will create consumer externality effect such that high-quality local goods and services will appear to serve them. Glaeser and Gottlieb (2006) document that people value urban amenities more in year 2000, compared to year 1970 (Figure 1-1, Figure 1-2); big cities attract a larger population, because they provide high amenities (Figure 1-3, Figure 1-4).

The rise of "consumer cities" is likely to be fueled by demographic changes: smaller households, later marriages, decisions not to have babies, the emergence of a huge and active baby boom population in its sixties and seventies. All these changes generate more households who are willing to live in a smaller house in a central location to enjoy urban life.

Although big cities are becoming the hubs for consumption amenities, some important research questions remain unanswered. In this thesis, we try to address some of them: how does the rise of "consumer cities" affect people's location choice across cities? How does it affect land use and urban structure? What are its implications for social welfare and inequality? How do fundamental factors, such as land supply, affect the rise of "consumer cities"? We contribute to the literature by focusing on the advantages of big cities in providing more diversified local service goods, while most of the previous studies only emphasize the advantage of big cities in improving productivity in manufacturing sector.

We proceed in following way. In Chapter 2, we reviewed the literature most relevant to our studies and find the research gaps. In Chapter 3, we develop a theory on skill sorting across cities. In Chapter 4, we develop a theory on skill sorting within a consumer city. We conclude the whole thesis in Chapter 5.

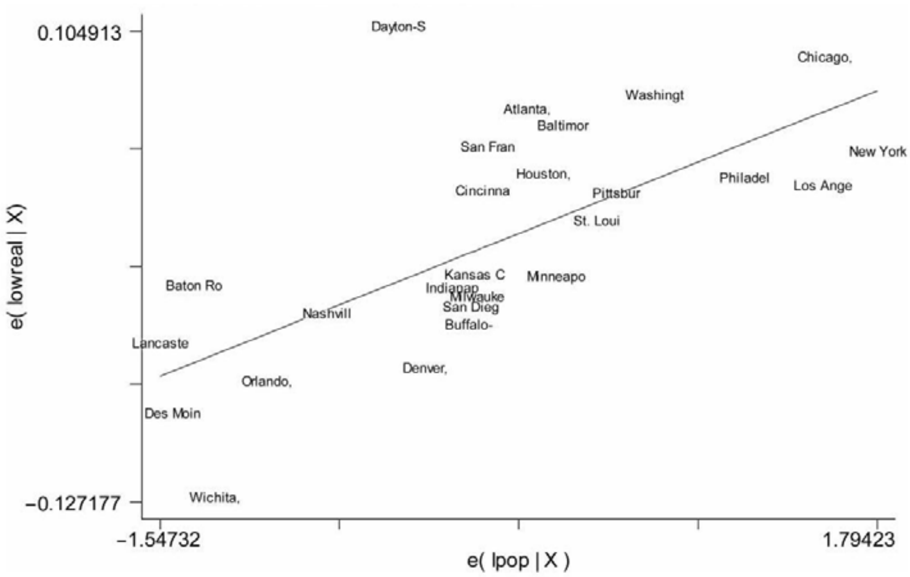

Figure 1-1: Log of real wages and city size, 1970

Source: Glaeser and Gottlieb (2006)

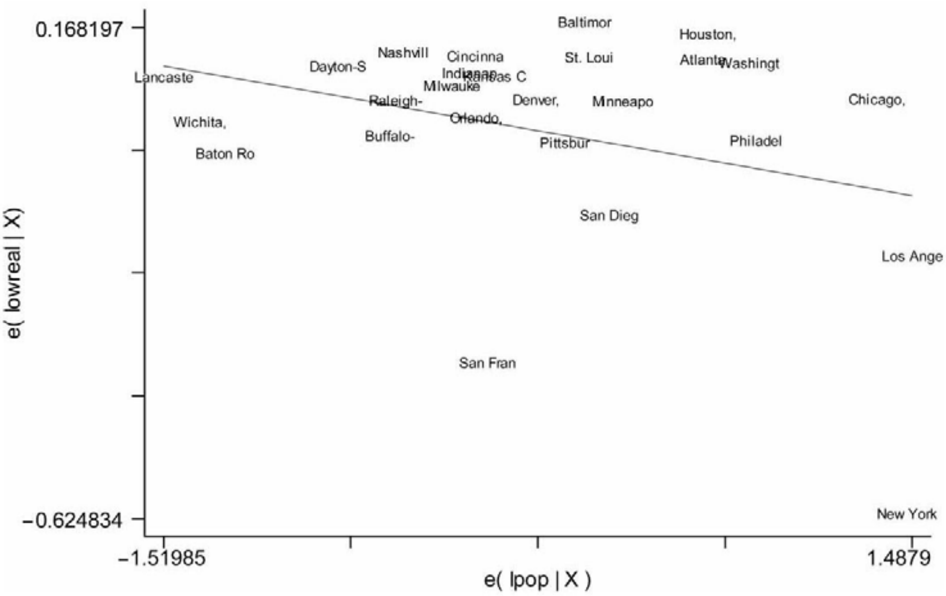

Figure 1-2: Log of real wages and city size, 2000

Source: Glaeser and Gottlieb (2006)

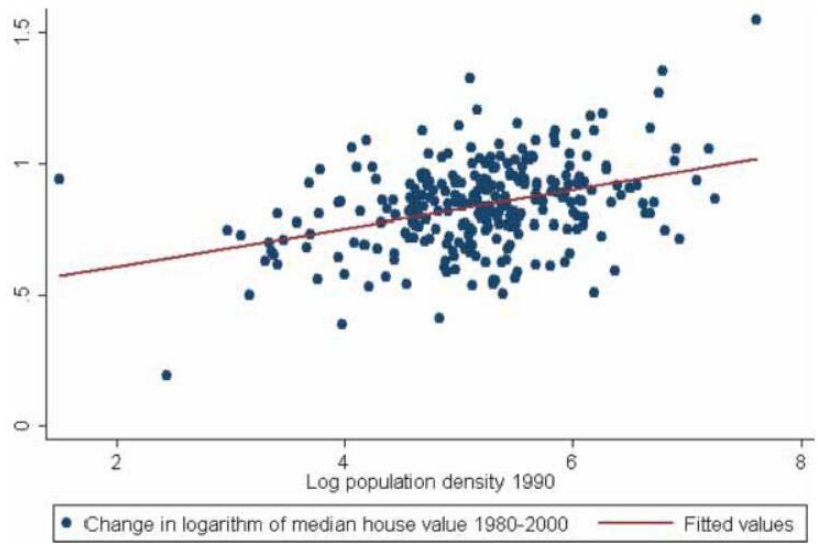

Figure 1-3: Housing value changes and urban density, 1980-2000

Source: Glaeser and Gottlieb (2006)

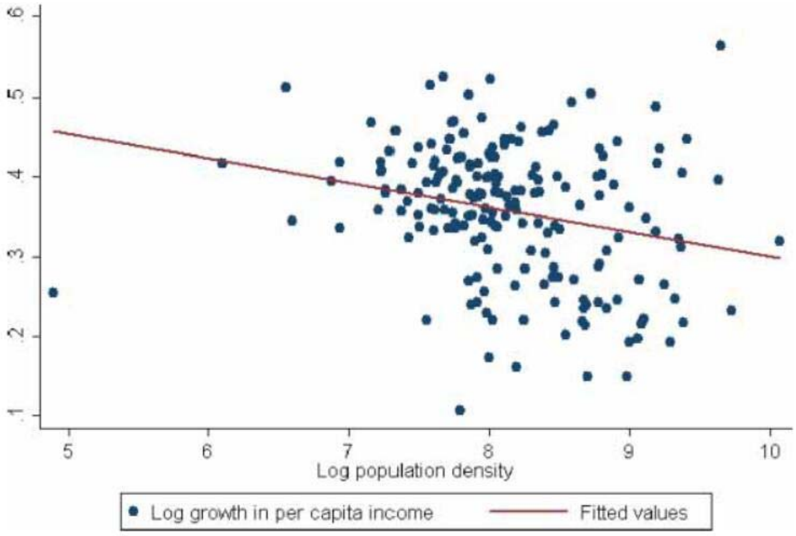

Figure 1-4: Income growth and population density, 1990-2000

Source: Glaeser and Gottlieb (2006)

2 Literature Review

In this section, we review the strands of literature that are most relevant to our studies. We first review the literature that focuses on consumer cities. In our first study, we provide a consumption-based theory for skill sorting across metropolitan areas. We review the empirical literature to outline the key stylized facts associated with the economic status of cities, including patterns of skill sorting and housing price dispersion. Following that, we review the theoretical literature on skill sorting to identify the research gaps. In our second study, we build a monocentric city model to study the internal structure and resource allocation efficiency of a consumer city. Hence, we review the empirical literature that focuses on the economic status of neighborhoods within the city. Then, we review the theoretical studies that focus on Income-residential pattern within the city, most of which are also under the framework of monocentric city model. Also, there is a new strand of literature that explores the urban implications of endogenous interactions among agents. These studies emphasize the role of the city center as a place for non-market interactions. While we focus on market interactions in our second study, we realize that our study shares some similarities with the literature on certain main assumptions. At the end of literature review, we summarize the key insights that we draw from the literature.

2.1 Consumer Cities

The rise of "consumer cities"

Glaeser, Kolko and Saiz (2001) document the rise of "consumer cities" in US economy. The “consumer cities” are characterized by educated, wealthy people choosing to live downtown even when they work in the suburbs. The rise of "consumer city" is associated with gentrification at city center. In the 10 largest MSAs, incomes of the residents within one mile of CBD rose 19% relative to the MSA average over 1980-1990 and incomes of the residents between one and three miles from CBD rise 9% over the same period. The cities with more attractive consumption amenities, such as more restaurants and live performance theaters, have grown more quickly, from 1977 to 1995.

Non-homothetic preference for consumption amenities

Many studies show that the high-income workers appreciate consumption amenities more than the low-income workers do. Glaeser and Gottlieb (2006) find that the central-city residents with high education consume more local service goods than the suburban residents with low education. Adamson, Clark, and Partridge (2004) show that returns to education for the high-skill workers decline with the urban scale and interpret the finding as that urban amenities primarily affect the high-skill workers. Lee (2010) also shows that urban wage premium decreases with worker's skill and interpret it as evidence for that varieties of local goods is more valuable to high-skill workers. Glaeser, Kolko and Saiz (2001) also notice that rents in US cities with more educated population have risen more quickly than wages since the 1970s, and interpret this as that while productivity has gone up in places with more educated workers, the quality of life has risen even faster. In a recent study, Fu and Liao (2014) find that willingness to pay for non-traded amenities increases with skill level, which is direct evidence that local consumption amenities are luxury goods.

Urban economists use non-homothetic utility functions to model housing consumption behavior. The non-homothetic utility makes it possible for the cost-of-living indices to vary across different income groups. Handbury (2013) estimates a non-homothetic utility function by using grocery data. The author argues that high-income households may find large cities to be more attractive because the large cities offer a wider range of groceries suited to the preference of the high-income. Albouy, Ehrlich and Liu (2015) also find the large cities are more attractive for the high-income because they spend a lower proportion of the income on housing.

The high-skill workers improve job opportunities for the low-skill

Other studies show that the presence of high-skill workers will improve employment outcomes for low-skill workers, especially for those employed in the non-traded service sector. In the context of US cities, Moretti (2010a) finds that each additional job in manufacturing sector generates 1.6 jobs in the non-traded sector in the same city. Also, the author finds that the local multiplier effect is even larger for skilled jobs, and one skilled job in the traded sector generates 2.5 jobs in the non-traded sector. In the context of Swedish cities, Moretti and Thulin (2013) finds that the local multiplier effect is approximately of equal magnitude with that uncovered in U.S: adding 1 skilled job in the traded sector creates 3 new jobs in the non-traded sector.

In the context of UK cities, Manning (2004) finds that the presence of retired educated workers raises the employment rate for unskilled workers. Because retired educated workers are unlikely to benefit the unskilled workers through knowledge spillover or any substitution effect in production, the author interprets the finding as that high-skill workers benefit the low-skill workers through raising demand for non-traded services. Also, the author also finds that the presence of high-skill workers in a city raises the share of low-skill workers in the non-traded sector but reduces the share in traded sector.

Kaplanis (2010b) finds that the presence of high-paid occupation workers raises wage for low-paid occupation workers, but not for middle-paid occupation workers, and interpret this finding as that high-paid workers raise demand for the low-skill workers through increasing local demand for non-traded goods. In another study, Kaplanis (2010a) documents that the presence of degree holders will improve the employment rate for the local population. And this effect is especially strong for the uneducated workers but weaker or even negative for other educational/ skill groups.

2.2 Empirical Evidence on Economic Status of Cities

The widening dispersion in housing price across cities

Gyourko, Mayer, and Sinai (2013) documents a widening dispersion in housing price across metropolitan areas. They plot the kernel density of average annual real house price growth rate between 1950 and 2000 for 280 US MSAs, as shown in Figure 2-1. The real housing price growth rate ranges widely, from 0.2 percent to over 3.8 percent. As a result, the dispersion in housing price also becomes wider. By 2000, the housing price in the most expensive cities is four times of the national average, as opposed to twice in 1950, as shown in Figure 2-2.

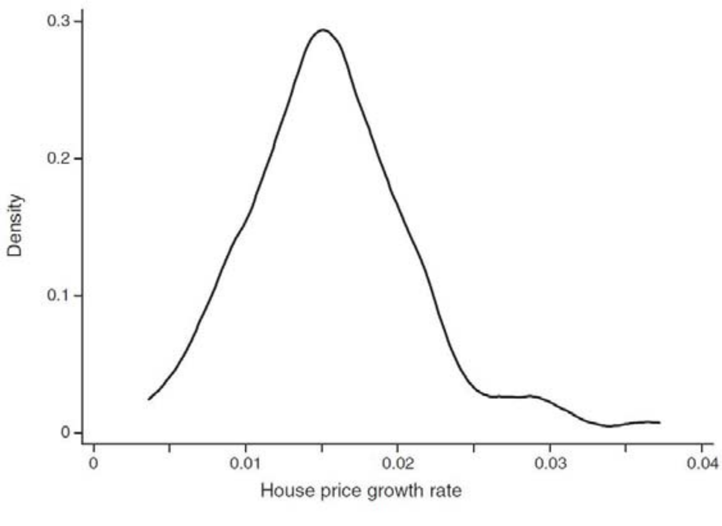

Figure 2-1: Density of 1950-2000 annualized real housing price growth rates across MSAs with 1950 population>50,000

Source: Gyourko, Mayer, and Sinai (2013)

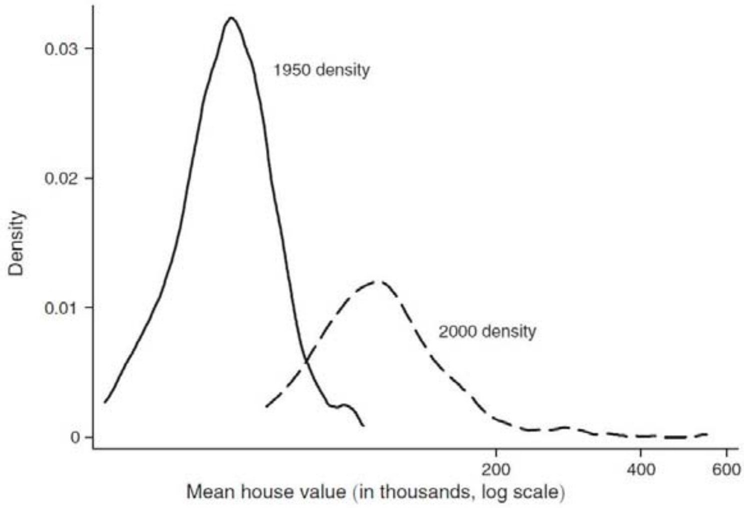

Figure 2-2: Density of mean house values across MSAs: 1950 vs. 2000

Source: Gyourko, Mayer, and Sinai (2013)

The widening dispersion in skill composition across cities

There are a growing number of empirical studies documenting that high-skill workers disproportionately sort in large cities (Bacolod, Blum, & Strange, 2009; Berry & Glaeser, 2005; S. Lee, 2010) and skill sorting accounts for a large fraction of urban wage premium (Baum-Snow & Pavan, 2012; Combes, Duranton, & Gobillon, 2008; Matano & Naticchioni, 2012). Also, this inequality of skill concentration was increasing in the past few decades (Fu & Liao, 2012)

Blum and Strange (2009) find that large cities pay a higher reward for worker's people skill and cognitive skill, while the reward for motor skill is no higher in the large cities. Matano and Naticchioni (2012) shows that agglomeration economies are stronger for individuals with high skill. Following the approach of Combes, Duranton and Gobillon (2008), they assess the time- and location-invariant part of worker's wage and use it to measure worker's skill. They show that the wage effect of spatial externality is increasing along skill distribution.

But, recent studies revealed that not only the most skilled but also the least skilled workers disproportionately sort in large cities. Eeckhout et al. (2014) are the first to document this stylized fact in the context of the US cities. Under the assumption that workers have Cobb-Douglas preference over a numéraire good and housing, the authors use wage, housing price and expenditure share of housing to calculate agent's utility. They use utility to measure individual's skill because the distribution of utility is isomorphic to the distribution of skills in a world with no market frictions. Their finding is robust to alternative measures of skill, e.g., educational qualifications and school years. In the context of French cities, Combes, Duranton, Gobillon and Roux (2012) also find a similar fat-tail skill distribution in the large cities. The authors follow the approach of Combes, Duranton, and Gobllon (2008) to assess the time- and location-invariant part of worker's wage, and then use it to measure worker's permanent skill.

2.3 Theories of Skill Sorting across Cities

Many theorists attempt to rationalize the observed patterns of skill sorting across cities. Most of them are successful in accounting for the pattern that high-skill workers disproportionately sort in large cities, but they neglect the stylized fact that the low-skill also disproportionately sort in large cities. The researchers provide a range of production-based explanations and usually assume that the high-skill worker’s productivity can be improved disproportionately through agglomeration economy in traded good sector. Hence, their focus is on the different channels that high-skill worker's productivity can be improved.

Skill-biased innovation

Berry and Glaeser (2005) document an increasing concentration of the college degree holders in the large cities, over the 1990s. They argue that this trend is induced by inelastic housing supply and skill-biased innovation, featuring that high-skill workers tend to innovate in ways that employ the high-skill.

They assume that high-skill and low-skill workers coexist in a city. In each skill group, a fixed proportion of workers will become new entrepreneurs and immobile. The high-skill entrepreneurs have an exogenous probability of producing new ideas that employ high-skill workers, but the low-skill workers can only produce ideas that employ low-skill workers. The presence of high-skill workers in a city will potentially benefit the low-skill through raising the demand for low-skill workers, but they will also harm the low-skill by raising the housing costs. Hence, the high-skill and the low-skill workers will live separately, when the high-skill workers live in a city with inelastic housing supply, and when they are more likely to invent technologies that only use the high-skill.

Quality of matching

Venables (2011) argues that, because low-quality workers are kept out by high housing price, expensive cities enhance the working efficiency for the high-quality workers by improving the average quality of their working partners. Hence, the high-quality workers will sort into large and expensive cities, while the low-quality workers will sort to small and cheap cities.

There are two cities, expensive and cheap, and two types of workers, high-quality and low-quality. Each worker must choose a city, pay an urban cost and collaborate with another partner in the city to produce a traded good, but the quality of potential partner is unobservable. Group productivity increases with the quality of both members, but good matching is worth more for high-quality workers (assumption of supermodularity of production technology). In equilibrium, all high-quality workers will sort into expensive cities, if the rent differential is sufficiently high to keep the low-quality workers out from the expensive cities, but not too high to be compensated by the benefits from better matching. In essence, the author argues that urban environment can serve as a self-selection mechanism that allows the high-quality workers to signal their ability by living in expensive cities.

Knowledge spillover as pure externalities

Giannetti (2003) argues that high-skill workers have a high propensity to move to the most productive regions. It happens because the high-skill workers received a higher reward for their skill in a city with a higher average stock of human capital.

The externalities stem from knowledge spillover and improved opportunities for learning (Jacobs, 1970; Lucas, 1988; Marshall, 1890; Rauch, 1993).

The author considers an overlapping generation model with infinite periods. In each period, workers with heterogeneous skills are randomly born in one of two cities, North and South, that the latter has lower initial stock of human capital. Workers can pay a migration cost to leave their origin cities. The total factor productivity (TFP) complements worker's skill in producing traded goods and it increases with the average level of human capital in the city. Hence, in equilibrium, skill premium increases with the local average level of human capital.

In the asymmetric equilibrium, the high-skill workers that are born in the South will move to the North, if the difference in reward for skill is sufficiently high to compensate their migration costs. The low-skill workers that are born in the North will move to the South, only when the differential in living costs is sufficiently high.

Their study is consistent with several studies on migration that show highly skilled workers tend to migrate to the regions where the concentration of human capital is highest (Giannetti, 2001). But, their study treats knowledge spillover as a pure externality, which is available freely to everyone in the city, it is close to assuming the conclusion in theoretical terms (Fujita, Krugman, & Venables, 2001).

Costly learning

Instead of treating learning effect as pure externalities, Davis and Dingel (2012) assume that learning is costly in time. They consider the role of cities as learning communities where workers spend time on learning to improve their productivity. The high-skill workers sort into the large and expensive city, because they benefit more from learning and the large and expensive city offer better opportunities for learning. Their "learning hypothesis" is supported by empirical studies (Fu & Liao, 2014).

In a simple version of their model, there are two cities with symmetric fundamentals and two sectors, traded and non-traded. Workers are heterogeneous in skills. Each worker allocates one unit of time between producing and learning. Learning can improve worker's productivity in the traded sector, but has no effect on that in the non-traded sector. The learning benefit is proportional to worker's skill, time spent on learning and local learning opportunities. Local learning opportunities will be improved, if there are more high-skill workers in the city and, if they spend a larger amount of time on learning. Each worker consumes a fixed amount of land and non-traded goods. Land price increases with city size. In equilibrium, the low-skill workers specialize in producing service goods, and they are indifferent between two cities. The high-skill workers will sort into the larger and more expensive city that has a better learning opportunity.

In general, Davis and Dingel (2012) contributes to the literature by showing that three important stylized facts can emerge from a spatial model with symmetric fundamentals, i.e., skill sorting of the high-skill into large cities, positive correlation between city size and skill premium, and higher migration rate of high-skill workers.

Complementarity between skills and business service

Distinguished from previous studies, Hendricks (2011) argues that it is the advantage of the business service sector that attracts the high-skill workers to the large cities. The author assumes that business services are produced subject to economics of scale. In equilibrium, because intermediate inputs (business service) complement skills, the high-skill workers sort to the cities with large service sectors and the land price differential keeps low-skill workers from entering the large cities. Their prediction is consistent with Davis and Henderson (2008) that firms locate their skill intensive headquarters in cities with large business services sectors.

Non-homothetic preference for consumption amenities

Lee (2010) provides a consumption-based theory for skill sorting across metropolitan areas. The author shows that high-skill workers will sort in large cities, provided that large cities offer more varieties of local goods and that the local goods are luxury goods.

There is a continuum of cities, where both land price and consumption varieties increase with the city population. Workers with heterogeneous skills can either choose to live in one of the cities or receive a reservation utility that increases with their skill. The author also assumes that each worker inelastically consumes one unit of land and spend the rest of the income on the local goods. This assumption makes the income elasticity of demand for local goods greater than one. Production of each variety of local goods requires all types of skill, implying that all skill types must be present in every city.

In equilibrium, workers are indifferent across cities. For high land rent in large cities, the workers are compensated through two channels: receiving an urban wage premium and enjoying a greater amount of varieties. The author shows that, because the benefits from the latter channel increase with skill, the urban wage premium decreases with skill. Because the wage of high-skill workers is relatively lower, firms in large cities will hire more high-skill workers. Hence, the model also predicts that high-skill workers tend to concentrate in large cities.

Idiosyncratic amenities, inelastic housing supply and population growth

Gyourko, Mayer, and Sinai (2013) show that when idiosyncratic tastes for locations are uncorrelated with income, asymmetric housing supply elasticity across cities is sufficient for population growth to drive dispersion in house price. In particular, cities with inelastic housing supply are "superstar" cities—they become more expensive, hence more exclusive to high-income households.

Imperfect substitution of skills and asymmetric total productivity factor

Eeckhout et al. (2014) is the only theoretical study that accounts for the fat-tail skill distribution in the large cities. The authors assume that cities are ex-ante asymmetric in the total factor of productivity (TFP), which is accessible to all local firms. High-skill, middle-skill and low-skill workers are imperfect substitutes in the production of tradable goods. Due to the scarcity of high-skill and low-skill workers, their marginal productivity is higher than that of the middle-skill worker. This marginal productivity advantage is further amplified in the large cities where the higher TFP complements the skills. Hence, high-skill workers and low-skill workers disproportionately sort into the large cities.

2.4 Economic Status of Neighborhoods

Income-residential pattern

In most of the metropolitan areas in the US, residents in the suburban area tend to be richer than those living in the central city (Brueckner & Rosenthal, 2009; Glaeser, Kahn, & Rappaport, 2008). Using census tract-level data from the 2000 decennial Census, Glaeser, Kahn and Rappaport (2008) show that average household income rises monotonically with distance to the city center in Atlanta, Los Angeles and Phoenix (Figure 2-3). But, important exceptions also exist. The income-distance relation is U-shaped in some large metropolitan areas, including New York, Chicago, Philadelphia (Figure 2-4). In these metropolitan areas, the central cities are home to affluent families.

Housing price pattern

Edlund, Machado and Sviatchi (2015) document that housing price premium at city center has been increasing, since 1980. We present their finding in Figure 2-5. Back in 1980, housing prices beyond 10 miles from city centers were higher than that within 10 miles of city centers. In 2010, housing price at city center was the highest, and it decreases dramatically with distance from city center.

