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# Isomorphisms of Direct Products of Finite Commutative Groups[1]

Hiroyuki Okazaki
Shinshu University
Nagano, Japan

Hiroshi Yamazaki
Shinshu University
Nagano, Japan

Yasunari Shidama
Shinshu University
Nagano, Japan

**Summary.** We have been working on the formalization of groups. In [1], we encoded some theorems concerning the product of cyclic groups. In this article, we present the generalized formalization of [1]. First, we show that every finite commutative group which order is composite number is isomorphic to a direct product of finite commutative groups which orders are relatively prime. Next, we describe finite direct products of finite commutative groups.

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The notation and terminology used in this paper have been introduced in the following articles: [2], [3], [19], [7], [13], [20], [8], [9], [10], [23], [24], [25], [26], [27], [14], [22], [17], [4], [5], [15], [16], [6], [11], [21], [18], [29], [28], and [12].

## 1. Preliminaries

Now we state the propositions:

(1) Let us consider sets $A$, $B$, $A_1$, $B_1$. Suppose

(i) $A$ misses $B$, and

(ii) $A_1 \subseteq A$, and

(iii) $B_1 \subseteq B$, and

(iv) $A_1 \cup B_1 = A \cup B$.

Then

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(v) $A_1 = A$, and

(vi) $B_1 = B$.

PROOF: $A \subseteq A_1$. $B \subseteq B_1$. $\square$

(2) Let us consider non empty finite sets $H$, $K$. Then $\overline{\overline{\prod\langle H, K\rangle}} = \overline{\overline{H}} \cdot \overline{\overline{K}}$.

Let us consider bags $p_2$, $p_1$, $f$ of Prime and a natural number $q$. Now we state the propositions:

(3) If support $p_2$ misses support $p_1$ and $f = p_2 + p_1$ and $q \in$ support $p_2$, then $p_2(q) = f(q)$.

(4) If support $p_2$ misses support $p_1$ and $f = p_2 + p_1$ and $q \in$ support $p_1$, then $p_1(q) = f(q)$.

Now we state the propositions:

(5) Let us consider a non zero natural number $h$ and a prime number $q$. If $q$ and $h$ are not relatively prime, then $q \mid h$.

(6) Let us consider non zero natural numbers $h$, $s$. Suppose a prime number $q$. Suppose $q \in$ support PrimeFactorization$(s)$. Then $q$ and $h$ are not relatively prime. Then support PrimeFactorization$(s) \subseteq$ support PrimeFactorization$(h)$. The theorem is a consequence of (5).

(7) Let us consider non zero natural numbers $h$, $k$, $s$, $t$. Suppose

(i) $h$ and $k$ are relatively prime, and

(ii) $s \cdot t = h \cdot k$, and

(iii) for every prime number $q$ such that $q \in$ support PrimeFactorization$(s)$ holds $q$ and $h$ are not relatively prime, and

(iv) for every prime number $q$ such that $q \in$ support PrimeFactorization$(t)$ holds $q$ and $k$ are not relatively prime.

Then

(v) $s = h$, and

(vi) $t = k$.

The theorem is a consequence of (6), (1), (3), and (4). PROOF: Set $p_2 =$ PrimeFactorization$(s)$. Set $p_1 =$ PrimeFactorization$(t)$. For every natural number $p$ such that $p \in$ support PFExp$(h)$ holds $p_2(p) = p^{p\text{-count}(h)}$. For every natural number $p$ such that $p \in$ support PFExp$(k)$ holds $p_1(p) = p^{p\text{-count}(k)}$. $\square$

Let $G$ be a non empty multiplicative magma, $I$ be a finite set, and $b$ be a (the carrier of $G$)-valued total $I$-defined function. The functor $\prod b$ yielding an element of $G$ is defined by

(Def. 1) There exists a finite sequence $f$ of elements of $G$ such that

(i) $it = \prod f$, and

(ii) $f = b \cdot \mathrm{CFS}(I)$.

Now we state the propositions:

(8) Let us consider a commutative group $G$, non empty finite sets $A$, $B$, a (the carrier of $G$)-valued total $A$-defined function $F_3$, a (the carrier of $G$)-valued total $B$-defined function $F_2$, and a (the carrier of $G$)-valued total $A \cup B$-defined function $F_1$. Suppose

(i) $A$ misses $B$, and

(ii) $F_1 = F_3 {+\cdot} F_2$.

