SU International Mathematics Research Meeting

Mathematical Biology Workshop

Stochastic Modeling of HIV Dynamics within an individual and its management

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Outline

- Brief overview
- Statement of the problem
- Objectives of the study
- Significance of the study
- Methodology
- Expected outcomes

Brief overview

- Why model HIV/ AIDS
- Interaction of the HIV virus and the immune system of an infected person
- Why stochastic processes

Problem statement

- Eradication of the HIV virus is not attainable with the current available drugs and now the focus is the management and control of the virus progression in an infected person.
- The allocation of the limited budget by the Government to combat the disease.

Research Objectives

Main objective

- To develop stochastic models for the study of HIV internal viral dynamics and its management.

Specific objectives

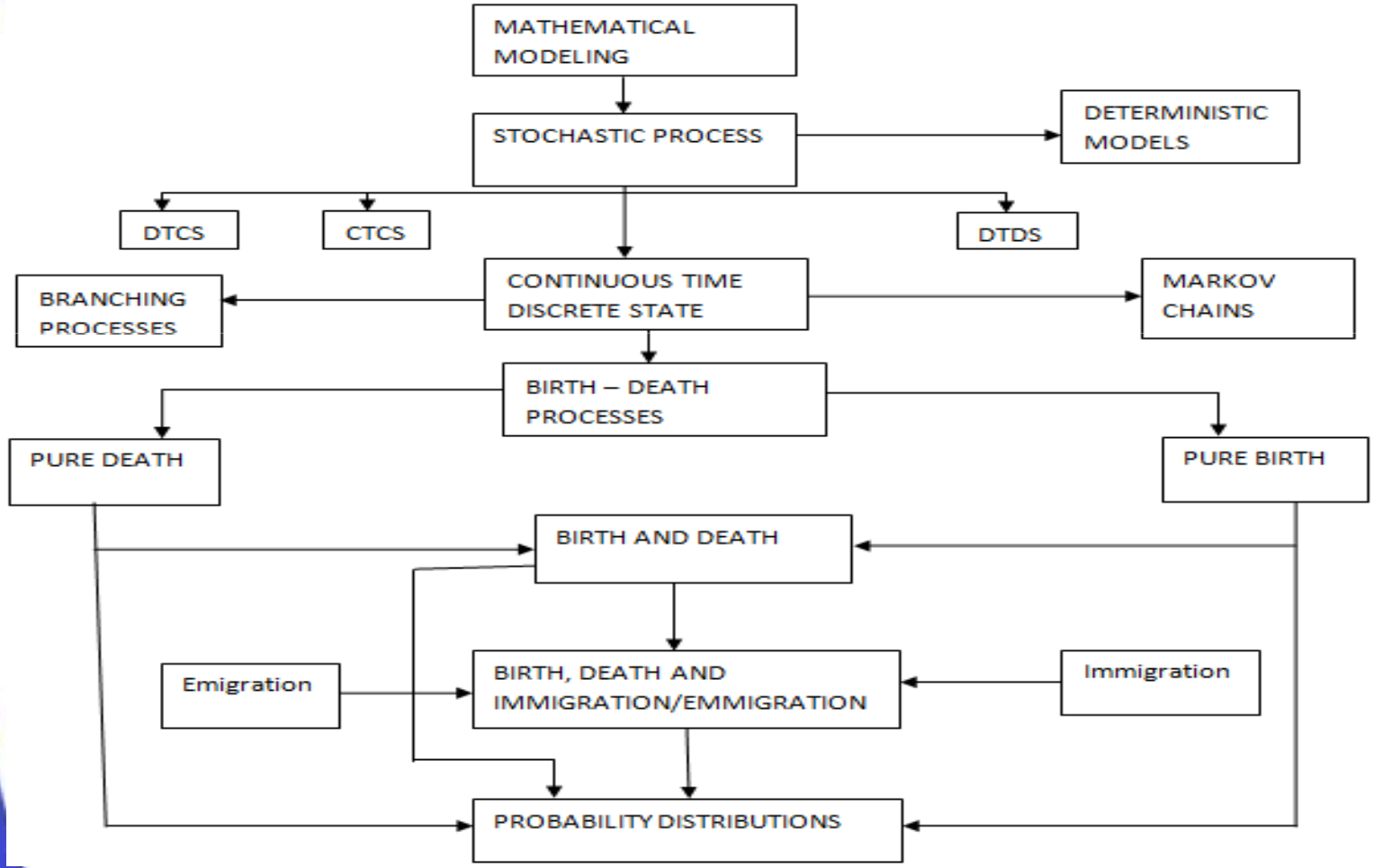
- Formulate a model for the Virus – host interaction
- Formulate stochastic Models for the progression of the disease
- Formulate a HIV Management Cost (HMC) Model
- Formulate a Cost Benefit Analysis (CBA) Model