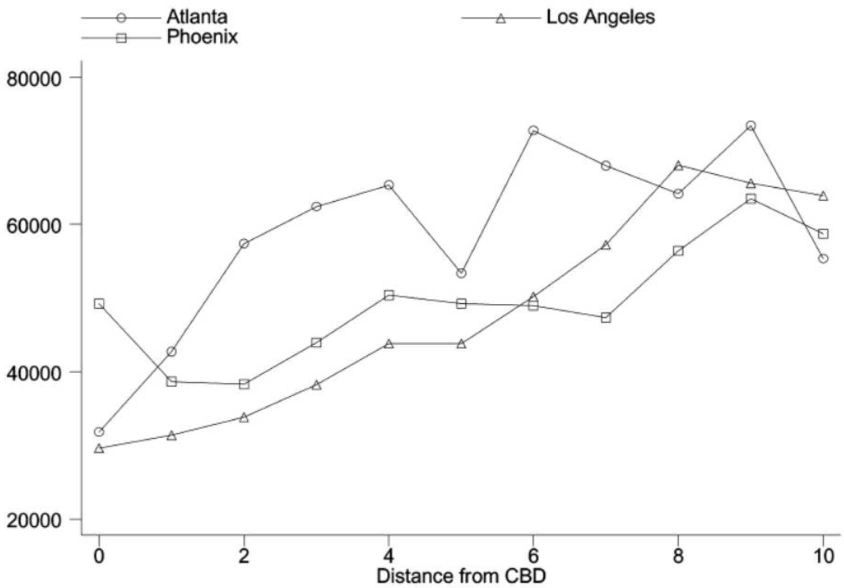

Figure 2-3: Income and distance from the CBD in three new cities

Source: Edlund, Machado and Sviatchi (2015)

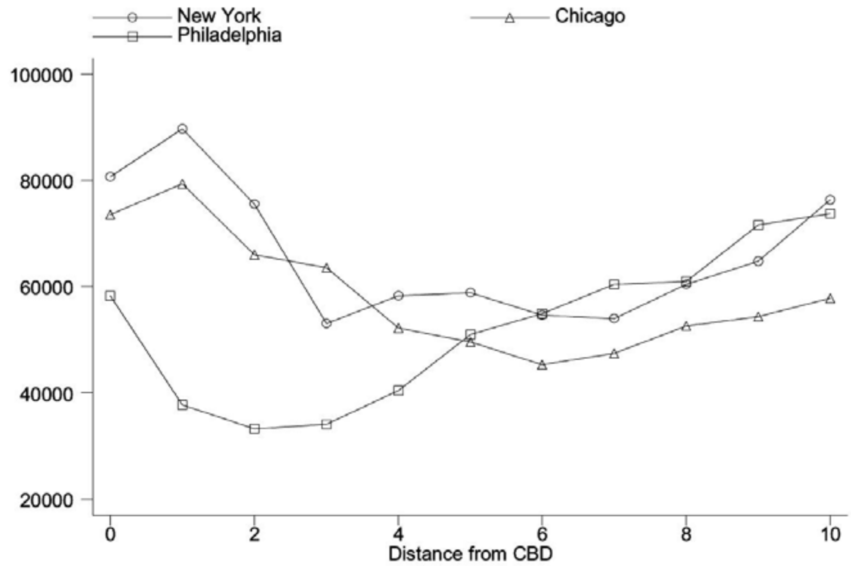

Figure 2-4: Income and distance from the CBD in three old cities

Source: Edlund, Machado and Sviatchi (2015)

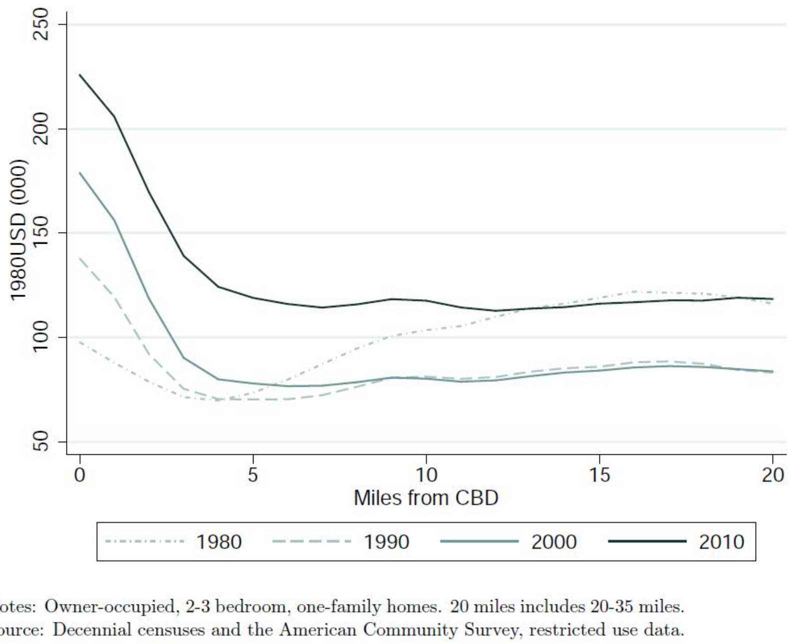

Figure 2-5: Housing price and distance from the CBD:

year 1980, 1990, 2000, 2010

Source: Edlund, Machado and Sviatchi (2015)

2.5 Theories of Income-Residential Pattern within Cities

Previous literature highlights several key determinants of income-residential pattern within the metropolitan area. They include 1) trade-off between commuting cost and land expenditure 2) access to public transportation 3) proximity to amenities 4) filtering process in housing market 5) fiscal externalities and local public goods. We will review them in sequence.

Trade-off between commuting cost and land/housing expenditure

Wheaton (1974) argues that the observed urban poverty can be attributed to rich people's incentive to consume more land, provided that land price is lower in suburban region. The author assumes that residents are heterogeneous in income. They derive utility from housing good and numéraire good that are both normal and have positive income effects. In spatial equilibrium, the bid rent offered by each resident is pinned down, such that any resident is indifferent about moving marginally away or toward to the CBD. The heterogeneous residents are ordered in the spatial dimension, such that the resident with steeper bid rent curve live closer to the CBD. The author shows that, if the income elasticity of demand for land is greater than the income elasticity of commuting costs, poor residents will occupy smaller dwellings close to the CBD, while the rich will live in the suburban area and occupy a larger parcel of land.

Because this explanation for the income-residential location pattern hinges greatly on consumer's preference patterns, in his later study (Wheaton, 1977), the author attempts to test empirically whether these preference patterns hold in US scenario. Using household data collected from the San Francisco metropolitan area, the author estimates the coefficients of consumer's utility function. He finds that income elasticity of demand for land and income elasticity of commuting costs are roughly equal. Hence, the author concludes that the flight of the rich from central city is less a result of the incentive to consume more land, as indicated by Wheaton (1974). The pattern is mostly due to other determinants, including fiscal liberalization, social preference, racial externalities.

Access to public transportation

Glaeser et al. (2008) also challenge the relevance of monocentric model with heterogeneous income in explaining the poverty in the central city. The authors argue that income elasticity of commuting costs must be close to one, since the commuting costly in time. Then, they estimate the income elasticity of demand for housing, by regressing the log parcel size on log income, using the data from 2003 American Housing Survey. They show that the income elasticity of demand for housing is between 0.1 to 0.3, and it is well below one. Hence, the authors argue that the income elasticity of demand for land is too low to justify the urban poverty under the framework suggested by Wheaton (1974).

Alternatively, following LeRoy and Sonstelie (1983), they suggest that better access to public transportation is one key determinant of residential location by income. The authors incorporate two modes of transportation: public transportation and automobile. Taking public transportation requires high fixed cost in time and is slow. Driving requires high fixed financial cost but is fast. The central city has the high population density that is required for public transit. Then, the author shows that, if the poor households have a comparative advantage in using public transit and public transit has a comparative advantage in commutes for a short distance, the poor households will live in the central city in equilibrium.

Access to amenities

Brueckner, Thisse and Zenou (1999) proposed an amenity-based explanation for the pattern of residential location by income in the city. There are two income groups in the economy, rich and poor. Residents derive utility from consuming housing goods, numéraire goods, and exogenous amenities. Marginal valuation of amenities rises sharply with income.

Under these assumptions, the relative location of the rich to the poor is not only determined by the tradeoff between housing expenditure and commuting costs, but also the spatial distribution of exogenous amenities. If exogenous amenities increases with the distance to the CBD, then it will work in the same direction as the conventional forces, further attract the rich to live in the suburban area. But, if amenities decreases with the distance to the CBD, the rich will live in the city center, provided that valuation of amenities increases rapidly with income and demand for land is not too income elastic.

The authors further extend their analysis to incorporate one type of endogenous amenity - the average income of neighborhood. The authors assume that the rich have a preference for the high-income neighborhood. Accounting for endogenous amenities can lead to multiple types of equilibrium. In particular, the rich might live in a neighborhood with low exogenous amenities, as long as endogenous amenities are sufficiently high to cover this shortage.

Filtering in housing market

Rosenthal (2008) and Brueckner and Rosenthal (2009) argue that gentrification pattern can stem from filtering process in the housing market. The authors emphasize that the rich has a preference for new housing units. Hence, as housing units at city center deteriorate, the rich will leave city center and move to suburban areas, where new housing developments occur. Their study can also potentially reconcile the varying Income-residential patterns across cities. In growing metropolitan areas, housing units in the suburban area tend to be newly built, hence, will attract the rich households. In old metropolitan areas, one or more rings at a different distance to city center experience redevelopments, hence, will be occupied by rich households. In a later study, Rosenthal (2014) shows that housing filters down with housing age and more quickly when the housing is new. Hence, the author confirms that lower income families tend to live in older homes, except for subsidized housing units.

Fiscal externalities and local public goods

DeBartolome and Ross (2003) highlight the role of local public services and fiscal in determining Income-residential pattern within a city. They embed fiscal competition (Tiebout, 1956) into the monocentric model. They assume that there are two income groups in the economy, rich and poor. All workers commute to work in the city center, and the commuting cost is higher for the rich. The consumers derive utility from a numéraire good and public services. Each consumer occupies a fixed amount of land. The city consists of two rings, i.e., inner city, and suburb. Within each jurisdiction, residents vote to determine the level of public services and finance it by residence tax.

Under these assumptions, the authors show that there are both income-sorting equilibrium and income mixing equilibrium. In the income-mixing equilibrium, the poor is the majority in the inner city, and they vote for a low level of public services that keeps the majority of the rich out of the inner city.

2.6 Spatial Interactions

There are also some studies that embed interaction among consumers or firms in the spatial model (Beckmann, 1976; Borukhov & Hochman, 1977; Fujita & Ogawa, 1982; Fujita & Thisse, 2013).

Interactions among firms

Lucas and Rossi-Hansberg (2002) propose a theory of urban land use that interactions among firms improve firm's productivity. Instead of taking the location of firms as exogenous, they allow workers and firms to compete for land at all locations in the city.

They assume that homogeneous agents consume traded good and residential land. Workers commute to work and incur a commuting cost regarding forsaken labor time. Production of traded goods not only requires labor but also land input. Hence, firms must compete with workers for land at all locations. Besides, there is an externality in the traded good sector. The level of externality available to a firm is determined by its relative location to the other firms, and it decays with distance. The assumption on externality conveys the idea that firms in a district with high employment density tend to benefit more from knowledge spillover.

In equilibrium, the location of firms and workers are simultaneously determined. Land at any location is allocated to either residential use or production use that offers higher bid rent. The authors identify a tradeoff faced by firms in choosing the production sites. If a firm chooses a location that is close to other firms but far from workers' residential location, it will benefit from higher production externality, but it has to pay a higher wage to workers to compensate for the higher commuting costs. On the contrary, if a firm chooses a location close to the residential location of workers but far from other firms, it can save on commuting costs, but it will suffer a loss from lower production externality.

They find that the configuration of city land use is sensitive to the level of commuting cost. When commuting cost is low, the city will have a central business area, surrounded by residential land. When commuting cost is high, the pattern of mixed land use will emerge.

Endogenous interactions among consumers

Although most of the studies assume that interactions among agents only depend on agents' locations, two recent studies assume that benefits from interactions also depend on the time devoted.

Helsley and Strange (2007) turn to explore the effect of endogenous non-market interactions on urban form and city efficiency. The authors assume that homogeneous consumers derive utility from a numéraire good, housing good and interaction with other consumers. Interactions among consumers only occur at an exogenous city center. To interact with others, an individual must visit the city center and pay a travel cost. The value of each visit depends on the interaction quality that increases with the total number of visits at the city level. Hence, when a consumer visits the city center, she also has a positive spillover effect on other consumers through improving the quality of interactions. Instead of assuming interaction among agents to be determined by agent’s location choice, the authors allow agents to choose their intensity of interaction with others. Urban interactions and spatial structure of city are jointly determined in equilibrium.

Because of the externalities, the market fails to achieve first-best outcomes. There are too few interactions occurring at city center in equilibrium because consumers do not fully account for their contribution to the quality of interactions. Then, the author proposes to subsidize transportation to achieve the first-best levels of visits and population density.

In another study, Helsley and Zenou (2014) explore how social network interacts with the residential location of heterogeneous agents. The authors assume that consumers derive utility from a numéraire good and interactions. Each consumer chooses the level of efforts that they put in interacting with other agents.

There are two locations in the economy, i.e., core and peripheral. All interactions occur at the core location. Agents who live at the peripheral location must visit the core location to interact with other agents and pay a travel cost. The author further assumes that agents are heterogeneous in their locations in the social network. Their centrality in the social network has a positive effect on the value of their each interaction.

Based on these assumptions, the authors find that the agents who are at the center of social network or live close to interaction center will choose a high level of interactions in equilibrium. Besides, the level of interactions in the whole economy also increases with the density of links in the social network and the density of population in geographic space. Hence, their study shows that high urban density and closer social connections among agents will favor the interactions in the city. Then, the authors allow agents to choose their residential locations endogenously. As expected, they find the agents who are located at the center of social network tend to live closer to the center of interaction.

2.7 Discussion

In this section, we highlight some key insights that we draw from the literature review and discuss them in sequence. First, recent studies document the rise of "consumer city", featuring that high-skill workers live in city center, even though they travel to work in suburbs. Second, preference for non-traded goods and services is non-homothetic. Local consumption amenities are more valuable to the high-skill, rich workers. Third, high-skill and low-skill workers are complementary through non-traded service market. The presence of high-skill workers will improve the job opportunities for the low-skill, and the low-skill workers can create the precious consumption amenities that attract the high-skill. Fourth, despite the salient empirical evidence on "consumer city", theoretical research on the topic is strikingly limited. Most of the previous studies that focus on skill sorting across cities emphasize the role of city as "manufacturing center." The only exception is Lee (2010). In that study, difference in consumption varieties across cities is taken as exogenous, rather than determined by consumer choices. The literature on the internal structure of the city is in a similar situation. As far as we know, there are no theories that explore the internal structure and efficiency of a consumer city. Fifth, previous studies are not able to explain the pattern of skill sorting revealed in recent empirical studies. Although most of the theoretical studies can capture the pattern that high-skill workers disproportionately sort into large cities, they neglect the fact that low-skill workers are also overrepresented in the large cities. The only exception is Eeckhout et al. (2014), who relies on a disputable assumption that cities have ex-ante asymmetric total productivity factors. Different from Eeckhout et al. (2014), our first study shows that asymmetric spatial equilibrium can emerge across perfectly symmetry locations in the presence of increasing returns in local consumer amenities and non-homothetic preferences for such amenities.

3 Emergent Superstar Cities

Abstracts

An important insight in Gyourko, Mayer and Sinai (*American Economic Journal: Economic Policy*5(4), 2013),that rising aggregate demand, rather than diverging local productivity, accounts for the widening house price dispersion across US cities after WWII, rests on the assumption of idiosyncratic location preferences and asymmetric housing supply elasticity across cities. Under such assumptions, cities with inelastic housing supply are "superstar" cities—they get more expensive, hence more exclusive to high-income households, as aggregate demand increases. We sharpen and extend this insight by presenting a model where "superstar" cities emerge from the interaction between increasing returns to local demand for differentiated non-traded services and non-homothetic preferences, instead of idiosyncratic location preferences and asymmetric housing supply elasticity. We consider an economy with heterogeneous workers differentiated by skill level, who earn income from employment either in the traded-good sector, where worker productivity depends on skill but not location, or in the non-traded-service sector, where worker productivity depends on local demand but not skill. A fixed cost is required for each variety of local service, giving rise to increasing return to local demand, which is income elastic. In equilibrium, high-skill workers share the location with a greater variety of local services and higher land rent, middle-skill workers prefer the location with less variety of local services and lower land rent, low-skill workers, who specialize in non-traded sector, are indifferent between locations. The model can also account for skill dispersion within cities, rising non-traded sector employment share, and a U-shaped welfare change across skill spectrum, as a result of increased skill disparity in the economy.

Key words: skill disparity, income sorting; house price dispersion; increasing return; taste for variety.

JEL classification: J3 R1 R3

3.1 Introduction

House price dispersion across US metropolitan areas has widened considerably since World War II. Gyourko, Mayer and Sinai (2013) offer a fundamental insight that the widened dispersion can be a result of aggregate demand increase rather than local productivity divergence. They show that, when idiosyncratic tastes for locations are uncorrelated with income, asymmetric housing supply elasticity across cities is sufficient for aggregate to drive house price dispersion. In particular, cities with inelastic housing supply are "superstar" cities—they become more expensive, hence more exclusive to high-income households, as aggregate demand rises. We sharpen and extend this insight by presenting a model where "superstar" cities emerge from the interaction between increasing returns to local demand for differentiated non-traded services and non-homothetic consumer preferences, instead of idiosyncratic location preferences and asymmetric housing supply elasticity.

I consider an economy where heterogeneous workers, differentiated by skill level, are perfectly mobile and earn income from employment either in the traded-good sector or in the non-traded-service sector. Worker productivity in the former sector depends on skill but not location, whereas in the latter sector, worker's wage depends on local demand but not skill. A fixed cost is required for each variety of local service, giving rise to increasing return to local demand. Workers derive their utility from the consumption of a numéraire traded good, housing, and differentiated non-traded services. The preferences are non-homothetic such that the demand for local service variety is income elastic. The equilibrium is characterized by worker choices of employment occupation and residential location, wage rates for non-traded service workers and land rent differential across cities clear the labor and housing markets. In equilibrium, low-skill workers choose non-traded service occupation according to comparative advantage. Also, high-skill traded-sector workers share the location with a greater variety of local services and higher land rent, middle-skill traded-sector workers choose the location with less variety of local services and lower land rent, low-skill non-traded service workers are indifferent between locations, and worker utility is convex, non-decreasing, in skill level. Increasing population skill disparity by raising the share of high-skill workers in the economy has the effect of elevating the demand for the variety of non-traded services, enabling the high-skill city to offer a greater variety of local services and thus become more attractive and more expensive.

Besides predicting widening house price dispersion as population skill disparity increases, the model accounts for several additional important features:1) more expensive cities tend to be larger in population and also have a wider skill spectrum, 2) non-traded sector employment share increases with population skill disparity, and3) increased population skill disparity produces U-shaped welfare changes across skill spectrum. While the evidence on the first two features is readily available in the literature, the last feature is broadly perceived but not fully appreciated. Our model predicts that increased population skill disparity actually benefits non-traded service workers, who have relatively low skills, and hurt traded-sector workers, who generally have high skills. The middle-skill traded-sector workers tend to suffer most. This happens because the increased skill disparity, as a result of rising share of high-skill workers in the economy, elevates the demand for non-traded services, raising the wage cost in the non-traded sector. The middle-skill workers, who do not benefit from the rising wage in the non-traded sector, suffer the most because they are hurt not only by the rising labor cost of non-traded services but also by getting pushed to smaller cities to have less variety of non-traded services to enjoy.

Our model is rooted in the tradition of the new economic geography literature (Fujita et al., 2001; Krugman, 1991) by emphasizing the role of increasing return at the city level in sustaining asymmetric spatial equilibrium. We focus on the increasing return with respect to local consumer amenity instead of that with respect to traded-sector productivity. Incorporating the latter is equivalent to augmenting the skill disparity, which reinforces the asymmetric equilibrium driven by the consumer amenity benefit. Imperfectly elastic housing supply is necessary to prevent the degenerate equilibrium with only one populated city. But relying not on asymmetric housing supply elasticity to drive asymmetric spatial equilibrium is important. Housing supply elasticity is not totally exogenous and can be altered by local land use regulations (Hilber & Robert-Nicoud, 2013). Moreover, restricting housing supply does not necessary give a city any advantage in attracting high-skill workers; indeed, doing so can hurt the city's attractiveness by limiting the local demand size and hence the variety of non-traded services.

The key premises of our model are consistent with empirical evidence. Increasing returns to local demand density for consumer amenities are documented by Couture (2013) and Schiff (2014). Handbury and Weinstein (2012) examine barcode data and find larger metropolitan areas in the US offer a larger variety of grocery goods and lower grocery retail price index. Glaeser, Kolko and Saiz (2001) also documented that large cities in Europe and US outperformed their smaller counterparts with respect to consumption benefits. The assumption of non-homothetic preferences is supported by the finding of increasing willingness to pay with skill level for non-traded amenities offered by large cities in Lee (2010) and Fu and Liao (2014).

Our model predicts a wider skill spectrum in the larger, more skilled cities, as these cities employ disproportionally more low-skill non-traded service workers. This prediction is consistent with the stylized fact that both high-skill and low-skill workers disproportionately sort into large cities (Combes et al., 2012; Eeckhout, Pinheiro, & Schmidheiny, 2010). Davis and Dingel (2012) also assume that non-traded service sector requires no formal skills and hence employ low-skill workers. Empirical evidence shows that the presence of high-skill workers improves employment outcomes for low-skill workers, especially for those employed in the non-traded service sector. Moretti (2010b), for example, finds that one additional skilled job in the traded sector generates 2.5 jobs in local goods and services sector in U.S. cities. Additional evidence can be found in Moretti and Thulin (2013), Manning (2004), and Kaplanis (2010a).

Skill sorting across cities is extensively documented in the literature (Bacolod et al., 2009; Combes et al., 2012; Henderson, 1974). Most studies focus on productive advantages of skill sorting, such as skill complementarity in production (Baum-Snow & Pavan, 2012, 2013; Berry & Glaeser, 2005; Combes et al., 2008; Giannetti, 2001, 2003; Glaeser & Resseger, 2010; Matano & Naticchioni, 2012; Mion & Naticchioni, 2009), learning externalities (D. R. Davis & Dingel, 2012), and sharing of intermediate inputs (J. C. Davis & Henderson, 2008; Hendricks, 2011). Behrens et al (2010), Venables (2011) and Davis and Dingel (2012) are recent examples that provide microfoundation for asymmetric spatial equilibrium and skill sorting across symmetric locations driven by agglomeration economies in traded-good production. Our present paper is in the same spirit as these examples but focuses instead on agglomeration economies with respect to non-traded service supply and consumption benefits. Adamson et al. (2004) and Gottlieb and Glaeser (2006) also highlight the consumption benefits of skill sorting, but they assume an exogenous distribution of consumer amenities.

Our model is presented in section (3.2). The sorting equilibrium is characterized in section (3.3). Section (3.4) provides an algorithm that searches for equilibrium solutions. Numerical examples are shown in section (3.5). Section (3.6) concludes.

3.2 The Model

I consider an economy with two cities at symmetric locations. The economy has a population of perfectly mobile workers with heterogeneous skill levels. They consume housing in one of the two cities, a numéraire traded good, and a bundle of differentiated non-traded services. They have a taste for variety of non-traded services, and their utility function is non-homothetic such that the income elasticity of demand for the non-traded services is greater than unity. The productivity of traded-good producers equals to their skill level but is independent of location, whereas the wage of non-traded-service producers is independent of their skill level but is subject to increasing return with respect to local demand (market thickness). The housing supply in each city is imperfectly elastic so that housing price dispersion widens as housing consumptions in two cities diverge. In such a setting, we show that the relatively low-skill workers will choose to specialize in producing non-traded services and cities will specialize with respect to different diversity of local services to cater to different income segments. The city that offers a greater diversity of local services (low non-traded-service price) and a higher compensating housing price—the superstar city—caters to high skill workers, who have greater willingness to pay for local service diversity, and also attracts a greater proportion of low-skill workers to provide non-traded services.

3.2.1 Consumption

Workers derive their utility from the consumption of a traded good, *X*, composite non-traded services, *S*, and housing, *H*. Previous studies have shown that the income elasticity of demand for housing expenditure is less than 1 (Albouy, 2008; Glaeser et al., 2008; Moretti, 2013).I assume that both housing and the traded good are necessity goods. Thus income elasticity of demand for non-traded services is greater than 1. This is a key assumption that drives spatial sorting of skills in our model.

Consumers' preference is defined by the indirect utility function,

$$V(I,G,P)=\frac{1}{\varepsilon}\left(\frac{I}{G}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P}{G}\right)^{\gamma} \quad (3\text{-}1)$$

[1]

where I is individual income, P is the price of a composite good of housing and traded good. G is composite price index of non-traded services. We let workers have a taste for variety of non-traded services, by defining G as,

$$G=\left(\int_0^n p\left(i\right)^{1-\sigma}di\right)^{1/\left(1-\sigma\right)} \quad (3\text{-}2)$$

where $p\left(i\right)$ is the price for variety i, n is the range of varieties produced and $\sigma>1$ is the elasticity of substitution between any two varieties.

The composite price index P is given by,

$$P=\alpha^{-\alpha}\left(1-\alpha\right)^{\alpha-1}P_h^{\alpha}P_X^{1-\alpha}, \quad (3\text{-}3)$$

where P_h is housing price and P_X is price of traded good price. The Cobb-Douglas

[1] In general, a direct utility function of (3-1) does not exist. When $\varepsilon=\gamma$, the direct utility of (3-1) is given by,

$$U(x_P,x_G)=\frac{1}{\varepsilon}x_G{}^{\varepsilon}-\frac{\left(\frac{x_P}{\beta}\right)^{\varepsilon/(1\text{-}\varepsilon)}-\beta}{\left(\left(\frac{x_P}{\beta}\right)^{1/(1\text{-}\varepsilon)}-x_P\right)^{\varepsilon}}$$

form of the composite price index implies that expenditure share of housing is a constant α of total expenditure on housing and traded good; $0<\alpha<1$. We use the traded good as numéraire good, thus setting P_X to $1. 0<\varepsilon<1$ measures the degree of non-homotheticity of the utility. If $\varepsilon=0$, the utility is homothetic. $\gamma>0$ will define the price elasticity of housing and traded-good consumption.

The specified indirect utility function (3-1) is a subclass of price independent generalized linearity (PIGL) preferences rooted in Muellbauer (1975, 1976). The utility function includes homothetic preferences as special cases. If $\gamma=\varepsilon=0$, I have Cobb-Douglas preferences, $V=\log\left(\frac{I}{P^{\beta}G^{1-\beta}}\right)$. If $\beta=0$, we have CRRA preferences, $V=\frac{1}{\varepsilon}\left(\frac{I}{G}\right)^{\varepsilon}$. Boppart (2014) proves that the indirect utility function is valid, if and only if $I^{\varepsilon}>\beta\left(\frac{1-\varepsilon}{1-\gamma}\right)P^{\gamma}G^{\varepsilon-\gamma}$. In the rest of paper, we choose parameters such that this condition is fulfilled. Following lemma demonstrates that function (3-1) is a valid utility representation.

Lemma I

Function (3-1) is a valid indirect utility function, if $\beta\left(\frac{P}{G}\right)^{\gamma}\leq\left(\frac{I}{G}\right)^{\varepsilon}$.

Proof. See the Appendix

By Roy's identity, the demand for traded good X and that for housing H by a worker are given by, respectively:

$$q_X = -\frac{\partial V / \partial P_X}{\partial V / \partial I} = (1-\alpha)\beta I \left(\frac{G}{I}\right)^{\varepsilon} \left(\frac{P}{G}\right)^{\gamma} \tag{3-4}$$

and

$$q_h = -\frac{\partial V / \partial P_h}{\partial V / \partial I} = \alpha\beta I \left(\frac{G}{I}\right)^{\varepsilon} \left(\frac{P}{G}\right)^{\gamma} \frac{1}{P_h} \tag{3-5}$$

The income elasticity of demand for housing and the traded good is $1-\varepsilon$. The price elasticity for housing is $\alpha\gamma - 1$. The demand for non-traded-service variety i by a worker is given by

$$q_{S,i} = -\frac{\partial V / \partial p(i)}{\partial V / \partial I} = \frac{I}{G}\left(\frac{G}{p(i)}\right)^{\sigma} \left[1-\beta\left(\frac{G}{I}\right)^{\varepsilon} \left(\frac{P}{G}\right)^{\gamma}\right] \tag{3-6}$$

Note that income needs to be sufficiently high to generate both a positive demand for non-traded services and a positive utility. We must have $\gamma \geq \varepsilon$ so that positive demand for non-traded services guarantees positive utility and the demands for the traded good and housing are non-increasing in the composite price index for non-traded services.

3.2.2 Production

3.2.2.1 Non-traded Service Sector

Non-traded services are produced by labor independent of skill and the production technology is identical for all varieties in all locations. Each worker supplies one unit of labor. The supply of each variety of non-traded services requires a fixed cost of F units of labor. The fixed cost can be in the form of research and development, setting up necessary equipment and shops, or obtaining necessary business licenses. In addition, each unit of service output also requires a constant marginal labor input c. Producing a quantity $z(i)$ of any variety thus requires l units of labor input:

$$l = F + cz(i) \tag{3-7}$$

Given a wage rate w for labor in non-traded service sector and price $p(i)$,the profit $\pi(i)$ for reach service variety is given by:

$$\pi(i) = p(i)z(i) - w\left[F + cz(i)\right] \tag{3-8}$$

Given a constant price elasticity of demand, profit maximization entails a constant mark-up pricing over the marginal cost of production:

$$p(i)^* = \frac{\sigma}{\sigma - 1} cw \tag{3-9}$$

Free entry drives the profit to zero,

$$\pi(i)^* = w\left(\frac{1}{\sigma - 1} cz(i)^* - F\right) = 0 \tag{3-10}$$

Thus, the equilibrium output for each variety is given by

$$z(i)^* = F(\sigma - 1) / c \tag{3-11}$$

which requires a labor input of

$$l^* = F\sigma \tag{3-12}$$

I choose the unit of measure for labor input such that $c = (\sigma - 1)/\sigma$. Thus,

$$z(i)^* = F\sigma \tag{3-13}$$

$$p(i)^* = w \; . \tag{3-14}$$

3.2.2.2 Traded Goods Sector

Work productivity in the traded-good sector benefits from formal training that produces skills. We define skill level such that each worker's productivity (employment income) in the traded-good sector equals his skill level, indicated by index *b*. The distribution of *b* in the worker population is described by a density function, $k(b)$,on a finite support $\left[\underline{b},\ \overline{b}\right]$. Workers are free to choose employment in any sector. Comparative advantage *a la* Roy (1951) allocates low-skill workers to the sector where skill does not benefit productivity. Specifically, given the wage rate *w* in the non-traded service sector, workers with skill level below w will choose employment in the non-traded sector and those with *b*>*w* will choose employment in the traded-good sector.

