THREE ESSAYS ON INNOVATION AND TECHNOLOGY TRANSFER

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Declaration

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Signed: 张续垚

Date: **November 16, 2016**

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Contents

Summary

This thesis consists of three independent chapters on innovation and technology transfer.[1]

The first chapter studies a model of research joint venture (RJV) competition where all firms, including firms in the RJV, independently choose their investments for process innovation before they compete in a Cournot market. Even with perfect spillovers between RJV firms, an industry-wide RJV does not lead to a better technological development and a higher consumer surplus, compared to the case without any RJV. Yet, every non-industry-wide RJV lead to strict improvements for both measures. Moreover, the improvements are larger when firms may license their technologies after making R&D investments. Government should encourage innovation through collaboration with technology transfer as an alternative to concerting an industry-wide cooperative effort.

The second chapter studies reverse licensing imposed by an upstream monopolist that requires downstream producers to surrender their patents so that the upstream monopolist may incorporate all the technologies into the inter-

[1]The first and second chapter is co-authored with my supervisor Professor Chiu Yu Ko, while the third chapter is co-authored with my supervisor, Professor Chiu Yu Ko, and Bo Shen.

mediate goods. Qualcomm, the world largest smartphone chip producer and the monopolist in the Chinese market, was ruled by Chinese government that its reverse licensing was anticompetitive, and that it must compensate downstream producers for patents surrendered. The chapter shows that reverse licensing yields the highest consumer surplus, aggregate profit, and hence social welfare, compared to the cases without licensing, with independent royalty licensing, and patent pool. Moreover, the remedy that requires compensation for surrendered patents will lead to a greater incentive to innovate, especially to firms with better technologies.

The third chapter studies the optimal environmental tax under the possibility of corruption and licensing of a clean technology. In an environment-oriented country, the firm with dirty technology may choose to bribe the bureaucrat to mislead the actual emission, rather than adopt the clean technology. Government should set a very high environmental tax, and corruption may improve social welfare in comparing with licensing. Higher wage for bureaucrat could effectively reduce corruption, but also hinder the incentive for the clean firm to license the technology. Technology transfer is more likely to occur in an output-oriented country. Government should set a low tax rate to induce high incentive for the license and adoption.

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Chapter 1

Research Joint Venture with Technology Transfer

1.1 Introduction

Research joint ventures (RJVs) are analyzed under the rule of reason in US since 1984. National Cooperative Research Act of 1984 (PL-98-42) states that "...the conduct of any person in making or performing a contract to carry out a joint research and development venture shall not be deemed illegal per se ...".[1] Antitrust authorities should be lenient towards the formation of a research joint venture (RJV) if it does not reduce competition. Since then, RJVs have becoming increasingly popular around the world.[2]

When RJV firms cooperate in research development, they are more will-

[1]Before 1984, antitrust authorities have prevented firms from forming RJVs (Grossman and Shapiro 1986). In 1993, a new amendment (PL-98-462) was passed to further reduce potential antitrust liability for a research joint venture. In 2004, the latest amendment (PL-108-237) included standard development organization into the Act.

[2]Hernan et al. (2003) document that there are 1229 and 892 RJVs formed in EU from 1986 to 1996 in information technology and aerospace industries respectively.

ing to spend resources on research due to positive spillovers. As RJV firms have lower costs of productions, there will be more intense competition in the product market and thereby leading to a lower price and a higher level of consumer surplus. However, in a seminal paper by Kamien et al. (1992), a RJV may fail to achieve any one of these objectives, and may even be worsen than no RJV at all. They consider a RJV competition where every firm simultaneously chooses research level followed by non-cooperative competition in the product market.[3] They show that the formation of an industry-wide RJV reduces effective investment level and consumer surplus comparing with firms doing individual research. First, this industry-wide RJV is not plausible in reality. Second, this anti-competitive nature of RJV is at odds with development of antitrust law in US. Their key assumption is that all firms either form a single RJV or no RJV. However, when all firms are in the RJV, they have very little incentive to do research given spillover to the other firms, and thus this rules out the important channel to promote competition in the research phase (Proposition 1.1).

We relax this assumption to allow an RJV formed by only some firms in the industry. We show that any RJV that is not industry-wide leads to strictly

[3] An RJV competition resembles the case that firms are assigned different tasks, and the whole project is the combination of the tasks. One example is software industry. They also consider RJV cartel where firms cooperate in their R&D activities to maximize the joint profit. Pharmaceutical industries seems to fall in this category.

lower total production costs and a higher level of consumer surplus (Theorem 1.1). This is consistent with the motivation that taking RJVs under the rule of reason in US.

With RJV firms possessing a more advanced technology than non-RJV firms, technology transfers through licensing is an important channel to recoup costs from research. As RJV firms do not share the full burden of research cost, they should have stronger incentive to innovate. However, licensing may reduce firms' incentive to innovate due to substitution between innovation and licensing.[4] We find that the timing for licensing is crucial for the welfare analysis due to strategic behavior of licensees.[5] On the one hand, if licensing agreement is signed before the research stage (ex-ante licensing), the licensees have no incentive to do any investment. On the other hand, if firms reach licensing agreement after research stage (ex-post licensing), licensees have incentive to do research to improve the bargaining position with the RJV firms. We show that ex-post licensing leads to more advanced technological development and improved consumer surplus, in comparison with no licensing. For ex-ante licensing, although technological investment is reduced compared with no licensing, consumer surplus could be improved compared to no licensing if

[4] Chang et al. (2013) show that when only one firm can do innovation, an (ex-post) licensing may reduce incentive to innovate and welfare.

[5] Gallini and Winter (1985) consider a dynamic duopoly model with different initial cost for firms, and stochastic process for the R&D. They show that when initial cost are close, ex-post licensing encourages R&D; while ex-ante agreement is unlikely to be formed.

(i) R&D cost is high, (ii) RJV size is small relative to the industry size, and (iii) number of licensee is few (Theorem 1.2).

To determine the equilibrium RJV size, we consider a simple RJV formation game. Without licensing, an industry-wide RJV does not maximize profits of its members, implying that if an industry features only one single RJV with closed membership, the RJV will not include all of the firms in the industry (Proposition 1.5). This suggests that policy implication based on the analysis on industry-wide RJV may require further scrutiny. For welfare analysis, our equilibrium RJV size is less than the social optimal one because RJV firms restrict innovation to lessen product market competition. Under ex-ante licensing, the equilibrium RJV is smaller because there will be technology transfer to non-RJV firms. For ex-post licensing, the equilibrium RJV size shrinks further. In particular, when research is not too efficient, the equilibrium size of RJV is always two regardless of number of firms in the industry. This implies that when institutional environment is conducive to licensing (for example, adequate protection of property right), then it may be difficult for the government to encourage an industry-wide agreement to consolidate research.

In the RJV literature, most focuses on RJV cartel and few studies RJV competition.[6] Katz (1986) shows that a RJV competition delivers better con-

[6] Our paper parallels Poyago-Theotoky (1995) that studies equilibrium and optimal size of RJV using numerical method under a setting of RJV cartel.

sumer surplus than the absence of a RJV, when RJV firms share their research costs according to some explicit rule. d'Aspermont and Jacquemin (1998) compare welfare consequence under RJV competition and RJV cartel in a duopoly, which is extended to a more general framework by Suzumura (1992).[7] Kaimen et al. (1992) show that in an oligopoly model, RJV cartel is consumer-surplus dominate no RJV which in turn dominates RJV competition. Greenlee (2005) studies RJV competition under some coalition formation games, and show that an industry-wide RJV improves welfare when spillover is low. To our knowledge, this is the first paper to formally study RJV competition for non-industry-wide RJV without cost sharing.

Our paper also contributes to the recent development in licensing literature to endogenzie the innovation process. Gallini and Winter (1986) study how ex-ante or ex-post licensing in a R&D game in a duopoly market. Recently, Sen and Tauman (2007) study how ex-post licensing affects incentive to innovate in an oligopolistic industry where only one innovator can do R&D. Ding and Ko (2016) study how ex-post licensing changes patent competition when all firms may invest in R&D.

The rest of the paper is organized as follows. Next section presents a motivating example. Section 1.3 presents a model of RJV competition. Section

[7]In RJV cartel, firms cooperate in their R&D activities to maximize the joint profit but still choose their product non-cooperatively.

1.4 extends the model with technology transfer, and Section 1.5 discusses the equilibrium size of RJV. Section 1.6 considers robustness and extensions, and Section 1.7 concludes. For exposition, some of precise statements of formal results and most of the proofs are relegated to Appendix.

1.2 A Motivating Example

Consider four firms competing in a Cournot market with positive marginal cost of production and zero fixed cost. Firms can invest in R&D to reduce this marginal cost. Consider a complete-information two-stage game where firms simultaneously decide their investments followed by production.

First consider a RJV formed by firms 1 and 2. Assume perfect spillover within the RJV that the reduction of marginal cost of any RJV firm is the aggregation of technological development of both firms 1 and 2. Compare the equilibrium with the case of four firms doing research individually, we have a two-firm RJV is superior to individual research in terms of both technological improvements and consumer surplus.[8] We will later show that this result holds in general in our Theorem 1.1.

As RJV firms spend more on R&D, they may license their advanced technology to non-RJV firms to increase their profits. Following Gallini and Winter

[8]The same result holds for the case where 3 firms form the RJV. However, in a simple RJV formation game in Section 3.3, the equilibrium RJV consists of two firms. Details of calculation can be found in Appendix A.2.1.

(1985), the licensing can be before research stage (**ex-ante licensing**) or after research stage (**ex-post licensing**). Consider a licensing auction by Katz and Shapiro (1986) where the licensor announces the number of license for sale. In equilibrium, RJV firms will license to two non-RJV firms in both ex-ante and ex-post licensing cases, if the research cost is sufficiently high. We can show an ex-post licensing further improves technology development and consumer surplus. RJV firms (firms 1 and 2) have more incentive to do R&D, because research cost can be recovered from the licensing fees. However, an ex-ante licensing reduces the technological development and consumer surplus, due to the free riding effect from the licensees. As our Theorem 1.2 shows, the result about ex-post licensing continues to hold in general setup but ex-ante licensing could still improves consumer surplus under some plausible conditions.

1.3 Model

We first study the benchmark case where all firms choose their research investment individually. Then we study the case when a single RJV formed by all firms in order to compare our result with Kaimen et al. (1992). Finally, we consider the cases when some firms are not in the RJV.

For tractability, there are two important departures from Kamien et al. (1992). First, we follow other papers (for example, d'Aspremont and Jacquemin

1988; Poyago-Theotoky 1995) in the literature to consider a standard linear demand function and a quadratic research cost function instead of a general demand function and a concave cost reduction function. Second, patent protection is perfect such that firms belonging to the RJV (referred as **RJV firms**) have a perfect information sharing, while firms outside RJV (referred as **non-RJV firms**) could not enjoy any spillover.[9] As discussed in Section 1.6, our main results remain valid when we remove the second departure.

1.3.1 Individual Research

There are $N \geq 3$ firms in a homogeneous good market with no fixed cost of production. Firms are indexed as $i \in \{1, \cdots, N\}$ and firm i has the marginal cost c_i. With a small abuse of notation, let the set $\{1, \ldots, N\}$ be denoted by N as well. Initially, all firms have the same production cost $c_i = c$ for all $i \in N$. The inverse demand function is $p = a - Q$ where $Q = \sum_{i \in N} q_i$ is the aggregate production and q_i is the production by firm i.

We consider the following two-stage game. In the first stage, each firm $i \in N$ simultaneously chooses to level of marginal cost reduction x_i so that the new marginal cost is $c_i = c - x_i$. To reduce marginal cost by x_i, firm i has to incur a research cost αx_i^2 where α captures research efficiency and

[9]Majewski (2008) documents that RJVs registered with US antitrust authority and found that RJVs exert huge effort to avoid unintended spillover to third parties.

the quadratic expression reflects the decreasing return in investment.[10] In the second stage, firms simultaneously choose their production. The profit for firm i is $\pi_i^{ind} = (p - c_i)q_i - \alpha x_i^2$. Throughout this paper, we assume α is sufficiently large such that production costs are non-negative after research. Following the standard assumption in the literature, all N firms remain active in the Cournot competition. Using backward induction, the equilibrium production by firm i is $q_i = \frac{a - Nc_i + \sum_{j \neq i} c_j}{N+1}$. The following lemma characterizes the subgame-perfect equilibrium.[11]

Lemma 1.1. *Under individual research, every firm $i \in N$ invests $x_i^{ind} = \frac{N}{\alpha(N+1)^2 - N}(a - c)$, produces $q_i^{ind} = \frac{\alpha(N+1)}{\alpha(N+1)^2 - N}(a - c)$ and earns a profit of $\pi_i^{ind} = \frac{\alpha(\alpha(N+1)^2 - N^2)}{(\alpha(N+1)^2 - N)^2}(a - c)^2$.*

1.3.2 Research Joint Venture

Consider a research joint venture (RJV) formed by $K \subseteq N$ firms. RJV firms share their research progress so that the cost reduction for firm $k \in K$ is $X_K \equiv \sum_{i \in K} x_i$. We first consider an industry-wide RJV ($K = N$) to compare our model with Kaimen et al. (1992). By backward induction, we have, for

[10] We require α to be not too small to guarantee non-negativity of production cost and second-order conditions as d'Aspremont and Jacquemin (1988) and Poyao-Theotoky (1995). The conditions can be found in the proof.

[11] From Lemma 1, it is evident that marginal costs are non-negative if and only if $\alpha > \alpha^*_{ind} \equiv \frac{aN}{c(N+1)^2}$.

all $i \in N$,

$$x_i^{all} = \frac{a-c}{\alpha(N+1)^2 - N} = \frac{x_i^{ind}}{N}, \text{ and } q_i^{all} = \frac{\alpha(N+1)(a-c)}{\alpha(N+1)^2 - N} = q_i^{ind}.$$

With the formation of the RJV, each firm is investing less than the case without the RJV because $\alpha(x_i^{all})^2 < \alpha(x_i^{ind})^2$, but they can achieve the same level of cost reduction as $X_N = \sum_{i \in N} x_i^{all} = x_i^{ind}$. Therefore, the formation of the RJV reduces overlapping research efforts due to spillover within RJV firms. Moreover, consumer surplus, measured by $(\sum_{i \in N} q_i)^2$, is the same for both individual research and industry-wide RJV cases. This results is consistent with Kamien et al. (1992) that they show an industry-wide RJV leads to no better technological improvement ($\sum_{i \in N} x_i^{all} \leq x_i^{ind}$) and consumer surplus ($(\sum_{i \in N} q_i^{all})^2 \leq (\sum_{i \in N} q_i^{ind})^2$), where equalities hold if and only if spillover within RJV is perfect. Our first proposition summarizes the above observation.[12]

Proposition 1.1. *An industry-wide RJV leads to the same technological improvement and consumer surplus compared to the case of individual research.*

Now we consider a K-firm RJV ($K \subsetneq N$), and will show that a K-firm RJV could achieve the higher level of technological development and consumer

[12] Throughout this paper, we will use the term technological improvements (or developments) as a short-hand notation for industrial level technological improvements.

surplus. The profit functions for RJV and non-RJV firms are

$$\pi_i^{no} = (p - (c - \sum_{k \in K} x_k))q_i - \alpha x_i^2 \text{ for all } i \in K, \text{ and}$$
$$\pi_j^{no} = (p - (c - x_j))q_j - \alpha x_j^2 \text{ for all } j \in N \backslash K.$$ [13]

By backward induction, technological improvements and quantity produced by RJV firms and non-RJV firms (assuming solution is interior) are

$$x_i^{no} = \frac{(N-K+1)[\alpha(N+1)^2 - N(N+1)]}{D^{no}}(a-c) \text{ for all } i \in K,$$
$$x_j^{no} = \frac{N(\alpha(N+1)^2 - K(N-K+1)(N+1))}{D^{no}}(a-c) \text{ for all } j \in N \backslash K,$$
$$q_i^{no} = \frac{\alpha(N+1)^2(\alpha(N+1) - N)}{D^{no}}(a-c) \text{ for all } i \in K,$$
$$q_j^{no} = \frac{\alpha(N+1)^2(\alpha(N+1) - k(N-K+1))}{D^{no}}(a-c) \text{ for all } j \in N \backslash K, \text{ and}$$

where $D^{no} = (\alpha(N+1)^2 - N(K+1))(\alpha(N+1)^2 - K(N-K+1)^2) - (N-K)(N-K+1)NK^2$. We assume α is large enough to guarantee non-negativity of production cost and production.[14] It is easy to check $q_i^{no} > q_j^{no}$ for all $i \in K$ and $j \in N \backslash K$. RJV firms will produce more than the non-RJV firms, due to the fact that they have advanced technology level, i.e. $\sum_{i \in K} x_i^{no} > x_j^{no}$ for all $j \in N \backslash K$.

Comparing equilibrium outcomes under a K-firm RJV, an industry-wide RJV and independent research, we can show that any K-firm RJV achieves

[13] The superscript “no” refers to no licensing to be distinguished from ex-post licensing “Ex-post” and ex-ante licensing “Ex-ante” in next section.

[14] The sufficient conditions are $\alpha > \alpha_{no}^*(a, c)$ and $\alpha \geq \frac{K(N-K+1)}{N+1}$ respectively where α_{no}^* is the larger root for the quadratic function $Kx_i^{no} = c$ for $i \in K$.

higher level of cost reduction and consumer surplus.

Theorem 1.1. *Every RJV formed by $K \subsetneq N$ firms yields a higher technological development and consumer surplus than the cases of individual research and an industry-wide RJV.*

Even though an industry-wide RJV does not improve the technological development and consumer surplus, a K-firm RJV leads to strict improvements on them. Comparing the cases of K-firm RJV and of independent research, RJV firms have additional incentive to spend more on research in aggregate level due to the advantageous position by information sharing nature of RJV. [15]However, as number of firms in a K-firm RJV increases, the advantageous position eventually diminished because more RJV-firms share advanced technology but non-RJV firms becomes fewer. Hence, after the number of firms belonging to the RJV exceeds some threshold, they will start decreasing investment, in order to reduce the production level competition. Eventually, when $K = N$, firms just want to maintain the aggregate level of technology as if they are doing individual research.[16]

[15]Even though individual firm is spending less in R&D, $x_i^{no} < x_i^{ind}$, due to free-riding nature of RJV. The aggregate R&D level is higher, $\sum_{i \in K} x_i^{no} > x_i^{ind}$, due to the information sharing nature of RJV.

[16]Whenever α is not too small, the profit of an RJV firm is single peak in the size of the RJV. The threshold value of α is around one-seventh of N. When α is very small, then the profit of an RJV firm has two peaks in the size of the RJV.

1.4 Technology Transfer

Since RJV firms tend to possesse superior technology to non-RJV firms, they may find it beneficial to transfer technology if the licensing fees exceed the negative spillover from technology diffusion. Under RJV competition, since RJV firms choose their investments independently, the licensing decision should be approved unanimously.[17] However, this does not pose a problem in our framework because all RJV firms are symmetric in the unique equilibrium. Following Gallini and Winter (1985), we consider licensing before (**ex-ante licensing**) and after (**ex-post licensing**) the research being done. An example of ex-ante licensing is package licensing where licensees are entitled to enjoy all the subsequent technology development. Ex-post licensing is a very common method to transfer technology in many industries such as smart phone industry. Formally, we define a dynamic game of three stages, namely licensing stage, research stage and production stage. In an ex-ante licensing scenario, the game proceeds from licensing stage to research stage and finally to production; while in ex-post licensing scenario, the game proceeds from research stage to licensing stage and finally to production stage. For the licensing mechanism, we follow Katz and Shapiro (1986) to consider licensing

[17]However, under RJV cartel, this is implicitly assumed as licensing decision is made to maximize joint profits of RJV firms under the same assumption imposed on investment levels. Majewski (2008) documents that technology transfers by RJVs require approvals from all members.

auction.

Due to the research sharing nature, RJV firms should always process higher technological level than the non-RJV firms. We assume the technologies developed by RJV firms and non-RJV firms are not compatible so that non-RJV firms will never have better technology after the transfer. Thus the aggregate technological development of the RJV represents the highest technological level in the entire industry.

1.4.1 Ex-ante Licensing

In the first stage, RJV firms transfers technology to $L \subseteq N \backslash K$ licensees.[18] Since all firms are symmetric, we assume the auction revenue $\sum_{j \in L} b_j$ is shared equally among RJV firms where b_j is the bid by firm j. In the second stage, all firms choose the level of research investment simultaneously. The profits of RJV firms, licensees and non-licensees are

$$\pi_i^{Ex-ante} = (p - (c - X^{Ex-ante}))q_i - \alpha x_i^2 + \frac{1}{K} \sum_{j \in L} b_j \text{ for all } i \in K,$$
$$\pi_j^{Ex-ante} = (p - (c - X^{Ex-ante}))q_j - \alpha x_j^2 - b_j \text{ for all } j \in L, \text{ and}$$
$$\pi_m^{Ex-ante} = (p - (c - x_m))q_m - \alpha x_m^2 \text{ for all } m \in N \backslash (L \cup K),$$

where $X^{Ex-ante} = \sum_{i \in K} x_i^{Ex-ante}$. It is clear to see that $x_j = 0$ for all $j \in L$ because no licensee has incentive to spend on research given that they would

[18] In our framework, the implicit form of this optimal number of licensees could be found. Numerical results on this could be found in Section 1.5.2.

have the technology transferred from RJV-firms.

In the last stage, firms engage in quantity competition. For the RJV firms to be licensors, we have assumed that RJV firms have more advanced technology, $\sum_{i\in K} x_i^{Ex-ante} = Kx_i^{Ex-ante} \geq x_m^{Ex-ante}$ for $i \in K$ and $m \in N\setminus(L \cup K)$, which is true if and only if $L \leq \frac{(N-K)(K-1)}{K}$. Our numerical analysis suggests that the equilibrium number of licensee always satisfy this relationship. [19]

By computing and comparing the investment level and total quantity produced, we reach the following conclusion:

Proposition 1.2. *Technological improvement under no licensing case is always higher than which under ex-ante licensing case.*

As RJV firms will develop a better technology, non-RJV firms are potential licensees. While licensing seems to reduce non-RJV firms incentive to innovate, the effect for RJV firms depends on marginal revenues from innovation. The increase in innovation reduces production costs and increases licensing fee, but also intensify competition due to technology transfer from RJV firms to the licensees. The competition effect is dominating in the ex-ante case, leading to a lower investment for RJV firms.

[19] It is possible for RJV firms to possess even lower technology level than non RJV firms, if there are too many licensees. The incentive for RJV firms to innovate will be significantly reduced, since the licensees will produce the same amount of output as the RJV firms. The competition level in production stage will be too intense such that the profit gain from licensing could not cover the profit loss from the production competition. In such case, the non-RJV firms actually become the licensor. But this is not equilibrium as suggested by our numerical analysis, and it is not plausible in reality.

Proposition 1.3. *Unless research efficiency is low, RJV size is small, and number of licensee is few, consumer surplus in no licensing case is always higher than that in ex-ante case.*

Even though RJV firms have lower incentive for technological development, ex-ante licensing could still improve consumer surplus, when cost of R&D is high, RJV size is small relative to the size of industry, and there are only few licensees.[20] First, with high cost of R&D, the gap in technology level between RJV firms and non-RJV firms are smaller under ex-ante licensing case than which under no licensing case, i.e. $Kx_i^{no} - x_m^{no} > Kx_i^{Ex-ante} - x_m^{Ex-ante}$ for all $i \in K$ and $m \in N \backslash (K \cup L)$. On the one hand, RJV firms will produce fewer due to lower technology level, comparing with no licensing case. On the other hand, non-RJV firms will produce more. Second, due to the small size of RJV and fewer number of licensees, the extra quantities produced by all the non-RJV firms will exceed the production reduction from the RJV firms; and hence, consumer surplus is improved.

1.4.2 Ex-post Licensing

Firms will do R&D competition in the first stage, followed by the licensing stage, and then production stage. The profits of RJV firms, licensees and

[20] As shown in the Appendix A.2.3, the condition is satisfied, for example, when $\alpha = 100$ and $K \in [2, 13]$.

non-licensees are

$$\pi_i^{Ex-post} = (p-(c-X^{Ex-post}))q_i - \alpha x_i^2 + \frac{1}{K}\sum_{j\in L} b_j(x_1,...,x_n) \text{ for all } i \in K,$$
$$\pi_j^{Ex-post} = (p-(c-X^{Ex-post}))q_j - \alpha x_j^2 - b_j(x_1,...,x_n) \text{ for all } j \in L, \text{ and}$$
$$\pi_m^{Ex-post} = (p-(c-x_m))q_m - \alpha x_m^2 \text{ for all } m \in N\backslash(L\cup K),$$

where $X^{Ex-ante} = \sum_{i\in K} x_i^{Ex-post}$. The difference between the ex-ante and ex-post case is that the bids depends on the investment levels in the first stage. The non-RJV firm's equilibrium bid is the difference of profits between winning and losing the auction. When $L \subsetneq N\backslash K$, the equilibrium bid of firm $j \in L$ is

$$\begin{aligned} b_j(x_1,...,x_N) &= \pi_j(\text{Winning}) - \pi_j(\text{Losing}) \\ &= ((p-c+\sum_{i\in J} x_i)q_i - \alpha x_j^2) - ((p-c+x_j)q_j - \alpha x_j^2) \\ &= (p-c+\sum_{i\in J} x_i)q_i - (p-c+x_j)q_j, \end{aligned}$$

so that profits of RJV firms and non-RJV firms are

$$\pi_i^{Ex-post} = \frac{L+K}{K}(p-c+\sum_{h\in K} x_h)q_i - \alpha x_i^2 - \frac{L}{K}(p-c+x_j)q_j, \text{ and} \tag{1.1}$$
$$\pi_j^{Ex-post} = (p-(c-x_j))q_j - \alpha x_j^2 \text{ for all } i \in K \text{ and } j \in N\backslash K. \tag{1.2}$$

The maximization problems above reflect that licensees and non-licensees are identical in the first stage. Unlike an ex-ante licensing, the winner(s) of the auction is unknown in the first stage. All the non-RJV firms will do some research, since they are not guaranteed winning the advanced technology in the second stage. In the case of losing, they are still able to compete in production stage, with their own R&D outcome. By computing and comparing the investment level and total quantity produced, we reach the following conclusion:

Proposition 1.4. *When research efficiency is low, the technological development and consumer surplus under ex-post licensing is higher than which under no licensing.*

For ex-post licensing, RJV firms have stronger incentives to do research than no licensing case as extra cost could be recovered from licensing fees. However, licensing may reduce incentive to do investment due to higher level of competition under production. Investment level is higher only when research cost is sufficiently high. With little investment, the aggregate technological development will be large. Non-RJV firms possess lower technologies, and hence are willing to bid higher in the auction, in order to place themselves at a better position in the production stage. This high bidding will raise the incentive for RJV firms to do research, as the costs will be recovered from licensing. These two effects will reinforce each other, so that the whole society ends up with a higher technology level and higher consumer surplus.

If research cost is low, non-RJV firms could do approximately the same level of technology development as RJV firms. The cost sharing nature of RJV will become less significant, and hence RJV firms will lose some of their advantages in research. We will conclude this section with the following theorem:

Theorem 1.2. *For every RJV formed by $K \subsetneq N$ firms, compared with no licensing case, ex-post licensing improves the technological development and consumer surplus; while an ex-ante licensing will always harm technological development but improve consumer surplus if (i) RJV size is small, (ii) number of licensee is few and (iii) research efficiency is low.*

1.5 RJV Formation

We will determine the equilibrium size of the RJV in a simple RJV formation game under the cases with and without licensing.

1.5.1 No Licensing

Consider a simple RJV formation game that each firm votes to change membership with no side payments made between firms.[21] The dynamic game with discrete time, $t = 1, 2, ..., \infty$ starts with a tentative K_1-firm RJV where $K_1 \subseteq N$ and $K_1 \neq \varnothing$. In round t with a K_t-firm RJV, a randomly chosen firm from the K_t-firm RJV may propose to admit new members or exclude existing members. Including new members requires approvals from all existing members and the candidate but excluding an existing member requires only the approvals from all members except the candidates to be excluded. Regardless

[21] Our equilibrium is outcome-equivalent to the coalition unanimity game considered in Greenlee (2005) if we impose a restriction that only one coalition can be formed.

of K_1, in a steady state, the equilibrium RJV must maximize the profit of each RJV firm.[22] The following proposition shows that the equilibrium RJV is a K-firm RJV where $K \subsetneq N$ and $|K| \neq 1$.

Proposition 1.5. *In a simple RJV formation game, individual research and an industry-wide RJV are never in an equilibrium.*

Due to the complexity of the analytical form of firm's profit function, we find the equilibrium RJV size by numerical simulations. The profit of an RJV firm is single peak in the size of the RJV whenever α is not too small.[23] As shown in Table 1, the equilibrium RJV size is around one-third of N, and increasing in N and non-decreasing in α.

Numerical results of welfare analysis of RJV size are presented in the Online Appendix. We only mention two interesting observations here. First, the equilibrium RJV size is too small for consumer surplus, producer surplus and social welfare. This is a natural consequence that consumers benefit from more technological innovation but firms would prefer less competition in the research market. Second, the size of social-welfare-maximizing RJV is very

[22] Our definition of equilibrium size requires that RJV firms do not want to admit more members or exclude some of its member while Poyago-Theotoky (1985) which only requires no single firm wanted to be admitted or excluded from the RJV. As will be discussed in the numerical analysis, the profit of each RJV firm is single-peaked in the size of the RJV so that the two definitions coincide in our setup.

[23] As discussed in footnote 16, the minimum α required for an RJV firm' profit to be single-peaked is close to 15 when $N = 100$. This is why Table 1.1 starts from $\alpha = 16$. Therefore, if an RJV can only admit or expel one member each round as in Poyago-Theotoky (1985), Proposition 1.5 remains intact whenever α is not small given the single-peakness.

α \ N	5	10	20	30	40	50	60	70	80	90	100
$[16, 25]$	3	4	8	11	14	18	21	24	28	31	34
$[26, 342]$	3	5	8	11	14	18	21	24	28	31	34
$[343, 1013]$	3	5	8	11	15	18	21	24	28	31	34
$[1014, 2041]$	3	5	8	11	15	18	21	25	28	31	34
$[2042, \infty)$	3	5	8	11	15	18	21	25	28	31	35

Table 1.1: Equilibrium RJV Size under various research efficiencies and industry size.

closed to the size of consumer-surplus-maximizing RJV. Regulation authorities with the goal to maximize social welfare could mainly focus on consumer surplus because the estimation of producer surplus is often noisy and difficult.

Our results in a RJV competition draw very different policy implication from numerical exercise on RJV cartel in Poyago-Theotoky (1995). First, it was suggested that producer surplus is a natural choice for social planner in a RJV cartel. However, in a RJV competition, it may not be appropriate if the social planner cares consumer surplus, social surplus or technology development, because the suggested RJV according to the producer surplus can be much larger than the optimal sizes under the other criteria. Second, Poyago-Theotoky (1985) suggests that consumer surplus can be assumed away for the social planner. However, our result suggests that the regulator should mainly focus on consumer surplus instead.

1.5.2 Under Licensing

		N							
		30	40	50	60	70	80	90	100
$\alpha = 30$	K^{no}	11	14	18	21	24	28	31	34
	$K^{ex-ante}$	6 (8)	8 (10)	18(0)	21(0)	24(0)	28(0)	31(0)	34(0)
	$K^{ex-post}$	2 (27)	2 (29)	2 (31)	2 (33)	2 (35)	2 (37)	2 (39)	2 (40)
$\alpha = 100$	K^{no}	11	14	18	21	24	28	31	34
	$K^{ex-ante}$	6 (9)	7 (12)	7 (17)	8 (20)	9 (24)	10 (28)	11 (31)	12 (35)
	$K^{ex-post}$	2 (27)	2 (37)	2 (47)	2 (57)	2 (67)	2 (77)	2 (87)	2 (90)
$\alpha = 1000$	K^{no}	11	15	18	21	24	28	31	34
	$K^{ex-ante}$	5 (10)	6 (13)	7 (17)	8 (21)	8 (26)	9 (30)	9 (34)	10 (38)
	$K^{ex-post}$	2 (27)	2 (37)	2 (47)	2 (57)	2 (67)	2 (77)	2 (87)	2 (97)

Table 1.2: Equilibrium RJV Size and Licensee under different research efficiencies.

We conduct simulation exercise to determine the equilibrium size of RJV, summarized in Table 1.2.[24] The equilibrium RJV sizes under no licensing, ex-ante licensing, and ex-post licensing are denoted by K^{no}, $K^{ex-ante}$, and $K^{ex-post}$ respectively. The number inside the brackets is the equilibrium number of licensees for each case. First, RJV size is much smaller under licensing because non-RJV firms can obtain innovation from technology transfer. It is because RJV firms prefer earning more licensing fee from non-RJV firms to sharing research cost by admitting more members. Second, the size of RJV under ex-ante licensing is larger than that under ex-post licensing. Compared with an ex-post licensee, an ex-ante licensee would not spend on R&D but

[24]Appendix A.2.3 contains additional numerical results.

wait for technology transfer. More licensing fees can be collected with a larger innovation which requires a larger RJV size to share research cost.

Third, similar to the case without licensing, the equilibrium size of RJV is non-decreasing in N. Forth, RJV size is now non-increasing in research efficiency α indicating that RJV prefers recovering research cost through licensing rather than expansion of the RJV size. As research cost becomes higher, a smaller RJV is sufficient to create the technology leadership, as the non-licensee firms have less incentive to do research.

Table 1.2 also shows some simulation results for the equilibrium number of licenses under ex-ante and ex-post licensing. First, the number of licenses is non-decreasing in α and N for both licensing schemes. When α increases, it is more expensive to conduct independent research for non-RJV firms compared with RJV firms, and RJV firms are more willing to recover research cost through licensing. As N increases, the competition effect becomes less severe as the market size expands. Hence, the pressure on price in the product market due to technology diffusion would be lessen so that RJV firms would be inclined to license to more firms. Second, ex-post licensing leads to more licensees than ex-ante licensing because RJV size is much smaller in ex-post licensing. Generally, an RJV with two firms are sufficient to create the optimal technology advantage when the research cost is covered by having more licensees.

Third, an ex-ante licensing may not occur when α is small and N is large while ex-post licensing always happens. Ex-ante licensing is less attractive because RJV firms want to retain the advantage of research given a large number of non-RJV firms. Finally, it is never optimal to licensee to all non-RJV firms except for the case when $N = 4$ as in the motivating example. Technology diffusion is not complete because RJV firms want to restricting the number of licensee so as to create a larger wedge between licensees and non-licensees.[25]

While the numerical results of welfare analysis of RJV size are presented in the Online Appendix, we only mention two interesting observations. First, ex-post licensing always leads to higher consumer surplus than ex-ante and no licensing regardless of the size of RJV. This suggests that govnerment should promote technology diffusion together with the encouraging the formation of RJVs. Second, when RJV is determiend by the simple RJV formation game, ex-ante licensing leads to a lower consumer surplus than no licensing but still higher than the case of an industry-wide RJV. This is because, as shown in Table 1.2, the RJV firms transfer technology to too many licensees so that technological development is reduced due to free-riding effect, and thus con-

[25]This is similar to Creane et al. (2013) that a complete technology transfer from one firm to another always increases joint profit when at least three firms remain in the industry.

sumer surplus is reduced in a less competitive product market.

1.6 Robustness and Extensions

We extend our model in several directions. First, we show that our main result remains valid in markets with multiple RJVs. Then we briefly discuss how our results may change when research outputs between RJV firms may not be fully compatible, and there is spillover between RJV firms and non-RJV firms. Detailed discussions are presented in the Appendix and Online Appendix.

1.6.1 Multiple RJVs

We have so far assumed that firms outside the RJV are doing individual research. We can show that Theorem 1 remains valid even when we allow those firms to overcome coordination problem by forming another RJV. Hence, there will be two competing RJVs in the market.

Proposition 1.6. *Suppose all firms belong to either one of the two RJVs. Technological development and consumer surplus are higher than those in individual research or an industry-wide RJV.*

This shows Theorem 1.1 is still valid even if we allow all non-RJV firms to form a second RJV. Greenlee (2005) studies a coalition formation game, and shows the equilibrium coalition structure is at most three RJVs, when the spillover effect is small. Theorem 1.1 and Proposition 1.6 show analytical result for one and two RJVs. For three competing RJVs, while the analytical result is not tractable, our numerical analysis show the result remains valid.

1.6.2 Imperfect Compatibility

Consider firms within the RJV are doing some overlapping researches. Let $\beta \in (0,1)$ measures the compatibility of the researches done by the RJV firms.[26] Note that $\beta = 0$ and $\beta = 1$ are equivalent to the cases of individual research and a K-firm RJV in section 1.3 respectively. The technological development for firm $k \in K$ is $X_K = x_k + \beta \sum_{i \in K \setminus \{k\}} x_i$.

Proposition 1.1*. *An industry-wide RJV with imperfect compatibility leads to strictly more technological improvement and consumer surplus compared to the case of individual research.*

The results above explain the relationship between research sharing incentive and free-riding effect among RJV firms. When technologies are perfectly incompatible, i.e. $\beta = 0$, firms are just doing individual research. There is no research sharing and free ride. As β increases, firms are more willing to do research due to the research sharing effect. However, the free-riding effect will become dominant, when β becomes large, i.e. beyond $\frac{1}{2}$. Eventually, when the

[26] In the Appendix A.2.4, we also consider the special cases of $\beta = 0$ and $\beta = 1$.

technology becomes perfectly compatible, an industry-wide RJV will act as if firms doing research individually.

Theorem 1.1*. *Every RJV formed by $K \subsetneq N$ firms under imperfect compatibility yields a higher technological development and consumer surplus than the cases of individual research and an industry-wide RJV. Furthermore, when RJV size is large and technologies are sufficiently compatible, technological development and consumer surplus are strictly higher than the case under perfect compatibility.*

We can still show that any K-firm RJV improves technological development and consumer surplus, thereby confirming the robustness of Theorem 1. Furthermore we can show that, when RJV size is large, slight incompatibility encourages RJV firms to do more research by reducing the free-riding effect within the RJV, resulting even higher technological development and consumer surplus.

1.6.3 Spillover

When patent protection is not perfect or imitation is easy, there will be spillover between RJV firms and non-RJV firms, and spillover among non-RJV firms.[27] Let $\gamma \in [0, 1]$ measures the spillover effect. Note that $\gamma = 0$

[27] RJV-firms have perfect spillover because they are sharing their research results.

corresponds to the model is the same as Section 1.3, whereas $\gamma = 1$ corresponds to the case of industry-wide RJV. The cost reduction for firm $k \in K$ is $X_K = x_k + \sum_{i \in K \backslash \{k\}} x_i + \gamma \sum_{i \in N \backslash K} x_i$ but firm $j \in N \backslash K$ is $x_j + \gamma \sum_{i \in N \backslash \{j\}} x_i$.

Similar to the imperfect compatibility case, Proposition 1 no longer holds unless $\gamma = 0$ or $\gamma = 1$, that an industry-wide RJV results in better technology improvement and consumer surplus than the case of individual research. Improvement is also not monotonic in γ due to positive sharing effect and free-riding effect under the same logic.

However, different from imperfect compatibility case, K-firm RJV may lead to lower cost reduction and consumer surplus if spillover γ is high due to free-riding effect. In particular, our numerical results suggest that free-riding is more serious when the size of RJV becomes larger due to increasing benefit from free-riding.

