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Formalization of the Data Encryption Standard[1]

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Summary. In this article we formalize DES (the Data Encryption Standard), that was the most widely used symmetric cryptosystem in the world. DES is a block cipher which was selected by the National Bureau of Standards as an official Federal Information Processing Standard for the United States in 1976 [15].

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The papers [14], [5], [12], [1], [16], [4], [6], [18], [11], [7], [8], [17], [20], [2], [3], [9], [21], [22], [13], [19], and [10] provide the terminology and notation for this paper.

1. Preliminaries

Let n be a natural number and let f be an n-element finite sequence. Note that $\mathrm{Rev}(f)$ is n-element.

Let D be a non empty set, let n be a natural number, and let f be an element of D^n. Then $\mathrm{Rev}(f)$ is an element of D^n.

Let n be a natural number and let f be a finite sequence. We introduce $\text{Op-Left}(f, n)$ as a synonym of $f \restriction n$. We introduce $\text{Op-Right}(f, n)$ as a synonym of $f_{\lfloor n}$.

Let D be a non empty set, let n be a natural number, and let f be a finite sequence of elements of D. Then $\text{Op-Left}(f, n)$ is a finite sequence of elements of D. Then $\text{Op-Right}(f, n)$ is a finite sequence of elements of D.

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Let D be a non empty set, let n be a natural number, and let s be an element of $D^{2 \cdot n}$. We introduce SP-Left s as a synonym of Op-Left(s, n). We introduce SP-Right s as a synonym of Op-Right(s, n).

Let D be a non empty set, let n be a natural number, and let s be an element of $D^{2 \cdot n}$. Then SP-Left s is an element of D^n.

One can prove the following propositions:

(1) For all non empty elements m, n of $\mathbb{N}$ and for every element s of D^n such that $m \leq n$ holds Op-Left(s, m) is an element of D^m.

(2) Let m, n, l be non empty elements of $\mathbb{N}$ and s be an element of D^n. If $m \leq n$ and $l = n - m$, then Op-Right(s, m) is an element of D^l.

Let D be a non empty set, let n be a non empty element of $\mathbb{N}$, and let s be an element of $D^{2 \cdot n}$. Then SP-Right s is an element of D^n.

Next we state the proposition

(3) For every non empty element n of $\mathbb{N}$ and for every element s of $D^{2 \cdot n}$ holds (SP-Left s) $\frown$ SP-Right $s = s$.

Let s be a finite sequence. The functor Op-LeftShift s yielding a finite sequence is defined by:

(Def. 1) Op-LeftShift $s = (s_{\lfloor 1}) \frown \langle s(1) \rangle$.

Next we state three propositions:

(4) For every finite sequence s such that $1 \leq \operatorname{len} s$ holds len Op-LeftShift $s = \operatorname{len} s$.

(5) If $1 \leq \operatorname{len} s$, then Op-LeftShift s is a finite sequence of elements of D and len Op-LeftShift $s = \operatorname{len} s$.

(6) For every non empty element n of $\mathbb{N}$ and for every element s of D^n holds Op-LeftShift s is an element of D^n.

Let s be a finite sequence. The functor Op-RightShift s yields a finite sequence and is defined by:

(Def. 2) Op-RightShift $s = (\langle s(\operatorname{len} s) \rangle \frown s) \restriction \operatorname{len} s$.

One can prove the following three propositions:

(7) For every finite sequence s holds len Op-RightShift $s = \operatorname{len} s$.

(8) If $1 \leq \operatorname{len} s$, then Op-RightShift s is a finite sequence of elements of D and len Op-RightShift $s = \operatorname{len} s$.

(9) For every non empty element n of $\mathbb{N}$ and for every element s of D^n holds Op-RightShift s is an element of D^n.

Let D be a non empty set, let s be a finite sequence of elements of D, and let n be an integer. Let us assume that $1 \leq \operatorname{len} s$. The functor Op-Shift$(s, n)$ yields a finite sequence of elements of D and is defined by:

(Def. 3) len Op-Shift$(s, n) = \operatorname{len} s$ and for every natural number i such that $i \in$ Seg len s holds $(\text{Op-Shift}(s, n))(i) = s((((i - 1) + n) \bmod \operatorname{len} s) + 1)$.

The following propositions are true:

(10) For all integers n, m such that $1 \leq \operatorname{len} s$ holds $\text{Op-Shift}(\text{Op-Shift}(s, n), m) = \text{Op-Shift}(s, n+m)$.

(11) If $1 \leq \operatorname{len} s$, then $\text{Op-Shift}(s, 0) = s$.

(12) If $1 \leq \operatorname{len} s$, then $\text{Op-Shift}(s, \operatorname{len} s) = s$.

(13) If $1 \leq \operatorname{len} s$, then $\text{Op-Shift}(s, -\operatorname{len} s) = s$.

(14) Let n be a non empty element of $\mathbb{N}$, m be an integer, and s be an element of D^n. Then $\text{Op-Shift}(s, m)$ is an element of D^n.

(15) If $1 \leq \operatorname{len} s$, then $\text{Op-Shift}(s, -1) = \text{Op-RightShift}\, s$.

(16) If $1 \leq \operatorname{len} s$, then $\text{Op-Shift}(s, 1) = \text{Op-LeftShift}\, s$.

Let x, y be elements of $\mathit{Boolean}^{28}$. Then $x \frown y$ is an element of $\mathit{Boolean}^{56}$.

Let n be a non empty element of $\mathbb{N}$, let s be an element of $\mathit{Boolean}^n$, and let i be a natural number. Then $s(i)$ is an element of *Boolean*.

Let n be a non empty element of $\mathbb{N}$, let s be an element of $\mathbb{N}^n$, and let i be a natural number. Then $s(i)$ is an element of $\mathbb{N}$.

Let n be a natural number. Observe that every element of $\mathit{Boolean}^n$ is boolean-valued.

Let n be an element of $\mathbb{N}$ and let s, t be elements of $\mathit{Boolean}^n$. We introduce $\text{Op-XOR}(s, t)$ as a synonym of $s \oplus t$.

Let n be a non empty element of $\mathbb{N}$ and let s, t be elements of $\mathit{Boolean}^n$. Then $\text{Op-XOR}(s, t)$ is an element of $\mathit{Boolean}^n$ and it can be characterized by the condition:

(Def. 4) For every natural number i such that $i \in \operatorname{Seg} n$ holds $(\text{Op-XOR}(s, t))(i) = s(i) \oplus t(i)$.

Let us notice that the functor $\text{Op-XOR}(s, t)$ is commutative.

Let n, k be non empty elements of $\mathbb{N}$, let R_1 be an element of $(\mathit{Boolean}^n)^k$, and let i be an element of $\operatorname{Seg} k$. Then $R_1(i)$ is an element of $\mathit{Boolean}^n$.

We now state the proposition

(17) For every non empty element n of $\mathbb{N}$ and for all elements s, t of $\mathit{Boolean}^n$ holds $\text{Op-XOR}(\text{Op-XOR}(s, t), t) = s$.

Let m be a non empty element of $\mathbb{N}$, let D be a non empty set, let L be a sequence of D^m, and let i be a natural number. Then $L(i)$ is an element of D^m.

Let f be a function from 64 into 16 and let i be a set. Then $f(i)$ is an element of 16.

Next we state the proposition

(18) For all natural numbers n, m such that $n + m \leq \operatorname{len} s$ holds $(s \restriction n) \frown (s_{\lfloor n} \restriction m) = s \restriction (n + m)$.

The scheme *QuadChoiceRec* deals with non empty sets $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, an element $\mathcal{E}$ of $\mathcal{A}$, an element $\mathcal{F}$ of $\mathcal{B}$, an element $\mathcal{G}$ of $\mathcal{C}$, an element $\mathcal{H}$ of $\mathcal{D}$, and a 9-ary predicate $\mathcal{P}$, and states that:

There exists a function f from $\mathbb{N}$ into $\mathcal{A}$ and there exists a function g from $\mathbb{N}$ into $\mathcal{B}$ and there exists a function h from $\mathbb{N}$ into $\mathcal{C}$ and there exists a function i from $\mathbb{N}$ into $\mathcal{D}$ such that $f(0) = \mathcal{E}$ and $g(0) = \mathcal{F}$ and $h(0) = \mathcal{G}$ and $i(0) = \mathcal{H}$ and for every element n of $\mathbb{N}$ holds $\mathcal{P}[n, f(n), g(n), h(n), i(n), f(n+1), g(n+1), h(n+1), i(n+1)]$

provided the following condition is satisfied:

- Let n be an element of $\mathbb{N}$, x be an element of $\mathcal{A}$, y be an element of $\mathcal{B}$, z be an element of $\mathcal{C}$, and w be an element of $\mathcal{D}$. Then there exists an element x_1 of $\mathcal{A}$ and there exists an element y_1 of $\mathcal{B}$ and there exists an element z_1 of $\mathcal{C}$ and there exists an element w_1 of $\mathcal{D}$ such that $\mathcal{P}[n, x, y, z, w, x_1, y_1, z_1, w_1]$.