Significance of the study

- Stochastic models for the management of the HIV epidemic
  - The analysis of the models will show what treatment combination is effective at what disease state.
  - The cost model will help the government and donors make informed decisions about resource allocation

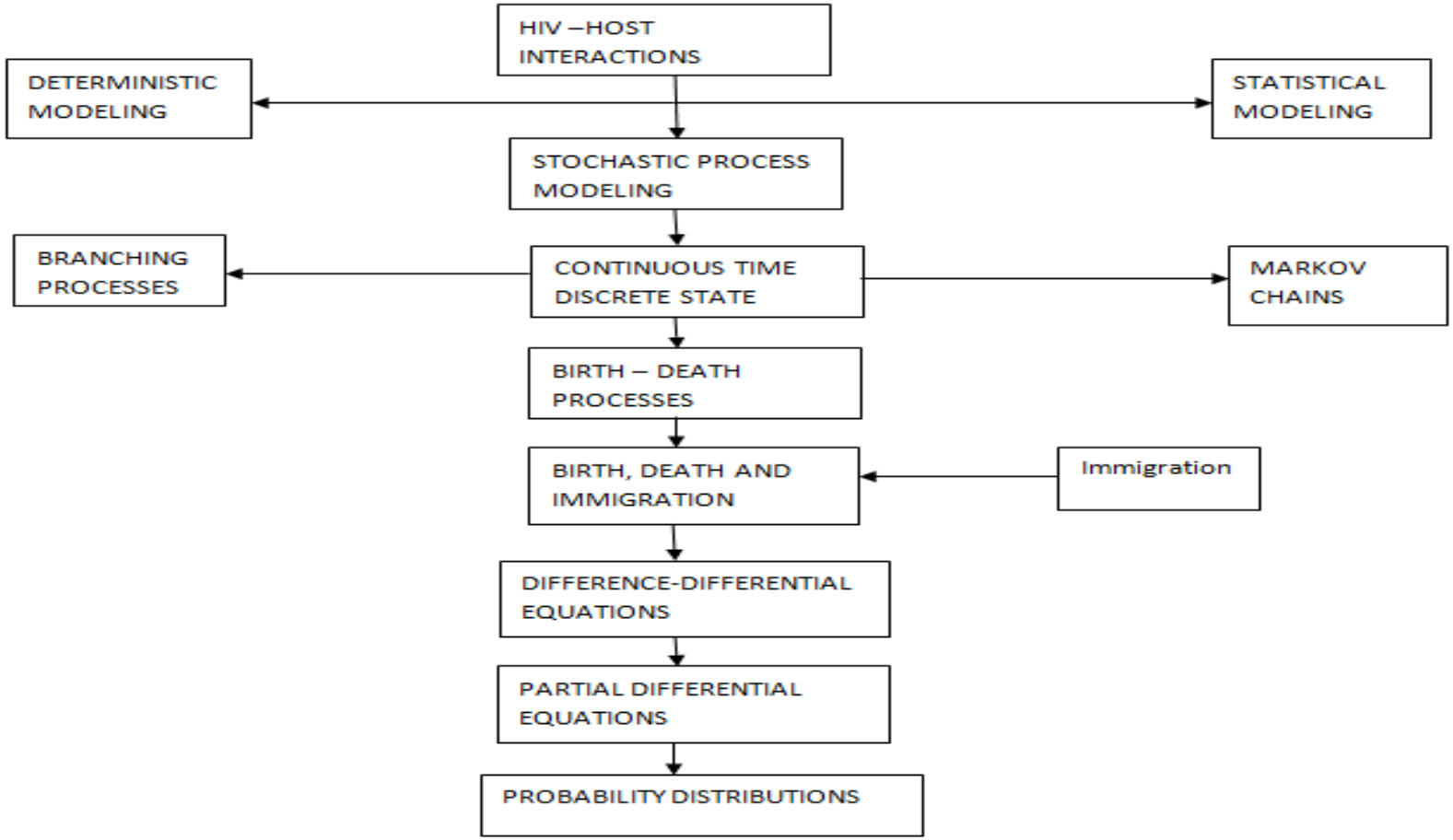
Methodology

Framework for stochastic processes



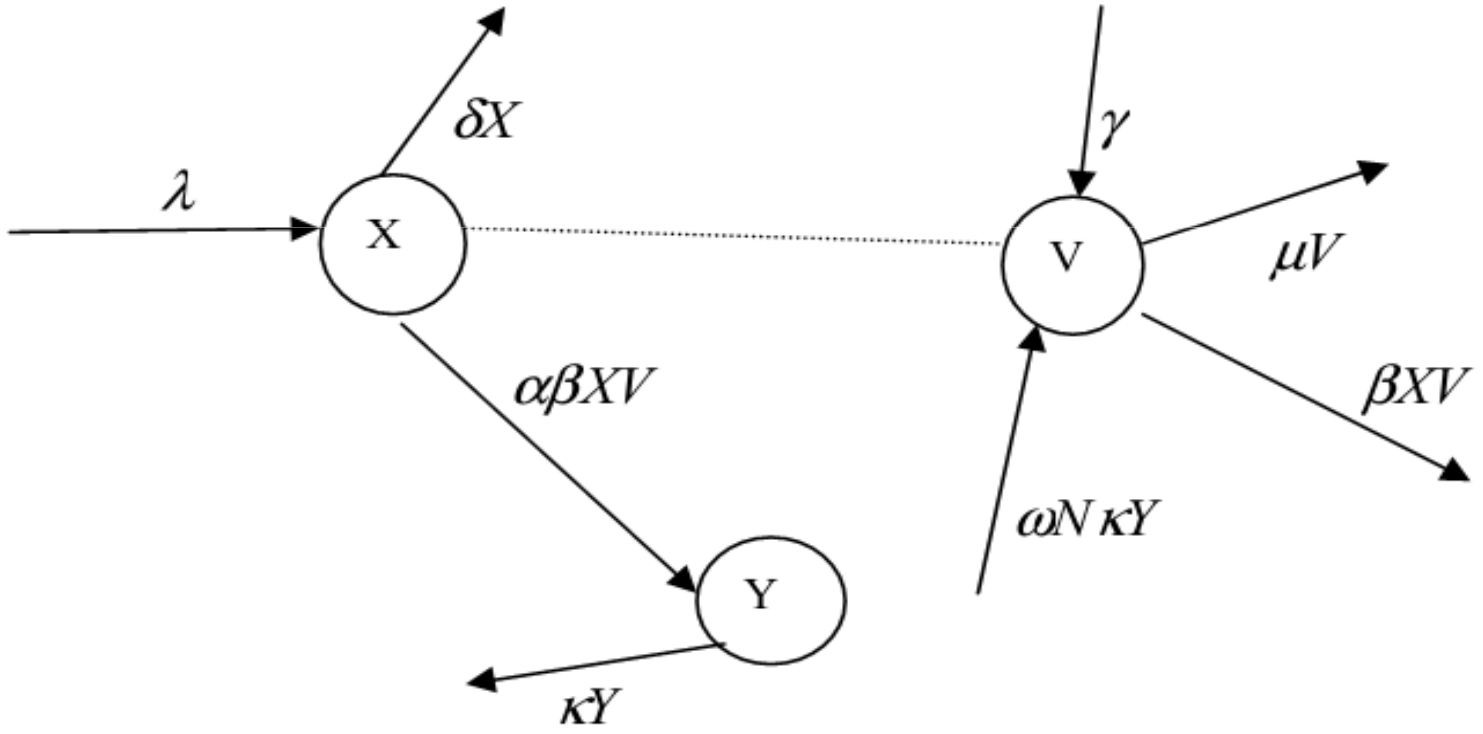
Methodology (Cont)

objective 1: HIV – Host interaction dynamics



Research Methodology(cont)

HIV virus –host Interaction dynamics



Rachel (2012)

Possible transitions in HIV - host interactions

Event	Population components (X,Y,V) at t	Population components (X,Y,V) at $(t, t+\Delta)$	probability of transition
Production of uninfected cell	$(x-1, y, v)$	(x, y, v)	$\lambda \Delta t$
Death of uninfected cell	$(x+1, y, v)$	(x, y, v)	$\delta(x+1)\Delta t$
Infection of uninfected cell	$(x+1, y-1, v+1)$	(x, y, v)	$\beta(x+1)(v+1)\Delta t$
Production of virons from the dying infected cell	$(x, y+1, v-1)$	(x, y, v)	$\kappa N(y+1)\Delta t$
Introduction of Virons due to re-infection because of risky behaviour	$(x, y, v-1)$	(x, y, v)	$\gamma \Delta t$
Death of virons	$(x, y, v+1)$	(x, y, v)	$\mu(v+1)\Delta t$

