Highly Accurate Scheme for the Cauchy Problem of the Generalized Burgers-Huxley Equation

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Abstract: In this paper, a weighted algorithm, based on the reduced differential transform method, is introduced. The new approach is adopted in the approximate analytical solution of the Cauchy problem for the Burgers-Huxley equation. The proposed scheme considers the initial and boundary conditions simultaneously for obtaining a solution of the equation. Several examples are discussed demonstrating the performance of the algorithm.

Keywords: Weighted reduced differential transform method; Burgers-Huxley equation; Cauchy problem.

1 Introduction

Obtaining solutions for nonlinear equations plays an important role in the study of many nonlinear phenomena. In this perspective, during the last years, seeking the solution of nonlinear models has been an important topic in mathematical physics. One important nonlinear equation is the generalized Burgers-Huxley equation [1, 2, 3, 4, 5]

$$\frac{\partial u}{\partial t} = \kappa\frac{\partial^2 u}{\partial x^2} - \alpha u^\delta \frac{\partial u}{\partial x} + \beta u(1-u^\delta)(\eta u^\delta - \gamma), \tag{1}$$

where κ, α, β and η are real constants, δ is a positive integer and $\gamma \in [0,1]$.
The equation (1) is a generalization of various well known nonlinear equations, such as the Burgers, Huxley, FitzHugh-Nagumo, Burgers-Huxley and Burgers-Fisher models [1, 2, 6, 7, 8]. These equations describe different phenomena in mathematical physics, biomathematics, chemistry and mechanics [9, 10, 11, 12, 13]. The Burgers equation characterizes the wave propagation in dissipative systems [1].

The reaction-diffusion FitzHugh-Nagumo equation is used for investigating the dynamical behavior near the bifurcation point for the Rayleigh-Benard convection of binary fluid mixtures [6]. The Huxley equation describes the dynamics of electric pulses propagation in nerve fibres [7]. The Burgers-Fisher equation has application in plasma physics, capillary-gravity waves, optics and chemical physics [9, 10, 11, 12, 13, 14].
The generalized Burgers-Huxley equation has been considered recently by researchers that developed some analytical and numerical methods for its solution. Schemes such as the adomian decomposition [14], homotopy perturbation [15], homotopy analysis [16], and reduced differential transform [17] were proposed to solve the initial value problem of the Burgers-Huxley equation. Moreover, some authors considered the initial boundary value problem of this equation and used spectral collocation [18], finite-difference [19, 20], Haar wavelet [21] and modified cubic B-spline differential quadrature [22] methods for its solution.
The generalized Burgers-Huxley equation (1) is considered with the conditions

$$u(x,0) = f(x), \tag{2}$$

and

$$u(0,t) = p(t), \qquad u_x(0,t) = q(t). \tag{3}$$

In this work, a weighted technique, according to the reduced differential transform method (RDTM), is introduced for solving (1)-(3).
The RDTM was adopted by researchers to obtain the analytical and approximate solutions for nonlinear problems [23, 24, 25]. Often, the differential transform method is considered according to the initial condition of the problem, but, here we use the initial and boundary conditions.
Bearing these ideas in mind, this paper is organized as follows. In Sections 2 and 3, the RDTM and a weighted algorithm are introduced, respectively. In Section 4, several prototype problems are solved in order to show the ability and efficiency of the new algorithm. Finally, in section 5 the main conclusions are outlined.

2 Reduced differential transform method

In this section, the fundamental definitions and operations of the RDTM are reviewed. Consider a function of $u(x,t)$ and suppose that the two-dimensional function $u(x,t)$ is separable as $u(x,t) = f(x)g(t)$. Based on the features of differential transform [23], we can represent this function as

$$u(x,t) = \sum_{k=0}^{\infty} F_i x^i \sum_{j=0}^{\infty} G_j t^j = \sum_{k=0}^{\infty} U_k(x) t^k = \sum_{k=0}^{\infty} V_k(t) x^k, \tag{4}$$

where $V_k(t)$ and $U_k(x)$ are called x-dimensional and t-dimensional spectrum functions of $u(x,t)$, respectively.

Definition 1. *Suppose that $u(x,t)$ is analytic and differentiated continuously with respect to t and x in their domains. Then*

Table 1
Some operations of the reduced differential transform.

