NOTES & DÉBATS

MATHEMATICS AND MORALITY ON THE CUSP OF MODERNITY

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ABSTRACT. — This note suggests that a fruitful way of investigating the history of mathematics lies in consideration of its pedagogical purposes. As a general illustration of the directions that such an approach might take, the paper discusses early-modern arguments for the practical utility of mathematics and its capacity to inculcate good habits of thought, as well as the appearance of new uses for mathematical training in the later eighteenth and early nineteenth centuries that served the purpose of the modernizing state, with its characteristic emphasis on impersonal criteria of evaluation and assessment of individuals. The paper encourages an understanding of mathematical pedagogy that refuses to treat it as unproblematic, and that seeks answers in social and cultural history.

RÉSUMÉ. — MATHÉMATIQUES ET MORALITÉ À LA POINTE DE LA MODERNITÉ. — Cette note suggère qu'une façon féconde d'étudier l'histoire des mathématiques est de considérer les visées pédagogiques de ces dernières. Afin d'illustrer les grandes orientations qu'une telle approche peut définir, l'article étudie les arguments qui ont été mis en avant au début de l'époque moderne en faveur de l'utilité pratique des mathématiques et de ses capacités à inculquer de bonnes habitudes de pensée. Il examine aussi l'apparition à la fin du XVIIIe et au début du XIXe siècles de nouveaux usages pour l'éducation mathématique, qui servent les intérêts de l'État en cours de modernisation, avec l'accent mis de manière caractéristique sur les critères impersonnels d'évaluation des individus. L'article vise une approche de la pédagogie mathématique, qui refuse de la traiter comme non problématique et qui cherche des réponses dans l'histoire sociale et culturelle.

The meaning of mathematics as a *pedagogical* discipline in the seventeenth and eighteenth centuries is one that seems strangely under-investigated. Perhaps this is because of an assumption that mathematics is a good thing to teach, presumably because of its associations with the rise of modern science. But there are other, more positive aspects of the teaching of mathematics in the early-modern period — a period in which the

nineteenth- and twentieth-century ideologies of modernity had not yet
been formed, and in which the value of an education in mathematics had
to be argued for and against in characteristically *pre*-modern terms. Early-
modern mathematical pedagogy needs to be understood in the terms of its
purported contributions to more dominant pedagogical aims of the period.
Such aims related to the formation of good character — mathematics as
a contributor to proper ways of behaving and thinking — and to broadly
humanist concerns with mathematics as a source of practical utility for
the good of the state. In general, therefore, it will be valuable to examine
arguments that presented mathematics as a program for the development
of moral virtue, whether individual or civic. This essay attempts a brief
overview of some of the issues that may emerge from such an examination.

First of all, it should be understood that "mathematics" here refers to
those disciplines that were regarded as constituting mathematics in this
period itself. The model of the medieval quadrivium still held sway, a
model comprising arithmetic, geometry, astronomy, and music. Privileg-
ing the first two, the branches of so-called "pure" mathematics, would do
violence to the understanding of the category that predominated in the
academic world of early-modern Europe. Mathematics was a way of doing
things as much as it was a particular domain of knowledge; it proceeded
by techniques of demonstration and construction, and it was concerned
with magnitudes, whether abstract or embodied in matter. The domains
in which mathematics was used themselves contributed to the value of a
mathematical education. In the eighteenth century, in D'Alembert's *Dis-
cours préliminaire* to the *Encyclopédie*, we read the following concerning
the physico-mathematical science of astronomy, the study of which

"est la plus digne de notre application par le spectacle magnifique
qu'elle nous présente. Joignant l'observation au calcul, et les éclairant
l'un par l'autre, cette science détermine avec une exactitude digne
d'admiration les distances et les mouvemens les plus compliqués des corps
célestes; elle assigne jusqu'aux forces mêmes par lesquelles ces mouve-
mens sont produits ou altérés. Aussi peut-on la regarder à juste titre
*comme l'application la plus sublime et la plus sûre de la géométrie et de
la mécanique réunies*; et ses progrès *comme le monument le plus incon-
testable du succès auquel l'esprit humain peut s'élever par ses efforts*"
[D'Alembert 1821, p. 27].