3.2.2.3 Housing Sector

Following Behrens et al. (2010) and Davis and Dingle (2012), we adopt a most stripped-down representation of the housing sector. Housing service is produced by capital only. A standard monocentric city model (Alonso, 1964a; Behrens et al., 2010; R. F. Muth, 1969) entails a constant cost of housing service (including commuting cost and land rent) throughout the city as land rent varies by location to compensate differential commuting cost. That cost of housing service in location *j*, denoted by $P_{h,j}$, must increase with city size in terms of total quantity of housing space consumed $Q_{h,j}$; the rate of increase, however, will depend on the city's housing supply elasticity, which regulates the residential density. We assume $P_{h,j} = \theta Q_{h,j}^{\rho}$, where parameters $\theta, \rho > 0$ and ρ represents the inverse of housing supply elasticity, which is invariant across locations.

3.3 Equilibrium

We first characterize equilibrium for the case of two *ex-ante* identical locations, labeled city 1 and city 2 respectively. Individual workers choose a city to live, an occupation and the consumption bundle to maximize their own utility. In equilibrium, non-traded sector wage rates in each city, w_1 and w_2, housing prices, $P_{h,1}$ and $P_{h,2}$, and composite non-traded service prices G_1 and G_2, clear the market for non-traded service workers and housing in each city. Spatial equilibrium requires any advantage of lower composite non-traded service price to be compensated by a higher housing price such that a marginal worker will be indifferent between two cities.

To build the intuition for the basic properties of an asymmetric equilibrium, Figure 3-1 shows the utility offered by each city for workers at different skill levels. Without loss of generality, we assume city 1 to have a lower composite non-traded service price and higher housing price: $G_1 < G_2$ and $P_{h,1} > P_{h,2}$. The utility offer curve of city 1 is steeper, with a slope of $G_1^{-\varepsilon}/\varepsilon$. With a higher composite non-traded service price G_2, the slope of the utility offer curve of city 2 is smaller, $G_2^{-\varepsilon}/\varepsilon$. A lower housing price $P_{h,2}$ shifts the city 2 utility offer curve to the left and determines the cutoff skill level b_1, above which skill the traded-sector workers will live in city 1. The non-traded sector wage rate in city 2, w_2, determines the cutoff skill level $b_2 = w_2$, below which skill the workers are better off employed in the non-traded sector. Non-traded service workers enjoy the same utility level represented by the horizontal line in Figure 3-1 that intersects the utility offer curve of city 2 at b_2. The intersection of this horizontal line with the utility offer curve of city 1 determines the non-traded service wage rate in city 1, w_1, which compensates the non-traded service workers in city 1 for the housing price premium $P_{h,1} - P_{h,2}$. The equilibrium utility across the skill spectrum is thus convex and non-decreasing in skill level; it is constant for low-skill workers in the non-traded service sector, it then rises with skill level above the cutoff point b_2 along the city 2 utility offer curve until the cutoff point b_1, it then rises more steeply along the utility offer curve of city 1 above the cutoff skill level b_1.

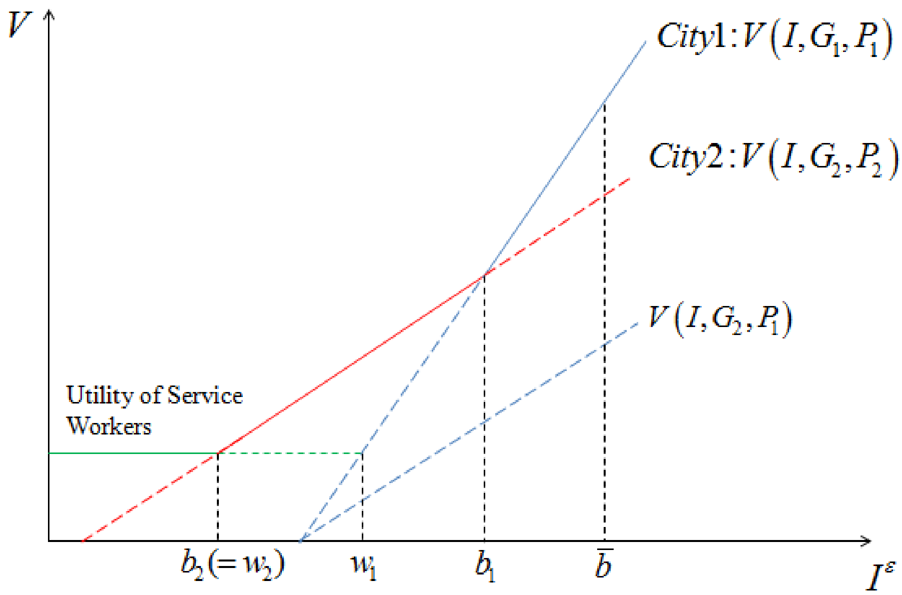

Figure 3-1: Utility offered by City 1 and City 2 at different skill level

Although the exact positions of two utility offer curves must be determined in general equilibrium, it is clear from Figure 3-1 that, as long as the composite non-traded service price is lower in city 1, city 2 must offer a lower housing price in order to have any positive number of workers to populate it. And as long as the composite non-traded service prices offered by the two cities are different, traded-sector workers sort themselves perfectly by skill levels between the two cities. High-skill workers outbid middle-skill workers in the city offering a lower composite non-traded service price. We formalize this result in proposition I.

Proposition I (skill sorting of traded-sector workers)

In asymmetric equilibrium, cities offer different levels of composite non-traded service price and different housing prices that compensate the difference in composite non-traded service price. Moreover, high-skill traded-sector workers sort into the city with a low composite non-traded service price but a higher housing price (City 1);the middle-skill traded-sector workers sort into the city with a high composite non-traded service price but a low housing price (City 2).

Proof. See the Appendix

Figure 3-1 shows that, given the population mass L and skill distribution $k(b)$, consumer preferences, production technologies, and housing supply elasticity, the asymmetric equilibrium is fully characterized by the two skill cutoff levels b_1 and b_2. The equations (3-15) through (3-25) below define these two cutoff skill levels. Equation (3-15) defines b_1, such that the traded-sector workers with skill b_1 are indifferent between two cities:

$$\frac{1}{\varepsilon}\left(\frac{b_1}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{b_1}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma} \tag{3-15}$$

Equation (3-16) defines the cutoff skill b_2, such that the workers with skill b_2 are indifferent between employment in the traded sector and employment in non-traded service sector,

$$b_2 = w_2 \tag{3-16}$$

Equation (3-17) describes the condition for non-traded service workers to be indifferent between two cities:

$$\frac{1}{\varepsilon}\left(\frac{w_1}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{w_2}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma} \tag{3-17}$$

The total population of non-traded service workers in the whole economy is $L\int_{\underline{b}}^{b_2} k(t)\,dt$. Let ϕ to denote the proportion of them who live in city 1. Equations (3-18) and (3-19) define the service price index in each city,

$$G_1 = \left[\frac{\phi L\int_{\underline{b}}^{b_2} k(t)\,dt}{F\sigma}\right]^{1/(1-\sigma)} w_1 \tag{3-18}$$

$$G_2 = \left[\frac{(1-\phi) L\int_{\underline{b}}^{b_2} k(t)\,dt}{F\sigma}\right]^{1/(1-\sigma)} w_2 \tag{3-19}$$

Equations (3-20) and (3-21) define the zero-profit conditions for non-traded service supply.

$$F\sigma = L\left(\frac{G_1}{w_1}\right)^{\sigma}\left\{\int_{b_1}^{\bar{b}}\frac{t}{G_1}\left[1-\beta\left(\frac{G_1}{t}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\right]dt+\phi K(b_2)\frac{w_1}{G_1}\left[1-\beta\left(\frac{G_1}{w_1}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\right]\right\}$$

(3-20)

$$F\sigma = L\left(\frac{G_2}{w_2}\right)^{\sigma}\left\{\int_{b_2}^{b_1}\frac{t}{G_1}\left[1-\beta\left(\frac{G_2}{t}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]dt+(1-\phi)K(b_2)\frac{w_2}{G_2}\left[1-\beta\left(\frac{G_2}{w_2}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]\right\}$$

(3-21)

On the right-hand side is the aggregate demand for individual variety in each city, which must equals $F\sigma$, to assure that the producers earn zero profit.

Equation (3-22) through (3-25) define the clearing of housing markets in both cities.

$$Q_{h,1} = \alpha\beta L\int_{b_1}^{\bar{b}} t\left(\frac{G_1}{t}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\frac{1}{P_{h,1}}k(t)dt+\alpha\beta\phi LK(b_2)w_1\left(\frac{G_1}{w_1}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\frac{1}{P_{h,1}} \quad (3\text{-}22)$$

$$Q_{h,2} = \alpha\beta L\int_{b_2}^{b_1} t\left(\frac{G_2}{t}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\frac{1}{P_{h,2}}k(t)dt+\alpha\beta(1-\phi)LK(b_2)w_2\left(\frac{G_2}{w_2}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\frac{1}{P_{h,2}} \quad (3\text{-}23)$$

$$P_{h,1} = \theta Q_{h,1}^{\rho} \quad (3\text{-}24)$$

$$P_{h,2} = \theta Q_{h,2}^{\rho} \quad (3\text{-}25)$$

In asymmetric equilibrium, our model predicts that non-traded sector employment in superstar city, *i.e.*, city 1 (with a low composite non-traded service price and a higher housing price), is always greater than that in city 2, as stated in the following proposition.

Proposition II (employment in non-traded service sector)

In asymmetric equilibrium, non-traded sector employment is larger in City 1 (with a *lower composite non-traded service price and a higher housing price)than in City 2.*

Proof. See the Appendix.

Intuitively, non-traded service workers in superstar city earn a lower wage than the cutoff traded sector workers, who are indifferent between the two cities. To compensate the low-skill service workers, who do not benefit very much from a low composite non-traded service price, the superstar city must pay a higher wage to compensate them for the higher housing price. Eventually, the population of low-skill workers, as well as the number of service varieties they produce, must grow to the extent that the service price index in the superstar city is lower despite the higher non-traded sector labor cost.

In summary, the asymmetric equilibrium emerging from the interaction between non-homothetic preferences and increasing return to local demand for non-traded consumer amenities, the supply of which employs low-skill workers, has richer implications beyond house price dispersion. The model also predicts the impact of aggregate skill distribution on income disparity within as well as between cities, on the employment of non-traded sector in the economy and across cities, and on the size distribution of cities. Since these predictions are based on structural parameters, such as income and price elasticity of demand, taste for variety, increasing return in non-traded service supply, housing supply elasticity, and skill distribution, the model can be calibrated to evaluate various counterfactuals, such as change in skill distribution and housing supply elasticity.

3.4 Algorithm

To illustrate the emergence of an asymmetric equilibrium, we provide an algorithm to find equilibrium cutoff skill levels b_1 and b_2. We adopt a bounded Pareto distribution to characterize the aggregate skill distribution. The Pareto distribution is a good approximation for income distribution observed in many countries, such as US. Its shape can be modified by a single parameter, a shape parameter ξ, which also determines inequality measures such as Gini coefficient. We adopt a support for the skill distribution from 1 to 100, to broadly reflect the reality of productivity spectrum across individuals in an economy like the US. Thus the skill probability density function is given by $k(b) = \xi b^{-\xi-1} / (1 - 0.01^{\xi})$, with $\xi > 1$, which has a mean value of approximately $\xi/(\xi-1)$ and a Gini coefficient of approximately $1/(2\xi-1)$.

The existence of the equilibrium can be demonstrated using a phase diagram for the two cutoff skill levels b_1 and b_2, as shown in Figure 3-2. The horizontal axis of the diagram is skill cutoff for service workers, b_2, and the vertical axis is the skill cutoff for traded-sector workers in city 1, b_1. Note that b_2 coincide with w_2, the non-traded sector wage rate in city 2. To determine how b_1 and b_2 will adjust when they deviate from the equilibrium levels, we construct two equilibrium curves. The first one traces the combination of b_1 and b_2 that clears the market for non-traded service employment. We refer to it as "zero excess employment demand" curve. The excess demand for non-traded service workers, or excess employment demand *EED*, is determined by the following equation:

$$EED = L\left(\frac{G_2}{w_2}\right)^{\sigma}\left\{\int_{b_2}^{b_1}\frac{t}{G_1}\left[1-\beta\left(\frac{G_2}{t}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]dt+\left(1-\phi\right)K(b_2)\frac{w_2}{G_2}\left[1-\beta\left(\frac{G_2}{w_2}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]\right\}-F\sigma.$$

To compute *EDD* all equilibrium conditions described by Eq (3-15) through Eq (3-25), except Eq (3-15) and Eq (3-21), are satisfied. The zero excess employment demand curve is thus defined by *EED* = 0. It is shown as the steeper curve in Figure 3-2. To the left of this curve, the non-traded service wage rate is too low, such that the supply of workers to the non-traded sector falls short of the demand (*EED*>0). As a result, the non-traded sector wage rate, hence b_2, will rise.

The second equilibrium curve traces the combination of b_1 and b_2 that clears the housing market in city 1 and city 2. The housing market clearance requires the marginal traded-sector worker in city 1 to obtain the same utility that City 2 can offer, so as to be indifferent between the cities. We refer to this curve as "equal utility for marginal worker" curve. The utility difference between City 1 and City 2 for the marginal traded worker in City, denoted by *UDM*, is determined by:

$$UDM = \frac{1}{\varepsilon}\left(\frac{b_1}{G_1}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma}=\frac{1}{\varepsilon}\left(\frac{b_1}{G_2}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma}.$$

Again, in computing *UDM*, all equilibrium conditions, Eq (3-15) through Eq (3-25), except Eq (3-15) and Eq (3-21), are satisfied. The equal marginal utility curve is thus defined by *MUD* = 0. It is shown as the flatter curve in Figure 3-2. Below this curve, the marginal traded-sector worker in city 1 will find city 1 too expensive (hence offering a lower utility than city 2, *MUD*< 0) and thus prefer to move to city 2. As the middle-skill marginal workers get pushed out of city 1, the skill cutoff for traded-sector workers in city 1 rises.

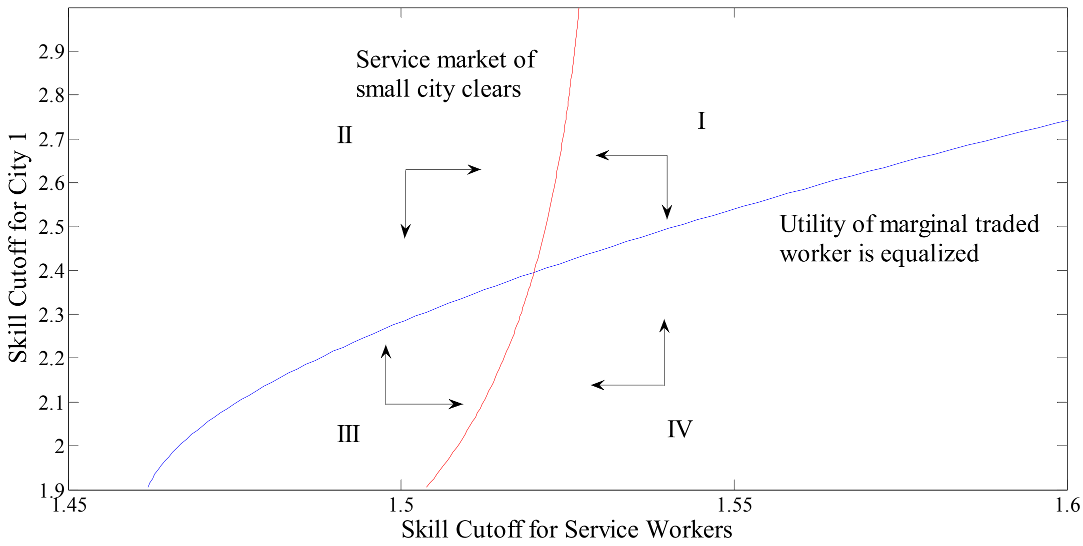

Figure 3-2: Phase diagram for equilibrium skill cutoff points

Notes: The model parameters are $L = 4$, $\alpha = 0.6$, $\beta = 0.6$, $\varepsilon = 0.1$, $\sigma = 7$, $\gamma = 0.1$, $\theta = 0.2$, $\rho = 0.5$, $F = 0.001$ and $\xi = 13/6$ (which gives a skill Gini coefficient of approximately 0.3).

The two equilibrium curves, the zero excess employment demand curve and equal marginal utility curve, divide the phase diagram into four regions, as shown in Figure 3-2. In each region, b_1 and b_2 will change due to market adjustment, as indicated by the arrows. The phase diagram shows that b_1 and b_2 will converge to the intersection of the two equilibrium curves, which defines the equilibrium.

The phase diagram provides two important insights. First, there exists a unique asymmetric equilibrium, as long as the preference for consumer amenity variety is not too strong (σ not too small) in relation to housing supply elasticity ($1/\rho$ not too large). Otherwise, city 1 will end up attracting everyone, and the equilibrium degenerates into a single-city outcome. Second, the asymmetric equilibrium is stable. Any deviation from the equilibrium skill cutoff combination b_1 and b_2 will be corrected by market adjustment.

It can be shown that in equilibrium, total income from traded sector exactly covers the aggregate housing expenditure in each city:

$$P_{h,1}Q_{h,1} = \alpha L \int_{b_1}^{\bar{b}} tk(t)\,dt \tag{3-26}$$

$$P_{h,2}Q_{h,2} = \alpha L \int_{b_2}^{b_1} tk(t)\,dt \tag{3-27}$$

These two equations are convenient for solving equilibrium, because together with Eq (3-24) and Eq (3-25) they determine housing prices and quantities based on two variables, b_1 and b_2, only. Thus, for any initial values b_1 and b_2, we can determine housing price and quantity in each city. Then, we use Eq (3-17), Eq (3-18), Eq (3-19), and Eq (3-22) to solve two remaining unknowns, ϕ and w_1. Subsequently, we calculate *EED* and *MUD* and adjust b_1 and b_2 in the direction that reduces the magnitude of *EED* and *MUD*. We repeat these steps until *EED* and *MUD* converge to zero.

3.5 Numerical Simulations

Using the algorithm described above, we numerically solve the asymmetric equilibrium for a two-city economy. We present the baseline case in Table 3-1.In equilibrium City 1 is more populous than City 2, attracts top skill traded-sector workers, offers a lower composite price index of non-traded services, has a higher housing price, and employs proportionally more workers in the non-traded sector. Unlike Gyourko, Mayer and Sinai (2013), where the superstar city is exclusive to high-income households, the equilibrium of our model entail the larger city (City 1) to have a wider spectrum of skills as documented in Eeckhout et al. (2010).

Table 3-1: Two-city Asymmetric Equilibrium

Main features	**City 1**	**City 2**
Population	2.0583	1.9417
Traded-sector workers' skill	2.3955-100	1.5199-2.3955
Non-traded service employment	1.4558	0.9295
Non-traded service wage	1.5855	1.5199
Composite price index of non-traded services	0.6514	0.6901
Housing price	0.4042	0.3588

Notes: The baseline case parameters are $L = 4$, $\alpha = 0.6$, $\beta = 0.6$, $\varepsilon = 0.1$, $\sigma = 7$, $\gamma = 0.1$, $\theta = 0.2$, $\rho = 0.5$, $F = 0.001$, and $\xi = 13/6$ (a skill Gini coefficient of approximately 0.3).

We next show how the equilibrium evolves as the aggregate skill disparity, indicated by the skill Gini coefficient, rises. We decrease the shape parameter of skill distribution such that its Gini coefficient increases from 0.3 to 0.6 (reflecting an increasing share of high-skill workers in the economy).The results are shown in Figure 3-3 through Figure 3-10. As the skill Gini coefficient increases to 0.6, city 1 grows even bigger and accounts for 62% of total population, as opposed to 51% when Gini coefficient is 0.3. During this process, the skill cutoff for the traded-sector workers in city 1 also increases, indicating that middle-skill traded-sector workers are pushed to City 2. As shown in Figure 3-3, the skill cutoff for non-traded service sector also increases, indicating that least skilled traded-sector workers are switching to the non-traded sector to cater to an increasing demand for non-traded services.

The employment share of service workers increases in both cities. Figure 3-4 shows the gain in non-traded service employment in the economy as a whole rise about 13 percentage points (with a corresponding loss of employment share by the traded goods sector) as the skill Gini coefficient doubles from 0.3. Interestingly, since 1960 U.S. manufacturing employment share declined by about 15 percentage points(Baily & Bosworth, 2014) as income Gini coefficient rose from 0.35 to 0.45. The results of our model suggest that the loss of low-skill manufacturing jobs in the U.S. is not entirely due to competition from China; growing domestic demand for non-traded services would play an important role.

Figure 3-5 shows that, as the aggregate skill inequality increases, City 1 becomes more attractive in terms of the variety of local consumer amenities it can offer. The composition price index of non-traded services in City 1 over than in City 2 declines from 0.94 when the skill Gini is 0.3 to 0.90 as the skill Gini rise to 0.6. The housing price premium in City 1 increases from 12% to 32%. Figure 3-6 shows that house price dispersion and city population size dispersion both increases with aggregate skill inequality.

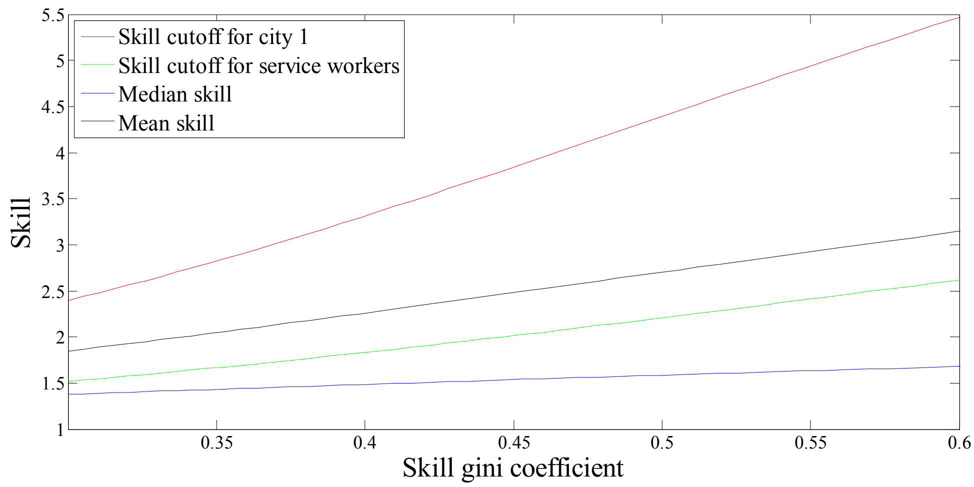

Figure 3-3: Skill cutoffs for city 1 and service employments over skill Gini coefficient

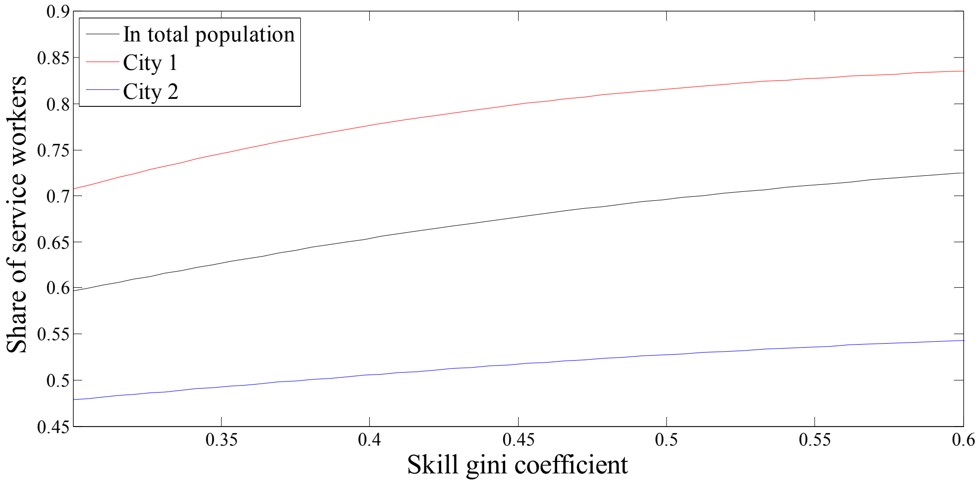

Figure 3-4: Share of service workers over skill Gini coefficient

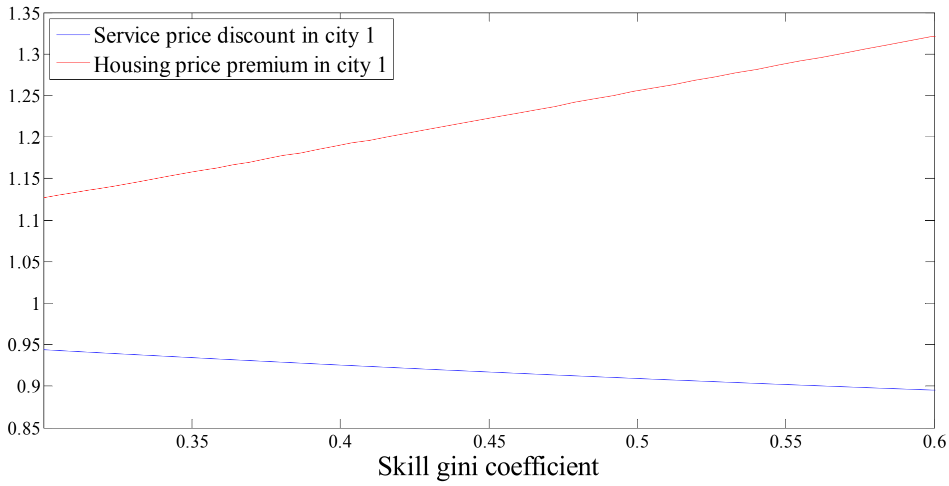

Figure 3-5: Service price discount and housing price premium in city 1 over skill Gini coefficient

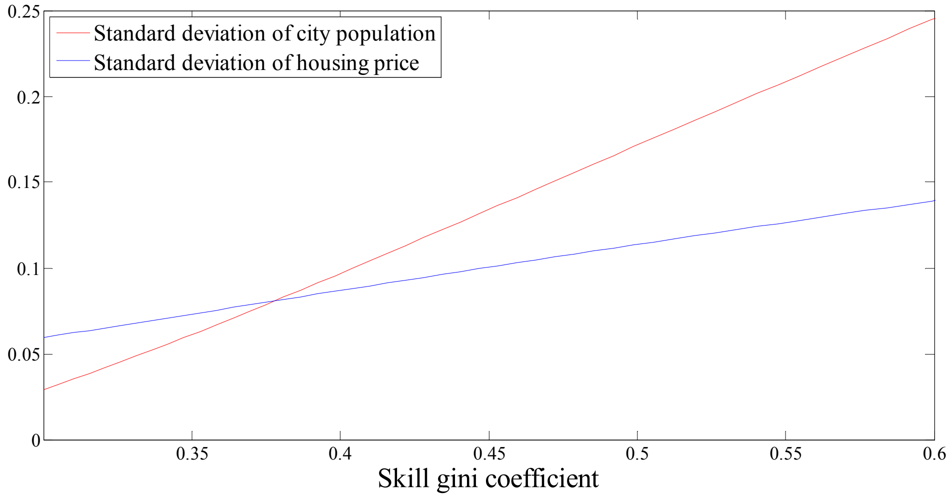

Figure 3-6: Standard deviation of city population and housing price over skill Gini coefficient

To explore the welfare implications of aggregate skill inequality for workers at different skill levels, we depict the utility paths of workers at skill level 1, 2, 4 and 8, respectively, in Figure 3-7 through Figure 3-10.The red lines in the figures represent the utility that workers can obtain working in the traded-good sector in city 1. The blue lines represent the utility offered by the traded-sector employment in city 2. The green lines represent the utility offered by non-traded service sector (in either city). Workers will choose the occupation and city that offer the highest utility.

