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RESEARCH ARTICLE

Formation control of road vehicles based on dynamic inversion and passivity

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Abstract

This paper proposes a hierarchical formation stabilization method for vehicles having nonlinear dynamics. Supposing that the formation control problem is already solved for the case of linear vehicle dynamics, the method proposes a dynamic inversion based low-level control, which linearizes, at least approximately, the original vehicle dynamics so that the formation control can be applied. In this way a hierarchical control system is obtained, which is then completed with a passivity based external stabilization procedure for the stability of the entire system can be guaranteed. The proposed algorithm is tested by simulation on a formation control problem of road vehicles.

Keywords

formation control · dynamic inversion · passivity · robust control

Acknowledgement

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1 Introduction

In the last years the increased computational capabilities of computer systems and the rapid development of the communication and sensor technologies have increased the interest in highly automated unmanned vehicles, which are able to cooperate with other vehicles and are able to perform, in the presence of uncertainties, disturbances and faults, complex tasks beyond the ability of the individual vehicles. This general concept has been realized in multiple applications [3]: Unmanned Aerial Vehicles (UAV-s), [1], Autonomous Underwater Vehicles (AUV-s) and automated highway systems (AHS) ([16]),

Although the application fields listed above are very different, in the control design several common points can be found. The control of autonomous vehicle groups is generally hierarchical, where the components on the lower levels are local, in the sense that they depend on the particular - and generally nonlinear - vehicle dynamics. These local controllers modify the original vehicle dynamics so that the dynamic behaviour of the closed loop can be modelled by a more simpler - e.g. linear - system. This simple model, which can even be the same for different vehicles, is then used in the design of the higher-level control components, where the prescribed cooperative tasks are taken into consideration. Due to this decoupling the complex, task-dependent control problems have to be solved for simplified vehicle models only, and the controllers obtained will be independent from the real vehicle dynamics. For the design of the high-level cooperative control several methods exist, depending on the prescribed task, the number of vehicles and the design constraints to be satisfied. In this paper we are focusing on the methods based on *artificial potential functions* ([6, 7, 14]). These methods construct a special potential energy function, which takes its minimum at the solution of the cooperative problem. Starting from an arbitrary initial state the controller then tries to steer the system along the gradient of the potential function until the energy reaches its minimum.

It is clear that the stability of the entire hierarchically controlled formation is a key issue in the controller design. Despite of this, the cooperative control literature concentrates mainly on the construction of a potential function and does not analyze the stability properties of the coupled system. Therefore in this paper we propose a hierarchical formation stabilization method comprising an arbitrary potential function based high-level controller and a dynamic inversion based low-level controller, which can be completed with a passivity based external feedback so that the stability of the entire formation is guaranteed.

The paper is organized as follows. In section 2 the vehicle model is presented and the necessary coordinate transformations are derived. In section 3 we build up the hierarchical control structure and design the external stabilizing feedback. Section 4 demonstrates the results via a cooperative control example and in section 5 the most important conclusions are drawn.

2 Vehicle models for cooperative control

This section presents the dynamic model of the vehicle and derives the elementary coordinate transformations that are necessary to formalize the formation control problem.

The simplified single-track model of a four-wheeled vehicle can be given in the following form [4],[12]:

$$\begin{aligned}
\dot{x} &= v\cos(\beta+\psi) = v\cos(\phi)\\
\dot{y} &= v\sin(\beta+\psi) = v\sin(\phi)\\
\dot{\phi} &= \dot{\beta}+\dot{\psi} = \frac{a_{11}}{v}\beta + \frac{a_{12}}{v^2}r + \frac{b_1}{v}\delta \qquad (1)\\
\dot{\beta} &= \frac{a_{11}}{v}\beta + (\frac{a_{12}}{v^2}-1)r + \frac{b_1}{v}\delta\\
\dot{r} &= a_{21}\beta + \frac{a_{22}}{v}r + b_2\delta\\
\dot{v} &= \alpha
\end{aligned}$$

