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THE USE OF COMBINED MULTICRITERIA METHOD FOR THE VALUATION OF REAL ESTATE

Summary

A considered problem is a valuation of real estate. It is important to specify their exact market value, which is the result of several factors. Valuation of property is made on the basis of information and transactions on the local market. Moreover, the valuation always is based on the data of the similar properties. A comprehensive set of data is needed for these reasons. It is quite confusing because the number of transactions on the local market often is not sufficient. The purpose of this paper is to present a method for multicriteria valuation of real estate. This procedure is based on the Analytic Hierarchy Process (AHP) and the Goal Programming (GP). It was designed especially for valuation in situation in which information are limited. The proposed method was used for the valuation of the real estate located on Warsaw.

Key words: real estate, value of real estate, AHP, GP, multicriteria agricultural valuation

1. Introduction

The considered problem is valuation of real estate as a subject of ownership. The study is limited only to the valuation of the market value of the property. In the paper are used definitions of a valuation and the market value of the real estate which are regulated by Real Estate Management Act [*Act of 21.08.1997*] and Council of Ministers On the Valuation of Property and Preparing the Appraisal [*Act of 21.08.2004*]. These definitions are based on recommendations of The International Valuation Standards, European Valuation Standards and EU directives [Mączyńska 2009; Trojanek, 2010, p. 66-75]. The valuation is defined as the process of estimating e.g. market value of property. The market value is defined as the price of a property that is available for purchase. This price is most likely to be concluded by buyers and sellers of the property [Mączyńska 2009; Trojanek, 2010, p. 66-75]. The correct valuation of real estate requires the fulfillment of certain conditions. Firstly, the correct valuation is based on comprehensive set of information and transactions from local real estate market. It means, that it is based on information of properties which were traded on the local market in the last two years. Therefore, all characteristics of real estate have a local nature. Secondly, the correct valuation is based on information of properties which are similar

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to the valued one. The similar property means, that it is comparable due to the type, location, legal status, purpose, method of use and other characteristics that affect its value. Also, in the valuation process is required the determining weights for the characteristics of properties which are compared [Mączyńska 2009; Trojanek, 2010, p. 66-75].

Unfortunately, on the local market often is not sufficient number of transactions that can be used for valuation. Moreover, values of characteristics of property mostly are intangible. Solution of this problem can be a method of valuation of real estate which does not need an extensive set of information to obtain reliable results.

The purpose of this paper is to present a combined multicriteria valuation method (CMVM). This procedure is based on a combination of two multicriteria decision-making methods i.e. the Analytic Hierarchy Process (AHP) and Goal Programming (GP). The mixed AHP and GP procedure for multicriteria real estate valuation has been designed especially for valuation in situation in which information are limited [*Mixed valuation methods...*, 2011, p. 221-238; Koziol-Kaczorek, 2012, p. 236-239; Koziol-Kaczorek, 2013, p. 87-93]. Originators of CMVM are J. Aznar, F. Guijarro and J.M. Moreno-Jimenez. They created this procedure during a research project of the Spanish Ministry of Education and Science. The application of the proposed methodology was illustrated by the example of the valuation of a peach plantation in the La Riberta district in Valencia in Spain.

The main aim of the CMVM is to extract the knowledge from specific characteristics of real estate during the valuation process. This technique can be used with both intangible and scarce information. The Analytic Hierarchy Process allows for the incorporate tangible and intangible aspects by means of using paired comparisons in the valuation procedure. The Goal Programming (GP) allows for the incorporate both the scarce information available (objective) and the individual appraiser's attitude with regards to the valuation process (subjective) [*Mixed valuation methods...*, 2011, p. 221-238; Koziol-Kaczorek, 2012, p. 236-239; Koziol-Kaczorek, 2013, p. 87-93]. The proposed method will be used for the valuation of the real estate located in Warsaw.

2. The Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) was developed by T. L. Saaty. It is discrete multicriteria method which combines a decisions theory and mathematics. AHP is used to making optimal choices for the multicriteria decisions problems by reducing them to a series of pairwise comparisons. These pairwise comparisons are carried out by experts and allow to designate a numerical measure of a validity of analysed variants. Other words, the optimal choice is made on the basis of weights of criteria and features determined by using AHP. These criteria and characteristics can be in the form of both metric or nonmetric (ordinal). AHP algorithm consists of few steps: definition of a problem, construction of a hierarchical model, pairwise comparison of the validity of analysed variants on each level of a hierarchy, construction of a vector of priorities

for analysed decisions elements, verification of a consistency of comparisons at each level of the hierarchy, analysis of results [Saaty 1980, Saaty 2001, Wysocki 2010]².

