

# SAFETY MARGIN SENSITIVITY ANALYSIS FOR MODEL SELECTION IN NUCLEAR POWER PLANT PROBABILISTIC SAFETY ASSESSMENT

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## ABSTRACT

The safety assessment of Nuclear Power Plants makes use of Thermal-Hydraulic codes for the quantification of the safety margins with respect to upper/lower safety thresholds, when postulated accidental scenarios occur. To explicitly treat uncertainties in the safety margins estimates within the Risk-Informed Safety Margin Characterization (RISMC) framework, we resort to the concept of Dynamic Probabilistic Safety Margin (DPSM). We propose to add to the framework a sensitivity analysis that calculates how much the **Thermal-Hydraulic (TH)** code inputs affect the DPSM, in support to the selection of the most proper probabilistic safety assessment method to be used for the problem at hand, between static or dynamic methods (e.g., Event Trees (ETs) or Dynamic ETs (DETs), respectively). Two case studies are considered: firstly a Station Black Out followed by a **Loss Of Coolant Accident (LOCA)** for a 3-loops **Pressurized Water Reactor (PWR)**, whose dynamics is simulated by a MAAP5 model and, secondly, the accidental scenarios that can occur in a U-Tube Steam Generator, whose dynamics is simulated by a SIMULINK model. The results show that the sensitivity analysis performed on the DPSM points out that an ET-based analysis is sufficient in one case, whereas a DET-based analysis is needed for the other case.

**KEYWORDS:** Dynamic Probabilistic Safety Margin, Risk-Informed Safety Margin Characterization, Integrated Deterministic Probabilistic Safety Assessment, Dynamic Event Tree, Sensitivity Analysis, Seal LOCA, U-Tube Steam Generator.

## LIST OF ACRONYMS

<b>AC</b>	Alternate Current
<b>AFW</b>	Auxiliary Feed Water
<b>BDDBA</b>	Beyond Design Basis Accident
<b>BE</b>	Best Estimate
<b>DBA</b>	Design Basis Accident
<b>DET</b>	Dynamic Event Tree
<b>DOE</b>	Department Of Energy
<b>DPSM</b>	Dynamic Probabilistic Safety Margin
<b>ET</b>	Event Tree
<b>FT</b>	Fault Tree
<b>IDPSA</b>	Integrated Deterministic Probabilistic Safety Assessment
<b>LOCA</b>	Loss of Coolant Accident
<b>LWRS</b>	Light Water Reactor Sustainability
<b>MAAP5</b>	Modular Accident Analysis Program version 5
<b>MCS</b>	Minimal Cut Set
<b>MVL</b>	Multiple Value Logic
<b>NM</b>	Near Miss
<b>NPP</b>	Nuclear Power Plant
<b>OS</b>	Order statistics
<b>PI</b>	Prime Implicant
<b>PID</b>	Proportional Integrative Derivative
<b>PSA</b>	Probabilistic Safety Assessment
<b>PWR</b>	Pressurized Water Reactor
<b>RCP</b>	Reactor Coolant Pump
<b>RCS</b>	Reactor Coolant System
<b>RISMC</b>	Risk Informed Safety Margin Characterization
<b>RPV</b>	Reactor Pressure Vessel
<b>SA</b>	Safety Assessment
<b>SBO</b>	Station Black Out

<b>SG</b>	Steam Generator
<b>TH</b>	Thermal-Hydraulic
<b>UTSG</b>	U-Tube Steam Generator

## LIST OF SYMBOLS

$a$	Accidental scenario
$M(y_j, a)$	Safety margin for the $j$ - $th$ safety parameter during the accidental scenario $a$
$y_j$	$j$ - $th$ safety parameter
$j$	Index of the safety parameter, $j = 1, 2, \dots, J$
$J$	Number of safety parameters
$y_j(a)$	$j$ - $th$ safety parameter for the accidental scenario $a$
$y_{j,ref}$	Nominal value of the safety parameter $y_j$ during normal operation
$U_j$	Upper threshold for the $j$ - $th$ safety parameter
$L_j$	Lower threshold for the $j$ - $th$ safety parameter
$y_{\gamma_1}$	Real value of the $\gamma_1^{th}$ percentile of the safety parameter
$y_t$	(Grace) time required to reach $y_j$
$\gamma_1$	Probability that $y$ is lower than $y_{\gamma_1}$
$y_{t_{\gamma_2}}$	Real value of the $\gamma_2^{th}$ percentile of the time $y_t$
$\gamma_2$	Probability that $y_t$ is lower than $y_{t_{\gamma_2}}$
$\hat{y}_{\gamma_1}$	Estimate of $y_{\gamma_1}$
$\hat{y}_{t_{\gamma_2}}$	Estimate of $y_{t_{\gamma_2}}$
$\beta$	Confidence value in the percentile estimation
$\beta_1$	Confidence in the estimation of $y_{\gamma_1}$
$\beta_2$	Confidence in the estimation of $y_{t_{\gamma_2}}$
$M(\gamma_1, \beta_1)$	Probabilistic Safety Margin estimated by the $\gamma_1^{th}$ percentile of $y$ with confidence $\beta_1$
$M(\gamma_1, \beta_1, \gamma_2, \beta_2)$	Dynamic Probabilistic Safety Margin estimated by the $\gamma_1^{th}$ percentile of $y$ with confidence $\beta_1$ and the $\gamma_2^{th}$ percentile of $y_t$ with confidence $\beta_2$
$\bar{x}$	Vector of a generic model inputs
$x$	Model input
$x_k$	$k$ - $th$ model input, $k = 1, 2, \dots$

$k$	Index of the model input
$x_{k,i}$	$i$ – $th$ value of the $k$ – $th$ model input
$\bar{y}$	Vector of the calculated safety parameter realizations
$y_n$	Safety parameter that is calculated during $n$ - $th$ calculation, $n = 1, 2, \dots, N$
$n$	Index of the simulations
$\bar{y}_t$	Output vector of the calculated times at which the values $\bar{y}$ are reached
$y_{t_n}$	Time at which $y_n$ is reached
$N$	Number of simulations
$\tilde{x}_{k,i}$	Normalized input value of $x_{k,i}$
$\Delta y_t$	Maximum variability range of the normalized output
$\Delta x_k$	Maximum variability range of the $k$ – $th$ input
$\tilde{y}_t   x_k = x_{k,i}$	Normalized value of $y_t$ computed for the subgroup with $x_k = x_{k,i}$ kept fixed
$I_{x_k}$	Sensitivity Index for the $k$ – $th$ input
$P_{CD}$	Core Damage Probability
$t_{rec}$	Recovery time
$P(x_k = x_{k,i})$	Probability that $x_k$ assumes the value $x_{k,i}$
$Q_e$	Feedwater in the UTSG
$P_O$	Operating Power in the UTSG
$P_n$	Nominal Power in the UTSG
$Q_v$	Exiting steam in the UTSG
$W_{rl}$	Wide Range Level in the UTSG
$N_{rl}$	Narrow Range Level in the UTSG
$P_F$	UTSG probability of failure

## 1. INTRODUCTION

The Safety Assessment (SA) of a Nuclear Power Plant (NPP) is based on the evaluation of the consequences of a number of postulated accidental scenarios and on the quantification of their probabilities of occurrence. This is done to verify that the plant design **satisfies prescribed** safety margins, i.e., that there is sufficient difference between the values reached by the pre-defined safety

parameters during the accidental scenarios and the pre-set thresholds that must not be exceeded in order not to endanger the NPP operability and safety.

Best Estimate (BE) Thermal-Hydraulic (TH) codes are used to simulate the dynamics of the safety parameters during the postulated accidental scenarios. Traditional (static) Probabilistic Safety Assessment (PSA) methods, such as Fault Trees (FTs) and Event Trees (ETs), are used to compute the probability of occurrence of the accidental scenarios.

Recently, Integrated Deterministic Probabilistic Safety Assessment (IDPSA) has been proposed as a way for explicitly embedding the deterministic TH analysis within the probabilistic analysis, by systematically treating both aleatory (stochastic) and epistemic (modelling) uncertainties in the accidental progression [Zio, 2014; Aldemir 2013].

IDPSA methods include Discrete Dynamic Event Tree [Karanki et al., 2011], Continuous Dynamic Event Tree [Smidts, 1994], Dynamic Event Tree [Metzroth et al., 2011; Karanki et al., 2015], Monte Carlo Dynamic Event Tree [Hofer et al., 2002; Hofer et al., 2004], DYNAMIC Logical Analytical Methodology [Cojazzi, 1996]. These methods are conceived to dynamically analyze the evolution of accidental scenarios and model the operational risk in complex dynamic systems, explicitly accounting for mutual interactions between failures of software and hardware components and their recovery, control and operator actions [Zio, 2014; Aldemir 2013].

Even though the safety margins quantification required by risk assessment within the Risk Informed Safety Margin Characterization (RISMC) initiated by the US Department Of Energy (DOE) within the Light Water Reactor Sustainability (LWRS) program [US DOE, 2009], is expected to be able to effectively catch the system dynamics and the uncertain TH codes assumptions and parameters, this work is the first effective attempt to achieve this goal.

We resort to the quantification of the Dynamic Probabilistic Safety Margin (DPSM), where Order Statistics (OS) is used to compute, with a given confidence, the estimate of a given percentile of the distribution of the safety parameter and a given percentile of the time required for the safety parameter to reach the considered parameter percentile value [Di Maio et al., 2016]. This allows giving due account to the dynamics of the system undergoing an accidental scenario.

The DPSM is, then, originally exploited within a novel sensitivity analysis approach to identify which input parameter affects most the safety margin and, in particular, how much dynamic inputs influence the safety margin. This helps understanding whether a dynamic probabilistic safety method (e.g., a Dynamic ET (DET)) or whether a static probabilistic method (e.g., a static ET) is needed for the NPP safety assessment. Indeed, the dynamic approach gives a more detailed description of the process, but at the expense of a large computational burden. In this respect, it would make no sense to waste resources on a dynamic analysis of a system when conventional static methods can provide adequate

results. As a matter of fact, the main goal of this paper is just to provide a framework for choosing which approach (whether static or dynamic) better fit to the system under analysis.

In order to show how the framework works, two case studies are considered. In the first case, a Station Black Out (SBO) accident followed by a Seal Loss Of Coolant Accident (LOCA) has been modelled and simulated with MAAP5 TH code [MAAP5]. Dynamic aspects such as time lag between SBO and LOCA and promptness of operators actions have been simulated. The DPSMs corresponding to the event of core uncovering have been computed and a sensitivity analysis has been performed on these time-dependent results. As we shall see, the results show that the dynamic aspects considered in TH simulations do not affect the calculated DPSMs and, thus, we conclude that the static probabilistic models are sufficient for the analysis and, therefore, no dynamic probabilistic models are developed for the Seal LOCA accident.

The second case study regards a U-Tube Steam Generator (UTSG), modelled with SIMULINK. In the dynamic model, four components (i.e., the outlet steam valve, the safety valve, the Proportional Integral Derivative (PID) controller and the communication between the sensor and the PID) can fail during the accident progression. Dynamic aspects such as the magnitude, the order and timing of the possible failure events have been included in the simulations. The DPSMs have been computed and the sensitivity analysis has been performed, showing the importance of including the dynamic aspects in the probabilistic model. Consequently, for the considered UTSG, a DET analysis is necessary for proper assessment and quantification of the probabilities of occurrence of the accidental scenarios and of the DPSMs.

The rest of the paper is organized as follows. In Section 2, the definition of the DPSM is given and the sensitivity analysis approach is described. In Section 3, the two case studies are presented and worked out. In section 4, some conclusions are drawn.

## **2. THE DPSM AND THE DPSM-BASED SENSITIVITY ANALYSIS**

### **2.1 The DPSM**

The safety margin is traditionally defined as the minimum distance between the system “loading” and its “capacity” [DOE, 2009]. Mathematically, considering a specific accidental scenario  $a$  and a safety parameter  $y_j$  ( $j=1\dots J$ ), the safety margin  $M(y_j, a)$  with respect to an upper threshold  $U_j$  can be written as:

$$M(y_j, a) = \left\{ \begin{array}{ll} \frac{U_j - y_j(a)}{U_j - y_{j,ref}}, & \text{if } y_j(a) \leq U_j \\ 0, & \text{if } U_j < y_j(a) \\ 1, & \text{if } y_j(a) < y_{j,ref} \end{array} \right\} \quad (1)$$

and with respect to a lower threshold  $L_j$  as:

$$M(y_j, a) = \left\{ \begin{array}{ll} \frac{y_j(a) - L_j}{y_{j,ref} - L_j}, & \text{if } L_j \leq y_j(a) \\ 0, & \text{if } y_j(a) < L_j \\ 1, & \text{if } y_j(a) > y_{j,ref} \end{array} \right\} \quad (2)$$

where  $y_{j,ref}$  is the nominal value of the safety parameter  $y_j$  during normal operation.

