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## Robustness- and complexity-oriented characterization of supply networks' structures

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### Abstract

In the past period the efficiency aspects of production were emphasized, sometimes even overemphasized. As a result, the vulnerability of production structures was put in the background, and consequently, by now, it is usually beyond its acceptable degree. The frequently changing and uncertain environment which manufacturing companies are facing in our days requires robustness on every level of the production hierarchy from the process / machine level, through the system and enterprise levels, up to the level of supply chains and networks. As to the supply networks, the question may arise, what level of complexity is required for achieving a certain degree of robustness while, naturally, keeping the efficiency aspects in mind as well. In order to be able to give appropriate answers to this question, it is indispensable to quantify the robustness and complexity of supply chains and networks. Structural (static) and operational (dynamic) robustness and complexity are distinguished in the paper, which focuses on the structural aspects. A complex network approach is used for this purpose, namely the structural – both robustness and complexity – nature of the networks is described by applying graph theoretical concepts. Appropriate, quantitative graph measures are introduced and their applicability for characterizing the robustness and complexity of supply chains and networks is investigated by using structures of three types, namely real and artificially generated ones, and structures taken from the literature. Finally, it is illustrated how a decision support system based on the approach described in the paper can contribute to the design and redesign of supply chains and networks striving for an appropriate balance between the robustness, complexity and efficiency aspects of the problem.

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### 1. Introduction

Efficiently managed supply chains represent one of the most important prerequisites for the success of today's manufacturing enterprises, sometimes even for their survival in the frequently changing and uncertain environment.

Striving for cost efficiency, companies streamlined their operations, by outsourcing auxiliary activities, introducing just-in-time, just-in-sequence and lean management concepts. The enterprises usually work with low level safety stocks, and as a consequence, they may be vulnerable to the turbulences occurring in their supply chains.

In order to be able to keep or to increase their appropriate market share, companies are forced to change their product

portfolios more frequently, or even to comply with the individual requirements of the customers. The growing number of product variants – parallel with the low stock levels – makes their dependence on their suppliers even stronger. To make the situation more complicated, most of the enterprises simultaneously participate in a number of supply chains, and as a result, supply networks emerge.

More and more frequently, supply chains spread over continents which fact itself makes their proper functioning more vulnerable. Let us only refer to the related consequences of the volcano eruption in Iceland, 2010, or the earthquake in March 2011 and the following tsunami in Japan, or other natural catastrophes, such as floods, not mentioning some political uncertainties.

All of the above tendencies highlight the importance of the robust functioning of supply chains and networks. A logical assumption is that the robustness of the supply chains can be increased by including, e.g. more suppliers, transport lines, distribution centers; in one word, by increasing their complexity. These steps, however, usually include some extra costs. Therefore, a key question is how to balance between robustness, complexity and efficiency aspects in the design and management of supply chains and networks.

**2. Robustness and complexity of supply chains and networks**

In the literature various definitions are given for the *robustness* of supply chains, moreover, some related concepts (resilience, responsiveness) are also in use [1]. In the paper the more comprehensive formulation introduced in [2] will be applied: “In the general sense, a supply chain is robust if it is able to comply with the most important key performance indicators (KPI) set towards it, at an acceptable level (i.e. remaining in a predefined robustness zone) during and after unexpected event(s) / disruption(s) which caused disturbances in one or more production or logistics processes”. Fig. 1 (a further developed version of the figure in [3]) illustrates this concept, also pointing to the possible outcome when the new stable state goes on with an even higher KPI.

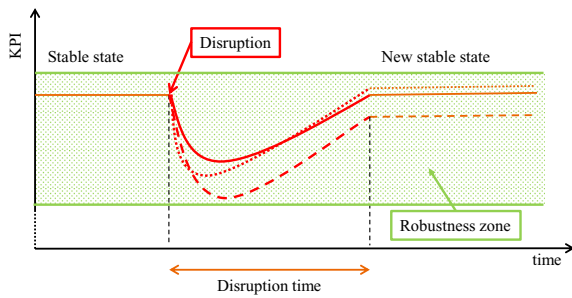


Fig. 1. Delineation of supply chains’ robustness used in the paper.

In the paper the concept *vulnerability* [3] is considered as a kind of feature opposite to the robustness, i.e. the more vulnerable a supply chain, the less robust it is.