Next we state a number of propositions:

(19) Let x be a set. Suppose $x \in \operatorname{Seg} 16$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$.

(20) Let x be a set. Suppose $x \in \operatorname{Seg} 32$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$.

(21) Let x be a set. Suppose $x \in \operatorname{Seg} 48$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$.

(22) Let x be a set. Suppose $x \in \operatorname{Seg} 56$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$ or $x = 49$ or $x = 50$ or $x = 51$ or $x = 52$ or $x = 53$ or $x = 54$ or $x = 55$ or $x = 56$.

(23) Let x be a set. Suppose $x \in \operatorname{Seg} 64$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or

$x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$ or $x = 49$ or $x = 50$ or $x = 51$ or $x = 52$ or $x = 53$ or $x = 54$ or $x = 55$ or $x = 56$ or $x = 57$ or $x = 58$ or $x = 59$ or $x = 60$ or $x = 61$ or $x = 62$ or $x = 63$ or $x = 64$.

(24) For every non empty natural number n holds $n = \{0\} \cup (\operatorname{Seg} n \setminus \{n\})$.

(25) For every non empty natural number n and for every set x such that $x \in n$ holds $x = 0$ or $x \in \operatorname{Seg} n$ and $x \neq n$.

(26) Let x be a set. Suppose $x \in 16$. Then $x = 0$ or $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$.

(27) Let x be a set. Suppose $x \in 64$. Then $x = 0$ or $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$ or $x = 49$ or $x = 50$ or $x = 51$ or $x = 52$ or $x = 53$ or $x = 54$ or $x = 55$ or $x = 56$ or $x = 57$ or $x = 58$ or $x = 59$ or $x = 60$ or $x = 61$ or $x = 62$ or $x = 63$.

(28) Let S be a non empty set and x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 be elements of S. Then there exists a finite sequence s of elements of S such that s is 8-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$.

(29) Let S be a non empty set and x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16} be elements of S. Then there exists a finite sequence s of elements of S such that
s is 16-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$.

(30) Let S be a non empty set and x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32} be elements of S. Then there exists a finite sequence s of elements of S such that
s is 32-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$

and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$ and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$ and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$ and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$.

(31) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}$ be elements of S. Then there exists a finite sequence s of elements of S such that

s is 48-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$ and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$ and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$ and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$ and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$ and $s(33) = x_{33}$ and $s(34) = x_{34}$ and $s(35) = x_{35}$ and $s(36) = x_{36}$ and $s(37) = x_{37}$ and $s(38) = x_{38}$ and $s(39) = x_{39}$ and $s(40) = x_{40}$ and $s(41) = x_{41}$ and $s(42) = x_{42}$ and $s(43) = x_{43}$ and $s(44) = x_{44}$ and $s(45) = x_{45}$ and $s(46) = x_{46}$ and $s(47) = x_{47}$ and $s(48) = x_{48}$.

(32) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}$ be elements of S. Then there exists a finite sequence s of elements of S such that

s is 56-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$ and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$ and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$ and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$ and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$ and $s(33) = x_{33}$ and $s(34) = x_{34}$ and $s(35) = x_{35}$ and $s(36) = x_{36}$ and $s(37) = x_{37}$ and $s(38) = x_{38}$ and $s(39) = x_{39}$ and $s(40) = x_{40}$ and $s(41) = x_{41}$ and $s(42) = x_{42}$ and $s(43) = x_{43}$ and $s(44) = x_{44}$ and $s(45) = x_{45}$ and $s(46) = x_{46}$ and $s(47) = x_{47}$ and $s(48) = x_{48}$ and $s(49) = x_{49}$ and $s(50) = x_{50}$ and $s(51) = x_{51}$ and $s(52) = x_{52}$ and $s(53) = x_{53}$ and $s(54) = x_{54}$ and $s(55) = x_{55}$ and $s(56) = x_{56}$.

(33) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}$,

x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}, x_{57}, x_{58}, x_{59}, x_{60}, x_{61}, x_{62}, x_{63}, x_{64} be elements of S. Then there exists a finite sequence s of elements of S such that

s is 64-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$ and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$ and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$ and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$ and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$ and $s(33) = x_{33}$ and $s(34) = x_{34}$ and $s(35) = x_{35}$ and $s(36) = x_{36}$ and $s(37) = x_{37}$ and $s(38) = x_{38}$ and $s(39) = x_{39}$ and $s(40) = x_{40}$ and $s(41) = x_{41}$ and $s(42) = x_{42}$ and $s(43) = x_{43}$ and $s(44) = x_{44}$ and $s(45) = x_{45}$ and $s(46) = x_{46}$ and $s(47) = x_{47}$ and $s(48) = x_{48}$ and $s(49) = x_{49}$ and $s(50) = x_{50}$ and $s(51) = x_{51}$ and $s(52) = x_{52}$ and $s(53) = x_{53}$ and $s(54) = x_{54}$ and $s(55) = x_{55}$ and $s(56) = x_{56}$ and $s(57) = x_{57}$ and $s(58) = x_{58}$ and $s(59) = x_{59}$ and $s(60) = x_{60}$ and $s(61) = x_{61}$ and $s(62) = x_{62}$ and $s(63) = x_{63}$ and $s(64) = x_{64}$.

Let n be a non empty natural number and let i be an element of n. We introduce $\mathrm{ntoSeg}\, i$ as a synonym of $\mathrm{succ}\, i$.

Let n be a non empty natural number and let i be an element of n. Then $\mathrm{ntoSeg}\, i$ is an element of $\mathrm{Seg}\, n$.

Let n be a non empty natural number and let f be a function from n into $\mathrm{Seg}\, n$. We say that f is NtoSeg if and only if:

(Def. 5) For every element i of n holds $f(i) = \mathrm{ntoSeg}\, i$.

Let n be a non empty natural number. One can check that there exists a function from n into $\mathrm{Seg}\, n$ which is NtoSeg.

Let n be a non empty natural number. Observe that every function from n into $\mathrm{Seg}\, n$ is bijective and NtoSeg.

We now state two propositions:

(34) Let n be a non empty natural number, f be an NtoSeg function from n into $\mathrm{Seg}\, n$, and i be a natural number. If $i < n$, then $f(i) = i + 1$ and $i \in \mathrm{dom}\, f$.

(35) Let S be a non empty set and x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}, x_{57}, x_{58}, x_{59}, x_{60}, x_{61}, x_{62}, x_{63}, x_{64} be elements of S. Then there exists a function f

from 64 into S such that
$f(0) = x_1$ and $f(1) = x_2$ and $f(2) = x_3$ and $f(3) = x_4$ and $f(4) = x_5$ and $f(5) = x_6$ and $f(6) = x_7$ and $f(7) = x_8$ and $f(8) = x_9$ and $f(9) = x_{10}$ and $f(10) = x_{11}$ and $f(11) = x_{12}$ and $f(12) = x_{13}$ and $f(13) = x_{14}$ and $f(14) = x_{15}$ and $f(15) = x_{16}$ and $f(16) = x_{17}$ and $f(17) = x_{18}$ and $f(18) = x_{19}$ and $f(19) = x_{20}$ and $f(20) = x_{21}$ and $f(21) = x_{22}$ and $f(22) = x_{23}$ and $f(23) = x_{24}$ and $f(24) = x_{25}$ and $f(25) = x_{26}$ and $f(26) = x_{27}$ and $f(27) = x_{28}$ and $f(28) = x_{29}$ and $f(29) = x_{30}$ and $f(30) = x_{31}$ and $f(31) = x_{32}$ and $f(32) = x_{33}$ and $f(33) = x_{34}$ and $f(34) = x_{35}$ and $f(35) = x_{36}$ and $f(36) = x_{37}$ and $f(37) = x_{38}$ and $f(38) = x_{39}$ and $f(39) = x_{40}$ and $f(40) = x_{41}$ and $f(41) = x_{42}$ and $f(42) = x_{43}$ and $f(43) = x_{44}$ and $f(44) = x_{45}$ and $f(45) = x_{46}$ and $f(46) = x_{47}$ and $f(47) = x_{48}$ and $f(48) = x_{49}$ and $f(49) = x_{50}$ and $f(50) = x_{51}$ and $f(51) = x_{52}$ and $f(52) = x_{53}$ and $f(53) = x_{54}$ and $f(54) = x_{55}$ and $f(55) = x_{56}$ and $f(56) = x_{57}$ and $f(57) = x_{58}$ and $f(58) = x_{59}$ and $f(59) = x_{60}$ and $f(60) = x_{61}$ and $f(61) = x_{62}$ and $f(62) = x_{63}$ and $f(63) = x_{64}$.