1.7 Conclusion

We have shown that improvement on technology level, producer surplus, and consumer surplus are higher for a K-firm RJV than an industry-wide RJV and individual research. We also consider the effect of technology transfer by introducing patent licensing. With licensing, the equilibrium size of RJV is much smaller than the case without licensing. This explains the phenomenon that RJV becomes increasingly popular, the size of RJV are small, and it is very difficult to establish an industry-wide RJV.[28] Hence, policymakers should also encourage the formation of RJV but need not facilitate an industry-wide agreement. Unde ex-post licensing, would further improve consumer surplus. Therefore, government should encourage technology transfer from RJV than forming a larger RJV. [29]

[28] In 2009, Taiwanese government establishes a new government-backed company, Taiwan Memory Company (TMC) to consolidate research development for all memory-chip producers in Taiwan. However, major producers including Micron Memory Taiwan, Nanya Technology Corp. and Powerchip Semiconductor Corp. refuse to join the venture.

[29] Even ex-ante licensing may not improve consumer surplus compared to no licensing, it is still better an industry-wide RJV.

Chapter 2

Reverse Licensing

2.1 Introduction

It has been a long discussion on how to license process innovation, i.e. the technology reducing production cost, since Katz and Shapiro (1986). In the most recent development, Lerner and Tirole (2004) and (2015) discuss about how patent pool is formed and such innovations should be priced under the patent pool and Standard Setting Organization. These analysis focus on horizontal markets. Even if the innovator is an outsider research lab, as in standard licensing case, there is no strategic connection between this outsider and the downstream market.

We would like to consider a market with both upstream manufacturers and downstream producers. The upstream firm not only sell intermediary input to the downstream firms, but also licensing process innovations used in downstream productions. For the downstream firms, we allow heterogeneity among them. Some firms may have done some R&D, thus possess some of the process innovation; while the others does not. Under this setup, we are considering not only the licensing strategies between upstream and downstream firms, but also the strategies among the downstream firms. Schmalensee (2009) discusses the solution to royalty-stacking and hold-up problem in standard setting, under this setup.

However, we would like to focus on "pass-through right" problem under this vertical structure. The pass-through rights refer as the patent holder gives the third party permission to license the patent without infringing the right of patent holder. A particular real life example we have in mind is the Qualcomm Incorporated's (Qualcomm) anti-trust case in China.

Qualcomm is the world's largest smartphone chipmaker[1], and a near monopoly in the Chinese market: 100% market share in the market for licensing of each relevant wireless communications standard essential patents ("SEPs"),[2] and above 50% market share in CDMA, WCDMA and LTE baseband chip markets. According to China's Anti-monopoly Law (AML) article 18, Qualcomm is considered to be in a market dominance position.

In November 2013, China's National Development and Reform Commis-

[1]According to Strategy Analytics report,Qualcomm is the leader with a share of 42 percent in the global smartphone processor market.

[2]By Shapiro (2001), a standard-essential patent is a patent that claims an invention that must be used to comply with a technical standard.

sion (”NDRC” who is responsible for price-related violations of China’s Anti-Monopoly Law) began the investigation on whether Qualcomm abused its dominant market position.[3] On March 2, 2015, NDRC published its decision regarding the anticompetitive conducts and ordered Qualcomm to cease these conducts and to pay a fine of RMB 6.088 billion (approx. US$975 million). [4] Qualcomm announced that it would not contest the NDRC decision and agreed to change certain of its patent licensing and baseband chip sales practices in China.

While NDRC ruled that Qualcomm has several anti-competitive conducts according to AML article 17, we focus on the charge of *reverse licensing*,[5] i.e. requiring Chinese licensees to cross-license their relevant SEPs and non-SEPs to Qualcomm and its customers without compensation and without offsetting royalties,[6] because it has not been studied formally in the literature.

In Chinese market, there are two major patent holders in the smartphone

[3]China is not the only country investigating Qualcomm’s anticompetitive conducts. Before reaching an agreement in 2008, Nokia had complaint Qualcomm for charging expired patent and high royalty rates. In 2009, South Korea fined Qualcomm Won 260 billion (USD 207 million), for abusing market dominance positions. More recently, in 2015, EU started an investigation on predatory pricing, i.e. Qualcomm is driving his competitors out of the market.

[4]According to AML article 47, the applicable antimonopoly enforcement authority may order the operator to cease the objectionable activities, confiscate its illegal gains and levy a fine between 1%-10% of the trunover of the business from the previous year.

[5]Also referred as “pass through rights” as we mentioned above.

[6]The anti-competitive conducts include bundling SEPs and non-SEPs without justification, imposing unreasonable sales terms on baseband chip customers, charging royalties for expired patents, etc.

production market: Huawei Technologies Co. Ltd ("Huawei") and Zhongxing Telecommunication Equipment Corporation ("ZTE"), with roughly 30,000 and 52,000 patents respectively. On contrary, Xiaomi Inc. ("Xiaomi") and OPPO Electronics Corp. ("OPPO") have only 10 and 103 patents in this market.[7] Qualcomm adopted the practice of *reverse licensing* without offsetting payment: (1) when Huawei and ZTE purchase chips and patents from Qualcomm, they have to surrender their own patents to Qualcomm for free, and (2) when Xiaomi and OPPO purchase the chips from Qualcomm, they get not only patents from Qualcomm, but also patents from Huawei and ZTE, without paying to them. Such a practice clearly hinders the incentive to innovate for not only Huawei and ZTE, as they gain nothing from their research; but also Xiaomi and OPPO, as they could free ride on the others.[8]

However, the practice of reverse licensing promotes the smartphone competition among the domestic Chinese producers. According to TrendForce, an analytical firm's report, the market share of Huawei and ZTE are 8.4% and 3.1% respectively in the year of 2015. Despite of few number of patents, Xiaomi and OPPO occupy 5.6% and 3.8% market share respectively.[9] Our model

[7]Source: Di Yi Cai Jing Ri Bao, 03 December 2014. Accessed online "http://tech.sina.com.cn/t/2014-12-03/02259846574.shtml" on 04 December 2014.

[8]Recently, Chinese government announced a new develop plan that stressed innovation. The main purpose of this investigation is to promote innovation and protect domestic smartphone producer. (According to newspaper article)

[9]Accessed online "http://press.trendforce.com/press/20160114-2265.html" on 08 June 2016

is consistent with data, by showing that reverse licensing improves consumer surplus.

We consider a model with an upstream monopolist (Qualcomm) selling an intermediate good (smartphone chips) to downstream firms who produce homogeneous final goods (smartphones) to consumers. Downstream firms are heterogeneous: two leading firms (such as Huawei and ZTE) have advanced technologies and the rest of firms have standard technologies of production. We consider the following three-stage model of reverse licensing.

In the first stage, the upstream monopolist collects technologies from downstream firms and incorporate the technology into the intermediate goods sold to all downstream firms. This resembles the Chinese smartphone chipset market. In the second stage, the upstream monopolist sets a uniform price for each smartphone chip and license all the collected technologies to all n downstream firms including those who possesses technology initially. We do not allow price discrimination against retailers as it is widely considered to be anticompetitive worldwide including US and China. In the third stage, downstream firms engage in a quantity competition.

We find that the upstream monopolist would charge uniform price and zero royalty. The intuition is straightforward: the upstream monopolist has a strong incentive to create a more competitive downstream market because of its full control of the intermediate product. He could, then, extract more profits from the downstream market.

Next, we study the model under the regulation that the upstream manufacturer cannot force downstream producer to surrender the technologies. To incentivize the downstream producers to provide the technologies, the upstream producer needs to compensate them adequately. We consider a general two-part tariff compensation scheme, showing that consumer surplus and aggregate producer surplus will always be improved compared to the reverse licensing case. In this way, the regulation by the Chinese government protects domestic industry without a sacrifice of consumer surplus and social welfare.[10]

Next we consider technology transfers in the first stage occurs other than reverse licensing. We will consider three different licensing regimes: (1) under no licensing, there is no technology transfer between any firms; (2) under independent licensing, the two leading firms could license their respective technologies through royalties as in Katz and Shapiro (1985) and Kamien and Tauman (1986); (3) under patent pool, the two leading firms combine their patents into a single bundle to be licensed to potential rivals where licensing fee is set to maximize profit of each firm.[11] In all four regimes, we find that reverse

[10]We propose four different fixed fee compensation schemes in Appendix B.2.

[11] We depart from Shapiro (2001) and Lerner and Tirole (2004). Instead assuming firms set royalty rate to maximize pool members' joint profit, we assume the royalty rate maximize pool member's individual profit. Because we focus on anti-trust studies, it may not be plausible for Huawei and ZTE to form a Cartel in the downstream market, as Cartel itself

licensing yields the highest consumer surplus, aggregate producer surplus, and hence social welfare. The intuition is similar to double marginalization. Since the upstream manufacturer is monopoly of the input and the two leaders in the downstream are monopoly of their respective technology, reverse licensing concentrates all market power into the hand of upstream manufacturer and thereby avoid the problem of double marginalization.

Alongside with this result, our paper also addresses the complements problem proposed by Cournot (1838), and studied by Shapiro (2001). Our paper is consistent with literature that when technologies are perfect complements, patent pool could solve the complement problem in the vertical structure we proposed above, i.e. the consumer surplus under patent pool is higher than which under independent licensing. We could further show with a small perturbation, when the technologies possess some degree of substitutability, patent pool may not be the best solution to the problem. However, reverse licensing could always improves consumer surplus, regardless of compatibility of the technologies.

Lastly, we study the effect of the compensation on research incentive of downstream firms. Clearly, under reverse licensing without offsetting payment, firms have the least incentive to do research. We show that offsetting payment

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is against anti-trust laws. Then we assume the two pool members will share the licensing profit equally.

will encourage firms to do research. Morevoer, to keep the leading position in the market, Huawei and ZTE have more incentive than the other downstream firms.

We will discuss the benchmark model of reverse licensing in section 2.2. Section 2.3 discuses remedy under licensing. Section 2.4 studies independent licensing and patent pool as alternatives to reverse licensing. Section 2.5 considers R&D decisions and Section 2.6 concludes.

2.2 Model

We consider a vertical structure with one upstream manufacturer (Qualcomm) of an intermediate good (chipset) and n downstream firms (smartphone producers).

Assume one unit production of final good requires one unit of input (producing one smartphone requires one chip). For simplicity, we assume the marginal cost of producing the chips is 0. The profit for the manufacturer is $\pi^m = dQ$, where Q is the market demand for the final product and d is the price for one unit of input.

The n downstream firms are heterogeneous: firms 1 and 2 have some process innovation, i.e. technologies reduce cost of production, ϵ_1 and ϵ_2, and thus they are the leader of the industry (for example, Huawei and ZTE in

Chinese market). For simplicity, we assume $\epsilon_1 = \epsilon_2 = \epsilon$. However, the two technologies may not be perfectly compatible. Combination of the two will reduce production cost by $\alpha\epsilon$, where $1 \leq \alpha \leq 2$. Note that α is the measure of compatibility, with $\alpha = 2$ indicating the technologies are perfect complements, and $\alpha = 1$ implies perfect substitutes.

For the downstream market, assume marginal cost of production of firm 1 and firm 2 are $c_1 = c_2 = c - \epsilon$, and for the rest of the firms, are $c_i = c$ for $i \neq 1, 2$. For tractability, we consider inverse demand function for the final good is linear, $P = a - Q = a - \sum_i q_i$ where $a > 0$. Therefore the profit for downstream firms is given by $\pi_i = (P - d - c_i)q_i$ for all $i \in N$ where N is the set of downstream firms.

2.2.1 No licensing

We consider the benchmark case where no licensing takes place. This case is relevant when property right protection is weak so that licensing is not possible. In this two-stage game, the upstream firm first chooses input price d and then every downstream firm i simultaneously chooses their production q_i. The following Lemma shows the subgame perfect equilibrium when all firms are active in the market.

Lemma 2.1. *When there is no technology transfer and all firms are active in* *the market, the price of the intermediary input is* $d^{no} = \frac{n(a-c)+2\epsilon}{2n}$ *and production of downstream firms are*

$$q_1^{no} = q_2^{no} = \frac{n(a-c)+2(n^2-n-1)\epsilon}{2n(n+1)},$$
$$q_3^{no} = ... = q_n^{no} = \frac{n(a-c)-2(2n+1)\epsilon}{2n(n+1)}.$$

Hence, total production $Q^{no} = \sum_{i\in N} q_i^{no} = \frac{n(a-c)+2\epsilon}{2(n+1)}$ and profits for the manufacturer and downstream producers are

$$\pi_1^{no} = \pi_2^{no} = (\frac{n(a-c)+2(n^2-n-1)\epsilon}{2n(n+1)})^2,$$
$$\pi_3^{no} = ... = \pi_n^{no} = (\frac{n(a-c)-2(2n+1)\epsilon}{2n(n+1)})^2,$$
$$\pi_m^{no} = d^{no}Q^{no} = (\frac{n(a-c)+2\epsilon}{2n})(\frac{n(a-c)+2\epsilon}{2(n+1)}) = \frac{(n(a-c)+2\epsilon)^2}{4n(n+1)}.$$

Note that we require $\epsilon < \frac{n}{2(2n+1)}(a-c)$ to ensure firms without technology are active in the market.

2.2.2 Reverse Licensing

We now consider a model where reverse licensing is imposed by an upstream manufacturer. The upstream manufacturer requires that the sale of input is conditional on surrender innovation. The upstream manufacturer combine the technologies to the intermediate input sold to the downstream producers. Hence, in first stage, the upstream manufacturer not only sets the price of the chip, d^r, but also the royalty fees r_i^r for the cost reducing technology to firm i. So the manufacturer's profit becomes $\pi_m^r = dQ + \sum_{i\in N} r_i^r q_i$.

In the second stage, firms make purchasing decision on both chips and technology, then engage in Cournot competition. If firm i purchases technology, the profit for downstream firms is $\pi_i^r = (P - d - c_i - r_i^r)q_i$, otherwise, $\pi_i^r = (P - d - c_i)q_i$. The following Lemma shows the subgame perfect equilibrium when all firms are active in the market.

Proposition 2.1. *When the upstream manufacturer engages in reverse licensing, the upstream manufacturer sets the price of chips to be* $d^r = \frac{1}{2}(a - c + \alpha\epsilon)$ *and royalty fee to be zero* ($r_i^r = 0$ *for all* $i \in N$) *and the production of downstream producers are* $q_i^r = \frac{1}{2(n+1)}(a - c + \alpha\epsilon)$ *for all* $i \in N$.

Note that the upstream manufacturer sets the same royalty fee for all downstream producers even if some of the downstream firms have already possessed part of the technology. The intuition is that the upstream manufacturer would be benefited from a more competitive downstream Cournot market due to the profits from the intermediary good. This explains why Qualcomm has treated downstream firms equally, despite the fact that Huawei and ZTE possess some technologies at the very beginning.

2.3 Remedy

Under the ruling of Chinese government, Qualcomm has to compensate downstream firms for patents surrendered. This implies that Qualcomm cannot exercise the market power to force downstream producers to surrender their patents, and it has to provide incentives for downstream producers to transfer their technologies.

Consider the manufacturer offers a two-part tariff compensation scheme to both firms 1 and 2.[12] Let r_i^t be the royalty fee of technologies $\alpha\epsilon$. Let r_j^d and F_j, for $j = 1$ and 2, be the royalty discount and fixed fee compensation for firms 1 and 2 respectively. Define $r_i = r_i^t - r_i^d + d$ be royalty fee and the price of chips for all $i \in N$. Note that $r_i^d = 0$ for all $i \in N \backslash \{1, 2\}$. Let Γ be the profit of firms 1 and 2, if they choose not to surrender the technologies to the manufacturer. Hence, Γ is the value of the outside option. The profits of firms are as follows,

$$\begin{aligned}
\pi_m &= \sum_i r_i q_i - F_1(r_1, ..., r_n) - F_2(r_1, ..., r_n),\\
\pi_1 &= (a - Q - (c - \alpha\epsilon + r_1))q_1 + F_1,\\
\pi_2 &= (a - Q - (c - \alpha\epsilon + r_2))q_2 + F_2,\\
\pi_j &= (a - Q - (c - \alpha\epsilon + r_j))q_j \text{ for } j = 3, 4, ..., n.
\end{aligned}$$

Using backward induction, the downstream firms will produce

$$\begin{aligned}
q_1 = q_2 &= \frac{(a - \alpha\epsilon) - (n-1)\underline{r} + (n-2)\bar{r}}{n+1},\\
q_3 = ... = q_n &= \frac{(a - \alpha\epsilon) + 2\underline{r} - 3\bar{r}}{n+1}.
\end{aligned}$$

[12]In appendix B.2, we consider compensation involving only fixed fee.

Note that, first, the fixed fee compensation is exogenous to firms 1 and 2. Second, by firms' symmetric property, we have $r_1 = r_2 \equiv \underline{r}$ and $r_3 = ... = r_n \equiv \bar{r}$ in equilibrium.

From the manufacturer's prospective, the fixed fee compensation is defined to be the profit difference between firms surrendering the technology and the outside option, i.e. $F_1(\underline{r}, \bar{r}) = \Gamma - \pi_1 = \Gamma - q_1^2$ and $F_2(\underline{r}, \bar{r}) = \Gamma - \pi_2 = \Gamma - q_2^2$. By symmetry, $F_1(\underline{r}, \bar{r}) = F_2(\underline{r}, \bar{r}) \equiv F(\underline{r}, \bar{r})$. Therefore the maximization problem of the manufacturer becomes

$$\max_{\underline{r}, \bar{r}} \underline{r}(q_1 + q_2) + \bar{r} \sum_{i=3}^{n} q_i - 2F(\underline{r}, \bar{r})$$

We have two cases to consider: $F > 0$ if and only if $\gamma \equiv \sqrt{\Gamma} > \frac{1}{4}(a - c + \alpha\epsilon)$; otherwise, $F \leq 0$.

If $F > 0$, solve manufacturer's problem to get

$$\underline{r} = \frac{a - c + \alpha\epsilon}{4},$$
$$\bar{r} = \frac{a - c + \alpha\epsilon}{2}.$$

And

$$q_1 = q_2 = \frac{a - c + \alpha\epsilon}{4},$$
$$q_3 = ... = q_n = 0$$

Due to the royalty compensation, firms 1 and 2 have a lower production cost. They will become duopoly in the downstream market. However, consumers are still better off. Comparing with reverse licensing case, note that consumer surplus in this case is measured by $\frac{a-c+\alpha\epsilon}{4} > \frac{n}{2(n+1)}(a-c+\alpha\epsilon) = Q^r$. Aggregate producer surplus is $\frac{(a-c+\alpha\epsilon)^2}{4} > \frac{n(n+2)(\alpha\epsilon+(a-c))^2}{4(n+1)^2} = \Pi^r$.

If $F \leq 0$, it is the corner solution, where only royalty discount is implemented by the manufacturer. In this case, let $\gamma = \sqrt{\Gamma} = q_1 = q_2$. And hence, manufacturer is

$$\max_{\underline{\mathrm{r}},\bar{r}} 2\underline{\mathrm{r}}\gamma + (n-2)\bar{r}\frac{(a-c+\alpha\epsilon)+2\underline{\mathrm{r}}-3\bar{r}}{n+1}$$
$$\text{subject to } \gamma = \frac{(a-c+\alpha\epsilon)-(n-1)\underline{\mathrm{r}}+(n-2)\bar{r}}{n+1}$$

Solve to get

$$\underline{\mathrm{r}} = \frac{n(a-c+\alpha\epsilon)-2(n+1)\gamma}{2(n-1)},$$
$$\bar{r} = \frac{a-c+\alpha\epsilon}{2},$$
$$q_1 = q_2 = \gamma,$$
$$q_3 = ... = q_n = \frac{a-c+\alpha\epsilon-4\gamma}{2(n-1)}$$

Note in this case, $\gamma = q_1 = q_2 < \frac{a-c+\alpha\epsilon}{4}$, as firms 1 and 2 are getting less royalty discount than the previous case. And hence $\underline{\mathrm{r}} > 0$ and $q_3 = ... = q_n > 0$. In this case, consumer surplus is measured by $2\gamma+(n-2)\frac{a-c+\alpha\epsilon-4\gamma}{2(n-1)} > \frac{a-c+\alpha\epsilon}{4} > \frac{n}{2(n+1)}(a-c+\alpha\epsilon)$. Because

$$2\gamma + (n-2)\frac{a-c+\alpha\epsilon-4\gamma}{2(n-1)}$$

$$
\begin{aligned}
&=2\gamma\frac{1}{n-1}+\frac{n-2}{2(n-1)}(a-c+\alpha\epsilon)\\
&>\frac{n-2}{2(n-1)}(a-c+\alpha\epsilon)\\
&>\frac{a-c+\alpha\epsilon}{4} \text{ as } n\geq 3.
\end{aligned}
$$

Aggregate producer surplus is

$$
\begin{aligned}
\Pi^{tt} \equiv &2\Gamma+(n-2)(\frac{a-c+\alpha\epsilon-4\gamma}{2(n-1)})^2+\gamma\frac{n(a-c+\alpha\epsilon)-2(n+1)\gamma}{(n-1)}\\
&+(n-2)\frac{a-c+\alpha\epsilon}{2}\frac{a-c+\alpha\epsilon-4\gamma}{2(n-1)}.
\end{aligned}
$$

The reverse licensing case should be the worst outside option for firms 1 and 2, i.e. they have to surrender the technologies to the manufacturer for free. Check that when $\gamma=q_i^r=\frac{1}{2(n+1)}(a-c+\alpha\epsilon)$, $\Pi^{tt}=\Pi^r$. When $\gamma=\frac{1}{4}(a-c+\alpha\epsilon)$, it reduces to the interior solution case, where $\Pi^{tt}=\frac{1}{4}(a-c+\alpha\epsilon)^2>\Pi^r$. Note that Π^{tt} is quadratic on γ, with leading coefficient $\frac{-4}{(n-1)^2}<0$. Therefore, for all $\gamma\in[\frac{1}{2(n+1)}(a-c+\alpha\epsilon),\frac{1}{4}(a-c+\alpha\epsilon)]$, the aggregate producer surplus under two-part tariff is weakly higher than which under reverse licensing case. We summarize the results as below:

Proposition 2.2. *Consider the upstream manufacturer offers two-part tariff compensation for the surrendered patents. The royalty discount is always positive. If the outside option is sufficiently large, i.e.* $\Gamma>\frac{1}{16}(a-c+\alpha\epsilon)^2$, *the upstream manufacturer also provides positive fixed compensation. Otherwise,* *the fixed compensation is zero. In both cases, consumer surplus and aggregate producer surplus are further improved than those under reverse licensing.*

From the two cases discussed above, we may infer that the manufacturer charges $\frac{1}{2}(a - c + \alpha\epsilon)$ for the smartphone chips, firms 1 and 2 will always get a royalty discount, while the other firms will not. The higher the royalty discounts, the higher the quantity produced, and the lower the fixed compensation is required. The manufacturer has to balance these three effects. When outside option for firms 1 and 2 is large, the manufacturer uses both royalty discount and fixed compensation to acquire the technologies, giving large production advantage for firms 1 and 2, and resulting all the other firms leaving the market. When the outside option is small, royalty discount is sufficient for the compensation. All firms will stay in market, as the difference of production cost between firms 1 and 2 and the rest of firms are not significant. With the positive compensation, firms will produce more smartphones in the production market, yielding higher consumer surplus. For the manufacturer's profit, the loss from the compensation is recovered from the increase in the output production. Since firms 1 and 2 are better-off, by earning the profit equals to the outside option, the overall aggregate producer surplus is also improved. Policy makers should encourage such compensation, as both consumers and the domestic leaders, i.e. Huawei and ZTE, will be better-off. Moreover, the aggregate producer surplus, and thus social welfare are also higher under this compensation scheme.

2.4 Alternative Licensing Regimes

In this section, we consider two alternative licensing regimes other than reverse licensing: (1) downstream producers may transfer their technologies to their rival; and (2) downstream producers with innovation may form a patent pool to transfer technology.

2.4.1 Independent licensing

Now we consider the standard independent royalty licensing case. The leading firms, i.e. firms 1 and 2, transfer technology through royalty licensing. Let the royalty rate be r_1^i and r_2^i respectively.[13] Consider the standard licensing game. In first stage, upstream firm set the price for intermediary input d^i. Firms 1 and 2 set the royalty rate for their technologies respectively in second stage. In the last stage, downstream firms purchase the input; make purchasing decision on the technologies, and then engage in Cournot competition.

The profits for firms are[14]

$$\pi_m^i = dQ,$$

[13] We use superscript i to denote the case independent licensing.

[14] According to Kaimen and Tauman (1986), all the firms will be purchasing the technology, as long as the royalty fee is no greater than the cost reduction by the technology.

$$\pi_1^i = (P - d - (c - \alpha\epsilon) - r_2^i)q_1 + r_1^i(q_2 + q_3 + ... + q_n),$$

$$\pi_2^i = (P - d - (c - \alpha\epsilon) - r_1^i)q_2 + r_2^i(q_1 + q_3 + ... + q_n),$$

$$\pi_j^i = (P - d - (c - \alpha\epsilon) - r_1^i - r_2^i)q_j \text{ for all } j \in N\backslash \{1, 2\}.$$

Using backward induction to solve for unconstraint optimal royalty rates r_1 and r_2:

$$r_1^i = r_2^i = \frac{2(n-1) + (n+1)(n-1)}{(5n-7)(n+1) - 2(n-1)(n-3)}(a - c - d + \alpha\epsilon) \equiv r^{i*}.$$

However, we have three restrictions on the general royalty rate r_i: (i) $q_3^i = ... = q_n^i \geq 0$ if and only if $r^i \leq \frac{1}{4}(a - c - d + \alpha\epsilon)$, (ii) For firms 1 and 2: they will purchase patent from each other if and only if $r^i < \epsilon(\alpha - 1)$, and (iii) For firms $3, 4...n$: they will purchase the patent from firm 1 and 2 if and only if $r^i < \frac{1}{2}\alpha\epsilon$.[15]

First of all, we note that $\frac{1}{2}\alpha\epsilon \geq \epsilon(\alpha - 1)$, with equality holds when $\alpha = 2$, i.e. the technologies are perfectly compatible. Intuitively, firms without any technology would like to pay more to improve their productions. It suffices to check the conditions (i) and (ii) only. Second, note that $r^{i*} > \frac{1}{4}(a - c - d + \alpha\epsilon)$ if and only if $n^2 + 2n + 1 > 0$, which is true for all n. Therefore r^{i*} cannot be the solution to the constraint optimization problem.

Hence, we only have corner solutions left, i.e. firms 1 and 2 charge the royalty fee such that firms without technology exit the market, or the royalty

[15] Standard royalty licensing results according to Kamien and Tauman (1986).

fee is the maximum firms 1 and 2 to pay each other. We have two cases to discuss, i.e. $r^i = \min\{\frac{1}{4}(a - c - d + \alpha\epsilon), \epsilon(\alpha - 1)\}$.

Lemma 2.2. *Under independent licensing, when $r^i = (\alpha - 1)\epsilon$, the upstream manufacturer sets the price of input to be $d^i = \frac{1}{2}(a-c+\alpha\epsilon) - \frac{(n-1)(\alpha-1)}{n}\epsilon$, and all the firms stay in the market with production $q_1^i = q_2^i = \frac{n(a-c+\alpha\epsilon)+2(n^2-2n-1)(\alpha-1)\epsilon}{2n(n+1)}$ and $q_3^i = ... = q_n^i = \frac{n(a-c+\alpha\epsilon)-2(3n+1)(\alpha-1)\epsilon}{2n(n+1)}$.*

Lemma 2.3. *When $r = \frac{1}{4}(a-c-d+\alpha\epsilon)$, the upstream manufacturer sets the price of input and royalty rates to be $d^i = \frac{1}{2}(a-c+\alpha\epsilon)$ and $r^i = \frac{1}{8}(a-c+\alpha\epsilon)$. Firms without initial technology will exit the market so that $q_1^i = q_2^i = \frac{1}{8}(a - c + \alpha\epsilon)$ and $q_3^i = ... = q_n^i = 0$.*

Proposition 2.3 below provides a sufficient condition for firms to exit the market. Under independent licensing, the market power of firms 1 and 2 increases as the technologies are more and more compatible, i.e. they are more of a complements. Furthermore, when technologies are efficient in reducing production cost, firms 1 and 2 would like to further increase their market power by cross licensing the technologies to each other, and hence achieving higher profits by driving all the other firms out of the market. Otherwise, they would like to license the technology to all the other firms, in order to achieve higher profit through the collection of licensing fees.

Proposition 2.3. *When technologies holding by firms 1 and 2 are compatible, i.e.* $\alpha > \frac{12n+2}{7n}$*, and the production cost reduction is sufficiently high, i.e.* $\frac{1}{7\alpha-8}(a-c) < \epsilon < \frac{n}{2(2n+1)}(a-c)$*, firms* $3, 4, ..., n$ *will exit the market.*

2.4.2 Patent Pool

Consider firms 1 and 2 form a pool, licensing the technology $\alpha\epsilon$ at a royalty fee r^p, and sharing the profit equally. The game structure is similar to independent licensing. In first stage, upstream firm set the price for intermediary input d^p. In the second stage, firms 1 and 2 charges r^p for the combined technology. In the last stage, downstream firms purchase the input; make purchasing decision on the combined technologies, and then engage in Cournot competition. We assume that firms 1 and 2 do not need to pay each other for their technologies respectively. Once they agree to combine the technologies, they have the knowledge of the new technology, and hence could use it for their own production.

Even though firms are forming a patent pool, we assume that they still choose the level of production to maximize own profit. It is not realistic for them (e.g. Huawei and ZTE) to form a collusion to maximize joint profits, i.e. there is no Cartel in the market. Pool members will agree to license the patent at some royalty rate r^p. By our model setup, since firms 1 and 2 symmetric, the royalty fee $r_1^p = r_2^p \equiv r^p$ should be the agreement outcome in equilibrium.

The profits for firms are[16]

$$\pi_m^p = dQ,$$

$$\pi_1^p = (P - d - (c - \alpha\epsilon))q_1 + \frac{1}{2} r^p (q_3 + ... + q_n),$$

$$\pi_2^p = (P - d - (c - \alpha\epsilon))q_2 + \frac{1}{2} r^p (q_3 + ... + q_n),$$

$$\pi_j^p = (P - d - (c - \alpha\epsilon) - r^p)q_j \text{ for all } j \in N\backslash \{1,2\}.$$

By backward induction,

$$q_1 = q_2 = \frac{(a-c-d) + \alpha\epsilon + (n-2)r^p}{n+1},$$
$$q_3 = ... = q_n = \frac{(a-c-d) + \alpha\epsilon - 3r^p}{n+1}.$$

Solving firm 1's maximization problem:

$$\max_r q_1^2 + \frac{1}{2} r^p (q_3 + ... + q_n)$$

gives us $r^{p*} = \frac{n+5}{2(n+7)}(a-c-d+\alpha\epsilon)$. Firms 3 and 4 will purchase the patent if and only if $r^p \leq \alpha\epsilon$. Positive production constraint is: $r^p \leq \frac{1}{3}(a-c-d+\alpha\epsilon)$. Check that $\frac{n+5}{2(n+7)} > \frac{1}{3}$ if and only if $n+1 > 0$. Therefore r^{p*} cannot be the solution of the constrained optimization problem. Combine the conditions, we have $r^p = \min\{\frac{1}{3}(a-c-d+\alpha\epsilon), \alpha\epsilon\}$.

Proposition 2.4. *Consider firms 1 and 2 form a patent pool. (a) When* $\frac{1}{5\alpha}(a-c) < \epsilon < \frac{n}{2(2n+1)}(a-c)$, *firms without initial technology will exit the*

[16] Following Kaimen and Tauman (1986).

market, the upstream manufacturer sets the price of chips $d^p = \frac{1}{2}(a - c + \alpha\epsilon)$ *and royal rate* $r^p = \frac{1}{6}(a - c + \alpha\epsilon)$*, and the production of downstream producers are* $q_1^p = q_2^p = \frac{1}{6}(a - c + \alpha\epsilon)$ *and* $q_3^p = ... = q_n^p = 0$*. (b) When* $\epsilon < \frac{1}{5\alpha}(a - c)$*, all the firms stay in the market, the upstream manufacturer sets the price of chips* $d^p = \frac{n(a-c)+2\alpha\epsilon}{2n}$ *and royal rate* $r^p = \alpha\epsilon$*, and the production of downstream producers are* $q_1^p = q_2^p = \frac{n(a-c)+2(n^2-n-1)\alpha\epsilon}{2n(n+1)}$*,and* $q_3^p = ... = q_n^p = \frac{n(a-c)-2(2n+1)\alpha\epsilon}{2n(n+1)}$*.*

In comparing with Proposition 2.3, under patent pool, compatibility between the technologies is no longer a factor for firms without initial technology to exit the market, as firms 1 and 2 will combine the technologies and license them as a whole. It is easy to check that $\frac{1}{5\alpha}(a - c) < \frac{1}{7\alpha - 8}(a - c)$, indicating firms 1 and 2 are more likely to foreclose the other firms under patent pool, even if the technologies are not very efficient in improving production efficiency. By forming a pool, there is no more competition between firms 1 and 2 in licensing the technologies, and hence market dominance is easily achieved. Our policy implication is that if the government wants to promote the downstream market competition, independent licensing should be preferred to patent pool.

2.4.3 Comparison

In section 2.2 and 2.4, we have two bounds for all firms producing in the downstream market and firms without initial technology leaving the market.

So there are 3 cases: (1) $0 < \epsilon < \frac{1}{5\alpha}(a-c)$, all firms will stay in market; (2) $\frac{1}{5\alpha}(a-c) \leq \epsilon < \frac{1}{7\alpha-8}(a-c)$, firms without initial technologies will exit the market under patent pool; and (3) $\frac{1}{7\alpha-8}(a-c) \leq \epsilon < \frac{n}{2(2n+1)}(a-c)$, firms without initial technologies will exit in both independent licensing and patent pool.

In the following analysis, we consider the reverse licensing without remedy. Since we have shown that remedy will further improve consumer surplus, aggregate producer surplus and social welfare, it is suffices for us to show without remedy, reverse licensing improves these three measures in comparing with independent licensing and patent pool. We will focus on the analysis of consumer surplus in this section, the proof for aggregate producer surplus could be found in Appendix B.1.

Lemma 2.4. *When all firms stay in the market, i.e.* $0 < \epsilon < \frac{1}{5\alpha}(a-c)$, *reverse licensing is the best for consumer.*

When firms exit under patent pool,

$$Q^{p'} = \frac{1}{3}(a-c+\alpha\epsilon) < \frac{n(a-c)+2\alpha\epsilon}{2(n+1)} = Q^p$$

because the competition becomes a duopoly with technology level $\alpha\epsilon$. Consumer surplus under duopoly is lower.

When firms exit under independent licensing,

$$Q^{i'} = \frac{1}{4}(a - c + \alpha\epsilon) < \frac{n(a - c + \alpha\epsilon) - 2(n-1)(\alpha - 1)\epsilon}{2(n+1)} = Q^i$$

because the competition becomes a duopoly with cross licensing, and hence consumer surplus is lower.

In all the cases, we can conclude that consumer surplus under reverse licensing is highest.[17] In fact, the aggregate producer surplus and hence, social welfare are also highest under reverse licensing.

Theorem 2.1. *Reverse licensing yields highest consumer surplus, aggregate producer surplus, and hence social welfare.*

Since the upstream manufacturer is monopoly of the input and the two leaders in the downstream are monopoly of their respective technology, reverse licensing concentrates all market power into the hand of upstream manufacturer and thereby avoids the problem of double marginalization.

Theorem 2.1 addresses also the “complements problem” which was proposed by Cournot in 1838, and studied by Shapiro (2001). When two (or more) perfectly complements are used as input for downstream firms’ production, patent pool could internalize the externality; and hence yields higher consumer surplus and producer surplus.

[17] More comparisons for consumer surplus are presented in Appendix B.1.

First of all, when $\alpha = 2$, i.e. the two technologies are perfect complements, patent pool could solve this complements problem. In addition, we show that reverse licensing is an even better way of solving the problem, as the consumer surplus is highest under reverse licensing. The upstream firm could internalize the complementary externality better than the two downstream leaders.

Second, we shall notice that patent pool fails to solve the complements problem, once the technology deviates a little from perfect compatibility (or perfect complements). Consider the case where all the firms stay in market in all the different licensing cases, i.e. $0 < \epsilon < \frac{1}{5\alpha}(a-c)$. When $\alpha < \frac{2(n-1)}{n}$, from the proof of Theorem 1, we have $Q^p < Q^i$. [18] However reverse licensing always yields highest consumer surplus, for all the compatibility level.

We further study the complements problem by varying the degree of compatibilities between the two technologies. Focusing on the case $0 < \epsilon < \frac{1}{5\alpha}(a-c)$, we have

$$\frac{\partial Q^i}{\partial \alpha} = \frac{(2-n)\epsilon}{2(n+1)} < 0;$$
$$\frac{\partial Q^p}{\partial \alpha} = \frac{2\epsilon}{2(n+1)} > 0;$$
$$\frac{\partial Q^r}{\partial \alpha} = \frac{n\epsilon}{2(n+1)} > 0.$$

Figure 2.1 shows the relationship of consumer surplus with respect to com-

[18] In Appendix B.1, we also show that When $\alpha < \frac{2(n-1)}{n}$, we have $\Pi^p < \Pi^i$, which is consistent with Shapiro (2001).

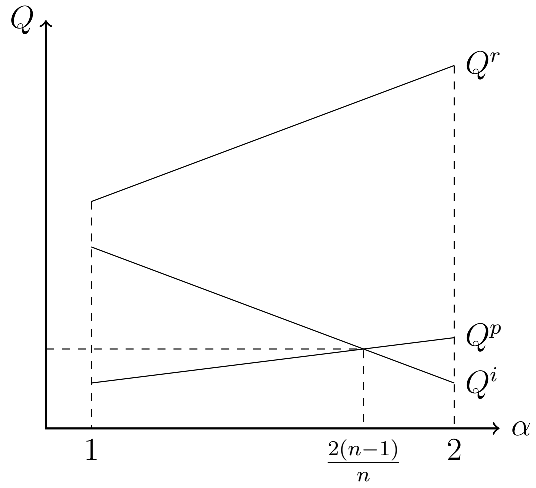

Figure 2.1: Consumer surplus varies with compatibility

patibility level α for the three different licensing regimes. As the technologies become more compatible, reverse licensing and patent pool could fully utilize these technologies, and hence leading to higher consumer surplus. Consumer surplus is decreasing as the technologies become more compatible under independent licensing, as firms 1 and 2 will raise the production cost of the downstream market, by charging higher royalty fees for their own technologies. Note that when the technologies are more of substitutes, firms would compete more intensively under independent licensing, leading to a higher consumer surplus than which under patent pool. The fact that reverse licensing solves the double marginalization problem is reflected by the highest consumer surplus curve, which also indicates the double marginalization problem is more serious in affecting consumer surplus, comparing with the complements problem.

2.5 Research and Development

Before surrendering the technologies to the upstream firm, downstream firms could invest fixed amount I to get a technology ϵ. We would like to study the investment decision of all the firms under the reverse licensing game. For simplicity, we assume all technologies are perfectly compatible in this section. But we also want to extent our study to k firms possess some initial technologies. Let K be the set of firms possess initial technology, and $N \backslash K$ be the set of firms does not possess any initial technology. Note that $K = 2$ is the case which we have studied in previous sections.