In the past years, handling *complexity* gained significant attention also in the production related literature [4,5]. Serdarasan distinguishes necessary and unnecessary complexities of supply chains on the one hand, and current and potential complexities, on the other [6]. By necessary complexity we mean the complexity level that the customer / market is willing to pay for and what would provide a significant competitive advantage. Unnecessary complexity brings no or not enough benefits for the company / supply chain, which would compensate for the additional costs. Fig. 2 summarizes the main approaches to dealing with supply chains’ complexity.

Unnecessary complexity	Manage	Manage
	Reduce / Eliminate	Prevent
	Current complexity	Potential complexity

Fig. 2. Approaches to dealing with supply chains’ complexity [6].

In the context of supply chains, both in the fields of robustness and complexity, we can speak of *structural (static)* and *operational (dynamic)* types. In course of structural investigations, the size of the network, its elements and the linkages between them are put in the focus, while operational investigations deal with the dynamic processes occurring in the supply chains, assuming unchanged structures [7,8,9].

Theoretically, the robustness of a supply chain can be influenced by changing its structural or operational properties. Remaining at the structure, generally, it is expected that the increase / decrease of the structural complexity – in tendency – should go hand in hand with the similar changes in the structural robustness. The challenge is to achieve the required level of robustness with the lowest possible level of complexity. The objectivity of the process for evaluating the different scenarios can be significantly enhanced by using quantitative measures of the structural robustness and complexity.

In contrast to most of the papers dealing with the structural properties of supply chains and networks, either from robustness or complexity point of view, here an attempt is made to characterize supply chains and networks from both the aspects of complexity and robustness.

**3. Graph theory based measures for describing the structure of supply chains and networks**

It is straightforward to use graph theoretical concepts for characterizing the structural properties of supply chains and networks. Elements (e.g. factories, warehouses, points of delivery) of the chains / networks can be represented by the vertices / nodes of the graph, while the connection of two elements (e.g. a supplier-buyer relationship) by its edges. For describing the relationships in the given field, directed graphs are more adequate than undirected ones.

*3.1. Fundamental complexity measures of graphs*

The most natural complexity measures are the *order of the graph* (the number of the vertices / nodes, *n*) and the *size of the graph* (the number of the edges, *m*). The number of edges incident to vertex *v* is the *degree of the vertex*, *deg(v)*.

Perhaps, the measure based on Shannon’s information theory [10], which considers the similarity between the vertex degrees in a graph, is one of the most frequently used measures of the graphs’ complexity [11,8]. The *entropy of a graph* derived accordingly is as follows:

$$H_{graph} = - \sum_{i=1}^n \frac{deg(v_i)}{m} * \log_2 \frac{deg(v_i)}{m}. \tag{1}$$

The value of the entropy lies in the interval  $[0, \log_2 n]$ . Its maximal value is taken if all the vertices have the same degree (largest complexity).

### 3.2. Some measures for describing graphs' robustness

For the characterization of the vulnerability and robustness of the graphs there are also some adequate measures.

*Betweenness centrality* of vertex  $v$  is defined as the proportion of the number of the shortest paths between vertices that pass  $v$  to the total number of the shortest paths in the graph [12]:

$$BC(v) = \sum_{u \neq w \in V} \frac{\sigma_{uw}(v)}{\sigma_{uw}}. \tag{2}$$

Here  $\sigma_{uw}$  represents the number of the shortest paths between any vertices  $u$  and  $w$  while  $\sigma_{uw}(v)$  is the number of the shortest paths within this set, which incorporate vertex  $v$ .  $V$  is the set of all vertices in the graph.

In order to compare the betweenness centrality of vertices which belong to graphs of different size, the betweenness centrality values are usually normalized with factors which are somehow related to the size of the given graph. In undirected graphs, an appropriate normalization factor is  $(n-1)*(n-2)/2$  and in directed graphs  $(n-1)*(n-2)$ . As for the description of supply chains and networks directed graphs are more adequate, in the investigations introduced in the paper, the normalized value of the betweenness centrality was determined by the following formula:

$$BC'(v) = \frac{BC(v)}{(n-1)*(n-2)}. \tag{3}$$

The value of the normalized betweenness centrality of a vertex lies between 0 and 1, and characterizes its importance. (The higher this value, the more important the given vertex, i.e. the graph is more vulnerable here; in other words it is less robust.)

A relatively new measure for graphs' robustness is *factor R* [13,14]:

$$R = \frac{1}{n} \sum_{Q=1}^n s(Q), \tag{4}$$

where  $s(Q)$  is the ratio of vertices in the largest connected subgraph after removing  $Q$  vertices, to the number of all vertices (order of the graph). (It is always the vertex with the highest degree in the largest subgraph which is eliminated.) The range of possible R values lies in the interval  $[1/n, 1/2]$ , it is  $1/n$  in star graphs and  $1/2$  in fully connected graphs.