2. S-Boxes

The function DES-SBOX1 from 64 into 16 is defined by the conditions (Def. 6).

(Def. 6) (DES-SBOX1)(0) = 14 and (DES-SBOX1)(1) = 4 and (DES-SBOX1)(2) = 13 and (DES-SBOX1)(3) = 1 and (DES-SBOX1)(4) = 2 and (DES-SBOX1)(5) = 15 and (DES-SBOX1)(6) = 11 and (DES-SBOX1)(7) = 8 and (DES-SBOX1)(8) = 3 and (DES-SBOX1)(9) = 10 and (DES-SBOX1)(10) = 6 and (DES-SBOX1)(11) = 12 and (DES-SBOX1)(12) = 5 and (DES-SBOX1)(13) = 9 and (DES-SBOX1)(14) = 0 and (DES-SBOX1)(15) = 7 and (DES-SBOX1)(16) = 0 and (DES-SBOX1)(17) = 15 and (DES-SBOX1)(18) = 7 and (DES-SBOX1)(19) = 4 and (DES-SBOX1)(20) = 14 and (DES-SBOX1)(21) = 2 and (DES-SBOX1)(22) = 13 and (DES-SBOX1)(23) = 1 and (DES-SBOX1)(24) = 10 and (DES-SBOX1)(25) = 6 and (DES-SBOX1)(26) = 12 and (DES-SBOX1)(27) = 11 and (DES-SBOX1)(28) = 9 and (DES-SBOX1)(29) = 5 and (DES-SBOX1)(30) = 3 and (DES-SBOX1)(31) = 8 and (DES-SBOX1)(32) = 4 and (DES-SBOX1)(33) = 1 and (DES-SBOX1)(34) = 14 and (DES-SBOX1)(35) = 8 and (DES-SBOX1)(36) = 13 and (DES-SBOX1)(37) = 6 and (DES-SBOX1)(38) = 2 and (DES-SBOX1)(39) = 11 and (DES-SBOX1)(40) = 15 and (DES-SBOX1)(41) = 12 and (DES-SBOX1)(42) = 9 and (DES-SBOX1)(43) = 7 and

(DES-SBOX1)(44) = 3 and (DES-SBOX1)(45) = 10 and (DES-SBOX1)(46) = 5 and (DES-SBOX1)(47) = 0 and (DES-SBOX1)(48) = 15 and (DES-SBOX1)(49) = 12 and (DES-SBOX1)(50) = 8 and (DES-SBOX1)(51) = 2 and (DES-SBOX1)(52) = 4 and (DES-SBOX1)(53) = 9 and (DES-SBOX1)(54) = 1 and (DES-SBOX1)(55) = 7 and (DES-SBOX1)(56) = 5 and (DES-SBOX1)(57) = 11 and (DES-SBOX1)(58) = 3 and (DES-SBOX1)(59) = 14 and (DES-SBOX1)(60) = 10 and (DES-SBOX1)(61) = 0 and (DES-SBOX1)(62) = 6 and (DES-SBOX1)(63) = 13.

The function DES-SBOX2 from 64 into 16 is defined by the conditions (Def. 7).

(Def. 7) (DES-SBOX2)(0) = 15 and (DES-SBOX2)(1) = 1 and (DES-SBOX2)(2) = 8 and (DES-SBOX2)(3) = 14 and (DES-SBOX2)(4) = 6 and (DES-SBOX2)(5) = 11 and (DES-SBOX2)(6) = 3 and (DES-SBOX2)(7) = 4 and (DES-SBOX2)(8) = 9 and (DES-SBOX2)(9) = 7 and (DES-SBOX2)(10) = 2 and (DES-SBOX2)(11) = 13 and (DES-SBOX2)(12) = 12 and (DES-SBOX2)(13) = 0 and (DES-SBOX2)(14) = 5 and (DES-SBOX2)(15) = 10 and (DES-SBOX2)(16) = 3 and (DES-SBOX2)(17) = 13 and (DES-SBOX2)(18) = 4 and (DES-SBOX2)(19) = 7 and (DES-SBOX2)(20) = 15 and (DES-SBOX2)(21) = 2 and (DES-SBOX2)(22) = 8 and (DES-SBOX2)(23) = 14 and (DES-SBOX2)(24) = 12 and (DES-SBOX2)(25) = 0 and (DES-SBOX2)(26) = 1 and (DES-SBOX2)(27) = 10 and (DES-SBOX2)(28) = 6 and (DES-SBOX2)(29) = 9 and (DES-SBOX2)(30) = 11 and (DES-SBOX2)(31) = 5 and (DES-SBOX2)(32) = 0 and (DES-SBOX2)(33) = 14 and (DES-SBOX2)(34) = 7 and (DES-SBOX2)(35) = 11 and (DES-SBOX2)(36) = 10 and (DES-SBOX2)(37) = 4 and (DES-SBOX2)(38) = 13 and (DES-SBOX2)(39) = 1 and (DES-SBOX2)(40) = 5 and (DES-SBOX2)(41) = 8 and (DES-SBOX2)(42) = 12 and (DES-SBOX2)(43) = 6 and (DES-SBOX2)(44) = 9 and (DES-SBOX2)(45) = 3 and (DES-SBOX2)(46) = 2 and (DES-SBOX2)(47) = 15 and (DES-SBOX2)(48) = 13 and (DES-SBOX2)(49) = 8 and (DES-SBOX2)(50) = 10 and (DES-SBOX2)(51) = 1 and (DES-SBOX2)(52) = 3 and (DES-SBOX2)(53) = 15 and (DES-SBOX2)(54) = 4 and (DES-SBOX2)(55) = 2 and (DES-SBOX2)(56) = 11 and (DES-SBOX2)(57) = 6 and (DES-SBOX2)(58) = 7 and (DES-SBOX2)(59) = 12 and (DES-SBOX2)(60) = 0 and (DES-SBOX2)(61) = 5 and (DES-SBOX2)(62) = 14 and (DES-SBOX2)(63) = 9.

The function DES-SBOX3 from 64 into 16 is defined by the conditions (Def. 8).

(Def. 8) (DES-SBOX3)(0) = 10 and (DES-SBOX3)(1) = 0 and (DES-SBOX3)(2) = 9 and (DES-SBOX3)(3) = 14 and (DES-SBOX3)(4) = 6 and (DES-SBOX3)(5) = 3 and (DES-SBOX3)(6) = 15 and (DES-SBOX3)(7) = 5 and (DES-SBOX3)(8) = 1 and (DES-SBOX3)(9) = 13 and (DES-SBOX3)(10) = 12 and (DES-SBOX3)(11) = 7 and (DES-SBOX3)(12) = 11 and (DES-SBOX3)(13) = 4 and (DES-SBOX3)(14) = 2 and (DES-SBOX3)(15) = 8 and (DES-SBOX3)(16) = 13 and (DES-SBOX3)(17) = 7 and (DES-SBOX3)(18) = 0 and (DES-SBOX3)(19) = 9 and (DES-SBOX3)(20) = 3 and (DES-SBOX3)(21) = 4 and (DES-SBOX3)(22) = 6 and (DES-SBOX3)(23) = 10 and (DES-SBOX3)(24) = 2 and (DES-SBOX3)(25) = 8 and (DES-SBOX3)(26) = 5 and (DES-SBOX3)(27) = 14 and (DES-SBOX3)(28) = 12 and (DES-SBOX3)(29) = 11 and (DES-SBOX3)(30) = 15 and (DES-SBOX3)(31) = 1 and (DES-SBOX3)(32) = 13 and (DES-SBOX3)(33) = 6 and (DES-SBOX3)(34) = 4 and (DES-SBOX3)(35) = 9 and (DES-SBOX3)(36) = 8 and (DES-SBOX3)(37) = 15 and (DES-SBOX3)(38) = 3 and (DES-SBOX3)(39) = 0 and (DES-SBOX3)(40) = 11 and (DES-SBOX3)(41) = 1 and (DES-SBOX3)(42) = 2 and (DES-SBOX3)(43) = 12 and (DES-SBOX3)(44) = 5 and (DES-SBOX3)(45) = 10 and (DES-SBOX3)(46) = 14 and (DES-SBOX3)(47) = 7 and (DES-SBOX3)(48) = 1 and (DES-SBOX3)(49) = 10 and (DES-SBOX3)(50) = 13 and (DES-SBOX3)(51) = 0 and (DES-SBOX3)(52) = 6 and (DES-SBOX3)(53) = 9 and (DES-SBOX3)(54) = 8 and (DES-SBOX3)(55) = 7 and (DES-SBOX3)(56) = 4 and (DES-SBOX3)(57) = 15 and (DES-SBOX3)(58) = 14 and (DES-SBOX3)(59) = 3 and (DES-SBOX3)(60) = 11 and (DES-SBOX3)(61) = 5 and (DES-SBOX3)(62) = 2 and (DES-SBOX3)(63) = 12.