Function Form	Transformed Form
$u(x,t)$	$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}$
$u(x,t) = c$ (c is a constant)	$U_k(x) = \delta(k) = \begin{cases}1 & k=0\\ 0 & k\neq 0\end{cases}$
$u(x,t) = v(x,t) + w(x,t)$	$U_k(x) = V_k(x) + W_k(x)$
$u(x,t) = cv(x,t)$	$U_k(x) = cV_k(x)$ (c is a constant)
$u(x,t) = x^m v(x,t)$	$U_k(x) = x^m V_k$
$u(x,t) = t^m v(x,t)$	$U_k(x) = V_{k-m}$
$u(x,t) = x^m t^n$	$U_k(x) = x^m \delta(k-n) = \begin{cases}x^m & k=n\\ 0 & k\neq n\end{cases}$
$u(x,t) = \frac{\partial^m}{\partial t^m}v(x,t)$	$U_k(x) = \frac{(k+m)!}{k!}V_{k+m}(x)$
$u(x,t) = \frac{\partial^m}{\partial x^m}v(x,t)$	$U_k(x) = \frac{\partial^m}{\partial x^m}V_k(x)$
$u(x,t) = v^2(x,t)$	$U_k(x) = \sum_{r=0}^{k-1} V(r)(x)V(k-r-1)(x)$
$u(x,t) = v^3(x,t)$	$U_k(x) = \sum_{s=0}^{k-1}\sum_{r=0}^{s} V(r)(x)V(k-s-1)(x)V(s-r)(x)$

- *The transformed function $U_k(x)$ is defined as*

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}. \tag{5}$$

Its inverse differential transformation of $U_k(x)$ is

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k. \tag{6}$$

- *The transformed function $V_k(t)$ is defined as*

$$V_k(t) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{x=0}. \tag{7}$$

The inverse differential transformation of $V_k(t)$ is

$$u(x,t) = \sum_{k=0}^{\infty} V_k(t)x^k. \tag{8}$$

The main operations of the reduced differential transform, according to the variable t, that can be deduced from Eqs. (5) and (6) [24, 25] are listed in Table 1. These operations can be obtained in a similar way for the reduced differential transforms according to the variable x.

Let us illustrate the fundamental concepts in more detail. Suppose L is a linear operator and N is a nonlinear operator. Consider a general nonlinear differential equation as

$$L[u(x,t)] + N[u(x,t)] = \phi(x,t), \tag{9}$$

with the initial condition

$$u(x,0) = u_0(x), \tag{10}$$

where $\phi(x,t)$ is an inhomogeneous term. We assume that $L = \frac{\partial}{\partial t}$. According to the properties of RDTM in Table 1, we get

$$(k+1)U_{k+1}(x) = \Phi_k(x) - N[U_k(x)], \tag{11}$$

where $U_k(x)$, $NU_k(x)$ and $\Phi_k(x)$ are the transformations of $Lu(x,t)$, $Nu(x,t)$ and $\phi(x,t)$.
If we consider $U_0(x) = u_0(x)$ as the transformation of (10), then $u(x,t)$ can be written as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k, \tag{12}$$

Similarly, the recurrence relation (11) and the expressions of Table 1 may be introduced for $L = \frac{\partial^2}{\partial x^2}$ as well. In this case, considering $V_0(t) = p(t)$ and $V_1(t) = q(t)$, we get

$$u(x,t) = \sum_{k=0}^{\infty} V_k(t)x^k. \tag{13}$$

3 The weighted method

A weighted method according to the RDTM is now presented for the solution of (1)-(3). We formulate the algorithm in two steps. In the first step, we consider (1) and we denote $L = \frac{\partial}{\partial t}$. Applying the basic properties of the differential transformations and Table 1, and substituting $U_0(x) = f(x)$ as the differential transformation of (2), we get the approximate solution

$$\hat{u}_n(x,t) = \sum_{k=0}^{n} U_k(x)t^k. \tag{14}$$

In the second step, we seek the approximate solution of the Eq. (1) according to the conditions (3). Suppose that $L = \frac{\partial^2}{\partial x^2}$. Taking the differential transformation of (1) and applying the basic properties listed in Table 1 with respect to x, the approximate solution

$$\check{u}_n(x,t) = \sum_{k=0}^{n} V_k(t)x^k, \tag{15}$$

is obtained. From the boundary conditions (3), we have

$$V_0(t) = p(t), \tag{16}$$

and

$$V_1(t) = q(t). \tag{17}$$

The approximate solutions (14) and (15) are not solutions of the problem (1)-(3), because expression (14) is obtained according to the initial condition (2) while expression (15) is obtained according to the boundary conditions (3). Thus, to obtain an approximate solution of the generalized Burgers-Huxley equation (1) that satisfies the conditions (2) and (3) simultaneously, we consider a convex combination of (14) and (15) as

