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Recursive Modelling in Predicting Excess Returns: Case of the Nairobi Securities Exchange

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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List of Abbreviations

dNSE	Change in the Value of the NSE Index
NSE	Nairobi Securities Exchange
M2	Broad Money Stock
Infl	Inflation rate
tbond	364 day Treasury Bond
fx	Foreign Exchange Rate

Abstract

The aim of this study was to test the applicability of a recursive modelling approach in modelling stock market returns in the Nairobi Securities Exchange. The dependent variable was the Nairobi Securities Exchange All Share Index, with a core assumption being that firms do not pay dividends. I test the applicability of recursive modelling using three returns models, each containing different regressors, and compare the performance of the models in predicting future values of the index, as well as the performance of the recursive forecasting model compared to a dynamic forecasting model. I find that a recursive model is capable of predicting future values of the index using all three models, with varying performance among the models, but fail to find conclusive evidence to suggest that the recursive forecasting model significantly outperforms a dynamic forecasting model.

Key Words

Recursive Modelling, Forecasting, Dynamic Forecasts, Static Forecasts, Model Performance, Nairobi Securities Exchange

1. INTRODUCTION

1.1 Background of the Study

Implicitly or explicitly, it is assumed in finance literature that historic results have at least some predictive ability of future returns. In this regard, most performance and forecasting measures are computed using historic data but are justified on the basis of predicted relationships (Sharpe, 1994). This means that few forecasting models give convincing solutions as to how they arrive at target return. Lately, quite a number of prominent analytic models of ex post relationships have been passed with regard to their ability and dependability in analyzing historic data. Ex ante models, however, are yet to come up with dependable forecasts of predicting performance, given the sociological and psychological backdrop that many of them possess.

As economic agents form expectations, they build up models of the economy and act on the basis of predictions generated by these models. These anticipative models need neither be explicit, nor coherent, nor mutually consistent and because of such difficulties, the mathematical tools economists customarily use, which exploit linearity, fixed points, and systems of differential equations, cannot provide a deep understanding of a constantly evolving market.

However, a promising step towards modelling stock returns has been through the simulation of investor's decisions in real time using publicly available information on a set of factors thought *a priori* to be relevant to predicting stock returns, and the formulation of a model that changes with changing expectations for each period. This approach develops into what is known as recursive learning and modelling as was introduced by Keim and Stambaugh (1986) and remodeled by Pesaran and Timmerman (2000). A recursive forecasting model is one where the initial estimation date is fixed, but additional observations are added or removed one at a time to the estimation period (Brooks, 2008).

Due to the time varying relationship between risk, return and risk premia, it seeks to encompass seasonality in business cycles as well as include shocks that may arise during the course of the forecast period. The model tries to mitigate any benefits that may arise from hindsight, or relying on historic information, in order to ensure a mostly forward looking approach. Recursive modelling would be useful for active fund managers who would prefer to have a forecasting function that allows for rebalancing according to the investors' expectations.

Prior to developing such a model, it is important to first investigate the aspect of predictability of stock returns if any. Studies such as (Lucas, 1978) have suggested that there may be predictable elements in stock returns. Tied to this, Pesaran and Timmerman (2000) found that the predictability of stock market returns tended to be particularly pronounced in periods of economic regime switches, where markets are relatively unsettled and investors are particularly uncertain as to which forecasting model to use. Recent shocks in our capital markets such as the global financial crisis, the 2007-2008 post-election violence and the sharp depreciation of the value of the Kenyan Shilling in 2011 have enhanced the need for more robust forecasting models that allow for shocks in the paths taken by capital asset prices. Factors such as continuous learning and the continuous growth in the number of sophisticated market participants illustrated by Andrew Lo, (2004) introduce the need for evolving models that cater for shocks in future periods. The evolution of forecasting models as time elapses reflects the learning processes of an investor or the changing nature of the underlying data generating process, or both as it is difficult to disentangle these two effects (Pesaran & Timmerman, 2000).

Another approach of evaluating the economic significance of stock market predictability would be to see if the evidence could have been exploited successfully in investment strategies. This could be through evaluating the track records of portfolio managers in real time and observe to see if these portfolios generate systematically any excess return (Pesaran & Timmerman, 1995). The key advantage of this is that it ensures that investors' portfolio decisions are based exclusively on historical information. However, this approach does not provide insight as to which specific factors may have been responsible for predicting stock returns, nor does it guarantee that the information used by the managers was publicly available. Pesaran and Timmerman (1995) suggest that an alternative approach would be that of simulating investors' decisions in real time using publicly available information on a set of factors thought *a priori* to have been relevant to forecasting stock returns.

1.2 Problem Statement

In light of all these developments, it becomes apparent that robust models that predict future performance or paths taken by excess returns *a priori* are either misunderstood, suffer gross misspecification errors, or are simply not popular among finance academics and practitioners. In addition, there is little to no compelling evidence suggesting that such models may be inaccurate and not dependable, if at all feasible. This study therefore sought to determine the effectiveness of such models with data from an emerging economy – Kenya – with emphasis on the movements of the All Share Index on its sole bourse, the Nairobi Securities Exchange.

1.3 Research Questions

The study sought to answer the following questions:

i. How feasible is the recursive modelling technique in modelling excess returns in frontier markets such as the Nairobi Securities Exchange?
ii. How appropriately does the model incorporate expected shocks in the economy?
iii. Does it offer any advantages over dynamic forecasting techniques?

1.4 Objectives of the Study

i. To determine the applicability and feasibility of the recursive modelling technique to modelling future returns in frontier markets such as the Nairobi Securities Exchange.
ii. To examine how effectively the model incorporates shocks in the economy.
iii. To determine advantages, if any, that such a model may carry over dynamic or multiperiod forecasting models.

1.5 Significance of the Study

The study offers valuable contribution to theory and practice by contributing to the limited literature on Kenya's stock market by evaluating how applicable a recursive modelling technique is to investors who wish to model their returns in real time. Given that the focus of major studies on recursive modelling has been on developed markets, this study aims to determine the predictability of excess returns on frontier markets such as the Nairobi Securities Exchange under a recursive framework while incorporating a few additions to both static and dynamic forecasting models and comparing the results.

The objective here is not to determine whether recursive modelling is doable or not. It is to test whether it can be a viable alternative to naïve or popular forecasting models such as the random walk model. At this point it would be useful for the reader to note that all theories can inevitably be found to have errors due to issues such as precision in the testing. Our emphasis is on an open minded investor with no strong beliefs in any particular model.

The next sections are ordered as follows: Chapter 2 discusses literature on predictability, forecasting, and recursive modelling; Chapter 3 discusses the methodology we wish to employ and Chapter 4 ends by stating our assumptions.

2. LITERATURE REVIEW

'It is now widely accepted that excess returns are predictable by variables such as dividend-price ratios, earnings-price ratios, dividend-earnings ratios, and an assortment of other financial indicators." (Lettau & Ludvigson, 2001). This view together with other significant contributions to financial literature have concluded that stock returns can be predicted by means of publicly available information, such as time series data on financial and macroeconomic variables with important business cycle components. Pesaran and Timmerman (1995) and Keim and Stambaugh (1986) list the important variables for predicting stock behavior to be interest rates, monetary growth rates, changes in industrial production, inflation rates, earnings-price ratios and dividend yields.

Predictability of excess returns may not necessarily imply stock market inefficiency, as stated by (Lo & Mackinlay, 1987), and can be interpreted only in conjunction with, and in relation to, an intertemporal equilibrium model of the economy. Inevitably however, all theoretical attempts at interpretation of excess return predictability will be model dependent and may turn inconclusive (Fama , 1991).