As shown in Figure 3-7, the bottom-skill workers will always be employed in the non-traded service sector. Their welfare increases as the aggregate skill inequality rises. Workers at skill level 2 are employed in the traded sector in City 2 when the aggregate skill inequality is low, as shown in Figure 3-8. Ata skill Gini coefficient of 0.45, these workers find it profitable to switch to employment in the non-traded service sector. Their welfare initially declines as the aggregate skill inequality rises and then improves with the aggregate skill inequality after they switch to employment in the non-traded sector.

Figure 3-9 and Figure 3-10 display the welfare paths for workers at skill level 4 and 8, who always work in the traded sector. Their welfare always declines with aggregate skill inequality. The middle-skill traded-sector workers enjoy City 1 when aggregate skill inequality is low and are pushed to City 2 when the aggregate skill inequality becomes sufficiently high. The high-skill traded-sector workers always choose City 1.

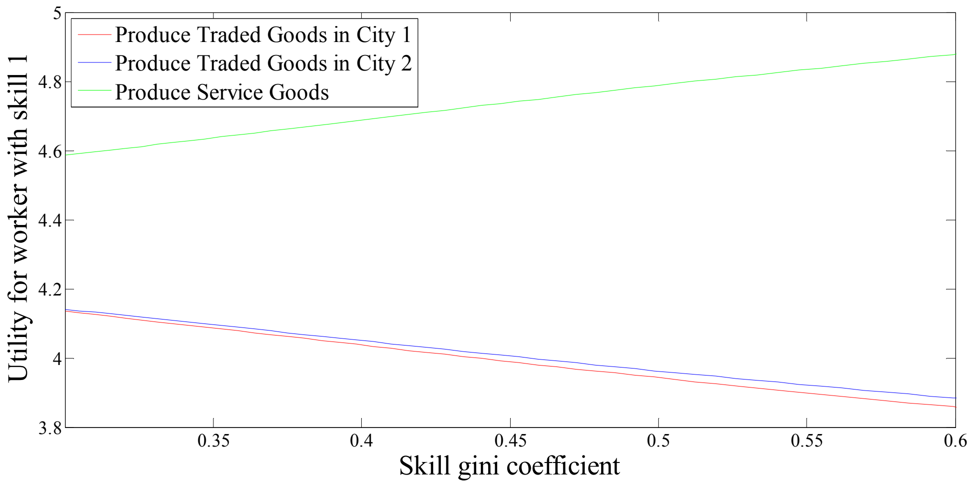

Figure 3-7: Utility of workers at skill level 1

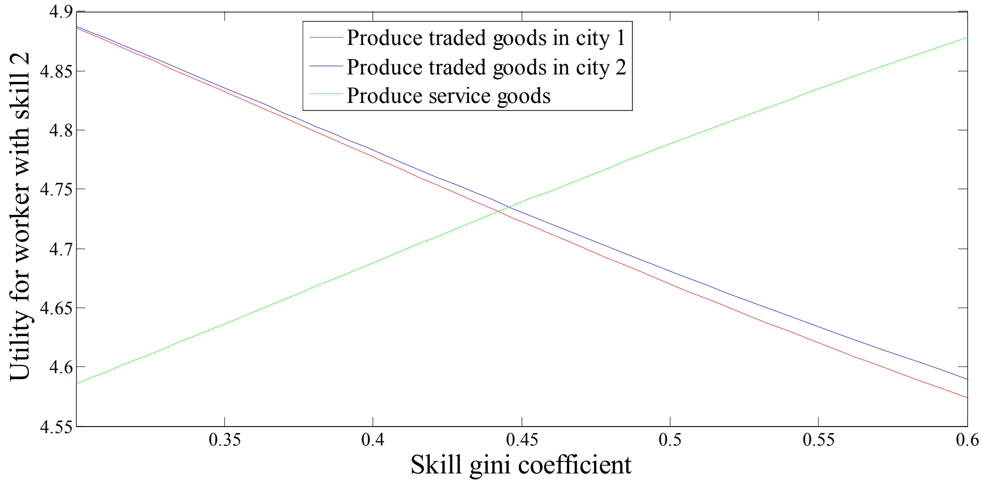

Figure 3-8: Utility of workers at skill level 2

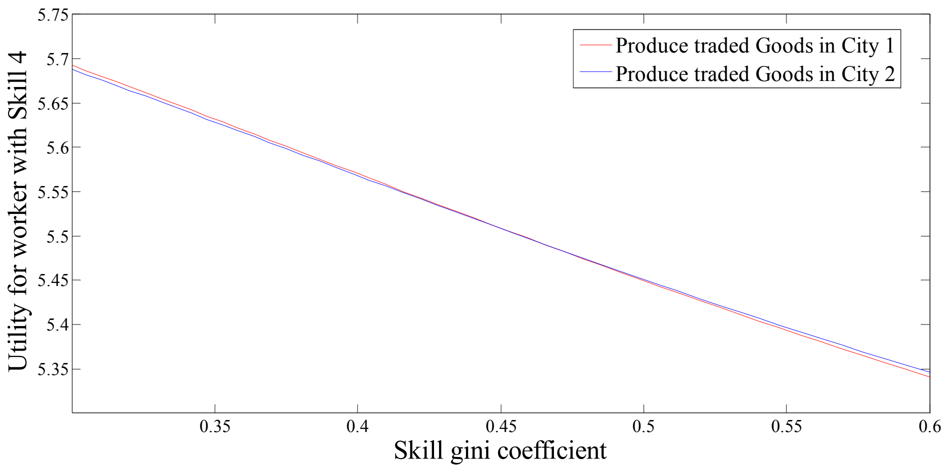

Figure 3-9: Utility of workers at skill level 4

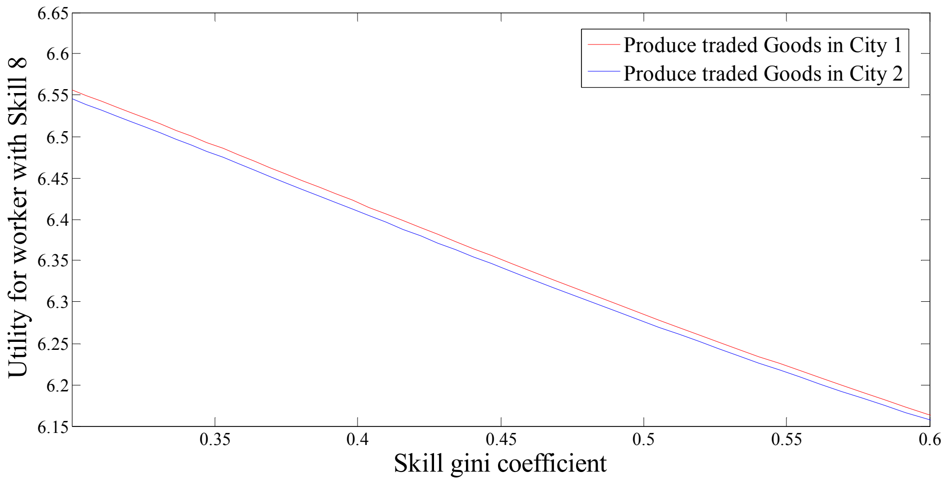

Figure 3-10: Utility of workers at skill level 8

In general, low-skill workers, who always choose employment in the non-traded sectors, always benefit from increased aggregate skill inequality, which elevates the demand for non-traded services. Low-skill traded sector workers are harmed by increased aggregate skill inequality initially but then benefit from it after they eventually move to the non-traded sector. Workers at the middle and high skill levels, who never find non-traded service employment to their advantage, are always harmed by an increase in aggregate skill inequality, which pushes up the labor cost of non-traded services. The middle-skill workers tend to lose more because they eventually also get pushed out of City 1, which offer more attractive consumer amenities. Thus, the welfare impact of rising aggregate skill inequality across the skill spectrum is U-shaped (more accurately, left-tilted L-shaped), as shown in Figure 3-11.

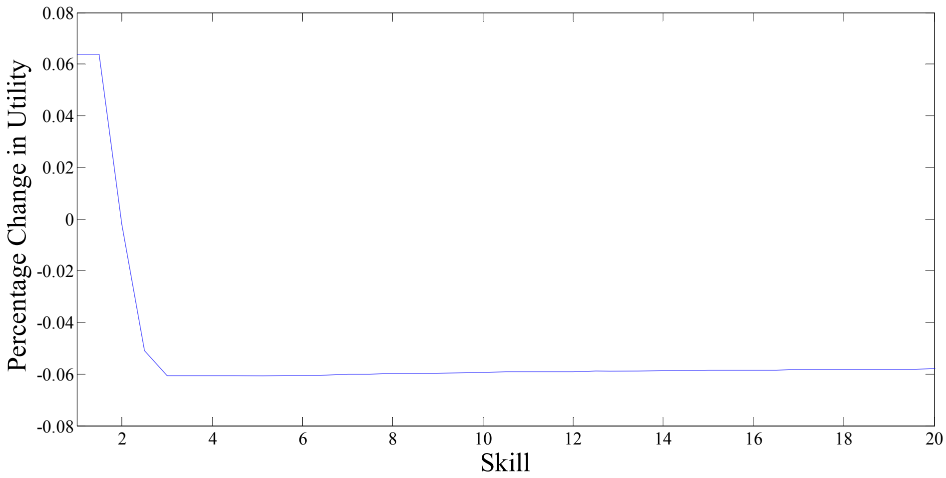

Figure 3-11: The welfare impact of rising aggregate skill Gini coefficient from 0.3 to 0.6

We further explore the effect of preference for consumer amenity variety and housing supply elasticity on spatial equilibrium outcomes. We examine two alternative scenarios. First, we reduce the consumer taste for non-traded service variety by increasing σ from 7 (baseline value) to 8. Second, we reduce the housing supply elasticity by increasing ρ from 0.5 (baseline value) to 0.75 (corresponding to a median housing supply elasticity across US cities reported by Saiz (2010)). The results are presented in Table 3-2. Column 1 shows the simulation results for the baseline scenario. Each cell displays two numbers, corresponding to the outcome associated with a skill Gini coefficient of 0.3 (top number) and a skill Gini coefficient of 0.6 (bottom number), respectively. Column 2 and 3 show the simulation results for the alternative scenarios.

At a weaker preference for consumer amenity variety, the composite price of non-traded services becomes higher in both cities in comparison with the baseline case. City 1 becomes smaller, as the lower-skill traded-sector workers no longer find the consumer amenity benefit in City 1 sufficiently attractive to justify the higher housing price in City 1. As City 1 share of worker population decline, its advantage in offering better local consumer amenity also declined, reflected by the convergence of its composite non-traded service price towards that of City 2. Housing price dispersion between the cities also decrease. Non-traded sector employment does not change much, as the effect of reduced demand on productivity is compensated by increased production scale for each of the smaller set of non-traded services. The difference in non-traded sector employment between City 1 and City 2, however, narrows. Total housing expenditure in the economy (an income leakage to friction) does not change much, although City 1's share of that decreases. The welfare diminishes for everyone because of the higher composite price of non-traded services. Moreover, the welfare inequality diminishes as the higher composite price of non-traded services hurt high-income workers more than low-income workers. Finally, we find that widening aggregate skill inequality has a smaller impact on the dispersion of mean skill level, composite non-traded service price, housing price, and population size across the cities.

Table 3-2: Comparative Static Analysis: Asymmetric Equilibrium

Column	1	2	3

Scenario	Baseline	$\sigma = 8$	$\rho = 0.75$
Skill cutoff for traded-sector workers in City 1, b_1	2.3955 5.4655	2.5073 6.9131	2.5589 7.7438
Skill cutoff for non-traded service employment, b_2	1.5199 2.6190	1.5203 2.6252	1.5096 2.5855
City 2 mean skill	1.5429 2.4901	1.5568 2.5940	1.5609 2.6337
City 1 to City 2 mean skill ratio	1.3851 1.4241	1.3875 1.3952	1.3967 1.3920
Non-traded-sector employment share	0.5963 0.7245	0.5965 0.7254	0.5904 0.7197
Non-traded service employment in city 2	0.9295 0.8252	0.9972 1.0473	1.0249 1.1513
City 1 to City 2 ratio of non-traded-sector employment	1.5662 2.5120	1.3931 1.7707	1.3041 1.5007
City 1 share of total population	0.5145 0.6201	0.4837 0.5375	0.4647 0.4952
City 2 composite non-traded service price	0.6901 1.2275	0.7848 1.3708	0.6775 1.1624
City 1 to City 2 ratio of composite non-traded service price	0.9439 0.8953	0.9643 0.9449	0.9646 0.9511
City 2 housing price	0.3588 0.3979	0.3677 0.4314	0.4469 0.5653
City 1 to City 2 housing price ratio	1.1264 1.3211	1.0787 1.1635	1.0679 1.1239
Total housing expenditure	2.8060 5.2072	2.8041 5.1901	2.8280 5.2261
Housing expenditure in city 1	1.6510 3.6320	1.5608 3.1746	1.5222 2.9670
Housing expenditure in city 2	1.1550 1.5752	1.2432 2.0155	1.3058 2.2591
Utility of workers at different skill levels			
Skill=1	4.5859 4.8781	4.5216 4.8082	4.5089 4.7785

Skill=2	4.8878 4.5900	4.8194 4.8082	4.8188 4.7785
Skill=4	5.6924 5.3465	5.6122 5.2693	5.6232 5.2641
Skill=8	6.5564 6.1637	6.4630 6.0732	6.4866 6.0807
Skill=20	7.7943 7.3421	7.6823 7.2318	7.7237 7.2575

Notes: The baseline case parameters are $L = 4$, $\alpha = 0.6$, $\beta = 0.6$, $\varepsilon = 0.1$, $\sigma = 7$, $\gamma = 0.1$, $\theta = 0.2$, $\rho = 0.5$, $F = 0.001$. The top number in each cell corresponds to the case of skill Gini=0.3, the bottom number, the case of skill Gini=0.6.

At a lower housing supply elasticity, $\rho = 0.75$, City 1 becomes more exclusive to higher-skill traded-sector workers, and its population share in the economy diminishes compared to the baseline case. The composite non-traded service price in City 2 decreases, and so does City 1's advantage in the composite price, as the demand for non-traded services is reduced by higher housing prices and housing expenditure leakage in the economy. The non-traded sector wage rate decreases, and so does the employment share of the non-traded service sector. Although the dispersion in mean skill level increases somewhat (at a low skill Gini coefficient), the dispersion in composite non-traded service price, housing price, and population size all decrease. Welfare for everyone is diminished due to higher housing expenditure (leakage). Again we note that widening aggregate skill inequality has a smaller impact on the dispersion of mean skill level, composite non-traded service price, housing price, and population size across the cities. This last result is in marked contrast with the result in Gyourko, Mayer and Sinai (2013), where more restrictive housing supply in a city helps strengthening its superstar-city status. In our case, more restrictive housing supply is an obstacle for the high-income city to take advantage of increasing returns in local consumer amenities.

3.6 Conclusion

We have presented a model to show asymmetric spatial equilibrium can emerge across perfectly symmetry locations in the presence of increasing returns in local consumer amenities and non-homothetic preferences for such amenities. Both premises are supported by empirical evidence recently documented in the literature. The model can account for widened housing price dispersion across cities solely by increased aggregate skill inequality (or increased share of high-skill workers) in the economy. A larger share of high-skill workers reinforces the increasing returns in local consumer amenities and income segregation among traded-sector workers across cities. The model helps sharpening an important insight in Gyourko, Mayer and Sinai (2013) but also clarifying the effect of local housing supply elasticity on asymmetric equilibrium outcome: restrictive housing supply may make the "superstar" city more exclusive but would moderate, rather than exacerbate, housing price dispersion across cities when aggregate skill inequality rises. This clarification has important policy implications—expanding housing supply in a "superstar" city can have unintended consequence of reinforcing its advantage in local consumer amenities and hence its high housing price.

More importantly, our model builds on a micro foundation that can be calibrated to quantify the contribution of aggregate skill inequality to housing price dispersion observed in a real economy. In addition, our model can also account for the rise of the employment share of non-traded service sector resulting from increased aggregate skill inequality, a significant feature of many economies like the US. Related to the impact of aggregate skill inequality on employment structure, our model reveals that widening aggregate skill inequality can benefit low-skill workers due to increased demand for non-traded services, which low-skill workers generally have a comparative advantage in producing. Moreover, the welfare gain of the low-skill non-traded service workers is at the expense of high-skill traded-sector workers, who, although enjoying a greater variety of non-traded services in the presence of a larger share of high-skill workers in the economy, nevertheless have to pay higher labor cost for each variety of non-traded services.

Our model can be extended to incorporate local agglomeration economies in the traded-sector employment and to cases with more than two locations (to study more realistic housing price dispersion across cities).

4 Land Use and Welfare in a Consumer City

Abstracts

The past three decades witnessed a resurgence of big cities in US and many developed economies driven by the growing demand for urban amenities like museums, restaurants and concerts (e.g. Glaeser & Gottlieb, 2006). What are the implications of the amenity-based urban agglomeration for the spatial structure and land-use efficiency of cities? We address the question using a monocentric-city model, where residents travel to the city center to consume amenities, which are produced locally with economies of scale.

The city is populated with workers with heterogeneous skills, who are employed either to produce a traded good at home and earn a wage according to their skill level, or to produce amenities at the city center and earn a wage according to the local demand. Workers consume the numeraire traded good, amenities, and housing, with the demand for amenities increasing with income. In equilibrium, low-skill workers choose to produce amenities and, when the commuting cost is not sufficiently high, live in the intermediate zone from the city center. High-skill workers live in the central zone to save the cost of their frequent trips to consume amenities and middle-skill workers in the outer zone to enjoy more housing consumption.

We show that, under monopolistic pricing of the amenities, innovations raising amenity quality would make the city more compact as a result of increased demand for traveling to the city center to consume amenities. Moreover, the competitive commercial rent at the city center, which sustains the monopolistic pricing of amenities, results in excessive rationing of amenity demand, urban sprawl, and a deadweight loss of welfare.

Keywords: monocentric city model; skill disparity; income sorting; house price premium; increasing return; taste for variety; efficiency; social welfare

JEL classification: J3 R1

4.1 Introduction

In past decades, US economy witnesses a rising economic status of the service sector. From 1950 to 2010, the share of consumption by Americans devoted to service goods has increased significantly from 40 percent to 65 percent (Timo Boppart, 2014). The booming development of service sector mirrors a decline in the manufacturing sector. The employment in manufacturing sector not only fall dramatically in a relative sense but also in absolute numbers (Baily & Bosworth, 2014; D. Lee & Wolpin, 2006).

Glaeser et al. (2001) document the rise of "consumer city", where the rich workers live in the central city, even though they work in suburbs. The trend is likely to continue because the demographic changes in U.S. also work in favor of the majority's choice for consumer city: smaller households, later marriages, decisions not to have babies, the emergence of a huge and active baby boom population in its sixties and seventies (Ehrenhalt, 2012). All these changes generate more households who are willing to live in a smaller house in a central location to enjoy urban life.

Despite the observed trend that service sector is gaining its importance in the economy and that people's location preference is shifting to central city that can provide more options for non-traded services, no theoretical studies explore its implications for the internal structure of city and resource allocation efficiency in the city. To fill this research gap, we develop a monocentric city model to explore the economics of consumer city.

We assume that the non-traded services are produced and consumed in a single central location, referred to as Central Commercial District, or CCD. This assumption captures positive externalities in non-traded service consumption, which arise from access to diversity by consumers when the services are concentrated in one location. Heterogeneous workers that are differentiated by skill level, earn income from employment either in the traded good sector or the non-traded service sector. Worker productivity in the former sector depends on skill but not location, whereas in the latter sector, worker's wage depends on aggregate local demand but not skill. A fixed cost is required for the production of local service, which rises with service quality, giving rise to increasing return to local demand. Workers derive their utility from the consumption of a numéraire traded good, housing, and non-traded services. The preferences are non-homothetic such that the demand for non-traded services is income elastic. The non-traded service employments are assumed to concentrate at the city center, referred to as a Central Commercial District (CCD), due to positive externalities in non-traded service consumption. There is a single CCD landlord, who tenders the space out to a commercial service operator capable of paying the highest land rent. We assume service workers travel to work in the CCD and pay a commuting cost that linearly increases with distance to CCD, to capture the "U" shape income profile with respect to distance to the CCD observed in major US cities. Consumers travel to the CCD to purchase services. On each trip, a consumer purchases one unit of service goods and pays a travel cost that linearly increases with distance to the CCD. In equilibrium, low-skill workers choose non-traded service occupation according to comparative advantage. High-skill traded-sector workers live in the central city to share the location with better access to local services and higher land rent, low-skill service workers occupy the region adjunct to the central city, while middle-skill traded-sector workers live in the suburban area with the worst access to non-traded services and lowest housing price. Hence, from the city center to city boundary, resident's income first decreases and then increase. Similarly, a U-shaped relation between resident's income and distance to the city center exists in many large US cities, as documented in previous literature (Glaeser et al., 2008).

By raising upper bound of skill distribution, we conduct a range of counterfactual experiments to understand the effect of increasing skill inequality on the spatial structure, economic structure of consumer city and social welfare. Our model shows that increasing skill dispersion will enhance gentrification at city center. More high-skill traded-sector workers will move into the central city, pushing middle-skill traded-sector workers to the suburbs. The bid rent curve of housing price will also become steeper at city center, indicating that housing price premium at city center will increase.

Also, as the dispersion of population skill increases, the city will convert to an economy that is more oriented by the non-traded service sector. First, non-traded service firm will improve the quality of its product to meet the growing demand for high-quality services, stemming from the increasing numbers of high-skill workers. Second, consumers will spend a greater share of their income on non-traded services, accompanied by a decrease in the share of expenditure on housing goods and traded goods. Third, a higher proportion of the total population will work in the service sector, featuring some middle-skill traded-sector workers will convert to produce non-traded services, driven by greater aggregate demand for non-traded services.

Our model also predicts that increased population skill dispersion do less harm to the non-traded service workers, who have little skills. While all workers in the city suffer from rising housing price in the central city, the low-skill service workers are partially compensated by rising wage, driven by increasing demand. In a city with more elastic housing supply and lower travel costs for service consumers, the high-skill traded-sector workers will suffer relatively less from rising skill dispersion. It happens because the housing price in the central city will remain relatively constant, even when total demand for housing increases. Also, when travel costs are low, the high-skill workers can substitute the non-traded service for housing consumption.

One of our main goals is to examine the resource allocation efficiency of the consumer city. We find that, when the city maximizes the aggregate commercial land rent in the CCD, the non-traded services are priced above the marginal production costs. Maximizing commercial land rent in the CCD is associated with a deadweight loss and it causes inefficiency in resource allocation--the share of employments in the non-traded service sector is below social optimal level and dispersed urban structure generates more frictions. Hence, we argue that the government should use the marginal-cost pricing regime by regulating the price of services and subsidizing the fixed costs changed on the service producer. By adopting the marginal-cost pricing regime, the government can encourage switch of the employments from traded goods sector to the non-traded service sector. Also, when consumption amenities are improved, the urban structure will become more compact, hence, associated with lower urban frictions. When the government adopts the marginal-cost pricing regime, all workers will experience welfare gains. But the policy will generate fewer benefits to the middle-skill traded-sector workers. It is because the middle-skill traded-sector workers do not experience a wage growth as what happens to the low-skill workers and they are constrained in the budget to take advantage of the lower service price.

The present study is related to the literature documenting the rise of “consumer city”. Glaeser et al. (2001) show that the cities with attractive non-traded services experience faster population growth and that educated and wealthy households choose to live in the central city, even when they work in suburbs. Previous studies also show that the mix of non-traded services is closely associated with the preference of local consumers (Handbury, 2013; Waldfogel, 2008). Consistently, our model predicts that, when the number of high-income households increases in the consumer city, quality of non-traded services will also be improved. Hence, our finding recalls the idea of "preference externalities" that, when a product's provision entails fixed costs, it will be made available only if a sufficient number of people want it(Waldfogel, 2008). Previous studies document that gentrification creates new jobs in non-traded service sectors (Lester & Hartley, 2014; Schuetz, Kolko, & Meltzer, 2012). Our model agrees with this argument and predicts that, as the dispersion of skill increases, gentrification will be enhanced, and a greater proportion of the population will work in the non-traded service sector.

Holian & Kahn (2013) shows that high-quality-of-life consumer cities are more likely to be low-carbon cities. They emphasize the importance of an attractive CBD in encouraging residents to visit and spend time in the center city, thus using more public transportation and reducing greenhouse gas emissions. Consistent with their results, our model predicts that higher quality of non-traded services is more sustainable in a city with a small commuting cost for non-traded service workers. Our model highlights that better public transportation system will induce the rise of high-quality "consumer city" through reducing the labor cost for service firms. When the commuting cost for the low-skill worker is lower, the service firm can pay a lower wage to the service workers. Hence, the service firm can hire a greater amount of low-skill service workers to improve the quality of non-traded service. Our model emphasizes the importance of enhancing the mobility of low-skill workers in creating an attractive consumer city.

From the theoretical perspective, our study is rooted in the traditional literature on monocentric city model (Alonso, 1964b; Mills, 1967; R. Muth, 1969) by emphasizing the tradeoff between access to the CBD and housing expenditure. Transitional models study resource allocation regarding housing and commuting in a "manufacturing city" setting, where all employment is in the traded goods sector and located at the city center. In this context, economic agents concentrate to form a city to make use of agglomeration economics in traded goods sector. But, the present paper emphasizes the idea of “consumer city" and examine a different set of tradeoffs when agglomeration economics is derived from the non-traded service sector and when the size of the sector is endogenously determined by local demand.

Our study is also related to the literature that focuses on income-residential pattern within a city. Previous studies consider how income affects the valuation of land, leisure foregone in commuting, access to amenities and access to public transportation, all of which contribute to determining the residential location (Brueckner et al., 1999; Glaeser et al., 2008; Rappaport, 2014; Wheaton, 1974). In particular, our paper is closely related to the study of Bruckner, Thisse, and Zenou (1999). Their study highlights the importance of exogenous amenities in determining residential location by income groups. Different from their study, we focus on consumption amenities that are endogenously created by the interaction between the tastes of high-skill workers and the labor supply of low-skill workers. Therefore, our model allows us to better understand the relation between skill inequality and consumption amenities in a unified framework. Also, the present theory rationalizes a non-monotonic relationship between resident income and distance to the city center, which is observed in old large US cities (Glaeser et al., 2008).

Our study is also associated with the recent studies that explore the implications of social interactions for urban structure. Helsley and Zenou (2014) present a monocentric city model to explore the implications of consumers' centrality in the social network for their physical locations in the city. The authors show that the consumers who are more central in the social network will live closer to the city center, provided that these consumers benefit more from social interactions. Different from their study, we think about workers of heterogeneous skills who sort within city according to their preferences for non-traded services. Helsley and Zenou (2014) assume that all social interactions occur at a single central location, and the present study assumes that production and consumption of non-traded services only occur at a single central location.

Our study is also related to the literature on club theory (Ng & Weisser, 1974; Scotchmer, 1985, 2002). The literature has two important insights. First, in the presence of externalities, it is efficient to price the club goods at marginal social cost and finance fixed cost of the club by collecting membership fees. Second, to maximize the participation of club members, it is optimal to collect higher membership fee from the consumers who have higher willingness to pay. In our model, the consumer city is a club where consumers cluster to share the fixed cost of non-traded services. Consumer purchases a house to acquire the membership of the club and pay additional service price and travel cost as a usage fee. Through choosing residential locations, heterogeneous agents will reveal their preference for the non-traded services. In equilibrium, the consumers with stronger preference purchase more expensive houses closer to the CCD, hence paying higher membership fees. Also, in the presence of pecuniary externalities, we argue that variable cost and fixed cost of the non-traded service sector should be financed separately. By adopting a marginal-cost pricing regime, the government can correct the distortion in resource allocation and improve social welfare.