## 4. Graph theoretical analysis of concrete supply chains and networks

The available literature dealing with structural properties of supply chains and networks investigates – nearly without exception – either their robustness or their complexity character. Contrarily, the results introduced in this section are derived from a series of investigations where a number of measures of both robustness and complexity nature were determined and analyzed. The investigated supply structures were of three types, namely real and artificially generated ones, and structures taken from the literature [2].

The following subsections focus on the main elements and results of these investigations.

### 4.1. A supply network consisting of OEM enterprises and their first tier suppliers producing a given part

The structure in Fig. 3 is based on real data from Japan [15]. The supply network consists of original equipment manufacturers (OEM) and some of their first tier suppliers, i.e. of 11 car manufacturers (A1-A11 assemblers for the final assembly of cars) and of 6 tire suppliers (S1-S6). The data are from 2002. (Obviously, the network illustrated in the figure represents only a small part of the whole automotive parts' supply network of Japan at that time.)

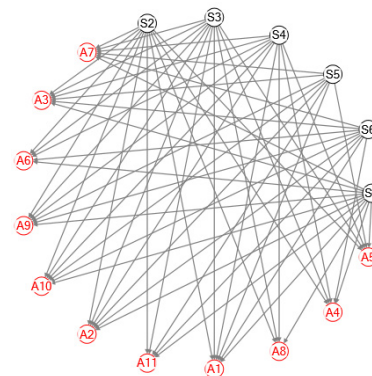


Fig. 3. Supply network of tires for automotive OEMs in Japan, 2002 (based on [15]).

In the network of Fig. 3 all of the suppliers S1-S6 deliver to a number of assemblers. S1-S3 supply all the 11 OEMs, while S4, S5 and S6 supply 10, 8 and 9 ones, respectively. Assemblers A1, A2, A3, A7, A9 and A10 are supplied by all the 6 tire manufacturers.

The graph which represents this supply network incorporates 17 nodes and 60 directed edges. Dense connections between the suppliers and the final assemblers can be observed.

As to the structure of this supply network, the authors of the referred paper [15] reported on a remarkable change in it in the period of 2002 – 2012. Focusing on the relation between the OEMs and the tire suppliers again, in 2012 the structure of Fig. 4 came into being.

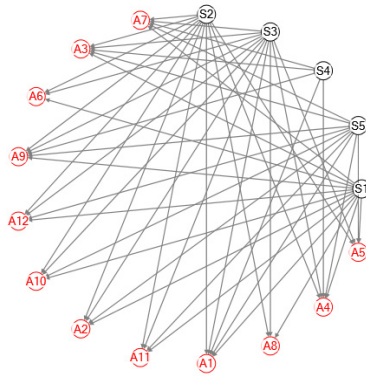


Fig. 4. Supply network of tires for automotive OEMs in Japan, 2012 (based on [15]).

Comparing the structures of Fig. 3 and Fig. 4, one can conclude that the number of assemblers increased from 11 to 12, the number of tire suppliers decreased from 6 to 5, and the number of supplier connections decreased from 60 to 51.

The real data from Japan gave the opportunity for a deep analysis in respect of the structural complexity and structural robustness of these networks. The most important results are summarized in Table 1.

Table 1. Change of the values of the complexity and robustness measures in the period of 2002-2012.

Year	No. of nodes	No. of edges	Average degree	Entropy	Maximum of the normalized betweenness centrality	Factor R
2002	17	60	3.529	4.012	0.106	0.27
2012	17	51	3	3.913	0.169	0.215

The 4 columns following the year data contain the values of the complexity measures. The order of the graphs (the number of the nodes / vertices) remained unchanged (17). The size of the graphs (the number of the edges) decreased from 60 to 51 in the investigated period. Accordingly, the average vertex degree changed from 3.529 to 3 (in this respect a directed edge was considered only once). The entropy (1) which measures the similarity of the nodes in the network in respect of their degree, decreased from 4.012 to 3.913. As a summary, all of the investigated complexity measures decreased in the period of 2002-2012. (Except for the number of the nodes, which remained unchanged.)

It is worth noting that in both investigated years nodes S1, S2 and S3 showed the largest normalized betweenness centrality (3) values, but the values in 2012 surpassed the earlier ones by 60%. This means that they became more vulnerable (their robustness decreased). Robustness factor R (4) decreased from 0.27 to 0.215.