Fama and French (1986) found that long holding period returns are significantly negatively serially correlated, implying that 25 to 45 percent of the variation in longer horizon returns is predictable from past returns. Lo and Mackinlay (1987) however found sharply contrasting results. They found significant positive serial correlation for weekly and monthly holding period returns. These findings, though contrasting, reflect that future returns can be predicted and are not entirely random. In addition to such findings, a returns fads model, they argue, may provide a likely explanation for the stochastic properties of short returns.

Models in support of the random walk hypothesis are considered redundant by Lo et al., (1987) who state that stock prices do not follow a random walk. For their period of testing, they found that the random walk model was strongly rejected throughout the entire sample period. This result, they asserted, could neither be attributed to the effects of infrequent trading nor time varying volatilities. The random walk hypothesis simply did not work. Given such evidence, they reiterate that their work on the failure of the random walk hypothesis should not be viewed as supporting a mean-reverting stationary model of asset prices, but rather as consistent with a specific non-stationary alternative hypothesis. In light of their recommendations, predictive regression of non-stationary variables may prove quite difficult to interpret. For our variables, we added lags to add stationarity and in consideration of (Lo & Mackinlay, 1987) we assumed a less stationary stance by predicting one period ahead returns.

More recently, work done by Torous, Valkanov, & Yan (2004) refutes the previous assertion by Lo and Mackinlay (1987) on the redundancy of the random walk hypothesis. Since rational expectations are not expected to change, they say, an expectation about the future quantity must follow a random walk if innovations in expectations are independently and identically distributed. In this light, future expectations form the basis of asset pricing, meaning that explanatory variables which are functions of asset pricing should also follow a random walk. This random element in the stochasticity of stock returns may therefore limit models to forecasting only one period ahead in order to get prudent estimates of future returns. In any case, Torous, Valkanov, & Yan, (2004) find little evidence of predictability in horizons greater than one year in the entire sample period they studied (1926-1994). Whether the random walk theory holds or not, these schools of thought may be viewed as supporting one period ahead forecasting as a reliable form of modelling.

Predictability of stock returns however remains a focus of research controversy because of (i) concerns about data mining, (ii) concerns that the statistical methods implemented to explore predictability indicate spurious results, (iii) concerns about the large gap between the in-sample evidence of predictability and the real-time performance of active investment management, and (iv) concerns about the poor out-of-sample performance of predictive regressions (Avramov & Chordia, 2005).

However, if such predictability of stock market returns was to exist, it would not guarantee that an investor could earn profits from a trading strategy based on such forecasts (Pesaran & Timmerman, 1995). This is because monthly stock returns do not follow a standard distribution but are instead more leptokurtic in nature. Transaction costs may also wade off any profits gained from such strategies, as compared to profits realized by managers who employ a buy-and-hold strategy. Pesaran and Timmerman, (1995) go around this drawback by employing a simple switching strategy, which asserts that investors should hold equity in periods where the business cycle indicators suggest that equity returns will outperform returns from holding bonds, and reverting their positions when it is expected that bonds will perform better.

Therefore, can we conclude that this model can work? Welch and Goyal (2008) assert that all predictive models constructed prior to 2005 are unstable and inconsistent both out-of-sample and in-sample. According to them, these models would not have helped an investor with access only to publicly available information to profitably time the market. Most models, they found, tended to predict poorly late in the samples, and not early enough to be able to enable investors reap profits. They further assert that as at 2005, most models developed early in literature have lost statistical significance both in-sample and out-of-sample. Those models not only failed to beat the conditional mean but underperformed it outright. However, the limited nature of the scope of the models they studied cannot be gainsaid, as it would be imprudent to rule out all existing models as implausible.

In comparison, Campbell and Thompson (2008) dispute Welsh's findings by arguing that many predictive regressions do in fact beat the historical average return, once weak restrictions are imposed on the signs of the coefficients and return forecasts. They find that even better results can be obtained by imposing the restrictions of steady state valuation models, thereby removing the need to estimate the average from a short sample of volatile stock returns. In short, restricted regressions perform relatively better than unrestricted regressions. These restrictions are that: (i) the regression coefficient has the theoretically expected sign and (ii) the fitted value of the equity premium is positive. Questions may however be raised about the validity of the second assumption in periods when the equity risk premium is expected to be negative. Out of sample explanatory power, however small, remains economically meaningful for mean-variance investors.

(Campbell & Thompson, 2008) further state that the performance of models does not depend sensitively on the particular valuation ratio that is used or the manner in which it is adjusted for long run growth. This result, they interpret, illustrates that even false theoretical restrictions can be helpful in forecasting if they reduce the variance of a predictor more than they increase its bias. It would therefore be healthy for a researcher to be careful and avoid falling into this trap.

Lucas (1978) encourages the use of recursive modelling to forecast returns provided that sufficient "impatience" of the investor is assumed. For our model, we will assume the investor instead of being impatient, assumes that the factors that affect market prices undergo constant change and evolution such that in order to continue making profits, one has to evolve with the market and its dynamics.

Pesaran and Timmerman (1995) found that recursive predictions based on the $\bar{R}^2$ and the recursive Sharpe criterion performed better than recursive predictions from the model that included all regressors. This means that it may be worthwhile for investors to engage in an active search process to find an adequate forecasting equation rather than just basing their forecasts on a fixed model specification that includes the entire set of regressors.

On model selection Pesaran and Timmerman (1995) strongly rebuke any analysis of stock market predictability that focuses on a particular forecasting model that is taken as known with certainty over the whole sample period. They criticize such an analysis for ignoring "model uncertainty" and the impact that "model uncertainty" is likely to have on investors in real time. This is largely true as it cannot be proven mathematically or otherwise, that an investor has all the accurate information required to forecast future returns, and that such information can be used publicly; for if such information were to leak to the public, any benefits that would be gained from acting on the information will end up being absorbed by the rest of the market participants and would possibly reestablish a new equilibrium.

Rather than assume that investors somehow had prior knowledge as to the specific model that was going to perform well ex ante, a much weaker assumption is made about investors' beliefs over the sort of business cycle and financial variables thought as being potentially important in forecasting portfolio returns. Based on these beliefs, Pesaran and Timmerman (2000) assume that agents establish a base set of potential forecasting variables and, at each point in time, search for a reasonable model specification, capable of predicting stock returns across that set. Notably, this procedure assumes that at each point in time, investors use only historically available information to select the model according to a predefined model selection criterion and then use the model to make one period ahead predictions of excess returns. This has the advantage of ensuring that any shocks that occur during a period are accounted for in the next period's recursive forecasts. It is also advantageous in that it allows constant rebalancing of a portfolio given the high volatility of equity markets and changing market dynamics.

Similarly on alternating variables a priori, Ferson, Sarkissian and Simin (2000) explored the implications of a persistent explanatory variable on the properties of predictive regressions. For their case, they treated expected returns as unobservable while demonstrating that, if expected returns are persistent, a spurious regression may be obtained if realized returns are regressed against an unrelated but persistent explanatory variable. Therefore, ideally, one regressor, should not be present throughout all the forecasting periods, unless it is a core variable (discussed in Section 2.3)

2.1 Forecasting Horizon

Torous, Valkanov, and Yan (2004) find that the statistical significance of return predictability is however much more pronounced at longer return horizons. This can be interpreted as reliable evidence of enhanced predictability at longer rather than at shorter horizons. According to them, it is better to rely on longer as opposed to shorter time horizons as this minimizes the noise inherent in returns, which in turn allows the posited predictive relation to be seen more clearly. Their subsequent findings however conclude that it is return horizons of less than one year for which statistically significant forecasting evidence of relation was found when the local-to-unity behavior of commonly used explanatory variables is taken to account. This suggests that the previous evidence reported at long horizons reflect that conventional statistics have long had the tendency of over-rejecting the null hypothesis of no predictability when the persistent behavior of the explanatory variables is not taken into account.