$$u_{approx[n]}(x,t) = c\hat{u}_n(x,t) + (1-c)\check{u}_n(x,t), \tag{18}$$

where $c \in [0,1]$. The limit of $u_{approx[n]}(x,t)$ is equal to $u(x,t)$ when n approaches infinity. For determining the value of the parameter c, we follow the scheme presented in [26] to minimize the discrepancy between $u_{approx[n]}(x,0)$, $u_{approx[n]}(0,t)$ and $\frac{\partial u_{approx[n]}}{\partial x}(0,t)$ with $f(x)$, $\varphi(t)$ and $\psi(t)$ in (2) and (3).

Theorem 1. *Suppose that $f(x) \in L^2[(0,L)]$, $\phi(t), \psi(t) \in L^2[(0,T)]$ and $\|.\|$ denotes the L^2-norm. Let*

$$c_1 = \|\hat{u}_n(0,t) - \phi(t)\|,$$
$$c_2 = \|\frac{\partial \hat{u}_n}{\partial x}(1,t) - \psi(t)\|,$$
$$c_3 = \|\check{u}_n(x,0) - f(x)\|.$$

Then the optimal value for c in (18) is

$$c = \frac{c_3^2}{c_1^2 + c_2^2 + c_3^2}, \qquad n \geq 0. \tag{19}$$

Proof. According to conditions (1)-(3), we define the following residual function on the domain $\{(x,t)|(x,t) \in [0,L] \times [0,T]\}$ as

$$F_n(x,t;c) = \|u_n(0,t) - \phi(t)\| + \|\frac{\partial u_n}{\partial x}(1,t) - \psi(t)\| + \|u_n(x,0) - f(x)\|. \tag{20}$$

Substituting (18) into (20), we have

$$\begin{aligned} F_n(x,t;c) &= \|c\hat{u}_n(0,t) + (1-c)\check{u}_n(0,t) - \phi(t)\|^2 \\ &+ \|c\frac{\partial \hat{u}_n}{\partial x}(1,t) + (1-c)\frac{\partial \check{u}_n}{\partial x}(1,t) - \psi(t)\|^2 \\ &+ \|c\hat{u}_n(x,0) + (1-c)\check{u}_n(x,0) - f(x)\|^2. \end{aligned}$$

From (14), (15) and (18), we get

$$\begin{aligned} F_n(x,t;c) &= \|c\hat{u}_n(0,t) + (1-c)\phi(t) - \phi(t)\|^2 + \|c\frac{\partial \hat{u}_n}{\partial x}(1,t) + (1-c)\psi(t) - \psi(t)\|^2 \\ &+ \|cf(x) + (1-c)\check{u}_n(x,0) - f(x)\|^2 \\ &= \|c\hat{u}_n(0,t) - c\phi(t)\|^2 + \|c\frac{\partial \hat{u}_n}{\partial x}(1,t) - c\psi(t)\|^2 \\ &+ \|(1-c)\check{u}_n(x,0) - (1-c)f(x)\|^2 = c^2c_1^2 + c^2c_2^2 + (1-c)^2c_3^2. \end{aligned}$$

The optimal value of c will minimize the residual function F_n. Thus, differentiating F_n with respect to c and setting the result equal to zero, yields

$$c = \frac{c_3^2}{c_1^2 + c_2^2 + c_3^2}, \qquad n \geq 0.$$

4 Applications

We analyze here the efficiency and applicability of the weighted reduced differential transform method (WRDTM). In this line of thought, we apply the WRDTM to Cauchy problems of some special cases of the generalized Burgers-Huxley equations in the areas of mathematical physics and mathematical biology. In the sequel we adopt n terms when evaluating the approximate solution $u_n(x,t)$.

Example 1. *Consider the following problem*

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x}, \qquad x > 0, \ \ t > 0, \tag{21}$$

with initial condition:

$$u(x,0) = \lambda \left(1 - \tanh\left(\frac{\lambda x}{2}\right)\right), \tag{22}$$

and boundary conditions:

$$u(0,t) = \lambda \left(\tanh\left(\tfrac{\lambda^2 t}{2}\right) + 1\right), \qquad u_x(0,t) = -\tfrac{1}{2}\lambda^2 \text{sech}^2\left(\tfrac{\lambda^2 t}{2}\right), \tag{23}$$

where $\lambda \in \mathscr{R}$ is an arbitrary parameter.