Summarizing the above observations, in the 10-year period both of the complexity and robustness measures of the investigated network decreased. The results support the statements of Kito and Ueda [15], who used one special measure (nestedness) for the structural characterization of the same supply networks.

#### 4.2. Supply chains consisting of a production company, its warehouse(s) and the regions where the products are to be delivered to

In this subsection artificially generated supply chains are analyzed from structural complexity and robustness points of view, starting with a simple star structure and continuing with its gradually enlarged versions. Fig. 5 illustrates the basic version of the supply chain, which consists of a production company with its in-house, central warehouse (W1) and 30 regions (R1-R30) to be served with its products. It is known that both the complexity and the robustness of this star structure are very low. The latter is easy to see, because if W1 – for whatever reasons – is not able to deliver, none of the regions can be served.

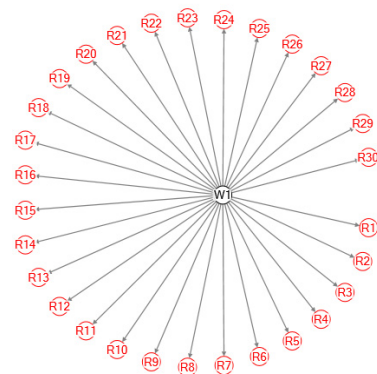


Fig. 5. Supply chain with 1 central warehouse (W1) and with 30 regions (R1-R30) to be served.

The structure was enlarged step by step in the following way: first, one external warehouse (H1) was added to the chain, which was supplied from W1. Three of the regions were served exclusively from H1, three other regions from both W1 and H1 and the remaining 24 regions solely from W1. The next two supply chains were distinguished from the previous one only in the numbers of how many regions were served by H1 only, and how many from both W1 and H1. These numbers were 6-6, and 9-9 respectively.

Finally, two other supply chains were generated both incorporating two external warehouses (H1 and H2). Similarly to the cases with one external warehouse, these external warehouses served some (3 and 6) regions solely, and the same numbers together with W1. The largest supply chain investigated is shown in Fig. 6.

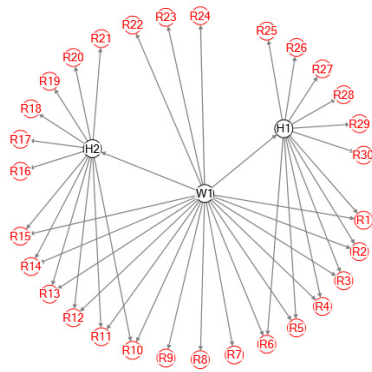


Fig. 6. Supply chain with 1 central warehouse (W1), with 2 external warehouses (H1, H2) and with 30 regions (R1-R30) to be served.

The same complexity and robustness measures which were used in the analysis of Subsection 4.1 were determined for these, artificially generated supply chains (Table 2).

Table 2. Complexity and robustness measures of the 6 investigated supply chains consisting of a production company, its warehouse(s) and the regions where the products are to be delivered to.

No. of ext. warehouses	No. of nodes	No. of edges	Average degree	Entropy	Maximum of the normalized betweenness centrality	Factor R
0	31	30	0.968	3.453	1	0.032
1	32	34	1.063	3.731	0.958	0.036
1	32	37	1.156	3.828	0.848	0.043
1	32	40	1.25	3.862	0.671	0.049
2	33	38	1.152	3.965	0.921	0.047
2	33	44	1.333	4.111	0.716	0.063

Comparing the values of the complexity measures (columns 2-5 in Table 2) of rows 2-6, with the measures of the first supply chain with one central warehouse (first row), one can see that all measures surpass their starting values. Within the blocks divided by horizontal lines in the table, i.e. in chains with the same number of external warehouses, the increase is monotonous. Moreover, the entropy values monotonically increase for all the consecutive supply chains, showing that the vertex degrees are more and more evenly distributed in the structures.

As to the two robustness-related measures (two columns of the right-hand side of the table) we can observe that – as it was expected for star graphs – the maximum of the normalized betweenness centrality is 1 (for node W1 in the supply chain of Fig. 5). It means that all the shortest paths go through this node, i.e. the supply chain vulnerability at this node is extremely high, contrary to the robustness which is very low. For this star structure with 31 nodes, robustness factor R takes its lowest possible value, i.e.  $1/31 = 0.032$ . In case of the supply chains with the same number of nodes (with the same number of external warehouses) their extension with additional edges resulted in growing robustness measures. This fact is in accordance with one’s natural perception that if a larger portion of the regions can be served from more warehouses, the supply chains’ robustness increases.