Our model is presented in section (4.2). We discuss the properties of equilibrium in section (4.3). We characterize the equilibrium in section (4.4) and show the existence of equilibrium by construction in section (4.5). We present a baseline scenario in section (4.6). Counterfactual experiments are shown in section (4.7). We discuss welfare distribution in section (4.8). Discussion on efficiency is presented in section (4.9). Section (4.10) concludes.

4.2 The Model

We consider a linear monocentric city[2]. Land of the city is represented by a segment on the positive real line that the boundary is endogenously determined. Within city boundary, the land is occupied for residential use and beyond the boundary is for

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[2] The model can also be extended to plain geography, like in Lucas and Rossi–Hansberg (2002). We leave it for our future work.

agriculture use. Absentee landlords own the city land. There is a Central Commercial District (CCD) in the city and its location is exogenous. Location of any place in the city is described by its distance to the CCD, x. Population in the city is exogenous. Workers are heterogeneous regarding skill b that is distributed according to a probability density function $k(b)$.

4.2.1 Consumption

Workers consume three types of goods: traded goods, non-traded services, and housing. Consumer's preference is given by indirect utility,

$$V = \frac{1}{\varepsilon}\left(\frac{I}{G/q}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P}{G/q}\right)^{\gamma} + \upsilon, 1 > \varepsilon > \gamma > 0, \beta > 0, \upsilon > 0 \tag{4-1}$$

where I is disposable income, G is the price of non-traded services, q is the quality of non-traded services, P is the composite price for traded goods and housing goods defined by,

$$P = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} P_h^{\alpha} P_X^{1-\alpha} \tag{4-2}$$

where P_h is housing price and P_X is the price of tradable goods. We normalize P_X to one. Because (4-2) corresponds to a sub-utility of Cobb-Douglas form, the expenditure share of housing goods is relatively constant to that of traded goods, which equals α. Consumers have a taste for service quality and they care about quality-adjusted service price, G/q.

The specified indirect utility function is a subclass of price independent generalized linearity (PIGL) preferences rooted in Muellbauer (1975, 1976). If we neglect the quality, the utility function includes homothetic preferences as special cases. If $\gamma = \varepsilon = 0$, we have Cobb-Douglas preferences, $V = \log\left(\frac{I}{P^{\beta} G^{1-\beta}}\right)$. If $\beta = 0$, we have CRRA preferences, $V = \frac{1}{\varepsilon}\left(\frac{I}{G}\right)^{\varepsilon}$. Lemma 1 shows that function (4-1) satisfies the standard properties of a utility function.

Lemma I

Function (4-1) is a valid indirect utility specification, if $\beta\left(\frac{Pq}{G}\right) \leq \left(\frac{Iq}{G}\right)^{\varepsilon}$

Proof. See Appendix

CCD is the center for production and consumption of non-traded services. Service workers go to work in the CCD, and they pay a commuting cost that linearly increases with the distance between their home and the CCD. Hence, disposable income for a service worker living at the location x is given by $w - \tau x$, where w is wage for service worker and τ is commuting cost per unit of distance.

Consumers travel to the CCD to purchase non-traded services. On each trip to the CCD, a consumer purchases one unit of service goods and pays a travel cost that linearly increases with her distance to the CCD. Hence, the total cost for one unit of services, inclusive of travel costs, is $G + kx$. We elaborate the indirect utility function by accounting for commuting cost and travel costs. Consumer preference is given by,

$$V=\frac{1}{\varepsilon}\left[\frac{I}{(G+kx)/q}\right]^{\varepsilon}-\frac{\beta}{\gamma}\left[\frac{P}{(G+kx)/q}\right]^{\gamma}+\upsilon \tag{4-3}$$

where $I=w-\tau x$ for service workers, $I=b$ for traded-sector workers and,

$$P=\alpha^{-\alpha}(1-\alpha)^{\alpha-1}P_h^{\alpha}P_X^{1-\alpha} \tag{4-4}$$

By Roy identity, we derive demand function for services,

$$Q_S(I,P_h,G,q,x)=\frac{I}{G+kx}\left[1-\beta\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma}\right] \tag{4-5}$$

and for housing goods,

$$Q_h(I,P_h,G,q,x)=\frac{\alpha\beta I}{P_h}\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma} \tag{4-6}$$

Parameter ε determines the degree of non-homotheticity of consumer's utility and $1-\varepsilon$ is the income elasticity of demand for housing goods and for traded goods. The price elasticity of housing demand is $\alpha\gamma-1$. Income elasticity of service demand is given by,

$$1-\varepsilon+\frac{\varepsilon}{1-\beta\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma}},$$

that is always greater than one, if $\varepsilon>0$.

Price elasticity of service demand is given by,

$$-\frac{G}{G+kx}\left(1+\gamma-\varepsilon+\frac{\varepsilon-\gamma}{1-\beta\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma}}\right)$$

that is always negative, if $\varepsilon>\gamma$.

And quality elasticity of service demand is given by,

$$\gamma-\varepsilon+\frac{\varepsilon-\gamma}{1-\beta\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma}}=\frac{(\varepsilon-\gamma)\beta\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma}}{1-\beta\left(\frac{G+kx}{qI}\right)^{\varepsilon}\left(\frac{qP}{G+kx}\right)^{\gamma}}$$

that is positive, if $\varepsilon>\gamma$.

4.2.2 Production

4.2.2.1 Traded goods sector

There are N individuals in the economy and each individual has one unit of labor, but different skill b, that is distributed according to a density function $k(b)$. Production of traded goods only requires skills as inputs and it subjects to constant return to scale. Traded worker's skill b is equivalent to her productivity in the sector.

4.2.2.2 Non-traded service sector

Non-traded services are only provided at the Central Commercial District (CCD), because of the positive externalities in providing non-traded services. The externalities arise, because social interaction is a key component of non-traded services. For example, concerts and entertainment shows are best enjoyed with a large audience.

There is a single landlord, who tenders the CCD to the service operator that is capable of paying the highest commercial rent. The production of non-traded services requires a fixed labor cost, F. The fixed cost can be in the form of employment training, setting up necessary equipment and shops, or obtaining necessary business licenses. Besides, producing each unit of service output also requires a constant marginal labor input m . Therefore, total labor cost of producing Q_S units of services is given by,

$$l = mQ_S + F \tag{4-7}$$

The producer could invest to improve service quality q. High quality requires high fixed input,

$$F(q) = \delta q^{\zeta}, \delta > 0, \zeta > 0 \tag{4-8}$$

In our model, we allow for labor specialization. Workers choose their employment sector to make the best use of their talents. By comparative advantage *a la* Roy (1951), high-skill workers will specialize in producing traded goods and the low-skill worker will specialize in producing non-traded services. In equilibrium, there is a marginal worker indifferent between two sectors. The workers, whose skill is higher than the marginal worker, will produce traded goods, and the workers, whose skill is lower, will produce non-traded services.

Our model setting is equivalent to that is used in monopolistic competition (Krugman, 1991). In the framework of monopolistic competition, consumers have a preference for varieties. Composite price index decreases with the number of available varieties in the economy. Producing a larger number of varieties requires a higher fixed cost.

4.2.2.3 Housing

Production of housing goods H takes land input, L and capital input, C,

$$H = L^{\mu}C^{1-\mu}, 0 < \mu < 1 \tag{4-9}$$

Capital is traded on the global market at a constant price, R_c. Given Cobb-Douglas technology, the production cost per unit of housing goods is,

$$\mu^{-\mu}(1-\mu)^{\mu-1} R_L^{\mu} R_c^{1-\mu}$$

where R_L is the land price. Because housing sector is perfectly competitive, firms earn zero profit. Hence, housing price must equals production cost,

$$P_h = \mu^{-\mu}(1-\mu)^{\mu-1} R_L^{\mu} R_c^{1-\mu} \tag{4-10}$$

Rearranging the equation, we find that land price R_L is driven by housing price P_h,

$$R_L = \left(\mu^{\mu}(1-\mu)^{1-\mu}\right)^{1/\mu} \left(P_h R_c^{\mu-1}\right)^{1/\mu} \tag{4-11}$$

Given the supply of land *L*, housing supply H_S is determined by,

$$H_S(P_h) = (1-\mu)^{\frac{1-\mu}{\mu}} \left(R_c^{\mu-1}\right)^{1/\mu} P_h^{1/\mu-1} L \tag{4-12}$$

Price elasticity of housing supply is $1/\mu - 1$. If μ is lower, housing construction will be less constrained by the inelastic land supply, therefore, housing supply will be more sensitive to changes of housing price.

4.3 Properties of Equilibrium

In this section, we first define equilibrium of the economy and then demonstrate its properties.

Definition of Equilibrium

An equilibrium for a population N with skill distribution $g(b)$ is a list of quantities $\{Q_s(b), Q_h(b), Q_X(b)\}$, prices $\{w, G, P_h(x), R(x)\}$, service quality q, worker's location choice $b(x)$, skill cutoff b_2 for service workers, such that,

(i) service firm maximizes its profit;

(ii) all consumers optimize (inclusive of employment choice, consumption choice and residential location choice);

(iii) service market, labor market, housing market and land market are all clear.

To demonstrate the properties of equilibrium, we first examine how traded-sector workers choose their residential locations in equilibrium. Because non-traded services are luxury goods, high-income traded-sector workers spend a greater proportion of their income on services. Hence, the high-skill traded-sector workers are more willing to pay for the central location that is close to service provider. As long as income elasticity of service demand is greater than that of housing demand, high-skill traded-sector workers will always choose to live closer to the CCD. The Lemma I formalizes this result.

Lemma II (Spatial sorting of traded-sector workers)

If income elasticity of non-traded service is greater than that of housing, for traded-sector worker *i* with skill b_i at location x_i, *i=j, k,* if and only if, $b_j > b_k$, that $x_j < x_k$.

Proof. See the Appendix

Because service workers must pay a commuting cost to work in city center, their willingness to pay for the central location is higher than the least skilled traded-sector workers. Hence, middle-skill traded-sector workers will occupy the suburban area and the least-skilled traded-sector worker will live at city boundary. When commuting cost for service workers is not sufficiently high, the benefits from saving commuting costs for low-skill service workers will be lower than the consumption benefits that high-skill traded-sector workers will receive from living at central location. Hence, high-skill traded-sector workers will have higher willingness to pay for the central location. In equilibrium, high-skill traded worker will occupy the central region, the middle-skill traded worker will live in suburban region, and the service workers will live in intermediate region. The Proposition II formalizes this result.

Proposition I (spatial structure of a consumer city)

If commuting cost is positive, i.e., $\tau > 0,$ but not sufficiently high,

(i) the least-skilled traded-sector workers (denoted by b_2) live at city boundary (denoted by x_3);

(ii) city center is occupied by most skilled traded-sector workers (denoted by $\bar{b}$); there exists a cutoff skill $b_1 < \bar{b}$, and distance cutoff x_1 and x_2, such that traded-sector workers with skill $[b_1, \bar{b}]$ live in the area $[0, x_1]$; service workers live in the area $[x_1, x_2]$;

(iii) wage received by the least-skilled traded-sector worker is higher than the wage received by service workers, i.e., $b_1 > w$.

Proof. See the Appendix

Proposition I shows that income of resident is non-monotonic in distance to the city center. The income of resident first decreases, and then increases with distance to the CCD. We summarize the findings of Proposition I in Figure 4-1.

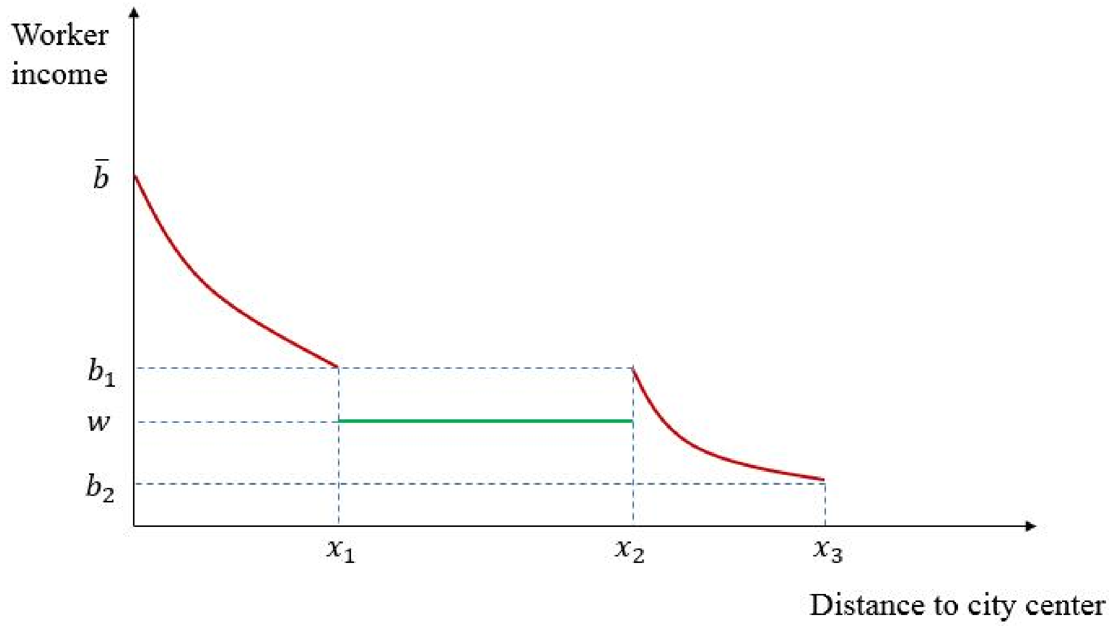

Figure 4-1: Skill sorting of heterogeneous workers in city

The spatial sorting of heterogeneous agents has two implications for the efficiency of resource allocation. First, sorting of the high-skill workers to central location helps to reduce urban frictions, because the consumers that travel more frequently live closer to the CCD. Second, the skill sorting of heterogeneous workers mimics the optimal two-part tariff for club goods. In our model, the consumer city is a club where consumers cluster to share the fixed cost of non-traded services. Each consumer purchases a house to acquire the membership of the club and pay additional service price and travel cost as a usage fee. Through choosing residential locations, heterogeneous agents will reveal their preference for the non-traded services. In equilibrium, the consumers with stronger preference purchase more expensive houses closer to the CCD, hence paying higher membership fees.

4.4 Characterization of Equilibrium

In this section, we characterize equilibrium of our economy. From Proposition I, we know that city can be divided into three areas, hence, we consider each area in sequence.

4.4.1 Housing market clearing in central area

We first consider central area $x \in [0, x_1]$. Because traded-sector workers' residential locations depend on their skills, we denote skill of traded-sector workers that live at location x by $b(x)$. Housing demand of traded-sector workers with skill $b(x)$ is given by,

$$h(b(x)) = \frac{\alpha\beta b(x)}{P_h(x)} \left(\frac{G+kx}{b(x)q}\right)^{\varepsilon} \left(\frac{P(x)q}{G+kx}\right)^{\gamma} \tag{4-13}$$

Service demand of traded-sector workers with skill $b(x)$ is given by,

$$s(b(x)) = \frac{b(x)}{G+kx}\left[1-\beta\left(\frac{G+kx}{b(x)q}\right)^{\varepsilon}\left(\frac{P(x)q}{G+kx}\right)^{\gamma}\right] \tag{4-14}$$

Between location x and $x+dx$, we will find individuals with skill between $b(x)$ and $b(x)+\frac{db(x)}{dx}dx$. The density of residents with skill between $b(x)$ and

$b(x)+\frac{db(x)}{dx}dx$ is $g(b(x))$. Hence, the aggregate housing demand at location x is given by,

$$H_D(x)=\left|hg(b(x))Ndb(x)\right| \tag{4-15}$$

Because supply of land between location x and $x+dx$ is dx, supply of housing is given by,

$$H_S(x)=(1-\mu)^{\frac{1-\mu}{\mu}}\left(R_c^{\mu-1}\right)^{1/\mu}P_h(x)^{1/\mu-1}dx \tag{4-16}$$

If housing market clears at location x, the total housing supply $H_S(x)$ must equal the total demand $H_D(x)$,

$$(1-\mu)^{\frac{1-\mu}{\mu}}\left(R_c^{\mu-1}\right)^{1/\mu}P_h(x)^{1/\mu-1}dx=\left|hg(b(x))Ndb(x)\right| \tag{4-17}$$

where h is individual housing demand.

Therefore, high-skill traded-sector workers' location choice must satisfy the following condition,

$$\frac{db}{dx}=-\frac{\left[\mu^{\mu}(1-\mu)^{1-\mu}\right]^{1/\mu}\left(P_hR_c^{\mu-1}\right)^{1/\mu}}{\alpha\mu\beta b\left(\frac{G+kx}{bq}\right)^{\varepsilon}\left(\frac{Pq}{G+kx}\right)^{\gamma}g(b)N} \tag{4-18}$$

In addition, housing price gradient at the location x is given by Alonso-Muth condition,

$$\frac{\partial P_h}{\partial x}=\frac{\partial L}{\partial x}=-\frac{1}{h}ks$$

Substitute s and h by using (4-5) and (4-6),

$$\frac{dP_h}{dx}=-\frac{1}{h}ks=-k\frac{b}{G+kx}\left[\frac{\alpha\beta b}{P_h}\left(\frac{G+kx}{bq}\right)^{\varepsilon}\left(\frac{Pq}{G+kx}\right)^{\gamma}\right]^{-1}\left[1-\beta\left(\frac{G+kx}{bq}\right)^{\varepsilon}\left(\frac{Pq}{G+kx}\right)^{\gamma}\right]$$

(4-19)

From (4-18) and (4-19), we obtain a system of differential equations for two unknown functions $b(x)$ and $P_h(x)$. Because the most skilled traded-sector workers live at city center, we have boundary condition,

$$b(0) = \bar{b} \tag{4-20}$$

Suppose housing price at city center is R_0, we have boundary condition,

$$P_h(0) = R_0 \tag{4-21}$$

Conditional on knowing R_0, we can solve this system of differential equations. We are able to know the skill of residents at any location and housing price at that location. Suppose we also know b_1, we can also use $b(x)$ to evaluate x_1,

$$x_1 = b^{-1}(b_1) \tag{4-22}$$

We can also evaluate housing price at location x_1,

$$R_1 = P_h(x_1) \tag{4-23}$$

Later, we will discuss how R_0 and b_1 are endogenously determined in equilibrium.

4.4.2 Housing market clearing in intermediate area

Then, we consider intermediate area $x \in [x_1, x_2]$ where service workers live. In general, within the group of service workers, our model remains exactly the same as a traditional monocentric model with homogeneous agents, such that the results are pinned down by utility equalization across locations.

The utility of service workers is determined by service wage w and housing price at location x_1,

$$\bar{u} = \frac{1}{\varepsilon}\left[\frac{(w-\tau x_1)q}{G+kx_1}\right]^{\varepsilon} - \frac{\beta}{\gamma}\left[\frac{P(x_1)q}{G+kx_1}\right]^{\gamma} \tag{4-24}$$

In a moment, we will discuss how service wage w is endogenously determined in equilibrium. Because service workers are indifferent among locations in the area $[x_1, x_2]$, they receive utility $\bar{u}$ at any location x. Hence,

$$\bar{u} = \frac{1}{\varepsilon}\left[\frac{(w-\tau x)q}{G+kx}\right]^{\varepsilon} - \frac{\beta}{\gamma}\left[\frac{P(x)q}{G+kx}\right]^{\gamma} \tag{4-25}$$

Hence,

$$\frac{1}{\varepsilon}\left[\frac{(w-\tau x_1)q}{G+kx_1}\right]^{\varepsilon} - \frac{\beta}{\gamma}\left[\frac{P(x_1)q}{G+kx_1}\right]^{\gamma} = \frac{1}{\varepsilon}\left[\frac{(w-\tau x)q}{G+kx}\right]^{\varepsilon} - \frac{\beta}{\gamma}\left[\frac{P(x)q}{G+kx}\right]^{\gamma} \tag{4-26}$$

Housing price at an arbitrary location $x \in [x_1, x_2]$ can be pinned down as,

$$P_h(x) = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{G+kx}{q}\right)\left(\frac{\gamma}{\beta}\left(\frac{1}{\varepsilon}\left(\frac{(w-\tau x)q}{G+kx}\right)^{\varepsilon} - \left(\frac{1}{\varepsilon}\left[\frac{(w-\tau x_1)q}{G+kx_1}\right]^{\varepsilon} - \frac{\beta}{\gamma}\left[\frac{P(x_1)q}{G+kx_1}\right]^{\gamma}\right)\right)\right)^{1/\gamma}\right)^{-\alpha}$$

(4-27)

and price of composite goods at the location $x \in [x_1, x_2]$ is given by,

$$P(x) = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} P_h^{\alpha}(x) \tag{4-28}$$

Hence, housing demand of individual service worker at location $x \in [x_1, x_2]$ is given by,

$$h(x) = \frac{\alpha\beta(w-\tau x)}{P_h(x)}\left(\frac{G+kx}{(w-\tau x)q}\right)^{\varepsilon}\left(\frac{P(x)q}{G+kx}\right)^{\gamma} \tag{4-29}$$

Furthermore, aggregate housing demand at location $x \in [x_1, x_2]$ is given by,

$$H_D(x) = h(x)\, g_{serv}(x)\, dx \tag{4-30}$$

where $g_{serv}(x)$ is population density of service workers at the location x.

Aggregate housing supply at location $x \in [x_1, x_2]$ is given by,

$$H_S(x) = (1-\mu)^{\frac{1-\mu}{\mu}} \left(R_c^{\mu-1}\right)^{1/\mu} P_h(x)^{1/\mu - 1}\, dx \tag{4-31}$$

Housing market clearing at location $x \in [x_1, x_2]$ implies that,

$$h(x)\, g_{serv}(x)\, dx = (1-\mu)^{\frac{1-\mu}{\mu}} \left(R_c^{\mu-1}\right)^{1/\mu} P_h(x)^{1/\mu - 1}\, dx \tag{4-32}$$

Hence, population density of service workers at location $x \in [x_1, x_2]$ can be pinned down as,

$$g_{serv}(x) = \frac{(1-\mu)^{\frac{1-\mu}{\mu}} \left(P_h(x) R_c^{\mu-1}\right)^{1/\mu}}{\alpha\beta(w-\tau x)} \left(\frac{(w-\tau x)q}{G+kx}\right)^{\varepsilon} \left(\frac{G+kx}{P(x)q}\right)^{\gamma} \tag{4-33}$$

Because service workers occupy the area, total population in area $x \in [x_1, x_2]$ must equal to population of service workers,

$$N\int_{\underline{b}}^{b_2} k(t)\, dt = \int_{x_1}^{x_2} g_{serv}(t)\, dt \tag{4-34}$$

The equation (4-35) defines a mapping between b_2 and x_2. Conditional on knowing b_2, we can find x_2 by solving equation (4-36). Furthermore, we can evaluate housing price at location x_2 by using (4-28),

$$R_2 = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{G+kx_2}{q}\right)\left(\frac{\gamma}{\beta}\left(\frac{1}{\varepsilon}\left(\frac{(w-\tau x_2)q}{G+kx_2}\right)^{\varepsilon} - \left(\frac{1}{\varepsilon}\left[\frac{(w-\tau x_1)q}{G+kx_1}\right]^{\varepsilon} - \frac{\beta}{\gamma}\left[\frac{P(x_1)q}{G+kx_1}\right]^{\gamma}\right)\right)\right)^{1/\gamma}\right)^{-\alpha}$$

(4-37)

This is useful for our purpose of finding the solutions for equilibrium.

4.4.3 Housing market clearing in suburban area

For suburban area $x \in [x_2, x_3]$, we have the same set of differential equations as area $x \in [0, x_1]$ that describe evolving housing price and skills across locations,

$$\frac{db}{dx} = -\frac{\left(\mu^{\mu}(1-\mu)^{1-\mu}\right)^{1/\mu}\left(P_h R_c^{\mu-1}\right)^{1/\mu}}{\alpha\mu\beta b\left(\frac{G+kx}{bq}\right)^{\varepsilon}\left(\frac{Pq}{G+kx}\right)^{\gamma} g(b) N} \tag{4-38}$$

$$\frac{dP_h}{dx} = -\frac{1}{h}ks = -k\frac{b}{G+kx}\left[\frac{\alpha\beta b}{P_h}\left(\frac{G+kx}{bq}\right)^{\varepsilon}\left(\frac{Pq}{G+kx}\right)^{\gamma}\right]^{-1}\left[1-\beta\left(\frac{G+kx}{bq}\right)^{\varepsilon}\left(\frac{Pq}{G+kx}\right)^{\gamma}\right] \tag{4-39}$$

Because traded-sector workers with skill b_1 lives at location x_2, we have boundary condition,

$$b(x_2) = b_1 \tag{4-40}$$

Suppose we also know housing price at location x_2, we have boundary condition,

$$P_h(x_2) = R_2 \tag{4-41}$$

After solving this system, we can evaluate city boundary x_3 by using $b(x)$,

$$x_3 = b^{-1}(b_2) \tag{4-42}$$

And find the housing price R_3 at city boundary x_3,

$$R_3 = P_h(x_3) \tag{4-43}$$

4.4.4 Utility equalization conditions

The marginal traded worker with skill b_1 must be indifferent between location x_1 and x_2,

$$\frac{1}{\varepsilon}\left[\frac{b_1}{(G+kx_1)/q}\right]^{\varepsilon}-\frac{\beta}{\gamma}\left[\frac{P(x_1)}{(G+kx_1)/q}\right]^{\gamma}=\frac{1}{\varepsilon}\left[\frac{b_1}{(G+kx_2)/q}\right]^{\varepsilon}-\frac{\beta}{\gamma}\left[\frac{P(x_2)}{(G+kx_2)/q}\right]^{\gamma}$$

(4-44)

In addition, the marginal traded worker with skill b_2 must be indifferent between two employment sectors,

$$\frac{1}{\varepsilon}\left[\frac{b_2}{(G+kx_3)/q}\right]^{\varepsilon}-\frac{\beta}{\gamma}\left[\frac{P(x_3)}{(G+kx_3)/q}\right]^{\gamma}=\bar{u}$$

(4-45)

Substitute $\bar{u}$ by (4-19), we have,

$$\frac{1}{\varepsilon}\left[\frac{w-\tau x_1}{(G+kx_1)/q}\right]^{\varepsilon}-\frac{\beta}{\gamma}\left[\frac{P(x_1)}{(G+kx_1)/q}\right]^{\gamma}=\frac{1}{\varepsilon}\left[\frac{b_2}{(G+kx_3)/q}\right]^{\varepsilon}-\frac{\beta}{\gamma}\left[\frac{P(x_3)}{(G+kx_3)/q}\right]^{\gamma}$$

(4-46)

4.4.5 Non-traded service market and land market clearing

Market of non-traded services must clear. Aggregate labor supply of service workers must equal to aggregate labor demand. To account for this condition, we need to consider aggregate service demand in each area. The aggregate service demand in area $x\in[0,x_1]$ is given by,

$$Q_{S,1}=N\int_{b_1}^{\bar{b}}\frac{t}{G+kb^{-1}(t)}\left(1-\beta\left(\frac{G+kb^{-1}(t)}{qt}\right)^{\varepsilon}\left(\frac{qP(t)}{G+kb^{-1}(t)}\right)^{\gamma}\right)dt$$

(4-47)

For area $x \in [x_1, x_2]$, the aggregate service demand is given by,

$$Q_{S,2} = \int_{x_1}^{x_2} \frac{w-\tau x}{G+kx} \left(1 - \beta \left(\frac{G+kx}{q(w-\tau x)} \right)^{\varepsilon} \left(\frac{qP(x)}{G+kx} \right)^{\gamma} \right) g_{serv}(x)\, dx \tag{4-48}$$

For area $x \in [x_2, x_3]$, the aggregate service demand is given by,

$$Q_{S,3} = N \int_{b_2}^{b_1} \frac{t}{G+kb^{-1}(t)} \left(1 - \beta \left(\frac{G+kb^{-1}(t)}{qt} \right)^{\varepsilon} \left(\frac{qP(t)}{G+kb^{-1}(t)} \right)^{\gamma} \right) dt \tag{4-49}$$

Service market clearing requires that,

$$N \int_{\underline{b}}^{b_2} g(t)\, dt = c \sum_{i=1}^{3} Q_{S,i} + \delta q^{\xi} \tag{4-50}$$

Last, land market must also clear. At city boundary, the residential land price must equal agriculture land price. From (4-11), it must be satisfied that,

$$R_{agr} = \left(\mu^{\mu} (1-\mu)^{1-\mu} \right)^{1/\mu} \left(R_3 R_c^{\mu-1} \right)^{1/\mu} \tag{4-51}$$

4.4.6 Service producer's optimization problem

The service producer has perfect information and chooses service price G and service quality q to maximize the profit, π,

$\pi = Q_S G - Q_S wm - wF$

where,

$F = \delta q^{\zeta}$

The first-order condition for service price G is given by,

$$\frac{\partial \pi}{\partial G} = Q_S + \frac{\partial Q_S}{\partial G} G - wm \frac{\partial Q_S}{\partial G} - \frac{\partial w}{\partial G} m Q_S - \frac{\partial w}{\partial G} F = 0 \tag{4-52}$$

and for service quality q,

$$\frac{\partial \pi}{\partial q}=\frac{\partial Q_S}{\partial q}G-\frac{\partial Q_S}{\partial q}wm-\frac{\partial w}{\partial q}mQ_S-\frac{\partial w}{\partial q}F-w\frac{dF}{dq}=0 \tag{4-53}$$

First-order condition (4-52) and (4-53) jointly determine the optimal choice of service price and quality.