2.5.1 Before remedy

We determine the threshold I^*, such that all firms will be doing research.

Suppose all firms are doing the investment, and hence, the aggregate technology level will be $(n + k)\epsilon$. Downstream firms' profits are $\Pi_1 = ... = \Pi_n = \frac{1}{4(n+1)^2}(a - c + (n + k)\epsilon)^2$. If firm $i \in K$ deviates, i.e. he does not invest. Then $\Pi_i' = \frac{1}{4(n+1)^2}(a - c + (n + k - 1)\epsilon)^2$. Therefore, the firms $i \in K$ will invest if and only if $I < \frac{(2(a-c)+(2n+3)\epsilon)\epsilon}{4(n+1)^2} \equiv I^*$.

If firm $j \in N \backslash K$ deviates, i.e. he does not invest. Then $\Pi_j' = \frac{1}{4(n+1)^2}(a - c + (n + k - 1)\epsilon)^2$. Therefore, firms $j \in N \backslash K$ will invest if and only if $I < \frac{(2(a-c)+(2n+2k-1)\epsilon)\epsilon}{4(n+1)^2} = I^*$.

Consider another symmetric equilibrium, where no firms are investing. Firms' profits are: $\Pi_1 = ... = \Pi_n = \frac{1}{4(n+1)^2}(a - c + k\epsilon)^2$. If firm $i \in K$ deviates, i.e. he invests. Then $\Pi'_i = \frac{1}{4(n+1)^2}(a-c+(k+1)\epsilon)^2$. Similarly, if firm $j \in N \backslash K$ deviates, his profit will be $\Pi'_j = \frac{1}{4(n+1)^2}(a-c+(k+1)\epsilon)^2$. Therefore, firms will deviate to invest if and only if $I < \frac{(2(a-c)+(2k+1)\epsilon)\epsilon}{4(n+1)^2} = I^{**}$.

Proposition 2.5. *If $I < I^{**} < I^*$, there is only one symmetric equilibrium where all firms invest. If $I^{**} < I < I^*$, either all firms invest or no firm invest.*

One implication of this proposition is that firms try to free ride each other, as $I^{**} < I^*$. When no one is doing investment, firms are reluctant to become the first one to do research. Before the anti-trust policy takes effect, reverse licensing harms the research incentive.

2.5.2 After remedy

We consider the simple case, where the manufacturer uses fixed fee as the compensation.[19] If one firm keeps his technology, he could only use it for himself. There is no licensing option. The manufacturer will pay the compensation as the profits difference between providing the technology and keeping it. Consider firm $i \in K$ will keep the technology, and he will do investment if and only if $(\frac{a-c+(3n+k-2)\epsilon}{2(n+1)})^2 - I > (\frac{a-c+(2n+k-2)\epsilon}{2(n+1)})^2$ which is equivalent to

[19]We depart from the two-part tariff compensation scheme in this section, as we need an explicit expression for the outside outside option, Γ. Otherwise, it is not possible for us to define the investment level under remedy.

$I < \frac{(2(a-c)+(5n+2k-4)\epsilon)n\epsilon}{4(n+1)^2} \equiv \hat{I}$. Consider firm $j \in N \backslash K$ will keep the technology, and he will do investment if and only if $(\frac{a-c+(2n+k-1)\epsilon}{2(n+1)})^2 - I > (\frac{a-c+(n+k-1)\epsilon}{2(n+1)})^2$ which is equivalent to $I < \frac{(2(a-c)+(3n+2k-2)\epsilon)n\epsilon}{4(n+1)^2} \equiv \bar{I}$.

Proposition 2.6. *Compensation from upstream firm encourages firms to invest. Leading firms have more incentive to research.*

First of all, $I^* < \bar{I} < \hat{I}$, indicating all firms have more incentive to research. Second, leading firms, $i \in K$, are more willing to invest than firms $j \in N \backslash K$ to keep their technological dominance in the market, because they could receive more compensation from the upstream firm. The Chinese mobile producers have indeed increased their R&D expenditure. Huawei has put USD9.2 billion in 2015, increasing 41.6% from 2014. In the research on the 5G technology, ZTE will spend USD400 million annually and adding 800 engineers to the research lab. [20] The cease of anti-competitive conducts of Qualcomm really stimulates the R&D incentive for the technology leaders in China.

Note that all the $I^*(k), I^{**}(k), \hat{I}(k)$ and $\bar{I}(k)$ are increasing in k. When there are more firms possessing initial technologies, all the firms are more willing to do investment, so that they could have more competitive power in the market.

[20] The R&D expenditure figure for Xiaomi and OPPO is not publicly announced.

2.6 Conclusion

Compared to patent pool or independent licensing among downstream firms, reverse licensing performs better in terms of consumer surplus, aggregate producer surplus and hence social welfare. A two-part tariff compensation from the manufacturer will further enhance the three measures. However, from long-run perspective, reverse licensing reduces incentive to innovate. Yet, under the pure fixed fee remedy, more incentive to innovate can be induced. Hence, for anti-trust regulation, this suggests that reverse licensing should be under rule of reason instead of per se illegal. In particular, the ruling of Qualcomm case in China exactly follows our policy suggestion.

Our model also sheds light on the classical complementary inputs problem by Cournot (1838). Shapiro (2001) considers the input to be technology such that patent pool can avoid this problem. In our vertical structure setup, we propose reverse license is a better solution than patent pool.

Chapter 3

Corruption, Pollution and Technology Transfer

3.1 Introduction

Corruption and pollution are the top two worried-list in China according to Spring 2015 Global Attitude survey. Beijing, the capital, is well-known for its haze and air pollution since 1998. [1] To ensure a clean environment for Beijing Olympic Games 2008, the most notorious polluter, Shougang Group, started moving its production from Beijng to Tangshan City, Hebei Province, since 2005. By 2010, the transfer of production is complete, and from 2013 onwards, the cities in Hebei Province became top polluted cities in China. The iron and steel production became one of the root problems for the air pollution in Hebei Province. World bank approved a loan of USD500 million to help

[1] According to World Health Organization's report 1998.

Hebei to fight for the air pollution in June 2016. [2] Hebei Province also put new regulations on improving the air quality since October 2016.

The presence of corruption will make the regulation even more difficult. Bribes between bureaucrats and firms could have devastating effect on undermining the environment. In a seminal paper, Liao and Geng (2014) have concluded that anti-corruption practices do have a significant negative effect on pollution suggesting corruption as one of the main factors of pollution in China. In June 2016, one officer in Tangshan Municipal Environmental Protection Bureau was arrested, by the charge that allowing hundreds of firms polluting. [3]

In Indonesia, corruption is often cited as the root cause of the haze problem in recent years.[4] Government officials turn a blind eye to (palm oil) companies who burn the crops generating devastating air pollution. Clean technology, such as using machines to clear the cultivated land, is not adopted by these firms due to the higher cost. In many empirical literatures, Fredriksson and Svensson (2003), and Damania et al. (2003) have shown that corruption

[2] According to "http://www.worldbank.org/en/news/press-release/2016/06/06/china-hebeis-efforts-to-curb-air-pollution-get-more-support-from-the-world-bank" Accessed online on 01/11/2016.

[3] According to "http://news.xinhuanet.com/legal/2016-06/14/c_1119042616.htm" Accessed online on 02/11/2016.

[4] Accessed online "http://www.eco-business.com/news/how-corruption-fuelling-singapores-haze/" on 02/01/2016.

has decreased the stringency of environmental regulations.[5] Ivanova (2010) has shown that under corruption, emissions tend to be significantly under-reported.[6]

Even there is indeed transferring of the clean technology, pollution may still increase due to higher production of the industries.[7] In the worst cases, the dirty firms may not be able to adopt the clean technology on their out-of-date machines. So shutting down these firms may be a better solution than transferring the technologies. In recent years, China had shut down thousands of heavily polluted firms to fight the pollution, and in 2016, Beijing is planning to shut down 2500 firms to improve the environment. To our knowledge, this is the first IO paper discussing government regulation in promoting technology transfer or shutting down dirty firms in the presence of corruption. We find that corruption may undermine the government policy in shutting down the heavily polluting firms, but also increases the likelihood of technology transfer. If we consider socially optimal taxation, even though corruption induces more pollution, it is possible to be the equilibrium outcome.

We consider two firms competing in a homogenous good market, with pol-

[5]Both papers uses tax as government regulation.

[6]Our model will be built on this point that firms could bribe the bureaucrat to under-report the emission.

[7]Takarada (2005) studies a trade model between two countries, and shows clean technology transfer may increase the pollution in both donor country and recipient country under certain conditions.

lution accompanied with production, which is not observed by the social planner. There is a clean technology, which could effectively reduce pollution. We consider one firm (firm 1) has the clean technology but the other firm (firm 2) does not.[8] Our paper departs from the literatures (for example, Acemoglu and Verdier (2000), and Stathopoulou and Varvarigos (2013)), i.e. they study homogeneous firms choosing clean or dirty technology. Under our setup, we could offer an alternative explanation for dirty firms not adopting the clean technology, despite of the low willingness to purchase. We find that firms with clean technology may not be willing to sell it to other firms; hence maintaining the dominance position in the market.

Firm 2 could make a private decision on purchasing the clean technology from firm 1 through fixed fee licensing. There is a bureaucrat inspecting the firms' technology adoption, and reporting these publicly. The environmental tax is then collected based on the report. The bureaucrat will always truthfully report firm 1 to be clean. However, he is corruptible, and will misreport the actual emission if he accepts bribe from the firm 2. Therefore, government needs to implement anti-corruption policy to monitor the bureaucrat.

Government cares about both consumer surplus and environment. We say a country is output-oriented, if government weighs consumer surplus more

[8] We consider the technology owner is an outsider in Discussion section and Appendix C.2.4.

than environment; otherwise, the country is environment-oriented. He will use environmental tax as an instrument to reduce pollution. Clearly, the tax has two opposing effects on social welfare. On the one hand, higher tax rate will reduce pollution inducing better environment. On the other hand, it will reduce firm's production, and hence lowers consumer surplus.

We study the model from three aspects. First of all, we find that there is substitution effect between corruption and technology transfer. In the absence of corruption, for an intermediary tax range, government taxation fails to promote technology transfer, leading to lower consumer surplus and higher pollution level. In this scenario, firm 1 would like to charge a high licensing fee for the technology to foreclose firm 2, thus gaining a monopoly position in the market. While under the presence of corruption, technology transfer will occur under such tax range. Intuitively, knowing the ease of bribing, firm 1 will set a lower fee on the clean technology, to induce firm 2 to come clean and compete in the open and equal terms. In this sense, allowing cheating disciplines firm 1, i.e. firm 1 could no longer forecloses firm 2 by charging high licensing fee, to reduce firm 2's incentive to cheat.

Second, strategic effect is not the driving force to the corruption equilibrium. We find that corruption may be the equilibrium when two heterogenous firms competing in the market. If the clean technology is owned by an outsider innovation, and there is only a monopoly in the production market. Corruption may still be the equilibrium under high environmental tax rate. In such case, the government would like to shut down the firm, but the firm will bribe the bureaucrat to remain active in the market. Therefore, corruption may occur regardless of competition among the firms.

Third, we have several policy implications, i.e. the regulatory effects. We show that a more output-oriented country would set a low tax rate to achieve highest social welfare. In such case, the competition effect will always ensure the technology transfer from firm 1 to firm 2, and hence consumer surplus will be higher and pollution will be lower. With low tax rate, firm 2 is competitive even without the clean technology. Firm 1, then, will always set a lower price on the technology, to ensure the transfer is profitable to him. Firm 2 will never bribe the bureaucrat, facing such a low price of the technology.

In a more environment-oriented country, the government should always set a high tax rate, aiming to shut down the heavily polluting firm 2, and hence reducing pollution in the market. With weak anti-corruption policy, when the bribing cost is low, firm 2 would choose from licensing or bribing in order to stay in market. In such case, bribing the bureaucrat may be socially optimal. Comparing with licensing, even though bribing will lower the total output, and hence lower consumer surplus; it lowers pollution level as well.

The later effect outweighs the former, and hence social welfare is improved. Nevertheless, firm 2 is not shut down, and government regulation fails in the presence of corruption.

Our paper is related to literatures in corruption and pollution. Lui (1985) and Beck and Maher (1986) have studied the efficiency-enhancing effect of corruption on resource allocation. Acemoglu and Verdier (2000) has studied the relationship between market failure and government intervention. In the Laissez-Faire equilibrium, all firms will choose the dirty technology, leading to market failure. government should both set an environmental tax and hire bureaucrats to monitor these firms, as ways of intervening the firms to adopt the clean technology. They have shown that the optimal intervention involves a fraction of bureaucrats accepting the bribes from the firms. In line with the insights, our paper suggests that in an environment-oriented country, social optimal taxation indeed induces corruption, if the clean technology is not very effective in reducing pollution.

The structure of our analysis are organized as follows: section 3.2 will be the model setup. We study the benchmark case without corruption in section 3.3, and include corruption in section 3,4. Section 3.5 is a discussion on outside innovator, i.e. technology is owned by domestic research lab. We conclude the paper in section 3.6. The technical details, proofs and detailed discussions are in Appendix.

3.2 Model

We consider a simple model in which taxation policy and corruption may affect firms' adoption of clean technology. Consider a homogeneous good production market, with linear demand function $P = 1 - Q$. The production of the good accompanies with pollution, E. Pollution is not observable by the government or social planner, thus is measured by production.[9] There are two types of technology: dirty technology and clean technology. For dirty technology, we assume each unit of production generates one unit of pollution, while for clean technology each unit of production generates only α unit of pollution, where $0 < \alpha < 1$. In other words, compared to dirty technology, clean technology could effectively reduce the pollution, but does not improve production efficiency.

There are two firms in the market that are engaged in Cournot competition. Both firms have the same production technology and the cost of production is normalized to zero. The two firms have different technologies in generating pollution: firm 1 has the clean technology while firm 2 has the dirty technology. The technology of each firm is common knowledge. Then we have $e_1 = \alpha q_1$

[9] Product tax is a form of environmental tax in the U.S., for example gas guzzler tax and tax on fertilizers.

and $e_2 = q_2$, where q_i is output of firm i and e_i is the amount of pollution generated by firm i. Denote the aggregate output by Q and the total amount of pollution by E, where $Q = q_1 + q_2$ and $E = e_1 + e_2$.

Though there is no cost of production, it is not costless for each firm to produce given that production generates pollution. Suppose government has set an environmental tax t for the production of each firm. And the total amount of tax paid by each firm depends on the technology level. Firm 2, then, may want to purchase the clean technology in effectively reducing the environmental tax. We assume that the technology transfer process is private so that whether firm 2 adopts the clean or dirty technology eventually is not publicly observed. Therefore, the government hires a bureaucrat, who may be corruptive, to check the technology of firm 2. The bureaucrat is risk-neutral and receives a fixed wage w (exogenous). If the bureaucrat is not corruptive, then he always reports the technology of firm 2 truthfully.[10] We also consider the possibility that the bureaucrat may be corruptible so that he may misreport the technology adopted by firm 2. This can be the case where the bureaucrat reports the technology of firm 2 be "clean", while in fact firm 2 has not purchased the clean technology from firm 1. The bureaucrat has an incentive to misreport if firm 2 has offered enough money as a bribe.

[10] We assume the bureaucrat will always truthfully report firm 1's technology to be clean, which is consistent with Ivanova (2011).

However, such a corruptive behavior is not costless. With a probability of σ, the bureaucrat's report will be checked, and thus be detected whether the report is true or not. If false report is discovered, the bureaucrat loses all his wage (including the bribe he has received, if any). Therefore, the bureaucrat is only willing to accept the bribe, denoted by b, if the expected gain from bribing is non-negative, i.e., $(1 - \sigma)(b + w) \geq w$. Thus the minimum bribe offer that the bureaucrat will accept is $\underline{b} = \frac{\sigma w}{1-\sigma}$.

We consider the following game. In stage 1, firm 1 sets a fixed licensing fee F for the clean technology and firm 2 decides whether to purchase the clean technology from the rival, or to bribe the bureaucrat by offering b, or to keep using the dirty technology. In stage 2, the bureaucrat inspects firm 2 and makes a report on the technology used by firm 2. Two firms compete in quantities and taxes are collected based on the report made by the bureaucrat.

We analyze two cases depending on whether the bureaucrat is corruptive or not. In each case, we first study firm 2' choice of whether to purchase the clean technology based on different taxation policies. We further discuss how different taxation policies affect pollution and consumer surplus. Lastly, we investigate the optimal taxation policy, which may depend on the nature of the government, i.e. output-oriented or environment-oriented.

3.3 Without Corruption

We first consider the benchmark case without corruption. In other words, the bureaucrat will always truthfully report the true technology of the firm being monitored (i.e., firm 2). In this sense, there is no scope for the firm with dirty technology to bribe the bureaucrat. Thus, we can just focus on two options of firm 2: purchasing the clean technology from the rival, or keeping using the dirty technology.

3.3.1 Equilibria

We analyze the equilibrium of this case using backwards induction.

3.3.1.1 Stage 2: Competition

In this stage, we just need to consider two possible cases depending on whether licensing takes place in stage 1.

Case 1: No Licensing

If licensing does not take place in stage 1, firm 1 produces with the clean technology which generates pollution $e_1 = \alpha q_1$, while firm 2 produces with the dirty technology which generates pollution $e_2 = q_2$. Therefore firms' profits are

$$\pi_1^N = (1 - q_1 - q_2)q_1 - \alpha t q_1,$$

$$\pi_2^N = (1 - q_1 - q_2)q_2 - tq_2.$$

Standard Cournot competition leads to

$$q_1^N = \frac{1 + (1-2\alpha)t}{3}, q_2^N = \frac{1 + (\alpha - 2)t}{3},$$
$$\pi_1^N = \left(\frac{1 + (1-2\alpha)t}{3}\right)^2, \pi_2^N = \left(\frac{1 + (\alpha - 2)t}{3}\right)^2,$$

if $0 \leq t < \frac{1}{2-\alpha}$; and

$$q_1^N = \frac{1 - \alpha t}{2}, q_2^N = 0,$$
$$\pi_1^N = \left(\frac{1 - \alpha t}{2}\right)^2, \pi_2^N = 0,$$

if $\frac{1}{2-\alpha} < t \leq 1$. In other words, when the tax rate is high, firm 2 will exit the market due to a high environmental tax.

Case 2: Licensing

If firm 1 licenses the clean technology to firm 2 at a fixed fee F, then firms' profits are

$$\pi_1^L = (1 - q_1 - q_2)q_1 - \alpha t q_1 + F,$$
$$\pi_2^L = (1 - q_1 - q_2)q_2 - \alpha t q_2 - F.$$

Then we have

$$q_1^L = q_2^L = \frac{1 - \alpha t}{3},$$
$$\pi_1^L = \left(\frac{1 - \alpha t}{3}\right)^2 + F, \pi_2^L = \left(\frac{1 - \alpha t}{3}\right)^2 - F.$$

3.3.1.2 Stage 1: Firm 2's choice

From Wang (1998), we know that firm 2 would choose to purchase the clean technology (i.e., licensing) if and only if the joint profit of the firms increases after technology transfer taking place. Therefore, licensing will be the equilibrium if and only if

$$\pi_1^L + \pi_2^L \geq \pi_1^N + \pi_2^N,$$

i.e., $t \leq \frac{2}{5-3\alpha}$; otherwise, no licensing will be the equilibrium.

The following Proposition summarizes the result.

Proposition 3.1. *Suppose there exits no corruption. When $t \leq \frac{2}{5-3\alpha}$, licensing takes place in equilibrium; otherwise, no licensing takes place in equilibrium.*

From Proposition 3.1, we know that a low tax rate leads to low cost of production. Firm 1 will have less advantage in the production level competition, and thus he will set a lower fee for the technology, which would lead to technology transfer among firms. In contrast, a high tax rate would prevent the adoption of clean technology of firm 2. In fact, firm 2 will exit in this case.

3.3.2 Consumer Surplus and Pollution

From Proposition 3.1, we know that firm 2's choice of whether to purchase the clean technology (i.e., licensing option) could affect competition between the two firms and result in different outputs (or consumer surplus) and the levels of pollution. The Proposition summarizes how licensing affects consumer surplus and pollution for any given taxation policy.

Proposition 3.2. *For any given tax rate, licensing always improves consumer surplus compared to no licensing. Licensing also reduces the total emission if and only if* $0 < t \leq \frac{1}{2}$.

Since technology transfer from licensing improves the technology used by firm 2, this further intensifies competition between the two firms and increases the total output produced by the firms. Therefore, licensing always results in higher consumer surplus regardless of the tax rate.

The effect of licensing on pollution depends on the tax rate. When the tax rate is low (i.e., $0 < t < \frac{1}{2}$), licensing improves the technology adopted by firm 2 and thus reduces the pollution generated by it, despite that firm 2 produces more. Furthermore, stronger competition between both firms lowers the output produced by firm 1, and reduces the pollution generated as well. As a result, the total pollution is reduced after technology transfer takes place. When tax rate is high (i.e., $\frac{1}{2} < t < 1$), firm 2 will exit the market in the absence of licensing (or technology transfer). In this case, firm 1 becomes the monopoly, and hence generates less total pollution. Therefore, compared to no licensing, licensing leads to both higher outputs and higher pollution.

However, our model also suggests that government policy alone may not be sufficient to achieve the first-best outcome. Consider the case where the clean technology is very efficient in reducing pollution, i.e. $\alpha < \frac{1}{3}$. In such case, for tax rate $\frac{2}{5-3\alpha} < t \leq \frac{1}{2}$, i.e. licensing will lead to higher consumer surplus and lower emission, but no licensing will be the equilibrium, following Proposition 3.1. Indeed, it is not jointly profitable for licensing to take place, and firm 1 would charge a high price for the technology to maintain its market dominance position by foreclosing firm 2. Since the private incentive does not perfectly align with the social incentive, government taxation policy may not be efficiently by inducing technology transfer.

3.3.3 Optimal Taxation Policy

Since the diffusion of clean technology between firms would effectively reduce pollution, a low tax rate seems more favorable. However, a lower tax rate will also lower the cost of production of firms, which in turn leads to higher outputs and higher levels of pollution. Thus, it is not clear what is the optimal tax rate that leads to the lowest level of pollution. Moreover, the social planner does not only care about the pollution level, but also the outputs produced by the firms. For the socially optimal tax level, it must maximize the social welfare.

We assume social welfare function takes the quadratic form in both the total output and the level of pollution, i.e., $W \equiv Q^2 - \beta E^2$.[11] Here β is an exogenous parameter that measures how government weighs environment in the country's development plans in comparing with consumer surplus. We say the country is output-oriented if β is low; otherwise the country is environment-oriented. The term E^2 assumes the marginal damage of the pollution on our environment is increasing.[12]

The welfare in each case can be written as

$$W(t) = \begin{cases} W^L(t) = \frac{4(1-\beta\alpha^2)(1-\alpha t)^2}{9}, & \text{if } t < \frac{2}{5-3\alpha} \\ W^N(t) = \frac{(2-(1+\alpha)t)^2}{9} - \frac{\beta(1+\alpha+2(\alpha-\alpha^2-1)t)^2}{9}, & \text{if } \frac{2}{5-3\alpha} < t < \frac{1}{2-\alpha} \\ W^{N'}(t) = \frac{(1-\beta\alpha^2)(1-\alpha t)^2}{4}, & \text{if } \frac{1}{2-\alpha} < t \leq 1. \end{cases}$$

The optimal taxation policy would depend on whether the country is more output-oriented or environment oriented. We consider these two cases separately.

3.3.3.1 Case 1: $\beta < \frac{1}{\alpha^2}$, output-oriented country

When $\beta < \frac{1}{\alpha^2}$, we aim to show that the optimal tax rate is zero. To show this, we first present the following lemma, which suggests that any tax rate $t \in [\frac{2}{5-3\alpha}, 1]$ can never be optimal.

Lemma 3.1. *$W^L(0) > W^N(t)$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$, and $W^L(0) > W^{N'}(t)$ for*

[11] We also show in Appendix C.2.3 that our results hold if we use a linear welfare function, i.e. $W = Q - \beta E$.

[12] In Damania (2002) and Ivanova (2010), it is assumed that the damage function by emission on social welfare is increasing and convex. Ivanova's study is on air pollution, and hence there is transboundary spillovers from (or to) the other countries. This explanation fits the Indonesia air pollution case, which firms burn the crops, generating pollution affecting other ASEAN countries.

$t \in [\frac{1}{2-\alpha}, 1]$.

Following Lemma 3.1, given that $W^L(t)$ is decreasing in t, $W^L(0) \geq W^L(t)$ for $t \in [0, \frac{2}{5-3\alpha}]$. Thus $t = 0$ is the optimal tax rate which maximizes the social welfare.

3.3.3.2 Case 2: $\beta > \frac{1}{\alpha^2}$, environment-oriented country

Similarly, we first present the following lemma which suggests that any tax rate $t \in [0, \frac{1}{2-\alpha}]$ can never be optimal.

Lemma 3.2. *$W^{N'}(1) > W^L(t)$ for $t \in [0, \frac{2}{5-3\alpha}]$, and $W^{N'}(1) > W^N(t)$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$.*

Following Lemma 3.2, given that $W^{N'}(t)$ is increasing in t, $W^{N'}(1) > W^{N'}(t)$ for $t \in [\frac{1}{2-\alpha}, 1)$. Thus $t = 1$ is the optimal tax rate which maximizes the social welfare.

The following theorem summarizes the main result on the optimal tax rate.

Theorem 3.1. *Suppose there exists no corruption. When $\beta < \frac{1}{\alpha^2}$, the optimal tax rate is zero and licensing takes place; when $\beta > \frac{1}{\alpha^2}$, the optimal tax rate is one and no licensing takes place.*

Government intervention is effective in implementing the environmental goal. When he is output-oriented, licensing is preferred as it would intensify competition and lead to higher outputs. From Proposition 3.1, we know that licensing takes place only when tax rate is low (i.e., $0 \le t < \frac{1}{2}$). In this case, a lower tax rate would not only raise the total output but also increase the total pollution. Since the country is output-oriented, the government cares more about the increase in total output. As a result, the tax rate would be set as low as possible and the optimal tax rate is zero.

When the government is environment-oriented, he will charge a high tax rate so as to prevent licensing to occur. This would in fact lower the total pollution, following Proposition 3.2. In this case, as firm 1 is the monopoly, the total output (or pollution) is decreasing in the tax rate. Since the government cares more about the total pollution, the tax rate should be set as high as possible and the optimal tax rate is one.

3.4 With Corruption

In the benchmark case, we assume that the bureaucrat is not corruptive so that there is no scope for bribing to take place. Now we consider the case where the bureaucrat is corruptive so that he may misreport the technology of the firm being inspected. In this case, we will show that it is possible for bribing to be the equilibrium, and licensing becomes more likely to occur. We still use backwards induction to analyze this game.

3.4.1 Equilibria

3.4.1.1 Stage 2: Competition

Since bribing may be the possible outcome in stage 1, we need to consider three cases depending on the choice of firm 2 in stage 1: licensing, bribing and no licensing and no bribing. The cases of licensing and no licensing and no bribing are exactly identical to those in the situation where the bureaucrat is non-corruptive. Thus we only need to focus on the case of bribing. If firm 2 has chosen to bribe the bureaucrat by paying the fixed amount b, then firms' profits become

$$\begin{aligned}\pi_1^B &= (1-q_1-q_2)q_1 - \alpha t q_1,\\ \pi_2^B &= (1-q_1-q_2)q_2 - \alpha' t q_2 - b,\end{aligned}$$

where $\alpha' = (1-\sigma)\alpha + \sigma$, and $\alpha < \alpha' < 1$. Note that after bribing, firm 2's effective marginal cost reduces to $\alpha' t$. Then we have

$$q_1^B = \frac{1+(\alpha'-2\alpha)t}{3}, q_2^B = \frac{1+(\alpha-2\alpha')t}{3}$$
$$\pi_1^B = \left(\frac{1+(\alpha'-2\alpha)t}{3}\right)^2, \pi_2^B = \left(\frac{1+(\alpha-2\alpha')t}{3}\right)^2 - b,$$

if $0 \leq t < \frac{1}{2\alpha'-\alpha}$;[13] and

$$q_1^B = \frac{1-\alpha t}{2}, q_2^B = 0$$

[13]Note that $q_1^B = \frac{1+(\alpha'-2\alpha)t}{3} > \frac{1+(\alpha-2\alpha')t}{3} = q_2^B > 0$, because $\alpha' > \alpha$.

$$\pi_1^B = \left(\frac{1-\alpha t}{2}\right)^2, \pi_2^B = 0,$$

if $\frac{1}{2\alpha'-\alpha} < t \leq 1$. Note that for $1/2 < \sigma < 1$, we have $2\alpha' - \alpha > 1$, which implies $\frac{1}{2-\alpha} < \frac{1}{2\alpha'-\alpha} < 1$. Otherwise $\frac{1}{2-\alpha} < 1 < \frac{1}{2\alpha'-\alpha}$, and firm 2 will never exit the market under bribing case, if the probability of being discovered is low.

3.4.1.2 Stage 1: Firm 2's Choice

Firm 2 has three possible choices: purchasing a clean technology from firm 1, bribing the bureaucrat or does nothing. Based these three choices, there are there possible equilibria, the conditions of which are summarized in the following lemma.

Lemma 3.3. *Consider the equilibria in stage 2.*

i) If $\pi_2^B < \pi_2^N$, then licensing is an equilibrium if $\pi_1^L + \pi_2^L \geq \pi_1^N + \pi_2^N$; otherwise no licensing and no bribing is an equilibrium.

ii) If $\pi_2^B \geq \pi_2^N$, then licensing is an equilibrium if $\pi_1^L + \pi_2^L \geq \pi_1^B + \pi_2^B$; otherwise bribing is an equilibrium.

Following Lemma 3.3, we further characterize the conditions under which each type of equilibrium arises in detail. In particular, our discussion will focus on the case $0 < \sigma \leq \frac{1}{2}$, under which firm 2 may exit the market when competing with firm 1 in stage 2, if he neither purchases the technology nor bribes the bureaucrat. In other words, firm 2 will always be active if bribing occurs. The analysis of the case $\frac{1}{2} < \sigma \leq 1$ is similar, and will be presented in the Appendix C.2.2.[14] We consider two cases as below.

Case 1: No licensing and no bribing is better than bribing ($\pi_2^N > \pi_2^B$)

When the bribing cost is too high, bribing will never arise in any equilibrium. Thus there are only two possible equilibria: licensing equilibrium, and no licensing and no bribing equilibrium. Clearly, the equilibrium depends on the tax rate t.

First, when tax rate is low (i.e., $0 \leq t \leq \frac{1}{2-\alpha}$), both firms will be active in stage 2, regardless of the choice of firm 2 in stage 1. In this case, $\pi_2^N > \pi_2^B$ is equivalent to

$$\begin{aligned} b &> \left(\frac{1+(\alpha-2)t}{3}\right)^2 - \left(\frac{1+(\alpha-2\alpha')t}{3}\right)^2 \\ &= \frac{4(1+(\alpha-\alpha'-1)t)(1-\alpha')t}{9} \equiv f(t). \end{aligned}$$

Then we know licensing is an equilibrium if $\pi_1^L + \pi_2^L \geq \pi_1^N + \pi_2^N$, i.e., $0 \leq t \leq \frac{2}{5-3\alpha}$. Note that we have $\frac{2}{5-3\alpha} < \frac{1}{2-\alpha}$. And no licensing and bribing is an equilibrium if $\frac{2}{5-3\alpha} < t \leq \frac{1}{2-\alpha}$.

Second, when the tax rate is high (i.e., $\frac{1}{2-\alpha} < t \leq 1$), firm 2 will exit the

[14]The only difference is that firm 2 may also exit the market if bribing occurs in stage 2, when tax rate is large enough.

market, if he neither purchases the clean technology from firm 1 nor bribes the bureaucrat in stage 1. In this case, $\pi_2^N > \pi_2^B$ is equivalent to

$$b > \left(\frac{1 + (\alpha - 2\alpha')t}{3}\right)^2 \equiv g(t).$$

Furthermore, we can easily check that $\pi_1^L + \pi_2^L < \pi_1^N + \pi_2^N$ always holds, indicating that no licensing and bribing is an equilibrium. Since the joint profit of the two firms is never improved after technology transfer, licensing will not take place in equilibrium. Figure 3.1 summarizes the discussion.

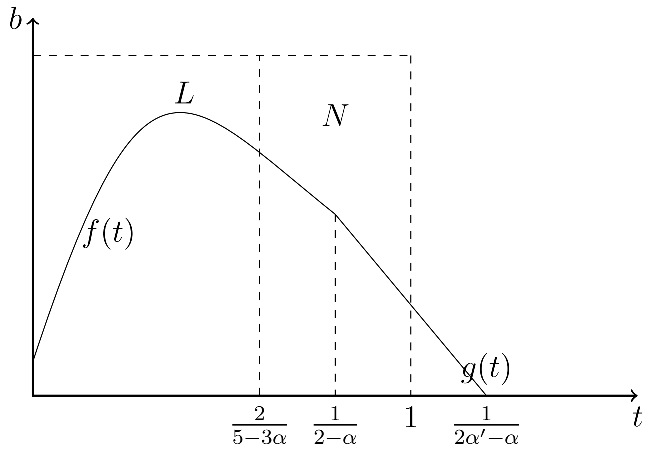

Figure 3.1: Equilibrium Region for Licensing and No

Case 2: Bribing is better than no licensing and no bribing ($\pi_2^B \geq \pi_2^N$)

Following the argument the in previous case, we know that $\pi_2^B \geq \pi_2^N$ is equivalent to $b \leq B(t)$, where

$$B(t) = \begin{cases} f(t), & \text{if } 0 \leq t \leq \frac{1}{2-\alpha}, \\ g(t), & \text{if } \frac{1}{2-\alpha} < t \leq 1. \end{cases}$$

When bribing cost is low, no licensing and no bribing can never be optimal for firm 2. In other words, there are two possible equilibria in stage 1: licensing equilibrium or bribing equilibrium. We know that bribing is better than licensing if and only if $\pi_1^L + \pi_2^L < \pi_1^B + \pi_2^B$, i.e.

$$b < \frac{(2 + (3\alpha - 5\alpha')t)(\alpha - \alpha')t}{9} \equiv h(t).$$

Therefore, under $\pi_2^B \geq \pi_2^N$, bribing is an equilibrium if $b \leq \min\{B(t), h(t)\}$; otherwise, licensing is an equilibrium.

3.4.1.3 Summary

Based on the analysis of the two cases above, we further summarize the equilibrium results in stage 1. The equilibria in stage 1 depend on the shapes of functions $f(t)$, $g(t)$ and $h(t)$. We first present the main properties of these functions, which are useful for the equilibrium analysis below.

Lemma 3.4. *The functions $f(t)$ and $g(t)$ have the following properties:*

1. *Consider $t \in [0, \frac{1}{2-\alpha}]$. Then $f(t) > 0$, $f'(t) > 0$ for $t \in [0, \frac{1}{2(1+\alpha'-\alpha)})$ and $f'(t) < 0$ for $t \in (\frac{1}{2(1+\alpha'-\alpha)}, \frac{1}{2-\alpha}]$.*

2. *Consider $t \in [\frac{1}{2-\alpha}, 1]$. $g(t) > 0$ and $g'(t) < 0$.*

3. *$f(\frac{1}{2-\alpha}) = g(\frac{1}{2-\alpha})$.*

Lemma 3.5. *The function $h(t)$ has the following properties:*

1. *$h(t) < 0$ for $0 < t < \frac{2}{5\alpha' - 3\alpha}$, and $h(t) > 0$ for $t > \frac{2}{5\alpha' - 3\alpha}$.*

2. *$h'(t) < 0$ for $t \in [0, \frac{1}{5\alpha' - 3\alpha})$, and $h'(t) > 0$ for $t > \frac{1}{5\alpha' - 3\alpha}$.*

3. *If $0 < \sigma \leq \frac{2}{5}$, $h(t) < 0$ for $0 < t \leq 1$; if $\frac{2}{5} < \sigma \leq \sqrt{2} - 1$, $h(t) < 0$ for $t < \frac{2}{5\alpha' - 3\alpha} < 1$, and $0 < h(t) < g(t)$ for $\frac{2}{5\alpha' - 3\alpha} < t < 1$; if $\sqrt{2} - 1 < \sigma \leq \frac{1}{2}$, $h(t) < 0$ for $t < \frac{2}{5\alpha' - 3\alpha}$, $0 < h(t) < g(t)$ for $\frac{2}{5\alpha' - 3\alpha} < t < t^*$, and $h(t) > g(t) > 0$ for $t^* < t \leq 1$, where t^* is uniquely defined by $g(t^*) = h(t^*)$ and $\frac{2}{5\alpha' - 3\alpha} < t^* < 1$.*

Note that the shapes of the functions depend on magnitude of σ, which implies that the equilibrium results should also depend on σ. For $0 < \sigma < \frac{1}{2}$, we need to further consider three cases separately: $0 < \sigma \leq \frac{2}{5}$, $\frac{2}{5} < \sigma \leq \sqrt{2} - 1$ and $\sqrt{2} - 1 < \sigma \leq \frac{1}{2}$. The following proposition summarizes the equilibria in the case $\sqrt{2} - 1 < \sigma \leq \frac{1}{2}$.[15]

Proposition 3.3. *Suppose $\sqrt{2} - 1 < \sigma \leq \frac{1}{2}$. Recall that t^* is defined such that $g(t^*) = h(t^*)$, where $\frac{2}{5\alpha' - 3\alpha} < t^* < 1$. The equilibria in stage 1 are:*

1. *If $t \leq \frac{2}{5 - 3\alpha}$, licensing is an equilibrium;*

2. *If $\frac{2}{5 - 3\alpha} < t \leq \frac{1}{2 - \alpha}$, then if $b > f(t)$, no licensing and bribing is an equilibrium; otherwise, licensing is an equilibrium;*

[15] The equilibrium results for the cases $0 < \sigma \leq \frac{2}{5}$ and $\frac{2}{5} < \sigma \leq \sqrt{2} - 1$ are presented in the Appendix C.2.1.

3. *If* $\frac{1}{2-\alpha} < t \leq \frac{2}{5\alpha'-3\alpha}$, *then if* $b > g(t)$, *no licensing and bribing is an equilibrium; otherwise, licensing is an equilibrium;*

4. *If* $\frac{2}{5\alpha'-3\alpha} < t \leq t^*$, *if* $b > g(t)$, *no licensing is an equilibrium; if* $g(t) \leq b < h(t)$, *licensing is an equilibrium; otherwise, bribing is an equilibrium;*

5. *If* $t^* < t \leq 1$, *then if* $b > g(t)$, *no licensing and bribing is an equilibrium; otherwise, bribing is an equilibrium.*

The equilibria are depicted in Figure 3.2. From Figure 3.2, we can see that the tax range which will result in licensing equilibrium has increased from $[0, \frac{2}{5-3\alpha}]$ to possibly $[0, t^*]$.[16] The choice of bribing will lower the licensing fee charged by firm 1. Knowing firm 2 could easily bribe the bureaucrat and hence compete with it in production, firm 1 has higher incentive to induce firm 2 to purchase the technology. When tax rate is relatively higher and cost of bribing is low, it is optimal for firm 2 to choose bribe. In such case, firm 1 could no longer effectively lower the price of technology, since the bribing is too attractive to firm 2. In any other cases, both purchasing the clean technology and bribing the bureaucrat are too costly, and firm 2 keeps using the dirty technology instead.