Stage I. This stage contains general definition of the problem being solved. It contains a determination of an overarching objective, a determination of main and partial criteria and also a determination of analysed variants of these criteria [Saaty, 1980; Saaty, 2001; Wysocki, 2010].

In the case of property valuation, the overarching objective is prioritization of those characteristics of property that affect its market value. It consists in assigning weights for each of characteristic. The main criterion are real estate, which have been the subject of a transaction on an analysed local market, along with the prices for 1 m². Partial criteria are characteristics of real estate and their values. The set of characteristics depends primarily on the type of property (undeveloped or developed property, premises realty etc.) and on the specificity of the local market. And so, the set of the characteristics of e.g. premises realty may consist position and location, the neighbourhood and the environment, surface, place, location of the premises in the building, the condition of the building, the standard finish of premises, etc. [Dydenko 2006]. Variants of partial criteria are values of the selected characteristics. Those values are determinants of both partial and the main criteria.

Stage II. This stage contains construction of a multicriteria hierarchical tree. The main criterion is always on the top of this structure. There are also below partial criteria. The rule indicates that in the tree structure cannot be less than two partial criteria but not more than seven. This rule is based on the psychological fact that human brain can compare, with no mistake, no more than 7 ± 2 objects [Saaty, 1980; Saaty, 2001; Wysocki, 2010].

In the case of property valuation, the form of the hierarchical model is depend on the type of real estate (undeveloped or developed property, premises realty etc.), as well as the peculiarities of the local market. The main criteria may include the following: the property with the lowest price (A), the property with the price somewhat below average (B), the property with the price slightly above average (C), the property with the highest price (D). Partial criteria are discussed above.

Stage III. The third stage involves the evaluation of all components of the hierarchical tree by comparing them in pairs on a "peer". Those comparisons are made on the basis of expert knowledge. Compared pair of elements of some level of hierarchical model is evaluated in terms of relative importance in relation to the criterion level located above (their weights are determined). For this purpose, a matrix of comparisons $\mathbf{A} = [a_{ij}]_{n \times n}$ is created, where

- a_{ij} are values of Saaty's fundamental scale;
- if $a_{ij} > 0$, then $a_{ij} = \frac{1}{a_{ji}}$;

² See also: [Zmienne decydujące..., 2011; Koziol-Kaczorek, 2012; Koziol-Kaczorek, 2013].

- if $i = j$, then $a_{ij} = 1$ $a_{ji} = 1$.

The Saaty's fundamental scale contains degrees of preference for the comparison of pairs of elements of decision-making. The scale of values and their descriptions are shown in table 1.

TABLE 1.

Saaty's fundamental scale

Numerical scale	Verbal scale
1	Same importance
3	One item moderately more important than another
5	One item significantly more important than another
7	One item much more important than another
9	One item very much more important than another
2, 4, 6, 8	Intermediate situations

Source: own study based on literature [Saaty 1980, Saaty 2001]

In the case of real estate, this stage involve construction of five matrices of comparisons:

- the matrix of comparisons of real estate due to prices,
- the matrix of comparisons of characteristics of property with the lowest price,
- the matrix of comparisons of characteristics of property with the price somewhat below average,
- the matrix of comparisons of characteristics of property with the price slightly above average,
- the matrix of comparisons of characteristics of property with the highest price [Zmienne decydujące..., 2011; Koziol-Kaczorek, 2012].

Stage IV. The fourth stage is the construction of the vector

$$\mathbf{W} = \mathbf{C}\bar{\mathbf{w}},$$

which is a vector of the priorities of the analysed objects. This vector is also called a vector of acceptable solutions. It expressed the weight of analysed criteria and alternatives due to the overarching objective. Columns in the matrix \mathbf{C} are vectors of priorities (scale) designated for the corresponding partial criteria. Vector $\bar{\mathbf{w}}$ is a vector of the scale fixed for the main criterion. The priorities vector is obtained by calculating the weights

$$w_{ij} = \frac{a_{ij}}{\sum_{i=1}^n a_{ij}},$$

and then by calculating elements of this vector

$$\bar{w}_i = \frac{\sum_{j=1}^n w_{ij}}{n}, i, j = 1, 2, \dots, n.$$

Elements of hierarchical model are arranged according to the size of elements of the priorities vector \mathbf{W} , in order of their importance. A higher value means more the significance of the item [Saaty 1980, Wysocki 2010].