The safety margins are, traditionally, computed by TH codes for a set of postulated scenarios, called Design Basis Accidents (DBAs). Conservatism in assumptions and parameters values is adopted in order to take into account uncertainties in models and parameters [Commission's White Paper, February 1999; IAEA, 2009].

Recently, safety margins are being evaluated also for Beyond Design Basis Accidents (BDBAs), thus, considering a larger set of scenarios. Moreover, Best Estimate (BE) TH codes are employed to evaluate the safety parameters with less conservatism and more realistic assumptions [Zio et al., 2010; Alvarenga et al., 2015]. To account and propagate all the aleatory and epistemic uncertainties [US D.O.E., 2009] affecting the BE TH codes, a Dynamic Probabilistic Safety Margin (DPSM) has been proposed in [Di Maio et al., 2016]. The DPSM extends the definitions of safety margin in Eq. (1) and Eq. (2), by replacing  $y_j$  with  $y_{\gamma_1}$ , and also by considering in the safety margin quantification  $y_{t_{\gamma_2}}$ . In particular,  $y_{\gamma_1}$  is the  $\gamma_1^{\text{th}}$  percentile of the distribution of the safety parameter  $y_j$  (usually the 95<sup>th</sup>, according to regulatory guidance), whereas  $y_{t_{\gamma_2}}$  is the  $\gamma_2^{\text{th}}$  percentile (usually the 5<sup>th</sup>) of the distribution of the earliest (grace) time  $y_t$  required to reach  $y_{\gamma_1}$  (i.e., the available time for recovering from the occurrence of accidental scenario  $a$ ).

Order Statistics (OS) is a non parametric approach that can be used for the estimation of  $y_{\gamma_1}$  and  $y_{t_{\gamma_2}}$ , as it allows evaluating the values of the required percentiles with confidences  $\beta_1$  and  $\beta_2$ , respectively, as explained in Appendix A, on the basis of a limited set of BE TH dynamic simulations of the system evolution.

In this work, two different approaches will be used for the computation of the estimates of  $y$  and  $y_t$ , namely the Bracketing and the Coverage approaches (see Appendix A). The former assumes the outputs  $y$  and  $y_t$  to be uncorrelated. Their corresponding percentile estimates ( $\hat{y}_{\gamma_1}$  and  $\hat{y}_{t_{\gamma_2}}$ ,

respectively) are, therefore, computed separately and independently ranked. On the other hand, since the coverage approach assumes correlation between the outputs, and, consequently,  $y$  is sorted corresponding to  $y_t$ , and they are required to simultaneously lie in the estimated percentile  $\gamma_1$  and  $\gamma_2$  [Di Maio et al., 2016].

We can, thus, define a DPSM with respect to an upper threshold  $U_j$  as in [Di Maio et al., 2016]:

$$M(\gamma_1, \beta_1, \gamma_2, \beta_2) = \left\{ \begin{array}{ll} \frac{U_j - \hat{y}_{\gamma_1}}{U_j - y_{j,ref}}, & \text{if } \hat{y}_{\gamma_1} \leq U_j \\ 0, & \text{if } U_j < \hat{y}_{\gamma_1} \\ 1, & \text{if } \hat{y}_{\gamma_1} < y_{j,ref} \end{array} \right\} \text{with grace time } \hat{y}_{t_{\gamma_2}} \quad (3)$$

where

$$\gamma_1 = Pr\{y < y_{\gamma_1}\} \quad (4)$$

$$\beta_1 = Pr\{y_{\gamma_1} < \hat{y}_{\gamma_1}\} \quad (5)$$

and

$$\gamma_2 = Pr\{y_t < y_{t_{\gamma_2}}\} \quad (6)$$

$$\beta_2 = Pr\{y_{t_{\gamma_2}} > \hat{y}_{t_{\gamma_2}}\} \quad (7)$$

where  $\hat{y}_\gamma$  is the estimate of the  $\gamma^{\text{th}}$  percentile.

As we shall see in what follows, it is worth mentioning that in those cases in which the number  $N$  of BE TH available simulations is low, the analyst should decide whether it is better to fix  $\beta$  or  $\gamma$  for providing the proper estimate of the DPSM that, in principle, should be given with  $\gamma_1 = \beta_1 = \beta_2 = 0.95$  and  $\gamma_2 = 0.05$ , as prescribed by regulation guidances [IAEA, 2009].

## 2.2 Sensitivity Analysis on the DPSM

Sensitivity analysis can help finding insights on the behaviour of a model, on its structure and on the way it responds to changes in the model input [Borgonovo et al., 2015; Zio, 2009; Saltelli et al., 2000]. It also allows ranking the model parameters according to the different contributions they give to the variability of the output [Di Maio et al., 2015a].

In this work, the model output of interest is the DPSM as defined in Section 2.1 and the aim is to determine whether the inputs of a dynamic simulation code influence (or not) the DPSM



quantification. A sensitivity index must be defined and, based on this, one can decide the appropriate probabilistic approach (static/dynamic) for the quantification of the probabilities of occurrence of the accidental scenarios to be analysed.

Sensitivity indexes can be classified into local and global [Zio, 2009; Saltelli et al., 2000]. However, since local sensitivity indexes provide information that is only valid locally, for the purpose of our work a global index seems to be more suitable. Global sensitivity indexes, indeed, aim to measuring the contribution of an input to the variability of the output over the entire range of both the input and the output, also accounting for the input interaction through dependences [Di Maio et al., 2015a; Zio, 2009; Saltelli et al., 2000]. Among popular global indexes are the Pearson and the Sobol indexes [Sobol, 1993], the latter being an extension of the Variance Decomposition method [Sobol, 1993; McKay, 1996; Borgonovo et al., 2014; Iooss et Lemaître, 2015]. However, Pearson and Sobol indexes are limited by the assumptions on the linearity and monotonicity of the model (Pearson) and by demanding computational costs (Sobol).

We therefore define a novel sensitivity index  $I_x$  (where  $x$  is a generic model input) that, even if approximate, is simple to calculate and does not require any hypothesis on the model. Without loss of generality, let assume that  $\bar{x} = \{x_1, x_2, x_3\}$  where  $x_1, x_2$  and  $x_3$  are three input variables, with instances  $\bar{x}_1 = \{x_{1,1}, x_{1,2}, x_{1,3}\}$ ,  $\bar{x}_2 = \{x_{2,1}, x_{2,2}\}$  and  $\bar{x}_3 = \{x_{3,1}, x_{3,2}, x_{3,3}, x_{3,4}\}$ . We also assume that one simulation is available for each combination of these values. Consequently a total of  $N = 24$  different simulations is available resulting in vectors of  $N$  possible values of  $y$  and  $y_t$ ,  $\bar{y} = \{y_1, y_2, \dots, y_N\}$ , and  $\bar{y}_t = \{y_{t_1}, y_{t_2}, \dots, y_{t_N}\}$ , respectively. By resorting to OS (see Appendix A), the DPSM for each input parameter and the corresponding time can be computed, as shown in Section 2.1, by using all the available simulations (that would account for the input variable interdependences) but also by considering the partitioning of the  $N$  available simulations in subgroups (that would account for the contribution of single inputs, independently). In other words, considering each input parameter separately, a DPSM is computed for each fixed value that the parameter can assume and using only the simulations in which the variable assumes the considered value. Thus, for example, when referring to  $x_1$ , we compute the DPSM considering only results of  $y_t$  collected for the subgroups of simulations done with  $x_1 = x_{1,1}$ ,  $x_1 = x_{1,2}$  and  $x_1 = x_{1,3}$ , (i.e.,  $y_t|x_1 = x_{1,1}$ ,  $y_t|x_1 = x_{1,2}$  and  $y_t|x_1 = x_{1,3}$ , respectively).

For the computation of the sensitivity index, both the input (e.g.,  $x_1$ ) and the output (e.g.,  $y_t|x_1 = x_{1,i}$  with  $i = 1,2,3$  when  $x = x_1$ ) are first normalized with respect to their maxima:

$$\tilde{x}_{1,i} = \frac{x_{1,i}}{\max(x_{1,1}, x_{1,2}, x_{1,3})}, \text{ with } i = 1,2,3 \quad (8)$$

$$\tilde{y}_t|x_1 = x_{1,1} = \frac{y_t|x_1 = x_{1,1}}{\max(y_t|x_1 = x_{1,1}; y_t|x_1 = x_{1,2}; y_t|x_1 = x_{1,3})} \quad (9)$$

Similarly,  $\tilde{y}_t|x_1 = x_{1,2}$  and  $\tilde{y}_t|x_1 = x_{1,3}$  can be obtained. The sensitivity index  $I_{x_1}$  can be, therefore, defined as:

$$I_{x_1} = \frac{\Delta y_t}{\Delta x_1} \quad (10)$$

where  $\Delta y_t$  is the range of variability of the normalized output, i.e, the difference between the maximum normalized output  $\max(\tilde{y}_t|x_1 = x_{1,1}; \tilde{y}_t|x_1 = x_{1,2}; \tilde{y}_t|x_1 = x_{1,3})$ , that is equal to 1 (according to the normalization), and the minimum normalized output  $\min(\tilde{y}_t|x_1 = x_{1,1}; \tilde{y}_t|x_1 = x_{1,2}; \tilde{y}_t|x_1 = x_{1,3})$ ; similarly,  $\Delta x_1$  is the range of variability of the normalized input  $x_1$ . Also,  $I_x$  can be computed for the other input variables  $x_2$  and  $x_3$ .

Notice that the index  $I_x$  is not constrained by any linearity and/or monotonic assumption on the input/output relationship (opposed to Pearson coefficient) with limited computational costs (opposed to Sobol indexes), and gives an indication on the variability of the output depending on the variability of the input: the larger  $I_x$ , the more the considered input  $x$  influences the output  $y_t$ .

As we shall see, the proposed strategy for deciding which probabilistic method should be used for the analysis is based on the ranking provided by  $I_x$ : when a dynamic input is ranked below others that are not time-dependent, a static probabilistic analysis of accident progression by ET is sufficient to compute the probabilities of occurrence of the considered accidental scenarios; when a dynamic input is among the top ranked, a dynamic probabilistic analysis, e.g., with a DET, is needed.

### 3. THE CASE STUDIES

In this Section, we present two applications of the proposed method for the selection of the probabilistic model to be used in a safety assessment. In the first case study, a Station BlackOut accident followed by a Seal Loss Of Coolant Accident is considered. The outcomes of the sensitivity analysis will suggest that a static probabilistic model for representing the accident progression to be enough for the purpose of the safety analysis. For this, ETs are built. In the second case study, a U-Tube Stam Generator is considered in which some components fail at different times and magnitudes. In this case, the calculated sensitivity indexes are larger for the dynamic inputs and, consequently, a framework for the DET is developed for the purpose of performing a safety analysis.

#### 3.1 SBO Seal LOCA

### 3.1.1 The accidental scenario

The accidental scenario considered is a station blackout accident followed by a Seal LOCA in a 3-loops Pressurized Water Reactor (PWR). During a SBO, a Loss Of Offsite Power (LOOP) is worsened by the failure of all the emergency diesel engines. Under these circumstances, the reactor trip is activated and the Reactor Coolant Pumps (RCPs) are shut off.

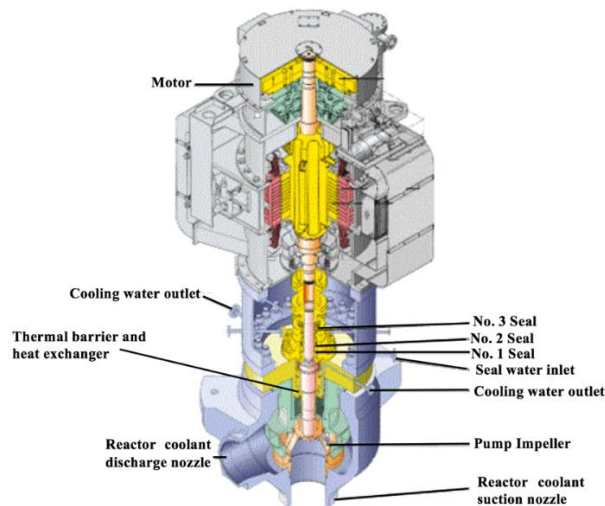


Figure 1: Typical design of a Reactor Coolant Pump [RCP]

In this sequence of events, we focus on the behaviour of the Reactor Coolant Pumps (shown in Figure 1), and in particular on its seal package. The RCPs are installed on the cold leg of each loop of the NPP between the Steam Generators (SGs) and the Reactor Pressure Vessel (RPV). They contain a seals package consisting in three different seals (i.e., seal stages), each made up by a primary and a secondary seal. These seals must continuously be cooled in normal operation as well as during the SBO [NUREG/CR-4948]. During normal operation, a seal injection system (that consists in a seal water inlet and a cooling water outlet) and a thermal barrier heat exchanger are the systems devoted to cool the seals. However, when a SBO occurs, seals cooling systems are lost and the seals overheat. Consequently, a Seal LOCA occurs at different leakage rates, that depend on the seal that fails. We consider leakage rates equal to 76, 182 and 480 gpm/RCP [WOG2000], corresponding to the failure of the first seal alone, both the second and third seals and all the three seals, respectively. Seals failure can occur according to three possible failure modes [WOG2000]: popping-open (i.e., opening of the seal faces due to hydraulic instability caused by fluid flashing), binding (i.e., binding failure of the seal ring against the housing inserts due to secondary seal extrusion), O-ring extrusion (i.e., overheating of the secondary sealing elastomers, allowing excessive leakage).