[16]In Appendix C.2.1, we shall show that licensing could occur when $t = 1$.

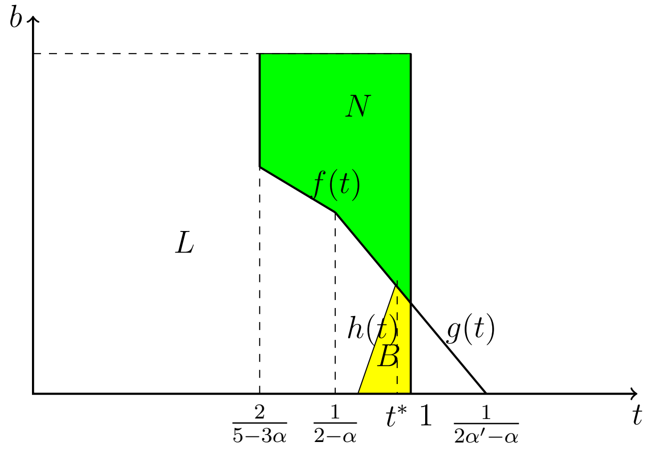

Figure 3.2: Equilibrium Region when $\sqrt{2} - 1 < \sigma < \frac{1}{2}$

3.4.2 Consumer Surplus and Pollution

We further investigate how firm 2's choice affects consumer surplus and total pollution. The following proposition summarizes the result for any given tax rate.

Proposition 3.4. *For any given tax rate, licensing will yield highest consumer surplus, followed by bribing (if possible), and then followed by no licensing and no bribing. When* $0 < t \leq \frac{1}{2}$*, bribing is not possible, emission under licensing is lower than that under no licensing and no bribing. When* $\frac{1}{2} < t \leq 1$*, emission under no licensing and no bribing is lowest. In this case, it is possible that bribing yield lower emission than which under licensing.*

Not surprisingly, licensing will always result in the highest consumer surplus, since the clean technology transfer will reduce firm's production cost and increase production level competition, and hence increase the total output. No licensing and no bribing will result in firm 2 exiting the market, and firm 1 being the monopoly, which leads to the lowest consumer surplus. Whenever bribing is an equilibrium, firm 2 will stay in market. Because of market competition, consumers are better-off than they are in the case where firm 1 is the monopoly.

Recall that from Proposition 3.1 and 3.2, we know that when technology is sufficiently good, licensing will improve consumer surplus, but no licensing will be equilibrium, when $\frac{2}{5-3\alpha} < t \leq \frac{1}{2}$. However, with corruption as a viable option for firm 2, when bribing cost is low, from Figure 3.2, we can see that licensing equilibrium is possible when $\frac{2}{5-3\alpha} < t \leq \frac{1}{2}$. Knowing the ease of bribing, even though the technology is very efficient, firm 1 would like to charge a lower price to ensure the technology transfer.

One possible implication from our model is how corruption may affect the technology transfer within a country. Consider two countries with the same taxation policy (i.e., the tax rate is the same, for example, $t = \frac{1}{2} - \epsilon$). Suppose technology is very efficient in reducing pollution, the country with corruption will result in technology transfer, while the other country without corruption does not. In this sense, corruption promotes the technology transfer.

3.4.3 Optimal Taxation Policy

We have analyzed the choice of firm 2, which depends on the tax rate. Now we proceed to investigate the optimal tax rate that maximizes the social welfare, taking into account the effect of tax rate on firm 2's choice.

We first calculate the welfare in each of the equilibria characterized in stage 1. Note that the welfare functions under the cases where licensing or no licensing and no bribing takes places are exactly identical to those in the case where the bureaucrat is not corruptive. In the case where bribing takes place, under $0 < \sigma < \frac{1}{2}$, we know that firm 2 will always be active in the market.[17] Thus the equilibrium social welfare is denoted by $W^B(t)$, where

$$W^B(t) = \frac{(2-(\alpha+\alpha')t)^2}{9} - \frac{\beta(1+\alpha+(\alpha\alpha'-2\alpha^2+\alpha-2\alpha')t)^2}{9}.$$

Following the equilibrium analysis in stage 1, we know that it is possible for licensing to be an equilibrium for any $0 \leq t \leq 1$. Thus, the largest domain for $W^L(t)$ to be the equilibrium welfare is $T^L = [0,1]$. Similarly, the largest domains for $W^N(t)$, $W^{N'}(t)$ and $W^B(t)$ to be the equilibrium welfare are $T^N = [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$, $T^{N'} = [\frac{1}{2-\alpha}, 1]$ and $T^B = [\frac{2}{5\alpha'-3\alpha}, 1]$ accordingly.

We then proceed by analyzing the optimal taxation policy. Since the social welfare depends on β, which measures whether the country is more output-

[17] For the case $\frac{1}{2} < \sigma < 1$, when firm 2 exit in the market, i.e., $t > \frac{1}{2\alpha'-\alpha}$, the social welfare is the same as the case of no licensing and bribing. We have detailed discussion of this case in Appendix C.2.2.

oriented or environment-oriented. We will consider two cases depending on the values of β similar to the analysis without corruption.

3.4.3.1 $\beta < \frac{1}{\alpha^2}$: output-oriented country

We want to show that the optimal tax rate is again zero. To show this, we aim to argue that none of the other cases can arise in equilibrium.

Lemma 3.6. *If $\beta < \frac{1}{\alpha^2}$, then $W^L(0) > W^{N'}(t)$ for $t \in [\frac{1}{2-\alpha}, 1]$, $W^L(0) > W^N(t)$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$, and $W^L(0) > W^B(t)$ for $t \in [\frac{2}{5\alpha'-3\alpha}, 1]$.*

From Lemma 3.6, we know that if we focus on the largest possible domain for $W^N(t)$, it is always smaller than $W^L(0)$. It suggests that $W^N(t)$ can never be the equilibrium welfare for any possible t.[18] Similar, $W^B(t)$ and $W^{N'}(t)$ can not be the equilibrium welfare either.

Because government puts more weight on consumer surplus, it should set a low tax rate to encourage technology transfer. The cost from pollution will reduce, and hence firms are willing to produce more.

We summarize the equilibrium result in the following Theorem.

Theorem 3.2. *Suppose there exists corruption. When $\beta < \frac{1}{\alpha^2}$, the optimal tax rate is zero and licensing takes place.*

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[18] Based on the proof of lemma 3.6, $W^N(t^*)$ is largest when the domain is largest. For any smaller domains, for example $\frac{1}{2-\alpha} < a < t < b < \frac{2}{5-3\alpha}$, we still have $W^L(0) > W^N(\frac{1}{2-\alpha}) > W^N(a)$, if the optimal $t^* = a$; and $W^L(0) > W^N(\frac{2}{5-3\alpha}) > W^N(b)$, if the optimal $t^* = b$.

Similar to the case without corruption, government intervention is efficient, when government's emphasis is on consumer surplus. He will set a zero tax to encourage the technology transfer, and production will be increased. In this case, the fixed fee charged by firm 1 for the clean technology is zero, since pollution is costless. In fact, it is more realistic to consider the case, where government charges a low tax $t = \epsilon_t > 0$, and firm 1 charges a low fee for the technology $F = \epsilon_F > 0$. Our result still holds that firm 2 will purchase the technology.[19] In fact, from figure 3.2, we can see that as long as $0 < t < \frac{2}{5-3\alpha}$, licensing will always be the equilibrium, regardless of bribing cost. It is socially optimal to charge $t = 0$, but in reality, to ensure the technology transfer, the government could set any tax in that range. Note also, according to Proposition 3.4, for any $0 < t < \frac{2}{5-3\alpha} < \frac{1}{2-\alpha}$, licensing will always yield highest consumer surplus and lowest pollution, comparing with bribing and no licensing and no bribing.

Due to the presence of competition, firm 2 will always purchase the technology to increase his competitive advantage in the production stage. Due to the low cost of the clean technology, there is no incentive for firm 2 to bribe the bureaucrat, even when the probability of being caught is low.

[19] Because of the continuity of the welfare functions.

3.4.3.2 $\beta > \frac{1}{\alpha^2}$: environment-oriented country

Before we provide general conditions for various equilibria outcomes in the following theorem, define $\tilde{t} = h^{-1}(b)$, and consider $\frac{2}{5\alpha' - 3\alpha} < \tilde{t} < 1$.[20] The following theorem summarizes the equilibrium results.

Theorem 3.3. *Suppose there exists corruption. When $\beta > \frac{1}{\alpha^2}$, we have*

1. *When $0 < \sigma \leq \frac{2}{5}$, the optimal tax rate 1. If $b > g(1)$, no licensing and no bribing will occur in equilibrium; otherwise, licensing will occur in equilibrium.*

2. *When $\frac{2}{5} < \sigma \leq \sqrt{2} - 1$, if $b > g(1)$, the optimal tax rate is 1 and no licensing and no bribing occurs in equilibrium; if $h(1) < b \leq g(1)$, the optimal tax rate is 1 and licensing occurs in equilibrium; if $b \leq h(1)$, and $W^L(\tilde{t}) > W^B(1)$, the optimal tax rate is $\tilde{t}$, and licensing occurs in equilibrium ; otherwise, the optimal tax rate is 1 and bribing occurs in equilibrium.*

3. *When $\sqrt{2} - 1 < \sigma \leq \frac{1}{2}$, if $b > g(1)$, the optimal tax rate is 1 and no licensing and no bribing occurs in equilibrium; if $b \leq g(1)$, and $W^L(\tilde{t}) > W^B(1)$, the optimal tax rate is $\tilde{t}$ and licensing occurs in equilibrium; otherwise, the optimal tax rate is 1 and bribing occurs in equilibrium.*

[20] $b > 0$ in such case.

On the one hand, Theorem 3 states that when firm 1 observes the bribing cost is high, he will set a high fixed fee for the clean technology, yielding the no licensing and no bribing equilibrium, and resulting firm 2 exit the market. In this case, government intervention on charging tax rate at one, and hence protecting the environment, is still effective. On the other hand, when firm 1 observes that the bribing cost is low, and bribing is easily a success, he will charge a low price for the technology, trying to induce higher incentive for firm 2 to purchase the technology. In this case, the government intervention fails. The result shows that bribing will make government regulation ineffective in shutting down the heavily polluting firms.

As the anti-corruptive effort increases, i.e. $\frac{2}{5} < \sigma \leq \frac{1}{2}$, and the bribing cost becomes lower, there will be two conflicting effects on both firm 1 and firm 2's decisions. On the one hand, it becomes more risky for firm 2 to bribe the bureaucrat. In such case, firm 1 will start raising the price of the technology. On the other hand, firm 2 has higher incentive to bribe due to the low bribing cost, and hence firm 1 wants to charge a lower price on the technology. Depending on government tax rate, it is possible for licensing or bribing to be the equilibrium outcome.

We present a sufficient condition for bribing or licensing to be the equilibrium in the propositions below.[21]

Proposition 3.5. *When $\frac{2}{5} < \sigma \leq \frac{1}{2}$, if (1) the bribing cost is low, (2) the clean technology is not efficient, and (3) government is even more environment-oriented, then government should set tax rate at one, and bribing will be the equilibrium.*

Proposition 3.6. *When $\frac{2}{5} < \sigma \leq \frac{1}{2}$, the bribing cost is low and the clean technology is very efficient, government should charge $t = \tilde{t}$, and licensing will be the equilibrium.*

When the government is environment-oriented, there is incentive for firm 2 to bribe the bureaucrat or purchase the clean technology in order for it to stay in market. When the clean technology is not effective in reducing pollution, and bribing cost is low, firm 2 will choose to bribe the bureaucrat instead of purchasing the technology. In this scenario, it is socially optimal to induce bribing equilibrium rather than licensing equilibrium.[22] Comparing with licensing at $t = \tilde{t}$, bribing at $t = 1$ will lower the output and hence pollution. The lower pollution effect dominates the lower output effect, due to the environment-oriented policy, i.e. higher value of β.

[21] A rigorous presentation of this proposition can be found in Appendix C.1.

[22] In Proposition 3.4, we have shown that it is possible $E^B(1) < E^L(1)$. Since emission under licensing is decreasing in t, we must have $E^L(\tilde{t}) > E^B(1)$, i.e. the equilibrium pollution under bribing at $t = 1$ is lower.

Knowing the bribing cost is low, either due to lack of anti-corruptive policy or lower wage for the bureaucrat, the lowest pollution level under no licensing and no bribing could never be reached. Government has two taxation mechanisms to reach the highest possible social welfare. On the one hand, if the clean technology is inefficient in reducing the pollution, government may want to charge tax rate at one, inducing bribing to be the equilibrium outcome. In this scenario, firms reduce pollution through fewer production. This effect dominates the effect from lowering consumer surplus. On the other hand, if the technology is very efficient, government should charge a high tax rate, $\tilde{t} \neq 1$, incentivizing firm 2 to purchase the technology. In this scenario, the effect of raising consumer surplus dominates the effect of higher pollution. In conclusion, even though government regulation at $t = 1$ fails to achieve the "first best" under corruption, bribing is still possible to be the "second-best" in reducing pollution and maximizing social welfare.

In the Appendix C.2.2, we have also shown that when $\frac{1}{2} < \sigma \leq 1$, i.e. government puts a lot of efforts on anti-corruptive policy, bribing will no long be the equilibrium when $\beta > \frac{1}{\alpha^2}$. In this case, it is optimal for government to set tax rate at one, and firm 2 will always choose no licensing and no bribing. When $\beta < \frac{1}{\alpha^2}$, government should still set tax rate at zero and licensing will always be the equilibrium. In this way, government intervention will become effective, when the anti-corruption policy is strong.

Higher wage is clearly another efficient way to reduce corruption, since it makes bribing more costly. As we can infer from Theorem 3.3, higher bribing cost may induce firm 1 to charge higher price on the clean technology. In such case, firm 2 will exit the market, and the environment-oriented government could achieve the no licensing and no bribing equilibrium. However, consider the non-optimal tax rate, i.e. $\frac{2}{5-3\alpha} < t \leq \frac{1}{2}$, as we have interpreted before, efficient clean technology will not be transferred. Therefore, higher wage may hinder the technology transfer, which may result in lower consumer surplus and higher pollution.[23]

In conclusion, we have shown that government intervention on shutting down the heavily polluting firms fails when the bribing cost is low, as both licensing or bribing could occur when government is environment-oriented. But bribing may be socially optimal under optimal taxation. Corruption could increase the likelihood of technology transfer.

3.5 Discussion: Outsider Innovator

It is natural to consider insider innovator and outsider innovator in licensing literature. We have discussed firm 1 to be the licensor in the main text. In

[23]Note that, as we have shown, when $\frac{2}{5-3\alpha} < t \leq \frac{1}{2}$, licensing yields higher social welfare than no licensing. But firm will always choose not to license, since it is more profitable.

this discussion, we assume that the technology is owned by domestic research lab, with a fixed price F, which is exogenous. So there will no strategic licensing effect in this model. Consider there is a monopoly producer in the market. Without competition, we just need to consider the three options for the monopoly, i.e. purchase the technology, bribing the bureaucrat and neither purchase nor bribe, and then consider the optimal tax rate from government point of view.

The detailed analysis are in Appendix C.2.4. Most of the analysis are the same to the insider innovator case. When the government is environment-oriented, it is possible that bribing is in the equilibrium. Therefore, production level competition and strategic licensing are not necessary in generating corruption. One important difference between outsider and insider innovator case is that when government is output-oriented, he can never reach the purchasing equilibrium by setting the tax rate at zero. Due to the uncompetitive nature of monopoly, if there is no pollution cost, there will be no incentive for the monopoly to bribe the bureaucrat, nor to purchase the technology. It is necessary for the government to raise the pollution cost, by setting some positive tax rate,to incentivize the transfer of the clean technology.

Clearly, given any positive tax rate, purchasing the technology will yield highest output level and hence consumer surplus. However, note also at a particular tax rate the emission under purchasing will also be higher than which under no purchasing. Therefore, government needs to balance the effect of raising consumer surplus and the effect of raising pollution.

3.6 Conclusion

When setting the optimal tax rate, the government always takes into account the trade-off of tax rate on production and pollution. Setting a higher tax rate has a negative effect on production and a positive effect on reducing production. When government weighs more on consumer surplus, it is optimal to charge lower tax rate in order to encourage technology transfer, and hence increasing the total production level. When government puts more emphasis on environment protection, it will always charge a higher tax rate, aiming to shut down the heavily polluting firm 2. However, in the presence of corruption, government regulation may fail. When bribing cost is low, firm 2 tries to purchase license or bribe the bureaucrat, in order to stay competitive in market. When technology is not efficient in reducing pollution, technology transfer may become an inferior option for firm 2, and bribing will be the equilibrium.

Comparing two countries with different environmental policy, where government 1 is more output-oriented, while government 2 is more environment-oriented. Our results imply that technology transfer is more likely to occur in country 1; while bureaucrat in country 2 are likely to be corrupted. If the two countries are both environment-oriented, at the same tax rate, the country with corruption may have lower social welfare and higher pollution. However, it is also possible that licensing occur in the country with corruption, while no licensing and no bribing occur in the other without corruption. In such case, corruption leads to technology transfer, and hence higher consumer surplus and lower pollution.

If the main goal of the government is to avoid or mitigate corruption, our policy implication for the government is to set a low tax rate or offering a high wage. On the one hand, when the tax rate is low, pollution does not lower firms' profits significantly and there is less incentive for the firms to bribe the bureaucrat. On the other hand, a higher wage will decrease the bureaucrat's willingness to accept the bribe.

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Appendix A

Proofs and Details of Chapter One

A.1 Proofs

For all our proofs, we are ignoring the term $a - c$, as it is constant and has no effect on the results.

A.1.1 Proof of Theorem 1.1

(i) For technology improvement, we need to show $Kx_1^{no} > x_i^{ind}$. Equivalently, we need to show $f(\alpha, K) = D^{ind}D^{no}(Kx_1^{no} - x_i^{ind}) > 0$. Note that $f(\alpha, K)$ is the numerator of $Kx_1^{no} - x_i^{ind}$. We will first show that f is increasing in α. Then, by the all firm participating condition $\alpha > \frac{K(N-K+1)}{N+1}$, it suffices to show that $h(K; N) \equiv f(\frac{K(N-K+1)}{N+1}, K)/[K(N+1)(N-K+1)] > 0$ for all $2 \leq K \leq N$.

Step 1: We need to show f is increasing in α. Let $X \equiv \alpha(N+1)^2$, and define

$g(X,K) \equiv f(\alpha, K)$. Then we have:

$$\begin{aligned} g(X,K) =& K(N-K+1)[X^2 - N(N+1)X + N^2(N+1)] - N[X^2 \\ & - [N(K+1) + K(N-K+1)^2]X + (N-K+1)NK(N+1)] \\ =& [K(N-K+1) - N]X^2 \\ & + [N^2(K+1) + NK(N-K+1)^2 - NK(N+2)(N-K+1)]X. \\ \frac{\partial g(X,k)}{\partial X} =& 2[K(N-K+1) - N]X \\ & + [N^2(K+1) + NK(N-K+1)^2 - NK(N+2)(N-K+1)] \end{aligned}$$

We have $\frac{\partial g(X,K)}{\partial X} > 0$ if and only if $X > \frac{N(K+1)}{2}$ or $\alpha > \frac{N(K+1)}{2(N+1)^2}$, which is guaranteed by the all-firm participating condition because $\alpha > \frac{K(N-K+1)}{N+1} > \frac{N(K+1)}{2(N+1)^2}$. Hence, f is increasing in α.

Step 2: Since f is increasing in α, it suffices to show that $f(\frac{K(N-K+1)}{N+1}, K) > 0$. Define $h(K;N) \equiv f(\frac{K(N-K+1)}{N+1}, K)/[K(N+1)(N-K+1)]$. Then we have

$$\begin{aligned} h(K;N) =& K^2(N+1)(N-K+1)^2 - N(N+2)K(N-K+1) \\ & - Nk(N+1)(N-K+1) + N[N(K+1) + K(N-K+1)^2]. \end{aligned}$$

It suffices to show $h(K;N) > 0$, for all $2 < K < N$. Note that $h(2;N) = (N-2)^2 + \frac{3N^2-12N+12}{2(N-1)} > 0$, and $h(N;N) = 0$ for all $N > 2$. It is easy to check that $h(2;3) > 0$. We just need to show $h'(K;N) = 0$ has only one root, when $N \geq 4$.

$$h'(K;N) = 2K(N+1)(N-K+1)^2$$

$$
\begin{aligned}
&- 2(N-K+1)K^2(N+1) - 2KN(N-K+1) \\
&+ NK^2 - N(N+2)(N-K+1) + N(N+2)K + N^2.
\end{aligned}
$$

Note that $h'(2;N) = (3N^2-2N-4)(N-4)+(6N-4) > 0$ and $h'(N;N) = 0$. Now

$$
\begin{aligned}
h''(K;N) =&(N+1)[2(N-K+1)^2 - 8K(N-K+1) + 2K^2] \\
&- 2N(N-K+1) + 4KN + 2N(N+2).
\end{aligned}
$$

and $h''(N;N) = 2(N+1)(N^2-4N+1)+6N^2+2N > 0$. Hence, $h'(N-\varepsilon;N) < 0$ and $h'(N+\varepsilon;N) > 0$ for some $\varepsilon > 0$. Since $h'(K;N)$ is a 3rd degree polynomial, there exists only one $k^* \in (2,N)$ such that $h'(K^*;N) = 0$.

(ii) For consumer surplus, It suffices to show $\sum q_i^{no} > \sum q_i^{ind}$. Let $f(\alpha,K) := \sum q_i^{no} - \sum q_i^{ind}$. Define $g(\alpha,K) \equiv\ D^{ind}D^{no}f(\alpha,K)$, i.e. $g(\alpha)$ is the numerator of $f(\alpha,K)$, where

$$
g(\alpha,K) = \alpha K(K-1)(N-K)(N+1)(\alpha(N+1)-N).
$$

Clearly g is increasing in α. By the all firm participating condition $\alpha > \frac{K(N-K+1)}{N+1}$, it suffices to show $g(\frac{K(N-K+1)}{N+1},K) > 0$. Now that $g(\frac{K(N-K+1)}{N+1}) = K^2(K-1)^2(N-K+1)(N-K)^2 > 0$, which completes the proof.

A.1.2 Propositions 1.2 and 1.3

Before proving Propositions 1.2 and 1.3, it is convenient to characterize the equilibrium first.

A.1.2.1 Equilibrium of Ex-ante Licensing

We will consider two cases: (1) $L < N - K$ and (2) $L = N - K$.

Case (1): Using backward induction, we can solve:

$$x_i^{Ex-ante} = \frac{((\alpha-1)N+\alpha)(N+1-K-L)}{D^{Ex-ante}}(a-c) \text{ for all } i \in K,$$
$$x_j^{Ex-ante} = 0 \text{ for all } j \in L,$$
$$x_m^{Ex-ante} = \frac{N(\alpha(N+1)-K(N+1-K-L))}{D^{Ex-ante}}(a-c) \text{ for all } m \in N\backslash(L \cup K),$$

and

$$q_i^{Ex-ante} = \frac{\alpha(N+1)((\alpha-1)N+\alpha)}{D^{Ex-ante}}(a-c) \text{ for all } i \in K \cup L,$$
$$q_m^{Ex-ante} = \frac{\alpha(N+1)(\alpha(N+1)-K(N+1-K-L))}{D^{Ex-ante}}(a-c)$$
$$\text{for all } m \in N\backslash(L \cup K),$$

where $D^{Ex-ante} = \alpha^2(N+1)^3 - \alpha(N+1)(K^3 - 2K^2(N-L+1) + K(N^2 + N(3-2L) + (L-1)^2) + NL + N) - KN(K-N+L-1)$.

In the first stage, non-RJV firms will compete in auction, so that they will be indifferent between winning or losing the bid, i.e. licensees and non-licensees will end up with the same level of profits. Therefore, the winning bid will be

$$B = (q_j^{Ex-ante})^2 - ((q_m^{Ex-ante})^2 - \alpha x_m^2) = (q_i^{Ex-ante})^2 - \pi_m^{Ex-ante}.$$

Check the "all-firm participating" condition, i.e. $q_m^{Ex-ante} \geq 0$ which holds if and only if $\alpha \geq \frac{K(N+1-K-L)}{N+1}$. This is guaranteed by Equation (3), since $\alpha \geq \frac{K(N-K+1)}{N+1} \geq \frac{K(N+1-K-L)}{N+1}$.

Case (2): Let the minimum bid be $\underline{B}$, and consider firm $j \in N \backslash K$ to be the representative from the non-RJV firms. If firm j rejects $\underline{B}$, and decides to do R&D by himself, we come to the case where the number of licensees become $(n-k-1)$, which is an interior case. By the condition $L \leq (N-K)(K-1)/K$, we then require

$$2K \geq N.$$

Therefore, ex-ante licensing to all non-RJV firms is possible if and only if $2K \geq N$. This condition requires the size of RJV to be large. One point worth noting is that licensing to all non-RJV firms will never improve consumer surplus comparing with no licensing case. The above inequality clearly violates the condition (the RJV size is small) in Proposition 1.3.

Define $\pi_j^{Ex-ante}(\alpha, N, K, N-K-1)$ be the firm j's profit in this case, where $N-K-1$ is the number of licensees.

If all non-RJV firms, i.e. firms belong to $N \backslash K$, accept the offer, $\underline{B}$, then firms will face the following maximization problem

$$\max_{x_i} \pi_i^{(C)Ex-ante} = (p - c + \sum_{i \in K} x_i) q_i - \alpha x_i^2 + \frac{N-K}{K} \underline{B} \text{ for all } i \in K,$$

$$\max_{x_j} \pi_j^{(C)Ex-ante} = (p - c + \sum_{i \in K} x_i) q_j - \alpha x_j^2 - \underline{B} \text{ for all } j \in L.$$

Backward induction implies that

$$x_i^{(C)Ex-ante} = \frac{1}{\alpha(N+1)^2 - K}(a-c) \text{ for all } i \in K,$$
$$q_j^{(C)Ex-ante} = \frac{\alpha(N+1)}{\alpha(N+1)^2 - K}(a-c) \text{ for all } j \in N,$$

$x_j^{(C)Ex-ante} = 0$ for all $j \in L$, and $\underline{B} = (q_i^{(C)Ex-ante})^2 - \pi_j^{Ex-ante}(\alpha, N, K, N - K - 1)$.

Now we are ready to proof proposition 1.2.

A.1.2.2 Proof of Proposition 1.2

Proof. We will consider two cases: (1) $L < N - K$ and (2) $L = N - K$.

Case (1): Let $f(L) = x_1^{Ex-ante}$. Since $f(0) = x_1^{no}$, it suffices to show

$$f'(L) = \frac{Q}{(D^{no})^2} < 0.$$

where $Q = -\alpha(N+1)((\alpha-1)N+\alpha)(\alpha(N+1)^2 + K^3 - 2K^2(N-L+1) + K(N-L+1)^2 - N(N+2))$. We have $f'(L) = 0$ if and only if $K^3 + \alpha(N+1)^2 - N(N+2) - 2K^2(N-L+1) + K(N-L+1)^2 = 0$. Solving the equation, we have

$$L^* = \frac{K - K^2 + KN \pm \sqrt{-\alpha K + 2KN(1-\alpha) + KN^2(1-\alpha)}}{K}.$$

Clearly the determinant is smaller than 0, because $\alpha > \frac{K(N-K+1)}{N+1}$. Hence $f'(L)$ has no real root. It is easy to see that the numerator of $f'(L)$ is a concave quadratic function of L. Therefore $f'(L) < 0$.

Case (2): We need to show $x_i^{no} - x_i^{(C)Ex-ante} > 0$ for all $i \in K$.

$$
\begin{aligned}
& x_i^{no} - x_i^{(C)Ex-ante} \\
=& \frac{\alpha(N+1)^2(N-K)(\alpha(N+1)^2 - K^2 + K(N+1) - N(N+2))}{(\alpha(N+1)^2 - K)D^{no}}.
\end{aligned}
$$

Let $f(\alpha) \equiv \alpha(N+1)^2 - K^2 + K(N+1) - N(N+2)$. It suffices for us to show $f(\alpha) > 0$. Clearly, $f(\alpha)$ is increasing in α. Take $\alpha = \frac{K(N-K+1)}{N+1}$. Define $g(K) \equiv f(\frac{K(N-K+1)}{N+1})$. Then after simplification, we have $g(K) = (N-K)(K-1)(N+2) > 0$. Therefore $f(\alpha) > 0$ for all $\alpha \geq \frac{K(N-K+1)}{N+1}$.

A.1.2.3 Proof of Proposition 1.3

We first restate Proposition 1.3 with technical details. Let $\alpha^*(N,K) \equiv \frac{K(N-K)(K-1)^2+K^3N+N^2K(1-K)}{(N+1)(N-K+K(2K-N))}$.

Proposition 3. *Unless* $\alpha \geq \max\{K, \alpha^*(N,K)\}$, $N > \frac{K(2K-1)}{K-1}$, *and* $L < \min\{L^*, N-K\}$. *, consumer surplus in no licensing case is always higher than that in ex-ante case.*

We will consider two cases: (1) $L < N-K$ and (2) $L = N-K$.

Case (1) Let $g(L) = \sum_{i\in N} q_i^{ex-ante}$. We have $g(0) = \sum q_i^{no}$ and

$$g'(L) = \frac{-a(N+1)((a-1)N+a)Y}{(D^{ex-ante})^2},$$

where $Y(L) \equiv a(N+1)(K(2K-N+2L-1)+N) + K(K^3 - 2K^2(N-L+1) + K(N-L+1)^2 - N(N+1))$. Now $g'(L) = 0$ if and only if $Y(L) = 0$. Let L_1 and L_2 be the roots of $Y(L) = 0$ such that

$$\begin{aligned}
L_1 =& \frac{1}{K^2}[-\alpha K + K^2 - K^3 - \alpha KN + K^2 N \\
& - \sqrt{(\alpha - K)K^2(N+1)(\alpha(N+1)-N)}] \\
L_2 =& \frac{1}{K^2}[-\alpha K + K^2 - K^3 - \alpha KN + K^2 N \\
& + \sqrt{(\alpha - K)K^2(N+1)(\alpha(N+1)-N)}].
\end{aligned}$$

Clearly, when $\alpha < K$, there is no real root to $g'(L) = 0$. It is easy to check the numerator of $g'(L)$ has a negative coefficient on the L^2 term. Therefore, no real root implies $g'(L) < 0$. And hence, $g(L) < g(0)$.

When $\alpha \geq K$, it is always $L_1 < 0$. Note that $L_2 < 0$ if and only if $N \leq \frac{K(2K-1)}{K-1}$, or $N > \frac{K(2K-1)}{K-1}$ with $\alpha < \alpha^*(N,K)$ where $\alpha^*(N,K) = \frac{-K^2+2K^3-K^4+KN-2K^2N+2K^3N+KN^2-K^2N^2}{-K+2K^2+N-2KN+2K^2N+N^2-KN^2}$. There are two subcases: (a) $L_2 \leq 0$, and (b) $L_2 > 0$. Subcase (a). We have $g(L) < g(0)$. Subcase (b). If $L_2 < N-K$, there exists $L^* \in [L_2, N-K)$ such that $g(L^*) = g(0)$ and $g(L) > g(0)$ for all $L \in [0, L^*]$. If $L_2 \geq N-K$, $g(L) > g(0)$ for all $L \in [0, N-K)$.

Therefore, ex-ante licensing improves consumer surplus if and only if $\alpha \geq \max\{K, \alpha^*(N,K)\}$, $N > \frac{K(2K-1)}{K-1}$ and $L < \min\{L^*, N-K\}$.

Case (2). We need to check $\sum q_i^{no} > \sum q_i^{(C)Ex-ante}$.

$$\begin{aligned}
&\sum q_i^{no} - \sum q_i^{(C)Ex-ante}\\
=&\frac{\alpha(N+1)(N-K)(-K(K^2-K(N+1)+N^2+N)+\alpha(N+1)((K-1)K+N))}{(\alpha(N+1)^2-K)D^{no}}.
\end{aligned}$$

Similar to the proof above, we just need $f(\alpha) \equiv -K(K^2-K(N+1)+N^2+N)+\alpha(N+1)((K-1)K+N) > 0$. Again, $f(\alpha)$ is increasing in α, and let $g(K) \equiv f(\frac{K(N-K+1)}{N+1}) = N^2(N-K)(K-1) > 0$. Therefore $f(\alpha) > 0$ for all $\alpha \geq \frac{K(N-K+1)}{N+1}$. □

A.1.3 Proposition 1.4

Before the proof of Proposition 1.4, we want to first state the equilibrium of ex-post licensing.

A.1.3.1 Equilibrium of ex-post licensing

We will consider two cases: (1) $L < N-K$ and (2) $L = N-K$.

Case (1): By backward induction, we have

$$\begin{aligned}
x_i^{Ex-post} &= \frac{(K+L)((\alpha(N+1)-N)(N-K+1)-NL)}{KD^{Ex-post}}(a-c) \text{ for all } i \in K,\\
x_j^{Ex-post} &= \frac{N(\alpha(N+1)+(K+L)(K-N+L-1))}{D^{Ex-post}}(a-c) \text{ for all } j \in N\backslash K,\\
q_i^{Ex-poat} &= \frac{\alpha(N+1)(\alpha(N+1)+L(K+L)-N)}{D^{Ex-post}}(a-c) \text{ for all } i \in K,
\end{aligned}$$

$$q_j^{Ex-post} = \frac{\alpha(N+1)(\alpha(N+1)+(K+L)(K-N+L-1))}{D^{Ex-post}}(a-c)$$

for all $j \in N \backslash K$,

where $D^{Ex-post} = -(-\alpha^2(N+1)^3+\alpha(N+1)(K^3-2K^2(N-L+1)+K(-4(N+1)L+(N+1)^2+L^2)-N(-K+N+2(L-1)L-L)+N^2(L+1)+N-2L^2+L)+N(K+L)(K-N+L-1))$. To ensure $q_j^{Ex-post} \geq 0$, the necessary and sufficient condition is

$$\alpha \geq \frac{N(K+N(K-1))}{K^2(N+1)}.$$

The condition $\alpha \geq K(N-K+1)/(N+1)$ is not sufficient for ex-post case, as we can check

$$\frac{N(K+N(K-1))}{K^2(N+1)} \leq \frac{K(N-K+1)}{N+1}$$

which is equivalent to $N \leq K^2$.

Case (2): Consider $j \in N \backslash K$. If he rejects the minimum bid $\underline{B}$, his profit will be

$$\pi_j^{Reject} := (p-c+x_j)q_j^{Reject} - \alpha x_j^2,$$

where $q_j^{Reject} = \frac{a-c+Nx_j-(N-1)\sum_{k\in K} x_k}{N+1}(a-c)$ in equilibrium. If he accepts $\underline{B}$, his profit will be

$$\pi_j^{Accept} := (p-(c-\sum_{k\in K} x_k))q_j^{Accept} - \alpha x_j^2 - \underline{B}.$$

Therefore $\underline{B}(x_1, ..., x_N) = (p - (c - \sum_{k\in K} x_k))q_j^{Accept} - (p - c + x_j)q_j^{Reject}$. Consider the following firms' maximization problem:[1]

$$\max_{x_i} \pi_i^{(C)Ex-post} = (p - (c - \sum_{k\in K} x_k))q_i - \alpha x_i^2 + \frac{N-K}{K}\underline{B}(x_1, ..., x_N) \text{ for all } i \in K,$$
$$\max_{x_j} \pi_j^{(C)Ex-ante} = (p - (c - \sum_{k\in K} x_k))q_j^{Accept} - \alpha x_j^2 - \underline{B}(x_1, ..., x_N) \text{ for all } j \in N\backslash K.$$

Solving for the equilibrium, we have

$$x_i^{(C)Ex-post} = \frac{(-N^3 + \alpha(N+1)^2(-KN + K + N^2))}{kD^{(C)Ex-post}}(a - c) \text{ for all } i \in K,$$
$$x_j^{(C)Ex-post} = \frac{N(\alpha(N+1)^2 - N^2)}{D^{(C)Ex-post}}(a - c) \text{ for all } j \in N\backslash K,$$
$$q_i^{(C)Ex-post} = \frac{\alpha(N+1)(\alpha(N+1)^2 + N(-(K+2)N + K + N^2)}{D^{(C)Ex-post}} \text{ for all } i \in N,$$

where $D^{(C)Ex-post} = (\alpha^2(N+1)^4 + \alpha(N+1)^2((N-3)N^2 - K(N-1)^2) + N^3)$.

Note that in equilibrium, $q_j^{Accept} = q_i^{(C)Ex-post}$, since all the firms are producing under the same technology level. The production quantity should be the same for all firms. By the condition $\alpha \geq K(N-K+1)/(N+1)$, $x_j^{(C)Ex-post} > 0$ for all $j \in N\backslash K$. It is easy to show that $q_i^{(C)Ex-post} > 0$ for all $i \in N$.

A.1.3.2 Proof of Proposition 1.4

We first restate Proposition 1.4 with technical details. Let $f(\alpha) = \alpha^2(N+1)^4 - \alpha(N+1)^2(K(-KN + K + 2N^2 - 1) + 2N) + N^2(K(N^2 + N - 1) + 1)$, and $g(\alpha) = \alpha^2(N+1)^3(K + N - 1) - \alpha(N+1)(K^2(-(N-1)) + K(N(N+$

[1] The superscript "(C)" refers to a corner solution in the number of licensees that $L = N - K$.

$1)^2-1)+(N-1)N^2)+KN^4$. Denote $\alpha^{(C)*}(N,K)$ and $\alpha^{(C)**}(N,K)$ be the larger root of $f(\alpha)=0$ and $g(\alpha)=0$.

Proposition 1.4. An ex-post licensing leads to (a) a higher investment level under if $\alpha > \max\{\alpha^{**}(N,K,L), \alpha^{(C)*}(N,K)\}$, and (b) a higher consumer surplus if $\alpha > \max\{\alpha^{(C)**}(N,K), \frac{K(K+L)((N-K)(N+1)+1)}{(N+1)((N+1)p+K(2N-L+2)-N-2K^2)}\}$

Proof. (a) We first show higher investment level under $\alpha > \max\{\alpha^{**}(N,K,L), \alpha^{(C)*}(N,K)\}$. We will consider two cases: (1) $L < N-K$ and (2) $L = N-K$. Case (1) The numerator of $x_i^{ex-post} - x_i^{no}$ for $i \in K$ is in the form of

$$\alpha(N+1)^2 L(a\alpha^2 + b\alpha + c),$$

where $a=(N-K+1)(N+1)^3$, $b=(1+N)(K^4+K^2(1+N)(2+2N-3L)+N(1+N)(2+N-L)+K^3(-3-3N+L)+K(-N(3+2N)+2(1+N)^2L))$ and $c=N(-2KN-K^3(2+N)+N(1+N-L)+K(2+N(2+N))L+K^2(2+2N+N^2-(2+N)L))$.

Clearly $a>0$. Therefore, $x_1^{ex-post} - x_1^{no} > 0$, if $\alpha > \alpha^{**}(N,K,L)$, where $\alpha^{**}(N,K,L)$ is the larger root of the quadratic equation.