where (x, y) denotes the position of the vehicle on the 2D plane in a fixed coordinate frame K_0 and v, β, r, ψ are the velocity, slideslip angle, yaw-rate and orientation respectively (see Fig. 2). The control inputs are the steering angle (δ) and acceleration (α). As outputs the position coordinates x and y were chosen, both are supposed to be measured by appropriate inertial and/or GPS sensors. The remaining parameters of the model are constant and can be calculated as follows:
$a_{11} = -\frac{c_f+c_r}{m}$, $a_{12} = \frac{c_r l_r - c_f l_f}{m}$, $a_{21} = \frac{c_r l_r - c_f l_f}{J}$, $a_{22} = -\frac{c_r l_r^2 + c_f l_f^2}{J}$, $b_1 = \frac{c_f}{m}$, $b_2 = \frac{c_f l_f}{J}$, where m is the mass of the vehicle, c_r, c_f are the rear and front cornering stiffness, J is the inertia, l_r, l_f are the distances of the center of mass from the rear and front axle. This single-track dynamics describes well the vehicle motion in case of normal operation i.e. when the lateral acceleration is not too high ($< 4\frac{m}{s^2}$).

In general the vehicle formations are formed and stabilized during motion, while the vehicle group is tracking a prescribed trajectory. The formation control problem can be formalized more conveniently if the dynamics of the vehicles are expressed relative to this common trajectory, namely if they are rewritten in a moving coordinate frame attached to the trajectory curve [2,13]. For this, let P be a point moving along the prescribed 2D trajectory curve $\mathcal{C}$ (see Fig. 2) and let the motion of P be defined by the dynamic equations $\dot{p} = \begin{bmatrix}\dot{x}_P(t)\\ \dot{y}_P(t)\end{bmatrix} = \begin{bmatrix}\dot{s}\cos\varphi(s)\\ \dot{s}\sin\varphi(s)\end{bmatrix}$,
where
$s(t) : \mathbb{R} \to \mathbb{R}$ is a continuous function and $\begin{bmatrix}x_P(t)\\ y_P(t)\end{bmatrix} \in \mathcal{C}$.

Let the coordinate system K be fixed to the point P and defined so that one of its axis is tangent to the trajectory curve. By applying the rules of *derivations in moving coordinate frame* [5] and the results of [13] the position and velocity of the vehicle and the related state variables can be expressed in K as follows:

$$\begin{aligned}
\dot{s}_1 &= v\cos\theta - \dot{s}(1-c(s)y_1)\\
\dot{y}_1 &= v\sin\theta - c(s)\dot{s}s_1 \qquad (2.a)\\
\dot{\theta} &= \dot{\phi}-\dot{\varphi} = \frac{a_{11}}{v}\beta + \frac{a_{12}}{v^2}r - c(s)\dot{s} + \frac{b_1}{v}\delta\\
\dot{v} &= \alpha \qquad (2.b)\\
\dot{\beta} &= \frac{a_{11}}{v}\beta + (\frac{a_{12}}{v^2}-1)r + \frac{b_1}{v}\delta\\
\dot{r} &= a_{21}\beta + \frac{a_{22}}{v}r + b_2\delta \qquad (2.c)
\end{aligned}$$

where $\theta = \phi - \varphi$, $c(s) = \frac{\partial\varphi(s)}{\partial s}$. By introducing new input and state variables so that

$$x_1 = \begin{bmatrix}s_1\\ y_1\end{bmatrix} \quad x_2 = \begin{bmatrix}\theta\\ v\end{bmatrix} \quad x_3 = \begin{bmatrix}\beta\\ r\end{bmatrix} \quad u = \begin{bmatrix}\delta\\ \alpha\end{bmatrix} \qquad (3)$$

the dynamics above can be rewritten in the following more compact form:

$$\begin{aligned}
\dot{x}_1 &= h(x_1, x_2, t)\\
\dot{x}_2 &= A_2(\rho)x_3 + B_2(\rho)u + f(t)\\
\dot{x}_3 &= A_3(\rho)x_3 + B_3(\rho)u \qquad (4)
\end{aligned}$$

where ρ is the vector comprising the time varying parameters of the matrices above, i.e. $\rho = \begin{bmatrix}1/v & 1/v^2\end{bmatrix}$.

3 Hierarchical passivity based control

As we have mentioned, our aim is to perform complex cooperative tasks (formation stabilization, trajectory tracking, etc.) with vehicles having nonlinear dynamics (4). To simplify this problem a hierarchical control framework is proposed in this section. The control structure will consist of two levels: the dynamic inversion based low-level controller linearizes – at least partially – the nonlinear vehicle dynamics. After the linearization the vehicle can be considered as a simple double integrator, for which the high-level formation controller can be easily designed. In order to have a Lyapunov function proving the stability of the entire closed-loop system a passivity based external feedback will be constructed at the end of the section.

