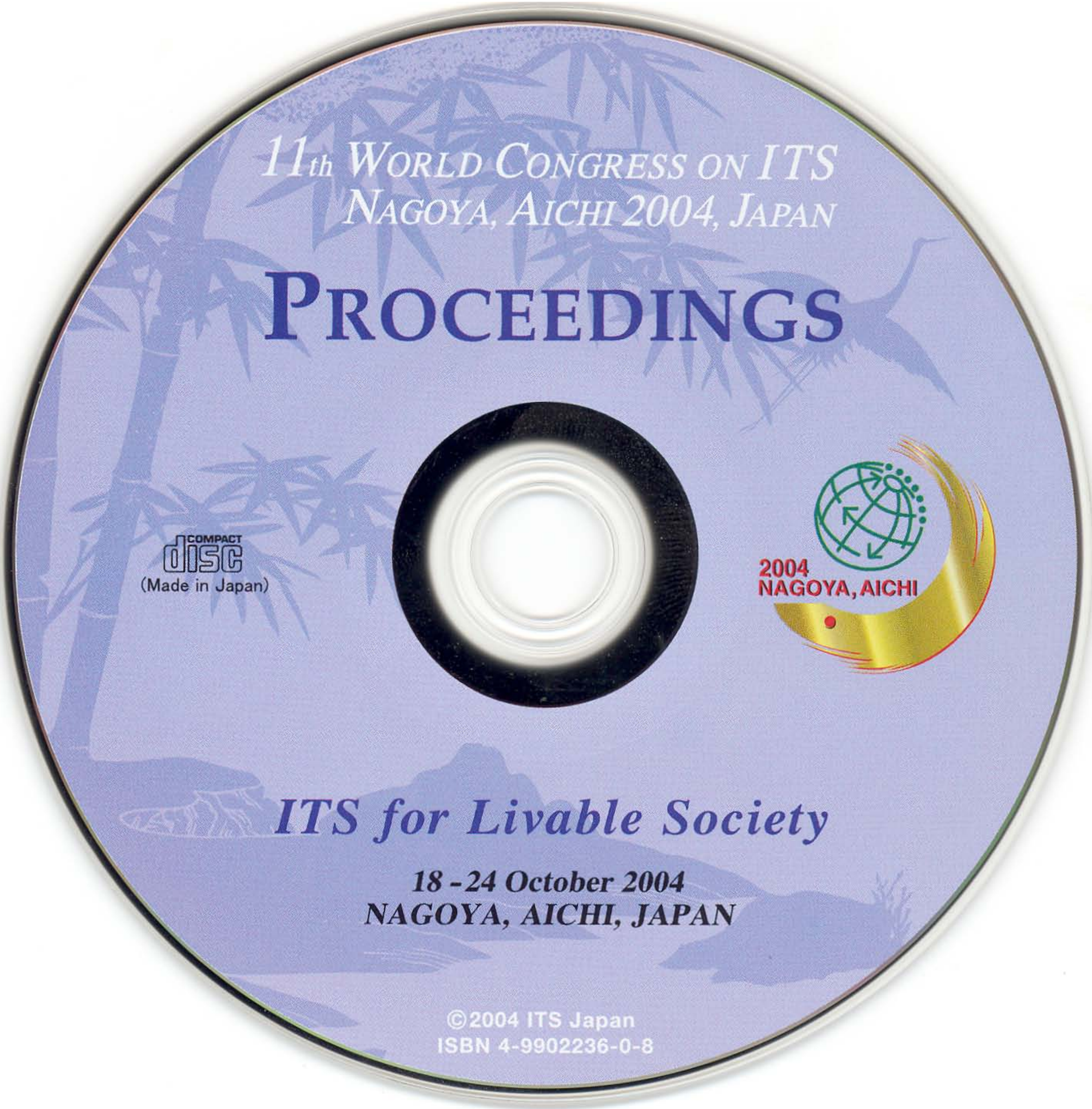
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A FOURIER PERSPECTIVE OF FREEWAY TRAFFIC

Lino Figueiredo
Professor, Institute of Engineering of Porto
Rua Dr. António Bernardino de Almeida, 4200-072 Porto, Portugal
TEL 351- 228340500, FAX 351-228321159, E-mail:lino@dee.isep.ipp.pt
J. A. Tenreiro Machado
Professor, Institute of Engineering of Porto
Rua Dr. António Bernardino de Almeida, 4200-072 Porto, Portugal
TEL 351-228340500, FAX 351-228321159, E-mail:jtm@dee.isep.ipp.pt
José Rui Ferreira
Professor, Faculty of Engineering of University of Porto
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
TEL 351-225081826, FAX 351 225081440, E-mail: jrf @fe.up.pt

ABSTRACT

This paper presents the Simulator of Intelligent Transportation Systems (SITS) and a dynamical analysis of several traffic phenomena. The SITS is based on a microscopic simulation approach to reproduce real traffic and considers different types of vehicles, drivers and roads. In order to process the resulting information, it is also introduced a new modelling scheme based on the embedding of statistics and Fourier transform. The results point out that it is possible to study traffic systems, taking advantage on the knowledge gathered with systems theory.