The problem (21)-(23) has the exact solution $u(x,t) = \lambda(1 - \tanh(\frac{1}{2}\lambda(x - \lambda t)))$. By using the properties of the differential transformation with respect to t, we can write

$$U_k(x) = \frac{1}{k}\left(\frac{\partial^2}{\partial x^2}U_{k-1}(x) - \sum_{r=0}^{k-1}\frac{dU_r(x)}{dx}U_{k-r-1}(x)\right). \tag{24}$$

Starting with $U_0(x) = \lambda\left(1 - \tanh\left(\frac{\lambda x}{2}\right)\right)$, from (24) we find

$$U_1(x) = \frac{1}{2}\lambda^3 \text{sech}^2\left(\frac{\lambda x}{2}\right),$$
$$U_2(x) = 2\lambda^5 \sinh^4\left(\frac{\lambda x}{2}\right)\text{csch}^3(\lambda x),$$
$$U_3(x) = \frac{1}{24}\lambda^7(\cosh(\lambda x) - 2)\text{sech}^4\left(\frac{\lambda x}{2}\right),$$
$$\cdots.$$

The differential inverse transform of $U_k(x)$ gives:

$$\hat{u}_n(x,t) = \sum_{k=0}^{n} U_k(x)t^k. \tag{25}$$

Now, we take the differential transformation of the Eq. (21) with respect to x. We apply the properties of Table 1 yielding

$$V_k(t) = \frac{1}{k(k-1)}\left(\frac{\partial}{\partial t}V_{k-2}(t) - V_{k-2} + \sum_{r=0}^{k-2}(r+1)V_{r+1}(t)V_{k-2-r}(t)\right). \tag{26}$$

After substituting

$$V_0(t) = \lambda\left(\tanh\left(\frac{\lambda^2 t}{2}\right)+1\right),$$

and

$$V_1(t) = -\frac{1}{2}\lambda^2 \operatorname{sech}^2\left(\frac{\lambda^2 t}{2}\right),$$

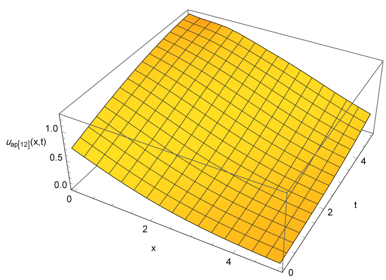
as the transformation of the boundary conditions in (23) , into (26), we obtain the next terms as

$$\begin{aligned} V_2(t) &= -2\lambda^3 \sinh^4\left(\frac{\lambda^2 t}{2}\right)\operatorname{csch}^3\left(\lambda^2 t\right), \\ V_3(t) &= -\frac{1}{24}\lambda^4\left(\cosh\left(\lambda^2 t\right)-2\right)\operatorname{sech}^4\left(\frac{\lambda^2 t}{2}\right), \\ &\cdots. \end{aligned}$$

Using the differential inverse transform of $V_k(x)$, we obtain

$$\breve{u}_n(x,t) = \sum_{k=0}^{n} V_k(t)x^k. \tag{27}$$

Suppose that $\lambda = 0.7$ and $n = 12$. According to (25), (27) and Theorem 1 we get $c = 0.999877$.
The approximate solution will be obtained by means of the expression (18). Figure 1 shows the exact and the approximate solutions of the problem for several values of t. The absolute error function $e_{12}(x,t) = |u(x,t) - u_{approx[12]}(x,t)|$ on the domain $\{(x,t)|(x,t) \in [0,5]\times[0,5]\}$, is shown in Figure 2.



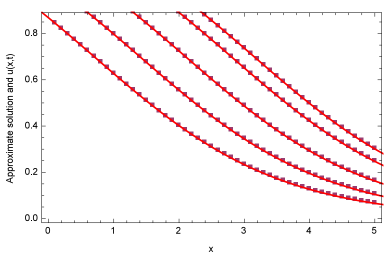


Figure 1
The exact and approximate solutions of example 1 with $n = 12$. Left: Plot of the approximate solution. Right: Exact solution (red line) and approximate solution (gray points) for various values of t.