3.1 Dynamic inversion based low-level controller design

The low-level part of the hierarchical control framework is based on the dynamic inverse of the vehicle model. The dynamic inverse can be obtained by applying the state transformation $z_1 = x_1 = y,\ z_2 = \dot{x}_1 = h(x_1, x_2),\ z_3 = x_3$ to (4) and expressing the control input from the dynamic equation of z_2. (For the details see [11]). Applying the same argument as [11] the dynamic inversion based controller can be obtained in the following form:

$$\begin{aligned}
u_c &= B_2^{-1} J_{x_2}^{-1}(-J_{x_1} z_2 - J_{x_2} A_2 z_{3c} - J_{x_2} f(t) - J_t + v) \\
\dot{z}_{3c} &= A_3 z_{3c} + B_3 u - w = \\
&= A_3 z_{3c} - B_3 B_2^{-1} J_{x_2}^{-1} J_{x_1} z_2 - B_3 B_2^{-1} A_2 z_{3c} - \\
&\quad - B_3 B_2^{-1} f(t) - B_3 B_2^{-1} J_{x_2}^{-1} J_t \\
&\quad + B_3 B_2^{-1} J_{x_2} v - w = (A_3 - B_3 B_2^{-1} A_2) z_{3c} + u_c^* - w
\end{aligned} \quad (5)$$

where $J_{x_2} = \frac{\partial h}{\partial x_2} = \begin{bmatrix} -v\sin\theta & \cos\theta \\ v\cos\theta & \sin\theta \end{bmatrix}$, $J_{x_1} = \frac{\partial h}{\partial x_1}$, $J_t = \frac{dh}{dt}$ and v and w are additional control inputs defined later and z_{3c} is the inner state of the controller used to estimate the unmeasured state z_3.

The controller above transforms the original vehicle dynamics into the following partially linear closed-loop system:

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= v + J_{x_2} A_2 (z_3 - z_{3c}) \\
\dot{z}_3 - \dot{z}_{3c} &= A_3 (z_3 - z_{3c}) + w
\end{aligned} \quad (6)$$

which, apart from the dynamics of the approximation error $z_3 - z_{3c}$, is equivalent to a double-integrator. The nonlinearity is caused by the parameter-dependence of matrices A_2, A_3 and state dependence of J_{x_2}. The controller (6) is applicable only if the following three conditions are satisfied: the internal dynamics $\dot{z}_{3c} = (A_3 - B_3 B_2^{-1} A_2) z_{3c}$ is stable, u_c^* is bounded and $z_3(t) - z_{3c}(t)$ tends to zero as t tends to infinity. The first two conditions are necessary for u_c to be bounded, the third guarantees perfect state estimation. We have already proved in [10] that condition 1 always holds irrespective of the vehicle parameters. Since u_c^* depends only on z_1 and z_2 it is always bounded if condition 3 holds and the linear subsystem $\dot{z}_1 = z_2, \dot{z}_2 = v$ is stabilized by an appropriate feedback $v = v_c(z_1, z_2)$. For the satisfaction of condition 3 we assume that the dynamics $\dot{e}_3 = \bar{A}_3 e_3$ with $\bar{A}_3 = A_3$ or $\bar{A}_3 = A_3 + K$, $K = \begin{bmatrix} 0 & k_1 \\ 0 & k_2 \end{bmatrix}$ is quadratically stable and $W(e_3, \rho) = e_3^T W(\rho) e_3$ is an appropriate Lyapunov function. The stabilizing feedback $K e_3$ is comprised in the additional control input w. Its special structure is motivated by the fact that from the two state variables in z_3 the yaw-rate can be well measured, so its estimation error can be feeded back to stabilize the error dynamics. It is clear that if this assumption holds then so does condition 3.

3.2 High-level formation control design

The goal of the high-level controller is to solve the formation control problem, i.e. to steer the group of vehicles into a prescribed spatial formation, while the entire group follows a predefined trajectory. This problem class comprises several special cooperative control problems, e.g. geometric formation shaping, obstacle avoidance or coordinated collective motion of high number of vehicles called 'flocking' [6]. Since the low-level controller has already linearized the dynamics, the high-level controller can be implemented as if the vehicles had double integrator dynamics.

