

Definition of First Order Language with Arbitrary Alphabet. Syntax of Terms, Atomic Formulas and their Subterms¹

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Summary. Second of a series of articles laying down the bases for classical first order model theory. A language is defined basically as a tuple made of an integer-valued function (adicity), a symbol of equality and a symbol for the NOR logical connective. The only requests for this tuple to be a language is that the value of the adicity in $=$ is -2 and that its preimage (i.e. the variables set) in 0 is infinite. Existential quantification will be rendered (see [11]) by mere prefixing a formula with a letter. Then the hierarchy among symbols according to their adicity is introduced, taking advantage of attributes and clusters.

The strings of symbols of a language are depth-recursively classified as terms using the standard approach (see for example [16], definition 1.1.2); technically, this is done here by deploying the ‘-multiCat’ functor and the ‘unambiguous’ attribute previously introduced in [10], and the set of atomic formulas is introduced. The set of all terms is shown to be unambiguous with respect to concatenation; we say that it is a prefix set. This fact is exploited to uniquely define the subterms both of a term and of an atomic formula without resorting to a parse tree.

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The papers [1], [3], [18], [5], [6], [12], [10], [7], [8], [9], [19], [14], [13], [2], [17], [4], [21], [22], [15], and [20] provide the terminology and notation for this paper.

We follow the rules: m, n are natural numbers, m_1, n_1 are elements of \mathbb{N} , and X, x, z are sets.

Let z be a zero integer number. One can check that $|z|$ is zero.

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Let us observe that there exists a real number which is negative and integer and every integer number which is positive is also natural.

Let S be a non degenerated zero-one structure. Observe that (the carrier of S) \setminus {the one of S } is non empty.

We introduce languages-like which are extensions of zero-one structure and are systems

\langle a carrier, a zero, a one, an adicity \rangle ,

where the carrier is a set, the zero and the one are elements of the carrier, and the adicity is a function from the carrier \setminus {the one} into \mathbb{Z} .

Let S be a language-like. The functor AllSymbolsOf S is defined by:

(Def. 1) AllSymbolsOf S = the carrier of S .

The functor LettersOf S is defined as follows:

(Def. 2) LettersOf S = (the adicity of S) $^{-1}$ ({0}).

The functor OpSymbolsOf S is defined by:

(Def. 3) OpSymbolsOf S = (the adicity of S) $^{-1}$ ($\mathbb{N} \setminus \{0\}$).

The functor RelSymbolsOf S is defined by:

(Def. 4) RelSymbolsOf S = (the adicity of S) $^{-1}$ ($\mathbb{Z} \setminus \mathbb{N}$).

The functor TermSymbolsOf S is defined as follows:

(Def. 5) TermSymbolsOf S = (the adicity of S) $^{-1}$ (\mathbb{N}).

The functor LowerCompoundersOf S is defined as follows:

(Def. 6) LowerCompoundersOf S = (the adicity of S) $^{-1}$ ($\mathbb{Z} \setminus \{0\}$).

The functor TheEqSymbOf S is defined as follows:

(Def. 7) TheEqSymbOf S = the zero of S .

The functor TheNorSymbOf S is defined as follows:

(Def. 8) TheNorSymbOf S = the one of S .

The functor OwnSymbolsOf S is defined by:

(Def. 9) OwnSymbolsOf S = (the carrier of S) \setminus {the zero of S , the one of S }.

Let S be a language-like. An element of S is an element of AllSymbolsOf S .

The functor AtomicFormulaSymbolsOf S is defined by:

(Def. 10) AtomicFormulaSymbolsOf S = AllSymbolsOf S \setminus {TheNorSymbOf S }.

The functor AtomicTermsOf S is defined by:

(Def. 11) AtomicTermsOf S = (LettersOf S) 1 .

We say that S is operational if and only if:

(Def. 12) OpSymbolsOf S is non empty.

We say that S is relational if and only if:

(Def. 13) RelSymbolsOf S \setminus {TheEqSymbOf S } is non empty.

Let S be a language-like and let s be an element of S . We say that s is literal if and only if:

(Def. 14) $s \in \text{LettersOf } S$.

We say that s is low-compounding if and only if:

(Def. 15) $s \in \text{LowerCompoundersOf } S$.