### 3.1.2 Dynamic MAAP5 Simulations

The Seal LOCAs scenarios have been simulated for all the considered leakage rates with a BE TH MAAP5 code [MAAP5], whose nodalization scheme is shown in Figure 2. This code reproduces, when fed with the set of initiating events and operators actions listed in Table 1, the responses of the reactor to the postulated accidental scenario. Although the code is able to provide the simulation results also beyond core damage, the analysis will focus on the core uncover. Then, the values of the safety parameter  $y_j$ , the threshold  $L$  and the value of  $y_t$  required to evaluate the DPSM can be retrieved from the simulations. In fact, we can simulate the evolution in time of the boiled-up water level of the primary coolant  $y_j$  (with  $j = 1$ , since this the only safety parameter considered) and the core uncover time  $y_t$ . In MAAP5 the core uncover corresponds to the condition when the boiled-up water level falls below the top of active fuel  $L = 6.6$  m. The corresponding core uncover time is therefore a result provided by the code. It is worth mentioning that, during normal operation, the reference level is equal to  $y_{ref} = 7$  m.

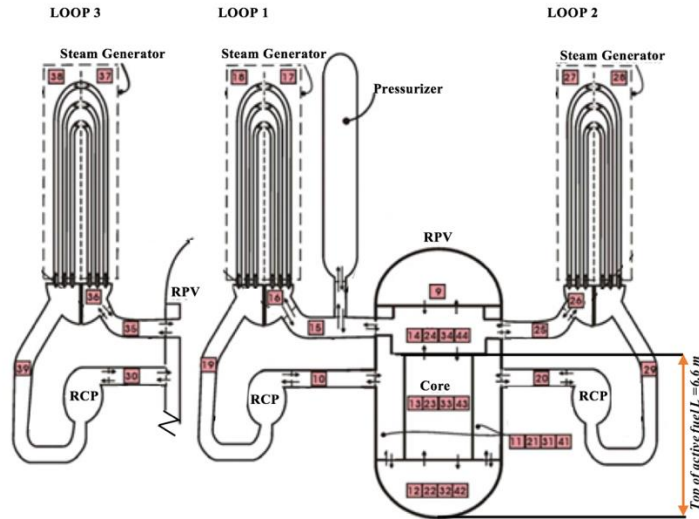


Figure 2: Water nodalization for 3-Loops PWR [MAAP5]

The scenario modelled is the following. The simulation starts with the SBO occurrence. All the seals cooling systems are, therefore, assumed to be lost, leading to the seals overheating. Seals can fail (due to popping-open) at different times ( $x_1$ ) from the onset of the SBO: immediately ( $x_{1,1} = 0$  min), after 13 min ( $x_{1,2} = 13$  min) or after 30 min ( $x_{1,3} = 30$  min). With the seal failure, the leakage rate increases from 21 gpm/RCP to a value that depends on the number of seals that have failed ( $x_{2,1} = 76$ ,  $x_{2,2} = 182$  and  $x_{2,3} = 480 \frac{gpm}{RCP}$ ). Operators are, thus, called to undertake counteracting measures to mitigate the possible consequences of this accident. They, first, activate a secondary cooling by opening a relief valve at different times ( $x_{3,1} = 20$ ,  $x_{3,2} = 30$ ,  $x_{3,3} = 40$ ,  $x_{3,4} = 50$ ,  $x_{3,5} = 60$  min from the

seal failure) and, then, regulate the Auxiliary Feed Water (AFW) system (by decreasing the target of the SG level) by following either strategy “A” ( $x_{4,1} = "A"$ ), that is the AFW regulation occurs at the same time of the relief valve opening, or strategy “B” ( $x_{4,2} = "B"$ ), that is the regulation of AFW occurs 3 h later than the leakage rate increase. Moreover, when the primary pressure falls below 4 MPa, the accumulators automatically inject water in the cold leg of each loop. The values of the input variables for the dynamic simulations are shown in Table 1.

$x_1$ [min] <i>Time of the leakage rate increase after SBO</i>	0
	13
	30
$x_2$ [gpm/RCP] <i>Leakage rate</i>	76
	182
	480
$x_3$ [min] <i>Time of activation of secondary cooling</i>	20
	30
	40
	50
	60
$x_4$ <i>Strategy</i>	A: $t$ (AFW operation) = $t$ (relief valve opening) B: $t$ (AFW operation) = 3 h after the leakage rate increase

Table 1: MAAP5 simulation inputs

Assuming  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  to be independent and described by discrete uniform distributions in the ranges of the simulation inputs of Table 1 (i.e., joint distributions are neglected), all the possible combinations of these inputs leads to  $N = 90$  alternative dynamic simulations for the case under analysis.

### 3.1.3 The DPSM-based Sensitivity Analysis results

#### 3.1.3.1 The DPSMs

The collected values of  $\bar{y}$  and  $\bar{y}_t$  (i.e., the boiled-up water level and the uncover time, respectively) have been employed for the calculation of the DPSM. Since the core uncover occurs when the boiled-up water level falls below the top of the active fuel length  $L$ , we assume the safety margins to be calculated with respect to  $L$  as lower threshold. Resorting to OS and using all the  $N = 90$  available

simulations as well as all the possible subgroups of simulation (that is, considering the simulations when each input variable is kept fixed), we have estimated the values of the  $\gamma_1^{th}$  and  $\gamma_2^{th}$  percentiles of the boiled-up water level and of the time at which such level is reached, with confidences  $\beta_1$  and  $\beta_2$ , respectively.

The DPSMs with “Bracketing” and “Coverage” approaches,  $N = 90$  (i.e., all the possible values of  $x_1, x_2, x_3$  and  $x_4$  have been considered simultaneously) and  $\gamma_2 = 0.05$  are shown in Table 2, whereas in Table 3 the DPSMs with  $N = 90$  and  $\beta_2 = 0.95$  are shown, again, for the both approaches.

It is worth mentioning that, even though the **regulatory guidances** prescribe the DPSM to be given with  $\gamma_1 = \beta_1 = \beta_2 = 0.95$  and  $\gamma_2 = 0.05$ , the limited number  $N$  of available simulations:

- limits the analysis on the estimation of the grace time available before core uncovering with the desired  $\gamma_2$  and  $\beta_2$ ;
- is not large enough to allow for an estimate of  $\hat{y}_t$  with  $\gamma_2 = 0.05$  and  $\beta_2 = 0.95$ .

Therefore, in Table 2, we provide the result of the  $\gamma_2^{th}$  percentile of  $y_b$ , with as large as possible  $\beta_2$ , according to the two different approaches of “Bracketing” and “Coverage”, as explained in Appendix A. Instead, in Table 3, we provide the results when the confidence in the estimation, i.e.,  $\beta_2$ , is fixed to at least equal to 0.95 whereas the value of the estimated percentile  $\gamma_2$  varies according to theory (Appendix A).

In general terms, all these simulations of  $y$  are “up to core uncovering”, i.e.,  $y$  reaches the lower threshold  $L$  in all the available simulations and, thus, the margin is estimated to be equal to 0. The DPSM of Eq. (3), however, provides the analyst with the additional information the grace time  $\hat{y}_t$  (before the core uncovering), which is a main benefit of the DPSM with respect to a traditional safety margin. When the  $\hat{y}_{\gamma_1}$  is equal to  $L$ , the value of the grace time  $\hat{y}_t$  is the time available for taking counteracting/mitigation actions before **core uncover**.

$\gamma_2$	$\beta_2$ “Bracketing”	$\beta_2$ “Coverage”	$\hat{y}_{\gamma_1}$ [m]	$\hat{y}_t$ [s]	DPSM
0.05	0.98	0.94	6.6 = L	7406	0 within a grace time of 7406 s

Table 2: DPSM ( $N=90$  and with fixed  $\gamma_2$ ) for the Seal LOCA

$\beta_2$	$\gamma_2$ “Bracketing”	$\gamma_2$ “Coverage”	$\hat{y}_{\gamma_1}$ [m]	$\hat{y}_t$ [s]	DPSM
0.95	0.04	0.03	6.6 = L	7406	0 within a grace time of 7406 s

Table 3: DPSM ( $N=90$  and with fixed  $\beta_2$ ) for the Seal LOCA

Results of the computation of the DPSM for the case of subgroups are shown hereafter. It is worth mentioning that: (i) being only “up to core uncover” transients available for the DPSM quantification; (ii) being the  $\hat{y}_{\gamma_1}$  equal to  $L$  (as shown before in Tables 2-3); (iii) being the number  $N$  of available simulations lower than 90 for each considered subgroup, we focus only on the estimation of the value  $\hat{y}_t$  that, incidentally, as already said, cannot be estimated with both  $\beta_2$  and  $\gamma_2$  values, as required by **regulatory guidances** (as shown in Tables 2-3). We first present in Table 4 the results when  $\gamma_2 = 0.05$ , even though with a reduced confidence  $\beta_2$  in its estimation. As we can see, for each case considered, we rely on a different number  $N$  of available simulations. This is due to the fact that when we consider a variable fixed at a given value, the number of simulations reduces to only those that have as inputs the selected fixed value of that input variable and the combination of all the other possible values of the other input variables. We show that:

- the “Bracketing” approach generally provides a larger confidence  $\beta_2$  for the estimation of the  $\gamma_2^{th}$  percentile of  $y_t$  than the “Coverage” approach;
- when we consider results obtained with fixed values of the inputs, the confidence  $\beta_2$  generally unfavourably increases, being the number  $N$  of available simulations lower than when all possible values of  $x_1, x_2, x_3$  and  $x_4$  are simultaneously considered; the values of  $\beta_2$ , indeed, go from 0.98 and 0.94 (“Bracketing” and “Coverage”, respectively), to 0.36 and 0.24 (when  $x_3$  is kept fixed and  $N = 18$ );
- the worst case (i.e., the shortest grace time available for counteracting the developing accident  $\hat{y}_t = 7406$  s) corresponds to the scenario in which a leakage rate  $x_2 = 480$  gpm/RCP occurs at the beginning of the scenario ( $x_1 = 0$  s) and operators act after twenty minutes ( $x_3 = 20$  min) with strategy  $x_4 = “A”$ .

		$N$	$\gamma_2$	$\beta_2$ “Bracketing”	$\beta_2$ “Coverage”	$\hat{y}_t$ [s]
	All possible values of $x_1, x_2, x_3$ and $x_4$ are considered simultaneously	90	0.05	0.98	0.94	7406
$x_1$ [min] is kept fixed	0	30	0.05	0.62	0.45	7406
	13	30	0.05	0.62	0.45	8516
	30	30	0.05	0.62	0.45	9555
$x_2$ [gpm/RCP] is kept fixed	76	30	0.05	0.62	0.45	30713
	182	30	0.05	0.62	0.45	15118
	480	30	0.05	0.62	0.45	7406
$x_3$ [min] is kept fixed	20	18	0.05	0.36	0.24	7406
	30	18	0.05	0.36	0.24	8298
	40	18	0.05	0.36	0.24	7608
	50	18	0.05	0.36	0.24	7544
	60	18	0.05	0.36	0.24	7599
$x_4$ is kept fixed	A	45	0.05	0.81	0.67	7406
	B	45	0.05	0.81	0.67	7544

Table 4: DPSM results for Seal LOCA simulations (fixed  $N$  and  $\gamma$ )

In a similar way, Table 5 presents the results of estimating the value of  $\gamma_2$  with  $\beta_2 = 0.95$ , and both “Bracketing” and “Coverage” approaches. The goal is to provide the  $\gamma_2$  estimates with the desired confidence, even though it cannot be the 95<sup>th</sup>. We show that:

- when  $\beta_2$  is fixed, the “Bracketing” approach provides an unfavorable estimate of the  $\gamma_2^{th}$  percentile with fixed  $\beta_2$  with respect to the “Coverage” approach.;
- when we consider results obtained with fixed values of the inputs, the value of the percentile that can be estimated unfavorably increases, being the number  $N$  of available simulations lower than when all possible values of  $x_1, x_2, x_3$  and  $x_4$  are simultaneously considered; the values of  $\gamma_2$ , indeed, go from 0.04 and 0.03 (“Bracketing” and “Coverage”, respectively), to 0.19 and 0.16 (when  $x_3$  is kept fixed and  $N = 18$ ).
- the worst case (i.e., the shortest grace time available for counteracting the developing accident  $\hat{y}_t = 7406$  s) corresponds to the scenario in which a leakage rate  $x_2 = 480$  gpm/RCP occurs at the beginning of the scenario ( $x_1 = 0$  s) and operators act after twenty minutes ( $x_3 = 20$  in) with strategy  $x_4 = “A”$ .