2.2 Excess Returns

Consider the following decomposition of excess returns

$$\rho_{t+1} = e_t + u_{t+1},$$

$$\text{Where } \rho_{t+1} = \Delta \ln(P_{t+1}) + \frac{D_{t+1}}{P_t} - rf_{t,}$$

And P_{t+1} is the end of period share price, D_{t+1} represents dividends per share paid during period $t+1$. e_t denotes the predictable part of excess returns, while u_{t+1} is a martingale difference process representing the unpredictable part. General asset pricing models, such as (Lucas, 1978) suggest that e_t need not be zero. This is largely the focus of our study, which is to determine the predictability of excess returns. Expected returns may vary over time but only to the extent to which they reflect a time varying covariance between investors' marginal rates of substitution and excess returns, relative to the variation in the conditional expectation of the marginal rates of substitution. This means that expected returns vary according to the rate at which investors are willing to forego consumption in order to invest. This can be seen from the equations below as we modify the standard first order Euler equation for a representative investor in a frictionless market, that is,

$$E_t(M_{t+1}\rho_{t+1}) = 0$$

where $E_t(.)$ is the conditional expectations operator with respect to the information available to the agent at time t and M_{t+1} is the stochastic discount factor representing the investor's marginal rate of substitution between future consumption in period $t+1$ and current consumption in period t.

Rearranging this equation we have

$$E_t(\rho_{t+1}) = e_t = \frac{-Cov_t(M_{t+1,}\rho_{t+1})}{E_t(M_{t+1})}$$

As developed by Hansen and Jagannathan (1991) in developing a lower bound on the volatility of the stochastic discount factor required for it to be consistent with a given sample of returns, we can rearrange the equation above using the definition of conditional correlation and the fact that this is bounded between plus and minus one. This gives us

$$\frac{E_t(\rho_t)}{\sigma_t(\rho_{t+1})} \leq \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})},$$

Where $\sigma_t(\rho_{t+1})$ and $\sigma_t(M_{t+1})$ are the conditional standard errors of excess returns and the stochastic discount factor respectively. The Sharpe ratio appearing on the left side of the above equation establishes a lower bound on the variation in the stochastic discount factor scaled by its conditional mean.

For the passive market portfolio, the equity premium puzzle shows that the Sharpe ratio of the passive market portfolio is already too high to be consistent with a wide range of utility functions. Findings of an even higher Sharpe ratio based on forecasting information suggest either that there are indeed exploitable predictable components in stock returns or that the discount factor varies even more than was previously thought (Pesaran & Timmerman, 2000).

2.3 The Recursive Modelling Strategy

Simulation of the forecasting model is done first by establishing a set of regressors over which a search is to be conducted to determine the functional form of the estimated models as well as the criteria used to select particular regression models.

The same set of base regressors may fail be re-used to construct intertemporal forecasting equations due to the restrictiveness they pose to the inclusion of new variables in the future. This is because future relevant variables cannot be known a priori. Expanding the number of base regressors on the other hand does not solve this problem as it reduces the computational feasibility of the model through having a multiplier effect on the number of models that would need to be recursively estimated according to the frequency of the data points, e.g. monthly, weekly etc. The major advantage of using a recursive modelling technique for data that exhibits large periods of non-stationary behavior is that it allows the forecasting model to change through time.

Pesaran & Timmerman, (2000) suggest the following three factors to be established in order to simulate historical processes through which investors attempt to forecast stock return: (i) the list of variables likely to be considered in modelling stock returns; (ii) the criteria to be adopted to select the appropriate forecasting model; (iii) the estimation procedure applied.

They further categorize regressors into three main groups: 'core variables',A_t, 'focal variables', B_t, and 'potentially relevant variables', C_t in declining order of importance. The core variables are believed to be important in forecasting stock returns and are always included in the forecasting equation. Focal variables are considered in the forecasting equation and are believed to be potentially important for capturing short term variations in the risk premia due to business cycle variations. Some or all of them may be left out of the forecasting model according to the model selection criteria in use. A_t and B_t combined form the base set. The third set of regressors, C_t, are considered as potentially relevant and are used only when investors discover clear evidence of the failure of the forecasting models obtainable from the base set –which can occur when the most recent residuals from excess returns equation using the variables in the base set exceed three (recursive) standard errors. Once a search for regressors in the C_t set has been triggered, the variables in C_t are only chosen by a particular model selection criterion that then includes those variables to B_t. The dimensions in B_t and C_t hence vary over time.

These sets of regressors are meant to reflect agents' *a priori* beliefs that a given regressor should be included in the forecasting equation. The set of potentially relevant variables are only considered in agents' modelling procedure after observing a residual which can be considered as an outlier.

C_t can also be used to deal with special set of circumstances in the data sets and events unlikely to be repeated for the period of study chosen e.g. recessions and other shocks, through introduction of a novel procedure for the recursive selection of dummy variables that can be under C_t. This however does not guarantee the inclusion of such dummy variables into the forecasting model. In any case, such a variable can be deselected at any point in time in the future. The dummy variable would have the value of unity in the period in question and zeros elsewhere.

In an attempt to reduce the effect that the benefit of hindsight might bring to the analysis, only regressors that can be safely argued to have been considered ex ante by investors are considered when searching for a return forecasting specification.

2.4 Choice of Variables

Given observations on returns and predictor variables, how should an investor allocate his wealth? One approach would be to estimate the predictability relation, treat the point estimates as known, and solve for the portfolio that maximizes utility. An alternative approach, adopted in Bayesian studies, is to specify prior beliefs on the parameters (Wachter & Wasusawitharana, 2005) (Campbell & Thompson, 2008).

As witnessed by the many exploratory studies in the literature on predictability of stock returns, financial theory only provides very limited guidance for which state variables should have predictive power over stock returns. Finance theories however suggest that in markets with risk averse agents, stock returns would vary with the state of the business cycle (Balvers, Cosimano, & Mcdonald, 1990) (Lucas, 1978).

Business cycle movements have been shown to have a significant impact on stock returns. The first author to publish this finding, Angas, (1936), writes "the major determinant of price movements on the stock exchange is the business cycle." Other variables suggested include short and long interest rates, dividend yields, industrial production, company earnings, liquidity measures and the inflation rate. In addition Balvers, Cosimano and Mcdonald (1990) suggest that if we forecast aggregate output, we can determine the general path that stock returns will follow given that the two variables are serially correlated. Since aggregate output is serially correlated and hence predictable, stock returns can also be predicted based on rational forecasts of output.

However, Kirui, Wawire, & Onono, (2014) find that for the Nairobi Securities Exchange, only exchange rates significantly affect stock returns. Given this recent finding, we will employ a bivariate model that contains only the aforementioned variable, and compare it against other multivariate recursive models.

3. METHODOLOGY

This chapter sets out the methodology used to achieve the objectives set out in chapter one. The succeeding section presents how forecasting was be done. The third section gives the framework under which regressors and variables were chosen. The last section mentions the sources from which we obtained data.