We say that s is operational if and only if:

(Def. 16) $s \in \text{OpSymbolsOf } S$.

We say that s is relational if and only if:

(Def. 17) $s \in \text{RelSymbolsOf } S$.

We say that s is termal if and only if:

(Def. 18) $s \in \text{TermSymbolsOf } S$.

We say that s is own if and only if:

(Def. 19) $s \in \text{OwnSymbolsOf } S$.

We say that s is of-atomic-formula if and only if:

(Def. 20) $s \in \text{AtomicFormulaSymbolsOf } S$.

Let S be a zero-one structure and let s be an element of (the carrier of S) \setminus {the one of S }. The functor $\text{TrivialArity } s$ yields an integer number and is defined by:

(Def. 21) $\text{TrivialArity } s = \begin{cases} -2, & \text{if } s = \text{the zero of } S, \\ 0, & \text{otherwise.} \end{cases}$

Let S be a zero-one structure and let s be an element of (the carrier of S) \setminus {the one of S }. Then $\text{TrivialArity } s$ is an element of \mathbb{Z} .

Let S be a non degenerated zero-one structure. The functor $S \text{ TrivialArity}$ yielding a function from (the carrier of S) \setminus {the one of S } into \mathbb{Z} is defined by:

(Def. 22) For every element s of (the carrier of S) \setminus {the one of S } holds $(S \text{ TrivialArity})(s) = \text{TrivialArity } s$.

Let us observe that there exists a non degenerated zero-one structure which is infinite.

Let S be an infinite non degenerated zero-one structure.

Observe that $(S \text{ TrivialArity})^{-1}(\{0\})$ is infinite.

Let S be a language-like. We say that S is eligible if and only if:

(Def. 23) $\text{LettersOf } S$ is infinite and $(\text{the adicity of } S)(\text{TheEqSymbOf } S) = -2$.

One can check that there exists a language-like which is non degenerated.

One can check that there exists a non degenerated language-like which is eligible.

A language is an eligible non degenerated language-like.

We follow the rules: S, S_1, S_2 are languages and s, s_1, s_2 are elements of S .

Let S be a non empty language-like. Then $\text{AllSymbolsOf } S$ is a non empty set.

Let S be an eligible language-like. Note that $\text{LettersOf } S$ is infinite.

Let S be a language.

Then $\text{LettersOf } S$ is a non empty subset of $\text{AllSymbolsOf } S$. Note that $\text{TheEqSymbOf } S$ is relational.

Let S be a non degenerated language-like. Then $\text{AtomicFormulaSymbolsOf } S$ is a non empty subset of $\text{AllSymbolsOf } S$.

Let S be a non degenerated language-like. Then $\text{TheEqSymbOf } S$ is an element of $\text{AtomicFormulaSymbolsOf } S$.

We now state the proposition

- (1) Let S be a language. Then $\text{LettersOf } S \cap \text{OpSymbolsOf } S = \emptyset$ and $\text{TermSymbolsOf } S \cap \text{LowerCompoundersOf } S = \text{OpSymbolsOf } S$ and $\text{RelSymbolsOf } S \setminus \text{OwnSymbolsOf } S = \{\text{TheEqSymbOf } S\}$ and $\text{OwnSymbolsOf } S \subseteq \text{AtomicFormulaSymbolsOf } S$ and $\text{RelSymbolsOf } S \subseteq \text{LowerCompoundersOf } S$ and $\text{OpSymbolsOf } S \subseteq \text{TermSymbolsOf } S$ and $\text{LettersOf } S \subseteq \text{TermSymbolsOf } S \subseteq \text{OwnSymbolsOf } S$ and $\text{OpSymbolsOf } S \subseteq \text{LowerCompoundersOf } S \subseteq \text{AtomicFormulaSymbolsOf } S$.

Let S be a language. One can verify the following observations:

- * $\text{TermSymbolsOf } S$ is non empty,
- * every element of S which is own is also of-atomic-formula,
- * every element of S which is relational is also low-compounding,
- * every element of S which is operational is also termal,
- * every element of S which is literal is also termal,
- * every element of S which is termal is also own,
- * every element of S which is operational is also low-compounding,
- * every element of S which is low-compounding is also of-atomic-formula,
- * every element of S which is termal is also non relational,
- * every element of S which is literal is also non relational, and
- * every element of S which is literal is also non operational.