This passage, from 1751, in a new, Newtonian universe, still sounds remarkably similar in spirit to corresponding passages from Plato and Aristotle, who also praised astronomy due to the "nobility" of its object, the heavens. D'Alembert was able simply to augment that judgement with appeals to the precision attainable by the new, physico-mathematical[1] science of Newtonian celestial mechanics; astronomy is still, nevertheless, suitably described by words such as "magnificent" and "sublime" [*ibid*].

I. MORAL WORTH AND INTELLECTUAL VALUE

Around the beginning of the seventeenth century, the prominent Jesuit mathematician and pedagogue Christopher Clavius had repeated an even more conventional praise of astronomy in his widely-used textbook on the subject, his commentary on Sacrobosco's *De sphaera*. Astronomy, he says [Clavius, *Opera* 3, p. 3], is the noblest of all the mathematical disciplines, because it fulfills Aristotle's criteria of excellence better than any other: not only does it use demonstrations from geometry of the greatest certainty, but it also deals with the most noble subject-matter, namely the heavens. Nobility, a moral evaluation, played a major role in Clavius's promotion of the mathematical sciences as a whole. Clavius wrote the following as part of his enormously influential attempts to raise the status of mathematical teaching in the Jesuit colleges:

"Since therefore the mathematical disciplines in fact require, delight in, and honor truth — so that they not only admit nothing that is false, but indeed also nothing that arises only with probability, and finally, they admit nothing that they do not confirm and strengthen by the most certain demonstrations — there can be no doubt that they must be conceded the first place among all the other sciences."[2]

Clavius was the prime mover in encouraging the teaching of mathematics as part of the curriculum in the European-wide network of Jesuit

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[1] On "physico-mathematical", see [Dear 1995, chap. 6].

[2] Clavius, "In disciplinas mathematicas prolegomena", in [Clavius, *Opera* 1, p. 5]: "Cum igitur disciplinae Mathematicae veritatem adeo expetant, adament, excolantque, ut non solum nihil, quod sit falsum, verum etiam nihil, quod tantum probabile existat, nihil denique admittant, quod certissimis demonstrationibus non confirment, corroborentque, dubium esse non potest, quin eis primus locus inter alias scientias omnes sit concedendus." My translation.

colleges, and the moral status of mathematical knowledge and its use clearly played an important role in the techniques by which Jesuit mathematicians continued to promote their subject in the colleges during the course of the seventeenth century.

The most important elaboration on Clavius's apologia for mathematics was written by a former student of his, Giuseppe Biancani, in a text of 1615, *De natura mathematicarum.* While largely a work of epistemology, the text makes powerful use of moral evaluations. Biancani describes earlier claims (including, especially, those of certain Jesuit philosophers) that attempted to downgrade the status of mathematical knowledge as "calumnies", and, like Clavius, he protests indignantly against them. Plato is a useful resource here; Biancani quotes Ficino on Plato's position concerning the educational value of mathematical training. Plato's Academy, of course, was said to have used the motto "Let no one ignorant of mathematics enter here", and Biancani writes the following:

"Therefore Socrates rightly said in the *Republic* that while the mind's eye is blinded, indeed, is gouged by other pursuits, the mathematical disciplines restore it and elevate it to the contemplation of Him Who Is, and from the imitations to the true things, for the beauty and order of mathematical reasonings, and the firmness and stability of contemplation join us and perfectly attach us to the intellects, which always remain the same, shine together with divine beauty, observing their mutual order." [3]