INTRODUCTION

Nowadays we have a saturation of the transportation infrastructures due to the growing number of vehicles over the last five decades. This situation affects substantially our lives, while people needs, more and more, to move rapidly between different places. The results are traffic congestion, accidents, transportation delays and larger vehicle pollution emissions.

The difficulties concerned with this subject motivated the research community to center their attention in the area of ITS (Intelligent Transportation Systems). ITS applies advanced communication, information and electronics technology to solve transportation problems such as, traffic congestion, safety, transport efficiency and environmental conservation. Therefore, we can say that the purpose of ITS is to take advantage of the appropriate technologies to create “more intelligent” roads, vehicles and users [1-2-3].

Computer simulation has become a common tool in the evaluation and development of ITS. The advantages of this tool are obvious. The simulation models can satisfy a wide range of requirements, such as: evaluating alternative treatments, testing new designs, training personal and analyzing safety aspects [4-5].
Bearing these facts in mind, this paper is organized as follows. Section 2 describes briefly the microsimulation model SITS. Section 3 presents simulation results related with the dynamic behaviour of a traffic system. Finally, section 4 presents some conclusions and outlines the perspectives towards future research.

THE SITS SIMULATION PACKAGE

SITS is a software tool based on a microscopic simulation approach, which reproduces real traffic conditions in an urban or non-urban network. The program provides a detailed modelling of the traffic network, distinguishing between different types of vehicles and drivers, and considering a wide range of network geometries. SITS uses a flexible structure that allows the integration of simulation facilities for any of the ITS related areas. This new simulation package is an object-oriented implementation written in C++. The overall model structure is represented on Figure 1.

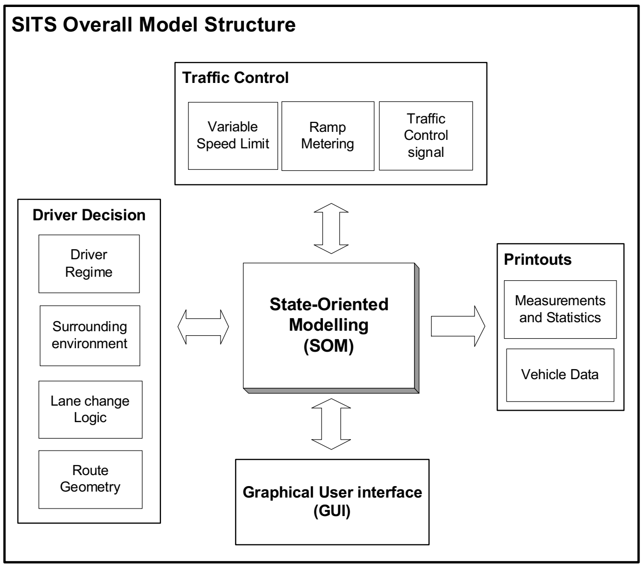


Figure 1. SITS overall model structure

SITS models each vehicle as a separate entity in the network according to the state diagram showing in figure 2. Therefore, are defined five states {1-aceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision} that represent the possible vehicle states in a traffic systems.

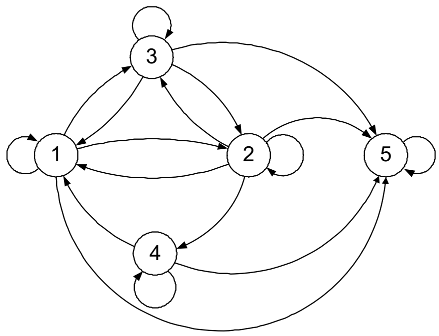


Figure 2. SITS state diagram (1-aceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision)

In this modelling structure, so called State-Oriented Modelling (SOM), every single vehicle in the network has one possible state for each sampling period. The transition between each state depends on the driver behaviour model and its surrounding environment. Some transitions are not possible; for instance, it is not possible to move from state #4 (stopped) to state #2 (braking), although it is possible to move from state #2 to state #4.