In the case of real estate, the vector of priorities includes weights of the individual characteristics of the property. This stage involve construction of five vectors of priorities:

- the vector of priorities for four chosen real estate;
- the vector of priorities for property with the lowest price;
- the vector of priorities for property with the price somewhat below average;
- the vector of priorities for property with the price slightly above average;
- the vector of priorities for property with the highest price [Zmienne decydujące..., 2011; Koziol-Kaczorek 2012].

Stage V. The fifth step involves checking compliance of comparisons for the whole hierarchical structure. To this end, is calculated the indicator of inconsistencies

$$CI = \frac{(\lambda_{max} - n)}{(n - 1)},$$

where

$$\lambda_{max} = \frac{1}{n} \left(\sum_{i=1}^n \frac{(\mathbf{A}\bar{\mathbf{w}})_i}{\bar{w}_i} \right), \lambda_{max} \geq n$$

is a maximal eigenvalue of the matrix \mathbf{A} . The value λ_{max} inform about the consistency of the results. The more closely related to the size of the n including the results of comparisons of pairs show greater compliance. When comparing pairs do not contain any incompatibilities $\lambda_{max} = n$. Similarly, if the value of the indicator $CI = 0$ and $\lambda_{max} = n$ it means that evaluations are completely consistent [Saaty, 1980; Saaty, 2001; Trzaskalik, 2006; Wysocki, 2010].

A coefficient of inconsistencies can be also computed by the formula

$$CR = \frac{CI}{RI},$$

where RI is a random index, which is dependent on the matrix order. If the value of the coefficient of inconsistencies $CR \leq 0.1$, then it can be concluded that comparisons are consistent. If the value of the coefficient of inconsistencies $CR = 0$, then it can be concluded that comparisons are completely consistent. When $CR > 0.1$ it is necessary to repeat (all or some) comparisons in pairs to remove the discrepancy [Saaty, 1980; Saaty, 2001; Trzaskalik, 2006; Wysocki, 2010].

3. The Goal Programming (GP)

The Goal Programming (GP) is an extension of linear programming. However, it is not focused on optimisation like linear programming. The Goal Programming is focused on satisfaction and searching of a number objectives at the same time. It is

mathematical nonlinear programming but it can be easily linearized. Objectives are formulated by specifying the desired values of the analyzed criteria. The main task of goal programming is to find the best solution in which all the conditions are satisfied. A lot of variants of GP exist in literature. In this paper two of them are applied: Weighted Goal Programming and MinMax. The form of the basic model for WGP is:

$$\begin{aligned} \min_x z_{[i1]} &= \sum_{j=1}^r \lambda_j (d_j^- + d_j^+) \\ z_j(x) + d_j^- - d_j^+ &= \hat{z}_j, j = 1, \dots, r \\ g_i(x) &\leq 0, i = 1, \dots, m \\ x &\geq 0, d_j^- \geq 0, d_j^+ \geq 0, \end{aligned}$$

where d_j^- is the negative deviation and d_j^+ is positive deviations with respect to the j -th goal (\hat{z}_j). Moreover, λ_j denotes a normalisation factor. Note that, since the values are already normalised in the distribution mode, there is no need for normalisation factor, so $\lambda_j = 1$ is taken [Mixed valuation methods..., 2011 pp. 221-238]. In the model occurs m strong constraints. They determine the feasible region i.e. $g_i(x), i = 1, \dots, m$. There are also r weak constraints for the goals considered ($\hat{z}_j, j = 1, \dots, r$).

The goal vector $\hat{Z} = (\hat{z}_1, \dots, \hat{z}_r)$ contains prices of each real estate comparable to the subject one during the valuation of real estate. The valuation function is on the form:

$$z(x^j) = a_0 + \sum_{l=1}^n a_l x_l^j, j = 1, \dots, r,$$

where x_l^j is the relative value in the l -th criterion of the j -th real estate, and $a_l > 0$ are estimated parameters of the model [Mixed valuation methods..., 2011, p. 221-238]. The basic model for MinMax (symbols are the same as in WGP) is:

$$\begin{aligned} \min_x z_{[\infty 1]} &= d_{max} = \max_j (d_j^- + d_j^+) \\ z_j(x) + d_j^- - d_j^+ &= \hat{z}_j, j = 1, \dots, r \\ d_j^- + d_j^+ &\leq d_{max}, j = 1, \dots, r \\ g_i(x) &\leq 0, i = 1, \dots, m \\ x &\geq 0, d_j^- \geq 0, d_j^+ \geq 0. \end{aligned}$$

Note that, this model uses L_∞ -metric to obtain the best solution [Mixed valuation methods..., 2011, p. 221-238; Koziol-Kaczorek, 2013].