3.1 Forecasting

To understand how to forecasting is done, the idea of conditional expectations is required. A conditional expectation would be expressed as

$$E(y_{t+1}|\Omega_t)$$

Which states that the expected value of y is taken for time $t+1$, conditional upon all the information available up to and including time $t(\Omega_t)$. Given that we are forecasting excess returns which follow a random walk process, the optimal forecast for the zero mean white noise process is zero, given as

$$E(y_{t+1}|\Omega_t) = 0 \; \forall \; s > 0$$

Since we are trying to forecast in real time with as little benefit from hindsight as possible, we will need to use a forecasting model with a finite memory but capable of giving us a historic mean value. A viable model would be the autoregressive [1] moving average [1] (ARMA [1, 1]) process, with lags of up to one previous period only. This is because through forecasting in real time, we are implying that tomorrow's value is determined by today's values of the parameters, with little consideration of other historic values. One period in this case is used to refer to one financial period. Let $f_{t,1}$ denote the forecast made using an ARMA (1,1) model at time t for 1 step into the future for some series y. The forecasts are generated by what is known as a forecast function, typically of the form

$$f_{t,1} = \sum_{i=1}^{1} a_i f_{t,1-i} + \sum_{j=1}^{1} b_j u_{t+1-j}$$

Where $f_{t,1} = y_{t+s}, s \leq 0;\ u_{t+1} = 0, s > 0$

$$= u_{t+s}, s \leq 0.$$

and a_i and b_j are the autoregressive and moving average coefficients respectively.

The MA (1) component, which has a memory only of length (1) is represented as

$$y_{t+1} = \mu + \theta_t + u_t$$

Parameter constancy over time is assumed. This means that if this relationship holds at time t it will hold at time $t+1, t+2, \ldots$, and so forth. This means that this model can be used repetitively, or recursively, through multiple periods. The forecast for time $t+1$ will be

$$f_{t,1} = E(y_{t+1}|\Omega_t) = \mu + \theta_1 + u_t$$

The forecast for y, 1 step ahead, made at time t is given by this linear combination of disturbance terms. It would not be appropriate to set the values of these disturbance terms to their unconditional mean of zero since it is the conditional expectation of their values that is of interest. Given that all information is known at time t, only the values of the error terms at time $t+1$ are unknown. u_{t+1} is not known at time t hence $E(u_{t+1}|\Omega_t) = 0$ and so on. Given that the MA (1) process has a memory of only 1 period, all forecasts two or more steps ahead, save for the constant term, collapse to the intercept.

However, since we will try as much as possible not to rely on the benefit of hindsight, we will only adopt an AR (1) process for our modelling procedure as earlier specified. The purpose of the autoregressive component stems from the fact that we are unable to ignore the stochastic element present in stock returns. Our AR (1) process will be estimated as follows

$$y_{t+1} = \mu + \phi_1 y_t + u_t$$

The assumption of parameter stability applies here as well. Hence, producing a one step ahead forecast would be easy, since all information required will be available at time t. Therefore, applying expectations to the previous equation, we will have

$$f_{t+1,1} = E(Y_{t+1}\Omega_t) = E(\mu + \phi_1 y_t + u_t|\Omega_t)$$

The Y_t function will now be broken down in the next section.

3.2 Regressors

In a predictive regression, rates of return are regressed against the lagged values of a stochastic explanatory variable (Torous, Valkanov, & Yan, 2004). Keim and Stambaugh (1986) shed light on the selection of three such predetermined and important variables.

Model 1

The first variable is the difference between long term low grade corporate bonds and short term treasury bills. The annual bond yield is divided by twelve, as the yield is stated on a monthly basis. This variable is used as it is believed to be a proxy for changes in expected risk premiums. The underlying proposition beneath it is that the level of prices is related to the level of expected risk premiums.

The second variable that we will regress in this model is minus the logarithm of the ratio of the real benchmark index to its previous long run level. Our benchmark index will be the NSE 20 index. This variable can be represented as $\{ -\log\{NSE_{t-1} / \overline{NSE}_{t-1}\}$, where NSE_{t-1} is the level of the index at the end of $t-1$, deflated by the consumer price index and $\overline{NSE}_{t-1}$ is the average of the year end real index over the period of data collection prior to period $t-1$. The significance of this variable is to reflect the variation in the NSE with changes in expected future discount rates or returns.

The third variable, also from the stock market, attempts to capture the most volatile element, which is small firms. Numerous finance literature report that small firms exhibit the greatest ex-post sensitivity to overall changes in expected risk premiums. This reveals that the expected risk premia for small firms are the most volatile. This can be done by either detrending NSE data on small firms backwards, or finding minus the natural logarithm of share price, averaged equally across the quintile of firms with the smallest market values in the NSE. This variable exhibits no detectable trend, but captures the variation in small-stock prices.

Model 2

For the second recursive model, we employ an econometric forecasting method. The regressors that are thought of to be the most recursive in forecasting models in terms of relevance and significance in most economic and finance literature can be broken down into the following function:

$$Y_t = \{YSP_{t-1}, EP_{t-1}, I1_{t-2}, I12_{t-1}, I12_{t-2}, \Pi_{t-2}, \Delta IP_{t-2}, \Delta M_{t-2}\}$$

Where YSP is the dividend yield, EP the earnings-price ratio, $I1$ is the 1 month T-bill rate, $I12$ is the 12 month T-bond rate, Π the year on year inflation, ΔIP the year on year rate of change in industrial output and ΔM is the year on year growth on narrow money stock. All variables computed using macroeconomic indicators such as ΔIP and ΔM are measured using 12 month moving averages in order to reduce the impact of historical data revisions on the results (Pesaran & Timmerman, 1995). This can be viewed as a point forecast (Brooks, 2008) since we intend to determine a single value from the variables of interest. This value is the mean value excess returns, since excess returns exhibit tendencies of long term mean reversion. Campbell and Shiller, (1988) find that a long moving average of real earnings helps to forecast future real dividends. If this is so, we can apply their conclusion to our study by inputting a long moving average of dividend yield to forecast excess returns in the Nairobi Securities Exchange.

Model 3

The third (recursive) model is bivariate. Kirui, Wawire, & Onono, (2014) found that of all macroeconomic variables, only exchange rates have a significant impact on stock returns in the Kenyan market.

4. TEST RESULTS

4.1 Unit root tests

Under the first recursive model we conducted, we found the variable "deflated NSE index" to be statistically significant upon differencing once; 'bond spread" was statistically significant upon inclusion of a one lag; and "small firms" was statistically significant upon differencing once.

Under the second model, the variable "T-bond" was statistically significant for a unit root test with a trend and intercept, upon first differencing; the variable "change in industrial output" was found to be statistically significant; the variable "change in the value of M2 broad money" was found to be statistically significant; and "inflation" was found to be statistically significant on the inclusion of a single lag.

On the third model, the variable "fx", the Foreign Exchange Variable, was found to be statistically significant after differencing once.

Our dependent variable, "dNSE" was statistically significant.

4.2 Johansen tests of cointegration

The key purpose of running these tests is to ascertain the credibility of the variables proposed by finance and economic theory, by testing whether they at all possess any long run relationship, before we develop our static ARMA forecasts.

Model 1

We find that there may be at most three cointegrating relationships at the 5% significance level, with evidence suggesting that there may be at least one cointegrating relationship between the variables at the same level of significance.

Model 2

Due to a limited amount of data on the Central Bank 364 day Tbond, we ejected it from this test as the data points were insufficient for regression purposes. We find that there may be at most 4 cointegrating relationships at the 5% significance level, with evidence suggesting that there may be at least one cointegrating relationship between the variables at the same level of significance.

Model 3

We do not perform a cointegration test on the third model, as it possesses only two variables, one of which is integrated to the order 0.

4.3 Regression and forecasts

We employ AR (4) and MA (4) components into our regression estimates given that our data is quarterly. This is because of existence of lower order correlations between the first three AR and MA terms resulting from the frequency of the data.