Included on the most important elements of SITS are the network components, travel demand, and driving decisions. Network components include the road network geometry, vehicles and the traffic control. To each driver is assigned a set of attributes that describe the drivers behavior, including desired speed, and his profile (*e.g.*, from conservative to aggressive). Likewise, vehicles have their own specifications, including size and acceleration capabilities. Travel demand is simulated using origin destination matrices given as an input to the model.

At this stage of development the SITS considers different types of driver behaviour models, namely car following, free flow and lane changing logic. SITS considers each vehicle in the network to be in one of two driver regimes: free flow and car-following. The free flow regime prevails when there is either (*i*) no lead vehicle in front of the subject vehicle or (*ii*) the leading vehicle is sufficiently far ahead that it does not influence the subject vehicles behavior. In the free flow case the driver travels at his desired maximum speed. Car-following regime dictates acceleration/deceleration decisions when a leading vehicle is near enough to the subject vehicle in order to maintain a safe following distance. Accelerations and decelerations are simulated using the Perception-Driver Model (PDM). According with the PDM, the driver decides to decelerate/accelerate depending on two factors: the difference between the distance to the leading vehicle and the critical distance, and his active state. The critical distance $d_{c,n}$ is defined as follows:

$$d_{c,n} = d_{sb,n} + d_{f,n} + L_{n+1} \quad (1)$$

where:

$d_{sb,n}$ is the safety braking distance for the vehicle n, given by equation (2),

$d_{f,n}$ is the following distance for the vehicle n, given by equation (5),
L_{n+1} is the length of the leading vehicle.

Figure 3 shows a schema of the critical distance for the n^{th} vehicle (assuming that the traffic conditions for both vehicles remain constant between time instants t_0 to t_1).

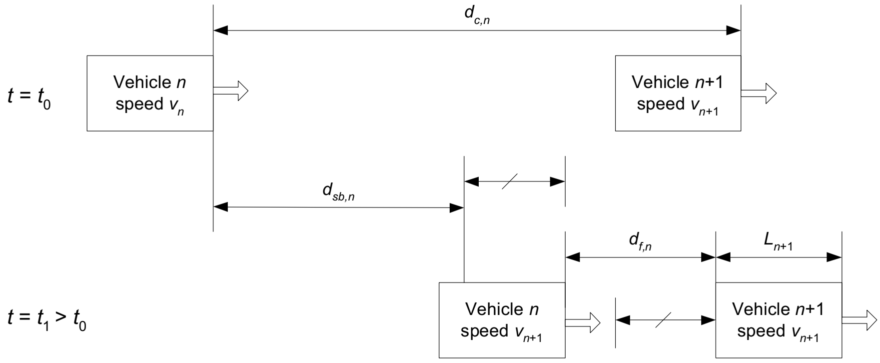


Figure 3. Critical distance schema

The safety braking distance $d_{sb,n}$ is given by:

$$d_{sb,n} = -\frac{\left(v_{n+1} - v_n\right)^2}{2\,a'_n} \left(1 + \frac{s_{n+1}}{a'_n}\right) \tag{2}$$

where:
v_n is the current speed of vehicle n.
v_{n+1} is the current speed of leading vehicle n+1.
a'_n is the deceleration of vehicle n given by equation (3).
s_{n+1} is the deceleration/acceleration of the leading vehicle n+1, given by equation (3) or (4) depending on his current state.

The driver reduces the speed by applying a deceleration a'_n. The model relates the vehicle performances with the driver characteristics.

$$a'_n = a'_{\max,c}\ \gamma_d \tag{3}$$

where:
$a'_{\max,c}$ is the maximum deceleration for a vehicle of type c.
γ_d is a parameter for driver type d ($0.1 < \gamma_d < 1.0$).

The value of γ_d can be changed at any time in order to prevent a collision. This parameter defines the driver profile (*e.g.*, from conservative $\gamma_d = 0.1$ to aggressive $\gamma_d = 1.0$).
The value of the deceleration/acceleration s_{n+1} depends on the state of the leading vehicle. If the vehicle is in state #2 then s_{n+1} is given by equation (3); otherwise if it is in state #1, s_{n+1} is given by equation (4). Therefore, $s_{n+1} = 0$ only when the vehicle is in one of the other states.

$$s_{n+1} = a_{\max,c}\,\gamma_d \tag{4}$$

where: $a_{\max,c}$ is the maximum acceleration for a vehicle of type c.