The Master equation for Virus-Host interaction

$$
\begin{aligned}
P'_{x,y,v}(t) &= -\{\lambda + \delta x + \alpha\beta xv + \mu v + \kappa y + \gamma\} P_{x,y,v}(t) \\
&+ \lambda P_{x-1,y,v}(t) \\
&+ \delta(x+1) P_{x+1,y,v}(t) \\
&+ \alpha\beta(x+1)(v+1) P_{x+1,y-1,v+1}(t) \\
&+ N\omega\kappa(y+1) P_{x,y+1,v-1}(t) \\
&+ \gamma P_{x,y,v-1}(t) \\
&+ \mu(v+1) P_{x,y,v+1}(t)
\end{aligned}
$$

The Lagrange Partial Differential Equation

$$\frac{\partial G}{\partial t} = \{(z_1 - 1)\lambda + (z_3 - 1)\gamma\}G + (1 - z_1)\delta\frac{\partial G}{\partial z_1} + (\omega N z_3 - z_2)\kappa\frac{\partial G}{\partial z_2}$$

$$+ (1 - z_3)\mu\frac{\partial G}{\partial z_3} + \alpha\beta(z_2 - z_1 z_3)\frac{\partial^2 G}{\partial z_1 \partial z_3}$$

Moments of X(t), Y(t) and V(t) from the pgf

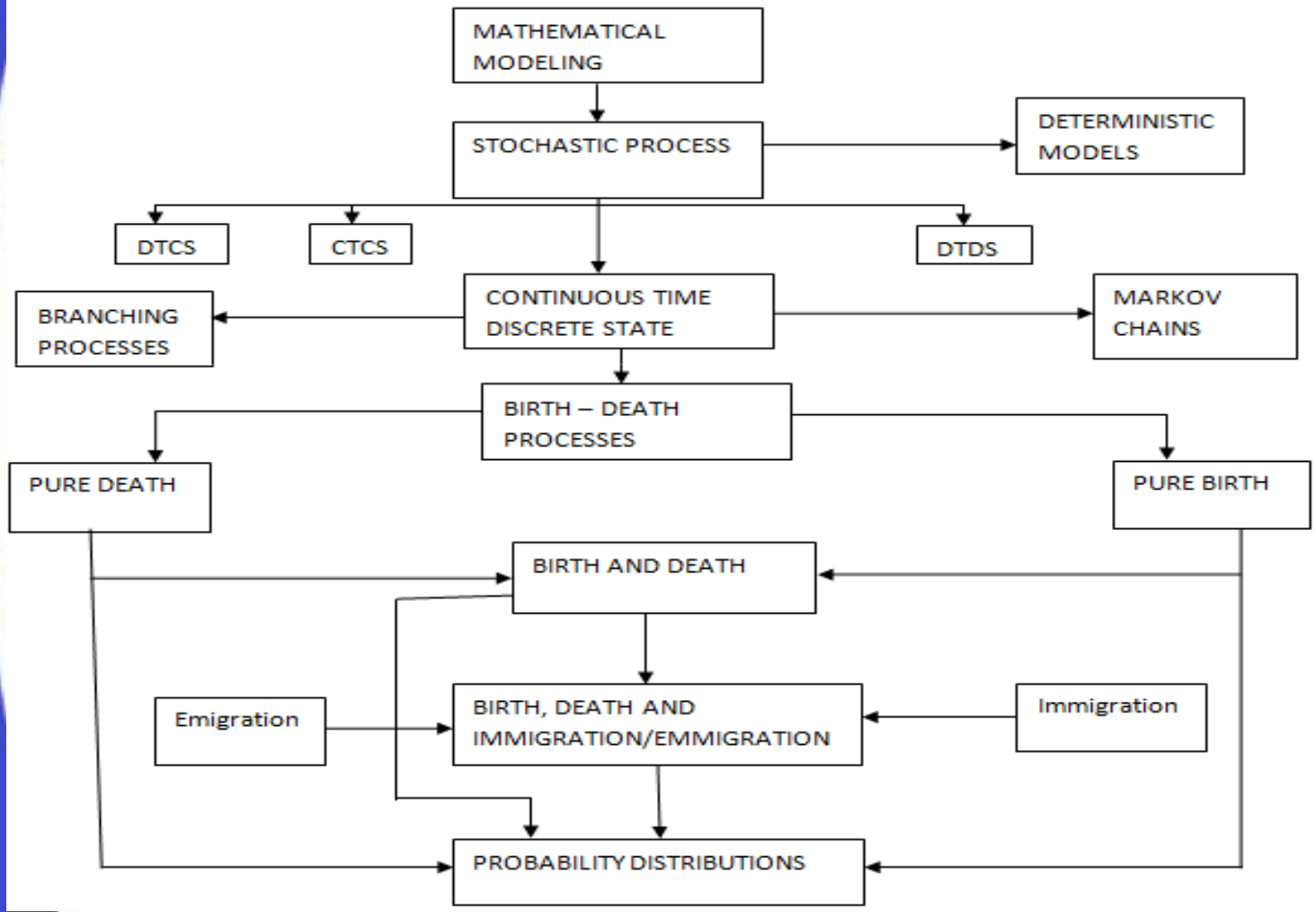
1. $\frac{dx}{dt} = \lambda - \delta x(t) - \alpha\beta x(t)v(t)$