Let S be a language. Note that there exists an element of S which is relational and there exists an element of S which is literal. Observe that every low-compounding element of S which is termal is also operational. One can check that there exists an element of S which is of-atomic-formula.

Let s be an of-atomic-formula element of S . The functor $\text{ar } s$ yielding an element of \mathbb{Z} is defined by:

(Def. 24) $\text{ar } s = (\text{the adicity of } S)(s)$.

Let S be a language and let s be a literal element of S . Note that $\text{ar } s$ is zero. The functor $S\text{-cons}$ yielding a binary operation on $(\text{AllSymbolsOf } S)^*$ is defined as follows:

(Def. 25) $S\text{-cons} = \text{the concatenation of AllSymbolsOf } S$.

Let S be a language.

The functor S -multiCat yields a function from $((\text{AllSymbolsOf } S)^*)^*$ into $(\text{AllSymbolsOf } S)^*$ and is defined by:

(Def. 26) S -multiCat = (AllSymbolsOf S)-multiCat .

Let S be a language. The functor S -firstChar yielding a function from $(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$ into AllSymbolsOf S is defined as follows:

(Def. 27) S -firstChar = (AllSymbolsOf S)-firstChar .

Let S be a language and let X be a set. We say that X is S -prefix if and only if:

(Def. 28) X is AllSymbolsOf S -prefix.

Let S be a language. Note that every set which is S -prefix is also

AllSymbolsOf S -prefix and every set which is AllSymbolsOf S -prefix is also S -prefix. A string of S is an element of $(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$.

Let us consider S . One can check that $(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$ is non empty. Note that every string of S is non empty.

Let us note that every language is infinite. Observe that AllSymbolsOf S is infinite.

Let s be an of-atomic-formula element of S , and let S_3 be a set. The functor Compound(s, S_3) is defined by:

(Def. 29) $\text{Compound}(s, S_3) = \{\langle s \rangle \wedge S\text{-multiCat}(S_4); S_4 \text{ ranges over elements of } ((\text{AllSymbolsOf } S)^*)^* : \text{rng } S_4 \subseteq S_3 \wedge S_4 \text{ is } |ar\ s|\text{-element}\}$.

Let S be a language, let s be an of-atomic-formula element of S , and let S_3 be a set. Then Compound(s, S_3) is an element of $2^{(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}}$. The functor S -termsOfMaxDepth yields a function and is defined by the conditions (Def. 30).

(Def. 30)(i) $\text{dom}(S\text{-termsOfMaxDepth}) = \mathbb{N}$,
 (ii) $S\text{-termsOfMaxDepth}(0) = \text{AtomicTermsOf } S$, and
 (iii) for every natural number n holds $S\text{-termsOfMaxDepth}(n + 1) = \bigcup\{\text{Compound}(s, S\text{-termsOfMaxDepth}(n)); s \text{ ranges over of-atomic-formula elements of } S : s \text{ is operational}\} \cup S\text{-termsOfMaxDepth}(n)$.

Let us consider S . Then AtomicTermsOf S is a subset of $(\text{AllSymbolsOf } S)^*$.

Let S be a language. The functor AllTermsOf S is defined as follows:

(Def. 31) AllTermsOf $S = \bigcup \text{rng}(S\text{-termsOfMaxDepth})$.

One can prove the following proposition

(2) $S\text{-termsOfMaxDepth}(m_1) \subseteq \text{AllTermsOf } S$.

Let S be a language and let w be a string of S . We say that w is ternal if and only if:

(Def. 32) $w \in \text{AllTermsOf } S$.

Let m be a natural number, let S be a language, and let w be a string of S . We say that w is m -ternal if and only if:

(Def. 33) $w \in S\text{-termsOfMaxDepth}(m)$.

Let m be a natural number and let S be a language. Note that every string of S which is m -terminal is also terminal.

Let us consider S . Then $S\text{-termsOfMaxDepth}$ is a function from \mathbb{N} into $2^{(\text{AllSymbolsOf } S)^*}$. Then $\text{AllTermsOf } S$ is a non empty subset of $(\text{AllSymbolsOf } S)^*$. Note that $\text{AllTermsOf } S$ is non empty.