The following distance d_f depends on the speed of vehicle n and the associated driver profile, yielding:

$$d_f = v_n^2\,\gamma_d \tag{5}$$

The lane changing model in SITS uses a methodology that tries to mimic a driver behaviour when producing a lane change. This methodology was implemented in three steps: (*i*) decision to consider a lane change; (*ii*) selection of a desired lane; (*iii*) execution of the desired lane change if the gap distances are acceptable. A driver produces a lane change maneuver in order to increase speed, to overtake a slower vehicle or to avoid the lane connected to a ramp. After selecting a lane, the driver examines the lead g_b and lag g_a gaps in the target lane in order to determine if the desired change can be executed, as shown in fig. 4.

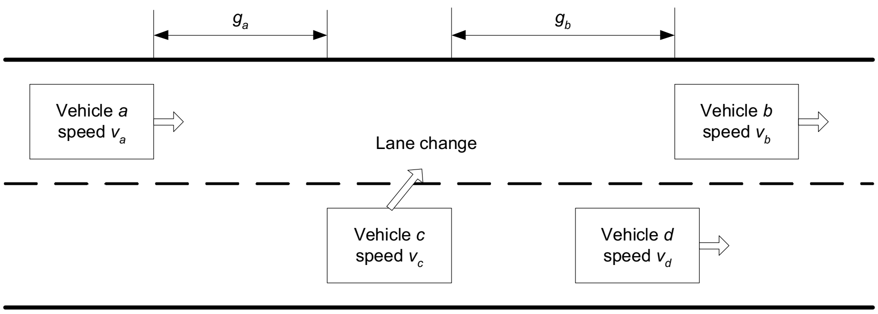


Figure 4. Lead g_b and lag g_a gaps for a lane change maneuver of vehicle n.

If g_a and g_b are higher than the critical distances between vehicle a and c, and c and b, respectively, then the desired lane change is executed in a single simulation sampling interval Δt.
SITS allows also the analysis of signal control devices and different road geometries considering road junctions and access ramps.
The simulation model adopted in the SITS is a stochastic one. Some of the processes include

random variables such as, individual vehicle speed and input flow. These values are generated randomly according to a pre-defined amplitude interval.

The main types of input data to the simulator are the network description, the drivers and vehicles specifications and the traffic conditions. The output of SITS consists not only in a continuously animated graphical representation of the traffic network but also the data gathered by the detectors, originating different types of printouts.

SITS tracks the movements of individual vehicles to a resolution of 10^{-2} sec and uses five different colours to represent the individual vehicle states; namely, stopped (red), acceleration (green), braking (yellow), cruise speed (blue) and collision (black), as represented on figure 5.

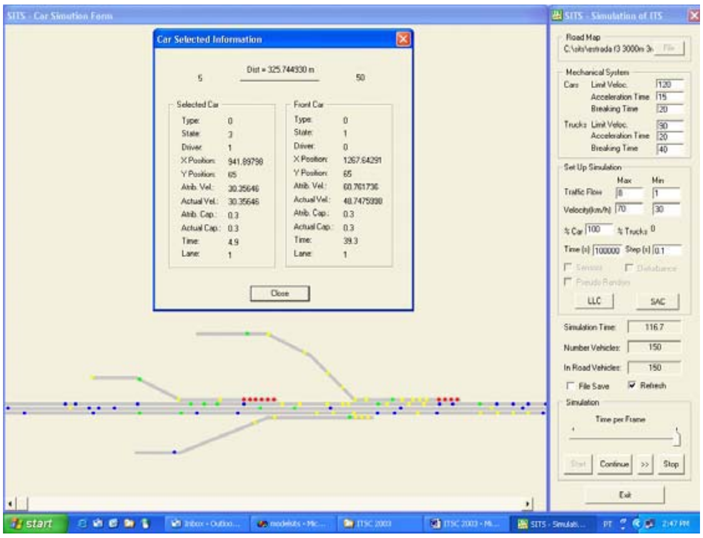


Figure 5. SITS animated graphical representation

SIMULATION RESULTS AND DYNAMICAL ANALYSIS

In the dynamic analysis are applied tools of systems theory. In this line of thought, a set of simulation experiments are developed in order to estimate the influence of the vehicle speed $v(t;x)$, the road length l and the number of lanes n_l in the traffic flow $\phi(t;x)$ at time t and road coordinate x. For a road with n_l lanes the Transfer Function (TF) between the flow measured by two sensors is calculated by the expression:

$$G_{r,k}\,(s;\, x_j,x_i) = \Phi_r(s;x_j)/\Phi_k(s;x_i) \qquad (6)$$

where k, r = 1,2,…, n_l define the lane number and, x_i and x_j represent the road coordinates ($0 \le x_i \le x_j \le l$), respectively. The Fourier transform for each traffic flow is:

$$\Phi_r(s;x_j) = \mathcal{F}\{\phi_r(t;x_j)\} \quad (7a)$$

$$\Phi_k(s;x_i) = \mathcal{F}\{\phi_k(t;x_i)\} \quad (7b)$$

Figure 6 show an overall schema of the notation adopted in the analysis of traffic dynamics.

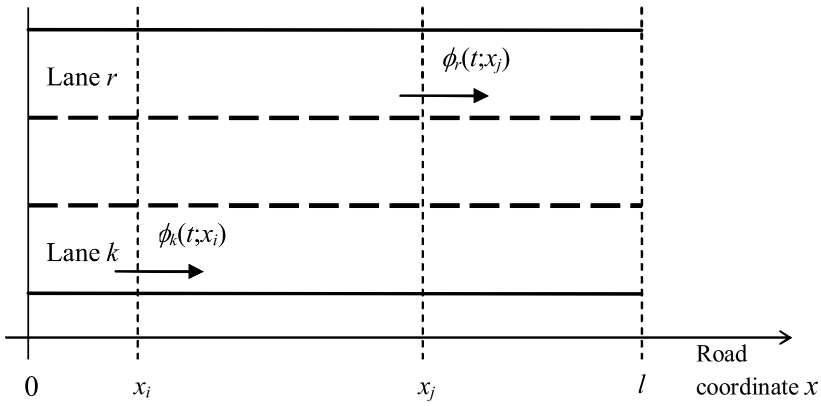


Figure 6. Overall schema of the notation adopted

It should be noted that, traffic flow is a time variant system but, in the sequel, it is shown that the Fourier transform can be used to analyse the system dynamics.

The first group of experiments considers a one-lane road (*i.e.*, $k = r = 1$) with length $l = 1000$ m. Across the road are placed n_s sensors equally spaced. The first sensor is placed at the beginning of the road (*i.e.*, at $x_i = 0$) and the last sensor at the end (*i.e.*, at $x_j = l$). Therefore, we calculate the *TF* between two traffic flows at the beginning and the end of the road such that, $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} for a vehicle speed $v_1(t;0) \in [30, 70]$ km h^{-1}, that is, for $v_1(t;0) \in [v_{av} - \Delta v, v_{av} + \Delta v]$, where $v_{av} = 50$ km h^{-1} is the average vehicle speed and $\Delta v = 20$ km h^{-1} is the maximum speed variation. These values are generated according to a uniform probability distribution function.

Figure 7 shows the Polar plot of the *TF* $G_{1,1}(s;1000,0) = \Phi_1(s;1000)/\Phi_1(s;0)$ between the traffic flow at the beginning and end of the one-lane road. It can be observed that the result is distinct from those usual in systems theory revealing a large variability. Moreover, due to the stochastic nature of the phenomena involved different experiments using the same input range parameters result in different *TF*s.

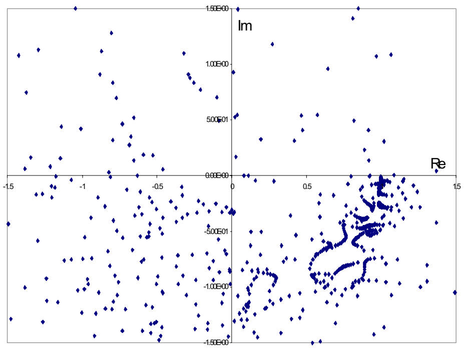


Figure 7. Polar plot of the *TF* $G_{1,1}(s;1000,0)$ for $n = 1$ experiment with $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} and $v_1(t;0) \in [30, 70]$ km h^{-1} ($v_{av} = 50$ km h^{-1}, $\Delta v = 20$ km h^{-1}, $l = 1000$ m and $n_l = 1$)

This phenomenon makes the analysis complex and experience demonstrates that efficient tools capable of rendering clear results are still lacking. Moreover, classical models are adapted to 'deterministic' tasks, and are not well adapted to the 'random' operation that occurs in systems with a non-structured and changing environment.

In order to overcome the problems, alternative concepts are required. Statistics is a mathematical tool well adapted to handle a large volume of data but not capable of dealing with time-dependent relations. Therefore, to overcome the limitations of statistics, it is adapted a new method, that takes advantage of the Fourier transform by embedding both tools.