Case (2): for firm $i \in K$, we have

$$x_i^{(C)Ex-post} - x_i^{no} = \frac{\alpha(N+1)^2(N-K)^2 f(\alpha)}{D^{no}D^{(C)Ex-post}}$$

where $f(\alpha) \equiv \alpha^2(N+1)^4 - \alpha(N+1)^2(K(-KN+K+2N^2-1)+2N) + N^2(K(N^2+N-1)+1)$. Note $f(\alpha)$ is a quadratic function in α with positive coefficient of α^2. Let $\alpha^{(C)*}(N, K)$ be larger root of $f(\alpha) = 0$. Therefore, when $\alpha > \alpha^{(C)*}(N, K)$, $x_1^{(C)Ex-post} > x_1^{no}$.

(b) Then we show that higher consumer surplus if $\alpha > \max\{\frac{K(K+L)((N-K)(N+1)+1)}{(N+1)((N+1)p+K(2N-L+2)-N-2K^2)}, \alpha^{(C)**}(N, K)\}$.

We will consider two cases: (1) $L < N - K$ and (2) $L = N - K$.

Case (1): Let $h(\alpha) = D^{ex-post}D^{no}(\sum q_i^{ex-post} - \sum q_i^{no})$, which is the numerator of the quantity difference. We have

$$\begin{aligned} h(\alpha) = &- \alpha(N+1)L((\alpha-1)N+\alpha)(\alpha(N+1)(2K^2+K(-2N+L-2) \\ &- (N+1)L+N) - K(K(N+1)-N(N+1)-1)(K+L)). \end{aligned}$$

Note that $h(\alpha) > 0$ if and only if $\alpha > \frac{K(K+L)((N-K)(N+1)+1)}{(N+1)((N+1)L+K(2N-L+2)-N-2K^2)} > 0$ where the last inequality holds because the denominator $(N+1)L+K(2N-L+2)-N-2K^2$ is increasing in L, and it becomes $(N-K+1)(2K-1)-(K-1) > 0$ when $L = 0$.

Case (2): We have

$$\sum q_i^{(C)Ex-post} - \sum q_i^{no} = \frac{\alpha(N+1)(K-N)^2 f(\alpha)}{D^{no}D^{(C)Ex-post}}$$

where $g(\alpha) = \alpha^2(N+1)^3(K+N-1) - \alpha(N+1)(K^2(-(N-1)) + K(N(N+1)^2 - 1) + (N-1)N^2) + KN^4$. Note $g(\alpha)$ is a quadratic function. Let $\alpha^{(C)**}(N, K)$ be larger root of $f(\alpha) = 0$. Therefore, when $\alpha > \alpha^{(C)**}(N, K)$, $\sum q_i^{(C)Ex-post} > \sum q_i^{no}$. $\square$

A.1.4 Proof of Proposition 1.5

Let $\pi(K)$ be the profit of a RJV firm in a RJV with K firms. We have

$$\pi(K) = \frac{\alpha\left(\alpha(N+1)-N\right)^2\left(\alpha(N+1)^2-(N-K+1)^2\right)}{D}$$

where $D = \left(\left(\alpha(N+1)^2 - N(K+1)\right)\left(\alpha(N+1)^2 - K(N-K+1)^2\right) - (N-K)(N-K+1)NK^2\right)^2$. When $K=2$, we have $\pi(2)-\pi(1) = \alpha((\alpha+(-1+\alpha)N)^2)\frac{f(N,\alpha)}{g(N,\alpha)^2}$, where $g(N,\alpha) = (\alpha+(-1+\alpha)N)^2(N-\alpha(1+N)^2)^2(2(-1+N)N+\alpha^2(1+N)^3-\alpha(2+N+N^2+2N^3))^2$ and $f(N,\alpha) = -4\alpha^3+3\alpha^4+(-8\alpha^2-4\alpha^3+14\alpha^4)N+(-4\alpha-14\alpha^2+11\alpha^3+23\alpha^4)N^2+(12\alpha-10\alpha^2+12\alpha^3+12\alpha^4)N^3+(3+3\alpha-6\alpha^2-7\alpha^3-5\alpha^4)N^4+(-6-4\alpha+4\alpha^2-12\alpha^3-2\alpha^4)N^5+(3+3\alpha+11\alpha^2-11\alpha^3+9\alpha^4)N^6+(-6\alpha+8\alpha^2-12\alpha^3+8\alpha^4)N^7+(3\alpha^2-5\alpha^3+2\alpha^4)N^8$. Note that $f(N,\alpha)>0$ when $\alpha\geq 1.5$.

Now consider $K=N$. We have $\pi(N)-\pi(N-1) = \alpha((\alpha+(-1+\alpha)N)^2)\frac{f(N,\alpha)}{g(N,\alpha)^2}$, where $g(N,\alpha) = (\alpha+(-1+\alpha)N)^2(N-\alpha(1+N)^2)^2(2(-1+N)N+\alpha^2(1+N)^3-\alpha(-4+N^2(5+N)))^2$ and $f(N,\alpha) = -16\alpha^2+8\alpha^3+11\alpha^4+(16\alpha-12\alpha^2-12\alpha^3+62\alpha^4)N+(-4-12\alpha+56\alpha^2-132\alpha^3+141\alpha^4)N^2+(8-36\alpha+120\alpha^2-242\alpha^3+160\alpha^4)N^3+(-17\alpha+97\alpha^2-168\alpha^3+85\alpha^4)N^4+(-6\alpha+20\alpha^2-24\alpha^3+6\alpha^4)N^5+(3\alpha-11\alpha^2+20\alpha^3-13\alpha^4)N^6+(-2\alpha^2+6\alpha^3-4\alpha^4)N^7$ where $f(N,\alpha)<0$ when $\alpha\geq 2$.

A.1.5 Proof of Proposition 1.6

Without loss of generality, we assume the K-firm RJV is larger, i.e. $K \geq N/2$. The other one is referred as $(N-K)$-firm RJV. By backward induction, we have for all $i \in K$ and $j \in N \backslash K$,

$$\begin{aligned}
x_i^{two} &= -\frac{(K-N-1)(\alpha(N+1)+(K+1)(K-N))}{D^{two}}(a-c),\\
x_j^{two} &= \frac{(K+1)(\alpha(N+1)+K(K-N-1))}{D^{two}}(a-c),\\
q_i^{two} &= \frac{\alpha(N+1)(\alpha(N+1)+(K+1)(K-N))}{D^{two}}(a-c),\\
q_j^{two} &= \frac{\alpha(N+1)(\alpha(N+1)+K(K-N-1))}{D^{two}}(a-c),
\end{aligned}$$

where $D^{two} = \alpha^2(N+1)^3 - \alpha(N+1)(N-K(N+4)(K-N)) + K(K+1)(K-N-1)(K-N)$. (i) We want to show $Kx_1^{two} - x_1^{ind} \geq 0$. First, $Kx_1^{two} - x_1^{ind} \geq 0$ is equivalent to $\alpha(K-1)(N+1)^2 + K(K(N+1)(K-N)+N-1)+N \geq 0$. Let $f(\alpha, K) = \alpha(K-1)(N+1)^2 + K(K(N+1)(K-N)+N-1)+N$. Clear f is increasing in α. By the all-firm participating condition $\alpha > \frac{K(N-K+1)}{N+1}$, it suffices to show $f(\frac{K(N-K+1)}{N+1}, K) > 0$ for $K \geq \frac{N}{2}$.

$$\begin{aligned}
f(\frac{K(N-K+1)}{N+1}, K) =& K(K-1)(N+1)(N-K+1)\\
&+ K(K(N+1)(K-N)+N-1)+N\\
=& N + K(N-1) + K(N+1)(2K-(N+1)) \equiv g(K)
\end{aligned}$$

Clearly $g(\frac{N+1}{2}) > 0$, and $g(\frac{N}{2}) = 0$. Since $g' > 0$, we have $Kx_i^{two} - x_i^{ind} \geq 0$ for $i \in K$.

(ii) Note that $\sum_i q_i^{two} - \sum_i q_i^{ind} \geq 0$ is equivalent to $(K-1)(2K-N) \geq 0$ which is true for $K \geq \frac{N}{2}$, which completes the proof.

A.2 Detailed Calculations and Extentions

We first details all calculations for the motivating example in the main text. Next, we show the welfare analysis of RJV size. Next, we consider the model when research outputs between RJV firms may not be fully compatible, and finally discuss the consequence of having spillover between RJV firms and non-RJV firms.

A.2.1 Motivating 4-firm Example

We first consider individual research, 2-firm RJV and then ex-ante and ex-post licensing.

A.2.1.1 Individual Research

First consider individual research case where no RJV has formed. The equilibrium technology development, production and profit for firm $i \in N = \{1, 2, 3, 4\}$ are

$$\begin{aligned} x_i^{ind} &= \frac{4}{25\alpha - 4}(a - c), \\ q_i^{ind} &= \frac{5\alpha}{25\alpha - 4}(a - c), \text{ and} \\ \pi_i^{ind} &= \frac{\alpha(25\alpha - 16)}{(25\alpha - 4)^2}(a - c)^2. \end{aligned}$$

A.2.1.2 Research Joint Venture

Now consider the case that firm 1 and firm 2 forms a RJV. Then, in equilibrium, we have

$$
\begin{aligned}
x_1^{no} = x_2^{no} &= \frac{3(25\alpha - 12) - 24}{(25\alpha - 12)(25\alpha - 18) - 96}(a - c),\\
x_3^{no} = x_4^{no} &= \frac{4(25\alpha - 18) - 48}{(25\alpha - 12)(25\alpha - 18) - 96}(a - c),\\
q_1^{no} = q_2^{no} &= \frac{5\alpha(25\alpha - 20)}{(25\alpha - 12)(25\alpha - 18) - 96}(a - c),\\
q_3^{no} = q_4^{no} &= \frac{5\alpha(25\alpha - 30)}{(25\alpha - 12)(25\alpha - 18) - 96}(a - c),\\
\pi_1^{no} = \pi_2^{no} &= \frac{\alpha(25\alpha - 9)(5\alpha - 4)^2}{(125\alpha^2 - 150\alpha + 24)^2}(a - c)^2, \text{ and}\\
\pi_3^{no} = \pi_4^{no} &= \frac{\alpha(25\alpha - 16)(5\alpha - 6)^2}{(125\alpha^2 - 150\alpha + 24)^2}(a - c)^2.
\end{aligned}
$$

First, we have $x_1 + x_2 > x_3 = x_4$ so that research done by RJV is higher than non-RJV firms. Second, to ensure non-negativity of production cost, we need to have

$$
\alpha > \frac{(3a + 12c) + \sqrt{3}\sqrt{3a^2 - 16ac + 48c^2}}{25c} \equiv Z.
$$

Then from the four second order conditions of the profits maximization, we have $\alpha > \frac{9}{25}$ and $\alpha > \frac{16}{25}$. Finally, for non-negativity of productions ($q_1 = q_2 > 0$ and $q_3 = q_4 > 0$), we have $\alpha > \frac{4}{5}$ and $\alpha > \frac{6}{5}$. Combining all the conditions, with $Z = \frac{15+\sqrt{105}}{25}$ when $a = c$, to ensure all firms will compete in the market, we have

$$\alpha > \max\{Z, \frac{6}{5}\}.$$

A.2.1.3 Licensing

We first consider ex-ante licensing, then ex-post licensing, and finally do a comparison.

A.2.1.3.1 Ex-ante licensing

There are two cases: (1) licensing to firm 3 only, and (2) licensing to both firms 3 and firm 4. Licensing to firm 4 only is the same as the case (1).

Case (1): Using backward induction, we have

$$\begin{aligned}
x_1^{Ex-ante} &= x_2^{Ex-ante} = \frac{2(25\alpha-16)-8}{(25\alpha-16)(25\alpha-8)-48}(a-c) = \frac{2}{25\alpha-4}(a-c), \\
x_3^{Ex-ante} &= x_4^{Ex-ante} = 0, \\
q_i^{Ex-ante} &= \frac{5\alpha}{25\alpha-4}(a-c) \text{ for all } i \in \{1,2,3,4\}, \\
\pi_1^{Ex-ante} &= \pi_2^{Ex-ante} = \frac{\alpha(25\alpha+4)}{(25\alpha-4)^2}(a-c)^2, \text{ and} \\
\pi_4^{Ex-ante} &= \pi_3^{Ex-ante} = \frac{\alpha(25\alpha-16)}{(25\alpha-4)^2}(a-c)^2.
\end{aligned}$$

The winning bid is

$$B^{Ex-ante} = \alpha(x_4^{A1})^2 = \frac{16\alpha}{(25\alpha-4)^2}(a-c)^2$$

Case (2): When the RJV licenses to both firms 3 and 4. First suppose firm 4 does not accept the license. Then

$$q_i = \frac{(a-c) + 2(x_1 + x_2) - x_4}{5} \text{ for } i \in \{1, 2, 3\}, \text{ and}$$
$$q_4 = \frac{(a-c) + 4x_4 - 3(x_1 + x_2)}{5}.$$

Profit maximization problems are

$$\max \pi_i = (q_i)^2 - \alpha x_i^2 + \underline{B} \text{ for } i \in \{1, 2\},$$

$$\max \pi_3 = (q_3)^2 - \alpha x_3^2 - \underline{B}, \text{ and}$$

$$\max \pi_4 = (q_4)^2 - \alpha x_4^2.$$

Solve the problems to get:

$$x_1 = x_2 = \frac{2}{25\alpha - 4}(a-c), \quad x_3 = 0, \quad x_4 = \frac{4}{25\alpha - 4}(a-c), \text{ and}$$
$$q_i = \frac{5\alpha}{25\alpha - 4}(a-c) \text{ for } i \in \{1, 2, 3, 4\}.$$

Therefore, firm 4's profit in the outside option is:

$$\pi_4 = \frac{\alpha(25\alpha - 16)}{(25\alpha - 4)^2}(a-c)^2$$

Now consider firm 4 accepts the license, all the firms will be having the same amount of technological development, and hence, producing the same amount of quantity:

$$q_i = \frac{(a-c) + (x_1 + x_2)}{5} \text{ for } i \in \{1, 2, 3, 4\}.$$

Then solving for the equilibrium we have

$$x_1^{(C)Ex-ante} = x_2^{(C)Ex-ante} = \frac{1}{25\alpha - 2}(a-c),$$
$$x_3^{(C)Ex-ante} = x_4^{(C)Ex-ante} = 0, \text{ and}$$
$$q_i^{(C)Ex-ante} = \frac{5\alpha}{25\alpha - 2}(a-c) \text{ for } i \in \{1,2,3,4\}.$$

Since $\pi_4^{(C)Ex-ante} = (q_4^{(C)Ex-ante})^2 - \underline{B}$, this profit is equal to the outside option, and hence:

$$\underline{B} = (\frac{5\alpha}{25\alpha - 2}(a-c))^2 - \frac{\alpha(25\alpha - 16)}{(25\alpha - 4)^2}(a-c)^2.$$

The profits for firms are:

$$\pi_1^{(C)Ex-ante} = \pi_2^{(C)Ex-ante} =$$
$$\frac{\alpha(25\alpha - 1)(25\alpha - 4)^2 + (25\alpha - 2)(300\alpha^2 - 32\alpha) + 100\alpha^2}{(25\alpha - 2)^2(25\alpha - 4)^2}(a-c)^2, \text{ and}$$
$$\pi_3^{(C)Ex-ante} = \pi_4^{(C)Ex-ante} = \frac{\alpha(25\alpha - 16)}{(25\alpha - 4)^2}(a-c)^2.$$

With the above, we can show the following results with ex-ante licensing.

Remark A.1. *First, we can show that the RJV will license to both firms 3 and 4 because* $\pi_1^{(C)Ex-ante} = \pi_2^{(C)Ex-ante} > \pi_1^{Ex-ante} = \pi_2^{Ex-ante}$ *and* $\pi_3^{(C)Ex-ante} = \pi_4^{(C)Ex-ante} = \pi_3^{Ex-ante} = \pi_4^{Ex-ante}$. *Second, we can show that* $\sum_i q_i^{(C)Ex-amte} < \sum_i q_i^{no}$ *so that consumers will always be worse off under licensing. Similarly, RJV firm's investment is also lower, i.e.* $x_1^{(C)Ex-ante} < x_1^{no}$. *Licensing will* *occurs as long as* $\alpha > 3.21273$ *because* $\pi_1^{(C)Ex-ante} > \pi_1^{no}$ *and* $\pi_4^{(C)Ex-ante} > \pi_4^{no}$

For producer surplus, we have $\sum \pi_i^{(C)Ex-ante} > \sum \pi_i$ *when* $\alpha > 1.6012$.

A.2.1.3.2 Ex-post licensing

There are two cases: (1) licensing to firm 3 only, and (2) licensing to both firms 3 and firm 4. Licensing to firm 4 is the same as the case (1).

Case (1): Using backward induction, we have

$$\begin{aligned}
x_1^{Ex-post} &= x_2^{Ex-post} = \frac{9(25\alpha-16)+24}{(25\alpha-16)(50\alpha-6)+144}(a-c),\\
x_3^{Ex-post} &= x_4^{Ex-post} = \frac{4(50\alpha-6)-216}{(25\alpha-16)(50\alpha-6)+144}(a-c),\\
q_1^{Ex-post} &= q_2^{Ex-post} = q_3^{Ex-post} = \frac{5\alpha(5\alpha-1)}{125\alpha^2-95\alpha+24}(a-c),\\
q_4^{Ex-post} &= \frac{5\alpha(5\alpha-6)}{125\alpha^2-95\alpha+24}(a-c),\\
\pi_1^{Ex-post} &= \pi_2^{Ex-post} = \frac{\alpha(2500\alpha^3-525\alpha^2+510\alpha-576)}{4(125\alpha^2-95\alpha+24)^2}(a-c)^2, \text{ and}\\
\pi_4^{Ex-post} &= \pi_3^{Ex-post} = \frac{(6-5\alpha)^2\alpha(25\alpha-16)}{(125\alpha^2-95\alpha+24)^2}(a-c)^2.
\end{aligned}$$

The winning bid is

$$B^{Ex-post} = \frac{125\alpha^2(10\alpha-7)}{(125\alpha^2-95\alpha+24)^2}(a-c)^2$$

Case (2): Now consider licensing to both firms 3 and 4. First suppose firm 4 accepts the fixed fee, the profit of firm 4 will be $(\frac{a-c+(x_1+x_2)}{5})^2 - \alpha x_4^2$. Otherwise, the profit is $(\frac{a-c+4x_4-3(x_1+x_2)}{5})^2 - \alpha x_4^2$. Therefore, the minimum bid is defined as:

$$\underline{B} = (\frac{a-c+(x_1+x_2)}{5})^2 - (\frac{a-c+4x_4-3(x_1+x_2)}{5})^2$$

The problems we need to solve are:

$$\begin{aligned}
&\max_{x_i}(\frac{a-c+(x_i+x_j)}{5})^2 - \alpha x_i^2 + \underline{B} \quad i=1,2; j\neq i \\
&\max_{x_3}(\frac{a-c+(x_1+x_2)}{5})^2 - \alpha x_3^2 - \underline{B} \\
&\max_{x_4}(\frac{a-c+4x_4-3(x_1+x_2)}{5})^2 - \alpha x_4^2
\end{aligned}$$

Solve to get:

$$\begin{aligned}
x_1^{(C)Ex-post} &= x_2^{(C)Ex-post} = \frac{125\alpha-32}{25\alpha(25\alpha-2)+64}(a-c), \\
x_3^{(C)Ex-post} &= x_4^{(C)Ex-post} = \frac{4(25\alpha-16)}{25\alpha(25\alpha-2)+64}(a-c), \\
q_i^{(C)Ex-post} &= \frac{5\alpha(25\alpha+8)}{25\alpha(25\alpha-2)+64}(a-c) \text{ for all } i=1,2,3,4, \\
\underline{B} &= \frac{1200\alpha^2(25\alpha-4)}{(625\alpha^2-50\alpha+64)^2}(a-c).
\end{aligned}$$

Therefore, firms' profits are:

$$\begin{aligned}
\pi_1^{(C)Ex-post} &= \pi_2^{(C)Ex-post} = \frac{\alpha(15625\alpha^3+24375\alpha^2+4800\alpha-1024)}{(625\alpha^2-50\alpha+64)^2}(a-c)^2 \\
\pi_3^{(C)Ex-post} &= \pi_4^{(C)Ex-post} = \frac{\alpha(25\alpha-16)^3}{(625\alpha^2-50\alpha+64)^2}(a-c)^2
\end{aligned}$$

With the above, we can show the following results with ex-post licensing.

Remark A.2. *First, we can show that the RJV will license to both firms 3 and 4 when when* $\alpha > 3.00731$. *Second, we have* $\sum_{i\in N} q_i^{(C)Ex-post} > \sum_{i\in N} q_i^{Ex-post} >$

$\sum_{i \in N} q_i^{no}$ *so that consumers will always be better off under ex-post licensing. For RJV firm's investment, we have* $x_1^{(C)Ex-post} > x_1^{no}$ *when* $\alpha > 1.74924$*. For producer surplus, we have* $\sum_{i \in N} \pi_i^{(C)Ex-post} > \sum_{i \in N} \pi_i^{no}$ *when* $\alpha > 1.45271$.

A.2.2 Equilibrium Size of RJV

As noted in the Appendix, the term $a - c$ only serves as a scaling factor. For the numerical analysis below, we assume $a - c = 1$ for the ease of exposition.

A.2.2.1 No Licensing

Figure B.1 illustrates how the RJV firms' profits changes with its size when $N = 30$ and $\alpha = 10$. From the graph, we can see there is a single peak, which is

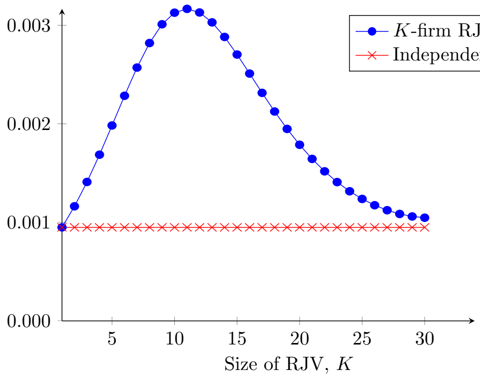

Figure A.1. Comparing firm's profit for a firm under independent licensing and K-firm RJV.

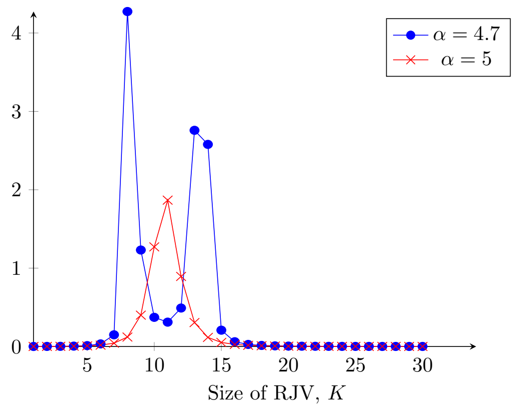

Figure A.2. Non-single peakedness of a RJV firm's profit for small α.

the equilibrium RJV size in the simple RJV formation game. However, single peaked will not hold when α is small, as in Figure A.2.

A.2.2.2 With Licensing

Figure A.3 shows how RJV firms' profits change with RJV size K, industry size N, research efficiency α, and number of licensee L.

Panel (a) is drawn under $N = 30$ and $\alpha = 20$. For each RJV size, we determine the equilibrium number of licensees, and therefore, plotting the equilibrium RJV firms' profits. The graph indicates that ex-post licensing will always improve RJV firms' profit; while ex-ante licensing will do so if the size of RJV is small. On the one hand, as K increases, the production level competition will become very high. On the other hand, the licensing fees are shared among more firms. These two effects will both reduce RJV firms' profits, which could explain the lower profit under ex-ante licensing. However, under ex-post licensing, RJV firms have more incentive to do research, as the cost of R&D could be covered by the licensing fees; and they are choosing number of licensees optimally. This will overcome the profit reducing effects, yielding higher profit for RJV firms.

Panel (b) is drawn under $\alpha = 20$. For each N varies from 8 to 30, we have determined the profit maximizing RJV size and the optimal number of licensees. Similar trend is observed as panel (a). As the industry size grows, the relative size of RJV becomes smaller. The intuition of decreasing in RJV firm's profit is the same as previous case.

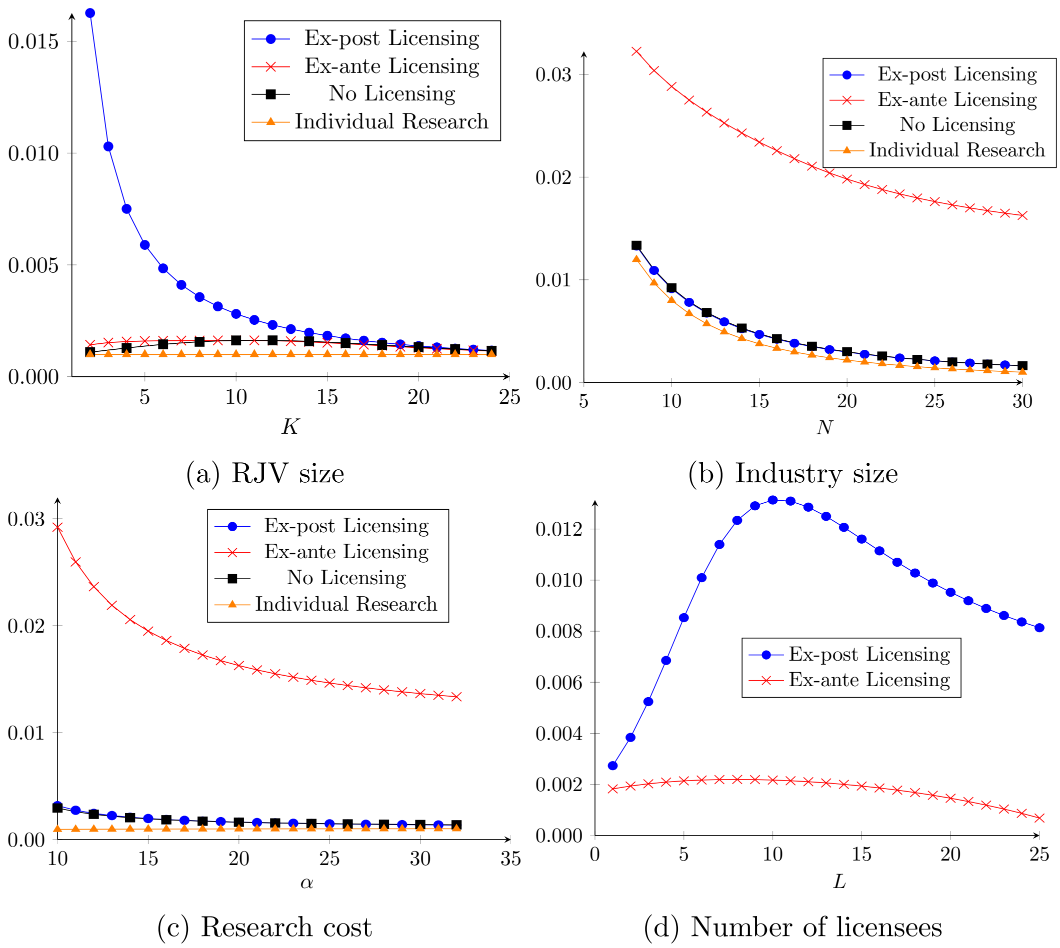

Figure A.3. Comparison for RJV Firms' Profits in 4 Cases. Vertical axes represents profits of a RJV firm. Horizontal axes are for RJV size, industry size, research cost and number of licensees respectively.

Panel (c) is drawn under $N = 30$, and α varies from 10 to 32. We can see as research cost increases, RJV-firms under ex-ante licensing will have higher profits comparing with the no licensing case. The result is confirming our previous conclusion that when research cost is high, RJV firms will have more advantage in developing technologies, due to the cost sharing nature of RJV. And hence they will earn higher profits.

Panel (d) is drawn under $N = 30$, $\alpha = 10$, and $K = 4$. We are able to find the optimal number of licensees numerically that there is a clear peak at about $L = 10$ for ex-post licensing. Interestingly, for the ex-ante licensing, RJV firms' profits do not vary too much with the number of licensees.

A.2.3 Welfare Analysis

A.2.3.1 No Licensing

We first present a very simple upper bound for the maximal technological development RJV size K^{TD}.

Remark A.3. *There exists $K^*(n, \alpha) \in [2, \frac{n+1}{2}]$, such that K^*-firm RJV has the highest technological development.*

Proof. Let $f(K) = Kx_1^{no}$. We just need to show $K^*(N, \alpha)$ solves $f'(K) = 0$. To find K^*, we need to solve the following equation:

$$g(K) = K^4 + N - 2KN + N^2 - 2K^3(N + 1)$$

$$- \alpha(N - 2K + 1)(N + 1)^2 + K^2(N^2 + N + 1) = 0.$$

Step 1: Note that $g(2) = 4 + (5N - \alpha(N+1)^2)(N - 3) < 0$, when $N > 3$. (Note $N = 3$ is a trivial case, because $K = 2$ is the only possible solution). Note that $g(\frac{N+1}{2}) = \frac{1}{16}(N^2 - 1)^2 > 0$.

Step 2: It suffices to show $g(K)$ is monotone when $K \in [2, \frac{N+1}{2}]$.

$$g'(K) = 2[-N + \alpha(n + 1)^2 + K(1 + 2K^2 + N + N^2 - 3K(N + 1))].$$

Check $g'(2) = 2(6 + \alpha(N + 1)^2 + N(2N - 11)) > 0$ and $g'(\frac{N+1}{2}) = 2\alpha(N + 1)^2 - N(N + 3) > 0$.

Step 3: Solve $g''(K) = 0$, gives us $\hat{K} = \frac{1}{6}(3(N + 1) - \sqrt{3(N^2 + 4N + 1)}$, where $\hat{K} \in [2, \frac{N+1}{2}]$. Note that $g'(\hat{K}) > 0$. As we have shown $g'(K) > 0$ for all $K \in [2, \frac{N+1}{2}]$, $g(K)$ will cut x-axis once and only once, which occurs at $K^*(N, \alpha)$, i.e. $g(K^*) = 0$. $\square$

Figure A.4 shows K^{TD} is quite close to the upper bound $(N+1)/2$ especially when α is large or N is small. From Figure A.5, consumer surplus has a higher impact on society, since the social optimal RJV size and the consumer optimal RJV size are very close, i.e. K^{SW} and K^{CS} are nearly identical.

Table A.1 summarizes the RJV size that maximize RJV firms' profits, producer surplus, social welfare, consumer surplus and technology level respectively. It is clear that there is too little incentive for firms to form RJV

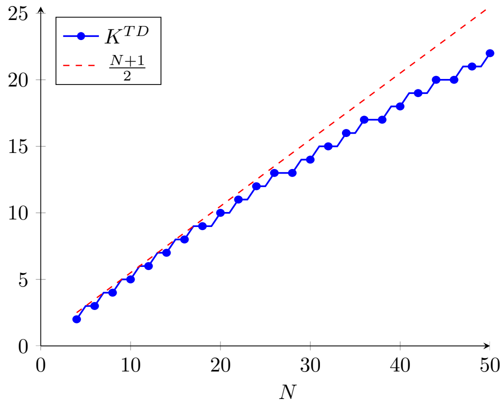

Figure.A.4 K^{TD} and its upper bound

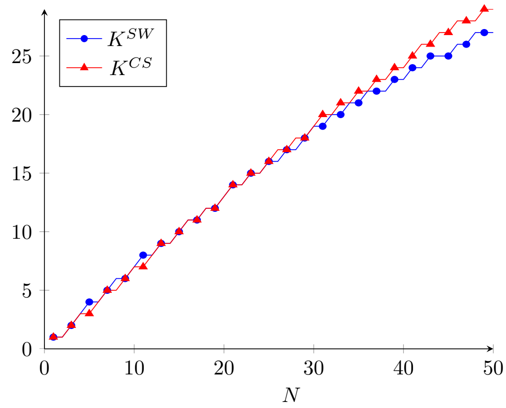

Figure A.5 K^{SW} and K^{CS}

		N							
		30	40	50	60	70	80	90	100
$\alpha = 30$	K^{no}	11	14	18	21	24	28	31	34
	K^{PS}	18	22	25	29	33	36	40	43
	K^{SW}	19	25	30	35	39	43	46	49
	K^{CS}	19	25	31	37	42	46	51	54
	K^{TD}	15	19	23	27	31	34	38	41
$\alpha = 100$	K^{no}	11	14	18	21	24	28	31	34
	K^{PS}	25	30	33	36	40	44	48	52
	K^{SW}	20	27	33	39	45	51	57	63
	K^{CS}	20	26	33	39	46	52	58	64
	K^{TD}	15	20	25	29	34	39	43	47
$\alpha = 1000$	K^{no}	11	15	18	21	24	28	31	34
	K^{PS}	27	36	44	53	61	69	77	84
	K^{SW}	20	27	34	40	47	54	60	67
	K^{CS}	20	27	33	40	47	53	60	67
	K^{TD}	15	20	25	30	35	40	45	50

Table A.1 RJV Size under the equilibrium simple formation game K^{no}, producer-surplus-maximization K^{PS}, social-welfare-maximization K^{SW}, consumer-surplus-maximization K^{CS} and technolgical-development-maximization K^{TD}.

on themselves, i.e. K^{no} is the smallest among all the RJV sizes. The social planner is maximizing both producer and consumer surplus. We could also conclude that firms doing individual research and industry-wide RJV are never optimal, even from social welfare perspective.

A.2.3.2 Under Licensing

Figure A.6 illustrates how consumer surplus changes with the size of RJV. For each RJV size, ex-post licensing delivers the highest consumer surplus, as in Proposition 1.4. For ex-ante licensing, consumer surplus is higher than individual research but is higher than no licensing if and only if RJV is small, consistent with Proposition 1.3. As shown in the above remark, ex-post licensing performs better than ex-ante licensing for consumer surplus.

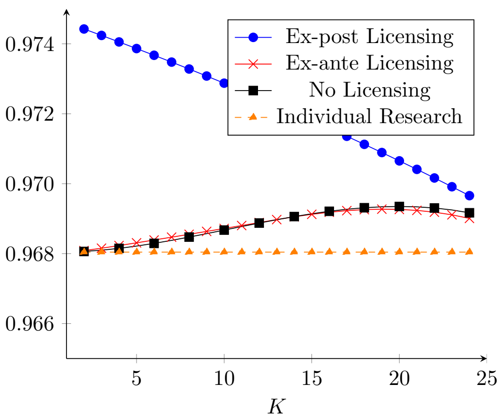

Figure A.6. Equilibrium consumer surplus varies with RJV size when $\alpha = 100$ and $N = 30$.

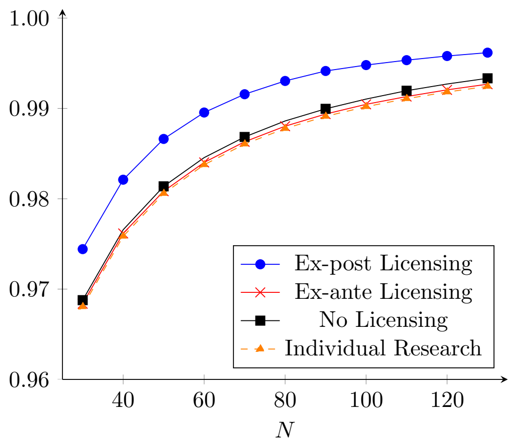

Figure A.7. Equilibrium consumer surplus varies with industry size when RJV is determeind by the simple RJV formation game and $\alpha = 100$.

Figure A.7 illustrates how consumer surplus changes with the size of industry when size of RJV is determeind by the simple RJV formation game. Consumer surplus can be ranked in the following descending order: ex-post licensing, no licensing, ex-ante licensing and finally individual research.

A.2.4 Imperfect compatibility

We consider the case of imperfect compatibility. Recall that the technological development for firm $k \in K$ is $X_K = x_k + \beta \sum_{i \in K \setminus \{k\}} x_i$ where $\beta \in [0,1]$ measures the degree of compatibility within the RJV. We use superscript im to represent this case. Since the case of individual research remain the same as the case with perfect compatibility, we now study industry-wide RJV.

A.2.4.1 Industry-wide RJV

First, the profit of firm $i \in N$ is $\pi_i^{im(all)} = (a - Q - (c - (x_i + \beta \sum_{j \neq i} x_j)))q_i - \alpha x_i^2$. Under quantity competition, the equilibrium production is

$$q_i^{im(all)} = \frac{(a-c) + N(x_i + \beta \sum_{j \neq i} x_j) - \sum_{d \neq i}(x_d + \beta \sum_{s \neq d} x_s)}{N+1}.$$

By backward induction, equilibrium technology improvement is

$$x_i^{im(all)} = \frac{(N - \beta(N-1))}{\alpha(N+1)^2 - (N - \beta(N-1))(1 + \beta(N-1))}(a-c).$$

Therefore, we have $q_i^{im(all)} = \frac{\alpha(N+1)}{\alpha(N+1)^2 - (N-\beta(N-1))(1+\beta(N-1))}(a-c)$. Since $(N - \beta(N-1))(1 + \beta(N-1)) \geq N$ if and only if $N + \beta(N-1)^2(1-\beta) \geq N$, we have the following:

Proposition 1.1*. *Under an industry-wide RJV, for all $i \in N$, we have $q_i^{im(all)} \geq q_i^{ind}$ and $\sum_{i \in N} x_i^{im(all)} \geq x_i^{ind}$ for all $\beta \in [0,1]$, with the equality at $\beta = 0$ and $\beta = 1$.*

The results above explain the relationship between research sharing incentive and free-riding effect among the RJV firms. When technologies are perfectly incompatible, i.e. $\beta = 0$, firms are just doing individual research. There is no research sharing and free riding. As β increases, firms are more willing to do research due to the research sharing effect. However, the free-riding effect will become dominant, when β becomes large, i.e. beyond $\frac{1}{2}$. Eventually, when the technology becomes perfectly compatible, the RJV will act as if firms doing research individually.

A.2.4.2 K-firm RJV

First, profits for firms $i \in K$ and $j \in N \backslash K$ are

$$\pi_i^{im} = (a - Q - (c - (x_i + \beta \sum_{d \neq i, d \in K} x_d)))q_i - \alpha x_i^2,$$
$$\pi_j^{im} = (a - Q - (c - x_j))q_j - \alpha x_j^2,$$

The equilibrium production is then

$$q_i^{im} = \frac{(a-c) + N(x_i + \beta \sum_{s \neq i} x_s) - \sum_{d \neq i}(x_d + \beta \sum_{m \neq d} x_m) - \sum_{j \in N \backslash K} x_j}{N+1};$$
$$q_j^{im} = \frac{(a-c) + N x_j - \sum_{s \neq j, s \in N \backslash K} x_s - \sum_{i \in K}(x_i + \beta \sum_{d \neq i} x_d)}{N+1}.$$

Backward induction implies

$$x_i^{im} = \frac{(N-\beta(K-1))(\alpha(N+1)-N)}{D^{im}}(a-c),$$
$$x_j^{im} = \frac{n(\alpha(N+1)-(1+\beta(K-1))(N-\beta(K-1)))}{D^{im}}(a-c),$$
$$q_i^{im} = \frac{\alpha(N+1)(\alpha(N+1)-N)}{D^{im}}(a-c),$$
$$q_j^{im} = \frac{\alpha(N+1)(\alpha(N+1)-(1+\beta(K-1))(N-\beta(K-1)))}{D^{im}}(a-c).$$

where $D^{im} = \alpha^2(N+1)^3 - \alpha(N+1)(\beta^2(K-1)^2(K-N-1) - \beta(K-1)(N-1)(K-N-1) + N(N+2)) + N(-\beta(K-1)-1)(\beta(K-1)-N)$.