Model 1

We find all variables as statistically significant with the model being specified as follows:

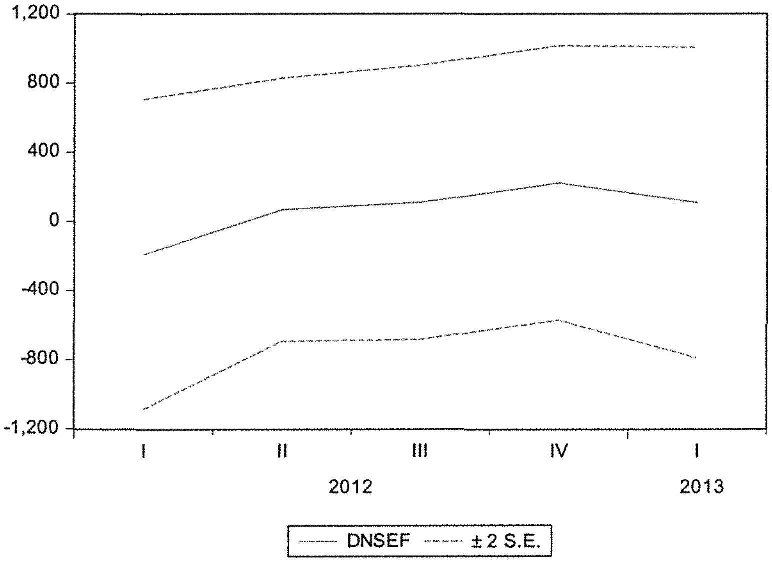
Equation 1: Model 1 Regression

$$dNSE = -8540.98 - 3663.10Smallfirms - 45116.89bondspread - 0.5846Lresid1 + 0.7845ar(4) - 0.9578ma(4)$$

Upon checking for serial correlation, we fail to reject the null hypothesis of no serial correlation, as we find a probability value of 0.1207 at a 5% significance level.

Static Forecast

Figure 1: Static Forecast under model 1

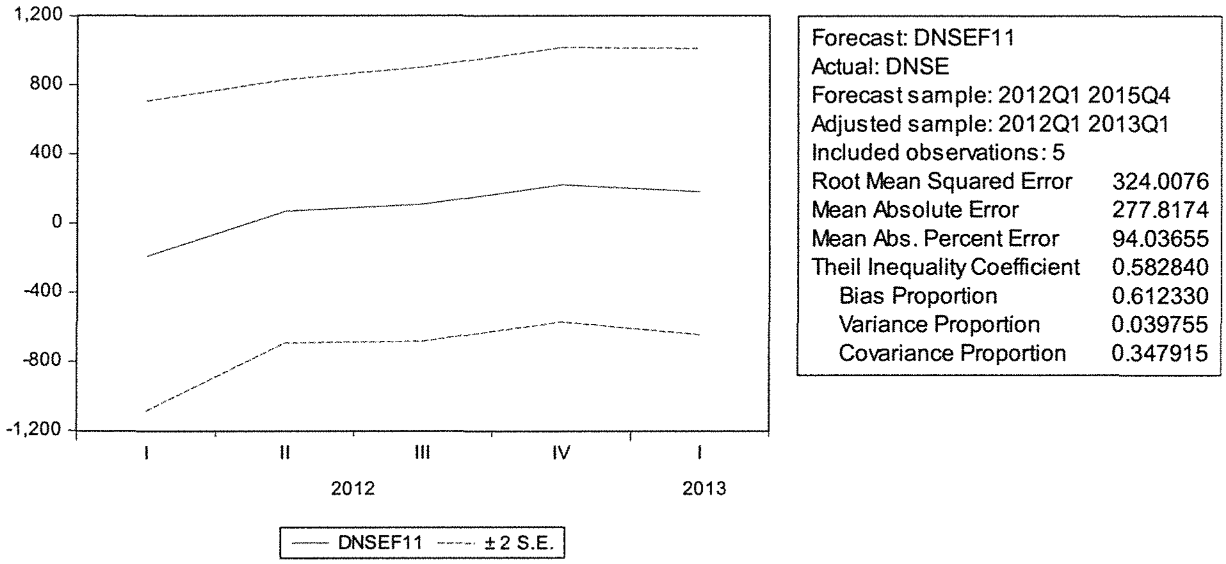


Forecast: DNSEF	
Actual: DNSE	
Forecast sample: 2012Q1 2015Q4	
Adjusted sample: 2012Q1 2013Q1	
Included observations: 5	
Root Mean Squared Error	349.4805
Mean Absolute Error	292.5250
Mean Abs. Percent Error	96.05735
Theil Inequality Coefficient	0.644318
Bias Proportion	0.589154
Variance Proportion	0.042878
Covariance Proportion	0.367968

Our forecast falls between the two standard deviations of the forecast as shown by figure 1, indicating a satisfactory predictive power of our model at the 95% confidence interval. We find Root Mean Squared Error of 349.58, signalling that the actual and the forecasted returns may move closely.

Dynamic Forecast

Figure 2: Dynamic Forecast under model 1



Our forecast falls between the two standard deviations of the forecast, as shown by figure 2, indicating a satisfactory predictive power of our model at the 95% confidence interval. We find Root Mean Squared Error of 324.00, which is lower than that of our static forecast.

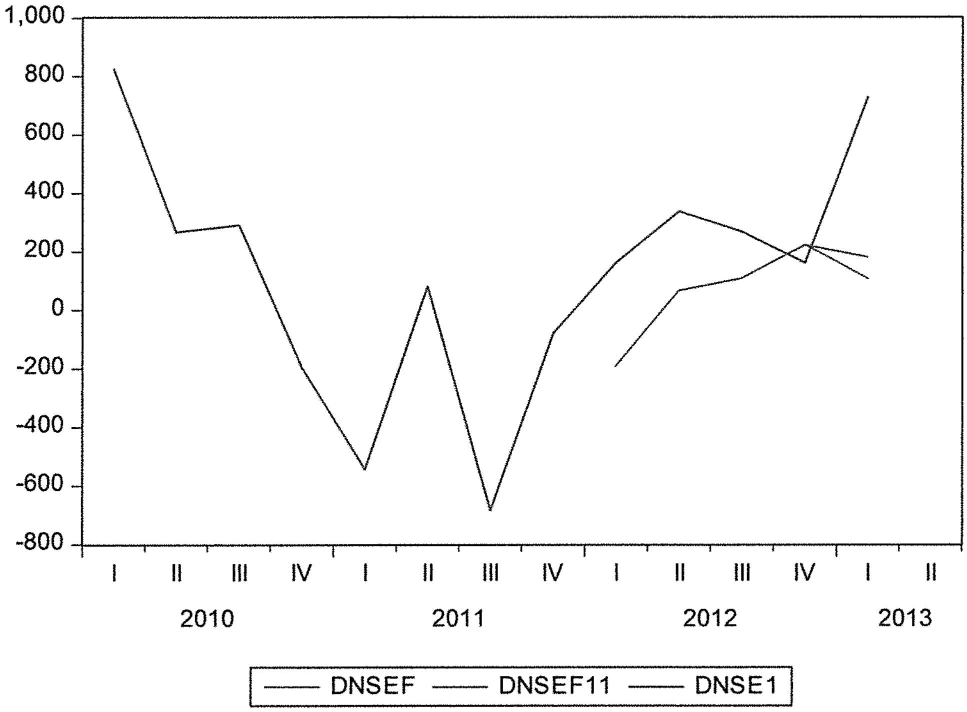
The tabular results comparison are shown in table 1 below:

Table 1: Comparison of Static and Dynamic Forecasts under model 1

Period	Static Forecast	Dynamic Forecast	Observed Values
2012Q1	-190.8031	-190.8031	161.8700
2012Q2	67.21772	67.21772	337.0500
2012Q3	108.6557	108.6557	268.0900
2012Q4	221.6812	221.6812	160.9900
2013Q1	107.8161	181.3540	727.8100