The function DES-SBOX4 from 64 into 16 is defined by the conditions (Def. 9).

(Def. 9) (DES-SBOX4)(0) = 7 and (DES-SBOX4)(1) = 13 and (DES-SBOX4)(2) = 14 and (DES-SBOX4)(3) = 3 and (DES-SBOX4)(4) = 0 and (DES-SBOX4)(5) = 6 and (DES-SBOX4)(6) = 9 and (DES-SBOX4)(7) = 10 and (DES-SBOX4)(8) = 1 and (DES-SBOX4)(9) = 2 and (DES-SBOX4)(10) = 8 and (DES-SBOX4)(11) = 5 and (DES-SBOX4)(12) = 11 and (DES-SBOX4)(13) = 12 and (DES-SBOX4)(14) = 4 and (DES-SBOX4)(15) = 15 and (DES-SBOX4)(16) = 13 and (DES-SBOX4)(17) = 8 and (DES-SBOX4)(18) = 11 and (DES-SBOX4)(19) = 5 and (DES-SBOX4)(20) = 6 and (DES-SBOX4)(21) = 15 and (DES-SBOX4)(22) = 0 and (DES-SBOX4)(23) = 3 and (DES-SBOX4)(24) = 4 and (DES-SBOX4)(25) = 7 and (DES-SBOX4)(26) = 2 and (DES-SBOX4)(27) = 12 and (DES-SBOX4)(28) = 1 and (DES-SBOX4)(29) = 10 and (DES-SBOX4)(30) = 14 and (DES-SBOX4)(31) = 9 and (DES-SBOX4)(32) = 10 and (DES-SBOX4)(33) = 6 and (DES-SBOX4)(34) = 9 and (DES-SBOX4)(35) = 0 and (DES-SBOX4)(36) = 12 and (DES-SBOX4)(37) = 11 and (DES-SBOX4)(38) = 7 and (DES-SBOX4)(39) = 13 and (DES-SBOX4)(40) = 15 and (DES-SBOX4)(41) = 1 and (DES-SBOX4)(42) = 3 and (DES-SBOX4)(43) = 14 and (DES-SBOX4)(44) = 5 and (DES-SBOX4)(45) = 2 and (DES-SBOX4)(46) = 8 and (DES-SBOX4)(47) = 4 and (DES-SBOX4)(48) = 3 and (DES-SBOX4)(49) = 15 and (DES-SBOX4)(50) = 0 and (DES-SBOX4)(51) = 6 and (DES-SBOX4)(52) = 10 and (DES-SBOX4)(53) = 1 and (DES-SBOX4)(54) = 13 and (DES-SBOX4)(55) = 8 and (DES-SBOX4)(56) = 9 and (DES-SBOX4)(57) = 4 and (DES-SBOX4)(58) = 5 and (DES-SBOX4)(59) = 11 and (DES-SBOX4)(60) = 12 and (DES-SBOX4)(61) = 7 and (DES-SBOX4)(62) = 2 and (DES-SBOX4)(63) = 14.

The function DES-SBOX5 from 64 into 16 is defined by the conditions (Def. 10).

(Def. 10) (DES-SBOX5)(0) = 2 and (DES-SBOX5)(1) = 12 and (DES-SBOX5)(2) = 4 and (DES-SBOX5)(3) = 1 and (DES-SBOX5)(4) = 7 and (DES-SBOX5)(5) = 10 and (DES-SBOX5)(6) = 11 and (DES-SBOX5)(7) = 6 and (DES-SBOX5)(8) = 8 and (DES-SBOX5)(9) = 5 and (DES-SBOX5)(10) = 3 and (DES-SBOX5)(11) = 15 and (DES-SBOX5)(12) = 13 and (DES-SBOX5)(13) = 0 and (DES-SBOX5)(14) = 14 and (DES-SBOX5)(15) = 9 and (DES-SBOX5)(16) = 14 and (DES-SBOX5)(17) = 11 and (DES-SBOX5)(18) = 2 and (DES-SBOX5)(19) = 12 and (DES-SBOX5)(20) = 4 and (DES-SBOX5)(21) = 7 and (DES-SBOX5)(22) = 13 and (DES-SBOX5)(23) = 1 and (DES-SBOX5)(24) = 5 and (DES-SBOX5)(25) = 0 and (DES-SBOX5)(26) = 15 and (DES-SBOX5)(27) = 10 and (DES-SBOX5)(28) = 3 and (DES-SBOX5)(29) = 9 and (DES-SBOX5)(30) = 8 and (DES-SBOX5)(31) = 6 and (DES-SBOX5)(32) = 4 and (DES-SBOX5)(33) = 2 and (DES-SBOX5)(34) = 1 and (DES-SBOX5)(35) = 11 and (DES-SBOX5)(36) = 10 and (DES-SBOX5)(37) = 13 and (DES-SBOX5)(38) = 7 and (DES-SBOX5)(39) = 8 and (DES-SBOX5)(40) = 15 and (DES-SBOX5)(41) = 9 and (DES-SBOX5)(42) = 12 and (DES-SBOX5)(43) = 5 and (DES-SBOX5)(44) = 6 and (DES-SBOX5)(45) = 3 and (DES-SBOX5)(46) = 0 and (DES-SBOX5)(47) = 14 and (DES-SBOX5)(48) = 11 and (DES-SBOX5)(49) = 8 and (DES-SBOX5)(50) = 12 and (DES-SBOX5)(51) =

7 and (DES-SBOX5)(52) = 1 and (DES-SBOX5)(53) = 14 and (DES-SBOX5)(54) = 2 and (DES-SBOX5)(55) = 13 and (DES-SBOX5)(56) = 6 and (DES-SBOX5)(57) = 15 and (DES-SBOX5)(58) = 0 and (DES-SBOX5)(59) = 9 and (DES-SBOX5)(60) = 10 and (DES-SBOX5)(61) = 4 and (DES-SBOX5)(62) = 5 and (DES-SBOX5)(63) = 3.

The function DES-SBOX6 from 64 into 16 is defined by the conditions (Def. 11).

(Def. 11) (DES-SBOX6)(0) = 12 and (DES-SBOX6)(1) = 1 and (DES-SBOX6)(2) = 10 and (DES-SBOX6)(3) = 15 and (DES-SBOX6)(4) = 9 and (DES-SBOX6)(5) = 2 and (DES-SBOX6)(6) = 6 and (DES-SBOX6)(7) = 8 and (DES-SBOX6)(8) = 0 and (DES-SBOX6)(9) = 13 and (DES-SBOX6)(10) = 3 and (DES-SBOX6)(11) = 4 and (DES-SBOX6)(12) = 14 and (DES-SBOX6)(13) = 7 and (DES-SBOX6)(14) = 5 and (DES-SBOX6)(15) = 11 and (DES-SBOX6)(16) = 10 and (DES-SBOX6)(17) = 15 and (DES-SBOX6)(18) = 4 and (DES-SBOX6)(19) = 2 and (DES-SBOX6)(20) = 7 and (DES-SBOX6)(21) = 12 and (DES-SBOX6)(22) = 9 and (DES-SBOX6)(23) = 5 and (DES-SBOX6)(24) = 6 and (DES-SBOX6)(25) = 1 and (DES-SBOX6)(26) = 13 and (DES-SBOX6)(27) = 14 and (DES-SBOX6)(28) = 0 and (DES-SBOX6)(29) = 11 and (DES-SBOX6)(30) = 3 and (DES-SBOX6)(31) = 8 and (DES-SBOX6)(32) = 9 and (DES-SBOX6)(33) = 14 and (DES-SBOX6)(34) = 15 and (DES-SBOX6)(35) = 5 and (DES-SBOX6)(36) = 2 and (DES-SBOX6)(37) = 8 and (DES-SBOX6)(38) = 12 and (DES-SBOX6)(39) = 3 and (DES-SBOX6)(40) = 7 and (DES-SBOX6)(41) = 0 and (DES-SBOX6)(42) = 4 and (DES-SBOX6)(43) = 10 and (DES-SBOX6)(44) = 1 and (DES-SBOX6)(45) = 13 and (DES-SBOX6)(46) = 11 and (DES-SBOX6)(47) = 6 and (DES-SBOX6)(48) = 4 and (DES-SBOX6)(49) = 3 and (DES-SBOX6)(50) = 2 and (DES-SBOX6)(51) = 12 and (DES-SBOX6)(52) = 9 and (DES-SBOX6)(53) = 5 and (DES-SBOX6)(54) = 15 and (DES-SBOX6)(55) = 10 and (DES-SBOX6)(56) = 11 and (DES-SBOX6)(57) = 14 and (DES-SBOX6)(58) = 1 and (DES-SBOX6)(59) = 7 and (DES-SBOX6)(60) = 6 and (DES-SBOX6)(61) = 0 and (DES-SBOX6)(62) = 8 and (DES-SBOX6)(63) = 13.