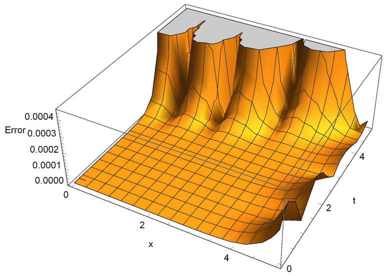


Figure 2
Absolute error for the approximate solution of example 1 with $n = 12$.

Example 2. *Consider the problem*

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} + u(1-u)(u-1), \qquad x > 0, \ t > 0, \tag{28}$$

with initial condition:

$$u(x,0) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right), \tag{29}$$

and boundary conditions:

$$u(0,t) = \tfrac{1}{2} - \tfrac{1}{2}\tanh\left(\tfrac{3t}{8}\right), \qquad u_x(0,t) = -\tfrac{1}{8}\operatorname{sech}^2\left(\tfrac{3t}{8}\right). \tag{30}$$

The equation (28) is called Chaffee-Infante equation representing a reaction Duffing model discussed in mathematical physics. The exact solution of this problem is as follow:

$$u(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{3t}{8} + \frac{x}{4}\right).$$

For applying the WRDTM we take the differential transform of (1) according to x and t, respectively, gives

$$\begin{aligned} U_k(x) = \tfrac{1}{k}\Bigg(& \tfrac{\partial^2}{\partial x^2}U_{k-1}(x) - U_{k-1}(x) + \sum_{r=0}^{k-1}\tfrac{\partial}{\partial x}U_r(x)U_{k-1-r}(x) \\ & +2\sum_{r=0}^{k-1}U_r(x)U_{k-1-r}(x) - \sum_{s=0}^{k-1}\sum_{r=0}^{s}U_r(x)U_{s-r}(x)U_{k-1-s}(x)\Bigg), \end{aligned} \tag{31}$$

$$\begin{aligned} V_k(t) = \tfrac{1}{k(k-1)}\Bigg(& \tfrac{\partial}{\partial t}V_{k-2}(t) + V_{k-2}(t) - \sum_{r=0}^{k-2}(r+1)\tfrac{\partial}{\partial x}V_{r+1}(t)V_{k-2-r}(t) \\ & -2\sum_{r=0}^{k-2}V_r(t)U_{k-2-r}(t) + \sum_{s=0}^{k-2}\sum_{r=0}^{s}V_r(t)V_{s-r}(t)V_{k-2-s}(t)\Bigg). \end{aligned} \tag{32}$$

For finding the solution of (28)-(30), we start the recursive relation (31) with $U_0(x) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)$ and the recursive relation (32) with $V_0(t) = \frac{1}{2} - \frac{1}{2}\tanh(\frac{3t}{8})$ and $V_1(t) = -\frac{1}{8}\operatorname{sech}^2(\dfrac{3t}{8})$. By using the relations (14), (15) and (18), the approximate solution will be obtained. Suppose that $n = 15$. From Theorem 1 we get

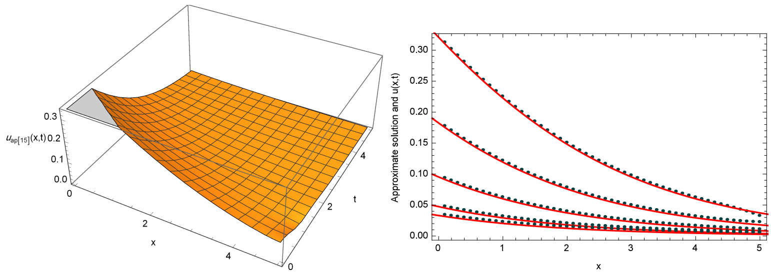


Figure 3
The exact and approximate solutions of example 2 with $n = 15$. Left: Plot of the approximate solution. Right: Exact solution (red line) and approximate solution (gray points) for various values of t.

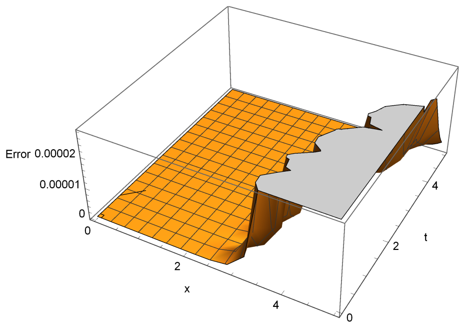


Figure 4
Absolute error for the approximate solution of example 2 with $n = 15$.