Bearing these facts in mind, the first stage of the new modelling formalism starts by comprising a set of input variables that are free to change independently (*ivs*) and a set of output variables that depend on the previous ones (*ovs*). In a traffic system the *ivs* and *ovs* are defined as $\phi_k(t;x_i)$ and $\phi_r(t;x_j)$, that is, the traffic flows at time t, at positions x_i and x_j respectively, for the k,r^{th} lanes ($k, r = 1,\ldots, n_l$).

The second stage of the formalism consists on embedding the statistical analysis into the Fourier transform through the algorithm:

i. A statistical sample is obtained by carrying out a large number (n) of experiments having appropriate time/space evolutions. All the *ivs* and *ovs* are calculated and sampled in the time domain.

ii. The Fourier transform is computed for each of the *ivs* and *ovs.*

iii. Statistical indices are calculated for the Fourier spectra obtained in *ii*).

iv. The values of the statistical indices calculated in *iii*) (for all the variables and for each frequency) are collected on a 'composite' frequency response entitled Statistical Transfer Function (*STF*) of each *TF*.

The previous procedure may be repeated for different numerical parameters (*e.g.*, traffic flow, vehicle speed, road geometry) and the partial conclusions integrated in a broader paradigm [6].

This *STF* scheme was compared with the adoption of the pseudoinverse $\mathbf{\Phi}^{\#}$ for the calculation of the transfer matrix $\mathbf{T}$:

$$\mathbf{T}\ \mathbf{\Phi}(s;0) = \mathbf{\Phi}(s;1000) \Rightarrow \mathbf{T} = \mathbf{\Phi}(s;1000)\ \mathbf{\Phi}^{\#}(s;0) \tag{8}$$

where $\mathbf{\Phi}(s;0) = [\Phi^1(s;0)|\ldots|\Phi^n(s;0)]$ and $\mathbf{\Phi}(s;1000) = [\Phi^1(s;0)|\ldots|\Phi^n(s;1000)]$, are the matrices composed by the results of the n different experiments. Nevertheless, with this method it was observed an inferior convergence due to the presence of a delay factor in the *TF* and, consequently, in the sequel is adopted the previous *STF* concept.

To illustrate the proposed modelling concept, the previous simulation was repeated for a sample of $n = 2000$ and it was observed the existence of a convergence of the *STF*, $T_{1,1}(s;1000,0)$, as show in Fig. 8, for a one-lane road with length l = 1000 m $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} and $v_1(t;0) \in [30, 70]$ km h^{-1}.

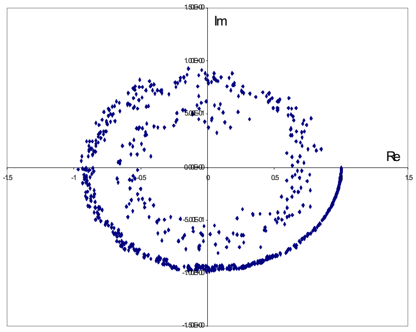


Figure 8. Polar plot of the *STF* $T_{1,1}(s;1000,0)$ for n = 2000 experiments with $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} and $v_1(t;0) \in [30, 70]$ km h^{-1} (v_{av} = 50 km h^{-1}, Δv = 20 km h^{-1}, l = 1000 m and n_l = 1).

Based on this result we can approximate numerically the *STF* to a fractional order [7] system with time delay yielding the approximate expressions of the type:

$$T_{1,1}(s;1000,0) = \frac{k_B\, e^{-\tau s}}{\left(\frac{s}{p}+1\right)^{\alpha}} \tag{9}$$

The parameters (k_B, τ, p, α) vary with the average speed v_{av}, the range of variation Δv, the road length l, the input vehicle flow ϕ_1 and the number of lanes n_l. For example, Figure 9 shows τ , p, and α *versus* Δv for $v_{av} = 50$ km h^{-1} and $n_l = 1$.