The function DES-SBOX7 from 64 into 16 is defined by the conditions (Def. 12).

(Def. 12) (DES-SBOX7)(0) = 4 and (DES-SBOX7)(1) = 11 and (DES-SBOX7)(2) = 2 and (DES-SBOX7)(3) = 14 and (DES-SBOX7)(4) = 15 and (DES-SBOX7)(5) = 0 and (DES-SBOX7)(6) = 8 and (DES-SBOX7)(7) =

13 and (DES-SBOX7)(8) = 3 and (DES-SBOX7)(9) = 12 and (DES-SBOX7)(10) = 9 and (DES-SBOX7)(11) = 7 and (DES-SBOX7)(12) = 5 and (DES-SBOX7)(13) = 10 and (DES-SBOX7)(14) = 6 and (DES-SBOX7)(15) = 1 and (DES-SBOX7)(16) = 13 and (DES-SBOX7)(17) = 0 and (DES-SBOX7)(18) = 11 and (DES-SBOX7)(19) = 7 and (DES-SBOX7)(20) = 4 and (DES-SBOX7)(21) = 9 and (DES-SBOX7)(22) = 1 and (DES-SBOX7)(23) = 10 and (DES-SBOX7)(24) = 14 and (DES-SBOX7)(25) = 3 and (DES-SBOX7)(26) = 5 and (DES-SBOX7)(27) = 12 and (DES-SBOX7)(28) = 2 and (DES-SBOX7)(29) = 15 and (DES-SBOX7)(30) = 8 and (DES-SBOX7)(31) = 6 and (DES-SBOX7)(32) = 1 and (DES-SBOX7)(33) = 4 and (DES-SBOX7)(34) = 11 and (DES-SBOX7)(35) = 13 and (DES-SBOX7)(36) = 12 and (DES-SBOX7)(37) = 3 and (DES-SBOX7)(38) = 7 and (DES-SBOX7)(39) = 14 and (DES-SBOX7)(40) = 10 and (DES-SBOX7)(41) = 15 and (DES-SBOX7)(42) = 6 and (DES-SBOX7)(43) = 8 and (DES-SBOX7)(44) = 0 and (DES-SBOX7)(45) = 5 and (DES-SBOX7)(46) = 9 and (DES-SBOX7)(47) = 2 and (DES-SBOX7)(48) = 6 and (DES-SBOX7)(49) = 11 and (DES-SBOX7)(50) = 13 and (DES-SBOX7)(51) = 8 and (DES-SBOX7)(52) = 1 and (DES-SBOX7)(53) = 4 and (DES-SBOX7)(54) = 10 and (DES-SBOX7)(55) = 7 and (DES-SBOX7)(56) = 9 and (DES-SBOX7)(57) = 5 and (DES-SBOX7)(58) = 0 and (DES-SBOX7)(59) = 15 and (DES-SBOX7)(60) = 14 and (DES-SBOX7)(61) = 2 and (DES-SBOX7)(62) = 3 and (DES-SBOX7)(63) = 12.

The function DES-SBOX8 from 64 into 16 is defined by the conditions (Def. 13).

(Def. 13) (DES-SBOX8)(0) = 13 and (DES-SBOX8)(1) = 2 and (DES-SBOX8)(2) = 8 and (DES-SBOX8)(3) = 4 and (DES-SBOX8)(4) = 6 and (DES-SBOX8)(5) = 15 and (DES-SBOX8)(6) = 11 and (DES-SBOX8)(7) = 1 and (DES-SBOX8)(8) = 10 and (DES-SBOX8)(9) = 9 and (DES-SBOX8)(10) = 3 and (DES-SBOX8)(11) = 14 and (DES-SBOX8)(12) = 5 and (DES-SBOX8)(13) = 0 and (DES-SBOX8)(14) = 12 and (DES-SBOX8)(15) = 7 and (DES-SBOX8)(16) = 1 and (DES-SBOX8)(17) = 15 and (DES-SBOX8)(18) = 13 and (DES-SBOX8)(19) = 8 and (DES-SBOX8)(20) = 10 and (DES-SBOX8)(21) = 3 and (DES-SBOX8)(22) = 7 and (DES-SBOX8)(23) = 4 and (DES-SBOX8)(24) = 12 and (DES-SBOX8)(25) = 5 and (DES-SBOX8)(26) = 5 and (DES-SBOX8)(27) = 11 and (DES-SBOX8)(28) = 0 and (DES-SBOX8)(29) = 14 and (DES-SBOX8)(30) = 9 and (DES-SBOX8)(31) = 2 and (DES-SBOX8)(32) = 7 and (DES-SBOX8)(33) = 11 and (DES-SBOX8)(34) = 4 and (DES-SBOX8)(35) = 1 and (DES-SBOX8)(36) = 9 and (DES-SBOX8)(37) = 12 and (DES-SBOX8)(38) = 14 and (DES-SBOX8)(39) = 2 and (DES-SBOX8)(40) = 0 and (DES-SBOX8)(41) = 6 and (DES-SBOX8)(42) = 10 and (DES-SBOX8)(43) = 13 and (DES-SBOX8)(44) = 15 and (DES-SBOX8)(45) = 3 and (DES-SBOX8)(46) = 5 and (DES-SBOX8)(47) = 8 and (DES-SBOX8)(48) = 2 and (DES-SBOX8)(49) = 1 and (DES-SBOX8)(50) = 14 and (DES-SBOX8)(51) = 7 and (DES-SBOX8)(52) = 4 and (DES-SBOX8)(53) = 10 and (DES-SBOX8)(54) = 8 and (DES-SBOX8)(55) = 13 and (DES-SBOX8)(56) = 15 and (DES-SBOX8)(57) = 12 and (DES-SBOX8)(58) = 9 and (DES-SBOX8)(59) = 0 and (DES-SBOX8)(60) = 3 and (DES-SBOX8)(61) = 5 and (DES-SBOX8)(62) = 6 and (DES-SBOX8)(63) = 11.

3. Initial Permutation

Let r be an element of $\mathit{Boolean}^{64}$. The functor DES-IP r yields an element of $\mathit{Boolean}^{64}$ and is defined by the conditions (Def. 14).

(Def. 14) (DES-IP r)(1) = r(58) and (DES-IP r)(2) = r(50) and (DES-IP r)(3) = r(42) and (DES-IP r)(4) = r(34) and (DES-IP r)(5) = r(26) and (DES-IP r)(6) = r(18) and (DES-IP r)(7) = r(10) and (DES-IP r)(8) = r(2) and (DES-IP r)(9) = r(60) and (DES-IP r)(10) = r(52) and (DES-IP r)(11) = r(44) and (DES-IP r)(12) = r(36) and (DES-IP r)(13) = r(28) and (DES-IP r)(14) = r(20) and (DES-IP r)(15) = r(12) and (DES-IP r)(16) = r(4) and (DES-IP r)(17) = r(62) and (DES-IP r)(18) = r(54) and (DES-IP r)(19) = r(46) and (DES-IP r)(20) = r(38) and (DES-IP r)(21) = r(30) and (DES-IP r)(22) = r(22) and (DES-IP r)(23) = r(14) and (DES-IP r)(24) = r(6) and (DES-IP r)(25) = r(64) and (DES-IP r)(26) = r(56) and (DES-IP r)(27) = r(48) and (DES-IP r)(28) = r(40) and (DES-IP r)(29) = r(32) and (DES-IP r)(30) = r(24) and (DES-IP r)(31) = r(16) and (DES-IP r)(32) = r(8) and (DES-IP r)(33) = r(57) and (DES-IP r)(34) = r(49) and (DES-IP r)(35) = r(41) and (DES-IP r)(36) = r(33) and (DES-IP r)(37) = r(25) and (DES-IP r)(38) = r(17) and (DES-IP r)(39) = r(9) and (DES-IP r)(40) = r(1) and (DES-IP r)(41) = r(59) and (DES-IP r)(42) = r(51) and (DES-IP r)(43) = r(43) and (DES-IP r)(44) = r(35) and (DES-IP r)(45) = r(27) and (DES-IP r)(46) = r(19) and (DES-IP r)(47) = r(11) and (DES-IP r)(48) = r(3) and (DES-IP r)(49) = r(61) and (DES-IP r)(50) = r(53) and (DES-IP r)(51) = r(45) and (DES-IP r)(52) = r(37)

and (DES-IP r)(53) $= r(29)$ and (DES-IP r)(54) $= r(21)$ and (DES-IP r)(55) $= r(13)$ and (DES-IP r)(56) $= r(5)$ and (DES-IP r)(57) $= r(63)$ and (DES-IP r)(58) $= r(55)$ and (DES-IP r)(59) $= r(47)$ and (DES-IP r)(60) $= r(39)$ and (DES-IP r)(61) $= r(31)$ and (DES-IP r)(62) $= r(23)$ and (DES-IP r)(63) $= r(15)$ and (DES-IP r)(64) $= r(7)$.