Assume that the formation control problem is prescribed for a group of N vehicles. Suppose that this problem can be solved by using artificial potential function, i.e. there exists an artificial potential function $V(\zeta_1)$, $\zeta_1 = [z_1^1, z_1^2, \ldots, z_1^N]$ so that $V(\zeta_1)$ has global minimum at the prescribed spatial formation. Consider now, the total energy of the point-mass system:

$$\mathcal{V}(\zeta_1, \zeta_2) = V(\zeta_1) + \frac{1}{2}\|\zeta_2\|^2 \quad (7)$$

where $\zeta_2 = [z_2^1, z_2^2, \ldots, z_2^N]$.

Let the control input v_c be chosen as follows:

$$\begin{aligned}
v_c(\zeta_1, \zeta_2) &= -\frac{\partial V(\zeta_1)}{\partial \zeta_1} - k\zeta_2 \quad k > 0 \\
v_c^i &= -\frac{\partial V(z_1^i)}{\partial z_1^i} - k z_2^i
\end{aligned} \quad (8)$$

It can be easily checked that this feedback stabilizes the formation by rendering the time derivative of $\mathcal{V}(\zeta_1, \zeta_2)$ negative:

$$\dot{\mathcal{V}}(\zeta_1, \zeta_2) = \frac{\partial V}{\partial \zeta_1}\zeta_2 - \zeta_2^T \frac{\partial V}{\partial \zeta_1} - k\zeta_2^T \zeta_2 = -k\|\zeta_2\|^2 \leq 0 \quad (9)$$

In order to calculate (8) every vehicle has to know the position and velocity of the others. This information has to be shared via appropriate communication channels.

3.3 Passivity based external feedback design

Now, being in possession of the high-level and the low-level controllers we can build up the hierarchical control structure. For this, let us substitute $v_c(\zeta_1, \zeta_2)$ into (6) to get the coupled vehicle dynamics:

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= v_c(\zeta_1, \zeta_2) + \mathcal{A}_2 \varepsilon_3 \\
\dot{\varepsilon}_3 &= \mathcal{A}_3 \varepsilon_3 + \omega
\end{aligned} \quad (10)$$

where $\mathcal{A}_2 = \mathrm{diag}(J_{x_2^1}^1 A_2^1, \ldots, J_{x_2^N}^N A_2^N)$,
$\mathcal{A}_3 = \mathrm{diag}(\bar{A}_3^1, \ldots, \bar{A}_3^N)$,
$\varepsilon_3 = [e_3^1, \ldots, e_3^N]$ and $\omega = [w^1, \ldots, w^N]$.

Notice that the Eqs. (10) realize a partial interconnection of the following two subsystems

1. $\dot{\varepsilon}_3 = \mathcal{A}_3 \varepsilon_3 + w$
2. $\dot{\zeta}_1 = \zeta_2$, $\quad \dot{\zeta}_2 = v_c(\zeta_1, \zeta_2)$

Our aim is to choose the external control input w in such a way that a Lyapunov function can be constructed for the entire controlled system. We solve this problem by using passivity-based technique in the following way: first new inputs and outputs are chosen for the subsystems with respect to which they will be passive. Then the control input w is set so that the dynamics (10) realizes a negative feedback interconnection of the subsystems, which consequently will be asymptotically stable [15].

Since Subsystem 2 is asymptotically stable with Lyapunov function $\mathcal{V}(\zeta_1, \zeta_2)$, then by calculating the time derivative of $\mathcal{V}$ we get hints for the choice of input u_2 and output y_2:

$$\frac{d\mathcal{V}}{dt} = \underbrace{\frac{\partial \mathcal{V}(\zeta_1,\zeta_2)}{\partial \zeta_1}\zeta_2 + \frac{\partial \mathcal{V}(\zeta_1,\zeta_2)}{\partial \zeta_2}v_c}_{<0} + \underbrace{\frac{\partial \mathcal{V}(\zeta_1,\zeta_2)}{\partial \zeta_2}\mathcal{A}_2}_{y_2^T}\underbrace{\varepsilon_3}_{u_2} = -k\|\zeta_2\|^2 + y_2^T u_2 \le y_2^T u_2 \quad (11)$$