Let us consider m . One can verify that $S\text{-termsOfMaxDepth}(m)$ is non empty. Observe that every element of $S\text{-termsOfMaxDepth}(m)$ is non empty. Observe that every element of $\text{AllTermsOf } S$ is non empty.

Let m be a natural number and let S be a language. Note that there exists a string of S which is m -terminal. Observe that every string of S which is 0-terminal is also 1-element.

Let S be a language and let w be a 0-terminal string of S . Observe that $S\text{-firstChar}(w)$ is literal.

Let us consider S and let w be a terminal string of S . Note that $S\text{-firstChar}(w)$ is terminal.

Let us consider S and let t be a terminal string of S . The functor $\text{ar } t$ yielding an element of \mathbb{Z} is defined as follows:

(Def. 34) $\text{ar } t = \text{ar } S\text{-firstChar}(t)$.

Next we state the proposition

(3) For every $m_1 + 1$ -terminal string w of S there exists an element T of $S\text{-termsOfMaxDepth}(m_1)^*$ such that T is $|\text{ar } S\text{-firstChar}(w)|$ -element and $w = \langle S\text{-firstChar}(w) \rangle \wedge S\text{-multiCat}(T)$.

Let us consider S, m . Note that $S\text{-termsOfMaxDepth}(m)$ is S -prefix.

Let us consider S and let V be an element of $(\text{AllTermsOf } S)^*$. Observe that $S\text{-multiCat}(V)$ is relation-like.

Let us consider S and let V be an element of $(\text{AllTermsOf } S)^*$. One can verify that $S\text{-multiCat}(V)$ is function-like.

Let us consider S and let p_1 be a string of S . We say that p_1 is 0-w.f.f. if and only if:

(Def. 35) There exists a relational element s of S and there exists an $|\text{ar } s|$ -element V of $(\text{AllTermsOf } S)^*$ such that $p_1 = \langle s \rangle \wedge S\text{-multiCat}(V)$.

Let us consider S . Note that there exists a string of S which is 0-w.f.f..

Let p_1 be a 0-w.f.f. string of S . Observe that $S\text{-firstChar}(p_1)$ is relational. The functor $\text{AtomicFormulasOf } S$ is defined as follows:

(Def. 36) $\text{AtomicFormulasOf } S = \{p_1; p_1 \text{ ranges over strings of } S: p_1 \text{ is 0-w.f.f.}\}$.

Let us consider S . Then $\text{AtomicFormulasOf } S$ is a subset of $(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$. Note that $\text{AtomicFormulasOf } S$ is non empty. Observe that every element of $\text{AtomicFormulasOf } S$ is 0-w.f.f.. Observe that $\text{AllTermsOf } S$ is S -prefix.

Let us consider S and let t be a termal string of S . The functor $\text{SubTerms } t$ yields an element of $(\text{AllTermsOf } S)^*$ and is defined by:

(Def. 37) $\text{SubTerms } t$ is $|\text{ar } S\text{-firstChar}(t)|$ -element and $t = \langle S\text{-firstChar}(t) \rangle \hat{\cap} S\text{-multiCat}(\text{SubTerms } t)$.

Let us consider S and let t be a termal string of S . One can verify that $\text{SubTerms } t$ is $|\text{ar } t|$ -element.

Let t_0 be a 0-termal string of S . Note that $\text{SubTerms } t_0$ is empty.

Let us consider m_1, S and let t be an $m_1 + 1$ -termal string of S . One can verify that $\text{SubTerms } t$ is $S\text{-termsOfMaxDepth}(m_1)$ -valued.

Let us consider S and let p_1 be a 0-w.f.f. string of S . The functor $\text{SubTerms } p_1$ yields an $|\text{ar } S\text{-firstChar}(p_1)|$ -element element of $(\text{AllTermsOf } S)^*$ and is defined as follows:

(Def. 38) $p_1 = \langle S\text{-firstChar}(p_1) \rangle \hat{\cap} S\text{-multiCat}(\text{SubTerms } p_1)$.

Let us consider S and let p_1 be a 0-w.f.f. string of S . Note that $\text{SubTerms } p_1$ is $|\text{ar } S\text{-firstChar}(p_1)|$ -element.