		$N$	$\beta_2$	$\gamma_2$ “Bracketing”	$\gamma_2$ “Coverage”	$\hat{y}_t$ [s]
	All possible values of $x_1, x_2, x_3$ and $x_4$ are considered simultaneously	90	0.95	0.04	0.03	7406
$x_1$ [min] is kept fixed	0	30	0.95	0.12	0.1	7406
	13	30	0.95	0.12	0.1	8516
	30	30	0.95	0.12	0.1	9555
$x_2$ [gpm/RCP] is kept fixed	76	30	0.95	0.12	0.1	30713
	182	30	0.95	0.12	0.1	15118
	480	30	0.95	0.12	0.1	7406
$x_3$ [min] is kept fixed	20	18	0.95	0.19	0.16	7406
	30	18	0.95	0.19	0.16	8298
	40	18	0.95	0.19	0.16	7608
	50	18	0.95	0.19	0.16	7544
	60	18	0.95	0.19	0.16	7599
$x_4$ is kept fixed	A	45	0.95	0.08	0.07	7406
	B	45	0.95	0.08	0.07	7544

Table 5: DPSM results for Seal LOCA simulations (fixed  $N$  and  $\beta$ )

### 3.1.3.2 Sensitivity analysis

On the basis of the results shown in Tables 4 and 5, the proposed sensitivity index of Eq. (10) has been computed for each input variable  $x_k$ ,  $k = 1, 2, 3, 4$  (see Section 2.2). Results are shown in Table 6, where in column 2 the possible values  $x_{k,i}$  of  $x_k$  are listed, in column 3 the estimated values of the grace time  $\hat{y}_t$  are listed, in columns 4 and 5 the variability range of the normalized input  $\Delta x_k$  (as explained in Section 2.2) and the variability range  $\Delta \hat{y}_t$  of the normalized  $\hat{y}_t$ , are listed respectively, for the simulations where each input variable is kept fixed. Finally, in the last column, the value of the sensitivity index  $I_{x_k}$  computed for each input variable is shown.

$x_k$	$x_{k,i}$	$\hat{y}_t$	$\Delta x_k$	$\Delta \hat{y}_t$	$I_{x_k}$
$x_1$ [min]	0	7406	1.00	0.23	0.23
	13	8516			
	30	9555			
$x_2$ [gpm/RCP]	76	30713	0.84	0.76	<b>0.90</b>
	182	15118			
	480	7406			
$x_3$ [min]	20	7406	0.67	0.11	0.16
	30	8298			
	40	7608			
	50	7544			
	60	7599			
$x_4$	A	7406	0.89	0.02	0.02
	B	7544			

Table 6: Results of the sensitivity analysis for the Seal LOCA case study

Based on the results of the sensitivity index  $I_x$  shown in Table 6, we can conclude that  $x_2$  (the leakage rate) is the input variable that most affects the grace time, followed by  $x_1$  (time of the leakage rate increase after the SBO). On the other hand,  $x_3$  (the time of the activation of the secondary cooling by operators) and  $x_4$  (the strategy they adopt for this) have little influence on the grace time and, thus, on the safety margin.

Since the dynamic variables  $x_3$  and  $x_4$  are not the most influencing the output, a static probabilistic safety assessment is considered sufficient for the analysis of a Seal LOCA following an SBO accident in a 3-loops PWR. The related following probabilistic analysis will focus only on the role of the leakage rate, and, eventually, of the time of the leakage rate increase. For this, we will resort to the models proposed by the Westinghouse Owners Group and named the WOG2000 and the WOG2000 (revised), the model revised by the US Nuclear Regulatory Commission (NRC), to build the static ETs for the computation of the probabilities of occurrence of the accident scenarios considered.

### 3.1.4 The Seal LOCA ETs models

For the safety assessment of the Seal LOCA accidental scenario considered, we calculate its probability of occurrence with static ETs, defined as WOG2000 and WOG2000 (revised) [WOG2000] that essentially differ in:

- the assumption on the time of the leakage rate increase after the SBO (30 min for the WOG2000, 0 and 13 min for the WOG2000 (revised));
- the value of the popping-open/binding failure probability of the third seal stage (this is explicitly accounted for the WOG2000, whereas it is not for the WOG2000 (revised) that assumes the failure can occur only when the second seal stage fails);
- the assumption on the seal failure probability due to O-ring extrusion (a value of 0.5 is assigned to the probability of occurrence of this failure mode in the WOG2000 (revised), whereas it is set equal to zero in the WOG2000, since this latter assumes the Reactor Coolant System (RCS) pressure is kept lower than 118 bar within two hours, avoiding O-ring extrusion).

These models define the first, second and third failure probabilities, for each seal failure mode (i.e., popping-open, binding, O-ring extrusion). However, a single probability is defined for the popping-open and binding failure mode as for the high-temperature O-ring failure mode (to which both the probabilistic models refer), and the binding failure mode is negligible. Each ET branch is characterized by a leakage rate probability that depends on the number of failed seals and by the probability of the different failure modes (i.e., popping-open, binding, O-ring extrusion), as listed in Table 7.

<b>Failure Mode</b>	<b>WOG2000 (<math>x_1=30</math> min)</b>	<b>WOG2000 (revised) (<math>x_1=13</math> min, <math>x_1=0</math> min)</b>
<b>Popping-open</b> <b>Binding</b>		
1 <sup>st</sup> Seal Stage	0.0125	0.0125
2 <sup>nd</sup> Seal Stage	0.2	0.2
3 <sup>rd</sup> Seal Stage	0.27	1 (only if the second seal stage fails)
<b>O-ring extrusion</b>	0	0.5

*Table 7: Probabilities of the branches of the RCP Seal LOCA ET models*

Figure 3 shows the static ET for the WOG2000 model, whereas Figure 4 the static ET for the WOG2000 (revised). Figure 3 and Figure 4 show the branches that arise from an initiating event of

SBO (with the consequent loss of the seal cooling), evolving according to the failure of the different seals. In the ETs, we can notice that O-ring extrusion is not considered for any of the models, since it has been demonstrated that the condition provided by the WOG2000 (revised) (depressurization resulting in a RCS pressure lower than 118 bar within two hours) leads to a probability of failure equal to zero.

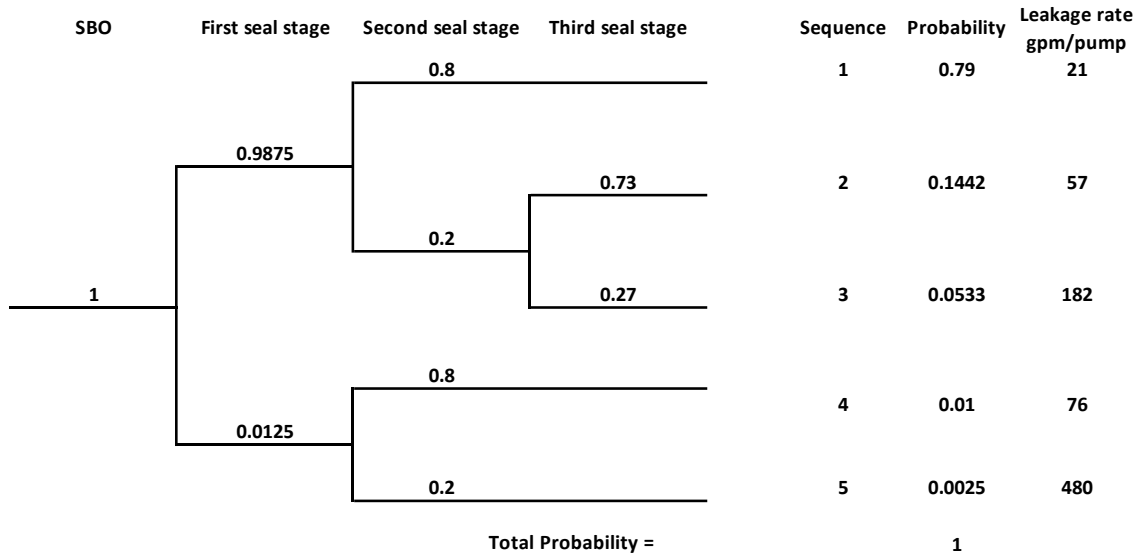


Figure 3: WOG2000 static ET [WOG2000]

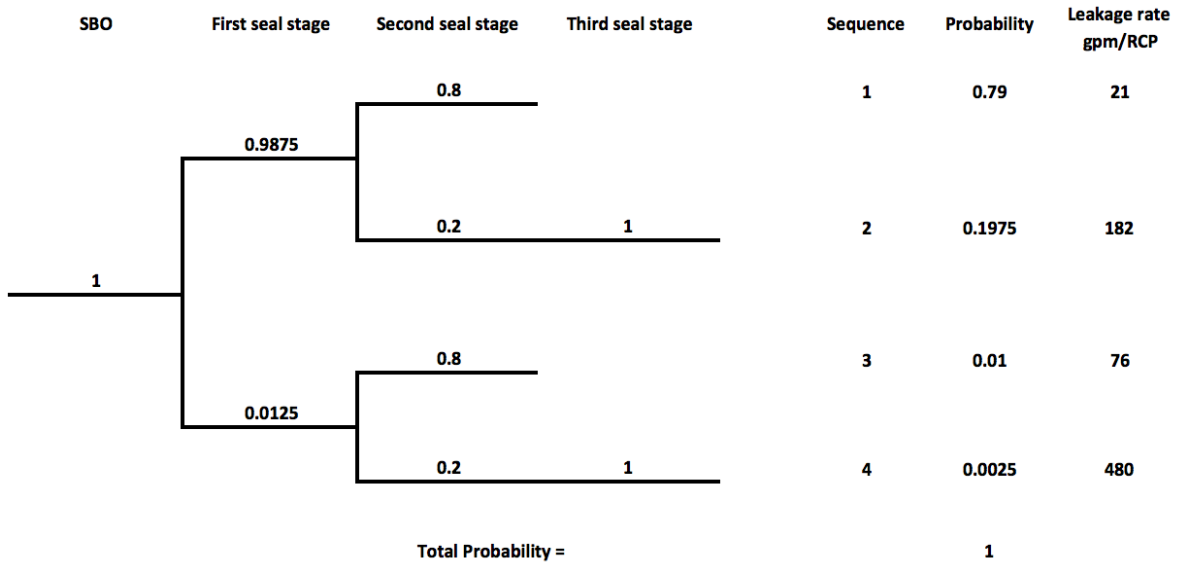


Figure 4: WOG2000 (revised) static ET [WOG2000]

The different number of possible sequences is due to the fact that the WOG2000 model considers five possible leakage rates (21, 57, 76, 182, 480 gpm/RCP), whereas four leakages (21, 76, 182, 480 gpm/RCP) are considered for the WOG2000 (revised) model because in this latter model, as already said, third seal fails with the second seal, otherwise it does not fail.

The static ETs can be used for the quantification of the different probabilistic models for the Seal LOCA. For this purpose, a probability has been assigned arbitrarily to each discrete possible value of the input variables of the dynamic simulations (in particular, for  $x_3$  and  $x_4$ ). Referring for example to  $x_3$ , we have arbitrarily assigned a value to the probability that  $x_3$  assumes the value  $x_{3,1}$  (i.e.,  $P(x_3 = x_{3,1})$ ),  $x_{3,2}$  (i.e.,  $P(x_3 = x_{3,2})$ ),  $x_{3,3}$  (i.e.,  $P(x_3 = x_{3,3})$ ),  $x_{3,4}$  (i.e.,  $P(x_3 = x_{3,4})$ ) and  $x_{3,5}$  (i.e.,  $P(x_3 = x_{3,5})$ ), where  $P(x_3 = x_{3,1}) + P(x_3 = x_{3,2}) + P(x_3 = x_{3,3}) + P(x_3 = x_{3,4}) + P(x_3 = x_{3,5}) = 1$ . Analogously, we have assumed  $P(x_4 = "A")$ ,  $P(x_4 = "B")$ . Whereas, for  $x_2$ , probabilities that  $x_2$  assumes its possible values (i.e., 21, 57, 76, 182 and 480 gpm/RCP) are defined by the different branches probabilities of the static ET (defined by the probabilistic model).