Let $X^{im} \equiv (1+\beta(K-1))x_i^{im}$ the aggregate technological development for the K-firm RJV with imperfect compatibility. Let x^{ind} and X^{no} technological developments for individual research and K-firm RJV with perfect compatible technology respectively. Note that when $\beta = 0$, $X^{im} = x_i^{im} = x_j^{im} = x^{ind}$. Note also $\lim_{\beta\to 1} x_i^{im} = x_i^{no}$ and $\lim_{\beta\to 1} x_j^{im} = x_j^{no}$. For RJV firms, the aggregate technological development when β approaches 1 is $\lim_{\beta\to 1} X^{im} = kx_i^{no} = X^{no}$.

Theorem 1.1*. *Consider a K-firm RJV under compatibility β. We have $x_i^{im} \geq x_j^{ind}$ and $(\sum_{k\in N} q_k^{im})^2 \geq (\sum_{k\in N} q_k^{ind})^2$ for all $i \in K$ and $j \in N$, where equalities hold when $\beta = 0$. When RJV size is large and β is large enough, $x_i^{im} > x_i^{no}$ and $(\sum_{k\in N} q_k^{im})^2 > (\sum_{k\in N} q_k^{no})^2$ for all $i \in K$.*

Proof. Define the aggregate cost reduction for the RJV firms, $X^{im}(\beta) \equiv (1+\beta(K-1))x_i^{im}$ to be a function of β.

$\dfrac{\partial X^{im}(\beta)}{\partial \beta} =$

$$\frac{\alpha(K-1)(N+1)(\alpha(N+1-N)(\alpha(N+1)^2-N(K+1))(N-1-2\beta(K-1))}{(D^{im})^2}(a-c)$$

Note that $sign(\frac{\partial X^{im}(\beta)}{\partial\beta}) = sign(N-1-2\beta(K-1))$, since the rest of terms are positive. There are two cases: (1) $N-1 \geq 2(K-1)$ and (2) $N-1 < 2(K-1)$. Case (1) : $\frac{\partial X^{im}(\beta)}{\partial\beta} \geq 0$ as $N-1-2\beta(K-1) \geq 0$, given $\beta \in [0,1]$. Since $X^{im}(0) = x^{ind}$ and $X^{im}(1) = X^{no}$, we have $x^{ind} \leq X^{im}(\beta) \leq X^{no}$. Case (2): These exist $\beta^* = \frac{N-1}{2(K-1)}$ such that when $\beta \in (\beta^*, 1], \frac{\partial X^{im}(\beta)}{\partial\beta} < 0$. Because $X^{im}(1) = X^{no}$, then there exists $\beta^{**} \in (0, \beta^*)$, such that $X^{im}(\beta^{**}) = X^{no}$. The existence of β^{**} is due to $X^{im}(0) = x^{ind}$ and $X^{no} > x^{ind}$.
Therefore, when $\beta \in [\beta^{**}, 1]$, then $X^{im}(\beta) \geq X^{no} > x^{ind}$. When $\beta \in [0, \beta^{**})$, then $X^{no} > X^{im}(\beta) \geq x^{ind}$.
For consumer surplus

$$Q^{im}(\beta) = \frac{\alpha(N+1)(\beta^2(K-1)^2(N-K)-\beta(K-1)(N-K)(N-1)+n(\alpha(N+1)-n))}{D^{im}}(a-c)$$

and we can show that

$$sign(\frac{\partial Q^{im}(\beta)}{\partial\beta}) = sign(N-1-2\beta(K-1))$$

The rest of the proof for consumer surplus is similar to technological development. □

A.2.5 Spill-over among firms

We consider the case where patent protection is not perfect. There is spill-over between RJV firms and non-RJV firms; there is also spill-over among non-RJV firms themselves. Note that RJV-firms have perfect spill-over, because they are sharing their research results.

Let $\gamma \in [0, 1]$ measures the spill-over effect. Note that $\gamma = 0$ corresponds to standard no licensing case as we considered in previous sections, and $\gamma = 1$ corresponds to all firms form a single RJV case, since firms could access all the other firms' technologies for free. We use superscript so (which stands for "spill-over") to represent this case.

A.2.5.1 Individual Research

First, under this setup, profit of firm $i \in N$ is

$$\pi_i^{so(ind)} = (a - Q - (c - (x_i + \gamma \sum_{s \neq i} x_s)))q_i - \alpha x_i^2$$

The equilibrium production is

$$q_i^{so(ind)} = \frac{(a - c) + N(x_i + \gamma \sum_{s \neq i} x_s) - \sum_{d \neq i}(x_d + \beta \sum_{m \neq d} x_m)}{N + 1}.$$

Note that this is essentially the same problem as in the imperfect compatibility. Therefore all relevant results directly applies.

A.2.5.2 K-firm RJV

Let $X \equiv \sum_{i \in K} x_i$ and $Y \equiv \sum_{j \in N \setminus K} x_j$ be the aggregate cost reduction for RJV-firms and non-RJV firms respectively. For all firms $i \in K$ and $j \in N \setminus K$, the profits are

$$\pi_i^{so} = (a - Q - (c - (X + \gamma Y)))q_i - \alpha x_i^2,$$

$$\pi_j^{so} = (a - Q - (c - (x_j + \gamma(X + Y - x_j))))q_j - \alpha x_j^2,$$

The equilibrium productions for firms $i \in K$ and $j \in N \backslash K$ are

$$q_i^{so} = \frac{(a-c) + (N-K+1)(X+\gamma Y) - \sum_{j \in N \backslash K}(x_j + \gamma(X+Y-x_j))}{N+1};$$

$$q_j^{so} = \frac{(a-c) + N(x_j + \gamma(X+Y-x_j)) - \sum_{d \neq j}(x_d + \gamma(X+Y-x_d)) - K(X+\gamma Y)}{N+1}.$$

We solve for $x_i^{so}, x_j^{so}, q_i^{so}$ and q_j^{so}, and then compare individual research case with spillover, i.e. we compare technological development and consumer surplus as in section A.2.4.

When we fix number of firms, N; size of RJV, K; and research cost parameter, α, our numerical results show that as spill-over effects becomes larger, i.e. γ is sufficiently high, individual research is better than K-firm RJV for both technological development and consumer surplus. When the patent protection for RJV-firms is very low, due to high spill-over, firms are reluctant to do research and tend to free-ride one other.

When we fix N and α, our numerical results, as in Tables A.2 and A.3, suggest that γ^* is decreasing in K, where γ^* is the threshold point, such that for all $\gamma \geq \gamma^*$, individual research is better than K-firm RJV both in terms of technological development and consumer surplus. As RJV size is larger, a small spill-over will benefit hugely to non-RJV firms. Therefore RJV-firms have less incentive to do research, and hence the consumer surplus will be reduced.

		N				
		30	40	50	60	70
	10	0.6893	0.7688	0.8160	0.8472	0.8693
	12	0.6204	0.7176	0.7751	0.8132	0.8403
K	14	0.5516	0.6663	0.7343	0.7793	0.8113
	16	0.4828	0.6151	0.6936	0.7454	0.7822
	18	0.4140	0.5640	0.6527	0.7115	0.7532

Table A.2. Threshold γ^* for technological development when $\alpha = 50$.

		N				
		30	40	50	60	70
	10	0.4082	0.4348	0.4494	0.4587	0.4651
	12	0.3830	0.4179	0.4368	0.4486	0.4567
K	14	0.3556	0.4000	0.4235	0.4381	0.4480
	16	0.3256	0.3810	0.4097	0.4271	0.4391
	18	0.2927	0.3606	0.3950	0.4158	0.4298

Table A.3. Threshold γ^* for consumer surplus when $\alpha = 50$.

Appendix B

Proofs and Details of Chapter Two

B.1 Proofs

B.1.1 Proof of Proposition 2.1

According to Kaimen and Tauman (1986), all the firms will be purchasing the technology, as long as the royalty fee is no greater than the cost reduction by the technology.

By backward induction, we have

$$q_i = \frac{a - c - d^r + \alpha\epsilon - nr_i^r + \sum_{j\neq i} r_j^r}{n+1} \text{ for all } i \in N.$$

The maximization of manufacturer's profit gives us $d^r = \frac{1}{2}(a - c + \alpha\epsilon)$ and $r_i^r = 0$ for all $i \in N$.

B.1.2 Proof of Proposition 2.2

The proof is in the main text.

B.1.3 Proof of Proposition 2.3

Under independent licensing, the equilibrium quantity is defined as follows:

$$
\begin{aligned}
q_1^i &= \frac{(a-c-d)+\alpha\epsilon+(n-1)r_1^i-2r_2^i}{n+1}, \\
q_2^i &= \frac{(a-c-d)+\alpha\epsilon+(n-1)r_2^i-2r_1^i}{n+1}, \\
q_3^i &= \frac{(a-c-d)+\alpha\epsilon-2r_1^i-2r_2^i}{n+1}.
\end{aligned}
$$

We have $r^i = \min\{\frac{1}{4}(a-c-d+\alpha\epsilon), \epsilon(\alpha-1)\}$. Therefore, for the firms 3,4, ..., n to exit the market, by lemma 2 and 3, we require:

$$
\begin{aligned}
&\frac{1}{8}(a-c+\alpha\epsilon) < (\alpha-1)\epsilon \\
\iff &(7\alpha-8)\epsilon > (a-c)
\end{aligned}
$$

First of all, we shall notice that the above inequality holds, if α is large, i.e. $7\alpha - 8 > 0$.

Then we also require $\epsilon < \frac{n}{2(2n+1)}(a-c)$. And hence, when α is sufficiently large, $\frac{1}{7\alpha-8}(a-c) < \epsilon < \frac{n}{2(2n+1)}(a-c)$. Now $\frac{1}{7\alpha-8} < \frac{n}{2(2n+1)}(a-c)$ if and only if $\alpha > \frac{12n+2}{7n} > \frac{8}{7}$.

Therefore, the first case is valid, only when both ϵ and α are sufficiently large.

Now we need to show that firms 3,4, ..., n exiting the market is indeed an equilibrium. Without loss of generality, it suffices for us to check firm 3 does not have profitable deviation, given all other firms' choices.

Given $d^i = \frac{1}{2}(a - c + \alpha\epsilon)$ and $r^i = \frac{1}{8}(a - c + \alpha\epsilon)$. The case where firm 3 does not buy the technologies from firms 1 and 2 while the others still do:

$$q_1 = \frac{(a-c-d) + 2\alpha\epsilon + (n-2)r_1 - 3r_2}{n+1},$$
$$q_2 = \frac{(a-c-d) + 2\alpha\epsilon + (n-2)r_2 - 3r_1}{n+1},$$
$$q_3 = \frac{(a-c-d) - (n-1)\alpha\epsilon + (n-2)r_1 + (n-2)r_2}{n+1},$$
$$q_4 = ... = q_n = \frac{(a-c-d) + 2\alpha\epsilon - 3r_1 - 3r_2}{n+1}.$$

And hence,$q_3 = \frac{(a-c)-3\alpha\epsilon}{6} > 0$ if and only if $\epsilon < \frac{1}{3\alpha}(a-c)$.

Now check that

$$\frac{1}{7\alpha - 8}(a-c) < \epsilon < \frac{1}{3\alpha}(a-c)$$
$$\Longleftrightarrow \alpha > 2$$

which violates $1 < \alpha \leq 2$.

Therefore, $q_3 = 0$ even if firm 3 does not purchase the technology.

B.1.4 Proof of Proposition 2.4

There are two cases: (1) $r = \frac{1}{3}(a - c - d + \alpha\epsilon)$ and (2) $r = \alpha\epsilon$.

Case (1): we have $\frac{1}{3}(a - c - d + \alpha\epsilon) < \alpha\epsilon$ if and only if

$$\epsilon > \frac{1}{2\alpha}(a-c-d) \tag{B.1}$$

For productions, we have

$$q_1 = q_2 = \frac{1}{3}(a - c - d + \alpha\epsilon),$$

$$q_3 = ... = q_n = 0.$$

Solve $\max_d \Pi_m = dQ$ gives us $d = \frac{1}{2}(a - c + \alpha\epsilon)$.

Thus

$$q_1 = q_2 = \frac{1}{6}(a - c + \alpha\epsilon),$$
$$q_3 = ... = q_n = 0.$$

and equation (B.1) becomes

$$\epsilon > \frac{1}{5\alpha}(a - c) \tag{B.2}$$

Check that firms $3, 4, ..., n$ **do not have profitable deviation**.

If firm 3 does not purchase the technology, while others do:

$$\begin{aligned} q_3 &= \frac{(a - c - d) - (n-1)\alpha\epsilon + (n-3)r}{n+1} \\ &= \frac{n(a-c) - 5n\alpha\epsilon}{6(n+1)} > 0 \\ \iff \epsilon &< \frac{1}{5\alpha}(a - c) \end{aligned}$$

which contradicts to equation (B.2).

Therefore, when $\frac{1}{5\alpha}(a - c) < \epsilon < \frac{n}{2(2n+1)}(a - c)$, firms 3,4, ..., n will exit the market.

$$d^p = \frac{1}{2}(a - c + \alpha\epsilon),$$
$$r^p = \frac{1}{6}(a - c + \alpha\epsilon),$$

$$q_1^p = q_2^p = \frac{1}{6}(a - c + \alpha\epsilon),$$

$$q_3^p = ... = q_n^p = 0.$$

Case (2): All firms stay in market . We have

$$d^p = \frac{n(a-c) + 2\alpha\epsilon}{2n},$$

$$r^p = \alpha\epsilon,$$

$$q_1^p = q_2^p = \frac{n(a-c) + 2(n^2 - n - 1)\alpha\epsilon}{2n(n+1)},$$

$$q_3^p = ... = q_n^p = \frac{n(a-c) - 2(2n+1)\alpha\epsilon}{2n(n+1)}.$$

Check that $q_3^p > 0$ if and only if $\epsilon < \frac{1}{5\alpha}(a-c) < \frac{n}{2(2n+1)\alpha}(a-c)$.

B.1.5 Proof of Theorem 2.1

Proof on consumer surplus:

First we have,

$$Q^{no} = \frac{n(a-c) + 2\epsilon}{2(n+1)},$$

$$Q^i = \frac{n(a - c + \alpha\epsilon) - 2(n-1)(\alpha - 1)\epsilon}{2(n+1)},$$

$$Q^r = \frac{n(a - c + \alpha\epsilon)}{2(n+1)},$$

$$Q^p = \frac{n(a-c) + 2\alpha\epsilon}{2(n+1)}.$$

Clearly $Q^p > Q^{no}$ and $Q^r > Q^i$.

Then,

$$Q^i > Q^p$$

$$\iff \alpha < \frac{2(n-1)}{n}$$

When there is still substitutability between the technologies. firms 1 and 2 will compete by setting lower prices for their patents, and hence consumer surplus is higher under independent licensing case.

Next, we check

$$Q^i > Q^{no}$$

$$\iff \alpha < 2.$$

In conclusion, when $\alpha < \frac{2(n-1)}{n}$, $Q^r > Q^i > Q^p > Q^{no}$. Otherwise, $Q^r > Q^p > Q^i > Q^{no}$

Proof on aggregate producer surplus:

First we have

$$\Pi^{no} = \frac{(a-c)^2n^2(n+2) + 4(a-c)\epsilon n(n+2) + 4\epsilon^2\left(2n^3 - 5n - 2\right)}{4n(n+1)^2},$$
$$\Pi^i = \frac{(-\epsilon(\alpha(n-2) - 2n + 2) + (a-c)n)(\epsilon((3\alpha - 2)n + 2) + (a-c)(n+2))}{4(n+1)^2},$$
$$\Pi^{i'} = \frac{3}{16}(a - c + \alpha\epsilon)^2,$$
$$\Pi^r = \frac{n(n+2)(\alpha\epsilon + (a-c))^2}{4(n+1)^2},$$
$$\Pi^p = \frac{(2\alpha\epsilon + (a-c)n)(2\alpha\epsilon n + (a-c)(n+2))}{4(n+1)^2},$$
$$\Pi^{p'} = \frac{2}{9}(a - c + \alpha\epsilon)^2.$$

where $\Pi^{i'}$ and $\Pi^{p'}$ represent the aggregate producer surplus, when firms 3,4,...,

n exit the market in independent licensing and patent pool cases respectively.

Lemma B.1. $\Pi^r > \Pi^{no}$

Proof.

$$\begin{aligned}
&\Pi^r - \Pi^{no} > 0 \\
\iff &\epsilon(2(a-c)n(2+n)(-2+\alpha n) + \epsilon(8 + n(20 - 8n^2 + \alpha^2 n(2+n)))) > 0.
\end{aligned}$$

Clearly $2(a-c)n(2+n)(-2+\alpha n) > 0$.

$\epsilon(8 + n(20 - 8n^2 + \alpha^2 n(2+n))) = 8\epsilon + 20n\epsilon + 2\alpha^2 n^2 \epsilon + n\epsilon(-8n^2 + \alpha^2 n^2)$.

It suffices for us to show $2n(n+2)(\alpha n - 2)(a-c) + n\epsilon(-8n^2 + \alpha^2 n^2) \geq 0$.

$$\begin{aligned}
&2n(n+2)(\alpha n - 2)(a-c) + n\epsilon(-8n^2 + \alpha^2 n^2) \\
>&2n^3(a-c) - 7n^3\epsilon - 4n(a-c) \text{ taking } \min \alpha = 1 \\
>&\frac{1}{4}n^3(a-c) - 4n(a-c) \text{ by taking } \max \epsilon = \frac{1}{4}(a-c) \\
>&(\frac{1}{4}n^2 - 4)(a-c) \geq 0 \text{ when } n \geq 4.
\end{aligned}$$

It is easy to check, when $n = 3$, $\Pi^r > \Pi^{no}$. $\square$

Lemma B.2. $\Pi^r > \Pi^i$ *and* $\Pi^r > \Pi^p$

Proof.

$$\begin{aligned}
&\Pi^r > \Pi^i \\
\iff &(\alpha - 1)\epsilon(n-1)((\alpha - 1)\epsilon n + (a-c) + \epsilon) > 0.
\end{aligned}$$

And

$$\Pi^r > \Pi^p$$

$$\iff \alpha\epsilon(n-2)(\alpha\epsilon n + 2(a-c)) > 0$$

Clearly, $\Pi^r > \Pi^{i'}$ and $\Pi^r > \Pi^{p'}$, since $\frac{n(n+2)}{4(n+1)^2} > \frac{2}{9} > \frac{3}{16}$. This completes the proof. □

B.1.6 More results on consumer surplus and aggregate producer surplus

More comparisons consumer surplus:

Proposition B.1. *When firms without initial technology exit the market under both independent licensing and patent pool, i.e.* $\frac{1}{7\alpha-8}(a-c) \leq \epsilon < \frac{n}{2(2n+1)}(a-c)$, *then* $Q^{i'} < Q^{no}$*; and there exists a threshold value* $\hat{\alpha}$*, such that when* $\alpha > \hat{\alpha}$*,* $Q^{p'} > Q^{no}$*.*

Proof.

$$Q^{i'} = \frac{1}{4}(a-c+\alpha\epsilon) > Q^{no}$$
$$\iff \epsilon > \frac{n-1}{(n+1)\alpha-4}(a-c)$$

Check that

$$\frac{n-1}{(n+1)\alpha-4}(a-c) > \frac{n}{2(2n+1)}(a-c)$$
$$\iff 4n^2-2n-2 > n^2\alpha+n\alpha-4n$$

$$\Leftarrow 4n^2 - 2n - 2 > 2n^2 + 2n - 4n \text{ as RHS is max at } \alpha = 2$$

$$\Longleftrightarrow 2n^2 - 2 > 0.$$

Therefore $Q^{i'} < Q^{no}$. This is intuitive. Drastic innovation in independent licensing leads to a duopoly in the market. With little market competition, consumers should be worse off than the case of no licensing, where the competition level is relatively high.

Now we show the second part of the proposition.

$$Q^{p'} = \frac{1}{3}(a - c + \alpha\epsilon) > Q^{no}$$

$$\Longleftrightarrow \epsilon > \frac{n-2}{2(n+1)\alpha - 6}(a-c)$$

Note that we require $\epsilon < \frac{n}{2(2n+1)}(a-c)$. Therefore

$$\frac{n-2}{2(n+1)\alpha - 6} < \frac{n}{2(2n+1)}$$

$$\Longleftrightarrow 2n^2 - 2 < \alpha n^2 + \alpha n$$

$$\Longleftrightarrow \alpha > \frac{2(n-1)}{n} \equiv \hat{\alpha}.$$

Therefore, when $\alpha > \hat{\alpha}$, $Q^{p'} > Q^{no}$. Otherwise $Q^{no} > Q^{p'}$. When technologies are compatible and drastic, even though the competition level is reduced by forming a pool, the technological improvement is too high, consumers are still better off. Compatibility does not affect consumer surplus in the case of no licensing. Given technology ϵ is significant, and α is large, the term $\alpha\epsilon$ will overcome the loss from the reduction in production level competition. $\square$

Proposition B.2. *When firms without initial technology exit the market under patent pool, independent licensing yields higher consumer surplus than the case of patent pool, i.e.* $Q^i > Q^{p'}$.

Proof.

$$
\begin{aligned}
& Q^{p'} < Q^i \\
\iff & (n-2)(a-c) - (5n-4)\alpha\epsilon + 6(n-1)\epsilon > 0.
\end{aligned}
$$

Note that LHS will reach minimum when $\alpha = 2$, therefore it suffices for us to show $(n-2)(a-c) - (4n-2)\epsilon > 0$.

$$
\begin{aligned}
& \epsilon < \frac{n-2}{2(n-1)} < \frac{n}{2(2n+1)} \\
\iff & n^2 - 2n - 2 > 0.
\end{aligned}
$$

Therefore, $Q^i > Q^{p'}$. $\square$

More on aggregate producer surplus:

Proposition B.3. $\Pi^p > \Pi^i$ *if and only if* $\alpha > \frac{2(n-1)}{n}$.

Proof.

$$\Pi^p > \Pi^i$$

$$
\begin{aligned}
&\iff \epsilon((\alpha-2)n+2)(\epsilon(\alpha(3n-2)-2n+2)+2(a-c))>0\\
&\iff (\alpha-2)n+2)>0\\
&\iff \alpha>\frac{2(n-1)}{n}.
\end{aligned}
$$

which completes the proof. (Consistent with Shapiro (2001).) $\square$

B.2 Fixed Fee Compensation Scheme

First, we will consider two different cases: (1) downstream producers may keep their technologies but does not transfer, and (2) downstream producers may keep and transfer their technologies. The first case is more relevant when imitation and/or patent protection is weak. Then, we consider the problem from normative perspective: fair compensation based on marginal contribution. This is important if the court decides compensation has to be fair as in Qualcomm case.

B.2.1 Firms keep the technology

We consider a game similar to the one described in the main text. Now firms 1 and 2 have a choice on giving (possibly with a compensation) their respective technologies to the manufacturer in the first stage. In the second stage,the manufacturer set the price of the chip d, and charge royalty rates for technologies, if he has any. Downstream firms engage in Cournot competition in the last stage. Let superscript rn to denote this reverse licensing without independent licensing game.

Proposition B.4. *Consider the upstream manufacturer engage in reverse licensing but the downstream producer may keep their technology. The upstream manufacturer will set the price of input to be* $d^{rn} = \frac{1}{2}(a - c + \alpha\epsilon)$*, charges zero* *royalty for licensing* $r_i^{rn} = 0$ *for all* $i \in N$*, and offer a fixed fee compensation* $F^{rn} = (\frac{(n-1)(a-c+\alpha\epsilon)+(2n^2-3n-1)(\alpha-1)\epsilon}{2(n^2-1)})^2 - (\frac{1}{2(n+1)}(a-c+\alpha\epsilon))^2$ *to downstream producers with advanced technology (firms 1 and 2). The production of downstream producers are* $q_1^{rn} = ... = q_n^{rn} = \frac{1}{2(n+1)}(a-c+\alpha\epsilon)$*.*

Hence, the profits of the upstream manufacturer is $\pi_m^{rn} = d^{rn}Q^{rn} - 2F^{rn}$, and the downstream producers are $\pi_1^{rn} = \pi_2^{rn} = (\frac{1}{2(n+1)}(a-c+\alpha\epsilon))^2 + F = (\frac{(n-1)(a-c+\alpha\epsilon)+(2n^2-3n-1)(\alpha-1)\epsilon}{2(n^2-1)})^2$, and $\pi_3^{rn} = ... = \pi_n^{rn} = (\frac{1}{2(n+1)}(a-c+\alpha\epsilon))^2$.

B.2.2 Firms may transfer their technologies

Now we consider the game similar to previous subsection, firms 1 and 2 have a choice on giving their respective technologies to the manufacturer. If they choose to keep their respective technologies, they could set royalty fees for the technologies independently in the second stage. Let superscript rl to denote this reverse licensing with independent licensing game.

Proposition B.5. *Consider the upstream manufacturer engage in reverse licensing but the downstream producer may transfer their technologies. The upstream manufacturer will set the price of input to be* $d^{rl} = \frac{1}{2}(a-c+\alpha\epsilon)$*, charges zero royalty for licensing* $r_i^{rl} = 0$ *for all* $i \in N$*, and offer a fixed fee compensation* $F^{rn} = (\frac{(2n^3-13n+13)(n(2n(n+2)-5)+1)(a-c+\alpha\epsilon)^2}{4(n(n(3n+4)-13)+2)^2} - (\frac{1}{2(n+1)}(a-c+\alpha\epsilon))^2$ *to downstream producers with advanced technology (firms 1 and 2). The pro-*

duction of downstream producers are $q_1^{rl} = ... = q_n^{rl} = \frac{1}{2(n+1)}(a - c + \alpha\epsilon)$.

Hence, the profits of the upstream manufacturer is $\pi_m^{rn} = d^{rl}Q^{rl} - 2F^{rl}$, and the downstream producers are $\pi_1^{rn} = \pi_2^{rn} = (\frac{1}{2(n+1)}(a - c + \alpha\epsilon))^2 + F = \frac{(2n^3-13n+13)(n(2n(n+2)-5)+1)}{4(n(n(3n+4)-13)+2)^2}(a - c + \alpha\epsilon)^2$, and $\pi_3^{rn} = ... = \pi_n^{rn} = (\frac{1}{2(n+1)}(a - c + \alpha\epsilon))^2$.

B.2.3 Fair Compensation

A normative analysis of fair compensation can be defined by the marginal contribution of the technologies (Hougaard, Ko and Zhang 2016). Marginal contribution of a downstream firm can be defined by the marginal increases of total profit due to the presence of the technologies. Then fair compensation to a downstream firm would be a share of it marginal contribution where the share is based on the relative marginal contribution of each agent. We will have two slightly different definition of fair compensation depending whether we include the upstream manufacturer into the calculation of marginal contribution. [1]

With the upstream manufacturer

In order to measure the marginal contribution, we consider a hypothetical case, where firm 1 is the only leader in the market, who possess a technology to reduce the production cost by ϵ. In this case, firms $2, 3, ..., n$ are identical.

Now consider the game described in the main text. The only difference is

[1] We have detailed discussion in Appendix E.3.

that firm 2 is no longer the leader, and the market technology level is only ϵ. Let superscript rm to denote this hypothetical marginal contribution game.

$$\pi_m^{rm} = dQ + \sum_{i \in N} r_i q_i,$$
$$\pi_i^{rm} = (P - d - (c - \epsilon) - r_i) q_i$$

In equilibrium, we have

$$r_1^{rm} = r_2^{rm} = ... = r_n^{rm} = 0,$$
$$d^{rm} = \frac{1}{2}(a - c - \epsilon),$$
$$q_1^{rm} = ... = q_n^{rm} = \frac{1}{2(n+1)}(a - c + \epsilon).$$

Comparing π_i^{rm} and π_i^r, for all $i = 1, 2, ..., n$. The marginal contribution of firm 2's technology to firms $1, 2, ..., n$'s profit is defined to be

$$\pi_i^{rm} - \pi_i^r = \frac{1}{4(n+1)^2}((a - c + \alpha\epsilon)^2 - (a - c + \epsilon)^2), \ \forall i = 1, 2, ..., n$$

Note that firm 2 is also benefited from his own technology.

The marginal contribution to the manufacturer is

$$\pi_m^{rm} - \pi_m^r = \frac{n}{4(n+1)}((a - c + \alpha\epsilon)^2 - (a - c + \epsilon)^2)$$

Define $\Phi = (a - c + \alpha\epsilon)^2 - (a - c + \epsilon)^2$. The compensation to firms 1 and 2 will be

$$F^{rm} = \frac{n(n+2)}{4(n+1)^2}\Phi.$$

Note that, for example, firm 2 will also need to "pay" the marginal contribution $\frac{1}{4(n+1)^2}\Phi$ to himself. And thus, the fee he actually collected from other firms (including the manufacturer) is given by $\frac{n(n+2)-1}{4(n+1)^2}\Phi$.

Lemma B.3. *All firms (including the manufacturer) are still making positive profit, if each firm is paying the marginal gain as a fee to firms 1 and 2.*

Proof. It suffices to show

$$2(a-c+\epsilon)^2 - (a-c+\alpha\epsilon)^2 > 0$$

$$\Longleftrightarrow (a-c)^2 + (4\epsilon - 2\alpha\epsilon)(a-c) + (2-\alpha^2)\epsilon^2 > 0$$

If $2-\alpha^2 > 0$, then the above inequality holds, as every term is positive.

If $2-\alpha^2 < 0$, because $\epsilon < \frac{n}{2(2n+1)}(a-c)$, then $(a-c)^2 > (\alpha^2-2)\epsilon^2$, as $\frac{1}{\sqrt{\alpha^2-2}} > \frac{n}{2(2n+1)}$. □

Without the upstream manufacturer

Another way to define the compensation is actually to consider the marginal contribution to *only* the downstream firms, i.e. $F = \frac{n}{4(n+1)^2}\Phi$. The reason being that only downstream firms are competing in the same market. However, since the upstream firm is also benefited from the additional piece of technology, he is required to share this lump-sum cost F.

Let us define the cost sharing rule. The total benefit of the economy is given by $\frac{n(n+2)}{4(n+1)^2}\Phi$. The marginal benefit of the upstream firm is $\frac{n}{4(n+1)}\Phi$. And hence, the proportion of his benefit in the whole economy is $\frac{n+1}{n+2}$. And similarly, the proportion of firms $1, 2, ..., n$'s benefit is $\frac{1}{n(n+2)}$.

Therefore, the upstream firm need to pay $\frac{n+1}{n+2}F$ to firms 1 and 2 respectively, and firms $1, 2, ..., n$ will only pay $\frac{1}{n(n+2)}F$ to firms 1 and 2 respectively. Note that firms 1 and 2 also pay to themselves.

Firms $3, 4, ..., n$ are better off, because they are paying less as compared with previous case. The upstream firm is also paying less, because $\frac{n+1}{n+2}F = \frac{n}{4(n+1)(n+2)}\Phi < \frac{n}{4(n+1)}\Phi$.

B.2.4 Comparison of the different compensation schemes

There are pros and cons for using fixed fees as compensation. On one hand, it greatly simplifies the computation, enables us to do comparisons with all different licensing schemes. On the other hand, however, it is non-distortionary. Consumer surplus is not affected by the various schemes of compensation. It is just the surplus transfer from the upstream firm to downstream firms.

Lemma B.4. *Consumer surplus is unchanged before and after the compensation.*

Lemma B.5. *For the manufacturer,* $\pi_m^{rn} > \pi_m^{rl}$ *and* $\pi_m^{rn} > \pi_m^{mc}$.

B.2.5. Proofs on Fixed Fee Compensation

B.2.5.1. Proof of Proposition B.4

Now we would like to check that surrendering the technologies in the first stage is not an equilibrium of the game, meaning that firms have profitable deviation by keeping the technologies for themselves. In order for the upstream firm to acquire the technologies, he must be paying some compensation, F^{rn}, to both of the leading firms. We will investigate this compensation in the following paragraphs.

Consider firm 1 still surrenders the technology to the manufacturer, while firm 2 does not. In this case, firm 2 should pay the same royalty rate as firms $3, 4, ..., n$. But he could enjoy a technology level of $\alpha\epsilon$, while firms 1,3,...,n will only have cost reduction of ϵ. So the profits for firms are: [2]

$$\pi_m = dQ + \sum_{i \in N} r_i q_i,$$
$$\pi_i = (P - d - (c - \epsilon) - r_i)q_i \text{ for } i \in N \backslash \{2\}, \text{ and}$$
$$\pi_2 = (P - d - (c - \alpha\epsilon) - r_2)q_2$$

Note that $r_2 = r_3 = ... = r_n \equiv r_j$ in equilibrium.

Using backward induction, we have:

$$q_1 = \frac{a - c - d + (2 - \alpha)\epsilon - nr_1 + (n - 1)r_j}{n + 1},$$

[2] We omit the superscript for this deviation analysis.

$$q_2 = \frac{a - c - d + n\alpha\epsilon - (n-1)\epsilon + r_1 - 2r_j}{n+1},$$
$$q_3 = ... = q_n = \frac{a - c - d + (2-\alpha)\epsilon + r_1 - 2r_j}{n+1}.$$

Solve $\frac{\partial \pi_m}{\partial d} = 0$, $\frac{\partial \pi_m}{\partial r_1} = 0$, $\frac{\partial \pi_m}{\partial r_j} = 0$. We have only 2 independent equations, and hence:

$$r_1 = -d + \frac{a - c + \epsilon}{2},$$
$$r_j = -d + \frac{(n-1)(a - c + \epsilon) + (\alpha - 1)\epsilon}{2(n-1)}.$$

As the manufacturer is charging fees for both chips and technologies, we can define $r_1^* \equiv r_1 + d = \frac{a-c+\epsilon}{2}$ and $r_j^* \equiv r_j + d = \frac{(n-1)(a-c+\epsilon)+(\alpha-1)\epsilon}{2(n-1)}$, for all $j \in N \backslash \{1\}$.

Check that $r_1^* < r_j^*$, as firm 1 provides the technology, the manufacturer should be giving him some discount.

For notations to be consistent with the rest of the paper, we assume $d = 0$, and therefore $r_1^* = r_1 = \frac{a-c+\epsilon}{2}$ and $r_j^* = r_j = \frac{(n-1)(a-c+\epsilon)+(\alpha-1)\epsilon}{2(n-1)}$. We conclude the equilibrium with the following lemma:

Lemma B.6. *If firm 2 chooses to keep the technology, the equilibrium is*

$$d = 0,$$
$$r_1 = \frac{a - c + \epsilon}{2},$$
$$r_j = \frac{(n-1)(a - c + \epsilon) + (\alpha - 1)\epsilon}{2(n-1)},$$

$$
\begin{aligned}
q_1 &= \frac{a-c+2\epsilon-\alpha\epsilon}{2(n+1)} > 0, \\
q_2 &= \frac{(n-1)(a-c+\alpha\epsilon)+(2n^2-3n-1)(\alpha-1)\epsilon}{2(n^2-1)} > 0, \\
q_3 &= ... = q_n = \frac{(n-1)(a-c)+(3n-1)\epsilon-2n\alpha\epsilon}{2(n^2-1)} \\
&= \frac{(n-1)(a-c+\alpha\epsilon)-(3n-1)(\alpha-1)\epsilon}{2(n^2-1)}.
\end{aligned}
$$

First, without loss of generality, we check $q_3 > 0$.

When $1 < \alpha < \frac{3n-1}{2n} < 2$, $3n-1-2n\alpha > 0$, then $q_3 > 0$.

Otherwise, $3n-1-2n\alpha < 0$. Then $q_3 > 0$ if and only if $\epsilon < \frac{n-1}{2n\alpha+1-3n}(a-c)$. The inequality holds, because $\epsilon < \frac{n}{2(2n+1)} < \frac{n-1}{2n\alpha+1-3n}$.

Next,we could check that $q_1 < q_2$ if and only if $\alpha > 1$. Therefore, firm 2 is indeed making higher profit than firm 1 by keeping the technology. He becomes the only leader in the market, with production cost reduced by $\alpha\epsilon$.

We compare firm 2's profit with the case that he must surrender the technology without any choice. He indeed wants to keep, if and only if $\pi_2 > \pi_2^r$, if and only if $(2n^2-3n-1)(\alpha-1)\epsilon > 0$.

Now we can define $F^{rn} = \pi_2 - \pi_2^r$ to be the compensation. Manufacturer needs to pay F^{rn} to both firms 1 and 2, so that they will be indifferent between surrendering the technology and keeping the technology.

Lemma B.7. *The manufacture is still making profit after paying the compen-*

sation to both firms 1 and 2, i.e. $\pi_m^r - 2F^{rn} > 0$.

The lemma shows that under the reverse licensing regime, the manufacturer is still making positive profit, after paying the compensation to both of the leading firms.

Proof of lemma B.7

Proof.

$$
\begin{aligned}
& \pi_m^r - 2F > 0 \\
\iff & \frac{n}{4(n+1)}(a-c+\alpha\epsilon)^2 - 2F > 0 \\
\iff & (a-c)^2(-1+n)^2 n(1+n) + 2(a-c)\epsilon(-1+n)(-2-6n+4n^2 \\
& + \alpha(2+5n-4n^2+n^3)) - \epsilon^2(2(1+3n-2n^2)^2 - 8\alpha(1+4n-5n^3+2n^4) \\
& + \alpha^2(6+19n-9n^2-15n^3+7n^4)) \\
& > 0
\end{aligned}
$$

First of all, note that $(2(1+3n-2n^2)^2 - 8\alpha(1+4n-5n^3+2n^4) + \alpha^2(6+19n-9n^2-15n^3+7n^4))$ is quadratic in α, with maximum occurs at $\alpha = 2$. The minimum point of the expression is at $\alpha = \frac{8(1+4n-5n^3+2n^4)}{2(6+19n-9n^2-15n^3+7n^4)} < 2$.

If we let $\alpha = 2$, the above equation simplifies to

$$
\begin{aligned}
& (a-c)^2(-1+n)^2 n(1+n) + 4(a-c)\epsilon(-1+n)(1+n(2+(-2+n)n)) \\
& - 2\epsilon^2(5+n(12+n(-13+2(-1+n)n)))
\end{aligned}
$$

With ϵ much smaller than $(a-c)$, the negative term is dominated by the first two positive. And hence, the manufacturer's profit will be positive, and he is willing to pay for the fixed fee to both firms 1 and 2. $\square$

B.2.5.2 Proof of Proposition B.5

Similar to the analysis of previous section, we would like to check that surrendering the technologies in the first stage is not an equilibrium of the game. In order for the upstream firm to acquire the technologies, he must be paying some compensation, F^{rl}, to both of the leading firms. We will investigate this compensation in the following paragraphs.

Without loss of generality, we consider firm 2 keep his technology, while firm 1 still surrender his to the manufacturer. Let r_i^{rl} for all $i \in N$ be the royalty fees charged by the upstream firm. Let r_2 be the royalty fee charged by firm 2 to firms $1, 3, ..., n$. Note that after purchasing patent from firm 2, all the downstream firms possess a technology which will reduce production cost by $\alpha\epsilon$.