Then $\text{AllTermsOf } S$ is an element of $2^{(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}}$. Note that every element of $\text{AllTermsOf } S$ is termal. The functor $S\text{-subTerms}$ yielding a function from $\text{AllTermsOf } S$ into $(\text{AllTermsOf } S)^*$ is defined by:

(Def. 39) For every element t of $\text{AllTermsOf } S$ holds $S\text{-subTerms}(t) = \text{SubTerms } t$.

We now state several propositions:

- (4) $S\text{-termsOfMaxDepth}(m) \subseteq S\text{-termsOfMaxDepth}(m + n)$.
- (5) If $x \in \text{AllTermsOf } S$, then there exists n_1 such that $x \in S\text{-termsOfMaxDepth}(n_1)$.
- (6) $\text{AllTermsOf } S \subseteq (\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$.
- (7) $\text{AllTermsOf } S$ is S -prefix.
- (8) If $x \in \text{AllTermsOf } S$, then x is a string of S .
- (9) $\text{AtomicFormulaSymbolsOf } S \setminus \text{OwnSymbolsOf } S = \{\text{TheEqSymbOf } S\}$.
- (10) $\text{TermSymbolsOf } S \setminus \text{LettersOf } S = \text{OpSymbolsOf } S$.
- (11) $\text{AtomicFormulaSymbolsOf } S \setminus \text{RelSymbolsOf } S = \text{TermSymbolsOf } S$.

Let us consider S . Observe that every of-atomic-formula element of S which is non relational is also termal.

Then $\text{OwnSymbolsOf } S$ is a subset of $\text{AllSymbolsOf } S$. Observe that every termal element of S which is non literal is also operational.

Next we state three propositions:

- (12) x is a string of S iff x is a non empty element of $(\text{AllSymbolsOf } S)^*$.
- (13) x is a string of S iff x is a non empty finite sequence of elements of $\text{AllSymbolsOf } S$.
- (14) $S\text{-termsOfMaxDepth}$ is a function from \mathbb{N} into $2^{(\text{AllSymbolsOf } S)^*}$.

Let us consider S . Note that every element of $\text{LettersOf } S$ is literal. One can check that $\text{TheNorSymbOf } S$ is non low-compounding.

Observe that $\text{TheNorSymbOf } S$ is non own.

Next we state the proposition

- (15) If $s \neq \text{TheNorSymbOf } S$ and $s \neq \text{TheEqSymbOf } S$, then $s \in \text{OwnSymbolsOf } S$.

For simplicity, we use the following convention: l, l_1, l_2 denote literal elements of S , a denotes an of-atomic-formula element of S , r denotes a relational element of S , w, w_1 denote strings of S , and t_2 denotes an element of $\text{AllTermsOf } S$.

Let us consider S, t . The functor $\text{Depth } t$ yielding a natural number is defined by:

- (Def. 40) t is $\text{Depth } t$ -termal and for every n such that t is n -termal holds $\text{Depth } t \leq n$.

Let us consider S , let m_0 be a zero number, and let t be an m_0 -termal string of S . Note that $\text{Depth } t$ is zero.

Let us consider S and let s be a low-compounding element of S . Note that $\text{ar } s$ is non zero.

Let us consider S and let s be a termal element of S . Observe that $\text{ar } s$ is non negative and extended real.

Let us consider S and let s be a relational element of S . Note that $\text{ar } s$ is negative and extended real.

Next we state the proposition

- (16) If t is non 0-termal, then $S\text{-firstChar}(t)$ is operational and $\text{SubTerms } t \neq \emptyset$.

Let us consider S . Observe that $S\text{-multiCat}$ is finite sequence-yielding.

Let us consider S and let W be a non empty $\text{AllSymbolsOf } S^* \setminus \{\emptyset\}$ -valued finite sequence. One can verify that $S\text{-multiCat}(W)$ is non empty.

Let us consider S, l . Note that $\langle l \rangle$ is 0-termal.

Let us consider S, m, n . One can check that every string of S which is $m + 0 \cdot n$ -termal is also $m + n$ -termal.

Let us consider S . One can check that every own element of S which is non low-compounding is also literal.

Let us consider S, t . One can check that $\text{SubTerms } t$ is $\text{rng } t^*$ -valued.