The probabilities assigned for each possible value of the inputs  $x_3$  and  $x_4$  are listed in Table 8.

$x_i$	$x_{k,i}$	$P(x_i = x_{k,i})$
$x_3$ [min]	20	0.42
	30	0.17
	40	0.15
	50	0.14
	60	0.12
$x_4$	"A"	0.6
	"B"	0.4

Table 8: Probabilities arbitrarily assigned for  $x_3$  and  $x_4$

Since each simulation, fed by a given input vector  $\bar{x} = \{x_1, x_2, x_3, x_4\}$ , leads to a core uncovering at time  $y_t$ , we can define the probability to have core uncovering at time  $y_t$  as the conditional probability that  $\bar{x}$  assumes the corresponding input values (considering fixed both the value of  $x_1$  by the probabilistic model and of  $x_2$ , as the different leakage rates have been considered separately). Correspondingly, for each scenario (and, thus, for each simulation), we can calculate the Core Damage Probability ( $P_{CD}$ ), defined, in general, as the probability of the occurrence of core uncovering at  $y_t$ , conditioned to the probability that the recovery of AC power (with the consequent restoration of safety systems to mitigate the consequences of the Seal LOCA) occurs at  $t_{rec}$  after the uncovering at  $y_t$  and, thus, the AC power recovery does not contribute to preventing the core damage. We assume the recovery time  $t_{rec}$  to follow a lognormal distribution with a mean value of 10800 s.

Tables 9 - 11 show the values of the  $P_{CD}$  and the accident sequence risk (computed by multiplying by the leakage initiating event probabilities) for the different probabilistic models, WOG2000, WOG2000 (revised) with  $x_1 = 13$  min, and the WOG2000 (revised) with  $x_1 = 0$  min, respectively.

For each probabilistic model, the  $P_{CD}$  for each leakage rate  $x_2$  is listed in the second column of the tables: this is defined as the sum of the  $P_{CD}$  calculated with fixed  $x_2$  and  $x_1$  (that is fixed by the probabilistic model) and all the possible combinations  $P_{CD}$  of  $x_3$  and  $x_4$ . For instance, for the WOG2000 model (corresponding to  $x_1 = 0$  min), the  $P_{CD}$  of  $x_2 = 480$  gpm/RCP has been computed as the sum of all the  $P_{CD}$  computed for sequences of vectors  $\bar{x} = \left\{ 0 \text{ min}, 480 \frac{\text{gpm}}{\text{RCP}}, x_3, x_4 \right\}$ , with  $x_3 = 20$  min, 30 min, 40 min, 50 min, 60 min and  $x_4 = \text{"A"}, \text{"B"}$ . It is worth mentioning that, even though no sequences were available for 21 gpm/RCP and 57 gpm/RCP we have assumed for the former negligible consequences, whereas for the latter the same core uncover time of the case  $x_2 = 76$  gpm/RCP, conservatively. The probability of occurrence of  $x_2$ , listed in the third column (as presented in Figure 3 and Figure 4), is multiplied by the computed  $P_{CD}$  to obtain the accident sequence probability related to each leakage rate listed in the fourth column. Finally, the total CD probability is the sum of the accident sequence probability related to all the leakage rates (i.e., 21, 57, 76, 182, 480 gpm/RCP for the WOG2000 and 21, 76, 182, 480 gpm/RCP for the WOG2000 (revised)) as reported in the fifth column.

Leakage rate $x_2$	$P_{CD}$	Probability of occurrence	Accident sequence	CD probability
21 gpm/RCP	0	0.79	0	1.35E-003
57 gpm/RCP	0	0.1442	0	
76 gpm/RCP	0	0.01	0	
182 gpm/RCP	1.15E-004	0.0533	6.14E-006	
480 gpm/RCP	5.36E-001	0.025	1.34E-003	

Table 9: WOG2000

Leakage rate $x_2$	$P_{CD}$	Probability of occurrence	Accident sequence	CD probability
21 gpm/RCP	0	0.79	0	1.93E-003
57 gpm/RCP	-	-	-	
76 gpm/RCP	0	0.01	0	
182 gpm/RCP	2.65E-004	0.1975	5.23E-005	
480 gpm/RCP	7.15E-001	0.025	1.88E-003	

Table 10: WOG2000 (revised) with  $x_1 = 13$  min

Leakage rate $x_2$	$P_{CD}$	Probability of occurrence	Accident sequence	CD probability
21 gpm/RCP	0	0.79	0	4.26E-003
57 gpm/RCP	-	-	-	
76 gpm/RCP	1.37E-008	0.01	1.37E-010	
182 gpm/RCP	9.86E-003	0.1975	1.95E-003	
480 gpm/RCP	9.26E-001	0.025	2.31E-003	

Table 11: WOG2000 (revised) with  $x_1 = 0$  min

We can conclude that WOG2000 (revised) with  $x_1 = 0$  min provides the largest estimation of the CD probability that is contributed mostly by  $x_2 = 480$  gpm/RCP and  $x_2 = 182$  gpm/RCP. For the other models, only  $x_2 = 480$  gpm/RCP really contributes to the CD probability, confirming the results of the DPSM-based sensitivity analysis of Section 3.1.3.2, that had identified  $x_2$  as the input variable that most affects the available grace time. Moreover, this probabilistic analysis also confirms that the worst case (i.e., the shortest grace time available for counteracting the developing accident  $\hat{y}_t = 7406$  s) corresponds to the scenario in which a leakage rate  $x_2 = 480$  gpm/RCP occurs at the beginning of the scenario ( $x_1 = 0$  s) and operators act after twenty minutes ( $x_3 = 20$  min) with strategy  $x_4 = "A"$ . As a last remark, it is worth mentioning that an increase of  $x_1 = 13$  min of the time of the leakage rate increase for the WOG2000 (revised) implies the total CD probability to be reduced of approximately 55% with respect to the same probabilistic model but with  $x_1 = 0$  min. Whereas, a further increase of the time of the leakage rate increase up to  $x_1 = 30$  min (WOG2000) implies a reduction of approximately 31% with respect to the WOG2000 (revised) model with  $x_1 = 13$  min. Therefore, the reduction is not significant for a relevant increase of  $x_1$ , that, again, confirms the results of the DPSM-based sensitivity analysis that ranks  $x_1$  lower than  $x_2$  with respect to its effects on the accident progression.

## 3.2 U-Tube Steam Generator

### 3.2.1 The system

A schematic of a U-Tube Steam Generator (UTSG) of a NPP is shown in Figure 5. This system has been chosen because several studies have shown that its malfunction can be considered as one of the major causes of NPP unavailability [Kothare et al., 2000; Habibiyan et al., 2004; Marseguerra et al., 2007].

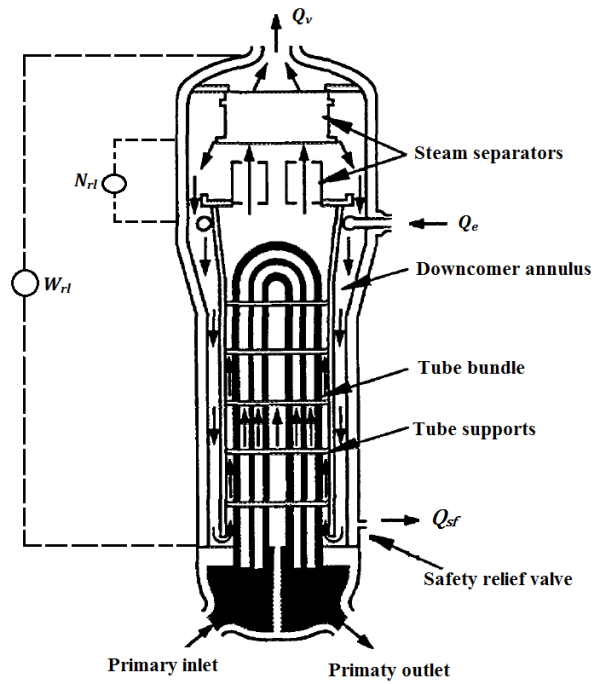


Figure 5: Schematic of the UTSG [IAEA-TECDOC-981, 1997]

The reactor coolant enters the UTSG at the bottom, moves upward and then downward in the inverted U-tubes, transferring heat to the secondary fluid before exiting at the bottom. The secondary fluid, the feed water ( $Q_e$ ), enters the UTSG at the top of the downcomer, through the space between the tube bundle wrapper and the SG shell. The value of  $Q_e$  is regulated by a system of valves: a low flow rate valve, used when the operating power ( $P_o$ ) is smaller than 15% of nominal power ( $P_n$ ), and a high flow rate valve when  $P_o > 0.15 P_n$  [Aubry et al., 2012]. In the secondary side of the tube bundle, water heats up, reaches saturation, starts boiling and turns into a two-phase mixture. The two-phase fluid moves up through the separator/riser section, where steam is separated from liquid water, and through the dryers, which ensure that the exiting steam ( $Q_v$ ) is essentially dry [Di Maio et al., 2014a]. The separated water is recirculated back to the downcomer. The balance between the exiting  $Q_v$  and the incoming  $Q_e$  governs the change in the water level in the SG. Because of the two-phase nature, two types of water level measurements are considered, as shown in Figure 5, each reflecting a different level concept: the Narrow Range Level ( $N_{rl}$ ) is calculated by pressure difference between two points close to the water level and indicates the mixture level, whereas, the Wide Range Level ( $W_{rl}$ ) is calculated by pressure difference between the two extremities of the SG (steam dome and bottom of the downcomer) and indicates the collapsed liquid level that is related with the mass of water in the SG [Di Maio et al., 2014a].



The goal of the system is to maintain the  $N_{rl}$  ( $y$ ) at a reference position  $y_{ref} = 151.67$  cm. The SG fails if  $y$  rises (falls) above (below) the threshold  $U = 177.417$  cm ( $L = 106.2$  cm), that triggers an automatic turbine trip. Indeed, if the  $y$  exceeds  $U$ , the steam separator and dryer lose their functionality and excessive moisture is carried in  $Q_v$ , degrading the turbine blades profile and the turbine efficiency; if  $y$  decreases below  $L$ , insufficient cooling capability of the primary fluid occurs. Similarly, the  $W_{rl}$  is relevant for the cooling capability of the primary circuit [Kothare et al., 2000]. A dedicated model of the system has been implemented in SIMULINK to simulate the dynamic response of the UTSG at different  $P_o$  values [Di Maio et al., 2014a]. In this model, both feedforward and feedback digital control schemes have been adopted. The feedback controller is a PID that provides a flow rate  $Q_{pid}$  resulting from the residuals between  $y$  and  $y_{ref}$ , whereas the feedforward controller operates a safety relief valve that is opened if and only if  $y$  exceeds the  $N_{hl}$  (which is an upper pre-alarm), and removes a constant flow rate ( $Q_{sf}$ ).

Multiple component failures considered can occur during system operation continuously at random time between  $[0, 4000]$  (s). This mission time ( $T_{miss} = 4000$  s) has been chosen in order to account for the complete development also of slow dynamic accident scenarios [Di Maio et al., 2014a].

Components can fail according to different magnitudes of the failure. In particular:

1. The outlet steam valve can fail in three different positions: i) closed; ii) stuck open at 50% of the nominal  $Q_v$  that should be provided at  $P_o$ ; iii) stuck open at 150% of the nominal  $Q_v$  that should be provided at  $P_o$ .
2. The communication between the sensor that monitors  $y$  and the PID controller can fail, returning the same input value of the previous time step.
3. The safety relief valve can fail at a uniform random value  $Q_{sf}$  in the range  $[0.5, 50.5]$  (kg/s).
4. The PID controller can fail providing a uniform random flow rate  $Q_{pid}$  belonging to  $[-18, 18]$  % of the nominal  $Q_e$  that should be provided at  $P_o$ .

Although in a real accident progression, timing, order and magnitude of failure events should assume continuous values, this would make the problem intractable within a classical PSA framework. To deal with this (and at the same time leveraging the computational demand) we have approximated the problem with discretized timing and magnitude values of the failure events in order to generate the dynamic scenarios. In particular, a Multiple Value Logic (MVL) for an approximated description of the continuous time of occurrence of component failures and their magnitudes has been adopted [Di Maio et al., 2014a]. The MVL allows describing a situation in which the components can fail at any (discrete) time along the scenario ( $x_{1,comp}$ , referring to a generic component  $comp$ ) with different

(discrete) magnitudes ( $x_{2,comp}$ ) [Di Maio et al., 2014a]. In the MVL vector, an additional variable ( $x_{3,comp}$ ) indicates the order of occurrence of the different failure events in the sequence.