2. $\frac{dy}{dt} = \alpha\beta x(t)v(t) - \kappa y(t)$

3. $\frac{dx}{dt} = \gamma + \omega N\kappa y(t) - \alpha\beta x(t)v(t)$

Methodology (Cont)

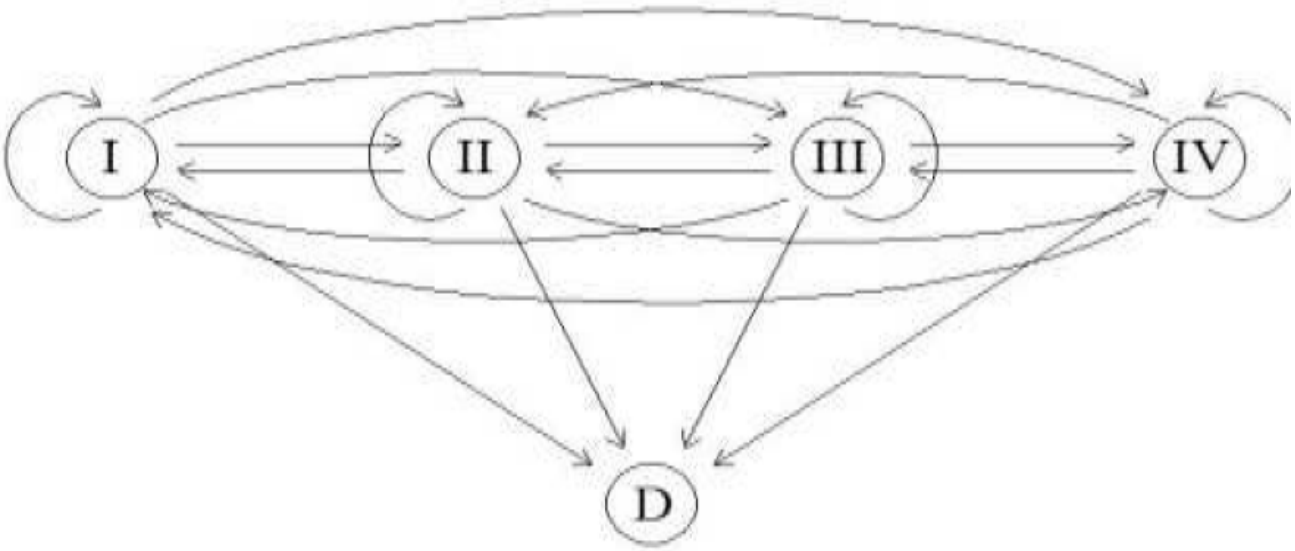
objective 2: HIV virus progression dynamics



Research Methodology(cont)

HIV progression dynamics

- We will assume that that the clinical course of untreated HIV infection proceeds through five states:



State I: (VL ≤ 400 cp/ml & CD4 < $200x10^6$ cells/ µL);

State 2: (VL ≤ 400 cp/ml &CD4 > $200x10^6$ cells/ µL);

State 3: (VL > 400 cp/ml & CD4 > $200\ x10^6$ cells/ µL);

State 4: (VL > 400 cp/ml &CD4 < $200\ x10^6$ cells/ µL);

State D: Absorbing state; (death of the patient).

Expected outcomes

- Stochastic model for Virus –host interaction
- Markov models to describe the disease internal progression in an infected person
- Transition probabilities that describe the various stages of the disease
- A Disease Management Cost (DMC) model
- Cost Benefit Analysis (CBA) model
- Future Scientific Research

Thank you