We rearrange (4-52) and find it has straightforward intuition,

$$Q_S+\frac{\partial Q_S}{\partial G}G=wm\frac{\partial Q_S}{\partial G}+\left(\frac{\partial w}{\partial G}mQ_S+\frac{\partial w}{\partial G}F\right)$$

If the producer asks for dG units of higher price, it has to account for two effects on revenue. First, its revenue increases by $Q_S dG$ because it asks for a higher price for each unit of current output. Second, because market demand decreases, the revenue also decreases by $GdQ_S=G\frac{\partial Q_S}{\partial G}dG$. In total, the additional revenue from asking for higher price is given by,

$$Q_S dG+GdQ_S=\left(Q_S+\frac{\partial Q_S}{\partial G}G\right)dG\ .$$

Besides, the producer must also consider two effects of higher price on production cost. First, because aggregate service demand decreases, the producer can benefit from saving labor costs at current service wage. This benefit is shown by,

$$wmdQ_S=wm\frac{\partial Q_S}{\partial G}dG$$

Second, higher service price has a general equilibrium effect on service wage that is measured by $\frac{\partial w}{\partial G}$. The change of service wage leads to a change of total labor represented by,

$$mQ_S dw + F dw = \left(\frac{\partial w}{\partial G} mQ_S + \frac{\partial w}{\partial G} F \right) dG$$

To choose the optimal service price, the additional revenue from asking higher price must exactly cover the additional costs, hence,

$$\left(Q_S + \frac{\partial Q_S}{\partial G} G \right) dG = wm \frac{\partial Q_S}{\partial G} dG + \left(\frac{\partial w}{\partial G} mQ_S + \frac{\partial w}{\partial G} F \right) dG$$

which is just the first-order condition for service price.

If service worker's wage is constant, the first-order condition for service price will boil down to,

$$Q_S + \frac{\partial Q_S}{\partial G} G - wm \frac{\partial Q_S}{\partial G} = 0$$

It is just the first-order condition for a textbook example of the monopoly. Our model is distinguished by including a general equilibrium effect of service price on service wage. If this effect is negligible, the second-order condition requires that the derivative of additional revenue be less than the derivative of extra costs.

We also rearrange the first-order condition for quality,

$$\frac{\partial Q_S}{\partial q} G - \frac{\partial Q_S}{\partial q} wm = \frac{\partial w}{\partial q} mQ_S + \frac{\partial w}{\partial q} F + w \frac{dF}{dq}$$

The intuition is straightforward. If the producer raises service quality, the aggregate service demand will also increase. Therefore, at current service price, the revenue will increase by,

$$G dQ_S = \left(\frac{\partial Q_S}{\partial q} G \right) dq$$

Yet, to raise service quality, the producer has to use more labor as fixed costs that is represented by,

$$wdF = w\frac{dF}{dq}dq$$

Rising quality also has a general equilibrium effect on service wage that is measured by $\frac{\partial w}{\partial q}$. As a response to service wage change, the labor cost will change by,

$$mQ_S dw + Fdw = \left(\frac{\partial w}{\partial q}mQ_S + \frac{\partial w}{\partial q}F\right)dq$$

To choose the optimal quality, the additional revenue from raising service quality must exactly cover the additional costs,

$$\left(\frac{\partial Q_S}{\partial q}G\right)dq = w\frac{dF}{dq}dq + \left(\frac{\partial w}{\partial q}mQ_S + \frac{\partial w}{\partial q}F\right)dq$$

It is just the first-order condition for service quality. If the general effect of service quality on service wage is negligible, the second-order condition requires that the derivative of additional revenue be less than the derivative of additional costs.

In sum, the equilibrium conditions are following:

(i) Workers with skill $b > b_1$ produce traded goods and live in the area $[0, x_1]$; workers with skill $b < b_2$ produce non-traded services and live in the area $[x_1, x_2]$; workers with skill $b_2 < b < b_1$ produce traded goods and live in the area $[x_2, x_3]$.

(ii) $P_h(x)$ and $b(x)$ satisfy (4-18) to (4-21) for $x \in [0, x_1]$;

(iii) $P_h(x)$ satisfies (4-28) for $x \in [x_1, x_2]$;

(iv) $P_h(x)$ and $b(x)$ satisfy (4-38) to (4-41) for $x \in [x_2, x_3]$;

(v) The marginal worker of skill b_1 is indifferent between location x_1 and x_2, (4-44) is satisfied;

(vi) The marginal worker of skill b_2 is indifferent between employment sectors, (4-46) is satisfied;

(vii) Non-traded service market clears, (4-50) is satisfied;

(viii) Land market clear, (4-51) is satisfied;

(ix) Service producer optimizes in choosing G and q, (4-51) and (4-51) are satisfied.

4.5 Existence of Equilibrium

In this section, we demonstrate that equilibrium exists by construction. Our idea is to show that economy will converge to equilibrium from an arbitrary initial state. Consider a state of economy that violates equilibrium condition, such that traded workers with skill b_1 gain higher utility by living at location x_1 than location x_2. It must be true that the workers with skill marginally lower than b_1 also find it beneficial to move from suburban area to central area. This movement will continue, until the increased congestion in central area neutralizes the utility gains. Consider another state that service workers are worse off than traded worker with skill b_2. The service workers with skill marginally lower than b_2 will convert to produce traded goods, inducing a decreasing population of service workers. If service goods are undersupplied, service firm must pay higher wage w to attract sufficient amount of workers. To search for equilibrium, we develop an algorithm that follows a similar intuition and it is shown in Appendix.

4.6 The Baseline Case

Based on our algorithm, we numerically solve the general equilibrium of our economy. We present the baseline case in Table 4-1.

Table 4-1: General Equilibrium of the Consumer City

Key features	Values
Skill cutoff b_1	12.7460
Skill cutoff b_2	1.4053
Service worker wage w	1.5594
Employment share of non-traded service workers	0.3438
Boundary of central city x_1	0.1659
Boundary of suburb area x_2	0.7224
Boundary of city x_3	4.2500
Service price G	9.9601
Service quality q	1.6806
Expenditure share of traded goods	0.2815
Expenditure share of housing goods	0.1515
Expenditure share of non-traded services	0.5322

Notes: The model parameters are $N = 10$, $\alpha = 0.35$, $\beta = 0.4$, $\gamma = 0.1$, $\varepsilon = 0.4$, $\mu = 0.5$, $k = 0.5$, $\tau = 0.1$, $m = 0.1$, $\delta = 1$, $\zeta = 1.1$, $R_c = 0.3$, $R_{agr} = 0.3$, and $\underline{b} = 1$, $\bar{b} = 30.6$, $\xi = 1.2$ (which gives a skill Gini coefficient of 0.45).

In equilibrium, the high-skill traded-sector workers with skill greater than 12.75 live in the central city. Boundary of the central city is at the location of 0.17. The middle-skill traded-sector workers with skill ranging from 1.41 to 12.75 live in the suburban region. The suburban region ranges from 0.72 to 4.25 of distance to the CCD. The workers with skill lower than 1.41 choose to produce non-traded services. Their residential locations range from 0.17 to 0.72 of distance to the CCD.

The service workers take up 34 percentage of total population in the city. Moretti (2010a) finds that for each additional job in manufacturing sector in a given city, 1.6 jobs are created in the nontradable sector in the same city and one additional skilled job in the tradable sector generates 2.5 jobs in local goods and services. These findings suggest the proportion of service workers should be around 60 to 70 percentage. But, since we ignore the service employments related to housing production in our model, it is reasonable that the proportion of service workers at equilibrium is below the level implied by Moretti (2010a). In equilibrium, the spending on non-traded services takes up 53 percentage of total expenditure.

The results of our model are consistent with many stylized facts. First, moving from the city center to suburbs, the income of the residents initially decreases and then increases, as shown in Figure 4-3. Similar to our finding, a U-shaped income curve is documented in superstar cities, including New York, Chicago and Philadelphia (Glaeser et al., 2008). Second, the bid rent curve of housing price and land price are both downward sloping, as shown in Figure 4-3 and Figure 4-4. Third, capital-to-land ratio is also downward sloping from CCD to suburbs, as shown in Figure 4-5. Because the land price is higher at city center, housing producer will substitute capital for land. This is consistent with our observation that buildings are usually higher at city center.

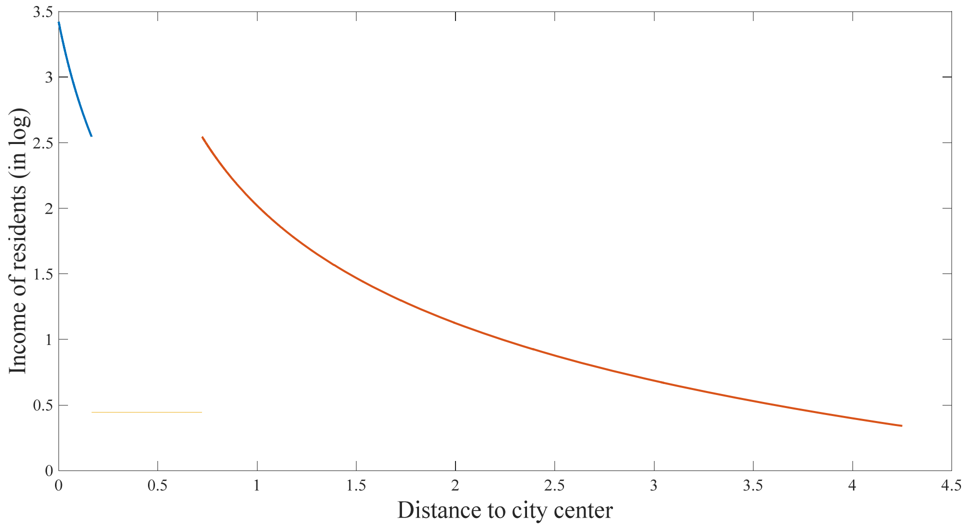

Figure 4-2: Income of residents and distance to the CCD

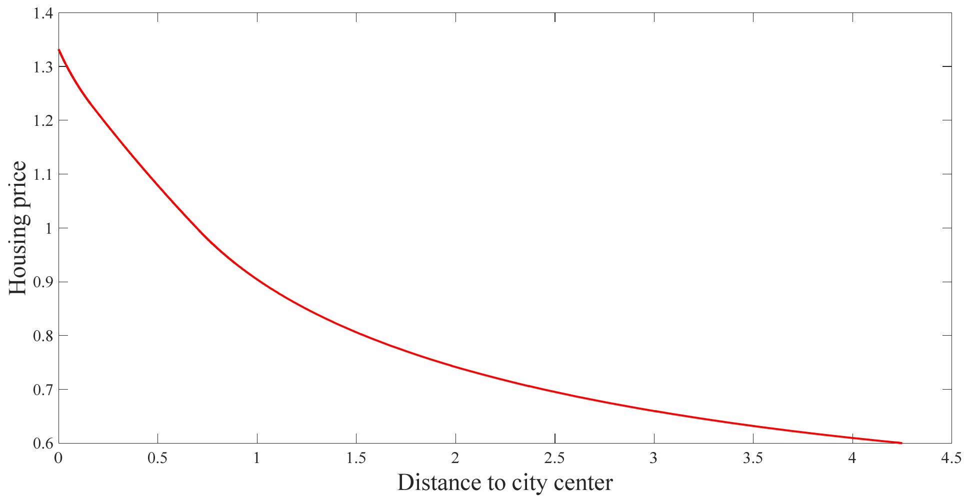

Figure 4-3: Housing price and distance to the CCD

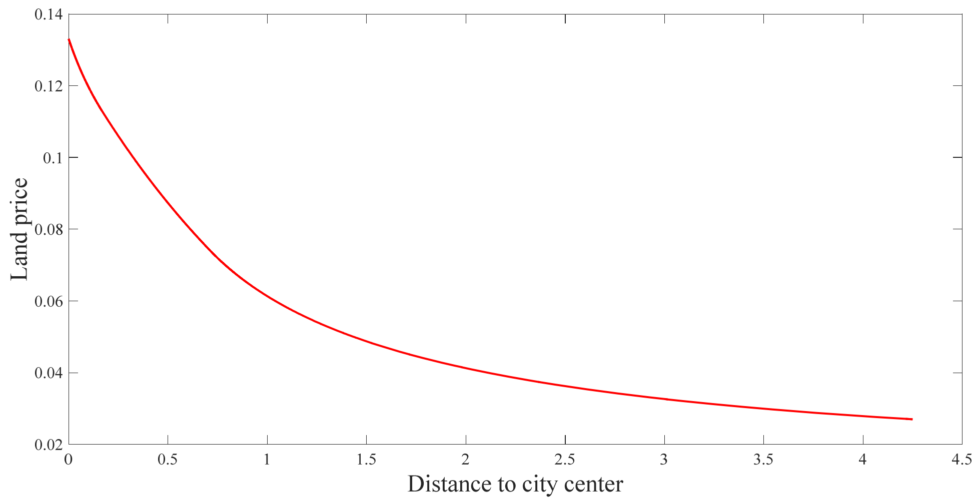

Figure 4-4: Land price and distance to the CCD

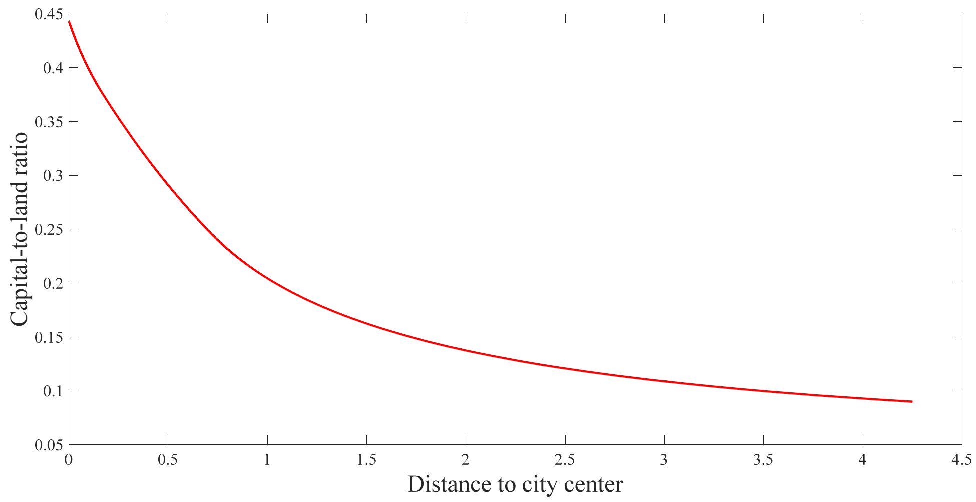

Figure 4-5: Capital-to-land ratio and distance to the CCD

4.7 Counterfactual Analysis

In past decades, economic wealth has become increasingly concentrated within a small group of wealthy Americans (Piketty, Goldhammer, & Ganser, 2014). In this section, we conduct a counterfactual analysis to understand the effects of increasing skill inequality on the economic structure of the city and the pattern of urban land use.

We refer to 2014 American Community Survey to choose a reasonable range of Gini coefficients for our counterfactual analysis. According to the survey data, the Gini coefficient is about 0.45 on average for the whole country. But, the degree of inequality can be much higher in some large cities. For example, the Gini coefficient is about 0.55 in New York and Los Angles. Accordingly, we increase Gini coefficient from 0.45 to 0.55, by raising upper bound of skill distribution from around 30 to 150.

4.7.1 Transition of economic structure

The rising income inequality has profound implications for economic structural of the city. In general, as skill distribution becomes more dispersed, the city will convert to an economy that is oriented by the non-traded service sector. The structural transition is reflected in many aspects, including producer's choice for service quality, the composition of employments and consumers' spending pattern.

First, the quality of non-traded services will become more appealing to residents. As the Gini coefficient increases, there will be more high-income traded-sector workers in the city. Because non-traded services are luxury goods, the larger group of high-income residents will necessarily lead to a higher demand for non-traded services, as shown in Figure 4-7. Since a larger market can sustain a higher fixed cost, service producer will find it profitable to improve the quality of services. Hence, as Gini coefficient increases from 0.45 to 0.55, the service quality increases from 1.68 to 1.80, as shown in Figure 4-6.

Figure 4-6: Service quality over skill Gini coefficient

Figure 4-7: Aggregate demand for non-traded services over skill Gini coefficient

Figure 4-8: Service worker wage over skill Gini coefficient

Figure 4-9: Skill cutoff for service employments over skill Gini coefficient

Second, there will be a greater proportion of workers employed in the service sector. Because the larger group of high-income consumers induce an increase in aggregate demand for services, service firm must offer a higher wage to attract more workers, as shown in Figure 4-8. Some traded-sector workers with relatively lower skill will convert to produce services, as indicated by rising skill cutoff b_2 from 1.41 to 1.47, shown in Figure 4-9. As a result, the total population of service workers increases from 3.44 to 3.72 and the share of service workers in city increases from around 34.39 percentage to 37.23 percentage, as shown in Figure 4-10 and Figure 4-11.

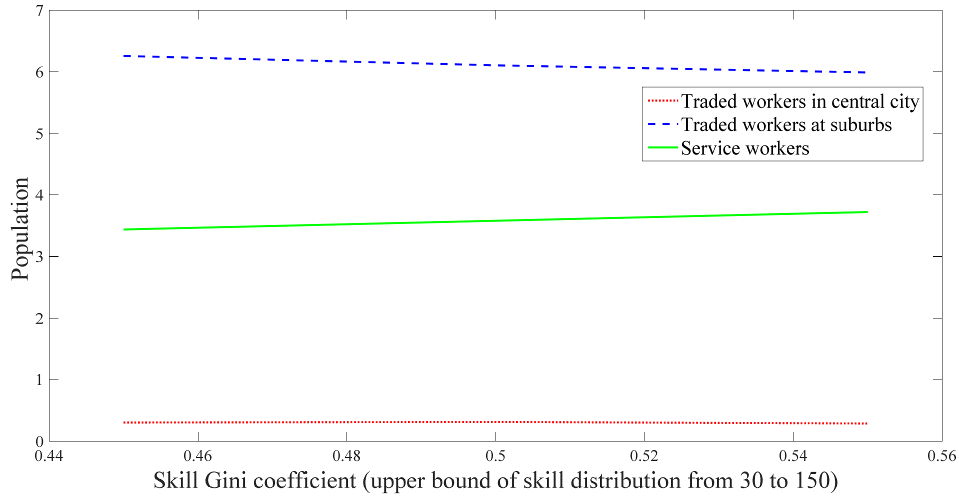

Figure 4-10: Population by occupations over skill Gini coefficient

Figure 4-11: Share of service employments over skill Gini coefficient

Third, the structural transition of the economy is also apparent from changes in consumer expenditure. As Gini coefficient increases from 0.45 to 0.55, the aggregate share of spending on services increases from 53.22 percentage to 57.97 percentage, as shown in Figure 4-12, while the expenditure share of traded goods and housing goods both decrease.

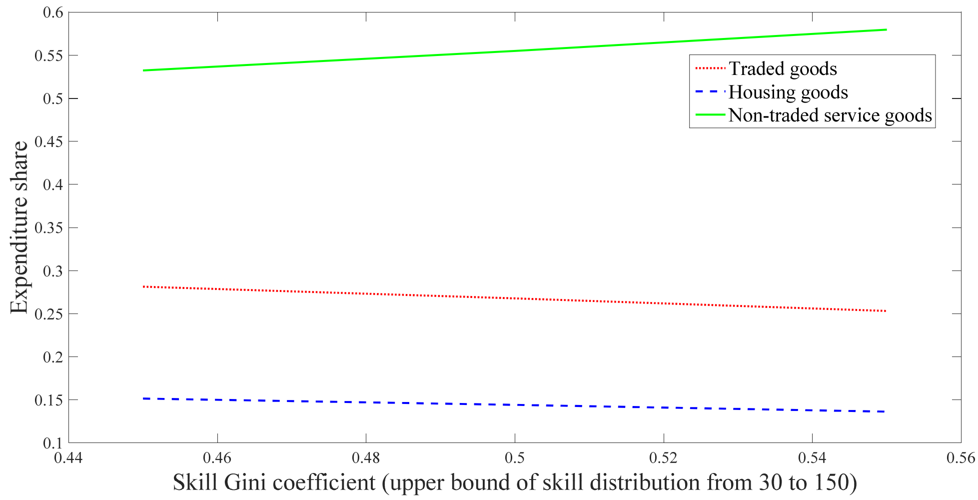

Figure 4-12: Expenditure shares over skill Gini coefficient

4.7.2 Changes of urban structure

The increasing skill dispersion not only leads to the economic transition of the city but also has an impact on the urban structure. First, the city will become larger in physical size. As shown in Figure 4-15, city boundary x_3 expands from 4.25 to 4.88, because higher aggregate income at city level drives up total housing demand. Similarly, the physical size of the central city x_1 also increases from around 0.17 to 0.25, as shown in Figure 4-13. As a result, population density in city decreases, as illustrated in Figure 4-16.

Figure 4-13: Boundary of central city over skill Gini coefficient

Figure 4-14: Boundary of service worker region over skill Gini coefficient

Figure 4-15: Boundary of city over skill Gini coefficient

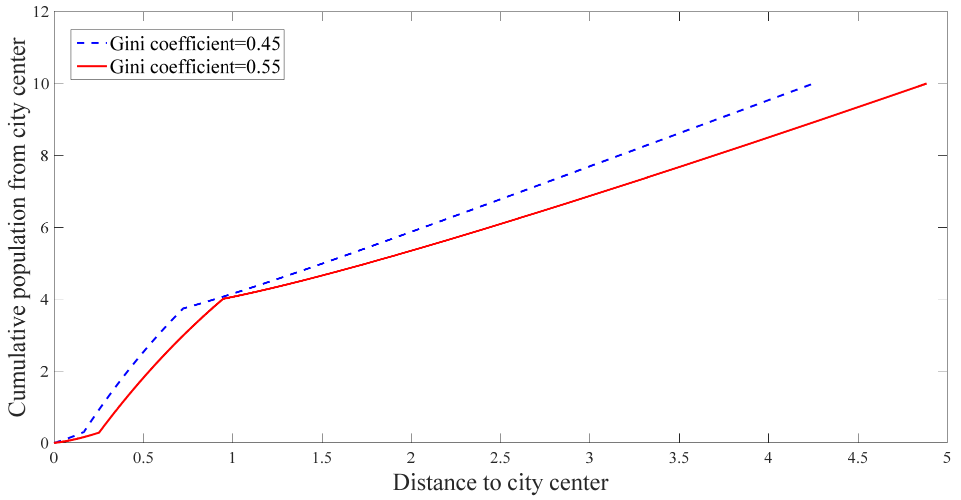

Figure 4-16: Cumulative population over distance to the CCD: Gini coefficient=0.45 vs. Gini coefficient=0.55

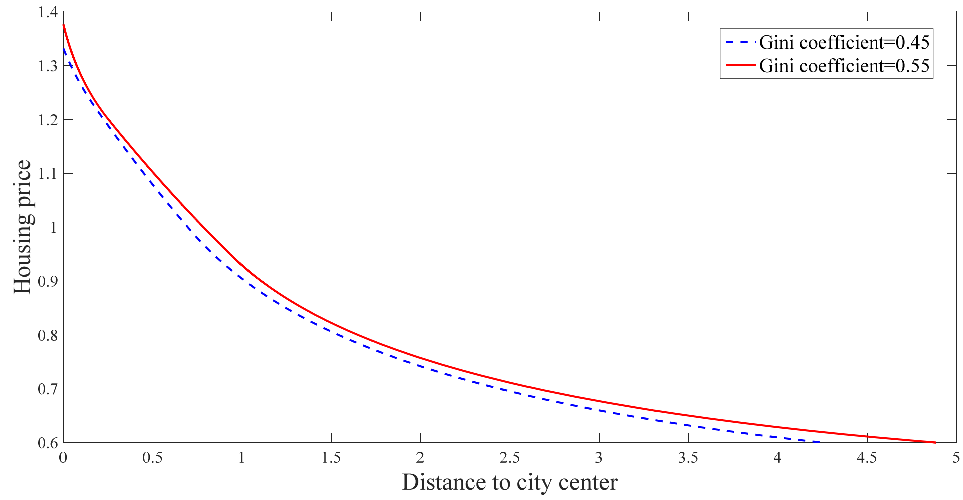

Figure 4-17: Housing price over distance to the CCD: Gini coefficient=0.45 vs. Gini coefficient=0.55

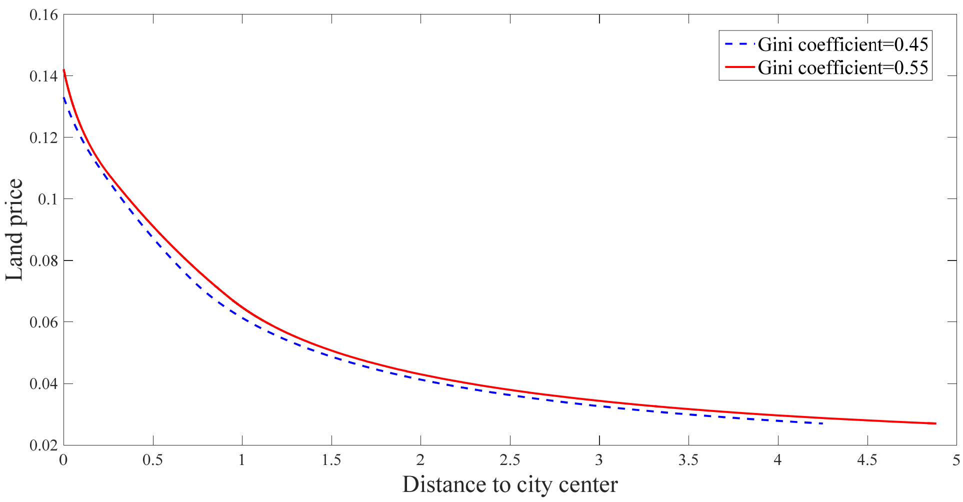

Figure 4-18: Land price over distance to the CCD: Gini coefficient=0.45 vs. Gini coefficient=0.55

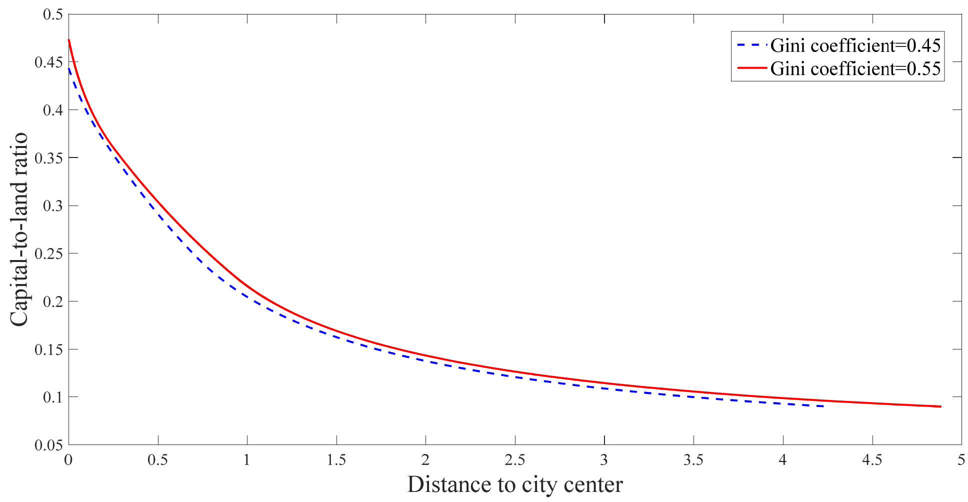

Figure 4-19: Capital-to-land ratio over distance to the CCD: Gini coefficient=0.45 vs. Gini coefficient=0.55

Second, the bid rent curves of housing price and land price both shift upward, as shown in Figure 4-17 and Figure 4-18. In particular, housing price at city center increases from 1.33 to 1.38. Because land is more expensive, the housing producer will substitute capital for land, as shown by the shift up of capital-to-land ratio in Figure 4-19. In addition, as more high-income traded-sector workers live in the central city, the competition for the housing at central location becomes more intensive. As a result, the bid rent curve of housing price becomes steeper at city center. The absolute value of the housing price slope at city center increases from 0.83 to 1.37, as shown in Figure 4-20. The prediction is consistent with the stylized fact that housing price premium at city center has increased in past decades (Edlund et al., 2015).