Another point that Biancani borrows from Clavius concerns the criticism that mathematics is inferior to other disciplines, and is not a true part of philosophy, because it "abstracts from the good" — that is, it fails to concern itself with "the good." Biancani cites Aristotle's *Metaphysics* in response, where Aristotle writes that "those who claim that mathematics says nothing about good or the beautiful speak falsely, for it does say, and it does show a great deal about them; for even if it does not mention them by name, by showing the works and reasons [of the good and the beautiful], does it not say anything about them? For the species of beauty are order, symmetry and shapeliness, which are shown especially

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[3] Translation adapted from [Mancosu 1996, p. 198]. This text is discussed more fully in [Dear 1995, chap. 2]. See also, on the general issue of the contemporary controversy over the scientific status of mathematical knowledge, [Jardine 1988, pp. 685–711], with many further references.

by mathematical sciences." [4]

There is much more along similar lines, including a description of algebra as equalling "no human ingenuity, but what you would rather call heavenly revelation" [*Ibid.*, p. 205]. And, naturally, Biancani notes that mathematics is relevant to things mentioned in the Scriptures [*Ibid.*, p. 207]. In general, Biancani, like Clavius, is concerned to stress the certainty of mathematical demonstrations in relation to the Aristotelian ideal, and in fact to characterize them as "perfect" demonstrations — a technical term, to be sure, that described the fact that they fitted all of Aristotle's criteria for demonstration; but also one that carried a valuable rhetorical function, in associating mathematics with perfection itself.

Biancani's text quickly became in the seventeenth century a standard source for discussions of the nature of mathematical knowledge, not just among Jesuit mathematicians but among mathematicians in general, including Protestant mathematicians (who were hardly able to ignore the widespread and influential Jesuit writings on the subject). So this Jesuit doctrine on the value of mathematical studies as part of a thorough liberal education was a widely-known attempt at selling mathematics for educational purposes, and resembles in many ways the by-then standard arguments for the moral value of a regular *humanist* education. [5] In that respect, it is of course no coincidence that Biancani made a point of citing classical authorities like Plato, whose pronouncements were largely irrelevant to the technical philosophical views of Aristotle on mathematics and demonstration.

Nonetheless, those Aristotelian views were of fundamental importance. In using mathematics (primarily geometry) to shape his account of deductive axiomatic systems in the *Posterior Analytics*, Aristotle had attempted to lay out the formal structure of any ideal science whatsoever, regardless of its subject matter. In the later sixteenth and seventeenth centuries, it had become a widely held belief that this Aristotelian deductive structure was in fact a representation of the best way of teaching a subject (this is also the view of most present-day scholars of Aristotle's philosophy). At the same time, the reverse of this kind of deductive inference, often referred to as "analysis," was held to be the best way

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[4] Translation Mancosu (adapted), see [Mancosu 1996, p. 202].

[5] See on this subject [Grafton and Jardine 1986].

of *discovering* new theorems — the results that deductive, or synthetic, procedures would then serve to prove in the usual way. This conception became common after 1589, when Pappus's writings on "analysis" and "synthesis" in geometry first became widely available in a printed Latin version. So mathematics as the model of good pedagogical procedure was often already implied in scholastic Aristotelianism by 1600.

Mathematics was also promoted as an intellectual discipline from which could be expected moral habits of a desirable kind. Juan Luis Vivès, in his *De tradendis disciplinis* of 1531, had written the following concerning the importance of mathematical training:

"The mathematical sciences are particularly disciplinary to flighty and restless intellects which are inclined to slackness, and shrink from or will not support the toil of a continued effort. For they engage these minds and compel them to action, and do not suffer them to wander" [Vivès 1913, p. 202].

Vivès concluded his remarks on mathematics with a jab against scholasticism:

"For these studies in the master and pupil there must be a calm intellect, and to a certain degree they must be steadfast, careful, attentive, intent, and keen upon the work. There is no need of disputations" [*Ibid.*, p. 207].