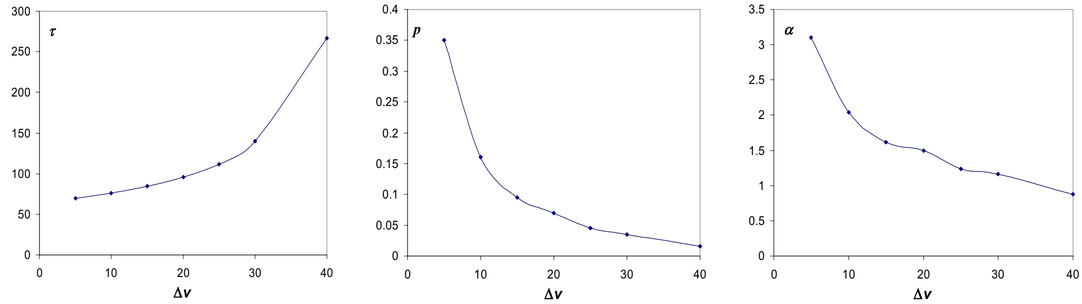


Figure 9. Time delay τ, pole p and fractional order α *versus* Δv for an average vehicle speed $v_{av} = 50$ km h^{-1}, $n_l = 1$, $l = 1000$ m and $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}

It is interesting to note that $(\tau, p) \to (\infty,0)$ when $\Delta v \to v_{av}$ and $(\tau, p) \to (l/v_{av}, \infty)$ when $\Delta v \to 0$. These results are consistent with our experience that suggests a pure transport delay $T(s) \approx e^{-\tau s}$ ($\tau = l/v_{av}$), when $\Delta v \to 0$, and $T(s) \approx 0$, when $\Delta v \to v_{av}$ (because of the existence of a blocking cars, with zero speed, on the road).

On the other hand, figure 10 show τ, p and α *versus* v_{av} for $\Delta v = 20$ km h^{-1} $n_l = 1$.

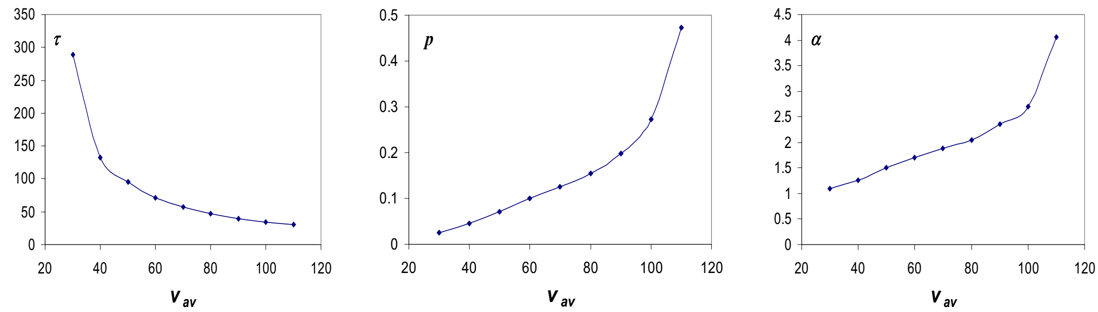
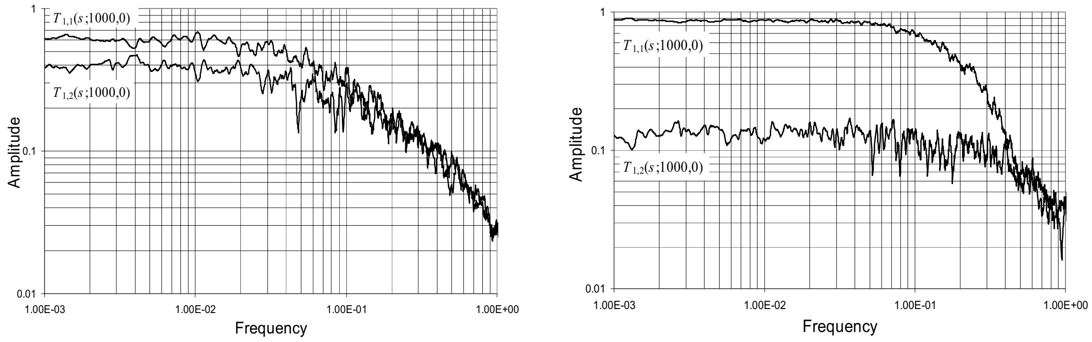


Figure 10. Time delay τ, pole p and fractional order α *versus* v_{av} for an range of variation $\Delta v = 20$ km h^{-1}, $n_l = 1$, $l = 1000$ m and $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}.

In this case we have $(\tau, p) \to (\infty,0)$ when $v_{av} \to \Delta v$ and $(\tau, p) \to (0,\infty)$ when $v_{av} \to \infty$, which has a similar intuitive interpretation.