Then $\prod F_1 = \prod F_3 \cdot \prod F_2$.

(9) Let us consider a non empty multiplicative magma $G$, a set $q$, an element $z$ of $G$, and a (the carrier of $G$)-valued total $\{q\}$-defined function $f$. If $f = q \,\dot{\longmapsto}\, z$, then $\prod f = z$.

## 2. Direct Product of Finite Commutative Groups

Now we state the propositions:

(10) Let us consider non empty multiplicative magmas $X, Y$. Then the carrier of $\prod\langle X, Y\rangle = \prod\langle$the carrier of $X$, the carrier of $Y\rangle$. Proof: Set $CarrX =$ the carrier of $X$. Set $CarrY =$ the carrier of $Y$. For every element $a$ such that $a \in \mathrm{dom}$ the support of $\langle X, Y\rangle$ holds (the support of $\langle X, Y\rangle)(a) = \langle$the carrier of $X$, the carrier of $Y\rangle(a)$. $\square$

(11) Let us consider a group $G$ and normal subgroups $A$, $B$ of $G$. Suppose (the carrier of $A$) $\cap$ (the carrier of $B$) $= \{\mathbf{1}_G\}$. Let us consider elements $a$, $b$ of $G$. If $a \in A$ and $b \in B$, then $a \cdot b = b \cdot a$.

(12) Let us consider a group $G$ and normal subgroups $A$, $B$ of $G$. Suppose

(i) for every element $x$ of $G$, there exist elements $a$, $b$ of $G$ such that $a \in A$ and $b \in B$ and $x = a \cdot b$, and

(ii) (the carrier of $A$) $\cap$ (the carrier of $B$) $= \{\mathbf{1}_G\}$.

Then there exists a homomorphism $h$ from $\prod\langle A, B\rangle$ to $G$ such that

(iii) $h$ is bijective, and

(iv) for every elements $a$, $b$ of $G$ such that $a \in A$ and $b \in B$ holds $h(\langle a, b\rangle) = a \cdot b$.

The theorem is a consequence of (11). Proof: Define $\mathcal{P}[\text{set}, \text{set}] \equiv$ there exists an element $x$ of $G$ and there exists an element $y$ of $G$ such that $x \in A$ and $y \in B$ and $\$_1 = \langle x, y\rangle$ and $\$_2 = x \cdot y$. For every element $z$ of $\prod\langle A, B\rangle$, there exists an element $w$ of $G$ such that $\mathcal{P}[z, w]$. Consider $h$ being a function from $\prod\langle A, B\rangle$ into $G$ such that for every element $z$ of $\prod\langle A, B\rangle$, $\mathcal{P}[z, h(z)]$. For every elements $a$, $b$ of $G$ such that $a \in A$ and $b \in B$ holds

$h(\langle a, b\rangle) = a \cdot b$. For every elements $z$, $w$ of $\prod\langle A, B\rangle$, $h(z \cdot w) = h(z) \cdot h(w)$. $\square$

Let us consider a finite commutative group $G$, a natural number $m$, and a subset $A$ of $G$. Now we state the propositions:

(13) Suppose $A = \{x$ where $x$ is an element of $G : x^m = \mathbf{1}_G\}$. Then

(i) $A \neq \emptyset$, and

(ii) for every elements $g_1$, $g_2$ of $G$ such that $g_1$, $g_2 \in A$ holds $g_1 \cdot g_2 \in A$, and

(iii) for every element $g$ of $G$ such that $g \in A$ holds $g^{-1} \in A$.

(14) Suppose $A = \{x$ where $x$ is an element of $G : x^m = \mathbf{1}_G\}$. Then there exists a strict finite subgroup $H$ of $G$ such that

(i) the carrier of $H = A$, and

(ii) $H$ is commutative and normal.

Now we state the propositions:

(15) Let us consider a finite commutative group $G$, a natural number $m$, and a finite subgroup $H$ of $G$. Suppose the carrier of $H = \{x$ where $x$ is an element of $G : x^m = \mathbf{1}_G\}$. Let us consider a prime number $q$. Suppose $q \in$ support PrimeFactorization($\overline{\overline{H}}$). Then $q$ and $m$ are not relatively prime.