4. The combined multicriteria valuation method (CMVM)

As it was written before, the combined multicriteria method (CMVM) is a combination of the Analytic Hierarchy Process (AHP) and Goal Programming (GP). The algorithm of CMVM contains three stages. The first stage is the use of AHP to quantify the subjective information about the elements being compared. The second stage is the use of GP for estimating the parameters of regression model $z(x^j)$ to obtain the

market value of real estate. The GP is used in two different ways, according to a Manhattan norm L_1 and a Tchebychef norm L_∞ . The Manhattan norm L_1 introduces scarce information available into the model. The Tchebychef norm L_∞ captures the subjective attitude with respect to the valuation process (i.e. the greater distance between the compared elements contribute to the greater subjectivity of the assessment). Furthermore, this norm allows consideration of the proximity of the subject asset to one of the comparable sets of assets that does not follow common or majority behaviour. These norms are used to determine the objective function to be optimised [*Mixed valuation methods...*, 2011, p. 221-238; Koziol-Kaczorek, 2013].

The third stage is the final valuation of the real estate. The final market value of real estate is a result of a convex combination of values obtained in stage two. The formula of the market value is:

$$CMM(X) = (1 - \alpha)V_1(X) + \alpha V_\infty(X), \alpha \in [0,1]$$

where $V_1(X)$ and $V_\infty(X)$ are the L_1 and L_∞ values. The level of α depends on the expert knowledge. The level $\alpha = 1$ means that the valued property is very similar to comparable properties. On the other hand, level $\alpha = 0$ means that the valued properties is significant different from comparable properties [*Mixed valuation methods...*, 2011, p. 221-238; Koziol-Kaczorek, 2013].

5. The valuation of real estate

The above-described methodology was applied to the valuation of premises realty located on Mokotów district in Warsaw. Therefore, the set of data also comes from the real estate market in the Mokotów district in Warsaw. It was collected by the property appraisers of Institute of Property Consultancy.

The first step of the CMVM is the use of AHP to determine the weights of characteristics of real estate. The overarching objective was the assigning weights for characteristics of valued premises realty. Main criteria were properties with the lowest price (A), the price somewhat below average (B), the price slightly above average (C), the highest price (D).

Partial criteria were characteristics of premises realty i.e.: the position and location, the neighbourhood and the environment, the surface, the location of the premises in the building, the condition of the building, the standard of finish of premises. Values of the selected characteristics of premises realty i.e.: very good, good, sufficient, bad were variants of partial criteria. The hierarchical model were then on the form presented in above table 2.

TABLE 2.

Hierarchical model

Main criterion	Partial criteria	Variants of partial criteria
The property A	the position and location	good
	the neighbourhood and the	bad
	the surface	sufficient
	the location of the premises in the	bad
	the condition of the building	sufficient
	the standard of finish of premises	bad
The property B	the position and location	good
	the neighbourhood and the	good
	the surface	very good
	the location of the premises in the	good
	the condition of the building	good
	the standard of finish of premises	sufficient
The property C	the position and location	good
	the neighbourhood and the	good
	the surface	very good
	the location of the premises in the	good
	the condition of the building	good
	the standard of finish of premises	sufficient
The property D	the position and location	very good
	the neighbourhood and the	very good
	the surface	good
	the location of the premises in the	very good
	the condition of the building	very good
	the standard of finish of premises	very good

Source: own study based on literature [Zmienne decydujące o wartości..., 2011; Koziol-Kaczorek, 2012].

The property A (the lowest price) is premises realty located on 11th floor in eleven storey building. The total area of the apartment is about 65 m². Requires overhaul of apartment. The building also needs global renovation due to its technical condition. The price of 1 m² of the property was PLN 4 443.

The property B (the price somewhat below average) is premises realty located on 5th floor in ten storey building. The total area of the apartment is about 37 m². Requires renovation of apartment. The building is in good technical condition. The price of 1 m² of the property was PLN 8 706.

The property C (the price slightly above average) is premises realty located on 5th floor in ten storey building. The total area of the apartment is about 31 m². Requires

renovation of apartment. The building is in good technical condition. The price of 1 m² of the property was PLN 8 710.