The profits for firms are

$$\pi_m = dQ + \sum_{i \in N} r_i^{rl} q_i,$$

$$\pi_i = (P - d - (c - \alpha\epsilon) - r_i^{rl} - r_2)q_i \text{ for } i \in N\backslash\{2\}, \text{ and}$$

$$\pi_2 = (P - d - (c - \alpha\epsilon) - r_2^{rl})q_2 + r_2(q_1 + q_3 + ... + q_n).$$

By backward induction, we have in equilibrium, $r_2^{rl} = r_3^{rl} = ... = r_n^{rl}$. For simplicity, we use r_2^{rl} as the royalty fee charged by the upstream firm to firm 2, 3,... , n.

$$q_1 = \frac{(a-c-d+\alpha\epsilon) - nr_1^{rl} - 2r_2 + (n-1)r_2^{rl}}{n+1},$$
$$q_2 = \frac{(a-c-d+\alpha\epsilon) - 2r_2^{rl} + r_1^{rl} + (n-1)r_2}{n+1},$$
$$q_3 = ... = q_n = \frac{(a-c-d+\alpha\epsilon) - 2r_2 - 2r_2^{rl} + r_1^{rl}}{n+1},$$
$$r_2 = \frac{(a-c)(n-1)(n+3) - n(n+2)(r_2^{rl} - \alpha\epsilon) - 3\alpha\epsilon - 4r_1^{rl} + 7r_2^{rl} + 3d}{2(n-1)(n+3)}.$$

Note that from the positive production condition, i.e. $q_3 = ... = q_n > 0$, we also require $r_2 \leq \frac{1}{2}(a-c-d+\alpha\epsilon - 2r_2^{rl} + r_1^{rl})$.

Solve the problem:

$$d = 0,$$
$$r_1^{rl} = \frac{(n(n(2n+3)-8)-1)}{2n(n(3n+4)-13)+4}(a-c+\alpha\epsilon),$$
$$r_2^{rl} = ... = r_n^{rl} = \frac{((n-1)n(n+3)-2)}{n(n(3n+4)-13)+2}(a-c+\alpha\epsilon).$$

Check that $r_1^{rl} < r_2^{rl}$, because firm 1 provides the technology, and he could enjoy some discount on the chips.

Check also

$$r_2 = \min\{\frac{(n-1)^2(n+3)}{n(n(3n+4)-13)+2}(a-c+\alpha\epsilon), \frac{(n(n(2n+7)-6)-19)(a-c+\alpha\epsilon)}{4(n(2n(n+4)-3)-23)}\}$$
$$= \frac{(n(n(2n+7)-6)-19)}{4(n(2n(n+4)-3)-23)}(a-c+\alpha\epsilon).$$

And therefore, in this case,

$$\begin{aligned}
q_1 &= \frac{(n-1)(n+3)}{2n(2n(n+4)-3)-46}(a-c+\alpha\epsilon),\\
q_2 &= \frac{(n(n(2n+7)-6)-19)}{4(n(2n(n+4)-3)-23)}(a-c+\alpha\epsilon),\\
q_3 &= ... = q_n = 0.
\end{aligned}$$

Firm 2 is unable to charge higher royalty rates to achieve highest profit by keeping and licensing the technology.

We can now define the compensation to be

$$F^{rl} = (q_2)^2 + r_2(q_1 + q_3 + ... + q_n) - \pi_2^r$$

Lemma B.8. *The manufacture is still making profit after paying the compensation to both firms 1 and 2, i.e.* $\pi_m^r - 2F^{rl} > 0$.

Proof. We need to check

$$\begin{aligned}
&\frac{n}{4(n+1)}(a-c+\alpha\epsilon)^2 - 2F > 0\\
\iff &4n^7 + 20n^6 + 9n^5 - 45n^4 - 56n^3 - 198n^2 - 25n + 547 > 0
\end{aligned}$$

the inequality holds with $n \geq 3$, because $4n^7 - 45n^4 > 0$, $20n^6 - 56n^3 > 0$ and $9n^5 - 198n^2 - 25n > 9n^5 - 223n^2 > (9 \times 27 - 223)n^2 > 0$. □

Under the reverse licensing regime, the manufacturer is still making positive profit, after paying the compensation to both of the leading firms.

B.2.5.3 Prove lemma 9 when $n = 3$

We can show the relationships by numerical methods, here we just present an analytical solution when $n = 3$,

$$\pi_m^{rn} = \frac{1}{16}(2(\alpha+2)(a-c)\epsilon + ((20-9\alpha)\alpha - 8)\epsilon^2 + 3(a-c)^2),$$
$$\pi_m^{rl} = \frac{7143((a-c)+\alpha\epsilon)^2}{70688},$$
$$\pi_m^{mc} = \frac{3}{16}(-(\alpha^2-2)\epsilon^2 - 2(\alpha-2)(a-c)\epsilon + (a-c)^2).$$

Now

$$\pi_m^{rn} - \pi_m^{mc} > 0$$
$$\iff \frac{1}{8}(-1+\alpha)\epsilon(4(a-c) + (7-3\alpha)\epsilon) > 0$$

The inequality holds, as $\alpha \in (1, 2]$.

$$\pi_m^{rn} - \pi_m^{rl} > 0$$
$$\iff 6111(a-c)^2 + 2(8836 - 2725\alpha)(a-c)\epsilon$$
$$-(35344 - 88360\alpha + 46905\alpha^2)\epsilon^2 > 0$$

Now $\max_\alpha(35344 - 88360\alpha + 46905\alpha^2) = 46244$ when $\alpha = 2$. Now $\max \epsilon = \frac{1}{4}(a-c)$. Therefore $\max(35344 - 88360\alpha + 46905\alpha^2)\epsilon^2 = 2890.25 < 6111$.

Hence, $\pi_m^{rn} > \pi_m^{rl}$.

Appendix C

Proofs and Details of Chapter Three

C.1 Proofs

C.1.1 Proof for Lemma 3.1

First, we want to show $W^L(0) > W^{N'}(t)$ for $t \in [\frac{1}{2-\alpha}, 1]$. Since both $W^L(t)$ and $W^{N'}(t)$ are decreasing in t, then $W^L(t)$ is maximized at $t = 0$ for $t \in [0, \frac{2}{5-3\alpha}]$, and $W^{N'}(t)$ is maximized at $t = \frac{1}{2-\alpha}$ for $t \in [\frac{1}{2-\alpha}, 1]$. Moreover, we have $W^L(t) > W^{N'}(t)$ for $t \in [0, 1]$, then $W^L(0) > W^L(\frac{1}{2-\alpha}) > W^{N'}(\frac{1}{2-\alpha})$. Thus we have shown $W^L(0) > W^{N'}(t)$ for $t \in [\frac{1}{2-\alpha}, 1]$.

Second, we aim to show that $W^L(0) > W^N(t)$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$. Define $\beta^N \equiv \frac{(1+\alpha)^2}{4(1+\alpha^2-\alpha)^2} < \frac{1}{\alpha^2}$, and we consider the optimal choice of the tax rate (t^*) depending on the value of β.

If $\beta < \beta^N$, then $W^N(t)$ is convex. It can be checked easily that $W^N(t)$ is decreasing in t for $\frac{2}{5-3\alpha} \leq t \leq \frac{1}{2-\alpha}$, and thus is maximized at $t = \frac{2}{5-3\alpha}$.

If $\beta = \beta^N$, then $W^N(t)$ is linear and decreasing in t, and thus is maximized at $t = \frac{2}{5-3\alpha}$.

If $\beta > \beta^N$, then $W^N(t)$ is concave. Let t^N solve $\frac{\partial W^N(t)}{\partial t} = 0$, where

$$t^N = \frac{2(1+\alpha)(1-\beta(1+\alpha^2-\alpha))}{(1+\alpha)^2 - 4\beta(1+\alpha^2-\alpha)^2}.$$

Then the optimal tax rate is $t^* = \frac{2}{5-3\alpha}$ if $t^N < \frac{2}{5-3\alpha}$; $t^* = t^N$ if $\frac{2}{5-3\alpha} < t^N < \frac{1}{2-\alpha}$; and $t^* = \frac{1}{2-\alpha}$ if $t^N > \frac{1}{2-\alpha}$. We know that $t^N < \frac{2}{5-3\alpha}$ if $\beta < \underline{\beta}^N$ and $t^N > \frac{1}{2-\alpha}$ if $\beta > \overline{\beta}^N$, where $\underline{\beta}^N \equiv \frac{4(1+\alpha)}{(1+7\alpha)(1-\alpha+\alpha^2)}$, $\overline{\beta}^N \equiv \frac{1+\alpha}{2\alpha(1-\alpha+\alpha^2)}$, and $\beta^N < \underline{\beta}^N < \overline{\beta}^N < \frac{1}{\alpha^2}$.

The following lemma summarizes the result on the optimal tax rate.

Lemma C.1. *When $\beta < \underline{\beta}^N$, the optimal tax rate is $t^* = \frac{2}{5-3\alpha}$; when $\underline{\beta}^N < \beta < \overline{\beta}^N$, the optimal tax rate is $t^* = t^N$; when $\overline{\beta}^N < \beta < \frac{1}{\alpha^2}$, the optimal tax rate is $t^* = \frac{1}{2-\alpha}$.*

Given the optimal tax rate t^*, the maximum of $W^N(t)$ is

$$W^N(t^*) = \begin{cases} W^N(\frac{2}{5-3\alpha}) = \frac{(1-\alpha)^2(64-(1+7\alpha)^2\beta)}{9(5-3\alpha)^2}, & \text{if } \beta < \underline{\beta}^N, \\ W^N(t^N) = \frac{(1-\alpha)^4\beta}{4(1-\alpha+\alpha^2)^2\beta-(1+\alpha)^2}, & \text{if } \underline{\beta}^N < \beta < \overline{\beta}^N, \\ W^N(\frac{1}{2-\alpha}) = \frac{(1-\alpha)^2(1-\beta\alpha^2)}{(2-\alpha)^2}, & \text{if } \overline{\beta}^N < \beta < \frac{1}{\alpha^2}. \end{cases}$$

Now we are ready to show $W^L(0) > W^N(t^*)$.

If $\beta < \underline{\beta}^N$, the optimal tax rate is $t^* = \frac{2}{5-3\alpha}$. Note that $W^N(\frac{2}{5-3\alpha})$ and $W^L(0)$ are both linear and decreasing in β. It is easy to check that when $\beta = 0$, $W^L(0) > W^N(\frac{2}{5-3\alpha})$; and when $\beta = \frac{1}{\alpha^2}$, $W^L(0) = 0 > W^N(\frac{2}{5-3\alpha})$. Therefore $W^L(0) > W^N(\frac{2}{5-3\alpha})$.

If $\overline{\beta}^N < \beta < \frac{1}{\alpha^2}$, the optimal tax rate is $t^* = \frac{1}{2-\alpha}$. We have $W^N(\frac{1}{2-\alpha}) < W^L(0)$, because $\frac{(1-\alpha)^2}{(2-\alpha)^2} < \frac{4}{9}$ if and only $\alpha > -1$.

If $\underline{\beta}^N < \beta < \overline{\beta}^N$, the optimal tax rate is $t^* = t^N$. It can be checked that $W^N(t^*) > W^L(0)$ if $\beta < \beta_1$ or $\beta > \beta_2$, where β_1 and β_2 are two thresholds such that $\beta_1 < \beta_2$; and $W^N(t^*) < W^L(0)$ if $\beta_1 < \beta < \beta_2$. Since we have $W^N(\underline{\beta}^N) < W^L(0)$ and $W^N(\overline{\beta}^N) < W^L(0)$, then for any $\underline{\beta}^N < \beta < \overline{\beta}^N$, $W^N(t^*) < W^L(0)$.

C.1.2 Proof for Lemma 2

First of all, we want to show $W^{N'}(1) > W^L(t)$ for $t \in [0, \frac{2}{5-3\alpha}]$. Note that $0 > W^{N'}(t) > W^L(t)$ for all $t \in [0,1]$. Since $W^{N'}(t)$ and $W^L(t)$ are both increasing in t, $W^L(t)$ is maximized at $t = \frac{2}{5-3\alpha}$, and $W^{N'}(t) > W^{N'}(\frac{2}{5-3\alpha}) > W^L(\frac{2}{5-3\alpha})$.

Second, we want to show $W^{N'}(1) > W^N(t)$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$. Following Lemma 1, we know that for $\beta > \frac{1}{\alpha^2}$, $W^N(t)$ is maximized at $t = \frac{1}{2-\alpha}$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$. Note also $W^N(\frac{1}{2-\alpha}) = W^{N'}(\frac{1}{2-\alpha})$ and $W^{N'}(t)$ is increasing in t, we must have $W^{N'}(1) > W^N(\frac{1}{2-\alpha})$.

C.1.3 Proof for Propositon 3.2

When $0 < t < \frac{1}{2-\alpha}$, $Q^N(t) = \frac{2-(1+\alpha)t}{3}$ and $E^N(t) = \frac{1+\alpha+\alpha(1-2\alpha)t+(\alpha-2)t}{3}$. When $\frac{1}{2-\alpha} < t < 1$, $Q^{N'}(t) = \frac{1-\alpha t}{2}$ and $E^{N'}(t) = \frac{\alpha(1-\alpha t)}{2}$. For any t, we have $Q^L(t) = \frac{2(1-\alpha t)}{3}$ and $E^L(t) = \alpha Q^L(t)$.

Clearly $Q^L(t) > Q^N(t)$ and $Q^L(t) > Q^{N'}(t)$, indicating licensing will always give us higher consumer surplus.

It is also easy to verify that $E^L(t) > E^{N'}(t)$.

Check that $E^L(t) < E^N(t)$ if and only if $2-(1+\alpha)t < 1+\alpha+\alpha(1-2\alpha)t+(\alpha-2)t$ if and only if $t < \frac{1}{2}$. Note that $\frac{1}{2-\alpha} < \frac{1}{2}$. This completes the proof.

C.1.4 Proof for Lemma 3.3

Consider the three possible equilibria in stage 2: licensing equilibrium, bribing equilibrium, and equilibrium with no licensing and no bribing.

1. Licensing equilibrium

In any licensing equilibrium, denote the equilibrium licensing fee by F^*. Then we have two cases:

First, if $\pi_2^B \geq \pi_2^N$, then the equilibrium conditions are

$$\begin{aligned} \pi_1^L &\geq \pi_1^B, \\ \pi_2^L &\geq \pi_2^B, \end{aligned}$$

where the equilibrium offer is $F^* = \left(\frac{1-\alpha t}{3}\right)^2 - \left(\frac{1+(\alpha-2\alpha')t}{3}\right)^2 + b > 0$. In other words, the equilibrium condition becomes

$$\pi_1^L + \pi_2^L \geq \pi_1^B + \pi_2^B.$$

Second, if $\pi_2^B < \pi_2^N$, then the equilibrium conditions are

$$\begin{aligned}\pi_1^L &\geq \pi_1^N, \\ \pi_2^L &\geq \pi_2^N,\end{aligned}$$

where the equilibrium offer is $F^* = \left(\frac{1-\alpha t}{3}\right)^2 - \left(\frac{1+(\alpha-2)t}{3}\right)^2 > 0$. We can also rewrite the condition as

$$\pi_1^L + \pi_2^L \geq \pi_1^N + \pi_2^N.$$

2. Bribing equilibrium

First, firm 2 does not want to deviate by not bribing, i.e.,

$$\pi_2^B \geq \pi_2^N.$$

Second, firm 1 cannot profitably deviate by licensing to firm 2, i.e. for any $F \geq 0$ such that $\pi_2^L > \pi_2^B$, we must have $\pi_1^L < \pi_1^B$. In equilibrium, $F^* = \left(\frac{1-\alpha t}{3}\right)^2 - \left(\frac{1+(\alpha-2\alpha')t}{3}\right)^2 + b$, and the conditions are

$$\begin{aligned}\pi_2^B &\geq \pi_2^N, \\ \pi_1^B + \pi_2^B &\geq \pi_1^L + \pi_2^L.\end{aligned}$$

3. Equilibrium with no licensing and no bribing

First, firm 2 does not want to deviate by bribing the bureaucrat, i.e.,

$$\pi_2^N \geq \pi_2^B.$$

Second, firm 1 cannot profitably deviate by licensing to firm 2, i.e., for any $F \geq 0$ such that $\pi_2^L > \pi_2^N$, we must have $\pi_1^L < \pi_1^N$. In equilibrium, $F^* = \left(\frac{1-\alpha t}{3}\right)^2 - \left(\frac{1+(\alpha-2)t}{3}\right)^2$, and the conditions are

$$\begin{aligned} \pi_1^N + \pi_2^N &\geq \pi_1^L + \pi_2^L, \\ \pi_2^N &\geq \pi_2^B - b. \end{aligned}$$

4. Summary

If $\pi_2^B \geq \pi_2^N$, then licensing is an equilibrium if $\pi_1^L + \pi_2^L \geq \pi_1^B + \pi_2^B$, otherwise bribing is an equilibrium. If $\pi_2^B < \pi_2^N$, then licensing is an equilibrium if $\pi_1^L + \pi_2^L \geq \pi_1^N + \pi_2^N$, otherwise no licensing and no bribing is an equilibrium.

C.1.5 Proof of Proposition 3.4

First of all, we have

$$\begin{aligned} Q^L(t) &= \frac{2}{3}(1-\alpha t), \\ Q^{N'}(t) &= \frac{1}{2}(1-\alpha t), \\ \text{and } Q^B(t) &= \frac{2-\alpha' t - \alpha t}{3}. \end{aligned}$$

Because of Proposition 3.2, it suffices for us to show $Q^L(t) > Q^B(t)$, which is trivial, and $Q^B(t) > Q^{N'}(t)$, if and only if $1+(\alpha-2\alpha')t > 0$. Note that bribing could be an equilibrium only when $0 < \sigma < \frac{1}{2}$ if and only if $1 + \alpha - 2\alpha' > 0$.

Next, we have

$$E^L(t) = \alpha\frac{2}{3}(1-\alpha t),$$
$$E^{N'}(t) = \alpha\frac{1}{2}(1-\alpha t),$$
$$\text{and } E^B(t) = \alpha\frac{1+\alpha' t - 2\alpha t}{3} + \frac{1-2\alpha' t + \alpha t}{3}.$$

We know $E^{N'}(t) < E^L(t)$.

Check $E^B(t) - E^{N'}(t) > 0$ if and only if $(2\alpha\alpha' - \alpha^2 + 2\alpha - 4\alpha')t + 2 - \alpha > 0$. Note $(2\alpha\alpha' - \alpha^2 + 2\alpha - 4\alpha') < 0$, therefore it suffices for us to show $E^{N'}(1) < E^B(1)$, if and only if $(2-\alpha)(1-\alpha)(1-2\sigma) > 0$. Note that bribing could be an equilibrium only when $0 < \sigma < \frac{1}{2}$. Therefore, for all t where bribing equilibrium is possible, we have $E^B(t) > E^{N'}(t)$.

Now we prove that it is possible that $E^B(t) < E^L(t)$. Consider $t = 1$, we have $E^B(1) < E^L(1)$ if and only if $1 - \alpha + \alpha\sigma - 2\sigma < 0$ if and only if $\alpha > \frac{1-2\sigma}{1-\sigma}$, which is one of the sufficient condition for bribing to be better than licensing in equilibrium. And hence, we conclude that when bribing is possible, we have, at least, $E^B(1) < E^L(1)$. Therefore, it is possible that bribing yield lower pollution than licensing.

C.1.6 Proof for Lemma 3.6

The proof for $W^L(0) > W^N(t)$ for $t \in [\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$ and $W^L(0) > W^{N'}(t)$ for $t \in [\frac{1}{2-\alpha}, 1]$ is identical to that in Lemma 3.1. We just need to show

$W^L(0) > W^B(t)$ for $t \in [\frac{2}{5\alpha'-3\alpha}, 1]$.

Let t^B solve $\frac{\partial W^B(t)}{\partial t} = 0$, where

$$t^B = \frac{2(\alpha'+\alpha) - \beta(1+\alpha)(-\alpha\alpha' + 2\alpha^2 - \alpha + 2\alpha')}{(\alpha'+\alpha)^2 - \beta(\alpha\alpha' - 2\alpha^2 + \alpha - 2\alpha')^2}.$$

Similarly, we define

$$\begin{aligned}
\beta^B &\equiv \frac{(\alpha'+\alpha)^2}{(\alpha\alpha' - 2\alpha^2 + \alpha - 2\alpha')^2}, \\
\underline{\beta}^B &\equiv \frac{8(\alpha'+\alpha)}{(1+7\alpha)(-\alpha\alpha' + 2\alpha^2 - \alpha + 2\alpha')}, \\
\bar{\beta}^B &\equiv \frac{(\alpha'+\alpha)(2-\alpha'-\alpha)}{(1+\alpha+\alpha\alpha' - 2\alpha^2 + \alpha - 2\alpha')(-\alpha\alpha' + 2\alpha^2 - \alpha + 2\alpha')}.
\end{aligned}$$

Then we have $0 < \beta^B < \underline{\beta}^B < \overline{\beta}^B < \frac{1}{\alpha^2}$.

The following lemma summarizes the result on the optimal choice of the tax rate (t^*) depending on the value of β.

Lemma C.2. *When $\beta < \underline{\beta}^B$, the optimal tax rate is $t^* = \frac{2}{5\alpha'-3\alpha}$; when $\underline{\beta}^B < \beta < \overline{\beta}^B$, the optimal tax rate is $t^* = t^B$; when $\overline{\beta}^B < \beta < \frac{1}{\alpha^2}$, the optimal tax rate is $t^* = 1$.*

Thus the maximum of $W^B(t)$ is

$$W^B(t^*) = \begin{cases} W^B(t^B) = \frac{\beta(1-\alpha)^2(\alpha'-\alpha)^2}{\beta(\alpha\alpha'-2\alpha^2+\alpha-2\alpha')^2-(\alpha'+\alpha)^2}, & \text{if } \underline{\beta}^B < \beta < \overline{\beta}^B, \\ W^B(\frac{2}{5\alpha'-3\alpha}) = \frac{(\alpha'-\alpha)^2(64-\beta(1+7\alpha)^2)}{9(5\alpha'-3\alpha)^2}, & \text{if } \beta < \underline{\beta}^B, \\ W^B(1) = \frac{(2-(\alpha+\alpha'))^2-\beta(1+\alpha+(\alpha\alpha'-2\alpha^2+\alpha-2\alpha'))^2}{9}, & \text{if } \beta > \overline{\beta}^B. \end{cases}$$

Following the same argument as in Lemma 3.1, $W^L(0) > W^B(t^*)$.

C.1.7 Proof of Theorem 3.3

We will focus on Part (3) of Theorem 3. The proof of Part (1) is trivial, and the proof of Part (2) is similar to that of Part (3). To show this, we follow three steps.

First of all, we want to show $W^L(t)$ can never be optimal. Note that $0 > W^{N'}(t) > W^L(t)$ for all $t \in [0,1]$. Note also $W^{N'}(t)$ and $W^L(t)$ are both increasing in t. Let $\tilde{t}^L \equiv \arg\max_t W^L(t)$, and clearly $W^{N'}(1) > W^L(\tilde{t}^L)$.

Second, $W^N(t)$ can never be optimal. Following the proof of Lemma 3.1, we know that for $\beta > \frac{1}{\alpha^2}$, within the largest domain $[\frac{2}{5-3\alpha}, \frac{1}{2-\alpha}]$, $W^N(t)$ is maximized at $t = \frac{1}{2-\alpha}$. Note also $W^N(\frac{1}{2-\alpha}) = W^{N'}(\frac{1}{2-\alpha})$ and $W^{N'}(t)$ is increasing in t, we must have $W^{N'}(1) > W^N(\frac{1}{2-\alpha})$.

Lastly, $W^B(t)$ can never be optimal. When $\beta > \frac{1}{\alpha^2}$, $W^B(t)$ is increasing in t. Let $\tilde{t}^B \equiv \arg\max_t W^B(t)$, then it suffices for us to show $W^{N'}(1) > W^B(1)$ and hence $W^{N'}(1) > W^B(\tilde{t}^B)$.

Lemma C.3. $W^{N'}(1) > W^B(1)$.

Proof. It is easy to show that when $\beta = \frac{1}{\alpha^2}$, $W^{N'}(1) = 0 > W^B(1)$.

Now consider the slope of $W^{N'}(1)$ and $W^B(1)$. Let

$$S^{N'}(\alpha) \equiv -\frac{1}{4}\alpha^2(1-\alpha)^2$$
$$\text{and } S^B(\alpha) \equiv -\frac{1}{9}(1+2\alpha+\alpha\alpha'-2\alpha^2-2\alpha')^2$$

Solve $S^{N'}(\alpha) = S^B(\alpha)$, we have $\alpha = 1$, $\alpha = 2$ and $\alpha = \frac{2(2\sigma-1)}{5+2\sigma} < 0$. Therefore $W^B(1)$ always has a steeper slope, and hence $W^{N'}(1) > W^B(1)$ for all $\beta > \frac{1}{\alpha^2}$. $\square$

Therefore, we can conclude that when $\sqrt{2}-1 < \sigma < \frac{1}{2}$, if $b > g(1)$, $W^{N'}(1)$ will be the equilibrium outcome.

The rest of the proof can be easily shown in figure 3.2.

C.1.8 Proof for Proposition 3.5

First, we express this proposition in detail:

Proposition 3.5*. *(Sufficient condition for bribing to be the equilibrium)*

Let $\beta^* = \frac{(\alpha'-\alpha)(4-3\alpha-\alpha')}{-(1-2\alpha'+\alpha\alpha')(1-2\alpha'+\alpha\alpha'+4\alpha-4\alpha^2)}$, *if* $\frac{1-2\sigma}{1-\sigma} < \alpha < 1$ *and* $\beta > \beta^*$,

(1) When $\frac{2}{5} < \sigma < \sqrt{2}-1$, *if* $b < h(1)$, *then government should set tax rate at 1, and bribing is the equilibrium.*

(2) When $\sqrt{2}-1 < \sigma < \frac{1}{2}$, *if* $b < g(1)$, *then government should set tax rate at 1, and bribing is the equilibrium.*

Next, we prove the theorem.

In both cases, define $\tilde{t} = h^{-1}(b) < 1$, we need to compare $W^B(1)$ with $W^L(\tilde{t})$. But we know that when $\beta > \frac{1}{\alpha^2}$, $W^L(t)$ is increasing in t. Therefore the sufficient condition for bribing to be the equilibrium is to show $W^B(1) > W^L(1)$.

It is easy to show that when $\beta = \frac{1}{\alpha^2}$, $W^L(1) = 0 > W^B(1)$.

Now consider the slope of $W^L(1)$ and $W^B(1)$. Let

$$S^L(\alpha) \equiv -\frac{4}{9}\alpha^2(1-\alpha)^2$$
$$\text{and } S^B(\alpha) \equiv -\frac{1}{9}(1 + 2\alpha + \alpha\alpha' - 2\alpha^2 - 2\alpha')^2$$

Solve $S^L(\alpha) = S^B(\alpha)$, we have $\alpha = 1$, $\alpha = \frac{1-2\sigma}{1-\sigma} < 1$ and $\alpha = \frac{2\sigma-1}{3+\sigma} < 0$. Therefore when $\frac{1-2\sigma}{1-\sigma} < \alpha < 1$, $0 > S^B(\alpha) > S^L(\alpha)$, indicating $W^L(1)$ has a steeper slope. Hence, there exists $\beta^* = \frac{(\alpha'-\alpha)(4-3\alpha-\alpha')}{-(1-2\alpha'+\alpha\alpha')(1-2\alpha'+\alpha\alpha'+4\alpha-4\alpha^2)} > \frac{1}{\alpha^2}$, which solves $W^L(1) = W^B(1)$. Thus $W^L(1) > W^B(1)$ when $\frac{1}{\alpha^2} < \beta < \beta^*$; otherwise, $W^B(1) > W^L(1)$.

C.1.9 Proof of Proposition 3.6

Proposition 5*. *(Sufficient condition for licensing to be the equilibrium)*

When $\frac{2}{5} < \sigma < \sqrt{2} - 1$, if $b < h(1)$; or when $\sqrt{2} - 1 < \sigma \leq \frac{1}{2}$, if $b < g(1)$; in addition, if $\beta > \frac{1}{\alpha^2}$ and $\alpha < \frac{15\sigma^2 + \sqrt{25\sigma^4 - 440\sigma^3 + 396\sigma^2 - 96\sigma + 4} - 14\sigma + 2}{2(5\sigma^2 + 3\sigma - 2)}$, government should charge $t = \tilde{t}$, and licensing will be the equilibrium.

We know that $W^L(t)$ is increasing with the minimum occurs at $W^L(\frac{2}{5\alpha' - 3\alpha})$. The sufficient condition for licensing to be the equilibrium requires $W^L(\frac{2}{5\alpha' - 3\alpha}) > W^B(1)$.

When $\beta = \frac{1}{\alpha^2}$, $W^L(\frac{2}{5\alpha' - 3\alpha}) = 0 > W^B(1)$.

Now consider the slope of $W^L(\frac{2}{5\alpha'-3\alpha})$ and $W^B(1)$. Let

$$\bar{S}^L(\alpha) \equiv -\frac{4}{9}\alpha^2(1-\frac{2\alpha}{5\alpha'-3\alpha})^2$$
$$\text{and } S^B(\alpha) \equiv -\frac{1}{9}(1+2\alpha+\alpha\alpha'-2\alpha^2-2\alpha')^2$$

Solve $\bar{S}^L(\alpha) = S^B(\alpha)$, we have $\alpha = \frac{15\sigma^2+\sqrt{25\sigma^4-440\sigma^3+396\sigma^2-96\sigma+4}-14\sigma+2}{2(5\sigma^2+3\sigma-2)} < 1$. Therefore when $\alpha < \frac{15\sigma^2+\sqrt{25\sigma^4-440\sigma^3+396\sigma^2-96\sigma+4}-14\sigma+2}{2(5\sigma^2+3\sigma-2)}$, $0 > \bar{S}^L(\alpha) > S^B(\alpha)$, indicating $W^B(1)$ has a steeper slope. And hence, $W^L(\frac{2}{5\alpha'-3\alpha}) > W^B(1)$.

C.2 Detailed Discussions

C.2.1 Detailed discussion on firms' equilibrium decision when $0 < \sigma < \frac{1}{2}$

We present the equilibrium decisions in the main text for the case $\sqrt{2} - 1 < \sigma < \frac{1}{2}$. Now we present the detail results for the whole range $0 < \sigma < \frac{1}{2}$ in the following proposition.

Proposition C.1. *Suppose $0 < \sigma < \frac{1}{2}$. Then the equilibria in stage 2 are summarized as below:*

- *Case 1:* $0 < \sigma < \frac{2}{5}$

  1. *If $t \leq \frac{2}{5-3\alpha}$, licensing is an equilibrium;*
  2. *If $\frac{2}{5-3\alpha} < t < \frac{1}{2-\alpha}$, then if $b > f(t)$, no licensing and no bribing is an equilibrium; if $b < f(t)$, licensing is an equilibrium.*
  3. *If $\frac{1}{2-\alpha} < t < 1$, then if $b > g(t)$, no licensing and no bribing is an equilibrium; if $b < g(t)$, licensing is an equilibrium.*

- *Case 2:* $\frac{2}{5} < \sigma < \sqrt{2} - 1$

  1. *If $t \leq \frac{2}{5-3\alpha}$, licensing is an equilibrium;*
  2. *If $\frac{2}{5-3\alpha} < t < \frac{1}{2-\alpha}$, then if $b > f(t)$, no licensing and no bribing is an equilibrium; if $b < f(t)$, licensing is an equilibrium;*

3. *If* $\frac{1}{2-\alpha} < t < \frac{2}{5\alpha'-3\alpha}$, *then if* $b > g(t)$, *no licensing and no bribing is an equilibrium; if* $b < g(t)$, *licensing is an equilibrium;*

4. *If* $\frac{2}{5\alpha'-3\alpha} < t < 1$, *then if* $b > g(t)$, *no licensing and no bribing is an equilibrium; if* $h(t) < b < g(t)$, *licensing is an equilibrium; if* $b < h(t)$, *bribing is an equilibrium.*

- *Case 3:* $\sqrt{2} - 1 < \sigma < \frac{1}{2}$

   1. *If* $t \leq \frac{2}{5-3\alpha}$, *licensing is an equilibrium;*

   2. *If* $\frac{2}{5-3\alpha} < t < \frac{1}{2-\alpha}$, *then if* $b > f(t)$, *no licensing and no bribing is an equilibrium; if* $b < f(t)$, *licensing is an equilibrium;*

   3. *If* $\frac{1}{2-\alpha} < t < \frac{2}{5\alpha'-3\alpha}$, *then if* $b > g(t)$, *no licensing and no bribing is an equilibrium; if* $b < g(t)$, *licensing is an equilibrium;*

   4. *If* $\frac{2}{5\alpha'-3\alpha} < t < t^*$, *if* $b > g(t)$, *no licensing and no bribing is an equilibrium is an equilibrium; if* $g(t) < b < h(t)$, *L;* $b < h(t)$, *bribing is an equilibrium;*

   5. $t^* < t < 1$, $b > g(t)$, *no licensing and no bribing is an equilibrium;* $b < g(t)$, *bribing is an equilibrium.*

We have the equilibrium regions indicated in the following figures.

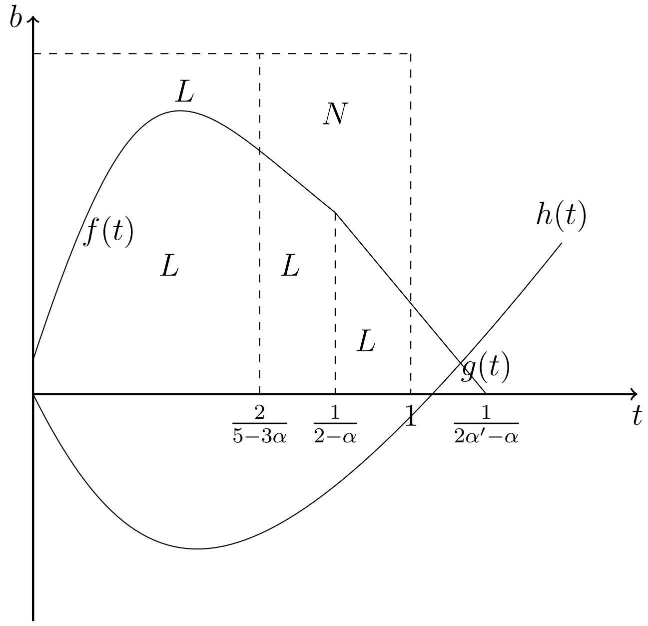

Figure C.2.1: Equilibrium Region for Case 1

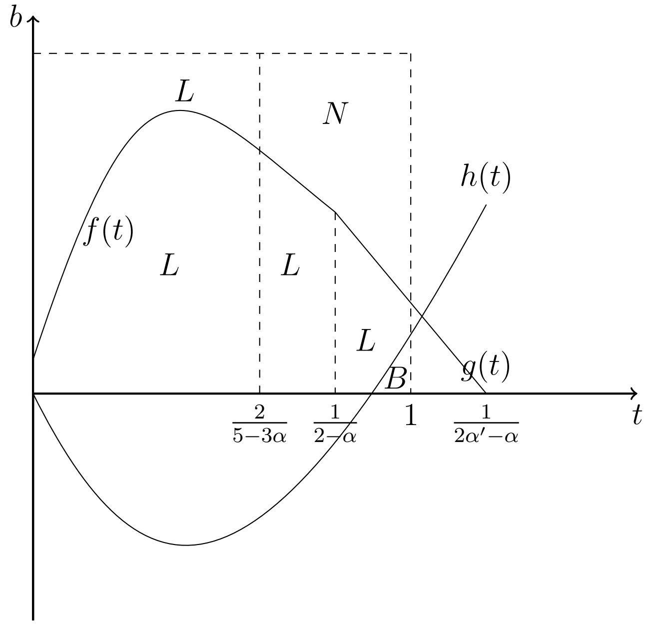

Figure C.2.2: Equilibrium Region for Case 2

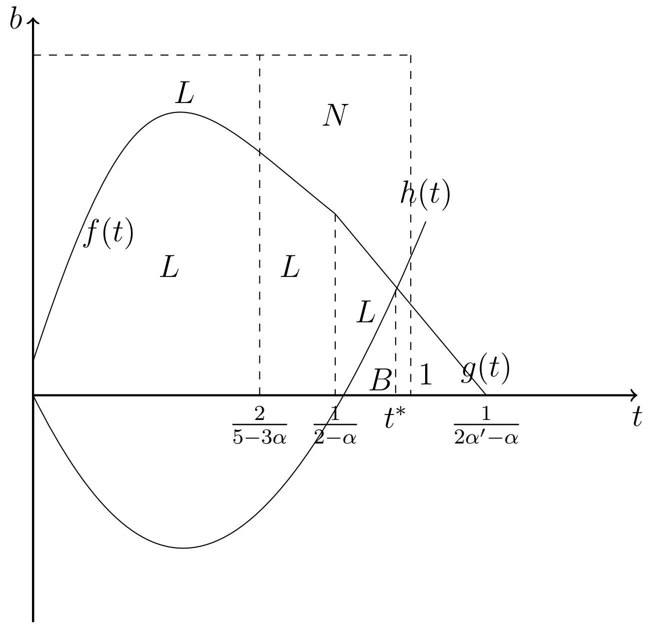

Figure C.2.3: Equilibrium Region for Case 3

C.2.2 Government puts effort on anti-corruption

Our discussion will focus on the case $\frac{1}{2} < \sigma < 1$, under which firm 2 may exit the market in No licensing and no bribing case and Bribing case.

The competition analysis between firm 1 and 2 are the same as presented in the main text. We will focus on firm 2's choice and government optimal tax.

C.2.2.1 Firm 2's Choice

In this case, firm 2 will exit the market in bribing case, when $\frac{1}{2\alpha'-\alpha} < t \leq 1$. Therefore, bribing could be an equilibrium, if

$$\hat{B}(t) = \begin{cases} f(t), & \text{if } 0 \le t \le \frac{1}{2-\alpha}, \\ g(t), & \text{if } \frac{1}{2-\alpha} < t \le \frac{1}{2\alpha'-\alpha}. \end{cases}$$

Note the difference between $B(t)$ in the main text and $\hat{B}(t)$ is the range of $g(t)$. The properties of functions $f(t)$, $g(t)$ and $h(t)$ remain the same. But the relationship among them changes due to the possible exit of firm 2 in the bribing case. Nevertheless we have Figure C.2.4 and C.2.5, analogous to Figure 3.1 and 3,2 in the main text. [1]

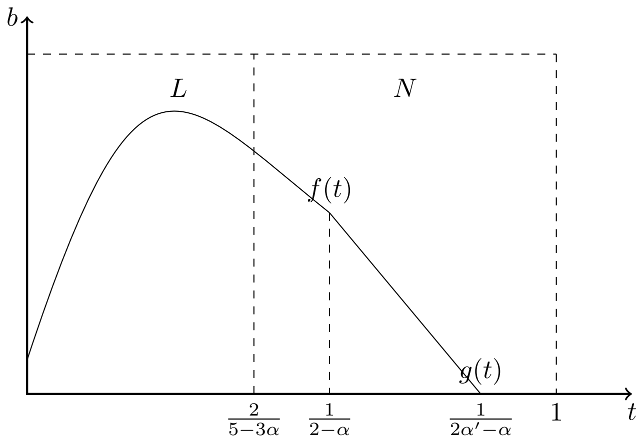

Figure C.2.4: Equilibrium Region for Licensing and No

C.2.2.2 Optimal Taxation Policy

The social welfare functions are exactly the same as in the main text. The only difference is the largest domain for $W^B(t)$ becomes to $\frac{2}{5\alpha'-3\alpha} < t < \frac{1}{2\alpha'-\alpha}$.

[1] We have three different cases depending on value of σ. Here we just present one possible case, where $\frac{1}{2} < \sigma < \frac{4}{5}$.. The detailed discussion is in the next section.