Magdalen and Corpus Christi Colleges at Oxford both adopted, in the early sixteenth century, a rule for attendance at mathematical lectures justified by a fear that Bachelors of Arts, for whom they were specifically prescribed, might "become listless through idleness, and slacken overmuch, not to say give a loose to their minds and abandon their studies in the vacations." (Quoted in [Feingold 1984, p. 37].) John Locke agreed [Locke 1996, pp. 179–180].

II. THE TOPOS OF "UTILITY"

Nonetheless, what were, from a modern perspective, more easily recognizable arguments in favour of teaching mathematics were also current. A number of humanist educators in the sixteenth century had been promoters of mathematical training, and they had laid some stress on the supposed *utility* of mathematics. Vivès, again, wrote that:

"From geometry we proceed to all measurement, proportion, movement and position of heavy weights. [...] Then follows the study how to measure fields, mountains, towers and buildings" [Vivès 1913, p. 204].

Petrus Ramus used similar kinds of arguments as part of his attempt to include a significant place for mathematics in his desired reform of the University of Paris; all this quite apart from arguments about the importance of mathematics in understanding many aspects of philosophy, including the works of Aristotle, that were emphasized in the educational reforms of sixteenth-century German humanists such as Melanchthon.

Melanchthon's earlier promotion of mathematics had occurred in the context of the educational reform of the Lutheran university at Wittenberg, and it had been conducted with an explicitly religious aim. This is therefore a particularly clear case of the utility of mathematics being emphasized in relation to matters beyond simply practical, *instrumental* utility. The stress by Melanchthon and by Wittenberg mathematicians themselves in the 1520s, '30s, '40s and later was on the value of mathematics for the natural sciences; in the specific case of the mathematical science of astronomy, they also emphasized its relevance to understanding the nature of God and of one's own immortality. See [Kusukawa 1995, p. 180] and [Methuen 1998, esp. chap. 3].

Towards the end of the sixteenth century, this Lutheran stress on the pedagogical significance of mathematics found, of course, its apotheosis in the work of Kepler. Within an Aristotelian taxonomy of the disciplines, mathematical sciences were traditionally taken to be distinct from natural philosophy, whereas in Melanchthon's particular reformed vision they were an integral part of it. Astronomy was, after all, an effective means of revealing God's Providence in the universe. Kepler surely stands as the clearest example of the Lutheran theological/natural-philosophical complex by the end of the sixteenth century, and we should also remember Kepler's ambition to develop a *physical*, not merely a mathematical, astronomy. This example shows that there is still a lot to be said about physics, mathematics, as well as God. In this period, precisely by treating these categories as flexible and always renegotiable: mathematics, historically, cannot be treated a natural kind, and neither can the reasons for its study, teaching, and practice.

The general stress on utility made its way, inevitably, into Jesuit

attitudes towards the subject, appearing in the Jesuits' *Ratio studiorum* in 1586, apparently thanks once again to Clavius. The *Ratio* lists uses of mathematics in various areas of intellectual endeavour such as physics, metaphysics, theology, and jurisprudence, as well as medicine, agriculture, navigation, and general uses to the state.[6] The Jesuit Hugo Sempilius's mathematical textbook of 1635 spends a good deal of space elaborating on Biancani's arguments in favour of mathematics, and it contains no less than thirty-two pages (the whole of Book II) detailing the "utility of the mathematical sciences" [Sempilius 1635, pp. 21–35].