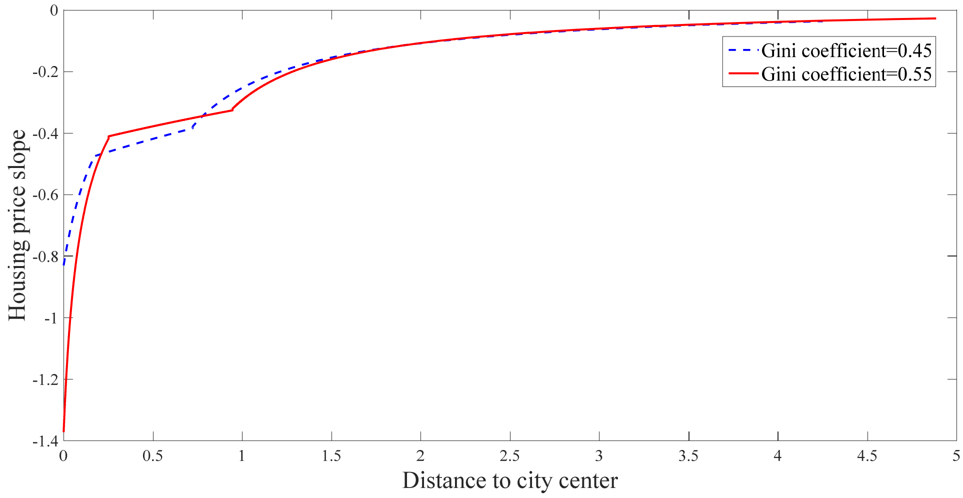

Figure 4-20: Housing price slope over distance to the CCD: Gini coefficient=0.45 vs. Gini coefficient=0.55

Third, the traded-sector workers with relatively lower skill are bid out from the central location, because of a more intensive competition for housing near the CCD. As Gini coefficient increases from 0.45 to 0.55, the skill cutoff b_1 increases from 12.75 to 17.57 (Figure 4-21). Because housing is more expensive at the central location, a traded worker has to be more skilled to find it desirable to live at the central location. Even for the traded-sector workers that stay in the central city, they are forced to live further from the CCD, as shown in Figure 4-22, because the high-skill traded-sector workers bid up housing price and occupy a greater amount of land. In general, our model predicts that, as skill distribution becomes more dispersed, city center will be increasingly occupied by the high-skill and high-income households.

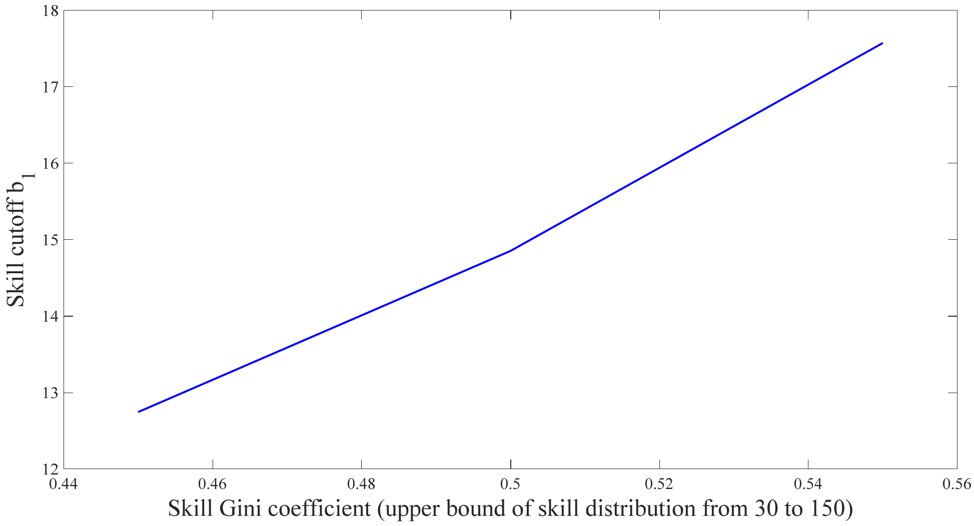

Figure 4-21: Skill cutoff for central-city residents over skill Gini coefficient

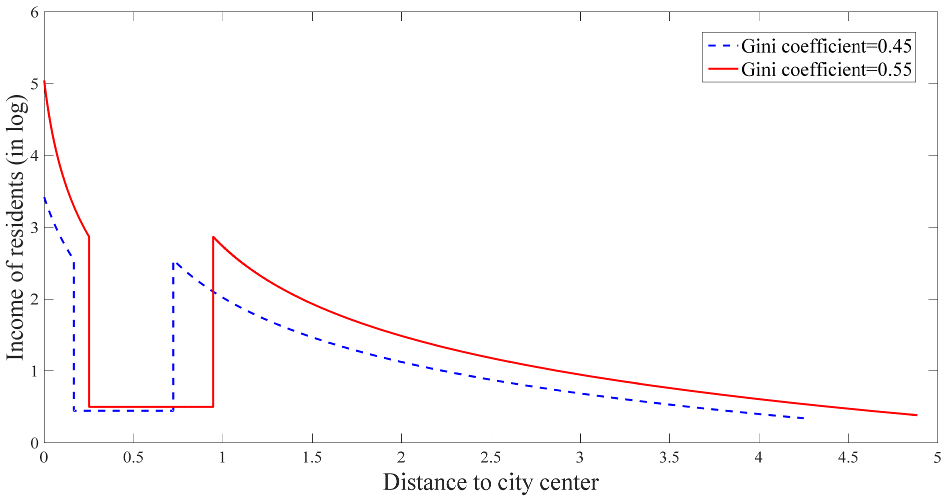

Figure 4-22: Income of residents (in log) over distance to the CCD: Gini coefficient=0.45 vs. Gini coefficient=0.55

4.7.3 Effect of different fundamentals

We further explore the effects of housing supply elasticity, service production technology and transportation costs on economic structure and land use of the city. We examine four scenarios. First, we make housing supply more elastic by reducing μ

from 0.6 to 0.3. Second, we reduce commuting cost for service workers by decreasing τ from 0.1 to 0.05. Third, we reduce the travel cost for service consumers by decreasing k from 0.5 to 0.3. Last, we reduce the fixed cost that is associated with service quality by reducing δ from 1 to 0.5. We present our results in Table 4-2 and column one is our baseline scenario. Each cell displays two numbers, corresponding to the outcome associated with a skill Gini coefficient of 0.45 (top number) and a skill Gini coefficient of 0.55 (bottom number), respectively. Column 2 to 5 show the simulation results for the alternative scenarios.

Our main results remain robust in a wide range of parameters. Besides, we could draw additional implications for skill sorting and urban land use of the city. When housing supply is more elastic, the physical size of the city will shrink in all dimensions including the sizes of the central city, suburb region, and service workers' residential region. Because the service workers can live closer to their workplace and save on commuting cost, the wage premium received by the service workers also becomes lower. More traded-sector workers will find it desirable to live in the central city, as indicated by the lower skill cutoff b_1. The aggregate demand for services also increases. Because housing supply is more elastic, the city center will lose its attractiveness relative to other locations in the central city. Hence, the slope of the housing bid rent becomes flatter at city center.

As commuting cost decreases, the service workers will lose their incentives to compete for the central location with the high-income traded-sector workers. Hence, the housing price and land price at city center become lower, and the housing price slope at city center also becomes flatter. With less incentive to save commuting costs, the service workers are willing to live further from city center, as indicated by rising x_1. The moving out of service workers allows the traded-sector workers to move from suburb region to central city, indicated by lower b_1. As a result, the suburb region occupies less land, but the central city expands. Besides, when the commuting cost is lower, the service firm will pay a lower wage premium to its employees.

When the travel cost for service purchases becomes lower, the high-skill traded-sector workers will have less incentive to bid for the central location. Hence, the housing price and land price at city center both become lower and the housing price slope at city center also becomes flatter. With lower willingness to pay for the central location, a proportion of traded-sector workers will move from central city to suburb region, as indicated by rising b_1. As a result, the central city will shrink in physical size, as shown by lower x_1, while the suburb area will expand.

Table 4-2: Comparative Static Analysis: the Consumer City

	Baseline	μ=0.3	τ=0.05	k=0.3	δ=0.5
Skill cutoff, b_1	12.7460 17.5734	12.4898 17.1067	5.5917 7.1354	28.9523 41.9465	15.7910 21.7326
Skill cutoff, b_2	1.4053 1.4658	1.4125 1.4745	1.4077 1.4688	1.3988 1.4588	1.3803 1.4346

Boundary of central city, x_1	0.1659 0.2514	0.0987 0.1483	0.5564 0.6596	0.0097 0.1313	0.1027 0.1835
Inner boundary of suburbs, x_2	0.7224 0.9461	0.4183 0.5468	1.3250 1.6020	0.6248 0.8982	0.5449 0.7324
City boundary, x_3	4.2500 4.8844	2.4322 2.7971	4.3306 5.0028	4.9309 5.5940	3.6879 4.2122
Physical size of suburbs, $x_3 - x_2$	3.5276 3.9383	2.0139 2.2503	3.0056 3.4008	4.3060 4.6958	3.1430 3.4798
Physical size of service worker region, $x_2 - x_1$	0.5564 0.6947	0.3195 0.3985	0.7686 0.9423	0.6151 0.7669	0.4422 0.5489
Housing price at CCD	1.3318 1.3765	0.8989 0.9167	1.2538 1.2946	1.1675 1.2045	1.3092 1.3517
Land price at CCD	0.1330 0.1421	0.0274 0.0293	0.1179 0.1257	0.1022 0.1088	0.1285 0.1370
Housing price slope at CCD	-0.8299 -1.3719	-0.5764 -0.9452	-0.7892 -1.3027	-0.4261 -0.7074	-0.8322 -1.3802
Price of non-traded services, G	9.9601 12.5296	9.8321 12.3153	9.9161 12.4760	10.1476 12.7113	11.4910 14.2512
Quality of service goods, q	1.6806 1.8004	1.6536 1.7739	1.6869 1.8083	1.6614 1.7790	3.0285 3.2398
Wage of service workers, w	1.5594 1.6477	1.5026 1.5807	1.5204 1.5998	1.5452 1.6345	1.5038 1.5808
Service wage premium, $w - b_2$	0.1541 0.1819	0.0901 0.1063	0.1127 0.1311	0.1463 0.1757	0.1236 0.1461
Aggregate demand for service goods	1.6686 1.8134	1.7409 1.8890	1.6749 1.8194	1.6528 1.8014	1.5985 1.7341
Proportion of service workers in economy	0.3439 0.3723	0.3480 0.3768	0.3452 0.3738	0.3401 0.3686	0.3290 0.3556
Expenditure share of non-traded service	0.5322 0.5797	0.5515 0.5973	0.5342 0.5818	0.5381 0.5851	0.5922 0.6349
Utility of workers at different skill levels					
Skill=1	3.8598 3.8568	3.8859 3.8818	3.8614 3.8585	3.8622 3.8589	3.9575 3.9460

Skill=2	4.0594 4.0235	4.0878 4.0498	4.0604 4.0244	4.0656 4.0293	4.2113 4.1624
Skill=5	4.7496 4.6699	4.7878 4.7072	4.7516 4.6717	4.7539 4.6756	5.0440 4.9465
Skill=10	5.4807 5.3538	5.5242 5.3982	5.4924 5.3616	5.4797 5.3561	5.9225 5.7729
Skill=30	7.1747 6.9339	7.2190 6.9862	7.1951 6.9545	7.1448 6.9165	7.9465 7.6726

Notes: parameters are same as in Table 1

4.8 Welfare Distribution

The increasing skill dispersion will affect the welfare of heterogeneous workers in different ways. The welfare impact of higher skill dispersion is in three folds. First, the larger group of high-income workers will raise the quality of service, offering a positive spillover effect on the other workers. Second, the larger group of high-income workers will raise the wage for service workers. Third, although these high-income newcomers can benefit the others through non-traded service market, they will put other workers in a worse situation on the housing market. They impose a negative spillover effect on other residents by bidding away central location and forcing the others to pay higher housing price and travel costs.

We present utility of workers by skill groups in Figure 4-23 to Figure 4-27. Besides, we also calculate the relative change in welfare across skill spectrum and show it in Figure 4-28. In general, we find that the high-skill traded-sector workers will suffer the most from the increasing skill dispersion, while the service-workers suffer the least.

Because of the negative spillover effect, all workers experience welfare loss. But, the low-skill worker suffers relatively less, as shown in Figure 4-28. It is because the positive spillover effect through rising wage is so strong that it partially covers the negative spillover effect through higher housing price. The traded-sector workers with relatively lower skill will suffer more from rising skill dispersion in an early stage, but as soon as they switch to produce non-traded services, they will be compensated by a higher wage. As an example, we analyze utility change of the worker with the skill of 1.43 (Figure 4-24). Initially, the worker produces traded goods and experiences a dramatic welfare loss from rising Gini coefficient. Yet, as Gini coefficient rises to around 0.49, the worker switches to produce non-traded services. Furthermore, as Gini coefficient rises from 0.49 to 0.55, the worker's welfare decreases at a much slower pace, because she receives a compensation through rising wage.

Next, we explore the welfare implications of housing supply elasticity and travel cost. We compare the relative change of utility in baseline and that in an alternative scenario. In general, changes in fundamental factors will rotate the curve of relative change of utility. Welfare benefits for high-skill traded-sector workers always mirror a welfare loss for the middle- and low-skill workers, vice versa.

When housing supply is more elastic, high-skill traded-sector workers will suffer relatively less from increasing skill dispersion, as opposed to the scenario that housing supply is inelastic, shown in Figure 4-29. When housing supply is more elastic, the central city could hold a greater amount of high-skill workers; hence, the negative spillover through housing market becomes weaker.

When travel cost for service consumers is lower, the high-skill traded-sector workers also suffer relatively less from increasing skill dispersion, as shown in Figure 4-30. The result is consistent with our expectation. Because the high-skill traded worker spends a greater proportion of their income on service goods, their welfare should be more sensitive to the change of travel costs.

Figure 4-23: Utility of service workers over skill Gini coefficient

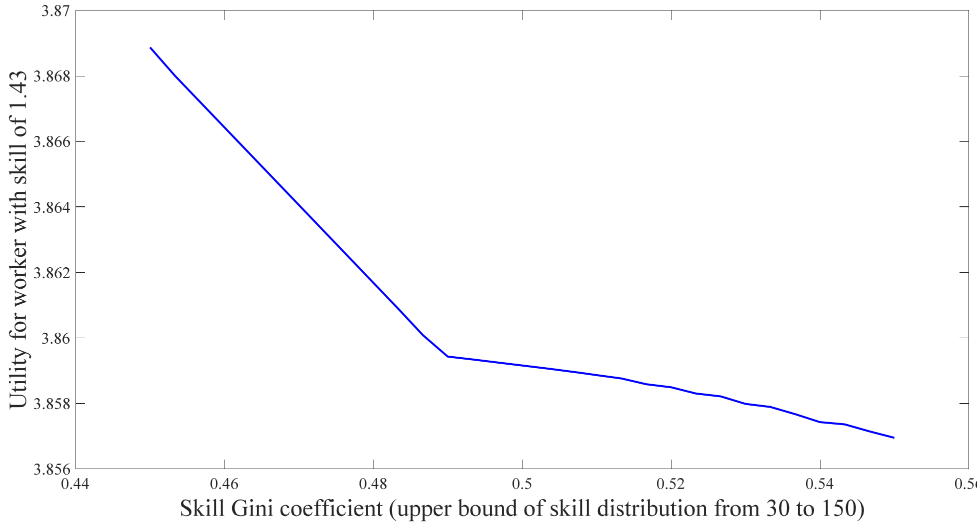

Figure 4-24: Utility of workers at skill level 1.43 over skill Gini coefficient

Figure 4-25: Utility of workers at skill level 5 over skill Gini coefficient

Figure 4-26: Utility of workers at skill level 10 over skill Gini coefficient

Figure 4-27: Utility of workers at skill level 30 over skill Gini coefficient

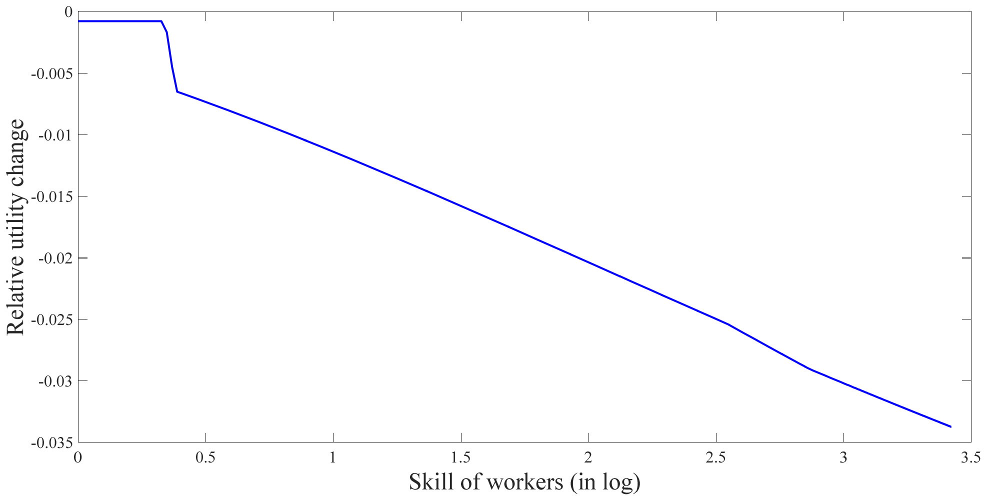

Figure 4-28: Relative change in utility by skill groups

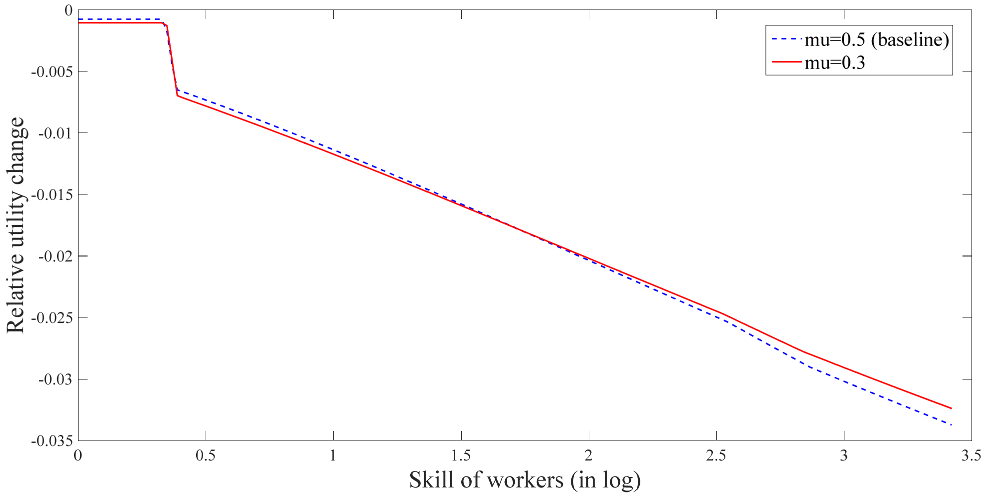

Figure 4-29: Relative change in utility by skill groups: mu=0.5 vs. mu=0.3

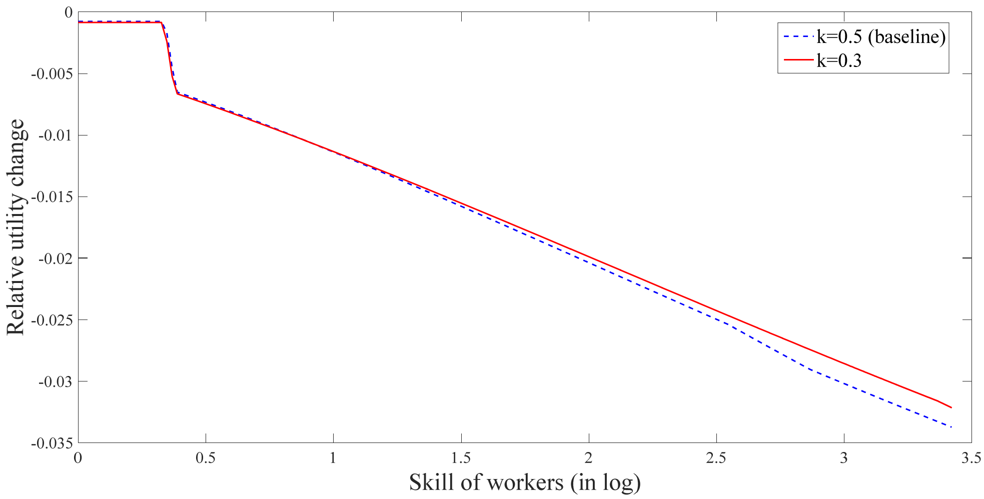

Figure 4-30: Relative change in utility by skill groups: k=0.5 vs. k=0.3

4.9 Efficiency of the Consumer City: Counterfactual Analysis with Marginal-cost Pricing

When the city maximizes the aggregate commercial land rent in the CCD, the non-traded services are priced above the marginal production costs. Hence, maximizing commercial land rent in the CCD is associated with a deadweight loss. Hence, we consider an alternative scenario that non-traded service is priced at the marginal production costs. We present the key results in Table 4-3. The baseline is given in column 1 and the marginal-cost pricing regime in column 2. To make them comparable, we assume that service quality is constant in two regimes.

Our results show that marginal-cost pricing favors the rise of "consumer city". When non-traded service becomes cheaper, consumers will substitute non-traded service for housing. Consumers will spend a higher share of their income on non-traded services. Driven by the increasing demand for services, a greater proportion of city employments will produce non-traded services.

When non-traded services are priced at marginal production costs, the consumer city will become more compact. The city boundary will shrink and the size of the internal city will also decrease. Hence, population density in the city will be higher, as shown in Figure 4-32. When non-traded service becomes cheaper, the central city will be made more attractive. Middle-skill traded-sector workers will move from suburban region to central city, as indicated by higher b_1. The benefits from improved consumption amenities are partially capitalized in housing price. Housing price in central city increases dramatically and housing price gradient at city center also becomes steeper, as shown in Figure 4-31 and Figure 4-33.

Table 4-3: Comparison between Two Pricing Regimes

	The baseline	Marginal-cost pricing
Service price/ marginal production cost	6.3870	1.0000
Skill cutoff, b_1	12.7460	8.7009
Skill cutoff, b_2	1.4053	5.0978
Boundary of central city, x_1	0.1659	0.0728
City boundary, x_3	4.2500	2.1581
Housing price at CCD	1.3318	2.3666
Housing price slope at CCD	-0.8299	-3.1995
Wage of service workers, w	1.5594	5.2785
Aggregate demand for service goods	1.6686	6.9836
Share of service workers in economy	0.3439	0.8754
Expenditure share of non-traded services	0.5322	0.6543
Total surplus	12.3470	30.3388

Notes: parameters are same as in Table 1.

Given the equilibrium prices in the marginal-cost pricing regime, we calculate "compensating income" for each individual that allow them to attain the level of utility as in the baseline scenario. Then, we compare the compensating income with the income received by the individual in the marginal-cost pricing regime. When the latter is higher, the individual experiences welfare gains, vice versa. For each skill group, we calculate worker's "willingness to pay" that is defined as the actual income minus the compensating income. We present our results in Figure 4-34.

In the marginal-cost pricing regime, all workers enjoy welfare gains, as shown by positive willingness to pay across skill groups. The willingness to pay of middle-skill workers are lower than that of high-skill and low-skill workers. It is because the middle-skill workers do not experience a wage growth, as it happens to the low-skill service workers, and they are poorer than the high-skill workers, thus constrained in their budget in taking advantage of the lower service price.

Also, we calculate the total surplus in the marginal-cost pricing regime that is defined as the sum of incremental land rent and aggregate willingness to pay minus the fixed cost in the service sector. Then, we compare it with the total surplus in the baseline that is defined as the sum of incremental land rent and service producer's profit. We find the former is much higher, hence showing that the deadweight loss associated with maximizing commercial land rent is significant (Table 4-3).

Our model implies that the marginal production costs and the fixed costs of non-traded service sector should be financed separately. While the marginal production costs should be covered by service price, the fixed cost charged on the service producer should be subsidized by the government. The policy implication is even more relevant for the Chinese cities where the high urban land rent dramatically drives up the fixed costs in the non-traded service sector. Our analysis implies that Chinese governments should subsidize the high commercial land rent that is charged on non-traded service producers, to accelerate the structural transition of the economy. By adopting the policy, the government will not only encourage the switch of employments from the traded sector to the non-traded service sector but also correct the distortion in resource allocation and improve social welfare.