The function DES-PIP from *Boolean*^{64} into *Boolean*^{64} is defined by:

(Def. 15) For every element i of *Boolean*^{64} holds (DES-PIP)(i) = DES-IP i.

Let r be an element of *Boolean*^{64}. The functor DES-IPINV r yields an element of *Boolean*^{64} and is defined by the conditions (Def. 16).

(Def. 16) (DES-IPINV r)(1) $= r(40)$ and (DES-IPINV r)(2) $= r(8)$ and (DES-IPINV r)(3) $= r(48)$ and (DES-IPINV r)(4) $= r(16)$ and (DES-IPINV r)(5) $= r(56)$ and (DES-IPINV r)(6) $= r(24)$ and (DES-IPINV r)(7) $= r(64)$ and (DES-IPINV r)(8) $= r(32)$ and (DES-IPINV r)(9) $= r(39)$ and (DES-IPINV r)(10) $= r(7)$ and (DES-IPINV r)(11) $= r(47)$ and (DES-IPINV r)(12) $= r(15)$ and (DES-IPINV r)(13) $= r(55)$ and (DES-IPINV r)(14) $= r(23)$ and (DES-IPINV r)(15) $= r(63)$ and (DES-IPINV r)(16) $= r(31)$ and (DES-IPINV r)(17) $= r(38)$ and (DES-IPINV r)(18) $= r(6)$ and (DES-IPINV r)(19) $= r(46)$ and (DES-IPINV r)(20) $= r(14)$ and (DES-IPINV r)(21) $= r(54)$ and (DES-IPINV r)(22) $= r(22)$ and (DES-IPINV r)(23) $= r(62)$ and (DES-IPINV r)(24) $= r(30)$ and (DES-IPINV r)(25) $= r(37)$ and (DES-IPINV r)(26) $= r(5)$ and (DES-IPINV r)(27) $= r(45)$ and (DES-IPINV r)(28) $= r(13)$ and (DES-IPINV r)(29) $= r(53)$ and (DES-IPINV r)(30) $= r(21)$ and (DES-IPINV r)(31) $= r(61)$ and (DES-IPINV r)(32) $= r(29)$ and (DES-IPINV r)(33) $= r(36)$ and (DES-IPINV r)(34) $= r(4)$ and (DES-IPINV r)(35) $= r(44)$ and (DES-IPINV r)(36) $= r(12)$ and (DES-IPINV r)(37) $= r(52)$ and (DES-IPINV r)(38) $= r(20)$ and (DES-IPINV r)(39) $= r(60)$ and (DES-IPINV r)(40) $= r(28)$ and (DES-IPINV r)(41) $= r(35)$ and (DES-IPINV r)(42) $= r(3)$ and (DES-IPINV r)(43) $= r(43)$ and (DES-IPINV r)(44) $= r(11)$ and (DES-IPINV r)(45) $= r(51)$ and (DES-IPINV r)(46) $= r(19)$ and (DES-IPINV r)(47) $= r(59)$ and (DES-IPINV r)(48) $= r(27)$ and (DES-IPINV r)(49) $= r(34)$ and (DES-IPINV r)(50) $= r(2)$ and (DES-IPINV r)(51) $= r(42)$ and (DES-IPINV r)(52) $= r(10)$ and (DES-IPINV r)(53) $= r(50)$ and (DES-IPINV r)(54) $= r(18)$ and (DES-IPINV r)(55) $= r(58)$ and (DES-IPINV r)(56) $= r(26)$ and (DES-IPINV r)(57) $= r(33)$ and (DES-IPINV r)(58) $= r(1)$ and (DES-IPINV r)(59) $= r(41)$ and (DES-IPINV r)(60) $= r(9)$ and (DES-IPINV r)(61) $= r(49)$ and (DES-IPINV r)(62) $= r(17)$ and

(DES-IPINV r)(63) = r(57) and (DES-IPINV r)(64) = r(25).

The function DES-PIPINV from *Boolean*^{64} into *Boolean*^{64} is defined by:

(Def. 17) For every element i of *Boolean*^{64} holds (DES-PIPINV)(i) = DES-IPINV i.

Let us note that DES-PIP is bijective.

Let us note that DES-PIPINV is bijective.

The following proposition is true

(36) DES-PIPINV = (DES-PIP)$^{-1}$.

4. Feistel Function

Let r be an element of *Boolean*^{32}. The functor DES-E r yielding an element of *Boolean*^{48} is defined by the conditions (Def. 18).

(Def. 18) (DES-E r)(1) = r(32) and (DES-E r)(2) = r(1) and (DES-E r)(3) = r(2) and (DES-E r)(4) = r(3) and (DES-E r)(5) = r(4) and (DES-E r)(6) = r(5) and (DES-E r)(7) = r(4) and (DES-E r)(8) = r(5) and (DES-E r)(9) = r(6) and (DES-E r)(10) = r(7) and (DES-E r)(11) = r(8) and (DES-E r)(12) = r(9) and (DES-E r)(13) = r(8) and (DES-E r)(14) = r(9) and (DES-E r)(15) = r(10) and (DES-E r)(16) = r(11) and (DES-E r)(17) = r(12) and (DES-E r)(18) = r(13) and (DES-E r)(19) = r(12) and (DES-E r)(20) = r(13) and (DES-E r)(21) = r(14) and (DES-E r)(22) = r(15) and (DES-E r)(23) = r(16) and (DES-E r)(24) = r(17) and (DES-E r)(25) = r(16) and (DES-E r)(26) = r(17) and (DES-E r)(27) = r(18) and (DES-E r)(28) = r(19) and (DES-E r)(29) = r(20) and (DES-E r)(30) = r(21) and (DES-E r)(31) = r(20) and (DES-E r)(32) = r(21) and (DES-E r)(33) = r(22) and (DES-E r)(34) = r(23) and (DES-E r)(35) = r(24) and (DES-E r)(36) = r(25) and (DES-E r)(37) = r(24) and (DES-E r)(38) = r(25) and (DES-E r)(39) = r(26) and (DES-E r)(40) = r(27) and (DES-E r)(41) = r(28) and (DES-E r)(42) = r(29) and (DES-E r)(43) = r(28) and (DES-E r)(44) = r(29) and (DES-E r)(45) = r(30) and (DES-E r)(46) = r(31) and (DES-E r)(47) = r(32) and (DES-E r)(48) = r(1).

Let r be an element of *Boolean*^{32}. The functor DES-P r yielding an element of *Boolean*^{32} is defined by the conditions (Def. 19).

(Def. 19) (DES-P r)(1) = r(16) and (DES-P r)(2) = r(7) and (DES-P r)(3) = r(20) and (DES-P r)(4) = r(21) and (DES-P r)(5) = r(29) and (DES-P r)(6) = r(12) and (DES-P r)(7) = r(28) and (DES-P r)(8) = r(17) and (DES-P r)(9) = r(1) and (DES-P r)(10) = r(15) and (DES-P r)(11) = r(23) and (DES-P r)(12) = r(26) and (DES-P r)(13) = r(5) and (DES-P r)(14) = r(18) and (DES-P r)(15) = r(31) and

(DES-P r)(16) = r(10) and (DES-P r)(17) = r(2) and (DES-P r)(18) = r(8) and (DES-P r)(19) = r(24) and (DES-P r)(20) = r(14) and (DES-P r)(21) = r(32) and (DES-P r)(22) = r(27) and (DES-P r)(23) = r(3) and (DES-P r)(24) = r(9) and (DES-P r)(25) = r(19) and (DES-P r)(26) = r(13) and (DES-P r)(27) = r(30) and (DES-P r)(28) = r(6) and (DES-P r)(29) = r(22) and (DES-P r)(30) = r(11) and (DES-P r)(31) = r(4) and (DES-P r)(32) = r(25).