(16) Let us consider a finite commutative group $G$ and natural numbers $h$, $k$. Suppose

(i) $\overline{\overline{G}} = h \cdot k$, and

(ii) $h$ and $k$ are relatively prime.

Then there exist strict finite subgroups $H$, $K$ of $G$ such that

(iii) the carrier of $H = \{x$ where $x$ is an element of $G : x^h = \mathbf{1}_G\}$, and

(iv) the carrier of $K = \{x$ where $x$ is an element of $G : x^k = \mathbf{1}_G\}$, and

(v) $H$ is normal, and

(vi) $K$ is normal, and

(vii) for every element $x$ of $G$, there exist elements $a$, $b$ of $G$ such that $a \in H$ and $b \in K$ and $x = a \cdot b$, and

(viii) (the carrier of $H$) $\cap$ (the carrier of $K$) $= \{\mathbf{1}_G\}$.

The theorem is a consequence of (14). PROOF: Set $A = \{x$ where $x$ is an element of $G : x^h = \mathbf{1}_G\}$. Set $B = \{x$ where $x$ is an element of $G : x^k = \mathbf{1}_G\}$. $A \subseteq$ the carrier of $G$. $B \subseteq$ the carrier of $G$. Consider $H$ being a strict finite subgroup of $G$ such that the carrier of $H = A$ and $H$ is commutative and $H$ is normal. Consider $K$ being a strict finite subgroup of $G$ such that the carrier of $K = B$ and $K$ is commutative and $K$ is normal. Consider $a$, $b$ being integers such that $a \cdot h + b \cdot k = 1$. (The carrier of $H$) $\cap$ (the carrier of $K$) $\subseteq \{\mathbf{1}_G\}$. For every element $x$ of $G$, there exist elements $s$, $t$ of $G$ such that $s \in H$ and $t \in K$ and $x = s \cdot t$. $\square$

(17) Let us consider finite groups $H$, $K$. Then $\overline{\overline{\prod\langle H, K\rangle}} = \overline{\overline{H}} \cdot \overline{\overline{K}}$. The theorem is a consequence of (10) and (2).

(18) Let us consider a finite commutative group $G$ and non zero natural numbers $h$, $k$. Suppose

- (i) $\overline{\overline{G}} = h \cdot k$, and
- (ii) $h$ and $k$ are relatively prime.

Then there exist strict finite subgroups $H$, $K$ of $G$ such that

- (iii) $\overline{\overline{H}} = h$, and
- (iv) $\overline{\overline{K}} = k$, and
- (v) (the carrier of $H$) $\cap$ (the carrier of $K$) $= \{\mathbf{1}_G\}$, and
- (vi) there exists a homomorphism $F$ from $\prod\langle H, K\rangle$ to $G$ such that $F$ is bijective and for every elements $a$, $b$ of $G$ such that $a \in H$ and $b \in K$ holds $F(\langle a, b\rangle) = a \cdot b$.

The theorem is a consequence of (16), (12), (17), (15), and (7).

## 3. Finite Direct Products of Finite Commutative Groups

Let us consider a group $G$, a set $q$, an associative group-like multiplicative magma family $F$ of $\{q\}$, and a function $f$ from $G$ into $\prod F$. Now we state the propositions:

(19) If $F = q \dot{\longmapsto} G$ and for every element $x$ of $G$, $f(x) = q \dot{\longmapsto} x$, then $f$ is a homomorphism from $G$ to $\prod F$.

(20) If $F = q \dot{\longmapsto} G$ and for every element $x$ of $G$, $f(x) = q \dot{\longmapsto} x$, then $f$ is bijective.

Now we state the propositions:

(21) Let us consider a set $q$, an associative group-like multiplicative magma family $F$ of $\{q\}$, and a group $G$. Suppose $F = q \dot{\longmapsto} G$. Then there exists a homomorphism $I$ from $G$ to $\prod F$ such that

- (i) $I$ is bijective, and
- (ii) for every element $x$ of $G$, $I(x) = q \dot{\longmapsto} x$.