i.e. the subsystem 2 is passive with storage function $\mathcal{V}$. A similar input/output selection procedure can be carried out for the subsystem 1 by introducing the Lyapunov function $\mathcal{W}(\varepsilon_3) = \frac{1}{2}\varepsilon_3^T \mathcal{W}\varepsilon_3$, $\mathcal{W} = \text{diag}(W^1, \dots, W^N)$:

$$\frac{d\mathcal{W}(\varepsilon_3)}{dt} = \underbrace{\varepsilon_3^T \mathcal{W}\mathcal{A}_3\varepsilon_3}_{<0} + \underbrace{\varepsilon_3^T}_{y_1^T}\underbrace{\mathcal{W}\omega}_{u_1} \le y_1^T u_1 \quad (12)$$

So, the subsystem 1 is also passive with respect to the chosen input u_1 and output y_1 with storage function $\mathcal{W}(e_3)$.

Notice that the partial interconnection of subsystem 1 and 2, coming from the original structure (10), can be expressed by the following relation $u_2 = y_1$. (The interconnected structure is depicted in Fig. 1) In order to achieve the negative feedback interconnection we have to set $u_1 = -y_2$ as it can be seen in Fig. 1. This means that the external control input ω has to be chosen as follows

$$\omega = -\mathcal{W}^{-1}\mathcal{A}_2^T \tfrac{\partial \mathcal{V}(\zeta_1,\zeta_2)}{\partial \zeta_2} = -\mathcal{W}^{-1}\mathcal{A}_2^T \zeta_2$$
$$\text{or} \quad w^i = -(W^i)^{-1} A_2^T (J_{x_2}^i)^T z_2^i \quad (13)$$

To prove the asymptotic stability of the entire system we prove first that the interconnected system is passive with storage function $S(\zeta_1, \zeta_2, \varepsilon_3) = \mathcal{V}(\zeta_1, \zeta_2) + \mathcal{W}(\varepsilon_3)$ and then we will see that this function can serve as Lyapunov function in our special case. Let us introduce two new, external inputs denoted by u_{e1} and u_{e2} respectively according to Fig. 1. By calculating the time-derivative of $S(\zeta_1, \zeta_2, \varepsilon_3)$

$$\dot{S} = \frac{d}{dt}\{\mathcal{V}(\zeta_1,\zeta_2) + \mathcal{W}(\varepsilon_3)\} =$$
$$\underbrace{\frac{\partial \mathcal{V}}{\partial \zeta_1}\zeta_2 + \frac{\partial \mathcal{V}}{\partial \zeta_2}v_c}_{<0} + \underbrace{\varepsilon_3^T \mathcal{W}\mathcal{A}_3\varepsilon_3}_{<0} + y_2^T u_{2e} + y_1^T u_{1e} \quad (14)$$

$$\le \begin{bmatrix} y_1^T & y_2^T \end{bmatrix} \begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix} \quad (15)$$