$c = 8.04462 \times 10^{-9}$. Figure 3 depicts the exact solution of (28)-(30) and its approximations in the domain $\{(x,t)|(x,t) \in [0,5] \times [0,5]\}$. The absolute error of the approximate solution, is shown in Figure 4.

Example 3. *Consider the equation (1) with $\alpha = \beta = \eta = \kappa = 1$, $\gamma = -1$ and $\delta = 2$. Also, in the conditions (2) and (3) assume that* $f(x) = \sqrt{\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{3}\right)}$, $p(t) = \sqrt{\frac{1}{2}\tanh\left(\frac{10t}{9}\right) + \frac{1}{2}}$ *and* $q(t) = -\dfrac{\operatorname{sech}^2\left(\frac{10t}{9}\right)}{12\sqrt{\frac{1}{2}\tanh(\frac{10t}{9}) + \frac{1}{2}}}$. *Under these assumptions, the exact solution of (1)-(3) is* $u(x,t) = \sqrt{\frac{1}{2}\tanh(\frac{10t}{9} - \frac{x}{3}) + \frac{1}{2}}$.
Taking the differential transform subject to x and t, we get the following recurrence relations

$$U_k(x) = \frac{1}{k}\left(\frac{\partial^2}{\partial x^2}U_{k-1}(x) + U_{k-1}(x) - \sum_{s=0}^{k-1}\sum_{r=0}^{s} U_r(x)U_{s-r}(x)U_{k-1-s}(x) - \sum_{s=0}^{k-1}\sum_{r=0}^{s} \frac{\partial}{\partial x}U_r(x)U_{s-r}(x)U_{k-1-s}(x)\right), \tag{33}$$

and

$$V_k(\mathrm{t}) = \frac{1}{k(k-1)}\Bigg(\frac{\partial}{\partial t}V_{k-2}(t) - V_{k-2}(t) + \sum_{s=0}^{k-2}\sum_{r=0}^{s}(r+1)V_r(t)V_{s-r}(t)V_{k-2-s}(t) + \sum_{s=0}^{k-2}\sum_{r=0}^{s}V_r(t)V_{s-r}(t)V_{k-2-s}(t)\Bigg). \tag{34}$$

Assuming that $n = 10$ and using (14), (15), (33) and (34), from (19) we get $c = 0.413211$. Substituting c in (18), yields an approximate solution. Figure 5 compares the approximate and the exact solution of the problem. The relative error function $r_{10}(x,t) = \frac{|u(x,t) - u_{approx[10]}(x,t)|}{|u(x,t)|}$ is shown in Figure 6.

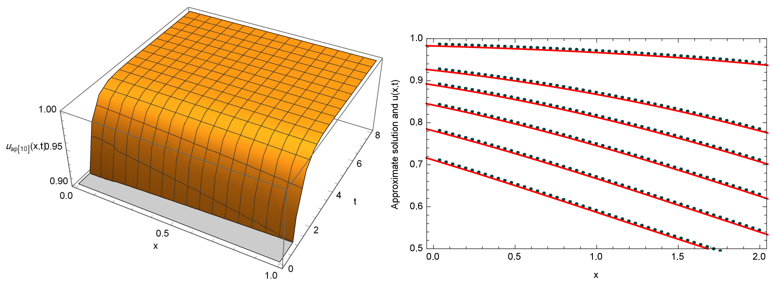


Figure 5
The approximate and exact solutions of example 3 with $n = 10$. Left: Plot of the approximate solution. Right: Exact solution (red line) and approximate solution (gray points) for various values of t.

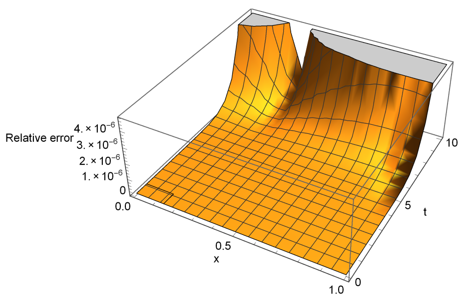


Figure 6
The relative errors for the approximate solution of example 3 with $n = 10$.

5 Conclusion

In this work a Cauchy problem of the generalized Burgers-Huxley equations was considered. Using the reduced differential transform method, a weighted algorithm to determine approximate-analytical solution was developed. To show the capability and reliability of the novel method, the solution of some special cases of the generalized Burgers-Huxley equation were obtained. The results confirm that the WRDTM is an efficient technique to solve such Cauchy problems.

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