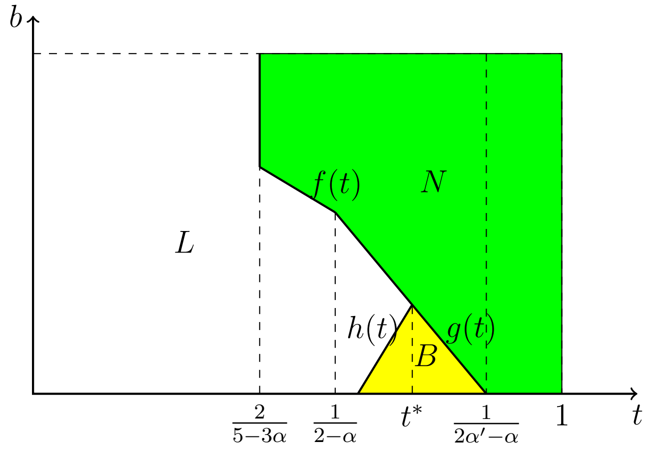

Figure C.2.5: Equilibrium Region when $\frac{1}{2} < \sigma < \frac{4}{5}$

Consider the t^B, which solves $\frac{\partial W^B(t)}{\partial t} = 0$, where

$$t^B = \frac{2(\alpha' + \alpha) - \beta(1+\alpha)(-\alpha\alpha' + 2\alpha^2 - \alpha + 2\alpha')}{(\alpha' + \alpha)^2 - \beta(\alpha\alpha' - 2\alpha^2 + \alpha - 2\alpha')^2}.$$

Similarly, we define $\beta^B \equiv \frac{(\alpha'+\alpha)^2}{(\alpha\alpha'-2\alpha^2+\alpha-2\alpha')^2}$, $\underline{\beta}^B \equiv \frac{8(\alpha'+\alpha)}{(1+7\alpha)(-\alpha\alpha'+2\alpha^2-\alpha+2\alpha')}$, and $\overline{\beta}^{B'} \equiv \frac{\alpha'+\alpha}{\alpha(-\alpha\alpha'+2\alpha^2-\alpha+2\alpha')}$. Then we have $0 < \beta^B < \underline{\beta}^B < \overline{\beta}^{B'} < \frac{1}{\alpha^2}$. Note the only difference with the proof of Lemma 3.6 is that $\bar{\beta}^{B'} < \bar{\beta}^B$.

The following lemma summarizes the result on the optimal choice of the tax rate (t^*) depending on the value of β.

Lemma C.4. *When $\beta < \underline{\beta}^B$, the optimal tax rate is $t^* = \frac{2}{5\alpha'-3\alpha}$; when $\underline{\beta}^B < \beta < \overline{\beta}^{B'}$, the optimal tax rate is $t^* = t^B$; when $\overline{\beta}^{B'} < \beta < \frac{1}{\alpha^2}$, the optimal tax rate is $t^* = \frac{1}{2\alpha'-\alpha}$.*

Thus then maximum of $W^B(t)$ is

$$W^B(t^*) = \begin{cases} W^B(t^B) = \frac{\beta(1-\alpha)^2(\alpha'-\alpha)^2}{\beta(\alpha\alpha'-2\alpha^2+\alpha-2\alpha')^2-(\alpha'+\alpha)^2}, & \text{if } \underline{\beta}^B < \beta < \overline{\beta}^B, \\ W^B(\frac{2}{5\alpha'-3\alpha}) = \frac{(\alpha'-\alpha)^2(64-\beta(1+7\alpha)^2)}{9(5\alpha'-3\alpha)^2}, & \text{if } \beta < \underline{\beta}^B, \\ W^B(\frac{1}{2\alpha'-\alpha}) = \frac{(\alpha'-\alpha)^2(1-\beta\alpha^2)}{(2\alpha'-\alpha)^2}, & \text{if } \beta > \overline{\beta}^{B'}. \end{cases}$$

We summarize this part with the two theorems below:

Theorem 3.2*. *When $\beta < \frac{1}{\alpha^2}$, in any equilibrium, the government sets the optimal tax rate to be zero and licensing takes place.*

Theorem 3.3*. *When $\beta > \frac{1}{\alpha^2}$, in an equilibrium, the optimal tax rate is 1. No licensing and no bribing takes place.*

Note that government regulation works again, when the anti-corruptive effort is strong. In this case, there is no room for corruption, as the bribing cost is too high for both the firm and the bureaucrat.

C.2.2.3 Detailed Discussion on Firms' Equilibrium Decision

First of all, we shall discuss the properties of $h(t)$. $h(t)$ is U-shape and $\arg\min_t h(t) = \frac{1}{5\alpha'-3\alpha} < \frac{1}{2-\alpha}$. When $0 < t < \frac{2}{5\alpha'-3\alpha}$, $h(t) < 0$. Note that $\frac{2}{5\alpha'-3\alpha} < \frac{1}{2-\alpha}$, when $\alpha' > \frac{4+\alpha}{5}$, if and only if $\sigma > \frac{4}{5}$.

Next, we discuss the relationship among $f(t)$, $g(t)$ and $h(t)$. $f\left(\frac{1}{2-\alpha}\right) > h\left(\frac{1}{2-\alpha}\right)$ if and only if $2\sqrt{2}-2+(3+2\sqrt{2})\alpha > \alpha'$, if and only if $\sigma < 2(\sqrt{2}-1)$. In this case, there exists a unique t^* such that $h(t^*) = g(t^*)$, where $\frac{1}{2-\alpha} < t^* < \frac{1}{2\alpha'-\alpha}$ and $h(t) > g(t)$ if $t > t^*$. Otherwise, when $\sigma > 2(\sqrt{2}-1)$,

$f\left(\frac{1}{2-\alpha}\right) < h\left(\frac{1}{2-\alpha}\right)$ and there exists a unique t^{**} such that $h(t^{**}) = f(t^{**})$, where $\frac{2}{5-3\alpha} < t^{**} < \frac{1}{2-\alpha}$ and $f(t) > h(t)$ if $t < t^{**}$.

Therefore, we have the following Proposition analogous to Proposition 7.

Proposition 7*. *Suppose $\frac{1}{2} < \sigma < 1$. Then the equilibria in stage 2 are summarized as below:*

- *Case 1: $\frac{1}{2} < \sigma < \frac{4}{5}$*

  1. *If $t \leq \frac{2}{5-3\alpha}$, licensing is an equilibrium;*
  2. *If $\frac{2}{5-3\alpha} < t < \frac{1}{2-\alpha}$, then if $b > f(t)$, no licensing and no bribing is an equilibrium; if $b < f(t)$, licensing is an equilibrium;*
  3. *If $\frac{1}{2-\alpha} < t < \frac{2}{5\alpha'-3\alpha}$, then if $b > g(t)$, no licensing and no bribing is an equilibrium; if $b < g(t)$, licensing is an equilibrium;*
  4. *If $\frac{2}{5\alpha'-3\alpha} < t < t^*$, then if $b > g(t)$, no licensing and no bribing is an equilibrium; if $h(t) < b < g(t)$, licensing is an equilibrium; if $b < h(t)$, bribing is an equilibrium;*
  5. *If $t^* < t < \frac{1}{2\alpha'-\alpha}$, then if $b > g(t)$, no licensing and no bribing is an equilibrium; if $b < g(t)$, bribing is an equilibrium;*
  6. *If $t > \frac{1}{2\alpha'-\alpha}$, no licensing and no bribing is an equilibrium.*

- *Case 2: $\frac{4}{5} < \sigma < 2(\sqrt{2}-1)$*

1. *If* $t \leq \frac{2}{5-3\alpha}$*, licensing is an equilibrium;*

2. *If* $\frac{2}{5-3\alpha} < t < \frac{2}{5\alpha'-3\alpha}$*, then if* $b > f(t)$*, no licensing and no bribing is an equilibrium; if* $b < f(t)$*, licensing is an equilibrium;*

3. *If* $\frac{2}{5\alpha'-3\alpha} < t < \frac{1}{2-\alpha}$*, then if* $b > f(t)$*, no licensing and no bribing is an equilibrium; if* $h(t) < b < f(t)$*, licensing is an equilibrium; if* $b < h(t)$*, bribing is an equilibrium;*

4. *If* $\frac{1}{2\alpha-\alpha} < t < t^*$*, then if* $b > g(t)$*, no licensing and no bribing is an equilibrium; if* $h(t) < b < g(t)$*, licensing is an equilibrium; if* $b < h(t)$*, bribing is an equilibrium;*

5. *If* $t^* < t < \frac{1}{2\alpha'-\alpha}$*, then if* $b > g(t)$*, no licensing and no bribing is an equilibrium; if* $b < g(t)$*, bribing is an equilibrium;*

6. *If* $t > \frac{1}{2\alpha'-\alpha}$*, no licensing and no bribing is an equilibrium.*

- *Case 3:* $2(\sqrt{2}-1) < \sigma < 1$

1. *If* $t \leq \frac{2}{5-3\alpha}$*, licensing is an equilibrium;*

2. *If* $\frac{2}{5-3\alpha} < t < \frac{2}{5\alpha'-3\alpha}$*, then if* $b > f(t)$*, no licensing and no bribing is an equilibrium; if* $b < f(t)$*, licensing is an equilibrium;*

3. *If* $\frac{2}{5\alpha'-3\alpha} < t < t^*$*, then if* $b > f(t)$*, no licensing and no bribing is an equilibrium; if* $h(t) < b < f(t)$*, licensing is an equilibrium; if* $b < h(t)$*, bribing is an equilibrium;*

4. *If* $t^* < t < \frac{1}{2-\alpha}$,*then if* $b > f(t)$, *no licensing and no bribing is an equilibrium; if* $b < f(t)$, *bribing is an equilibrium;*

5. *If* $\frac{1}{2-\alpha} < t < \frac{1}{2\alpha'-\alpha}$, *then if* $b > g(t)$, *no licensing and no bribing is an equilibrium; if* $b < g(t)$, *bribing is an equilibrium;*

6. *If* $t > \frac{1}{2\alpha'-\alpha}$, *no licensing and no bribing is an equilibrium.*

We have the equilibrium regions indicated in the following figures.

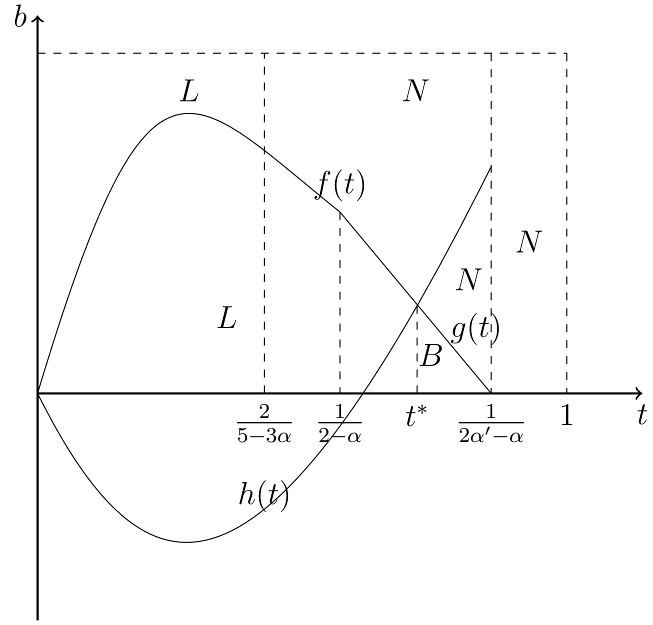

Figure C.2.6: Equilibrium Region for Case 1

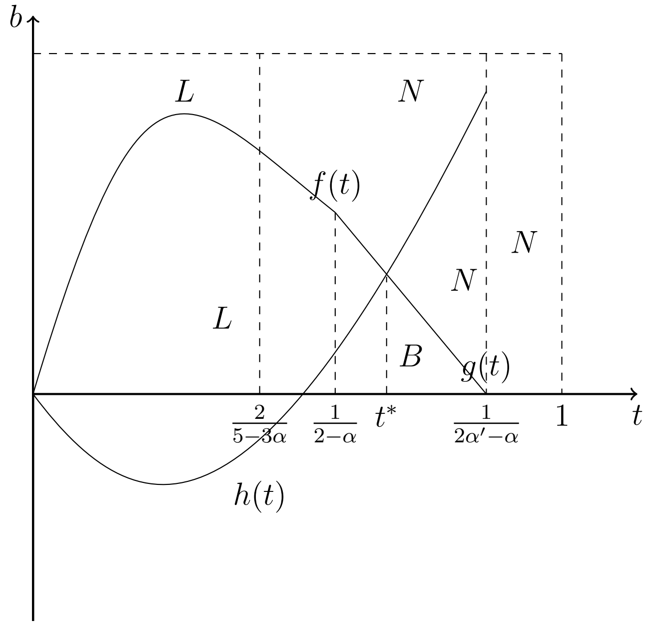

Figure C.2.7: Equilibrium Region for Case 2

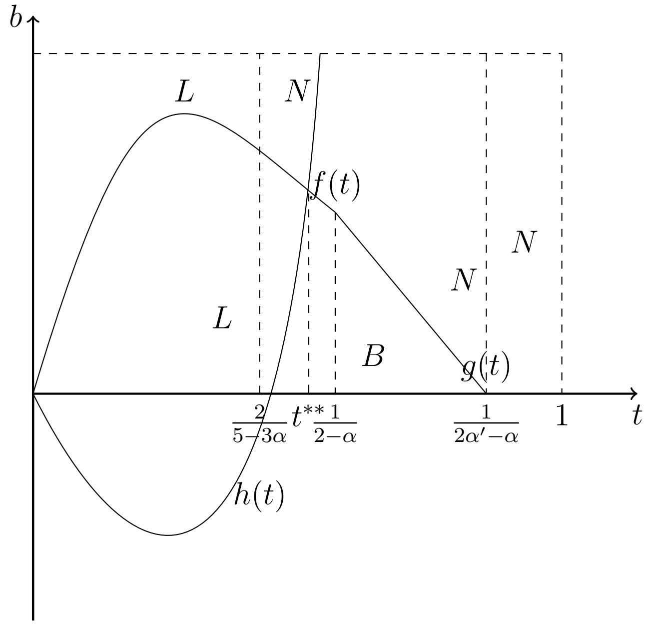

Figure C.2.8: Equilibrium Region for Case 3

C.2.3 Linear Welfare

C.2.3.1 Analysis

Consider the case where the welfare function is linear in both output and the level of pollution, i.e., $W = Q - \beta E$, where $\beta > 0$. Welfare expressions for different types of equilibria under the largest possible domains are as below

$$\begin{cases} W^L(t) = \frac{2(1-\beta\alpha)(1-\alpha t)}{3} & \text{if } 0 < t < 1; \\ W^N(t) = \frac{2-(1+\alpha)t}{3} - \frac{\beta(1+\alpha+2(\alpha-\alpha^2-1)t)}{3} & \text{if } \frac{2}{5-3\alpha} < t < \frac{1}{2-\alpha}; \\ W^{N'}(t) = \frac{(1-\beta\alpha)(1-\alpha t)}{2} & \text{if } \frac{1}{2-\alpha} < t < 1; \\ W^B(t) = \frac{2-(\alpha+\alpha')t}{3} - \frac{\beta(1+\alpha+(\alpha\alpha'-2\alpha^2+\alpha-2\alpha')t)}{3} & \text{if } \frac{2}{5\alpha'-3\alpha} < t < \frac{1}{2\alpha'-\alpha}. \end{cases}$$

Given $W^L(t)$, $W^N(t)$, $W^{N'}(t)$ and W^B, we have the following properties:

1. $W^N(\frac{1}{2-\alpha}) = W^{N'}(\frac{1}{2-\alpha})$.

2. If $\frac{1}{2} < \sigma < 1$, then $W^B(\frac{1}{2\alpha'-\alpha}) = W^{N'}(\frac{1}{2\alpha'-\alpha})$.

3. $\frac{dW^N(t)}{dt} > 0$ if and only if $\beta > \beta_N = \frac{1+\alpha}{2(1+\alpha^2-\alpha)}$, where $0 < \beta_N < \frac{1}{\alpha}$.

4. $\frac{dW^B(t)}{dt} > 0$ if and only if $\beta > \beta_B = \frac{\alpha+\alpha'}{2\alpha^2+2\alpha'-\alpha\alpha'-\alpha}$, where $0 < \beta_B < \frac{1}{\alpha}$.

5. If $\beta < \frac{1}{\alpha}$, then $W^L(t) > W^{N'}(t) > 0$ for $t \in [0,1]$, and $\frac{dW^L(t)}{dt} < 0$ and $\frac{dW^{N'}(t)}{dt} < 0$; if $\beta > \frac{1}{\alpha}$, then $W^L(t) < W^{N'}(t) < 0$ for $t \in [0,1]$, and $\frac{dW^L(t)}{dt} > 0$ and $\frac{dW^{N'}(t)}{dt} > 0$.

The equilibrium results are summarized in the following theorem:

Theorem C.1. *Suppose welfare is linear in total output and pollution. When the government puts less emphasis on environment ($\beta < \frac{1}{\alpha}$), the optimal tax rate is* 0 *and licensing is always an equilibrium.*

When the government puts more emphasis on environment ($\beta > \frac{1}{\alpha}$), then:

(1) If $0 < \sigma < \frac{2}{5}$, the optimal tax rate is always 1. *When $b > g(1)$, no licensing and no bribing is an equilibrium; when $b < g(1)$, licensing is an equilibrium.*

(2) If $\frac{2}{5} < \sigma < \sqrt{2} - 1$, when $b > g(1)$, the optimal tax rate is 1 *and no licensing and no bribing is an equilibrium; when $h(1) < b < g(1)$, the optimal tax rate is* 1 *and licensing is an equilibrium; when $b < h(1)$, if $W^L(\tilde{t}) > W^B(1)$, then the optimal tax rate is $\tilde{t} \in (\frac{1}{2-\alpha}, 1)$ and licensing is an equilibrium; otherwise, the optimal tax rate* 1 *and bribing is an equilibrium.*

(3) If $\sqrt{2} - 1 < \sigma < \frac{1}{2}$, when $b > g(1)$, the optimal tax rate is 1 *and no licensing and no bribing is an equilibrium; when $b < g(1)$, if $W^L(\tilde{t}) > W^B(1)$, then the optimal tax rate is $\tilde{t} \in (\frac{1}{2-\alpha}, 1)$ and licensing is an equilibrium; otherwise, the optimal tax rate* 1 *and bribing is an equilibrium.*

(4) If $\frac{1}{2} < \sigma < 1$, the optimal tax rate is always 1 and no licensing and no bribing is an equilibrium.

C.2.3.2 Proof of the Theorem

To show the above theorem, we consider two different cases depending the value of β.

Case 1: $\beta < \frac{1}{\alpha}$

(1) Consider the case $0 < \sigma < 1/2$. Given that $W^L(t) > W^{N'}(t)$, and $\frac{W^{N'}(t)}{dt} < 0$, $\frac{W^L(t)}{dt} < 0$, we must have $W^L(0) > W^{N'}(t)$ for $t \in (\frac{1}{2-\alpha}, 1]$. Thus it is never optimal to set $t \in (\frac{1}{2-\alpha}, 1]$. Then we need to compare $W^L(0)$ with $W^N(t)$ and $W^B(t)$.

First, for $W^N(t)$, we just need to focus on the largest domain, i.e., $t \in (\frac{2}{5-3\alpha}, \frac{1}{2-\alpha})$. We know that $\frac{dW^N(t)}{dt} < 0$ if and only if $\beta < \beta_N < \frac{1}{\alpha}$. Then when β_N, $W^N(t)$ is maximizes at $t = \frac{2}{5-3\alpha}$. Comparing $W^L(0)$ with $W^N(\frac{2}{5-3\alpha})$, it can be shown that $W^L(0) > W^N(\frac{2}{5-3\alpha})$. If $\beta_N < \beta < \frac{1}{\alpha}$, $W^N(t)$ is maximized at $t = \frac{1}{2-\alpha}$. Comparing $W^L(0)$ with $W^N(\frac{1}{2-\alpha})$, we know that $W^N(\frac{1}{2-\alpha}) = W^{N'}(\frac{1}{2-\alpha}) < W^L(0)$. Therefore, $W^L(0)$ is always better than $W^N(t)$.

Second, for $W^B(t)$, we also need to focus on the largest domain, i.e., $t \in (\frac{2}{5\alpha'-3\alpha}, 1]$. We know that $\frac{dW^B(t)}{dt} < 0$ if and only if $\beta < \beta_B < \frac{1}{\alpha}$. Then if $\beta < \beta_N$, $W^B(t)$ is maximized at $t = \frac{2}{5\alpha'-3\alpha}$. Comparing $W^L(0)$ with $W^B(\frac{2}{5\alpha'-3\alpha})$, and it can be shown that $W^L(0) > W^N(\frac{2}{5\alpha'-3\alpha})$. If $\beta_B < \beta < \frac{1}{\alpha}$, then $W^B(t)$ is maximized at $t = 1$. Comparing $W^L(0)$ with $W^B(1)$, it can be shown $W^L(0) > W^B(1)$. Therefore, $W^L(0)$ is always better than $W^B(t)$.

In sum, the equilibrium tax rate is 0 and licensing takes place.

(2) Consider the case $\frac{1}{2} < \sigma < 1$

The analysis is similar to the case of $0 < \sigma < \frac{1}{2}$, except that the largest domain for $W^B(t)$ becomes $t \in (\frac{2}{5\alpha' - 3\alpha}, \frac{1}{2\alpha' - \alpha})$. All the previous analysis is still valid, except that if $\beta_B < \beta < \frac{1}{\alpha}$, we need to compare $W^L(0)$ with $W^B(\frac{1}{2\alpha' - \alpha})$. And it can be shown easily that $W^L(0) > W^B(\frac{1}{2\alpha' - \alpha})$.

In sum, the optimal tax rate is also 0 and licensing takes place in equilibrium.

Case 2: $\beta > \frac{1}{\alpha}$

(1) Consider $0 < \sigma < \frac{1}{2}$

Then we know $W^L(t) < W^{N'}(t) < 0$ for all t, and $\frac{dW^L(t)}{dt} > 0$ and $\frac{dW^{N'}(t)}{dt} > 0$. Then we have $W^{N'}(1) > W^L(t)$ for $t \in [0, 1]$. We need to consider three cases:

If $0 < \sigma < \frac{2}{5}$, when $b > g(1)$, $W^{N'}(1)$ is optimal; when $b < g(1)$, we know that licensing is always an equilibrium for $t \in [0, 1]$, and $W^L(1)$ is optimal.

If $\frac{2}{5} < \sigma < \sqrt{2} - 1$, when $b > g(1)$, $W^{N'}(1)$ is optimal; when $h(1) < b < g(1)$, $W^L(1)$ is optimal; when $b < h(1)$, we need to compare $W^L(\tilde{t})$ w-ith $W^B(1)$. If $W^L(\tilde{t}) > W^B(1)$, then licensing is an equilibrium and the optimal tax rate is $\tilde{t} \in (\frac{1}{2-\alpha}, 1)$; otherwise, bribing is an equilibrium and the optimal tax rate is 1.

If $\sqrt{2}-1 < \sigma < \frac{1}{2}$, when $b > h(t^*) = g(t^*)$, $W^{N'}(1)$ is optimal; when $g(1) < b < h(t^*)$, we need to compare $W^{N'}(1)$ with $W^B(t)$. Since it can be shown easily that $W^{N'}(1) > W^B(1)$, thus $W^{N'}(1)$ must be optimal. When $b < g(1)$, we need to compare $W^L(\tilde{t})$ with $W^B(1)$. And the result is the same as the case with $\frac{2}{5} < \sigma < \sqrt{2}-1$.

For the case $\frac{2}{5} < \sigma < \sqrt{2}-1$ and $\sqrt{2}-1 < \sigma < \frac{1}{2}$, we provide sufficient conditions below under which bribing is an equilibrium.

(2) Consider the case $\frac{1}{2} < \sigma < 1$

We know that $\frac{dW^L(t)}{dt} > 0$, $\frac{dW^N(t)}{dt} > 0$, $\frac{dW^{N'}(t)}{dt} > 0$ and $\frac{dW^B(t)}{dt} > 0$. Given that $W^L(t) < W^{N'}(t) < W^{N'}(1)$ for $t \in (0,1)$, $W^N(t) < W^N(\frac{1}{2-\alpha}) = W^{N'}(\frac{1}{2-\alpha}) < W^{N'}(1)$, and $W^B(t) < W^B(\frac{1}{2\alpha-\alpha}) = W^{N'}(\frac{1}{2\alpha'-\alpha}) < W^{N'}(1)$, we must have $W^{N'}(1)$ is always optimal.

The following lemma provide sufficient conditions under which bribing is an equilibrium.

Lemma C.5. *(Sufficient conditions for bribing to be an equilibrium) Suppose* $\frac{1-2\sigma}{1-\sigma} < \alpha < 1$ *and* $\beta > \beta^*$*, where* $\beta^* = \frac{\alpha'-\alpha}{2\alpha'-\alpha\alpha'-1} > \frac{1}{\alpha}$*. Then (a) when* $\frac{2}{5} < \sigma < \sqrt{2}-1$ *and* $b < h(1)$*, the optimal tax rate is* 1 *and bribing is an equilibrium; (b) when* $\sqrt{2}-1 < \sigma < \frac{1}{2}$ *and* $b < g(1)$ *the optimal tax rate is* 1 *and bribing is an equilibrium.*

In both cases, let $\tilde{t} = h^{-1}(b) < 1$. We just need to compare $W^L(\tilde{t})$ with

$W^B(1)$. To show bribing is an equilibrium, a sufficient condition is $W^B(1) > W^L(1)$ given that $W^L(t)$ is increasing in t for $\beta > \beta^*$.

Then we have

$$W^L(1) - W^B(1) = \frac{1}{3}(\alpha' - \alpha + (1 - 2\alpha' + \alpha\alpha')\beta).$$

We know that the slope $1 - 2\alpha' + \alpha\alpha'$ is negative if and only if $\frac{1-2\sigma}{1-\sigma} < \alpha < 1$. In this case, we know that $W^L(1) < W^B(1)$ if $\beta > \beta^* = \frac{\alpha'-\alpha}{2\alpha'-\alpha\alpha'-1} > \frac{1}{\alpha}$.

C.2.4 Outsider Innovator

There is clean technology for sale at fixed price F, which is exogenous.[2] We do not have the strategic licensing effect in this case. There is one monopoly in the market with zero marginal cost of production. The production is Q and pollution emission is $E = Q$. Firm's profit is measured by $\pi = PQ - tE$. If the monopoly purchases the technology, his pollution will become $E = \alpha Q$, where $\alpha \in (0, 1)$, which measures the efficiency of technology in reducing pollution. If the monopoly chooses to bribe the bureaucrat, with the minimum bribe fee $\frac{\sigma w}{1-\sigma}$, the bureaucrat will report the monopoly possessing clean technology, even it does not. With probability σ, the bureaucrat will be discovered, and the monopoly still have to pay pollution tax tQ. Otherwise, the bribe is successful, the monopoly only needs to pay $t\alpha Q$.

[2] We assume the technology is owned by domestic research lab, so that the incurred cost F does not enter the social welfare function.

First, consider the monopoly is making decision on purchasing, bribing or neither. And then consider the government optimal choice on tax.

C.2.4.1 Monopoly's Choice

Consider the three options for the monopoly. If he neither purchases the technology, nor bribe, the profit function, equilibrium production and social welfare will be[3]

$$\pi^N = (1-Q)Q - tQ\ ,Q^N = \frac{1-t}{2},\ \text{and}\ W^N(t) = (1-\beta)(\frac{1-t}{2})^2.$$

If he purchases the technology at the fee F,[4]

$$\pi^L = (1-Q)Q - t\alpha Q - F,\ Q^L = \frac{1-\alpha t}{2},\ \text{and}\ W^L(t) = (1-\alpha^2\beta)(\frac{1-\alpha t}{2})^2.$$

If he bribes the bureaucrat, his expected profit will be[5]

$$\pi^B = \sigma((1-Q)Q - tQ - b) + (1-\sigma)((1-Q)Q - t\alpha Q - b).$$

Let $\alpha' \equiv \sigma + (1-\sigma)\alpha$, and note that $\alpha < \alpha' < 1$, we have

$$\pi^B = (1-Q)Q - t\alpha' Q - b,\ Q^B = \frac{1-\alpha' t}{2},\ \text{and}\ W^B(t) = (1-\beta)(\frac{1-\alpha' t}{2})^2.$$

We have three equilibria in different subgames to compare.

[3] let superscript N denote this case.
[4] let superscript L denote this case.
[5] let superscript B denote this case.

C.2.4.1.1 Compare Purchasing and No purchasing and no bribing.

Purchasing is better than No if and only if[6]

$$F \leq (\frac{1-\alpha t}{2})^2 - (\frac{1-t}{2})^2 = \frac{1}{4}(1-\alpha)t(2-\alpha t - t) \equiv c^L(t).$$

Note that $c^L(t)$ is quadratic with a negative leading coefficient. Since $\arg\max_t c^L(t) = \frac{1}{1+\alpha}$, $c^L(t) \leq c^L(\frac{1}{1+\alpha}) = \frac{1-\alpha}{4(1+\alpha)}$ for all $t \in [0,1]$. Thus if $F > \frac{1-\alpha}{4(1+\alpha)}$, we always have $F > c^L(t)$ and No is always better than purchasing. If $F \leq \frac{1-\alpha}{4(1+\alpha)}$, by solving $F = c^L(t)$, we have

$$t^L_{min} = \frac{(1-\alpha) - \sqrt{(1-\alpha-4(1+\alpha)F)(1-\alpha)}}{(1-\alpha^2)}, \text{and}$$
$$t^L_{max} = \frac{(1-\alpha) + \sqrt{(1-\alpha-4(1+\alpha)F)(1-\alpha)}}{(1-\alpha^2)}.$$

Then for all $t \in [t^L_{min}, t^L_{max}]$, $c^L(t) > F$ and purchasing is better than No; otherwise, No is better than purchasing.[7]

C.2.4.1.2 Compare Bribing and No.

Bribing is better than No if and only if

$$b \leq (\frac{1-\alpha' t}{2})^2 - (\frac{1-t}{2})^2 = \frac{1}{4}(1-\alpha')t(2-\alpha' t - t) \equiv c^B(t).$$

Similar to the previous case, solving $b = c^B(t)$, we have

$$t^B_{min} = \frac{(1-\alpha') - \sqrt{(1-\alpha'-4(1+\alpha')b)(1-\alpha')}}{(1-(\alpha')^2)}, \text{and}$$

[6]Subsequently, we will use "No" as a short hand notation for "No purchasing and no bribing"

[7]Note that $0 \leq t^L_{min} \leq \frac{1}{1+\alpha} \leq t^L_{max}$, and $t^L_{max} \leq 1$ if and only if $\frac{(1-\alpha)^2}{4} \leq F \leq \frac{1-\alpha}{4(1+\alpha)}$.

$$t_{max}^{B} = \frac{(1-\alpha') + \sqrt{(1-\alpha'-4(1+\alpha')b)(1-\alpha')}}{(1-(\alpha')^2)}.$$

If $b < \frac{1-\alpha'}{4(1+\alpha')}$, for all $t \in [t_{min}^{B}, t_{max}^{B}]$, we have $c^B(t) > b$ and bribing is better than No.[8] Otherwise, No is better.

C.2.4.1.3 Compare Purchasing and Bribing.

Purchasing is better than Bribing if and only if

$$F - b \leq c^L(t) - c^B(t) = \frac{1}{4}(\alpha' - \alpha)t(2 - \alpha' t - \alpha t) \equiv c^{LB}(t).$$

Similar to the previous cases, solving $F - b = c^{LB}(t)$, we have

$$t_{min}^{LB} = \frac{(\alpha'-\alpha) - \sqrt{(\alpha'-\alpha-4(\alpha'+\alpha)(F-b))(\alpha'-\alpha)}}{(\alpha'-\alpha)(\alpha'+\alpha)}, \text{and}$$
$$t_{max}^{LB} = \frac{(\alpha'-\alpha) + \sqrt{(\alpha'-\alpha-4(\alpha'+\alpha))(F-b))(\alpha'-\alpha)}}{(\alpha'-\alpha)(\alpha'+\alpha)}.$$

If $F - b < \frac{\alpha'-\alpha}{4(\alpha'+\alpha)}$, then for all $t \in [t_{min}^{LB}, t_{max}^{LB}]$, we have $c^{LB}(t) > F - b$ and licensing is always better than Bribing. Otherwise, Bribing is better.

C.2.4.2 Choice of Tax Rate

Depending on the government environmental policy β, we have three cases to discuss: (1) government puts little emphasis on environment, $0 < \beta \leq 1$, (2) the emphasis is moderate, $1 < \beta \leq \frac{1}{\alpha^2}$ and (3) the emphasis is strong, $\beta > \frac{1}{\alpha^2}$.

[8] Note that $0 \leq t_{min}^{B} \leq \frac{1}{1+\alpha'} \leq t_{max}^{B}$, and $t_{max}^{B} \leq 1$ if and only if $\frac{(1-\alpha')^2}{4} \leq b \leq \frac{1-\alpha'}{4(1+\alpha')}$.

C.2.4.2.1 $0 < \beta \leq 1$**, government is more output-oriented**

In this case, $W^N(t)$, $W^L(t)$ and $W^B(t)$ are all decreasing in t. We have

$$\arg\max_t W^N(t) = 0,$$

$$\arg\max_t W^L(t) = t^L_{min},$$

$$\arg\max_t W^B(t) = t^B_{min}.$$

Lemma C.6. *It is possible that* $W^L(\max\{t^L_{min}, t^{LB}_{min}\}) > W^N(0)$.

Proof. We solve:

$$W^L(t) = W^N(0)$$
$$\Longrightarrow \hat{t}^L = \frac{1 - \sqrt{(1-\beta)/(1-\alpha^2\beta)}}{\alpha}$$

Because $W^L(t)$ is decreasing in t, when $\max\{t^L_{min}, t^{LB}_{min}\} < \hat{t}^L$, $W^L(\max\{t^L_{min}, t^{LB}_{min}\}) > W^N(0)$. □

Lemma C.7. $W^B(t^B_{min}) < W^N(0)$.

Proof. It is trivial to see that $W^B(t^B_{min}) = (1-\beta)(\frac{1-\alpha' t^B_{min}}{2})^2 < \frac{1}{4}(1-\beta) = W^N(0)$. □

Proposition C.2. *If* $F < \min\{\frac{\alpha'-\alpha}{4(\alpha'+\alpha)} + b, \frac{1-\alpha}{4(1+\alpha)}\}$ *and* $\max\{t^L_{min}, t^{LB}_{min}\} < \hat{t}^L$, *we have* $W^L(\max\{t^L_{min}, t^{LB}_{min}\}) > W^N(0)$, *i.e. government should charge*

$t = \max\{t_{min}^{L}, t_{min}^{LB}\}$, *and the monopoly will purchase the clean technology. In all other cases, government should charge* $t = 0$ *and the monopoly will neither purchase the technology nor bribe. Bribing never appears in equilibrium.*

When government puts less emphasis on environment protection, it is most likely to charge zero tax rate, leading to the monopoly neither purchase the clean technology nor bribe the bureaucrat. But still, under reasonable conditions, government should charge a positive tax rate, inducing the monopoly to purchase the clean technology. The conditions are as follows: first, the technology is cheap; second, the bribing cost should be sufficiently high, which could be ensured by the high probability of being discovered or higher wage; and third, the tax rate may not be too high.

C.2.4.2.2 $1 < \beta \leq \frac{1}{\alpha^2}$, government favors neither output nor environment

In this case, $W^N(t)$ and $W^B(t)$ are increasing in t, while $W^L(t)$ is decreasing in t. We have $W^N(1) = 0 > W^B(t_{max}^{B})$, and $W^L(t_{min}^{L}) > 0 = W^N(1)$. Therefore, purchasing is always the socially optimal outcome. The equilibrium tax is $t = t_{min}^{L}$.

C.2.4.2.3 $\beta > \frac{1}{\alpha^2}$, government is more environment-oriented

In this case, $W^N(t)$, $W^L(t)$ and $W^B(t)$ are all increasing in t. We have $W^N(1) = 0 > W^B(t_{max}^{B})$, and $W^N(1) = 0 > W^L(t_{max}^{L})$.

Therefore, when government is more environment-oriented, it is socially optimal for the monopoly neither purchases the clean technology nor bribes the bureaucrat. And hence, he will set the tax, $t = 1$. However, it is possible that $t^B_{max} = 1$ when $b < \frac{(1-\alpha')^2}{4}$; and $t^L_{max} = 1$, when $F < \frac{(1-\alpha)^2}{4}$. We have the following proposition to conclude this case.

Proposition C.3. *When $F < \frac{(1-\alpha)^2}{4}$, government should set tax rate at one, and the monopoly purchasing the technology is socially optimal. When $b > \frac{(1-\alpha')^2}{4}$ and $F > \frac{(1-\alpha)^2}{4}$, government should set tax rate at one, and no purchasing and no bribing is socially optimal. When $b < \frac{(1-\alpha')^2}{4}$ and $F > \frac{(1-\alpha)^2}{4}$, government should set tax rate at one, and bribing is socially optimal.*

There is another interpretation of Proposition C.3. In any of the cases, government should always set tax at one. It is the monopoly, who will make decision optimally, i.e. maximize his profit. It is easy to see $\pi^L(1) > \pi^B(1) > \pi^N(1)$, and hence, when price of the technology is low, he will purchase the technology; when bribing cost is low, he will bribe; otherwise he does nothing. This explanation shows that government intervention may fail. As the government goal is to reach the no purchasing and no bribing equilibrium, i.e. driving the monopoly out of the market, but the monopoly may still choose purchasing or bribing to stay in the market, as which maximizes his profit.

Theorem C.2 below summarizes the equilibrium of this monopoly game.

Theorem C.2. *Consider a monopoly is in the production market. 1) If the government is output-oriented ($0 < \beta < 1$), we have equilibrium summarized in Proposition C.2. 2) If the government is neither output-oriented nor environment-oriented ($1 < \beta < \frac{1}{\alpha^2}$), government should charge a positive tax, and the firm will always purchase the clean technology. 3) If the government is environment-oriented ($\beta > \frac{1}{\alpha^2}$), we have equilibrium summarized in Proposition C.3.*

Even if there is no competition in the market, firm still have incentive to bribe the bureaucrat when the government is environment-oriented, bribing cost is low and technology price is high. In this case, firm will remain active in the market. Otherwise, if he chooses no purchasing and no bribing, he will have to leave the market, due to the high cost of pollution tax.

Purchasing the technology will yield highest output level. When government is output-oriented, he can never reach the purchasing equilibrium by setting the tax rate at zero. There will be no pollution cost, and hence there is no incentive for the monopoly to bribe the bureaucrat, nor to purchase the technology. It is necessary for the government to raise the pollution cost, by setting some positive tax rate, to incentivize the transfer of the clean technology, .

Clearly, consumer surplus under firm using clean technology is higher than the case using a dirty one. So our policy implication is that comparing governments with different environment policies. The one with output-oriented policy will always lead to a transfer of clean technology, leading to a better environment through technology diffusion. Consumers will be better-off, due to higher total production level.

The interpretation for environment-oriented government is similar to the insider innovator case. If the country sets a very high goal on environment protection, and hence a high tax rate,it is possible that bribing is the equilibrium. In such case, government intervention on tax fails. Especially when the government is aiming to shut down heavily polluting industry, it may not come true.