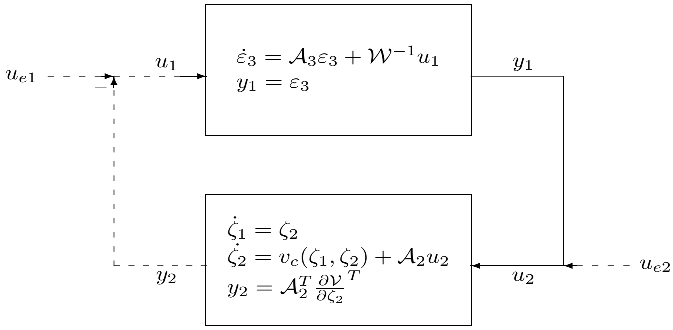


Fig. 1. Interconnection of passive subsystems

we can see that the interconnected system is passive with respect to input $\begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix}$ and output $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ with storage function $S(\zeta_1, \zeta_2, \varepsilon_3)$. In our case the external inputs u_{e1} and u_{e2} are 0 thus $\dot{\mathcal{V}}(\zeta_1, \zeta_2) + \dot{\mathcal{W}}(\varepsilon_3) \le 0$. Consequently the positive definite function $\mathcal{V}(\zeta_1, \zeta_2) + \mathcal{W}(\varepsilon_3)$ is an appropriate Lyapunov function candidate for the coupled dynamics (10). In order to prove that S is a valid Lyapunov function we can apply LaSalle's theorem. Since $\dot{S} \le 0$ the trajectories of system (10) will converge to the maximal invariant subset of $\Omega = \{(\zeta_1, \zeta_2, \varepsilon_3) \mid \dot{S}(\zeta_1, \zeta_2, \varepsilon_3) = 0\}$. By examining the dynamics (10) we can easily see that Ω contains only the origin. Thus, the trajectories of the system tend to zeros as t tends to infinity. This proves that the system is globally asymptotically stable with Lyapunov function S.

4 Formation control of road vehicles

As an illustrative example for the presented method we solve in this section a formation reconfiguration problem with five road vehicles. In the beginning the vehicles are in a column formation that is perpendicular to the trajectory. Then they are ordered to change their formation. The new formation is a line, which is tangential to the trajectory (according to Fig. 2). Of course, during the reconfiguration the vehicles must not collide and the entire group has to track a prescribed trajectory.

The vehicles in the formation have the following identical modelling parameters obtained by identifying a heavy-duty vehicle [12]:

$$a_{11} = -147.1481 \quad a_{12} = 0.0645 \quad a_{21} = 0.0123$$
$$a_{22} = -147.1494 \quad b_1 = 66.2026 \quad b_2 = 31.9835$$

If $1 \le v \le 25$ we found - by solving the appropriate LMI [10] - that the estimation error dynamics $\dot{e}_3 = A_3 e_3$ in (6) is quadratically stable with the following Lyapunov function

$$W = e_3^T \begin{bmatrix} 246.7608 & -4.7350 \\ -4.7350 & 247.7231 \end{bmatrix} e_3 \quad \forall i \quad (16)$$

Thus, in our case, there is no need for additional stabilizing feedback ($K = 0$).

To solve the formation control problem by using the presented hierarchical control concept we have to find first an appropriate potential function. If the positions of the vehicles in the new configuration are defined a-priori, and are given by the vectors r_i, $i = 1...5$, the following potential function candidate can be constructed:

$$V(\zeta_1) = k_V \cdot \sum_{i=1}^{N} \left(\mu(\|\zeta_1^i - r_i\|) + \sum_{j,j\neq i} \mu(d - \|\zeta_1^i - \zeta_1^j\|) \right) \quad (7)$$

where d denotes the prescribed inter-vehicle distance, k_V is a scaling factor and $\mu(\cdot) : \mathbb{R} \to \mathbb{R}^+$ is an appropriate continuous scaling function satisfying the following conditions: $\mu(x) = 0$ if $x \leq 0$ and $\mu'(x) > 0$ if $x > 0$. In our simulations μ is defined as follows: $\frac{1}{2}mx^2$ if $0 \leq x \leq \frac{M}{m}$ and $Mx - \frac{1}{2}\frac{M^2}{m}$ if $\frac{M}{m} < x$. The first term in V takes its minimum if the vehicles reach their prescribed positions inside the formation. The second term penalizes the small inter-vehicle distances to force collision avoidance. By using V the high level control input v_c was calculated by using the formula (8).

Along the prescribed trajectory curve (see Fig. 2) a constant reference velocity $v_s = 15\frac{m}{s}$ was prescribed for the entire formation. More precisely, the origin of the moving reference frame K was computed by: $s(t) = v_s t + s_0$, $\dot{s}(t) = v_s$, where the position offset s_0 was $5m$. The remaining parameters of the trajectory ($\varphi(s(t))$ and its derivatives) were computed at each simulation time step by evaluating the spline function.

The vehicles were started from the following initial state:

$s_1(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$y_1(0) = \begin{bmatrix} 20 & 10 & 0 & -10 & -20 \end{bmatrix}$,

$\theta(0) = \begin{bmatrix} 0 & -0.1 & 0.1 & 0 & 0.1 \end{bmatrix}$,

$v(0) = \begin{bmatrix} 17 & 14 & 15 & 16 & 13 \end{bmatrix}$,

$\beta(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$r(0) = \begin{bmatrix} 0 & 0 & 0.1 & 0.1 & 0 \end{bmatrix}$,

$\beta_c(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$r_c(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

where $z_{3c}(0) = \begin{bmatrix} \beta_c(0) & r_c(0) \end{bmatrix}^T$.