In a second group of experiments are analyzed the characteristics of the *STF* matrix for roads with several lanes considering identical traffic conditions (*i.e.*, $\phi_k(t;0) \in [1, 8]$ vehicles s^{-1}, $k = 1,2$, $l = 1000$, $\Delta v = 20$ km h^{-1}). Figure 11a depicts the amplitude Bode diagram of $T_{1,1}(s;1000,0)$ and $T_{1,2}(s;1000,0)$ for $v_{av} = 50$ km h^{-1} (*i.e.*, $v_k(t;0) \in [30, 70]$ km h^{-1}).



a) b)

Figure 11. Amplitude Bode diagram for $T_{r,k}(s;1000,0)$, $n_l = 2$, $l = 1000$ m, $\phi_k(t;0) \in [1, 8]$ vehicles s^{-1}, $\Delta v = 20$ km h^{-1}, $k = 1,2$: a) $v_{av} = 50$ km h^{-1} b) $v_{av} = 90$ km h^{-1}.

We verify that $T_{1,1}(s;1000,0) \approx T_{2,2}(s;1000,0)$ and $T_{1,2}(s;1000,0) \approx T_{2,1}(s;1000,0)$. This property occurs because SITS uses a lane change logic where, after the overtaking, the vehicle tries to return to the previous lane. Therefore, lanes 1 and 2 have the same characteristics leading to identical *STF*.

Figure 11b presents the amplitude Bode diagram of $T_{1,1}(s;1000,0)$ and $T_{1,2}(s;1000,0)$ for $v_{av} = 90$ km h^{-1} (*i.e.*, $v_k(t;0) \in [70, 110]$ km h^{-1}). Comparing Fig. 11a and these results, we conclude that the transfer matrix elements vary significantly with v_{av}. Moreover, the *STF* parameter dependence is similar to the one-lane case represented previously. Figure 12 shows the variation parameters of the k_B, p, and α for $T_{1,1}(s;1000,0)$ *versus* v_{av} with $\Delta v = 20$ km h^{-1} and $n_l = 2$.

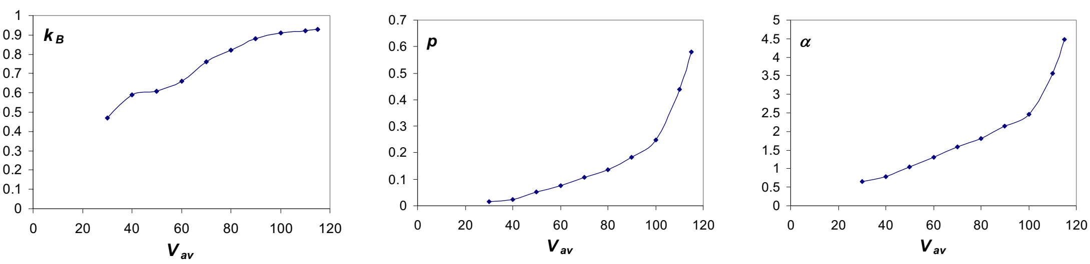


Figure 12. Gain k_B, pole p and fractional order α *versus* v_{av} for $T_{1,1}(s;1000,0)$ with $\Delta v = 20$ km h^{-1}, $n_l = 2$, $l = 1000$ m and $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}.

We conclude that:

i. The time delay τ is independent of the number of lanes n_l.

ii. For a fixed set of parameters we have for each *STF* gain $\times$ bandwidth $\approx$ constant.

iii. For each row of the transfer matrix, the sum of the *STF* gains is the unit.

iv. The gains and the poles of the diagonal elements of the *STF* matrix are similar. The gain of the non-diagonal elements, that represent dynamic coupling between the lanes, are lower (due to *iii*), but the corresponding pole are higher (due to *ii*).

v. The fractional order α increases with v_{av}. Nevertheless, the higher the number of lanes the lower the low-pass filter effect, that is, the smaller the value of α.

CONCLUSIONS

Based on the SITS package several experiments were carried out in order to analyse the dynamics of the traffic systems. The results of using classical system theory tools point out that it is possible to develop traffic systems, including the knowledge gathered with automatic control algorithms. In his line of thought it was also presented a new modelling formalism based on the embedding of statistics and Fourier transform. The analysis leads to simple and clear conclusions compatible with the common knowledge in systems theory. Therefore, we verify that the proposal modelling scheme, based on the new software simulation package SITS, leads to comprehensible results, compatible with common control systems engineering practice. Furthermore, the adoption of the fractional calculus tools is straightforward in the perspective of capturing dynamic characteristics that vary continuously with the system operating conditions.

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