The discretization of the time and magnitudes values is as follows:

- time discretization: we refer to  $x_{1,comp} = 1$ ,  $x_{1,comp} = 2$ ,  $x_{1,comp} = 3$  and  $x_{1,comp} = 4$ , for failures occurring in the intervals  $[0, 1000]$  (s),  $[1001, 2000]$  (s),  $[2001, 3000]$  (s),  $[3001, 4000]$  (s), respectively; if the label  $x_{1,comp} = 0$ , the component does not fail within the time of the whole scenario,  $T_{miss}$ .
- Magnitude discretization:
  - the steam valve magnitude ( $x_{2,steam}$ ) is indicated as 1, 2 or 3 for failure states corresponding to stuck at 0%, stuck at 50% and stuck at 150% of the  $Q_e$  value that should be provided at  $P_o$ , respectively; if  $x_{2,steam} = 0$ , the component does not fail in  $T_{miss}$ ;
  - the communication between the sensor measuring  $y$  and the PID controller  $x_{2,comm} = 0$  if the communication works,  $x_{2,comm} = 1$  otherwise;
  - the safety relief valve fails with magnitude  $x_{2,safety}$  that can assume the values 1, 2, 3 and 4, if it is stuck between  $[0.5, 12.6]$  (kg/s),  $(12.6, 25.27]$  (kg/s),  $(25.27, 37.91]$  (kg/s) and  $(37.91, 50.5]$  (kg/s), respectively;  $x_{2,safety} = 0$ , indicates that the component does not fail in  $T_{miss}$ ;
  - the PID controller failure magnitude range is discretized into 8 equally spaced magnitude intervals,  $x_{2,PID}$  labeled from 1 to 8, representative of failure states corresponding to discrete intervals of output value belonging to  $[-18,18]\%$  of the  $Q_e$  value that should be provided at  $P_o$ ; if  $x_{2,PID} = 0$ , the component does not fail in  $T_{miss}$ .

The possible failure events combinations give rise to 100509 possible sequences, each described by a MVL vector. Among these sequences, in order to reproduce the same conditions of the previous case study for the comparison of the results, an arbitrary number  $N = 90$  of simulations have been randomly selected to challenge the DPSM sensitivity analysis method presented in Section 2 (i.e., we assume independent inputs and, therefore, as in Section 3.1.2, no joint distributions). For each dynamic simulation, the value  $y$  of the level that is reached along the scenario progression and the corresponding grace time  $y_t$  have been collected,  $\bar{y}$  and  $\bar{y}_t$  respectively. In particular, when  $y = U = 177.417$  cm is reached (and, thus, the corresponding safety margin is 0), we record the grace time  $y_t$  for reaching the threshold; otherwise, for the scenarios in which  $y < U$ , during the entire

accidental progression, the maximum value reached by  $y$  and the corresponding grace time  $y_t$  at which it is reached are recorded.

### 3.2.2 DPSMs and Sensitivity Analysis results

#### 3.2.2.1 The DPSMs

The collected values of  $\bar{y}$  and  $\bar{y}_t$  (i.e., the Narrow Range Level  $N_{rl}$  and the corresponding time, respectively) have been employed for the calculation of the DPSMs (see Section 2.2). Since the alarm triggers either when  $N_{rl}$  falls below the lower threshold  $L$  or rises above the upper threshold  $U$ , we focus, without loss of generality, on the upper threshold  $U$ . Resorting to OS, we have estimated the values of the  $\gamma_1^{th}$  and  $\gamma_2^{th}$  percentiles, with confidences  $\beta_1$  and  $\beta_2$ , of the Narrow Range Level  $y$  and of the time at which this level is reached. This has been determined using all the  $N = 90$  simulations, as well as all the subgroups, that is, considering the simulations when each input variable is kept fixed, and components are analyzed independently.

In Tables 12 and 13, we report  $\hat{y}_{\gamma_1}$ , the value of  $\hat{y}_t$  and the values of the DPSM for two different cases ( $\gamma_2$  and  $\beta_2$  is fixed, respectively), when all the  $N = 90$  simulations results are considered.

Results of the computation of DPSM for the case in which all the  $N = 90$  available simulations are used simultaneously is shown in Tables 12 and 13. The estimation of the value of the Narrow Range Level  $\hat{y}$  results equal to  $U$ . The DPSM of Eq. (3) provides the additional information of the grace time  $\hat{y}_t = 213$  s (to reach the alarm triggering).

DPSM provides the analyst with the additional information of the grace time  $\hat{y}_t$  (before the alarm triggering), which is a main benefit of the DPSM with respect to the traditional safety margin. This means that, when  $\hat{y}_{\gamma_1}$  is equal to  $U$ , the value of the grace time  $\hat{y}_t$  is the time available for taking counteracting/mitigation actions before alarm triggering. It is worth mentioning that, even though the regulation guidances prescribe the DPSM to be given with  $\gamma_1 = \beta_1 = \beta_2 = 0.95$  and  $\gamma_2 = 0.05$ , the limited number  $N$  of available simulations:

- limits the analysis on the estimation of the grace time available before core uncover with the desired  $\gamma_2$  and  $\beta_2$ ;
- is not large enough to allow for an estimate of  $\hat{y}_t$  with  $\gamma_2 = 0.05$  and  $\beta_2 = 0.95$ .

Therefore, in Table 12, we provide the results when we aim at estimating exactly the  $\gamma_2^{th}$  percentile of  $y_t$ , with the as large as possible  $\beta_2$  according to the two different approaches (“Bracketing” and “Coverage” as explained in Appendix A). On the contrary, in Table 13, we provide the result when the confidence in the estimation, i.e.,  $\beta_2$ , is fixed to at least equal to 0.95 whereas the value of the estimated percentile  $\gamma_2$  varies as theory.

$\gamma_2$	$\beta_2$ “Bracketing”	$\beta_2$ “Coverage”	$\hat{y}_{\gamma_1} [cm]$	$\hat{y}_t [s]$	DPSM
0.05	0.98	0.94	177.417 = $U$	213	0 within a grace time of 213 s

Table 12: DPSM ( $N=90$  and with fixed  $\gamma_2$ ) for the UTSG

$\beta_2$	$\gamma_2$ “Bracketing”	$\gamma_2$ “Coverage”	$\hat{y}_{\gamma_1} [cm]$	$\hat{y}_t [s]$	DPSM
0.95	0.04	0.03	177.417 = $U$	213	0 within a grace time of 213 s

Table 13: DPSM ( $N=90$  and with fixed  $\beta_2$ ) for the UTSG

Results of the computation of DPSM for the case of subgroups are shown hereafter. It is worth mentioning that: (i) being the computed  $\hat{y}_{\gamma_1}$  equal to  $U$  (as shown before in Tables 12 and 13); (ii) being the number  $N$  of available simulations lower than 90 for each considered subgroup, and each component separately, we focus only on the estimation of the value  $\hat{y}_t$  that, incidentally, cannot be estimated with both  $\beta_2$  and  $\gamma_2$  with the values, as required by regulation guidances. The reduced number  $N$  of simulations results is due to the fact that, when we consider a variable to be kept fixed at a given value of its possible instances, the number of available simulations reduces to only those that have as inputs the selected fixed value of that input variable and the combination of all the other possible values of the other input variables.

Results of the computation of DPSM for the case of subgroups (and focusing on the single component) are shown hereafter. Tables 14-17 show the DPSM quantification when the steam valve, the communication, the safety valve and the PID fail independently, respectively.

In particular, in Table 14, the results of the estimated grace time  $\hat{y}_t$ , when the steam valve fails, are shown, when a different number  $N$  of simulations are available due to the input  $x_{1,steam}$ ,  $x_{2,steam}$ ,  $x_{3,steam}$  being kept fixed at given values. As already said, both the case when  $\gamma_2$  is fixed and  $\beta_2$  is fixed are presented.

Similarly to the first case study, we find that:

- the “Bracketing” approach generally provides a larger confidence  $\beta_2$  for the estimate of the  $\gamma_2^{th}$  percentile of  $y_t$  than the “Coverage” approach. Contrarily, when  $\beta_2$  is fixed, the “Bracketing” approach provides unfavorable estimated value of the  $\gamma_2^{th}$  percentile with fixed  $\beta_2$  with respect to the “Coverage” approach.

- When we consider results obtained with fixed values of the inputs, the value of the percentile that can be estimated unfavorably increases, being the number  $N$  of available simulations lower than when all simulations are simultaneously considered; the values of  $\gamma_2$ , indeed, go from 0.04 and 0.03 (“Bracketing” and “Coverage”, respectively), to 0.28 and 0.25 (when  $x_{1,steam} = 4$  is kept fixed and  $N = 11$ ). Similarly, we can see that the confidence  $\beta_2$  unfavourably decreases, when the value of  $\gamma_2$  is fixed to 0.05. The values of  $\beta_2$ , indeed, go from 0.98 and 0.94 (“Bracketing” and “Coverage”, respectively) for the best case with  $N = 90$ , to 0.19 and 0.11 (when  $x_{1,steam} = 4$  is kept fixed and  $N = 11$ );
- The worst case (i.e., the shortest grace time available for counteracting the developing accident  $\hat{y}_t = 213$  s) corresponds to the scenario in which the failure of the steam valve occurs at time  $x_{1,steam} = 1$ , with magnitude  $x_{2,steam} = 2$  when the failure is the second event of the progression,  $x_{3,steam} = 2$ .

		$N$	$\beta_2$ “Bracketing”	$\beta_2$ “Coverage”	$\gamma_2$ “Bracketing”	$\gamma_2$ “Coverage”	$\hat{y}_t$
			$\gamma_2 = 0.05$		$\beta_2 = 0.95$		
	<i>All simulations values of <math>x_{1,steam}</math>, <math>x_{2,steam}</math> and <math>x_{3,steam}</math> accounted</i>	90	0.98	0.94	0.04	0.03	213
$x_{1,steam}$ Time is kept fixed	1	17	0.34	0.22	0.19	0.17	213
	2	26	0.54	0.38	0.13	0.11	544
	3	24	0.5	0.35	0.14	0.12	266
	4	11	0.19	0.11	0.28	0.25	544
$x_{2,steam}$ Magnitude is kept fixed	0	12	0.21	0.13	0.26	0.23	312
	1	19	0.39	0.25	0.18	0.15	318
	2	32	0.65	0.49	0.11	0.09	213
	3	27	0.56	0.40	0.13	0.11	253
$x_{3,steam}$ Order is kept fixed	1	19	0.39	0.25	0.18	0.15	253
	2	29	0.6	0.44	0.12	0.1	213
	3	18	0.36	0.24	0.18	0.16	307
	4	12	0.21	0.13	0.26	0.23	266

Table 14: DPSM for the steam valve

In Table 15, the results of the estimated grace time  $\hat{y}_t$  when the communication between the sensor and the PID fails, are shown, for a different number  $N$  of simulations with the input  $x_{1,comm}$ ,  $x_{2,comm}$ ,  $x_{3,comm}$  kept fixed at given values. As already said, both the case when  $\gamma_2$  is fixed and  $\beta_2$  is fixed are presented. The insight from the results obtained are similar to the ones obtained before.

		$N$	$\beta_2$	$\beta_2$	$\gamma_2$	$\gamma_2$	$\hat{y}_t$ [s]
			“Bracketing”	“Coverage”	“Bracketing”	“Coverage”	
		$\gamma_2 = 0.05$		$\beta_2 = 0.95$			
<i>All simulations values of <math>x_{1,comm}</math>, <math>x_{2,comm}</math> and <math>x_{3,comm}</math> accounted</i>		90	0.98	0.94	0.04	0.03	213
$x_{1,comm}$ Time is kept fixed	1	21	0.43	0.29	0.16	0.14	213
	2	18	0.36	0.24	0.18	0.16	318
	3	18	0.36	0.24	0.18	0.16	307
	4	20	0.41	0.27	0.17	0.15	335
$x_{2,comm}$ Magnitude is kept fixed	0	13	0.24	0.14	0.25	0.22	213
	1	77	0.96	0.90	0.05	0.04	213
$x_{3,comm}$ Order is kept fixed	1	19	0.39	0.25	0.18	0.15	544
	2	26	0.54	0.38	0.13	0.12	266
	3	19	0.39	0.25	0.18	0.15	213
	4	13	0.24	0.14	0.25	0.22	307

Table 15: DPSM for the communication between the sensor and the PID controller

In Table 16, the results of the estimated grace time  $\hat{y}_t$  when the safety valve fails, are shown, for a different number  $N$  of simulations, with the input  $x_{1,safety}$ ,  $x_{2,safety}$ ,  $x_{3,safety}$  kept fixed at given values. As already said, both the case when  $\gamma_2$  is fixed and  $\beta_2$  is fixed are presented. Also in this case, the insight from the results obtained are similar to the ones obtained before.