Such stress on the usefulness of mathematics became a standard trope in writings on the subject. Utility in general was, of course, a major element of Francis Bacon's programmatic approach to knowledge in general, but especially natural philosophy, in the first three decades of the seventeenth century. His concern for utility was, once again, explicitly based on religious grounds, his basic moral arguments for it being founded on a particular notion of Christian charity. He criticized existing philosophical approaches by identifying their goals and procedures as unworthy. In the case of Aristotelian philosophy, Bacon asserted that the fault lay above all in a misconstrual of the *purpose* of natural philosophy. By showing contempt for *practical* knowledge, Aristotelians were acting immorally: Aristotle's unproductive philosophy is a dereliction of the Christian duty of charity towards others. Because, in Bacon's view, natural philosophy could in principle help people, it was a duty to direct its pursuit towards that purpose. He wrote that "The true and legitimate goal of the sciences is to endow human life with new discoveries and resources" [Bacon 2000, Book I, aph. 81]; "Just let man recover the right over nature which belongs to him by God's gift, and give it scope; right reason and sound religion will govern its use" [*Ibid.*, aph. 129]. Pursuing power over nature was a matter of "right reason and sound religion."

This kind of Baconian rhetoric also found its way into discussion of mathematics and its practical uses. During the period of the Civil Wars

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[6] *Ratio studiorum et institutiones scholasticae Societatis Iesu per Germaniam olim vigentes collectae*, 4 vols., Berlin, 1887–1894, vol. 2, pp. 141–2. *Cf.* Clavius, "Modus quo disciplinae mathematicae in scholis Societatis possent promoveri," in Lukácz (Ladislaus), ed., *Monumenta Paedagogica Societatis Iesu, nova editio retractata, VII*: *Collectanea de Ratione Studiorum Societatis Iesu (1588–1616)*, Rome: Institutum Historicum Societatis Iesu, 1992, pp. 115–117.

in the 1640s, and especially the Interregnum of the 1650s between Charles I's execution and the Restoration in 1660, a number of works were published that criticized the established teachings of the universities. Their chief criticisms focused on the form of scholastic learning, which they tended to associate with Catholicism, and also on the alleged *uselessness* of university teachings. Mathematics was promoted by such people as a prime example of potentially useful learning that was being ignored. But sometimes the very formalism of deductive mathematical techniques themselves led to their being associated with scholastic logic and decried for their barrenness, their uselessness. A valuable precedent for English claims about the uselessness of university learning came from attacks on scholasticism by J.B. Van Helmont, the Dutch iatrochemist.

Among a number of would-be educational reformers around the middle of the seventeenth century, Van Helmont in fact criticized the Aristotelianism of the schools for being too wrapped up in what he explicitly described as "mathematics" [Debus 1978, p.127]. Presumably, in using this term, Van Helmont meant to include above all the rote procedures of syllogistic logic. This criticism of scholastic logic had of course been one of Francis Bacon's chief objections to the received learning of his time, and although Bacon did not explicitly align it with mathematics, we are reminded here of Bacon's famous antipathy towards mathematics properly so-called. Bacon tended to regard mathematics as essentially nothing other than measurement, and while he acknowledged the value of measurement in doing natural philosophy, he said that its value was limited: it failed to address the central issues concerning the forms of things, and should only "give limits to natural philosophy" [Bacon 2000, aph. 96]. From this perspective, and the even stronger one of Van Helmont, "mathematics," taken as formal, mechanical reasoning, is actually to be seen as having a negative moral valence, associated with the hated universities. For defenders of the universities against these sorts of charges, therefore, mathematics was one of the subjects that needed protecting in just this moral sense.

III. ENGLISH CONTROVERSIES

Perhaps the most famous of the English attacks on the universities at this time is Thomas Hobbes' great work *Leviathan* of 1651. He advocated

an approach to higher education that emphasized the role of the universities in supporting the state, and condemned their scholasticism as being too much in thrall to the traditions inherited from the Catholic church. Even in the specific case of mathematical disciplines, Hobbes stressed the importance of authority as the fundamental bedrock on which mathematical truths should rest. His medium for discussing this question was the category of "reason," which for him, as for others in seventeenth century England, had to be specified in the terms of so-called "right reason."

"Right reason" was a form of reason that was correctly oriented with accepted orthodoxy. For most users of the term, the authority was that of religion; "right reason" always supported, and never contradicted, orthodox religious teaching. For Hobbes, however, who was determined that civil authority not be threatened by an autonomous church, it was the civil authority whose interests should be promoted by "right reason," and that, in effect, defined what "right reason" was: it was reason that had been morally purified.[7] When he discussed even so basic a subject as arithmetic, Hobbes required such authority.