A graphical comparison of the static and dynamic forecasts and actual values is shown in figure 3 below:

Figure 3: Graphical Comparison of Static and Dynamic forecasts and Observed Series, under model 1



Where DNSEF is the static forecast, DNSEF11 is the dynamic and DNSE1 are the observed values

Model 2

We find all variables in this regression, with the exception of the AR and MA Terms, to be insignificant, with the model being specified as follows:

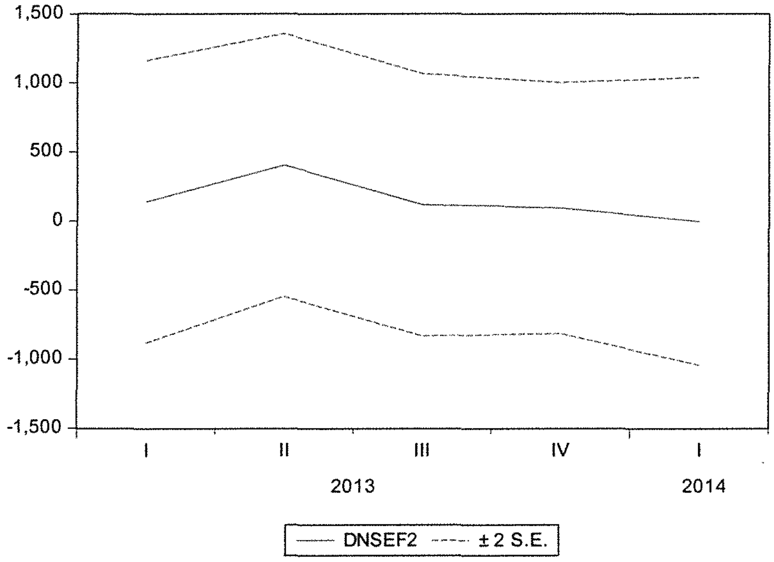
Equation 2: Model 2 Regression

$$dNSE = 91.36 - 1623.32m2 + 547.38infl + 2114.24output + 0.2658Lresid2 + 0.7112ar(4) - 0.9999ma(4)$$

Upon checking for serial correlation, we fail to reject the null hypothesis of no serial correlation between the residuals of the variables, with a probability of 0.1207 at a 5% confidence interval. The forecast is shown below

Static Forecast

Figure 4: Static Forecast under Model 2

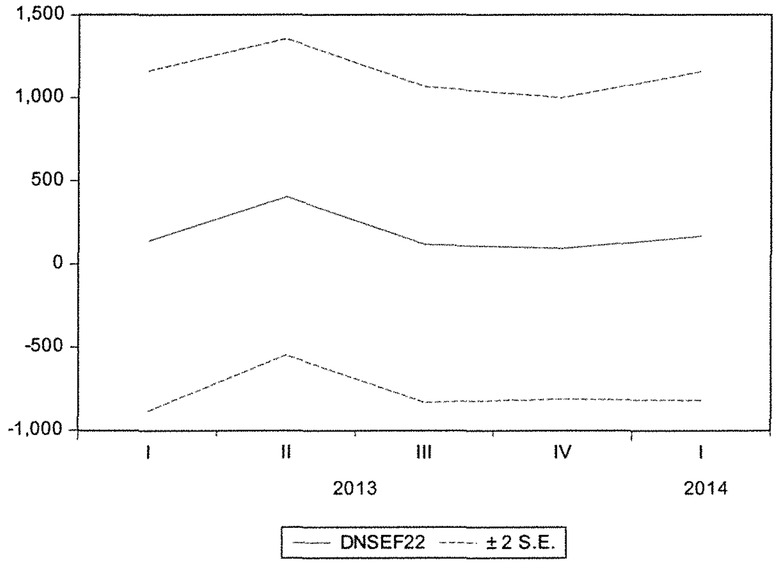


Forecast: DNSEF2	
Actual: DNSE	
Forecast sample: 2013Q1 2015Q4	
Adjusted sample: 2013Q1 2014Q1	
Included observations: 5	
Root Mean Squared Error	399.9751
Mean Absolute Error	279.4047
Mean Abs. Percent Error	103.9805
Theil Inequality Coefficient	0.708633
Bias Proportion	0.000988
Variance Proportion	0.219347
Covariance Proportion	0.779665

Our forecast falls between the two standard deviations of the forecast, indicating a satisfactory predictive power of our model at the 95% confidence interval, as shown in figure 4 above. We find a Root Mean Squared Error of 399.98, meaning that the actual and the forecasted returns move closely.

Dynamic Forecast

Figure 5: Dynamic Forecast under Model 2



Forecast: DNSEF22	
Actual: DNSE	
Forecast sample: 2013Q1 2015Q4	
Adjusted sample: 2013Q1 2014Q1	
Included observations: 5	
Root Mean Squared Error	405.3410
Mean Absolute Error	304.8256
Mean Abs. Percent Error	239.1259
Theil Inequality Coefficient	0.701503
Bias Proportion	0.002793
Variance Proportion	0.269514
Covariance Proportion	0.727692

Our forecast falls between the two standard deviations of the forecast, indicating a satisfactory predictive power of our model at the 95% confidence interval, as shown on figure 5 above. We find a Root Mean Squared Error of 405.34, which is higher than that of the static forecast.

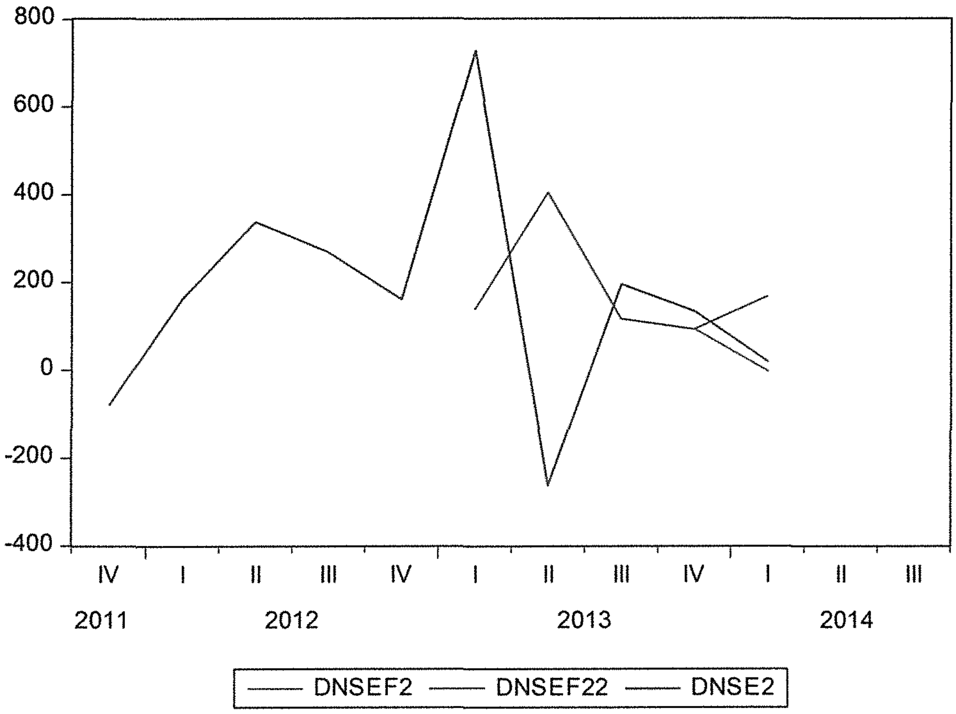
The tabular results are shown in table 2 below