The target positions of the vehicles inside the formation were chosen according to the configuration depicted in Fig. 2: $s_{1d} = \begin{bmatrix} 10 & 20 & 0 & -20 & -10 \end{bmatrix}$, $y_{1d} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ where the inter vehicle distance was $d = 10m$.

The simulation results in case of controller parameters $M = 4$, $m = 0.1$, $k = 1$, $k_V = 4$ can be found in Figs. 3, 4, 5. It can be seen that the vehicles follow the prescribed trajectory in the intended formation while the control inputs remain in a realizable range. The right subfigure of Fig. 5 depicts the minimal inter-vehicle distance measured during the simulation. As it can be seen every vehicle moved far enough from the others, so no collisions occured.

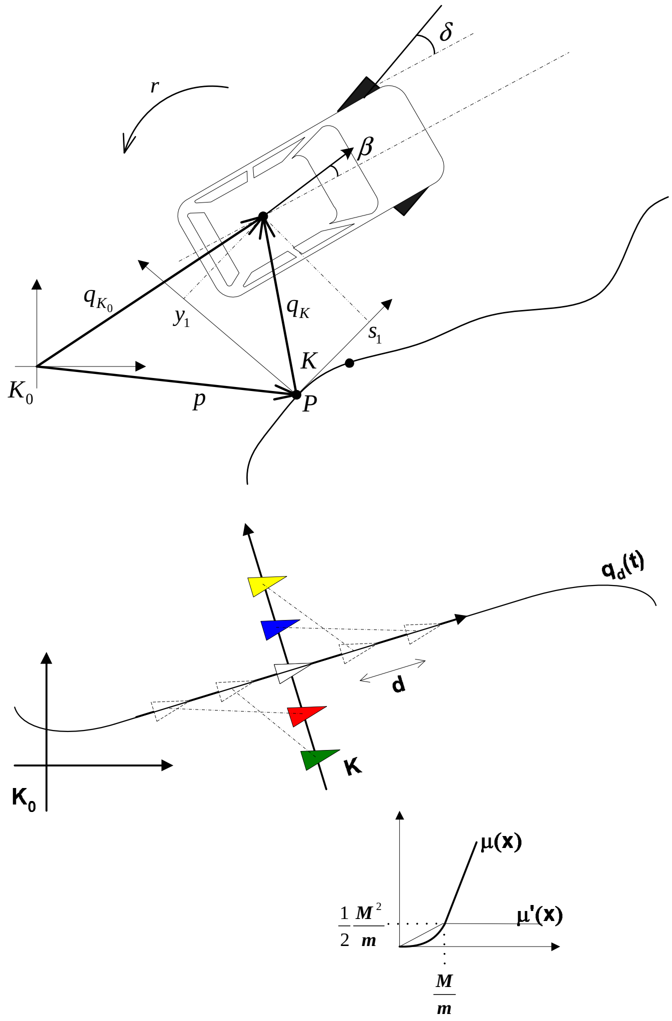


Fig. 2. Vehicle model and its parameters (top). Intended formation and scaling function $\mu(\cdot)$. (bottom)

5 Conclusions

A hierarchical, dynamic inverse and passivity based control structure has been proposed for the stabilization of vehicle formation. The control structure contains a dynamic inversion based low-level controller, which linearizes, at least partially the nonlinear vehicle dynamics. We have shown that the internal dynamics of the inverse system is globally stable, irrespective of the physical parameters, thus the inversion based controller can always be constructed. After linearizing the vehicle dynamics the formation control can be designed by using an arbitrary method based artificial potential functions. In order to guarantee the stability of the entire formation and to obtain an appropriate Lyapunov function we have designed an external feedback by exploiting the passivity property of the coupled controlled system. At the end of the chapter we have examined the robustness properties of the control structure by giving a class of perturbation models, against which, the system remains globally stable.

In this paper we have solved the formation control problem in the unconstrained case, although constraints on states and inputs are often prescribed in real applications. In the example above, the control inputs could be kept in an acceptable range

Fig. 3. Simulation results. The motion of the vehicles along the prescribed trajectory.