Let r be an element of $Boolean^{48}$. The functor DES-DIV8 r yielding an element of $(Boolean^6)^8$ is defined by the conditions (Def. 20).

(Def. 20) (DES-DIV8 r)(1) = Op-Left($r, 6$) and (DES-DIV8 r)(2) = Op-Left(Op-Right($r, 6$), 6) and (DES-DIV8 r)(3) = Op-Left(Op-Right($r, 12$), 6) and (DES-DIV8 r)(4) = Op-Left(Op-Right($r, 18$), 6) and (DES-DIV8 r)(5) = Op-Left(Op-Right($r, 24$), 6) and (DES-DIV8 r)(6) = Op-Left(Op-Right($r, 30$), 6) and (DES-DIV8 r)(7) = Op-Left(Op-Right($r, 36$), 6) and (DES-DIV8 r)(8) = Op-Right($r, 42$).

Next we state the proposition

(37) Let r be an element of $Boolean^{48}$. Then there exist elements s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 of $Boolean^6$ such that s_1 = (DES-DIV8 r)(1) and s_2 = (DES-DIV8 r)(2) and s_3 = (DES-DIV8 r)(3) and s_4 = (DES-DIV8 r)(4) and s_5 = (DES-DIV8 r)(5) and s_6 = (DES-DIV8 r)(6) and s_7 = (DES-DIV8 r)(7) and s_8 = (DES-DIV8 r)(8) and $r = s_1 \frown s_2 \frown s_3 \frown s_4 \frown s_5 \frown s_6 \frown s_7 \frown s_8$.

Let t be an element of $Boolean^6$. The functor B6toN64 t yielding an element of 64 is defined by:

(Def. 21) B6toN64 $t = 32 \cdot t(1) + 16 \cdot t(6) + 8 \cdot t(2) + 4 \cdot t(3) + 2 \cdot t(4) + 1 \cdot t(5)$.

The function N16toB4 from 16 into $Boolean^4$ is defined by the conditions (Def. 22).

(Def. 22) (N16toB4)(0) = $\langle 0,0,0,0\rangle$ and (N16toB4)(1) = $\langle 0,0,0,1\rangle$ and (N16toB4)(2) = $\langle 0,0,1,0\rangle$ and (N16toB4)(3) = $\langle 0,0,1,1\rangle$ and (N16toB4)(4) = $\langle 0,1,0,0\rangle$ and (N16toB4)(5) = $\langle 0,1,0,1\rangle$ and (N16toB4)(6) = $\langle 0,1,1,0\rangle$ and (N16toB4)(7) = $\langle 0,1,1,1\rangle$ and (N16toB4)(8) = $\langle 1,0,0,0\rangle$ and (N16toB4)(9) = $\langle 1,0,0,1\rangle$ and (N16toB4)(10) = $\langle 1,0,1,0\rangle$ and (N16toB4)(11) = $\langle 1,0,1,1\rangle$ and (N16toB4)(12) = $\langle 1,1,0,0\rangle$ and (N16toB4)(13) = $\langle 1,1,0,1\rangle$ and (N16toB4)(14) = $\langle 1,1,1,0\rangle$ and (N16toB4)(15) = $\langle 1,1,1,1\rangle$.

Let R be an element of $Boolean^{32}$ and let R_2 be an element of $Boolean^{48}$. The functor DES-F(R, R_2) yields an element of $Boolean^{32}$ and is defined by the condition (Def. 23).

(Def. 23) There exist elements D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8 of $Boolean^6$ and

there exist elements x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 of $Boolean^4$ and there exists an element C_{32} of $Boolean^{32}$ such that
$D_1 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(1)$ and
$D_2 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(2)$ and
$D_3 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(3)$ and
$D_4 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(4)$ and
$D_5 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(5)$ and
$D_6 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(6)$ and
$D_7 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(7)$ and
$D_8 = (\text{DES-DIV8}\,\text{Op-XOR}(\text{DES-E}\,R, R_2))(8)$ and
$\text{Op-XOR}(\text{DES-E}\,R, R_2) = D_1 \frown D_2 \frown D_3 \frown D_4 \frown D_5 \frown D_6 \frown D_7 \frown D_8$ and $x_1 = (\text{N16toB4})((\text{DES-SBOX1})(\text{B6toN64}\,D_1))$ and $x_2 = (\text{N16toB4})((\text{DES-SBOX2})(\text{B6toN64}\,D_2))$ and
$x_3 = (\text{N16toB4})((\text{DES-SBOX3})(\text{B6toN64}\,D_3))$ and
$x_4 = (\text{N16toB4})((\text{DES-SBOX4})(\text{B6toN64}\,D_4))$ and
$x_5 = (\text{N16toB4})((\text{DES-SBOX5})(\text{B6toN64}\,D_5))$ and
$x_6 = (\text{N16toB4})((\text{DES-SBOX6})(\text{B6toN64}\,D_6))$ and
$x_7 = (\text{N16toB4})((\text{DES-SBOX7})(\text{B6toN64}\,D_7))$ and
$x_8 = (\text{N16toB4})((\text{DES-SBOX8})(\text{B6toN64}\,D_8))$ and $C_{32} = x_1 \frown x_2 \frown x_3 \frown x_4 \frown x_5 \frown x_6 \frown x_7 \frown x_8$ and $\text{DES-F}(R, R_2) = \text{DES-P}\,C_{32}$.

The function DES-FFUNC from $Boolean^{32} \times Boolean^{48}$ into $Boolean^{32}$ is defined as follows:

(Def. 24) For every element z of $Boolean^{32} \times Boolean^{48}$ holds $(\text{DES-FFUNC})(z) = \text{DES-F}(z_{\mathbf{1}}, z_{\mathbf{2}})$.

5. Key Schedule

Let r be an element of $Boolean^{64}$. The functor DES-PC1 r yields an element of $Boolean^{56}$ and is defined by the conditions (Def. 25).

(Def. 25) $(\text{DES-PC1}\,r)(1) = r(57)$ and $(\text{DES-PC1}\,r)(2) = r(49)$ and $(\text{DES-PC1}\,r)(3) = r(41)$ and $(\text{DES-PC1}\,r)(4) = r(33)$ and $(\text{DES-PC1}\,r)(5) = r(25)$ and $(\text{DES-PC1}\,r)(6) = r(17)$ and $(\text{DES-PC1}\,r)(7) = r(9)$ and $(\text{DES-PC1}\,r)(8) = r(1)$ and $(\text{DES-PC1}\,r)(9) = r(58)$ and $(\text{DES-PC1}\,r)(10) = r(50)$ and $(\text{DES-PC1}\,r)(11) = r(42)$ and $(\text{DES-PC1}\,r)(12) = r(34)$ and $(\text{DES-PC1}\,r)(13) = r(26)$ and $(\text{DES-PC1}\,r)(14) = r(18)$ and $(\text{DES-PC1}\,r)(15) = r(10)$ and $(\text{DES-PC1}\,r)(16) = r(2)$ and $(\text{DES-PC1}\,r)(17) = r(59)$ and $(\text{DES-PC1}\,r)(18) = r(51)$ and $(\text{DES-PC1}\,r)(19) = r(43)$ and $(\text{DES-PC1}\,r)(20) = r(35)$ and $(\text{DES-PC1}\,r)(21) = r(27)$ and $(\text{DES-PC1}\,r)(22) = r(19)$ and $(\text{DES-PC1}\,r)(23) = r(11)$ and $(\text{DES-PC1}\,r)(24) = r(3)$ and $(\text{DES-PC1}\,r)(25) = r(60)$ and

(DES-PC1 r)(26) = $r(52)$ and (DES-PC1 r)(27) = $r(44)$ and (DES-PC1 r)(28) = $r(36)$ and (DES-PC1 r)(29) = $r(63)$ and (DES-PC1 r)(30) = $r(55)$ and (DES-PC1 r)(31) = $r(47)$ and (DES-PC1 r)(32) = $r(39)$ and (DES-PC1 r)(33) = $r(31)$ and (DES-PC1 r)(34) = $r(23)$ and (DES-PC1 r)(35) = $r(15)$ and (DES-PC1 r)(36) = $r(7)$ and (DES-PC1 r)(37) = $r(62)$ and (DES-PC1 r)(38) = $r(54)$ and (DES-PC1 r)(39) = $r(46)$ and (DES-PC1 r)(40) = $r(38)$ and (DES-PC1 r)(41) = $r(30)$ and (DES-PC1 r)(42) = $r(22)$ and (DES-PC1 r)(43) = $r(14)$ and (DES-PC1 r)(44) = $r(6)$ and (DES-PC1 r)(45) = $r(61)$ and (DES-PC1 r)(46) = $r(53)$ and (DES-PC1 r)(47) = $r(45)$ and (DES-PC1 r)(48) = $r(37)$ and (DES-PC1 r)(49) = $r(29)$ and (DES-PC1 r)(50) = $r(21)$ and (DES-PC1 r)(51) = $r(13)$ and (DES-PC1 r)(52) = $r(5)$ and (DES-PC1 r)(53) = $r(28)$ and (DES-PC1 r)(54) = $r(20)$ and (DES-PC1 r)(55) = $r(12)$ and (DES-PC1 r)(56) = $r(4)$.