The theorem is a consequence of (19) and (20). Proof: Define $\mathcal{P}[\text{set}, \text{set}] \equiv$ $\$_2 = q \dot{\longmapsto} \$_1$. For every element $z$ of $G$, there exists an element $w$ of $\prod F$ such that $\mathcal{P}[z, w]$. Consider $I$ being a function from $G$ into $\prod F$ such that for every element $x$ of $G$, $\mathcal{P}[x, I(x)]$. $\square$

(22) Let us consider non empty finite sets $I_0$, $I$, an associative group-like multiplicative magma family $F_0$ of $I_0$, an associative group-like multiplicative magma family $F$ of $I$, groups $H$, $K$, an element $q$ of $I$, an element $k$ of $K$, and a function $g$. Suppose

(i) $g \in$ the carrier of $\prod F_0$, and

(ii) $q \notin I_0$, and

(iii) $I = I_0 \cup \{q\}$, and

(iv) $F = F_0 +\cdot (q \dot{\longmapsto} K)$.

Then $g +\cdot (q \dot{\longmapsto} k) \in$ the carrier of $\prod F$. PROOF: Set $HK = \langle H, K \rangle$. Set $w = g +\cdot (q \dot{\longmapsto} k)$. For every element $x$ such that $x \in \operatorname{dom}$ the support of $F$ holds $w(x) \in$ (the support of $F)(x)$. $\square$

Let us consider non empty finite sets $I_0$, $I$, an associative group-like multiplicative magma family $F_0$ of $I_0$, an associative group-like multiplicative magma family $F$ of $I$, groups $H$, $K$, an element $q$ of $I$, a function $G_0$ from $H$ into $\prod F_0$, and a function $G$ from $\prod \langle H, K \rangle$ into $\prod F$. Now we state the propositions:

(23) Suppose $G_0$ is a homomorphism from $H$ to $\prod F_0$ and $G_0$ is bijective and $q \notin I_0$ and $I = I_0 \cup \{q\}$ and $F = F_0 +\cdot (q \dot{\longmapsto} K)$. Then suppose for every element $h$ of $H$ and for every element $k$ of $K$, there exists a function $g$ such that $g = G_0(h)$ and $G(\langle h, k \rangle) = g +\cdot (q \dot{\longmapsto} k)$. Then $G$ is a homomorphism from $\prod \langle H, K \rangle$ to $\prod F$.

(24) Suppose $G_0$ is a homomorphism from $H$ to $\prod F_0$ and $G_0$ is bijective and $q \notin I_0$ and $I = I_0 \cup \{q\}$ and $F = F_0 +\cdot (q \dot{\longmapsto} K)$. Then suppose for every element $h$ of $H$ and for every element $k$ of $K$, there exists a function $g$ such that $g = G_0(h)$ and $G(\langle h, k \rangle) = g +\cdot (q \dot{\longmapsto} k)$. Then $G$ is bijective.

Now we state the propositions:

(25) Let us consider a set $q$, a multiplicative magma family $F$ of $\{q\}$, and a non empty multiplicative magma $G$. Suppose $F = q \dot{\longmapsto} G$. Let us consider a (the carrier of $G$)-valued total $\{q\}$-defined function $y$. Then

(i) $y \in$ the carrier of $\prod F$, and

(ii) $y(q) \in$ the carrier of $G$, and

(iii) $y = q \dot{\longmapsto} y(q)$.

(26) Let us consider a set $q$, an associative group-like multiplicative magma family $F$ of $\{q\}$, and a group $G$. Suppose $F = q \dot{\longmapsto} G$. Then there exists a homomorphism $H_0$ from $\prod F$ to $G$ such that

(i) $H_0$ is bijective, and

(ii) for every (the carrier of $G$)-valued total $\{q\}$-defined function $x$, $H_0(x) = \prod x$.

The theorem is a consequence of (21), (25), and (9). Proof: Consider $I$ being a homomorphism from $G$ to $\prod F$ such that $I$ is bijective and for every element $x$ of $G$, $I(x) = q \dot{\longmapsto} x$. Set $H_0 = I^{-1}$. For every (the carrier of $G$)-valued total $\{q\}$-defined function $y$, $H_0(y) = \prod y$. □

(27) Let us consider non empty finite sets $I_0$, $I$, an associative group-like multiplicative magma family $F_0$ of $I_0$, an associative group-like multiplicative magma family $F$ of $I$, groups $H$, $K$, an element $q$ of $I$, and a homomorphism $G_0$ from $H$ to $\prod F_0$. Suppose

- (i) $q \notin I_0$, and
- (ii) $I = I_0 \cup \{q\}$, and
- (iii) $F = F_0 {+\cdot} (q \longmapsto K)$, and
- (iv) $G_0$ is bijective.