		$N$	$\beta_2$	$\beta_2$	$\gamma_2$	$\gamma_2$	$\hat{y}_t$ [s]
			“Bracketing”	“Coverage”	“Bracketing”	“Coverage”	
		$\gamma_2 = 0.05$		$\beta_2 = 0.95$			
<i>All simulations values of <math>x_{1,safety}</math>, <math>x_{2,safety}</math> and <math>x_{3,safety}</math> accounted</i>		90	0.98	0.94	0.04	0.03	213
$x_{1,safety}$ Time is kept fixed	1	18	0.36	0.24	0.18	0.16	213
	2	27	0.56	0.40	0.13	0.11	253
	3	19	0.39	0.25	0.18	0.15	312
	4	23	0.48	0.33	0.15	0.13	213
$x_{2,safety}$ Magnitude is kept fixed	0	3	0.02	0.01	0.71	0.67	9000
	1	13	0.24	0.14	0.25	0.22	1253
	2	23	0.48	0.33	0.15	0.13	213
	3	28	0.58	0.42	0.12	0.11	312
	4	23	0.48	0.33	0.15	0.13	213
$x_{3,safety}$ Order is kept fixed	1	25	0.52	0.37	0.14	0.12	1221
	2	17	0.34	0.22	0.20	0.17	307
	3	29	0.6	0.44	0.12	0.1	213
	4	16	0.31	0.2	0.11	0.18	213

Table 16: DPSM for the safety valve

In Table 17, the results of the estimated grace time  $\hat{y}_t$ , when the PID controller fails, are shown, for a different number  $N$  of simulations, with the input  $x_{1,PID}$ ,  $x_{2,PID}$ ,  $x_{3,PID}$  kept fixed at given values. As already said, both the case when  $\gamma_2$  is fixed and secondly, the  $\beta_2$  is fixed are presented. Results obtained are similar to the previous ones.

		N	$\beta_2$	$\beta_2$	$\gamma_2$	$\gamma_2$	$\hat{y}_t$ [s]
			“Bracketing”	“Coverage”	“Bracketing”	“Coverage”	
		$\gamma_2 = 0.05$		$\beta_2 = 0.95$			
All simulations values of $x_{1,PID}$ , $x_{2,PID}$ and $x_{3,PID}$ accounted		90	0.98	0.94	0.04	0.03	213
$x_{1,PID}$ Time is kept fixed	1	18	0.36	0.24	0.19	0.16	213
	2	27	0.56	0.4	0.13	0.11	544
	3	19	0.39	0.25	0.18	0.15	503
	4	23	0.48	0.33	0.15	0.13	1253
$x_{2,PID}$ Magnitude is kept fixed	0	0	0	0	0	0	0
	1	10	0.16	0.09	0.31	0.28	544
	2	10	0.16	0.09	0.31	0.28	213
	3	12	0.21	0.13	0.26	0.23	307
	4	19	0.39	0.25	0.18	0.15	266
	5	10	0.16	0.09	0.31	0.28	253
	6	9	0.14	0.08	0.34	0.3	1503
	7	15	0.29	0.18	0.22	0.19	1253
8	5	0.05	0.03	0.52	0.48	1253	
$x_{3,PID}$ Order is kept fixed	1	21	0.43	0.29	0.16	0.14	213
	2	23	0.48	0.33	0.15	0.13	253
	3	14	0.26	0.16	0.23	0.2	503
	4	32	0.65	0.49	0.11	0.09	1253

Table 17: DPSM for the PID

### 3.2.2.2 Sensitivity analysis

On the basis of the results shown from Table 14 to Table 17, the proposed sensitivity index of Eq. (10) has been computed for each input variable  $x_k$ ,  $k = 1, 2, 3$  (see Section 2.2), considering the components independently. Results, referred to each component, are shown in Tables 18-21, where in column 2 the possible values  $x_{k,i}$  of  $x_k$  are listed, in column 3 the estimated values of the grace time  $\hat{y}_t$  are listed, in columns 4 and 5 the variability range of the normalized input  $\Delta x_k$  (as explained in Section 2.2) and the variability range  $\Delta \hat{y}_t$  of the normalized  $\hat{y}_t$ , are listed respectively, when only the simulations when each input variable is kept fixed are considered. Finally, in the last column, the value of the sensitivity index  $I_{x_k}$  computed for each input variable is shown.



In Table 18, the results of the sensitivity analysis referred to the failure events of the steam valve are presented. The steam valve failure time  $x_{1,steam}$  is the input that most affects the grace time  $\hat{y}_t$  available, whereas the failure magnitude  $x_{2,steam}$  and the order of the failure  $x_{3,steam}$  along the accidental sequence of events have almost the same (low) influence on the output  $\hat{y}_t$ .

$x_k$	$x_{k,i}$	$\hat{y}_t$	$\Delta x_k$	$\Delta \hat{y}_t$	$I_{x_k}$
$x_{1,steam}$	1	213	0.75	0.61	<b>0.81</b>
	2	544			
	3	266			
	4	544			
$x_{2,steam}$	1	318	0.67	0.33	0.49
	2	213			
	3	253			
$x_{3,steam}$	1	253	0.75	0.31	0.41
	2	213			
	3	307			
	4	266			

Table 18: Sensitivity analysis for the steam valve

In Table 19, the results of the sensitivity analysis referred to the failure of the communication between the sensor and the PID controller are shown.

The failure magnitude  $x_{2,comm}$  appears to be irrelevant for the  $\hat{y}_t$ , ( $\Delta \hat{y}_t = 0$ ), in both cases of communication working ( $x_{2,comm} = 0$ ) or failed ( $x_{2,comm} = 1$ ). However, the order  $x_{3,comm}$  at which communication fails along the accidental sequence is the most important input for the quantification of  $\hat{y}_t$ .

$x_k$	$x_{k,i}$	$\hat{y}_t$	$\Delta x_k$	$\Delta \hat{y}_t$	$I_{x_k}$
$x_{1,comm}$	1	213	0.75	0.36	0.49
	2	318			
	3	307			
	4	335			
$x_{2,comm}$	0	213	1	0	0
	1	213			
$x_{3,comm}$	1	544	0.75	0.61	<b>0.81</b>
	2	266			
	3	213			
	4	307			

Table 19: Sensitivity analysis for the communication between the sensor and the PID controller

The results of sensitivity analysis referred to the safety valve failure are shown in Table 20. The failure magnitude  $x_{2,safety}$  and the failure order along the accidental sequence  $x_{3,safety}$  have a large influence on  $\hat{y}_t$ , whereas the failure time  $x_{1,safety}$  has smaller influence on  $\hat{y}_t$ .

$x_k$	$x_{k,i}$	$\hat{y}_t$	$\Delta x_k$	$\Delta \hat{y}_t$	$I_{x_k}$
$x_{1,safety}$	1	213	0.75	0.32	0.42
	2	253			
	3	312			
	4	213			
$x_{2,safety}$	1	1253	0.75	0.83	<b>1.10</b>
	2	213			
	3	312			
	4	213			
$x_{3,safety}$	1	1221	0.75	0.83	<b>1.10</b>
	2	307			
	3	213			
	4	213			

Table 20: Sensitivity analysis for the safety valve

The results for the failure of the PID are shown in Table 21. Although the failure magnitude  $x_{2,PID}$  is relevant for the quantification of  $\hat{y}_t$ , the failure time  $x_{1,PID}$  and the failure order along the accidental sequence  $x_{3,PID}$  affect  $\hat{y}_t$  with a larger extent.

$x_k$	$x_{k,i}$	$\hat{y}_t$	$\Delta x_k$	$\Delta \hat{y}_t$	$I_{x_k}$
$x_{1,PID}$	1	213	0.75	0.83	<b>1.10</b>
	2	544			
	3	503			
	4	1253			
$x_{2,PID}$	1	544	0.875	0.86	0.98
	2	213			
	3	307			
	4	266			
	5	253			
	6	1503			
	7	1253			
	8	1253			
$x_{3,PID}$	1	213	0.75	0.83	<b>1.10</b>
	2	253			
	3	503			
	4	1253			

Table 21: Sensitivity analysis for the PID

In conclusion, we can claim that dynamic aspects are critical for the estimation of  $\hat{y}_t$  for the UTSG system: sensitivity analysis results, indeed, show that, among the different inputs considered ( $x_{1,comp}$ ,  $x_{2,comp}$ ,  $x_{3,comp}$ ), the failure time  $x_{1,steam}$  affects the quantification of  $\hat{y}_t$  when the failure of the steam valve is considered, the failure magnitude  $x_{2,safety}$  and the failure order  $x_{3,safety}$ , affects  $\hat{y}_t$  when the failure of the safety valve is considered, the failure time  $x_{1,PID}$  and the failure order  $x_{3,PID}$  affects  $\hat{y}_t$  when the failure of the PID is considered and finally, the failure order  $x_{3,comm}$  influences  $\hat{y}_t$  when the failure of the communication between the sensor and the PID controller is taken into account. Although a comprehensive analysis should consider all the number of possible sequences of events originated by the MVL approximation of Section 3.2.1, the results here provided based on only  $N = 90$  randomly selected sequences of events are sufficient to conclude that a DET analysis is needed rather than a static ET analysis. In next Section, we will briefly show the advantages of a DET, that accounts for the different times, magnitude and order of failure events, with respect to a static ET for the case study here considered.

### 3.2.3 The Dynamic Event Tree

In this Section, supported by the outcomes of the sensitivity analysis of Section 3.2.2.2, a DET methodology is used to account for the system dynamics along the accidental sequences progression (considering the timing  $x_{1,comp}$ , the magnitude  $x_{2,comp}$  and the order  $x_{3,comp}$  of failure events for the different components) and to compute the probability of occurrence of the accidental scenarios.

A DET is in principle similar to a static ET (where the sequence of system responses following an initiating event is predetermined by the analyst) except that, in a DET, both the timing and sequence of system responses are determined by a time-dependent model of system evolution, and only the branching rules are determined by the analyst [Aldemir, 2013]. The time-dependent model of the systems allows for a MVL description of components states which, therefore, accounts for different timing, order and magnitude of the possible failure events, leading, consequently, to a multitude of possible scenarios much larger than for a static ET analysis.

Among the scenarios, the identification of Prime Implicants (PIs) is fundamental for the calculation of the probability of failure of the system and for the analysis of such a system. PIs correspond, indeed, to the Minimal Cut Sets (MCS) of the static ET, but are also supplied with the information of time, sequence and magnitude of failures occurrences. [Quine, 1952; Di Maio et al., 2014b; Di Maio et al., 2015b]. Moreover, PIs identification allows for a more accurate characterization of failure sequences of non-coherent systems, that are those systems where both the failed or the working states of a component can lead to the system failure [Di Maio et al., 2014a], as it is for the UTSG hereafter analyzed.

Another important category of sequences arises when dynamics is considered in the analysis that are the so called Near Misses, which are sequences (among safe scenarios), whose sequences of events lead the safety parameter values close to, but not exceeding, the corresponding acceptable thresholds [Zio et al., 2009]. For the UTSG, we rely on the classification of the sequences in Safe, Near Misses and PIs as presented in [Di Maio et al., 2014a].

In what follows, we compare the advantages of a DET with respect to a static ET by assuming a failure probability equal to  $10^{-3}$  for the PID ( $P_{fail,PID}$ ) and for the communication between the sensor and the PID failure ( $P_{fail,comm}$ ),  $10^{-2}$  for the steam valve ( $P_{fail,steam}$ ) and  $10^{-4}$  for the safety valve ( $P_{fail,safety}$ ). Figure 6 shows a static ET for the scenario events related to safety valve, communication, PID and steam valve. The probability of each branch of the ET is given in Figure 6. Dashed branches are the MCS for this scenario as identified in [Di Maio et al., 2014a], whose total probability of occurrence turns out to be equal to 1.097E-02.