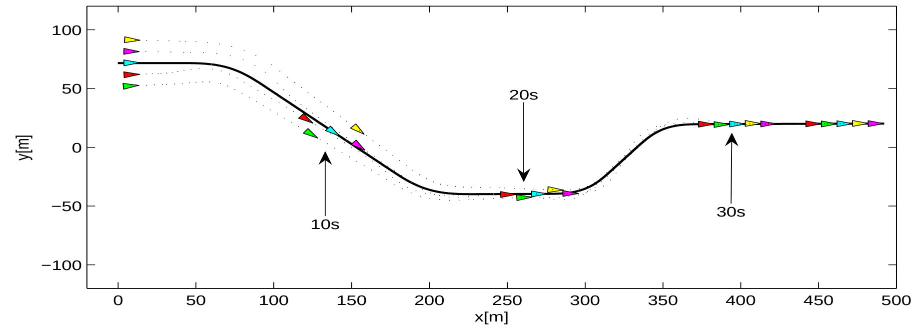


Fig. 4. State variables θ, v and β, r of the vehicles during simulation.

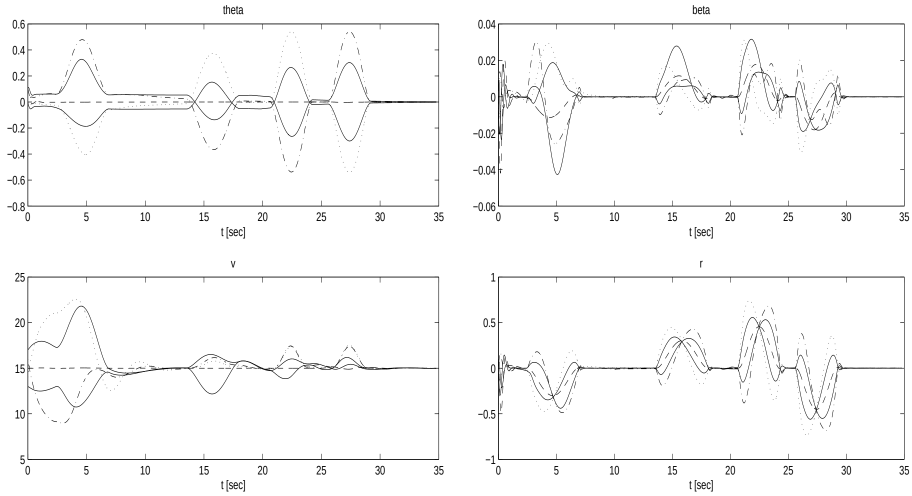
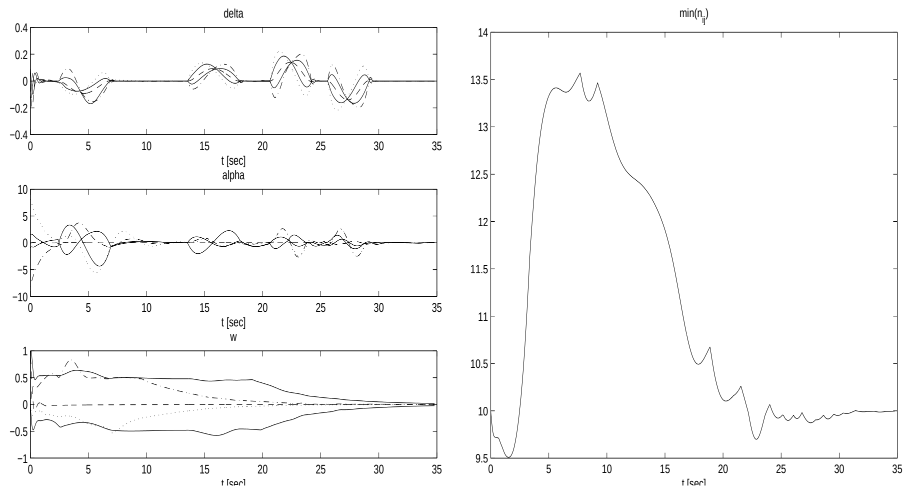


Fig. 5. Control inputs δ, α, w and the minimal inter-vehicle distance



by appropriately scaling the potential function via k, M, m and modifying the scaling factor k_V. However it is important to note that avoiding saturation in case of dynamic inversion based control is a hard problem in general [8, 9] which is currently under intensive research. Our research can be continued in this direction.

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