Then there exists a homomorphism $G$ from $\prod\langle H, K\rangle$ to $\prod F$ such that

- (v) $G$ is bijective, and
- (vi) for every element $h$ of $H$ and for every element $k$ of $K$, there exists a function $g$ such that $g = G_0(h)$ and $G(\langle h, k\rangle) = g {+\cdot} (q \dot{\longmapsto} k)$.

The theorem is a consequence of (22), (23), and (24). Proof: Set $HK = \langle H, K\rangle$. Define $\mathcal{P}[\text{set}, \text{set}] \equiv$ there exists an element $h$ of $H$ and there exists an element $k$ of $K$ and there exists a function $g$ such that $\$_1 = \langle h, k\rangle$ and $g = G_0(h)$ and $\$_2 = g {+\cdot} (q \dot{\longmapsto} k)$. For every element $z$ of $\prod\langle H, K\rangle$, there exists an element $w$ of the carrier of $\prod F$ such that $\mathcal{P}[z, w]$. Consider $G$ being a function from $\prod\langle H, K\rangle$ into $\prod F$ such that for every element $x$ of $\prod\langle H, K\rangle$, $\mathcal{P}[x, G(x)]$. For every element $h$ of $H$ and for every element $k$ of $K$, there exists a function $g$ such that $g = G_0(h)$ and $G(\langle h, k\rangle) = g {+\cdot} (q \dot{\longmapsto} k)$. □

(28) Let us consider non empty finite sets $I_0$, $I$, an associative group-like multiplicative magma family $F_0$ of $I_0$, an associative group-like multiplicative magma family $F$ of $I$, groups $H$, $K$, an element $q$ of $I$, and a homomorphism $G_0$ from $\prod F_0$ to $H$. Suppose

- (i) $q \notin I_0$, and
- (ii) $I = I_0 \cup \{q\}$, and
- (iii) $F = F_0 {+\cdot} (q \longmapsto K)$, and
- (iv) $G_0$ is bijective.

Then there exists a homomorphism $G$ from $\prod F$ to $\prod\langle H, K\rangle$ such that

- (v) $G$ is bijective, and
- (vi) for every function $x_0$ and for every element $k$ of $K$ and for every element $h$ of $H$ such that $h = G_0(x_0)$ and $x_0 \in \prod F_0$ holds $G(x_0 {+\cdot} (q \dot{\longmapsto} k)) = \langle h, k\rangle$.

The theorem is a consequence of (27). PROOF: Set $L0 = {G_0}^{-1}$. Consider $L$ being a homomorphism from $\prod\langle H, K\rangle$ to $\prod F$ such that $L$ is bijective and for every element $h$ of $H$ and for every element $k$ of $K$, there exists a function $g$ such that $g = L0(h)$ and $L(\langle h, k\rangle) = g{+\cdot}(q \dot{\longmapsto} k)$. Set $G = L^{-1}$. For every function $x_0$ and for every element $k$ of $K$ and for every element $h$ of $H$ such that $h = G_0(x_0)$ and $x_0 \in \prod F_0$ holds $G(x_0{+\cdot}(q \dot{\longmapsto} k)) = \langle h, k\rangle$. $\square$

(29) Let us consider a non empty finite set $I$, an associative group-like multiplicative magma family $F$ of $I$, and a total $I$-defined function $x$. Suppose an element $p$ of $I$. Then $x(p) \in F(p)$. Then $x \in$ the carrier of $\prod F$.

(30) Let us consider non empty finite sets $I_0$, $I$, an associative group-like multiplicative magma family $F_0$ of $I_0$, an associative group-like multiplicative magma family $F$ of $I$, a group $K$, an element $q$ of $I$, and an element $x$ of $\prod F$. Suppose

- (i) $q \notin I_0$, and
- (ii) $I = I_0 \cup \{q\}$, and
- (iii) $F = F_0{+\cdot}(q \dot{\longmapsto} K)$.