The property D (the highest price) is premises realty located on 3rd floor in five storey building. The total area of the apartment is about 45 m². The apartment was completely renovated, and the standard of it is very high. The building is new and in very good technical condition. The price of 1 m² of the property was PLN 13 283 .

Presented in table 2 hierarchical tree was established by analysis of the local market and the appraiser's knowledge. Then it was constructed five matrices of comparisons:

- the matrix of comparisons of real estate due to prices;
 - the matrix of comparisons of characteristics of property with the lowest price;
 - the matrix of comparisons of characteristics of property with the price somewhat below average;
 - the matrix of comparisons of characteristics of property with the price slightly above average;
 - the matrix of comparisons of characteristics of property with the highest price;
- and also five vectors of priorities:
- the vector of priorities for four chosen real estate;
 - the vector of priorities for property with the lowest price;
 - the vector of priorities for property with the price somewhat below average;
 - the vector of priorities for property with the price slightly above average;
 - the vector of priorities for property with the highest price [Zmienne decydujące..., 2011; Koziol-Kaczorek, 2012].

The another step involved checking compliance of comparisons for the whole hierarchical structure. For this reason the coefficients of inconsistencies were computed:

- for comparisons of real estate due to prices ($CR = 0.08$);
- for comparisons of characteristics of property with the lowest price ($CR_A = 0.01$);
- for comparisons of characteristics of property with the price somewhat below average ($CR_B = 0.01$);
- for comparisons of characteristics of property with the price slightly above average ($CR_C = 0.01$);
- for comparisons of characteristics of property with the highest price ($CR_D = 0.01$).

The obtained values of the coefficients of inconsistencies indicate that every comparison was consistent and compatible. A detailed description of the analysis and detailed results are on Koziol-Kaczorek [2012]. Weights obtained as a result of the analysis are presented in the Table 3.

Letter V denotes the valued property. It is premises realty located on 2nd floor in five storey building. The total area of the apartment is about 39 m². The apartment was completely renovated, and the standard of it is very high. The building is in very good technical condition.

TABLE 3.

Weights of characteristics of premises realty

Characteristic	Properties				
	A	B	C	D	V
the position and location	0.3324	0.1425	0.2520	0.3615	0.3037
the neighbourhood and the environment	0.1525	0.3675	0.2520	0.0740	0.1588
the surface	0.0609	0.0563	0.0471	0.1316	0.1007
the location of the premises in the building	0.3324	0.0563	0.2520	0.0740	0.1146
the condition of the building	0.0609	0.1425	0.0471	0.1316	0.1154
the standard of finish of premises	0.0609	0.2349	0.1498	0.2272	0.2068

Source: own study based on literature [Zmienne decyzje..., 2011; Koziol-Kaczorek, 2012].

The second stage of the CMVM contains the GP application. The GP was used in two different ways (the Manhattan norm L_1 and the Tchebychef norm L_∞) for estimating the regression parameter. In this paper the mentioned regression model was on the form

$$z(x^j) = a_0 + a_1x_1^j + a_2x_2^j + a_3x_3^j + a_4x_4^j + a_5x_5^j + a_6x_6^j, \quad j = 1,2,3,4$$

where x^j denotes weight of one of six characteristics of j -th premises realty and $z(x^j)$ denotes their prices. Obtained parameters were used to calculate the values of valued property by:

- WGP: $V_1(X) = 10\,564$ PLN/m²;
- MinMax: $V_\infty(X) = 11\,861$ PLN/m².

The obtained market value of the valued premises realty (V) is within the range (10 564; 11 862) PLN/m². This market value is considerably in excess of the average price of 1 m² of premises realty on this local real estate market. This is due to the fact, that the valued premises realty (V) was more similar to the property with the highest price than to those with the average price.

6. Conclusions

The use of usual valuation method often is not possible because of the common problem with insufficient number of transactions on local real estate market. Described in the paper method of valuation (CMVM) can be solution of this problem. It is the combination of two multicriteria methodologies i.e. Analytic Hierarchy Process (AHP) and Goal Programming (GP). The AHP enables to quantify qualitative variables and include the weight of the importance of preferences. The GP captures the information from the limited information and the attitude of the appraiser in the valuation process. The calculated value range enables the expert to define the final market value.

This method was primarily created for valuation of agricultural real estate. In this study, however, showed that it is appropriate solution also for situations with a lack of information and a limited number of transactions in other real estate markets. In present paper it was adapted to premises realty market. The obtained market value of the valued premises realty is within the range (10 564; 11 862) PLN/m². It is higher price from average price but the valued property is much more similar to the property with the highest price not to the property with average price.

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