Table 2: Comparison of Static and Dynamic forecasts under model 2

Period	Static Forecast	Dynamic Forecast	Observed Values
2013Q1	139.1171	139.1171	727.8100
2013Q2	404.4061	404.4061	-262.6700
2013Q3	116.2490	116.2490	195.0400
2013Q4	92.74868	92.74868	133.7700
2014Q1	-2.632351	167.3566	18.81000

The graphical comparison of the forecasted and the observed is as shown on figure 6 below:

Figure 6: Graphical Comparison of the Static and Dynamic Forecasts and Observed values, under model 2



Where DNSEF2 is the static forecast, DNSEF22 is the dynamic forecast and DNSE2 is the observed series

Model 3

The output of the regression between the dependent and the explanatory variables is represented in the following equation:

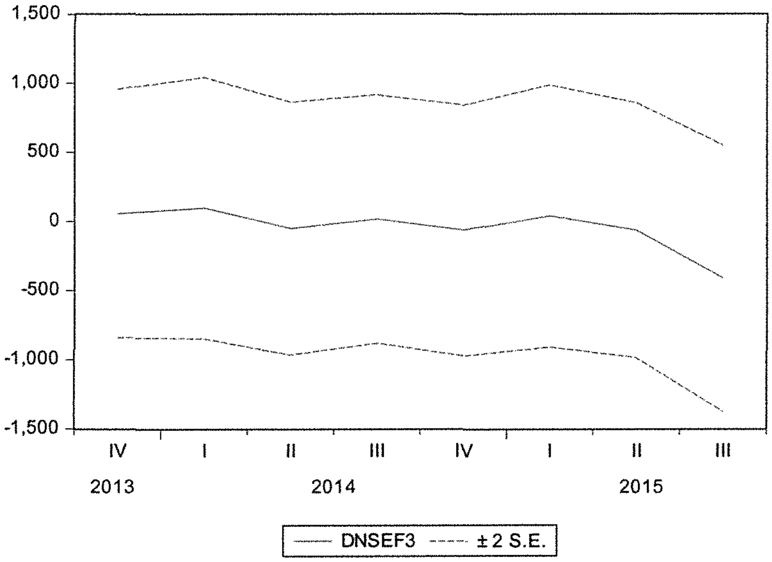
Equation 3: Model 3 Regression

$$dNSE = 47.17 - 38.905dFX + 0.4482ar(4) - 0.9081ma(4)$$

All regressors were found to be statistically significant.

Static Forecast

Figure 7: Static Forecast under Model 3



Forecast: DNSEF3	
Actual: DNSE	
Forecast sample: 2013Q4 2015Q4	
Adjusted sample: 2013Q4 2015Q3	
Included observations: 8	
Root Mean Squared Error	202.9873
Mean Absolute Error	160.5484
Mean Abs. Percent Error	103.9938
Theil Inequality Coefficient	0.420954
Bias Proportion	0.019844
Variance Proportion	0.677483
Covariance Proportion	0.302673

Figure 7 shows that our forecast falls between the two standard deviations of the forecast, indicating a satisfactory predictive power of our model at the 95% confidence interval. We find a Root Mean Squared Error of 515.45, meaning that the actual and the forecasted returns move closely.

For this particular model, the software employed was unable to simulate the dynamic forecast, perhaps due to limited data points on which it could form a viable forecast, based on the number of iterations carried out.

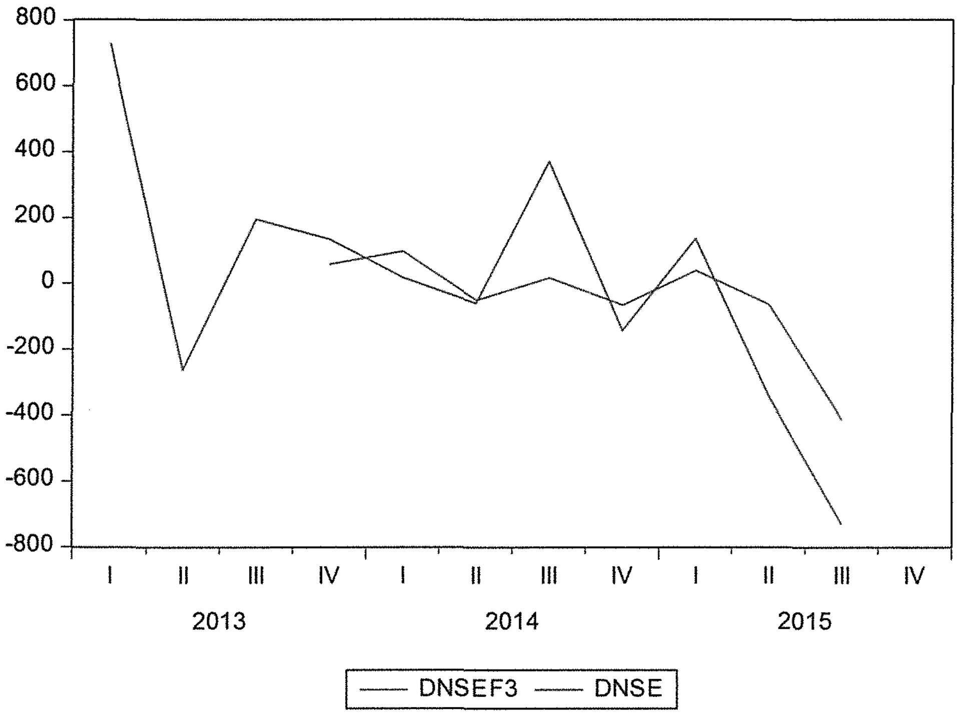
The tabular results are shown in table 3.

Table 3: Comparison of Static forecast and Observed series under model 3

Period	Forecast	Observed
2013Q4	57.83206	133.7700
2014Q1	96.90455	18.81000
2014Q2	-51.74883	-60.74000
2014Q3	15.97107	370.5800
2014Q4	-66.55215	-142.9700
2015Q1	38.24178	135.5100
2015Q2	-65.19273	-342.0900
2015Q3	-413.3087	-729.4800

Figure 8 shows a graphical comparison

Figure 8: Graphical Comparison of Static Forecast and Observed Series under model 3



Where DNSEF3 is the forecasted series and DNSE is the actual series

4.4 Model Performance

We compare the model performance of the three models based on their observed Root Mean Squared Error Values. We find that Model 3 has the best forecasting power given that it has a Root Mean Squared Error of 202.99, which is significantly lower than that of the other two models 349.48 and 324.00 (for model 1) and 399.98 and 405.34 (for model 2). This finding concurs with what (Kirui, Wawire, & Onono, 2014) had found.

However, we obtain mixed results in terms of the performance for both the one step ahead and multiple period ahead forecasts, as observed from the Mean Squared Errors for both the static and dynamic forecasts of model 1 and model 2, with the Dynamic Model outperforming the Static Model in model 1 while underperforming the Static Model in model 2.

In light of these findings, we conclude that the multivariate models that incorporate suggestions from economic theory used in this study are less powerful than the bivariate model in terms of reducing the variability of forecasts.

5. Data Sources

Published time series data were sourced from the Kenya National Bureau of Statistics (KNBS), the Central Bank of Kenya (CBK), the Nairobi Securities Exchange Limited (NSE) databases and the Kenya Statistical Abstracts.

6. Assumptions

A major assumption in the study is that prices, to a large extent, reflect all available information and that there exists rational expectations by investors.

The intuition underlying the theoretical model arises from wealth maximization for a rational investor. This goes beyond the precipice of utility maximization whereby we will assume that a rational and sophisticated investor will continuously look for good trades and will not get satisfied after a number of successful profits. Tied to this is another assumption of non-dividend issuing firms, as shareholders are assumed to prefer firms that do not issue dividends compared to those that do due to tax disincentives that arise from issuing of dividends. We tie this theoretical assumption to (Black, 1976).