Let p_0 be a 0-w.f.f. string of S . Observe that $\text{SubTerms } p_0$ is $\text{rng } p_0^*$ -valued. Then $S\text{-termsOfMaxDepth}$ is a function from \mathbb{N} into $2^{(\text{AllSymbolsOf } S^* \setminus \{\emptyset\})}$.

Let us consider S, m_1 . Observe that $S\text{-termsOfMaxDepth}(m_1)$ has non empty elements.

Let us consider S, m and let t be a termal string of S . One can verify that $t \text{ null } m$ is $\text{Depth } t + m$ -termal. One can check that every string of S which is termal is also $\text{TermSymbolsOf } S$ -valued. Observe that $\text{AllTermsOf } S \setminus (\text{TermSymbolsOf } S)^*$ is empty.

Let p_0 be a 0-w.f.f. string of S . Observe that $\text{SubTerms } p_0$ is $\text{TermSymbolsOf } S^*$ -valued. One can verify that every string of S which is 0-w.f.f. is also

$\text{AtomicFormulaSymbolsOf } S$ -valued. One can check that $\text{OwnSymbolsOf } S$ is non empty.

In the sequel p_0 is a 0-w.f.f. string of S .

The following proposition is true

- (17) If $S\text{-firstChar}(p_0) \neq \text{TheEqSymbOf } S$, then p_0 is $\text{OwnSymbolsOf } S$ -valued.

Let us observe that there exists a language-like which is strict and non empty.

Let S_1, S_2 be languages-like. We say that S_2 is S_1 -extending if and only if:

- (Def. 41) The adicity of $S_1 \subseteq$ the adicity of S_2 and $\text{TheEqSymbOf } S_1 = \text{TheEqSymbOf } S_2$ and $\text{TheNorSymbOf } S_1 = \text{TheNorSymbOf } S_2$.

Let us consider S . One can verify that S null is S -extending. Observe that there exists a language which is S -extending.

Let us consider S_1 and let S_2 be an S_1 -extending language. Observe that $\text{OwnSymbolsOf } S_1 \setminus \text{OwnSymbolsOf } S_2$ is empty.

Let f be a \mathbb{Z} -valued function and let L be a non empty language-like. The functor $L \text{ extendWith } f$ yields a strict non empty language-like and is defined by the conditions (Def. 42).

- (Def. 42)(i) The adicity of $L \text{ extendWith } f = f \upharpoonright (\text{dom } f \setminus \{\text{the one of } L\}) + \text{the adicity of } L$,
- (ii) the zero of $L \text{ extendWith } f = \text{the zero of } L$, and
- (iii) the one of $L \text{ extendWith } f = \text{the one of } L$.

Let S be a non empty language-like and let f be a \mathbb{Z} -valued function. Note that $S \text{ extendWith } f$ is S -extending.

Let S be a non degenerated language-like. Observe that every language-like which is S -extending is also non degenerated.

Let S be an eligible language-like. One can check that every language-like which is S -extending is also eligible.

Let E be an empty binary relation and let us consider X . Note that $X \upharpoonright E$ is empty.

Let us consider X and let m be an integer number. Note that $X \mapsto m$ is \mathbb{Z} -valued.

Let us consider S and let X be a functional set.

The functor $S \text{ addLettersNotIn } X$ yields an S -extending language and is defined as follows:

- (Def. 43) $S \text{ addLettersNotIn } X =$
 $S \text{ extendWith}((\text{AllSymbolsOf } S \cup \text{SymbolsOf } X)\text{-freeCountableSet} \mapsto$
 $0 \text{ qua } \mathbb{Z}\text{-valued function}).$

Let us consider S_1 and let X be a functional set.

Note that $\text{LettersOf}(S_1 \text{ addLettersNotIn } X) \setminus \text{SymbolsOf } X$ is infinite.

Let us note that there exists a language which is countable.

Let S be a countable language. Observe that $\text{AllSymbolsOf } S$ is countable.

One can verify that $(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$ is countable.

Let L be a non empty language-like and let f be a \mathbb{Z} -valued function. Note that $\text{AllSymbolsOf}(L \text{ extendWith } f) \div (\text{dom } f \cup \text{AllSymbolsOf } L)$ is empty.

Let S be a countable language and let X be a functional set. One can check that $S \text{ addLettersNotIn } X$ is countable.

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