Let r be an element of *Boolean*^{56}. The functor DES-PC2 r yielding an element of *Boolean*^{48} is defined by the conditions (Def. 26).

(Def. 26) (DES-PC2 r)(1) = $r(14)$ and (DES-PC2 r)(2) = $r(17)$ and (DES-PC2 r)(3) = $r(11)$ and (DES-PC2 r)(4) = $r(24)$ and (DES-PC2 r)(5) = $r(1)$ and (DES-PC2 r)(6) = $r(5)$ and (DES-PC2 r)(7) = $r(3)$ and (DES-PC2 r)(8) = $r(28)$ and (DES-PC2 r)(9) = $r(15)$ and (DES-PC2 r)(10) = $r(6)$ and (DES-PC2 r)(11) = $r(21)$ and (DES-PC2 r)(12) = $r(10)$ and (DES-PC2 r)(13) = $r(23)$ and (DES-PC2 r)(14) = $r(19)$ and (DES-PC2 r)(15) = $r(12)$ and (DES-PC2 r)(16) = $r(4)$ and (DES-PC2 r)(17) = $r(26)$ and (DES-PC2 r)(18) = $r(8)$ and (DES-PC2 r)(19) = $r(16)$ and (DES-PC2 r)(20) = $r(7)$ and (DES-PC2 r)(21) = $r(27)$ and (DES-PC2 r)(22) = $r(20)$ and (DES-PC2 r)(23) = $r(13)$ and (DES-PC2 r)(24) = $r(2)$ and (DES-PC2 r)(25) = $r(41)$ and (DES-PC2 r)(26) = $r(52)$ and (DES-PC2 r)(27) = $r(31)$ and (DES-PC2 r)(28) = $r(37)$ and (DES-PC2 r)(29) = $r(47)$ and (DES-PC2 r)(30) = $r(55)$ and (DES-PC2 r)(31) = $r(30)$ and (DES-PC2 r)(32) = $r(40)$ and (DES-PC2 r)(33) = $r(51)$ and (DES-PC2 r)(34) = $r(45)$ and (DES-PC2 r)(35) = $r(33)$ and (DES-PC2 r)(36) = $r(48)$ and (DES-PC2 r)(37) = $r(44)$ and (DES-PC2 r)(38) = $r(49)$ and (DES-PC2 r)(39) = $r(39)$ and (DES-PC2 r)(40) = $r(56)$ and (DES-PC2 r)(41) = $r(34)$ and (DES-PC2 r)(42) = $r(53)$ and (DES-PC2 r)(43) = $r(46)$ and (DES-PC2 r)(44) = $r(42)$ and (DES-PC2 r)(45) = $r(50)$ and (DES-PC2 r)(46) = $r(36)$ and (DES-PC2 r)(47) = $r(29)$ and

(DES-PC2 r)(48) $= r(32)$.

The finite sequence $\mathrm{bitshift}_{\mathrm{DES}}$ of elements of $\mathbb{N}$ is defined by the conditions (Def. 27).

(Def. 27) $\mathrm{bitshift}_{\mathrm{DES}}$ is 16-element and $(\mathrm{bitshift}_{\mathrm{DES}})(1) = 1$ and $(\mathrm{bitshift}_{\mathrm{DES}})(2) = 1$ and $(\mathrm{bitshift}_{\mathrm{DES}})(3) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(4) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(5) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(6) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(7) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(8) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(9) = 1$ and $(\mathrm{bitshift}_{\mathrm{DES}})(10) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(11) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(12) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(13) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(14) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(15) = 2$ and $(\mathrm{bitshift}_{\mathrm{DES}})(16) = 1$.

Let K_1 be an element of $Boolean^{64}$. The functor DES-KS K_1 yielding an element of $(Boolean^{48})^{16}$ is defined by the condition (Def. 28).

(Def. 28) There exist sequences C, D of $Boolean^{28}$ such that

- (i) $C(0) = \text{Op-Left}(\text{DES-PC1}\, K_1, 28)$,
- (ii) $D(0) = \text{Op-Right}(\text{DES-PC1}\, K_1, 28)$, and
- (iii) for every element i of $\mathbb{N}$ such that $0 \le i \le 15$ holds $(\text{DES-KS}\, K_1)(i + 1) = \text{DES-PC2}(C(i + 1) \frown D(i + 1))$ and $C(i + 1) = \text{Op-Shift}(C(i), (\mathrm{bitshift}_{\mathrm{DES}})(i))$ and $D(i + 1) = \text{Op-Shift}(D(i), (\mathrm{bitshift}_{\mathrm{DES}})(i))$.

6. Encryption and Decryption

Let n, m, k be non empty elements of $\mathbb{N}$, let R_1 be an element of $(Boolean^m)^k$, let F be a function from $Boolean^n \times Boolean^m$ into $Boolean^n$, let I_1 be a permutation of $Boolean^{2 \cdot n}$, and let M be an element of $Boolean^{2 \cdot n}$. The functor DES-like-CoDec(M, F, I_1, R_1) yields an element of $Boolean^{2 \cdot n}$ and is defined by the condition (Def. 29).

(Def. 29) There exist sequences L, R of $Boolean^n$ such that

- (i) $L(0) = \text{SP-Left}\, I_1(M)$,
- (ii) $R(0) = \text{SP-Right}\, I_1(M)$,
- (iii) for every element i of $\mathbb{N}$ such that $0 \le i \le k - 1$ holds $L(i + 1) = R(i)$ and $R(i + 1) = \text{Op-XOR}(L(i), F(R(i), (R_1)_{i+1}))$, and
- (iv) $\text{DES-like-CoDec}(M, F, I_1, R_1) = {I_1}^{-1}(R(k) \frown L(k))$.

The following proposition is true

(38) Let n, m, k be non empty elements of $\mathbb{N}$, R_1 be an element of $(Boolean^m)^k$, F be a function from $Boolean^n \times Boolean^m$ into $Boolean^n$, I_1 be a permutation of $Boolean^{2 \cdot n}$, and M be an element of $Boolean^{2 \cdot n}$. Then $\text{DES-like-CoDec}(\text{DES-like-CoDec}(M, F, I_1, R_1), F, I_1, \text{Rev}(R_1)) = M$.

Let R_1 be an element of $(Boolean^{48})^{16}$, let F be a function from $Boolean^{32} \times Boolean^{48}$ into $Boolean^{32}$, let I_1 be a permutation of $Boolean^{64}$, and let M be an element of $Boolean^{64}$. The functor DES-CoDec(M, F, I_1, R_1) yielding an element of $Boolean^{64}$ is defined by:

(Def. 30) There exists a permutation I_2 of $Boolean^{2 \cdot 32}$ and there exists an element M_1 of $Boolean^{2 \cdot 32}$ such that $I_2 = I_1$ and $M_1 = M$ and DES-CoDec$(M, F, I_1, R_1) =$ DES-like-CoDec(M_1, F, I_2, R_1).

The following proposition is true

(39) Let R_1 be an element of $(Boolean^{48})^{16}$, F be a function from $Boolean^{32} \times Boolean^{48}$ into $Boolean^{32}$, I_1 be a permutation of $Boolean^{64}$, and M be an element of $Boolean^{64}$.
Then DES-CoDec(DES-CoDec$(M, F, I_1, R_1), F, I_1, \mathrm{Rev}(R_1)) = M$.

Let p_1, s_9 be elements of $Boolean^{64}$. The functor DES-ENC(p_1, s_9) yields an element of $Boolean^{64}$ and is defined by:

(Def. 31) DES-ENC$(p_1, s_9) =$ DES-CoDec$(p_1,$ DES-FFUNC, DES-PIP, DES-KS $s_9)$.

Let c_1, s_9 be elements of $Boolean^{64}$. The functor DES-DEC(c_1, s_9) yields an element of $Boolean^{64}$ and is defined as follows:

(Def. 32) DES-DEC$(c_1, s_9) =$
DES-CoDec$(c_1,$ DES-FFUNC, DES-PIP, Rev(DES-KS $s_9))$.

The following proposition is true

(40) For all elements m_1, s_9 of $Boolean^{64}$ holds
DES-DEC(DES-ENC$(m_1, s_9), s_9) = m_1$.

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