"Reason," for Hobbes, could always be reduced to a matter of adding and subtracting (a kind of early-modern computer science). Arithmetic was therefore the most fundamental of sciences, because it represented formalized "reckoning," as he put it, which was all that reason truly was. But not everyone could do it correctly. Hobbes wrote the following:

"[. . .] the ablest, most attentive, and most practised men, may deceive themselves, and inferre false Conclusions; Not but that Reason it selfe is always Right Reason, as well as Arithmetique is a certain and infallible Art: But no one mans Reason, nor the Reason of any one number of men, makes the certaintie; no more than an account is therefore well cast up, because a great many men have unanimously approved it. And therfore, as when there is a controversy in an account, the parties must by their own accord, set up for right Reason, the Reason of some Arbitrator, or Judge, to whose sentence they will both stand, or their controversie must either come to blowes, or be undecided, for want of a right Reason constituted by Nature; so is it also in all debates of what kind soever" [Hobbes 1914, p.19].

Reason, in other words, needed to be policed correctly; its virtue,

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[7] See, on this term, Lotte Mulligan, "'Reason,' 'Right Reason', and 'Revelation' in Mid-Seventeenth-Century England", in [Vickers 1984, pp. 375–401]; also [Vickers 1994].

represented above all in the strict procedures of mathematics, could not stand by itself. Otherwise, exactly as with religious enthusiasm, the social fabric would fall apart, as Hobbes goes on to explain:

"And when men think themselves wiser than all others, clamor and demand right Reason for judge; yet seek no more, but that things should be determined, by no other mens reason but their own, it is as intolerable in the society of men, as it is in play after trump is turned, to use for trump on every occasion, that suite whereof they have most in their hand. For they do nothing els, that will have every of their passions, as it comes to bear sway in them, to be taken for right Reason, and that in their own controversies: bewraying their want of right Reason, by the claym they lay to it." [*Ibid.*]

Needless to say, few people went as far as Hobbes in stressing the social disciplining necessary for right reason to be successfully exercised, for correct behaviour to be ensured, still less in stressing civil authoritarianism as the only solution. But Hobbes's attack on the abilities of the universities to promulgate safe and proper knowledge of value to the state, even in the case of mathematics, was vigorously emulated by other writers.

In the turmoil of the 1650s in England there were a fair number of literary attacks on the practices of the English universities. Bacon had set a useful, and influential, precedent earlier in the century with his own disparagements of the kind of education that he had received at Cambridge in the late sixteenth century, but Oxford seems to have been the more sensitive of the two universities in responding to the new attacks of the 1650s. One of the most famous critiques appeared in 1654 from the pen of the Puritan reformer John Webster. In his book *Academiarum examen*, or *The Examination of Academies*, Webster devotes a chapter to the mathematical sciences, having in the previous chapter disposed of scholastic logic as something that only encourages wordy disputations and the preservation of ignorance. Mathematics, by contrast, ought to do much better: Webster refers to "the *Mathematical sciences*, the superlative excellency of which transcends the most of all other Sciences, in their perspicuity, veritude and certitude." Despite this, he says, "in the general they are but either sleightly and superficially handled in definitions, divisions, axiomes, and argumentations, without any solid practice, or true demonstrations, either artificial or mechanical; or else the most abstruse,

beneficial, and noble parts are altogether passed by, and neglected."[8]

Webster dislikes the intellectual game of mathematical demonstrations, and wants instead "solid practice" and "true demonstrations," by which he means things that can be put into practice, as in the case of mathematics for navigation. Arithmetic, says Webster, is disregarded by the university masters, who leave it to "Merchants and Mechanicks," as not being worthy of themselves. He makes similar criticisms of the academic handling of geometry, and then devotes most of his attention to the mixed mathematical sciences, especially astronomy, where his main criticism is that people in the universities simply talk in imprecise terms and false assertions. All in all, Webster's critique is not particularly subtle, and improves little on the kinds of things that Bacon had said. But it did provoke strong response.