Initiating Event	Safety valve	Communication	Steam valve	PID	Branch Probability
	No failure <b>9,999E-01</b>	No failure <b>9,990E-01</b>	No failure <b>9,900E-01</b>	No failure <b>9,990E-01</b>	<b>9,879E-01</b>
				Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,889E-04</b>
			Failure @time = 0 with maximum magnitude <b>1,000E-02</b>	No failure <b>9,990E-01</b>	<b>9,979E-03</b>
				Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,989E-06</b>
		Failure @time = 0 with maximum magnitude <b>1,000E-03</b>	No failure <b>9,900E-01</b>	No failure <b>9,990E-01</b>	<b>9,889E-04</b>
				Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,899E-07</b>
			Failure @time = 0 with maximum magnitude <b>1,000E-02</b>	No failure <b>9,990E-01</b>	<b>9,989E-06</b>
				Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,999E-09</b>
	Failure @time = 0 with maximum magnitude <b>1,000E-04</b>	No failure <b>9,990E-01</b>	No failure <b>9,900E-01</b>	No failure <b>9,990E-01</b>	<b>9,880E-05</b>
				Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,890E-08</b>
			Failure @time = 0 with maximum magnitude <b>1,000E-02</b>	No failure <b>9,990E-01</b>	<b>9,980E-07</b>
				Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,990E-10</b>
	Failure @time = 0 with maximum magnitude <b>1,000E-03</b>	No failure <b>9,900E-01</b>	No failure <b>9,990E-01</b>	<b>9,890E-08</b>	
			Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>9,900E-11</b>	
		Failure @time = 0 with maximum magnitude <b>1,000E-02</b>	No failure <b>9,990E-01</b>	<b>9,990E-10</b>	
			Failure @time = 0 with max magnitude <b>1,000E-03</b>	<b>1,000E-12</b>	

Figure 6: Event Tree

However, it is important to notice that the results of the static ET might be invalidated when a different combination of failures times, order, and magnitudes occur. As we will show, indeed, a single branch (scenario) of the static ET corresponds effectively to more different branches in the DET, which are all the possible combinations of timing, order and magnitude of the components that fail during the considered static scenario. Consequently, a branch of static ET lumps together safe, near miss and PI scenarios, that would be otherwise spooned out if the system dynamics was modelled within a DET. To show this, without loss of generality, we focus on a selection of MVL sequences of the UTSG that, incidentally, are three safe sequences, four NMs and four PIs, among the 100509 possible sequences. Table 22 lists the selected sequences (defined in terms of order and discrete magnitude and timing of failure events of each component).

For each dynamic sequence, times of failures and magnitudes have been sampled from the corresponding intervals that have been introduced in Section 3.2.1 (e.g., if the time of failure is labeled as 1, it is sampled from the interval [1, 1000]; similarly, for magnitudes), and the SIMULINK model has been run.

Figure 9 plots the dynamic sequences listed in Table 22 as branches of a DET (where dashed branches are left unsolved and only the failure magnitudes  $m$  generate alternative paths, whereas the effects of timing and order are not shown, for the sake of the Figure clarity and to avoid the combinatorial explosion of the number of branches to be plotted). Figure 22 reports, for each sequence, the values

of the probability of the corresponding branch calculated in the static ET of Figure 8. It is important to mention that, for the DET, we have considered the PID failure probability  $P_{fail,PID}$  equal to  $1.25E-04$  (i.e., 1/8 of the  $P_{fail,PID}$  in the static ET), the safety valve failure probability  $P_{fail,SV}$  equal to  $2.5E-05$  (i.e., 1/4 of the  $P_{fail,SV}$  in the static ET), the steam valve failure probability  $P_{fail,ST.V}$  equal to  $3.33E-03$  (i.e., 1/3 of the  $P_{fail,ST.V}$ ), and  $P_{fail,COMM}$  has been assumed equal to  $10^{-2}$ , being only one the possible magnitude of the communication.

Sequence	Safety valve			Communication			PID			Steam valve		
	Time	Mag	Order	Time	Mag	Order	Time	Mag	Order	Time	Mag	Order
SAFE	0	0	0	4	1	2	4	8	1	0	0	0
SAFE	1	3	2	1	1	1	4	3	3	0	0	0
SAFE	4	3	4	2	1	3	2	5	1	2	2	2
NM	3	3	2	4	1	3	2	4	1	0	0	0
NM	4	4	3	4	1	2	1	4	1	0	0	0
NM	4	3	4	4	1	3	1	4	1	2	1	2
NM	4	2	3	2	1	2	1	4	1	0	0	0
PI	1	1	1	0	0	0	3	4	2	0	0	0
PI	1	4	1	3	1	3	1	2	2	0	0	0
PI	2	3	1	4	1	2	4	3	3	0	0	0
PI	2	4	1	3	1	4	3	4	3	2	3	2

Table 22: The MVL dynamic sequences considered for the DET

In Figure 9, the evidence of the effects of timing and failure order on the accidental scenarios consequences is shown. For example, let us consider the sequence with safety valve failure magnitude equal to 3, failure magnitude of the communication equal to 1, steam valve not failed (failure magnitude equal to 0) and, finally, failure magnitude of the PID controller equal to 3. These events are resumed into the second and the tenth sequence of Table 22, that means that the originated DET branch correspond to two different sequences differing only for timing and order of failure events (and, thus, not shown) but leading to two opposite consequences, i.e., in one case safe and in the other PI.

Moreover, results of Figure 9 show that each failure sequence evolves with a larger probability in the ET, than in the DET. However, further efforts should be devoted to investigate whether a static ET branch envelopes (or not) all the possible dynamic sequences that would be generated by a DET. It may happen, in fact, that, when considering all the dynamic sequences (lumped in the same branch in an ET) the ET may not necessarily overestimate, but it may even underestimate the failure probability (if the system modelled in not-coherent, for which both failed and working states of the same components can lead the system to failure [Di Maio et al., 2015a]).

Initiating Event	Safety valve	Communication	Steam valve	PID	Type of sequence	Branch Probability of the static ET	Branch Probability of a DET, taking into account magnitudes			
	No failure	No failure	No failure	No failure						
					$x_{2,PID} = 1$					
					$x_{2,PID} = 2$					
					$x_{2,PID} = 3$					
					$x_{2,PID} = 4$					
					$x_{2,PID} = 5$					
					$x_{2,PID} = 6$					
					$x_{2,PID} = 7$					
					$x_{2,PID} = 8$					
					$x_{2,steam} = 1$	No failure				
							$x_{2,PID} = 1$			
							$x_{2,PID} = 8$			
			$x_{2,steam} = 2$	No failure						
				$x_{2,PID} = 1$						
				$x_{2,PID} = 8$						
			$x_{2,steam} = 3$	No failure						
				$x_{2,PID} = 1$						
				$x_{2,PID} = 8$						
		$x_{2,comm} = 1$	No failure							
				$x_{2,PID} = 8$	SAFE	9.90E-07	1.24E-07			
	$x_{2,safety} = 1$	No failure	No failure							
				$x_{2,PID} = 4$	PI	9.89E-07	3.09E-09			
	$x_{2,safety} = 2$		No failure							
		$x_{2,comm} = 1$		$x_{2,PID} = 4$	NM	9.90E-11	3.09E-012			
	$x_{2,safety} = 3$		No failure							
		$x_{2,comm} = 1$		$x_{2,PID} = 3$	SAFE	9.90E-11	3.09E-12			
				$x_{2,PID} = 4$	PI	9.90E-11	3.09E-12			
			$x_{2,steam} = 1$							
				$x_{2,PID} = 4$	NM	9.90E-11	3.09E-12			
				$x_{2,PID} = 4$	NM	1.00E-12	1.04E-14			
			$x_{2,steam} = 2$		SAFE	1.00E-12	1.04E-14			
				$x_{2,PID} = 5$						
	$x_{2,safety} = 4$		No failure							
		$x_{2,comm} = 1$		$x_{2,PID} = 2$	PI	9.90E-11	3.09E-12			
				$x_{2,PID} = 4$	NM	9.90E-11	3.09E-12			
			$x_{2,steam} = 3$							
				$x_{2,PID} = 4$	PI	1.00E-12	1.04E-14			

Fig. 9: Comparison of ET and DET results for the dynamic sequences of Table 22

## 4. Conclusions

In this work, a sensitivity analysis method has been presented, to determine which input variables of a TH codes affect most the safety margin quantification of a dynamic system. Safety margin has been

computed according to a dynamic probabilistic definition, which allows for the quantification of the uncertainties affecting the safety parameter evolution along an accidental scenario and, at the same time, accounts also for the earliest time needed to reach the estimated safety margin. Based on the results of this analysis, it is possible to conclude whether dynamic aspects influence (or not) the safety margin quantification. This can be used to decide the most correct probabilistic model to be used for the system safety assessment and the computation of the probability of occurrence of the accidental scenarios considered: a static analysis (i.e., a static ET) or a dynamic analysis (i.e., a DET). The method has been applied to two different cases of study: an SBO followed by a Seal LOCA and the UTSG in which four components are supposed to fail. Concerning the first case of study, the sensitivity analysis allows us to conclude that simulated boiled up water level in the core whose dynamic evolution with MAAP5 TH code is not sensitive to the dynamic inputs (i.e., time of operators actions, time of the onset of the increased leakage rate). Therefore, a static analysis has been performed to evaluate the probabilities of core damage based only on the failure probabilities of static events (i.e., seal stages). On the contrary, in the second case of study, dynamic inputs of the SIMULINK model of the UTSG (i.e., time, magnitude and order of the steam valve, the communication, the safety valve and the PID) result to be relevant for the variability of the water narrow range level fluctuation during the accidental scenarios considered. Consequently, a simplified DET has been built and a comparison between the results of a static ET and the DET have been presented. The comparison has shown, in particular, that a single branch of the static ET corresponds to many branches of the DET and that, consequently, a static scenario of the ET can correspond, in reality, simultaneously to safe scenario, a NM, or a PI, in the DET, that would lead to a wrong quantification of the system failure if dynamic aspects are neglected.

From the applications to the two cases of study, some limits for the proposed method arise:

1. The estimation of the values of  $y_{\gamma_1}$  and  $y_{t_{\gamma_2}}$  through OS suffers the limited number of available TH code simulations, when the analyst is required to ensure both the value of the percentile and of the confidence on its estimate. One possible solution can consist in utilizing other methods, such as Finite Mixture Models [McLachlan et al. 2000] that have been proven in [Di Maio et al., 2015a] to require a lower number of simulations than OS while, at the same time, to allow for estimating the entire distribution of the parameter rather than only few given percentiles, as it is for OS.
2. The sensitivity index  $I_x$ , here defined gives an indication of which is the input that most affects the DPSM. However, it does not consider any non-linear system response to the accidental progressions that are simulated and, therefore, further efforts should be devoted to fill this gap, in future research works.



Therefore, we can conclude that further studies are necessary in order to overcome these limits and improve, consequently, the methodology presented for the selection of the probabilistic model to be used for the safety assessment of a NPP.

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## Appendix A

The real values of  $y_{\gamma_1}$  and  $y_{t_{\gamma_2}}$  are unknown, because the distributions of  $y$  and  $y_t$  are also unknown. Order Statistics (OS) is a non parametric model used to determine  $\hat{y}_{\gamma_1}$  and  $\hat{y}_{t_{\gamma_2}}$ , which are the estimation of  $y_{\gamma_1}$  and  $y_{t_{\gamma_2}}$ , with a confidence  $\beta_1$  and  $\beta_2$ , respectively [Nutt et al., 2004]. A limited number  $N$  TH simulations is available, where  $\bar{y} = \{y_1, \dots, y_N\}$  is the vector of the output safety variable and  $\bar{y}_t = \{y_{t_1}, \dots, y_{t_N}\}$  is the vector containing the time to reach the corresponding output values of  $\bar{y}$ . OS ensures that the  $m^{\text{th}}$  value (usually the first) of the  $N$  sorted output has a certain probability  $\beta$  of exceeding (undershooting) the unknown true value of the  $\gamma^{\text{th}}$  percentile of its distribution. This is valid for both  $y$  and  $y_t$  [Di Maio et al., 2016].

Two different approaches, namely “Bracketing” and “Coverage”, are possible, depending on the assumptions done on the relationship between  $\gamma$ ,  $\beta$  and  $N$ . “Bracketing” approach [Nutt et al., 2004], with Assuming  $m = 1$  and referring to uncorrelated outputs  $y$  and  $y_t$ , defines:

$$\beta = (1 - \gamma^N)^2 \quad (\text{A}_1)$$

And, thus, ensures  $\gamma_1$  to be the probability that  $y$  lies below  $y_{\gamma_1}$  in any of the  $N$  runs, whatever the value of  $y_t$ , and  $\gamma_2$  to be the corresponding probability for  $y_t$  [Di Maio et al., 2016].

“Coverage” approach, with uncorrelated output  $y$  and  $y_t$ , defines [Nutt et al., 2004]:

$$\beta = 1 - \gamma^N + N\gamma^N \ln(\gamma) \quad (\text{A}_2)$$

Usually, Eqs. (A<sub>1</sub>) and (A<sub>2</sub>) are used to get the optimal number  $N$  of simulations needed to estimate the  $\gamma=0.95$  percentile with a confidence  $\beta = 0.95$ , as required by the regulation guidance: with this settings two approaches give  $N= 72$  for “Bracketing” and  $N = 89$  for “Coverage”. As we can notice, “Coverage” approach requires a larger number of runs as compared to the “Bracketing” approach. This is because in the “Coverage” approach (contrarily to the “Bracketing” approach) one output (e.g.,  $y$ ) is sorted jointly with the other output (e.g.,  $y_t$ ) and both percentiles  $y_{\gamma_1}$  and  $y_{t_{\gamma_2}}$  are required to simultaneously lie within the estimated percentiles  $\hat{y}_{\gamma_1}$  and  $\hat{y}_{t_{\gamma_2}}$  to guarantee the confidence  $\beta_1$  and  $\beta_2$  [Di Maio et al., 2016].