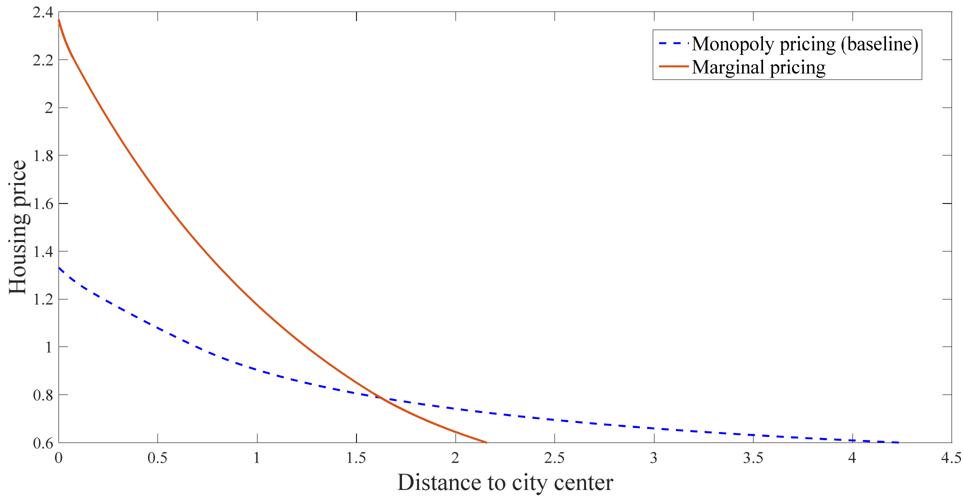

Figure 4-31: Housing price over distance to the CCD: maximizing commercial land rent vs. marginal-cost pricing

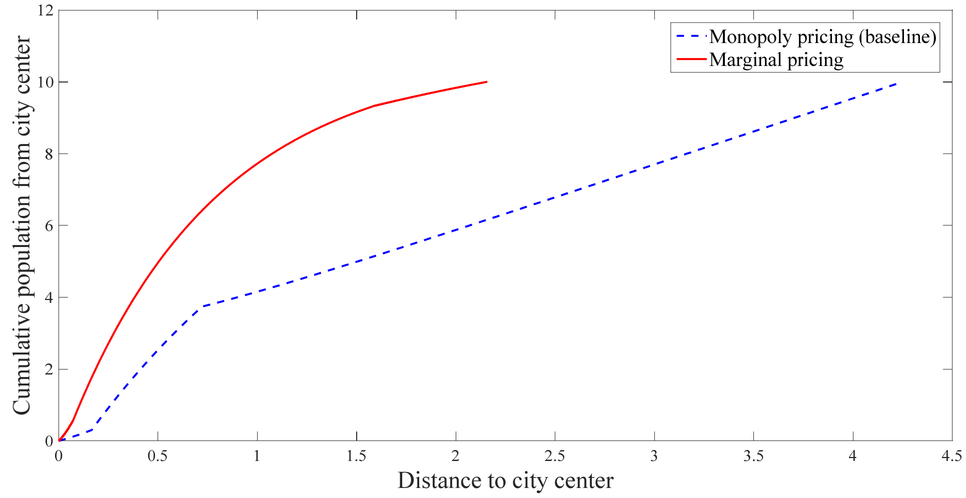

Figure 4-32: Cumulative population over distance to the CCD:

maximizing commercial land rent vs. marginal-cost pricing

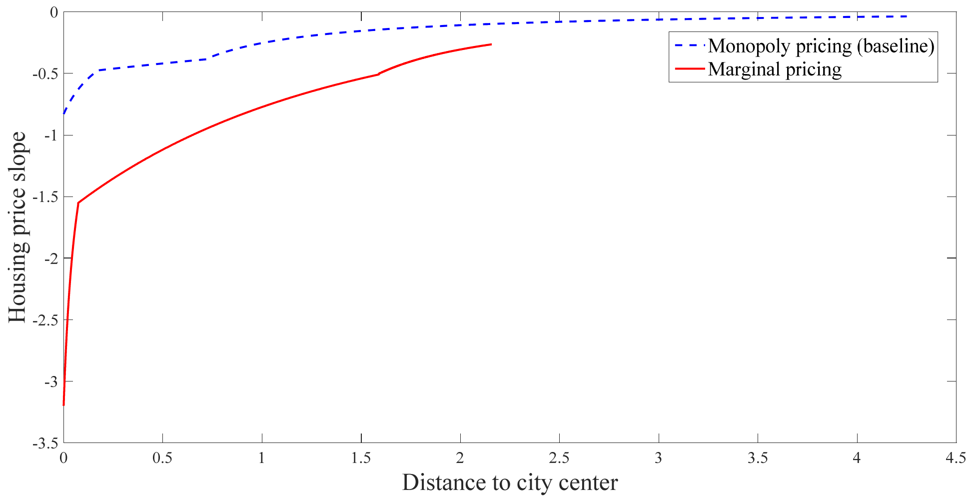

Figure 4-33: Housing price slope over distance to the CCD:

maximizing commercial land rent vs. marginal-cost pricing

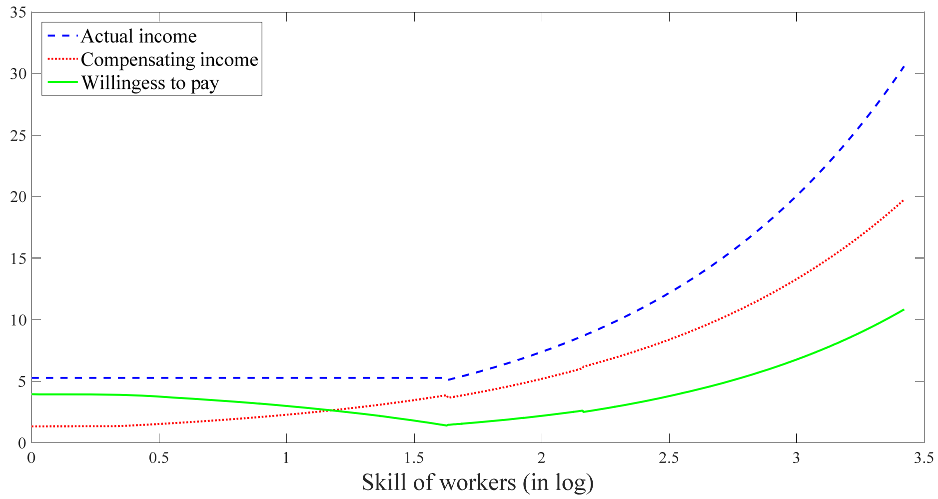

Figure 4-34: Willingness to pay by skill groups

4.10 Conclusion

As the service sector takes up greater share of the US economy, it is important to understand its implications for the cities. In this thesis, we explore a few research questions associated with consumer cities. Different from most of previous studies that examine the concept of "manufacturing cities", we emphasize the cities as consumption centers where people can enjoy the diversity of local services.

We contribute to the literature on skill sorting, by providing a consumption-based theory of skill sorting across cities. While most of previous studies argue that skill sorting can be driven by agglomeration benefits in traded sector, our study shows that it can also be a result of consumer's desire to enjoy more varieties of non-traded services. Also, our studies can provide important implications for welfare inequality. The presence of the high-skill workers in the economy can benefit the low-skill through raising the wage in service sector, but it may not benefit the middle-skill workers.

Our studies also contribute to the literature on urban land use. While previous studies have done an excellent job in examining the functions of cities as manufacturing centers, few of them explores the internal structure of a city that serves as a consumption center for local services. We fill this research cap in the literature, by examining internal structure of a consumer city where people cluster to share the externalities in non-traded service sector. Also, we examine efficiency of resource allocation in a consumer city.

The researches on the topic of consumer cities are far from sufficient. Even though we abstract from the agglomeration economics in traded sector in this thesis, it will be interesting to construct a more general framework to understand the interactions between the increasing returns in traded sector and non-traded sector. Also, it will be interesting to embed the monocentric consumer city model in a broader framework that accounts for skill sorting across cities.

5 Conclusion of the Thesis

This thesis consists of two studies that focus on consumer cities. In the first study, we present a model to show that asymmetric spatial equilibrium can emerge from symmetric locations in the presence of increasing returns in local consumer amenities and non-homothetic preferences for such amenities. In the model, the workers are fully mobile across cities and their location choices determine the skill distribution in each city endogenously.

The model can account for widened housing price dispersion across cities solely by increased aggregate skill inequality in the economy. A larger share of high-skill workers reinforces the increasing returns in local consumer amenities and income segregation among traded-sector workers across cities. The model also clarifies the effect of local housing supply elasticity on asymmetric equilibrium outcome: restrictive housing supply may make the "superstar" city more exclusive but would moderate, rather than exacerbate, housing price dispersion across cities when aggregate skill inequality rises. This clarification has important policy implications—expanding housing supply in a "superstar" city can have unintended consequence of reinforcing its advantage in local consumer amenities and hence its high housing price.

More importantly, our model builds on a micro foundation that can be calibrated to quantify the contribution of aggregate skill inequality to housing price dispersion observed in a real economy. In addition, our model can also account for the rise of the employment share of non-traded service sector resulting from increased aggregate skill inequality, a significant feature of many economies like US. Related to the impact of aggregate skill inequality on employment structure, our model reveals that widening aggregate skill inequality can benefit low-skill workers due to increased demand for non-traded services, which low-skill workers generally have a comparative advantage in producing. Moreover, the welfare gain of the low-skill non-traded service workers is at the expense of high-skill traded-sector workers, who, although enjoying a greater variety of non-traded services in the presence of a larger share of high-skill workers in the economy, nevertheless have to pay higher labor cost for each variety of non-traded services.

In the second study, we examine the internal structure and efficiency of a consumer city, conditional on skill distribution of the workers in the city. Traditional monocentric city model studies resource allocation in terms of housing and commuting in a “manufacturing city” setting, where all employment is in the traded sector and located at the city center. In this the study, we present a model to examine the urban form and resource allocation of a "consumer city", where agglomeration is derived by the non-traded sector and when the size of the non-traded sector is endogenous. The consumer city is a club for non-traded services, where agents cluster to share the fixed cost. Through spatial skill sorting, heterogeneous agents reveal their preference for the non-traded services in equilibrium and the urban form of the consumer city mimics the optimal two-part tariff.

When the inequality of worker's skill becomes wider, our model predicts that the status of consumer city will be reinforced: service producer will improve the quality of its products and hire a greater proportion of total workforce; consumers will spend a larger percentage of their income on non-traded services. We also highlight that better public transportation system will improve the quality of life in the consumer city when it reduces commuting cost for low-skill workers and allow the service firms to hire those workers at lower wages.

Our model also predicts a downward-sloping welfare change across skill spectrum. Because the high-skill workers will bid up housing price at the central location, all workers will be harmed by higher housing price in the city. But the low-skill workers will suffer relatively less, due to increased demand for non-traded services, which low-skill workers have a comparative advantage in producing.

One of our main goals is to examine efficiency of the consumer city. We find that maximizing commercial land rent in the central commercial district is associated with a deadweight loss, and it causes inefficiency in resource allocation--the share of employments in the non-traded service sector is below social optimal and dispersed urban form generates more frictions. Hence, we argue that government should use the marginal-cost pricing regime by regulating the price of services and subsidizing the fixed costs changed on the service producer. By adopting the marginal-cost pricing regime, the government will encourage the switch of employments from traded goods sector to the non-traded service sector. Also, as consumption amenities are improved, the urban form will become more compact, hence, associated with fewer urban frictions.

In the future work, we will examine the implications of plain geography for the urban form and resource allocation efficiency of a consumer city. Also, it will be interesting to embed the monocentric consumer city model in the model of skill sorting across cities to endogenize the skill distribution in the consumer city.

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Appendices

Proof of Lemma I in Essay I

The indirect utility corresponds to the expenditure function,

$$I = \left(\varepsilon \left(V + \frac{\beta}{\gamma} \left(\frac{P}{G} \right)^{\gamma} \right) \right)^{\frac{1}{\varepsilon}} G$$

First, the expenditure function is homogenous of degree one in *G* and *P*. Second, the expenditure function must be non-decreasing in *G* and *P*. $\frac{\partial I}{\partial P}$ and $\frac{\partial I}{\partial G}$ are given by,

$$\frac{\partial I}{\partial P} = \beta \left(\frac{I}{G} \right)^{1-\varepsilon} \left(\frac{G}{P} \right)^{1-\gamma}$$

$$\frac{\partial I}{\partial G} = \left(\frac{I}{G} \right)^{1-\varepsilon} \left(\left(\frac{I}{G} \right)^{\varepsilon} - \beta \left(\frac{P}{G} \right)^{\gamma} \right)$$

If and only if $\left(\frac{I}{G}\right)^{\varepsilon} - \beta \left(\frac{P}{G}\right)^{\gamma} \geq 0$, the expenditure function is non-decreasing in *G*.

Third, according to the integrability theorem, the utility function represents a locally non-satiated preference relation, if and only if the Slutsky matrix $\boldsymbol{H}$ is symmetric and negative semidefinite and satisfies $\boldsymbol{H} \cdot \boldsymbol{P} = \boldsymbol{0}$, where $\boldsymbol{P}$ is the vector of prices. The Hessian of the expenditure function is,

$$H = \chi \begin{bmatrix} \frac{G}{P} & -1 \\ -1 & \frac{P}{G} \end{bmatrix}$$

where,

$$\chi = \beta \left(\frac{I}{G} \right)^{1-2\varepsilon} \left(\frac{P}{G} \right)^{-1+\gamma} \frac{1}{G} \left(\beta (1-\varepsilon) \left(\frac{P}{G} \right)^{\gamma} - \left(\frac{I}{G} \right)^{\varepsilon} (1-\gamma) \right)$$

Eigenvalues of the Hessian matrix are 0 and

$$\beta I\left(\frac{I}{G}\right)^{-2\varepsilon}\left(\frac{P}{G}\right)^{\gamma}\left[\beta(1-\varepsilon)\left(\frac{P}{G}\right)^{\gamma}-(1-\gamma)\left(\frac{I}{G}\right)^{\varepsilon}\right]\left(\frac{1}{P^2}+\frac{1}{G^2}\right)$$

The second eigenvalue is less or equal to zero, if and only if,

$$\beta(1-\varepsilon)\left(\frac{P}{G}\right)^{\gamma}-(1-\gamma)\left(\frac{I}{G}\right)^{\varepsilon}\leq 0.$$

Because $\varepsilon \geq \gamma$, $\left(\frac{I}{G}\right)^{\varepsilon}-\beta\left(\frac{P}{G}\right)^{\gamma}\geq 0$ is a sufficient condition for $\beta(1-\varepsilon)\left(\frac{P}{G}\right)^{\gamma}-(1-\gamma)\left(\frac{I}{G}\right)^{\varepsilon}\leq 0.$

Q.E.D

Proof of Proposition I in Essay I (skill sorting of traded-sector workers)

First, given the indirect utility function Eq (3-1) and the assumption that worker productivity in the traded sector is independent of location, traded-sector workers will always prefer living in the city with low housing price and low composite non-traded service price. Therefore, the city with high housing price and high composite non-traded service price will attract no traded-sector workers and hence has no income to support non-traded service employment. Therefore, any equilibrium with positive population in both locations must have housing price differences across locations compensating the differences in composite non-traded service price.

Second, from the indirect utility function Eq (3-1),we have,

(A1) $$\frac{\partial^2 V}{\partial I\,\partial G}=-\varepsilon I^{\varepsilon-1}\left(\frac{1}{G}\right)^{\varepsilon+1}<0.$$

High income and low composite non-traded service price are complementary. If there exists a traded worker with skill level b^*, who is indifferent between two cities, i.e.,

$$\frac{1}{\varepsilon}\left(\frac{b^*}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{b^*}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma},$$

then the single-crossing condition (A1) ensures that the traded-sector workers with skill $b > b^*$ will all prefer the city with a lower composite non-traded service price and higher housing price.

Q.E.D

Proof of Proposition II in Essay I (non-traded sector employment)

We prove this proposition in two steps:

Step 1. City 1 pays higher wage to the workers in non-traded service sector, i.e., $w_1 > w_2$.

Suppose that $w_1 \le w_2$. Because $w_2 = b_2 < b_1$, proposition I says that non-traded service workers will strictly prefer city 2, and this conflicts with the condition that non-traded service workers are indifferent between two cities. Therefore, City 1 must pay higher wage to the non-traded service workers to make the non-traded service workers indifferent between two cities.

Step 2. City 1 must employ a larger number of non-traded service workers.

Because City 1 pays higher wage to the workers in non-traded service sector, from (3-18), the price of a single service variety is higher. To maintain a lower composite non-traded service price, City 1 must produce a greater variety of services. Therefore, City 1 must employ a larger number of non-traded service workers.

Q.E.D

Proof of Lemma I in Essay II

The indirect utility corresponds to the expenditure function,

$$I = \left(\varepsilon \left(V + \frac{\beta}{\gamma} \left(\frac{P}{G/q} \right)^{\gamma} \right) \right)^{\frac{1}{\varepsilon}} \frac{G}{q}$$

First, the expenditure function is homogenous of degree one in *G* and *P*. Second, the expenditure function must be non-decreasing in *G* and *P*. $\frac{\partial I}{\partial P}$ and $\frac{\partial I}{\partial G}$ are given by,

$$\frac{\partial I}{\partial P} = \beta \left(\frac{Iq}{G} \right)^{1-\varepsilon} \left(\frac{G}{Pq} \right)^{1-\gamma}$$

$$\frac{\partial I}{\partial G} = \left(\frac{Iq}{G} \right)^{1-\varepsilon} \left(\left(\frac{Iq}{G} \right)^{\varepsilon} - \beta \left(\frac{Pq}{G} \right)^{\gamma} \right) \frac{1}{q}$$

If and only if $\left(\frac{Iq}{G}\right)^{\varepsilon} - \beta \left(\frac{Pq}{G}\right)^{\gamma} \geq 0$, the expenditure function is non-decreasing in *G*.

Third, according to the integrability theorem, the utility function represents a locally non-satiated preference relation, if and only if the Slutsky matrix H is symmetric and negative semidefinite and satisfies $H \cdot P = 0$, where P is the vector of prices. The Hessian of the expenditure function is,

$$H = \chi \begin{bmatrix} \frac{G}{P} & -1 \\ -1 & \frac{P}{G} \end{bmatrix}$$

where,

$$\chi = \beta \left(\frac{Iq}{G} \right)^{1-2\varepsilon} \left(\frac{Pq}{G} \right)^{-1+\gamma} \frac{1}{G} \left(\beta(1-\varepsilon)\left(\frac{Pq}{G} \right)^{\gamma} - \left(\frac{Iq}{G} \right)^{\varepsilon} (1-\gamma) \right)$$

Eigenvalues of the Hessian matrix are 0 and

$$\beta I \left(\frac{Iq}{G} \right)^{-2\varepsilon} \left(\frac{Pq}{G} \right)^{\gamma} \left[\beta(1-\varepsilon)\left(\frac{Pq}{G} \right)^{\gamma} - (1-\gamma)\left(\frac{Iq}{G} \right)^{\varepsilon} \right] \left(\frac{1}{P^2} + \frac{1}{G^2} \right)$$

The second eigenvalue is less or equal to zero, if and only if,

$$\beta(1-\varepsilon)\left(\frac{Pq}{G} \right)^{\gamma} - (1-\gamma)\left(\frac{Iq}{G} \right)^{\varepsilon} \le 0.$$

Because $\varepsilon \ge \gamma$, $\left(\frac{Iq}{G}\right)^{\varepsilon} - \beta\left(\frac{Pq}{G}\right)^{\gamma} \ge 0$ is a sufficient condition for $\beta(1-\varepsilon)\left(\frac{Pq}{G}\right)^{\gamma} - (1-\gamma)\left(\frac{Iq}{G}\right)^{\varepsilon} \le 0.$

Q.E.D

Proof of Lemma II in Essay II (skill sorting of traded-sector workers)

Suppose consumer's preference is represented by a utility function in a general form,

$$U(X, S, H)$$

Where X is consumption of traded goods, S is nontraded services and H is housing goods. At each location, the housing bid rent is the highest price that individuals are willing to pay, subject to the constraint that such price leaves the individual no worse off than others of equal skill. Therefore, housing bid rent can be determined by solving the following optimization problem, in which consumers with income I choose consumption bundles to maximize housing price, subject to that he receives a reservation utility, $\bar{u}$,

$$\max_{X,S,H} P_h = \frac{I - X - S(G + kx)}{H}$$
$$s.t. U(X,S,H) = \bar{u}$$

Lagrange function of this optimization problem is,

$$L = \frac{I - X - S(G + kx)}{H} + \lambda\left(\bar{u} - U(X,S,H)\right)$$

We can obtain Alonso-Muth condition by using envelope theorem,

$$\frac{\partial P_h}{\partial x} = \frac{\partial L}{\partial x} = -\frac{1}{H} kS$$

Calculating the derivative with respect to the worker's income, we have,

$$\frac{\partial^2 P_h}{\partial x \partial I} = \frac{\partial H}{\partial I}\frac{kS}{H^2} - \frac{\partial S}{\partial I}\frac{k}{H} = \frac{kS}{IH}(\varepsilon_H - \varepsilon_S)$$

If income elasticity of demand for housing ε_H is lower than income elasticity of demand for services ε_S, $\frac{\partial^2 P_h}{\partial x \partial I} < 0$,and high-skill traded-sector workers live closer to the city center. **Q.E.D.**

Proof of Proposition I in Essay II (spatial structure of a consumer city)

We denote city boundary by x_3 and denote the skill of the least skilled traded-sector workers by b_2.

I prove Proposition I by contradiction. Suppose the least-skilled traded-sector workers do not live at city boundary x_3 but at a location $x_l < x_3$. According to the Lemma I, it must be that non-traded service workers occupy the area $[x_l, x_3]$. Because utility of service worker and trade worker of skill b_2 must be equal at location x_l, the service worker and the traded worker of skill b_2 must earn equal net income,

$$w - \tau x_l = b_2$$

Also, because of utility equalization, they consume equal amount of housing goods and service goods, i.e., $Q_{S,trd} = Q_{S,ser}$ and $Q_{h,trd} = Q_{h,ser}$. For service workers, the gradient of bid rent at location l is given by,

$$gradient_{ser} = \frac{-\tau - kQ_{S,serv}}{Q_{h,ser}}$$

For the least skilled traded-sector workers, gradient of bid rent at location x_l is given by,

$$gradient_{trd} = \frac{-kQ_{S,trd}}{Q_{h,trd}}$$

Therefore, $|gradient_{ser}| > |gradient_{trd}|$, which means that service workers have higher wiliness to pay for more central location and they will move closer to the city center. Therefore, in equilibrium, the traded worker with skill b_2 will live at city boundary. The part (i) is proved.

Next, I prove part (ii). First, it is straight to prove that $Q_S(I,P,G)/Q_h(I,P,G)$ decreases in income *I*. Then, suppose city center is occupied by most-skilled traded worker (denoted $\bar{b}$), housing price gradient at city center is given by,

$$|gradient_{\bar{b}}| = \frac{kQ_S(\bar{b}, P_0, G_0)}{Q_h(\bar{b}, P_0, G_0)}$$

Suppose city center is occupied by service worker, the gradient is given by,

$$|gradient_{ser}| = \frac{\tau + kQ_S(w, P_0, G_0)}{Q_h(w, P_0, G_0)}$$

If commuting cost $\tau \to 0$, service worker's wage, $w \to b_2$. Because $\bar{b} > b_2$, $\bar{b} > w$.

Besides, if $\tau \to 0$,

$$|gradient_{ser}| \to \frac{kQ_S(w, P_0, G_0)}{Q_h(w, P_0, G_0)} < |gradient_{\bar{b}}|$$

Therefore, there must exists positive τ, such that

$$|gradient_{ser}| < |gradient_{\bar{b}}|$$

and the most-skilled traded worker occupies city center. Further, there must exists a skill cutoff $b_1 < \bar{b}$ and distance cutoff x_1, x_2, such that traded worker with skill $[b_1, \bar{b}]$ live in area $[0, x_1]$ and service workers live in the area $[x_1, x_2]$. Part (ii) is proved.

Then, I prove part (iii). In equilibrium, service workers must be indifferent between location x_1 and x_2,

$$\frac{1}{\varepsilon}\left(\frac{w-\tau x_1}{(G+kx_1)/q}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P_1}{(G+kx_1)/q}\right)^{\gamma}=\frac{1}{\varepsilon}\left(\frac{w-\tau x_2}{(G+kx_2)/q}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P_2}{(G+kx_2)/q}\right)^{\gamma}$$

Traded workers with skill b_1 are indifferent between location x_1 and x_2,

$$\frac{1}{\varepsilon}\left(\frac{b_1}{(G+kx_1)/q}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P_1}{(G+kx_1)/q}\right)^{\gamma}=\frac{1}{\varepsilon}\left(\frac{b_1}{(G+kx_2)/q}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P_2}{(G+kx_2)/q}\right)^{\gamma}$$

From above two equations, we have,

$$\left(\frac{w-\tau x_1}{G+kx_1}\right)^{\varepsilon}-\left(\frac{b_1}{G+kx_1}\right)^{\varepsilon}=\left(\frac{w-\tau x_2}{G+kx_2}\right)^{\varepsilon}-\left(\frac{b_1}{G+kx_2}\right)^{\varepsilon}$$

Rearrange the equation,

$$b_1^{\varepsilon}=\frac{\left(\frac{G_2}{G_1}\right)^{\varepsilon}(w-\tau x_1)^{\varepsilon}-(w-\tau x_2)^{\varepsilon}}{\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1}$$

It is sufficient to compare b_1^{ε} and w^{ε},

$$b_1^{\varepsilon}-w^{\varepsilon}=\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1\right)^{-1}\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}(w-\tau x_1)^{\varepsilon}-(w-\tau x_2)^{\varepsilon}-\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1\right)w^{\varepsilon}\right)$$

Because $G_2 > G_1$,

$$\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1\right)^{-1}>0$$

Hence, we only need to discuss the sign of,

$$\left(\frac{G_2}{G_1}\right)^{\varepsilon}\left(1-\frac{\tau x_1}{w}\right)^{\varepsilon}-\left(1-\frac{\tau x_2}{w}\right)^{\varepsilon}-\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1\right)$$

When commuting cost is only a small proportion of total income of service workers, $\frac{\tau x_1}{w}$ and $\frac{\tau x_2}{w}$ are close to zero. Then, using first-order Taylor approximation, we have,

$\left(1-\frac{\tau x_1}{w}\right)^{\varepsilon} \approx -\frac{\varepsilon \tau x_1}{w}$ and $\left(1-\frac{\tau x_2}{w}\right)^{\varepsilon} \approx -\frac{\varepsilon \tau x_2}{w}$

Hence,

$$\left(\frac{G_2}{G_1}\right)^{\varepsilon}\left(1-\frac{\tau x_1}{w}\right)^{\varepsilon}-\left(1-\frac{\tau x_2}{w}\right)^{\varepsilon}-\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1\right)\approx-\left(\left(\frac{G_2}{G_1}\right)^{\varepsilon}-1\right)\frac{\varepsilon\tau x_1}{w}+\frac{\varepsilon\tau(x_2-x_1)}{w}$$

Substitute G_1 and G_2 using $G_1 = G + kx_1$ and $G_2 = G + kx_2$,

$$-\left(\left(\frac{G+kx_2}{G+kx_1}\right)^{\varepsilon}-1\right)\frac{\varepsilon\tau x_1}{w}+\frac{\varepsilon\tau(x_2-x_1)}{w}=-\left(\left(1+\frac{k(x_2-x_1)}{G+kx_1}\right)^{\varepsilon}-1\right)\frac{\varepsilon\tau x_1}{w}+\frac{\varepsilon\tau(x_2-x_1)}{w}$$

Using first-order Taylor approximation,

$$\left(1+\frac{k(x_2-x_1)}{G+kx_1}\right)^{\varepsilon}-1\approx\frac{\varepsilon k(x_2-x_1)}{G+kx_1}$$

Hence,

$$-\left(\left(1+\frac{k(x_2-x_1)}{G+kx_1}\right)^{\varepsilon}-1\right)\frac{\varepsilon\tau x_1}{w}+\frac{\varepsilon\tau(x_2-x_1)}{w}\approx\frac{\varepsilon\tau}{w}(x_2-x_1)\left(1-\frac{\varepsilon k x_1}{G+kx_1}\right)>0$$

Hence, $b_1^{\varepsilon} > w^{\varepsilon}$ and $b_1 > w$ **Q.E.D.**

Algorithm for Searching Equilibrium

We develop an algorithm to search for equilibrium in three steps.

Step 1: Starting with an initial guess of $\{b_1, b_2, w, R_0, G, q\}$, we find the set of functions $\{P_h(x), b(x)\}$ that satisfy the equilibrium conditions (i) to (iv).

Step 2: We develop four indicators to guide our search for equilibrium $\{b_1, b_2, w, R_0\}$. First, we compare the utility of traded worker with skill b_1 at location x_1 and x_2. If utility at x_1 is higher than x_2, traded worker will move to the location x_1. Hence, we must lower the guess for b_1 to allow more traded-sector workers to move to the central area, vice versa. Second, we calculate the utility of trade worker with b_2 and compare it with the utility of service workers. If the traded worker with skill b_2 can gain higher utility in the service sector, it means the service wage w is too high, hence, we must lower the guess for service wage w, vice versa. Third, we calculate aggregate demand of services and compare it with the total output implied by service worker population. If there is an oversupply of services, we lower the guess for b_2 to reduce the population of service workers, vice versa. Last, based on the housing price at city boundary, we calculate the land price at city boundary and compare it with agriculture land price R_{agr}. If the land price at city boundary is higher than R_{agr}, we lower the guess for R_0, vice versa.

Step 3: Repeat Step 1 and 2, until condition (i) to (viii) are satisfied.

Step 4: Calculate $\frac{\partial \pi}{\partial G}$ and $\frac{\partial \pi}{\partial q}$ numerically. If $\frac{\partial \pi}{\partial G} > 0$, raise initial guess for G, vice versa. If $\frac{\partial \pi}{\partial q} > 0$, raise initial guess for *q*, vice versa.

Step 5: Repeat Step 1 to 4, until all equilibrium conditions are satisfied.