Then there exists a total $I_0$-defined function $x_0$ and there exists an element $k$ of $K$ such that $x_0 \in \prod F_0$ and $x = x_0{+\cdot}(q \dot{\longmapsto} k)$ and for every element $p$ of $I_0$, $x_0(p) \in F_0(p)$. PROOF: Reconsider $y = x$ as a total $I$-defined function. Reconsider $k = y(q)$ as an element of $K$. Reconsider $y0 = y \restriction I_0$ as an $I_0$-defined function. For every element $i$ of $I_0$, $y0(i) \in$ (the support of $F_0$)$(i)$ and $y0(i) \in F_0(i)$. $\square$

(31) Let us consider a group $G$, a subgroup $H$ of $G$, a finite sequence $f$ of elements of $G$, and a finite sequence $g$ of elements of $H$. If $f = g$, then $\prod f = \prod g$. PROOF: Define $\mathcal{P}$[natural number] $\equiv$ for every finite sequence $f$ of elements of $G$ for every finite sequence $g$ of elements of $H$ such that $\$_1 = \operatorname{len} f$ and $f = g$ holds $\prod f = \prod g$. $\mathcal{P}[0]$. For every natural number $k$ such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. $\square$

(32) Let us consider a non empty finite set $I$, a group $G$, a subgroup $H$ of $G$, a (the carrier of $G$)-valued total $I$-defined function $x$, and a (the carrier of $H$)-valued total $I$-defined function $x_0$. If $x = x_0$, then $\prod x = \prod x_0$. The theorem is a consequence of (31).

(33) Let us consider a commutative group $G$, non empty finite sets $I_0$, $I$, an element $q$ of $I$, a (the carrier of $G$)-valued total $I$-defined function $x$, a (the carrier of $G$)-valued total $I_0$-defined function $x_0$, and an element $k$ of $G$. Suppose

- (i) $q \notin I_0$, and
- (ii) $I = I_0 \cup \{q\}$, and

(iii) $x = x_0 {+}{\cdot} (q \mapsto\!\cdot\!\!\longrightarrow k)$.

Then $\prod x = \prod x_0 \cdot k$. The theorem is a consequence of (8) and (9). Proof: Reconsider $y = q \mapsto\!\cdot\!\!\longrightarrow k$ as a (the carrier of $G$)-valued total $\{q\}$-defined function. $I_0$ misses $\{q\}$. $\square$

Let us consider a finite commutative group $G$. Now we state the propositions:

(34) Suppose $\overline{\overline{G}} > 1$. Then there exists a non empty finite set $I$ and there exists an associative group-like commutative multiplicative magma family $F$ of $I$ and there exists a homomorphism $H_0$ from $\prod F$ to $G$ such that $I =$ support PrimeFactorization($\overline{\overline{G}}$) and for every element $p$ of $I$, $F(p)$ is a subgroup of $G$ and $\overline{\overline{F(p)}} =$ (PrimeFactorization($\overline{\overline{G}}$))($p$) and for every elements $p$, $q$ of $I$ such that $p \neq q$ holds (the carrier of $F(p)) \cap$ (the carrier of $F(q)) = \{\mathbf{1}_G\}$ and $H_0$ is bijective and for every (the carrier of $G$)-valued total $I$-defined function $x$ such that for every element $p$ of $I$, $x(p) \in F(p)$ holds $x \in \prod F$ and $H_0(x) = \prod x$.

(35) Suppose $\overline{\overline{G}} > 1$. Then there exists a non empty finite set $I$ and there exists an associative group-like commutative multiplicative magma family $F$ of $I$ such that $I =$ support PrimeFactorization($\overline{\overline{G}}$) and for every element $p$ of $I$, $F(p)$ is a subgroup of $G$ and $\overline{\overline{F(p)}} =$ (PrimeFactorization($\overline{\overline{G}}$))($p$) and for every elements $p$, $q$ of $I$ such that $p \neq q$ holds (the carrier of $F(p)) \cap$ (the carrier of $F(q)) = \{\mathbf{1}_G\}$ and for every element $y$ of $G$, there exists a (the carrier of $G$)-valued total $I$-defined function $x$ such that for every element $p$ of $I$, $x(p) \in F(p)$ and $y = \prod x$ and for every (the carrier of $G$)-valued total $I$-defined functions $x_1$, $x_2$ such that for every element $p$ of $I$, $x_1(p) \in F(p)$ and for every element $p$ of $I$, $x_2(p) \in F(p)$ and $\prod x_1 = \prod x_2$ holds $x_1 = x_2$.

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