### 4.3. Multitier supply chains

An important part of the investigations reported here focused on complexity and robustness analysis of multitier supply chains. Each of the concrete structures taken from [8] consists of 22 nodes including the one which assembles the final product. The 6 analyzed supply chains differ only in the number of the edges which varies from 23 to 59, in such a way that a given structure incorporates the previous one as a subgraph (it has been extended with additional edges only).

Because of the limited space, only the smallest and the largest chains are shown in Fig. 7.

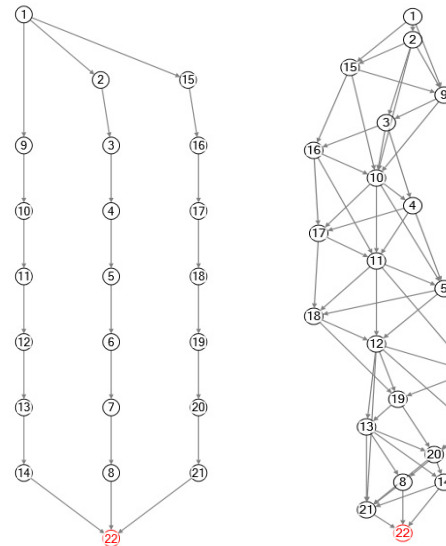


Fig. 7. The smallest and the largest analyzed multitier supply chains (based on [8]).

The results of the investigations are summarized in Table 3.

Table 3. Complexity and robustness measures of the 6 investigated multitier supply chains.

No. of nodes	No. of edges	Average degree	Entropy	Maximum of the normalized betweenness centrality	Factor R
22	23	1.045	4.447	0.317	0.145
22	28	1.273	4.414	0.415	0.213
22	34	1.545	4.373	0.518	0.254
22	40	1.818	4.379	0.389	0.273
22	47	2.136	4.376	0.412	0.318
22	59	2.682	4.411	0.339	0.376

The described way of generation of the consecutive structures provided opportunity for investigating the effect of additional links on the structural complexity and the structural robustness measures of multitier supply chains.

It is obvious that the average degree of the nodes increases with the inclusion of additional edges. However, if the increased number of the edges does not contribute to a more even distribution of the vertex degrees in the supply chain, in the given step, the entropy does not necessarily go hand in hand with the average vertex degree.

The maximum of the normalized betweenness centrality refers to the most vulnerable node within the structure, and in this sense is a local feature. It is understandable that not every additional edge in the supply chain influences the vulnerability of a given node in the targeted direction.

The best – positive – correlation was found between two global structural features, namely between the average vertex degree (as complexity measure) and the factor R (as robustness measure) (Fig. 8). (The same was experienced in the investigations described in Subsection 4.2.)

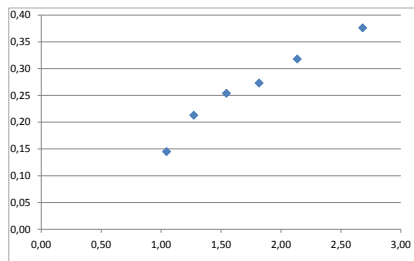


Fig. 8. The robustness factor R (y axis) in the function of the average vertex degree (x axis) in case of the investigated multitier supply chains.

## 5. Conclusions

In the paper graph theoretical measures were introduced for the characterization of structural robustness and structural complexity of supply chains and networks. Comparing the approach with the ones available in the literature, the novelty lies in the joint analysis of the robustness- and complexity-related features.

The investigations showed that, though the increase of structural complexity – as is to be expected, at least, in tendency – increases the robustness level, appropriate caution is needed when steps of increasing the supply chains' complexity with the aim of strengthening of their robustness level are considered, because it may happen that only the unnecessary complexity will increase.

Moreover, the deeper analysis showed that depending on some graph features, e.g. graph diameter, different robustness and complexity measures come into the limelight [2].

The practical applicability of the quantitative approach presented in the paper for analyzing supply chains both in their design and functional phases is straightforward. The values of the introduced robustness and complexity measures for the considered supply chain scenarios can be determined, graphically represented and compared. The approach can be an important part of a managerial decision support system for (re)designing supply chains. By this way, supply chains can be implemented and operated in which the complex relation of efficiency, robustness and complexity can be handled according to the management priorities.

The paper presented only the first results of a longer research period. As further steps, e.g. the elaboration of more detailed structural models of supply chains and networks, the inclusion of the operational measures into the investigations and, moreover, the joint use of the structural and operational measures are planned.

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