FORMALIZED MATHEMATICS
Vol. 17, No. 4, Pages 245–248, 2009
DOI: 10.2478/v10037-009-0031-9

Basic Properties of Periodic Functions

Bo Li
Qingdao University of Science and Technology
China

Yanhong Men
Qingdao University of Science and Technology
China

Dailu Li
Qingdao University of Science and Technology
China

Xiquan Liang
Qingdao University of Science and Technology
China

Summary. In this article we present definitions, basic properties and some examples of periodic functions according to [5].

MML identifier: `FUNCT_9`, version: `7.11.04 4.130.1076`

The papers [2], [6], [3], [10], [11], [9], [8], [1], [4], and [7] provide the terminology and notation for this paper.

1. Basic Properties of a Period of a Function

We use the following convention: x, t, t_1, t_2, r, a, b are real numbers and F, G are partial functions from $\mathbb{R}$ to $\mathbb{R}$.

Let F be a partial function from $\mathbb{R}$ to $\mathbb{R}$ and let t be a real number. We say that t is a period of F if and only if:

(Def. 1) $t \neq 0$ and for every x holds $x \in \operatorname{dom} F$ iff $x+t \in \operatorname{dom} F$ and if $x \in \operatorname{dom} F$, then $F(x) = F(x+t)$.

Let F be a partial function from $\mathbb{R}$ to $\mathbb{R}$. We say that F is periodic if and only if:

(Def. 2) There exists t which is a period of F.

We now state a number of propositions:

ISSN 1426–2630(p), 1898-9934(e)

(1) t is a period of F iff $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t$, $x-t \in \operatorname{dom} F$ and $F(x) = F(x+t)$.

(2) If t is a period of F and a period of G, then t is a period of $F+G$.

(3) If t is a period of F and a period of G, then t is a period of $F-G$.

(4) If t is a period of F and a period of G, then t is a period of $F\,G$.

(5) If t is a period of F and a period of G, then t is a period of F/G.

(6) If t is a period of F, then t is a period of $-F$.

(7) If t is a period of F, then t is a period of $r\,F$.

(8) If t is a period of F, then t is a period of $r+F$.

(9) If t is a period of F, then t is a period of $F-r$.

(10) If t is a period of F, then t is a period of $|F|$.

(11) If t is a period of F, then t is a period of F^{-1}.

(12) If t is a period of F, then t is a period of $F^{\mathbf{2}}$.

(13) If t is a period of F, then for every x such that $x \in \operatorname{dom} F$ holds $F(x) = F(x-t)$.

(14) If t is a period of F, then $-t$ is a period of F.

(15) If t_1 is a period of F and t_2 is a period of F and $t_1+t_2 \neq 0$, then t_1+t_2 is a period of F.

(16) If t_1 is a period of F and t_2 is a period of F and $t_1-t_2 \neq 0$, then t_1-t_2 is a period of F.

(17) Suppose $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t$, $x-t \in \operatorname{dom} F$ and $F(x+t) = F(x-t)$. Then $2 \cdot t$ is a period of F and F is periodic.

(18) Suppose $t_1+t_2 \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t_1$, $x-t_1$, $x+t_2$, $x-t_2 \in \operatorname{dom} F$ and $F(x+t_1) = F(x-t_2)$. Then t_1+t_2 is a period of F and F is periodic.

(19) Suppose $t_1-t_2 \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t_1$, $x-t_1$, $x+t_2$, $x-t_2 \in \operatorname{dom} F$ and $F(x+t_1) = F(x+t_2)$. Then t_1-t_2 is a period of F and F is periodic.

(20) Suppose $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t$, $x-t \in \operatorname{dom} F$ and $F(x+t) = F(x)^{-1}$. Then $2 \cdot t$ is a period of F and F is periodic.

Let us observe that there exists a partial function from $\mathbb{R}$ to $\mathbb{R}$ which is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$. One can check that $-F$ is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$ and let r be a real number. One can check the following observations:

* $r\,F$ is periodic,
* $r+F$ is periodic, and

* $F - r$ is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$. One can check the following observations:

* $|F|$ is periodic,
* F^{-1} is periodic, and
* $F^{\mathbf{2}}$ is periodic.

2. Some Examples

Let us note that the function sin is periodic and the function cos is periodic.

We now state two propositions:

(21) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function sin.

(22) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function cos.

Let us observe that the function cosec is periodic and the function sec is periodic.

We now state two propositions:

(23) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function sec.

(24) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function cosec.

Let us mention that the function tan is periodic and the function cot is periodic.

Next we state a number of propositions:

(25) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of the function tan.

(26) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of the function cot.

(27) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of |the function sin|.

(28) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of |the function cos|.

(29) For every element k of $\mathbb{N}$ holds $\frac{\pi}{2} \cdot (k+1)$ is a period of |the function sin| + |the function cos|.

(30) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function sin)$^{\mathbf{2}}$.

(31) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function cos)$^{\mathbf{2}}$.

(32) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function sin) (the function cos).

(33) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function cos) (the function sin).

(34) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of $b + a$ (the function sin).

(35) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of a (the function sin)$-b$.

(36) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of $b + a$ (the function cos).

(37) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of a (the function cos)$-b$.

(38) If $\operatorname{dom} F = \mathbb{R}$ and for every real number x holds $F(x) = a$, then for every element k of $\mathbb{N}$ holds $k+1$ is a period of F.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.
[2] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.
[3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[4] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[5] Chuanzhang Chen. *Mathematical Analysis.* Higher Education Press, Beijing, 1978.
[6] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[7] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[8] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[9] Peng Wang and Bo Li. Several differentiation formulas of special functions. Part V. *Formalized Mathematics*, 15(**3**):73–79, 2007, doi:10.2478/v10037-007-0009-4.
[10] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[11] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

Received October 10, 2009