Given that we are studying data from a Kenyan context prior to the setting up the derivatives market, we assume the absence of short selling and that investors are unable to use leverage when selecting their portfolios.

The final assumption that we take account of as we simulate investors' portfolio decisions in "real time" is the existence of low or medium transaction costs to shed light on whether the predictable elements in stock returns are economically exploitable net of transaction costs.

7. CONCLUSION

The study attempted to establish the viability of recursive modelling and forecasting techniques in predicting excess equity returns using movement in the Nairobi Securities Exchange All Share Index as a proxy for excess returns through capital gains. A major assumption to this effect was that firms within this index do not pay dividends, and when they do, they pay very little due to the tax disincentives of issuing dividends, with shareholders preferring companies to retain earnings through slack – a factor that can maximize their utility functions through increasing the value of their stock.

We used three models to this effect, each with different regressors and forecasted both one period ahead and multiple periods ahead. We found that the model that engaged exchange rates was indeed more powerful than those that used macroeconomic variables or variables that proxied agents' expectations through including various risk premia from various asset classes.

In comparing the performance of the one period ahead returns Vis a Vis those of multiple periods ahead, we obtained mixed results. In our first model, the dynamic forecasts outperformed the static forecasts, as the former had a lower Root Mean Squared Error. In our second model, the static forecasts outperform the dynamic forecasts. This could be due to model specification as to which variables to include in the regressions. We therefore fail to find conclusive evidence that one step ahead (recursive) forecasts outperform multiple step ahead (dynamic) forecasts.

7.1 Recommendations for Future Study

Market agents may benefit significantly from future studies that aim to determine the variables that have the highest power in modelling and predicting stock returns. Once the core factors are determined, future model to model comparison of the performance of static and dynamic forecasts may yield more conclusive evidence of the superiority of one over the other, as at now their use would depend largely on the information that an investment manager seeks to obtain.

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Appendices

Appendix 1

Johansen Cointegration Tests

Model 1

Sample (adjusted): 2004Q4 2014Q1

Included observations: 38 after adjustments

Trend assumption: Linear deterministic trend

Series: DNSE BONDSPREAD SMALLFIRMS DEFNSE

Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.617435	76.64562	47.85613	0.0000
At most 1 *	0.403468	40.13311	29.79707	0.0023
At most 2 *	0.359524	20.50147	15.49471	0.0081
At most 3	0.089688	3.570796	3.841466	0.0588

Trace test indicates 3 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.617435	36.51251	27.58434	0.0028
At most 1	0.403468	19.63164	21.13162	0.0800
At most 2 *	0.359524	16.93067	14.26460	0.0185
At most 3	0.089688	3.570796	3.841466	0.0588

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unit Root Test for residuals on first regression

Null Hypothesis: RESID1 has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.051041	0.0419
Test critical values:	1% level	-3.679322	
	5% level	-2.967767	
	10% level	-2.622989	

Model 2

Date: 11/30/15 Time: 11:18

Sample (adjusted): 2005Q3 2014Q3

Included observations: 37 after adjustments

Trend assumption: Linear deterministic trend

Series: DNSE M2 OUTPUT INFL

Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.686439	91.62200	47.85613	0.0000
At most 1 *	0.463838	48.71085	29.79707	0.0001
At most 2 *	0.357831	25.64806	15.49471	0.0011
At most 3 *	0.221423	9.260621	3.841466	0.0023

Trace test indicates 4 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.686439	42.91115	27.58434	0.0003
At most 1 *	0.463838	23.06279	21.13162	0.0264
At most 2 *	0.357831	16.38744	14.26460	0.0227
At most 3 *	0.221423	9.260621	3.841466	0.0023

Max-eigenvalue test indicates 4 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unit root test for regression on second model

Null Hypothesis: RESID2 has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.218038	0.0002
Test critical values:	1% level	-3.661661	
	5% level	-2.960411	
	10% level	-2.619160	

Appendix 2

Regression output

Model 1

Dependent Variable: DNSE

Method: Least Squares

Date: 11/30/15 Time: 09:52

Sample (adjusted): 2006Q3 2012Q4

Included observations: 26 after adjustments

Convergence achieved after 31 iterations

MA Backcast: 2005Q3 2006Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-8540.979	2341.702	-3.647338	0.0017
SMALLFIRMS	-3663.102	889.8373	-4.116597	0.0006
BONDSPREAD	-45116.89	24547.35	-1.837953	0.0818
DEFNSE	-1761.424	537.8427	-3.274980	0.0040
RESID1(-1)	-0.584603	0.279911	-2.088530	0.0504
AR(4)	0.784508	0.099070	7.918749	0.0000
MA(4)	-0.957780	0.047382	-20.21415	0.0000

R-squared	0.660648	Mean dependent var	-4.902692
Adjusted R-squared	0.553484	S.D. dependent var	491.1702
S.E. of regression	328.2089	Akaike info criterion	14.64998
Sum squared resid	2046701.	Schwarz criterion	14.98870
Log likelihood	-183.4498	Hannan-Quinn criter.	14.74752
F-statistic	6.164848	Durbin-Watson stat	1.247047
Prob(F-statistic)	0.001010		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.341314	Prob. F(1,18)	0.1434
Obs*R-squared	2.408169	Prob. Chi-Square(1)	0.1207

Model 2

Dependent Variable: DNSE

Method: Least Squares

Date: 11/30/15 Time: 11:42

Sample (adjusted): 2007Q2 2012Q4

Included observations: 23 after adjustments

Convergence achieved after 28 iterations

MA Backcast: 2006Q2 2007Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	91.36309	535.8698	0.170495	0.8668
M2	-1623.319	4596.205	-0.353187	0.7286
INFL	547.3827	2607.289	0.209943	0.8364
OUTPUT	2114.237	4344.357	0.486663	0.6331
RESID2(-1)	0.265782	0.236893	1.121947	0.2784
AR(4)	0.711229	0.142576	4.988406	0.0001
MA(4)	-0.999986	1.66E-06	-601266.1	0.0000

R-squared	0.432478	Mean dependent var	-43.50652
Adjusted R-squared	0.219657	S.D. dependent var	464.9594
S.E. of regression	410.7312	Akaike info criterion	15.11955
Sum squared resid	2699202.	Schwarz criterion	15.46513
Log likelihood	-166.8748	Hannan-Quinn criter.	15.20646
F-statistic	2.032124	Durbin-Watson stat	1.884393
Prob(F-statistic)	0.120452		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.108864	Prob. F(1,15)	0.7460
Obs*R-squared	0.000000	Prob. Chi-Square(1)	1.0000

Model 3

Dependent Variable: DNSE

Method: Least Squares

Date: 11/30/15 Time: 11:08

Sample (adjusted): 2006Q2 2013Q3

Included observations: 30 after adjustments

Convergence achieved after 54 iterations

MA Backcast: 2005Q2 2006Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	47.17936	65.72508	0.717829	0.4793
DFX	-38.90513	16.79872	-2.315958	0.0287
AR(4)	0.448186	0.171182	2.618181	0.0145
MA(4)	-0.908145	0.140415	-6.467591	0.0000

R-squared	0.316720	Mean dependent var	23.05200
Adjusted R-squared	0.237880	S.D. dependent var	479.8658
S.E. of regression	418.9203	Akaike info criterion	15.03680
Sum squared resid	4562849.	Schwarz criterion	15.22363
Log likelihood	-221.5521	Hannan-Quinn criter.	15.09657
F-statistic	4.017257	Durbin-Watson stat	2.244192
Prob(F-statistic)	0.017869		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.442943	Prob. F(1,25)	0.5118
Obs*R-squared	0.000000	Prob. Chi-Square(1)	1.0000