In the same year, 1654, two future Fellows of the Royal Society, Seth Ward and John Wilkins, both at Oxford, replied to Webster in defense of Oxford University. In the usual style of early-modern polemic, their text, called *Vindiciae Academiarum*, spends a lot of time abusing Webster for his ignorance, but on the specific topic of the mathematical sciences, indignation is the dominant tone. In detailing the mathematical expertise to be found in the English universities, chiefly taking the form of naming those prominent mathematicians who had been associated at some point in their careers with Oxford or Cambridge colleges, Ward and Wilkins mention among other things "the promotion of the Doctrine of *Indivisibilia*."[9] This is an interesting instance in which the mathematical activity cited was indeed abstruse and therefore lent itself to being seen as a mark of the high level of mathematical work being done in the universities, but its very abstruseness, combined with no exemplification of what the new techniques could accomplish, might have seemed only to confirm Webster's criticisms.

Steven Shapin's observations on the status of mathematical and natural-philosophical knowledge in English gentlemanly culture in the seventeenth century are relevant here. Shapin notes that particularly abstruse and formalistic kinds of knowledge and argument, of the kind generally

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[8] John Webster, *Academiarum Examen* (1654), pp. 40–41; facsimile reprint in [Debus1970].

[9] John Wilkins and Seth Ward, *Vindiciae Academiarum* (1654), p. 30; facsimile reprint in [Debus 1970].

associated with scholasticism and the traditional practices of the universities, were frequently treated as inappropriate for a gentleman not only because they were difficult and intellectually demanding, but also because they were inappropriate topics of polite discussion. Mathematical demonstrations and arguments were, above all, *dogmatic*, which was therefore the negative side of their much-praised *certainty*. Mathematical arguments did not allow divergent opinions, at least in the ideal, and lent themselves poorly to civil conversation. Accordingly, as Shapin notes, it was frequently said in seventeenth century England that too much mathematical education was not suitable for a gentleman, and this was reflected to a considerable degree in the lowly place held by mathematics in the universities [Shapin 1994, esp. chap. 7]. Mordechai Feingold [1984] has argued that this standard historical view about university mathematics in England is exaggerated, and that quite a few competent mathematicians were fellows of Oxford and Cambridge colleges, making it possible for some students to gain quite a lot of mathematical instruction at those places. However, the fact remains that mathematics was not prominent in the official curricula of Oxford and Cambridge — even less prominent at Cambridge than at Oxford, with its Savilian professorships. Its lack of attention in the curricula would seem to confirm the view that mathematics was accorded a low pedagogical value, despite the possibility of studying it at the universities.

The chief place for the teaching of mathematical sciences in England was not at either of the two universities, but at Gresham College in London, where the emphasis was firmly on their practical utility. Members of the Gresham mathematical circle exerted their main influence through their private teaching, however, in areas such as navigation and surveying.[10]

IV. PEDAGOGICAL MEANINGS OF MATHEMATICS IN THE EIGHTEENTH CENTURY: A ROUTE TO MODERNITY

A similar story obtained to a considerable extent in France: in the early part of the seventeenth century, the curricular requirements for mathematics at the University of Paris in 1601 were restricted to lectures on Euclid

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[10] See for further references, [Ames-Lewis 1999]; also [Neal 1997].

and on Sacrobosco's *De sphaera*; Laurence Brockliss [1987] notes that even this rule was less-than-scrupulously observed. Mathematics found its way more seriously into the instruction of the university in the second half of the century, when its major selling-point was the use of geometry in the study of physics, particularly as the impact of Cartesian teachings began to be felt. At the beginning of the eighteenth century, Jean Le Clerc described, in his *Opera philosophica*, the advances made in mathematics during the seventeenth century. During this period, he wrote, "men arose excelling in ingenuity, who, having been instructed especially in the mathematical disciplines, opened to posterity a new way of philosophizing. They were most skilled in the beautiful methods of geometry, which were used most fortunately in the investigation of truth..."[11] This view of the career of mathematics in the seventeenth century stressed it as a new approach to philosophy, and its supreme virtue was its provision of truth. Le Clerc indicated his specifically Cartesian direction when he noted that mathematical discussions concerned themselves especially with clear and distinct ideas (*Ibid.*, p. 190).

Such praise of mathematics as an exalted path to truths both strictly mathematical and physical was still, of course, continuous with the older commonplaces about mathematics that had existed before the close association with physics. As noted above, D'Alembert still spoke, in the 1750s, of the value of mathematics in quite traditional ways. But the pedagogical value of mathematics could also manifest itself in entirely distinct values, ones that we can associate with the development of nineteenth- and twentieth-century modernity. See [Porter 1995]. University mathematics teaching had been esteemed, when it was esteemed at all, for the virtues of its content and procedures, and for the good things that could flow from its proper uses, including its use for natural philosophy. But at the University of Cambridge in the second half of the eighteenth century, another virtue of mathematics became evident: mathematics as a subject lent itself well to university examinations.

The testing of students, and their precise ranking in terms of their performance in written examinations, could be justified in its own terms

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[11] [Le Clerc 1704, vol. 1, 2nd-3rd pp. (u.p.)], my translation: "exorti sunt viri ingenio excellentes, qui Mathematicis potissimùm disciplinis instructi novam posteris philosophandi viam aperverunt. Ii cùm pulcherrimae Geometrarum methodi peritissimi essent, ea felicissimè in investigatione Veri usi sunt..."

by reference to the close reasoning that it required, a virtue that showed a continuity with the stress on logic in the old *scholastic* curricular structure. But the procedural advantages of the new examination system in the prestigious Mathematical Tripos at Cambridge meant that academic advancement became associated to a remarkable degree with success in that easily-evaluated exercise. John Gascoigne [1989, p. 272] quotes a Cambridge don, writing in 1792, who complained that "the public honours of the University [...] are distributed merely according to mathematical merit, unless one evening dedicated to an examination in morality to which no attention is paid in ranking the candidates, may be called an exception."[12] Gascoigne argues that this idea of advancement-through-examination, to which mathematics was so suited, provided a direct model for the efforts of nineteenth-century social reformers eager to provide an alternative to advancement through personal patronage; much the same point applies to post-Revolutionary France and the growing power of academically-trained engineers. See [Alder 1997] and [Picon 1992]. The training of accredited experts is a typical feature of the liberal modern state, and mathematics has played a very important role in that development.

That mathematics was something worth teaching in universities was not, then, a self-evident proposition throughout most of the early-modern period. The arguments both for and against it related above all to the pedagogical purposes that universities were intended to fulfill. When those purposes were related to such issues as the training of a gentleman, or a future member of the clergy, the only way to promote mathematics was to stress its intellectual and moral virtues for the individual, and its potential utility for other endeavors that such a person might be destined to pursue. Real success in the incorporation of mathematics into the curriculum only appears to have come about when mathematics became accepted as crucial to natural philosophy. That acceptance simultaneously involved the development of natural philosophy itself from a purely contemplative science to a science that engaged with such practical, and often state-approved, matters as engineering. Hence the spectacular growth of French mathematical training at new institutions such as the

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[12] See also [Gascoigne 1984].

École Polytechnique at the end of the eighteenth century,[13] and the equally utilitarian *social* engineering of the later eighteenth-century Cambridge mathematical examination system. By that time, the characters of both mathematics and the universities had changed markedly since 1600.

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[13] See, besides [Picon 1992], [Belhoste 1994] with further references.

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