Stochastic model for In-Host HIV dynamics with Therapeutic intervention

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Outline

- Brief overview
- Background to the Research problem
- Methodology
- Expected Outcome/Result

Brief overview

- Why model HIV/ AIDS?
- Interaction of the HIV virus and the immune system of an infected person
- Why stochastic models

Background to the research Problem

- Eradication of the HIV virus is not attainable with the current available drugs and now the focus is the management and control of the virus progression in an infected person.

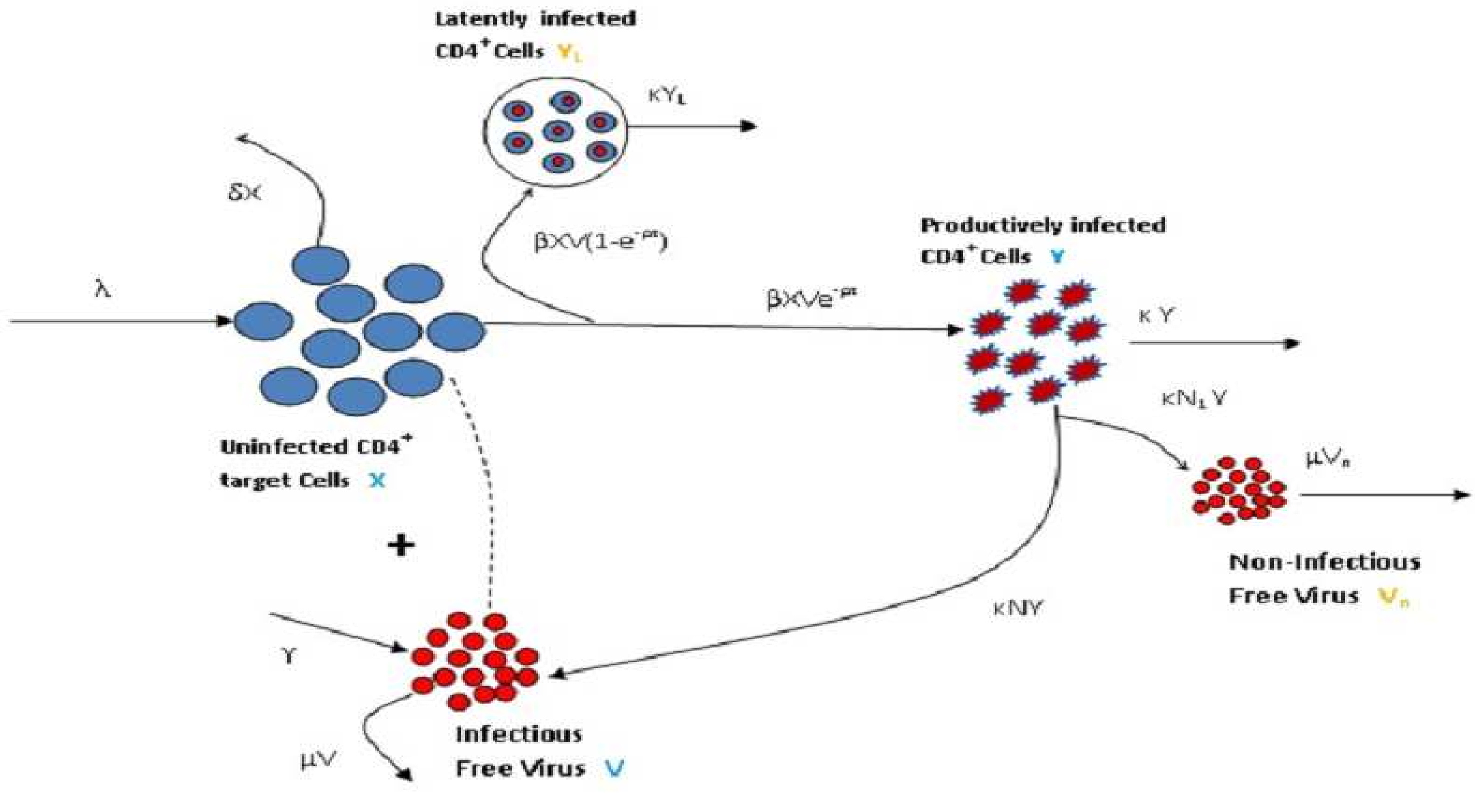
Methodology

The Approaches include

- Developing stochastic models for the study of the In-host virus dynamics.
- Formulating a model for the disease management with treatment therapies.
- Analyzing the models in effect of combined treatment

Methodology(cont)

HIV – CD4+ cells Interaction dynamics



Source: Rachel (2012)

Possible transitions in host interaction of HIV and Immune system Cells and corresponding probabilities			
Event	Population components (X,Y,V) at t	Population components (X,Y,V) at $(t, t+\Delta)$	probability of transition
Production of uninfected cell	$(x-1, y, v)$	(x, y, v)	$\lambda\Delta t$
Death of uninfected cell	$(x+1, y, v)$	(x, y, v)	$\delta(x+1)\Delta t$
Infection of uninfected cell	$(x+1, y-1, v+1)$	(x, y, v)	$\beta(x+1)(v+1)e^{-\rho\tau}\Delta t$
Production of virons from the bursting infected cell	$(x, y+1, v-1)$	(x, y, v)	$\kappa N(y+1)\Delta t$
Introduction of Virons due to re-infection because of risky behaviour	$(x, y, v-1)$	(x, y, v)	$\gamma\Delta t$
Death of virons	$(x, y, v+1)$	(x, y, v)	$\mu(v+1)\Delta t$

Table 1: transitions in HIV – CD4 + cell interactions

The Master equation for Virus-Host interaction

$$
\begin{aligned}
P'_{x,y,v}(t) &= -\{\lambda + \delta x + \alpha\beta x v + \mu v + \kappa y + \gamma\} P_{x,y,v}(t) \\
&+ \lambda P_{x-1,y,v}(t) \\
&+ \delta(x+1) P_{x+1,y,v}(t) \\
&+ \beta(x+1)(v+1) e^{-\rho\tau} P_{x+1,y-1,v+1}(t) \\
&+ N\kappa(y+1) P_{x,y+1,v-1}(t) \\
&+ \gamma P_{x,y,v-1}(t) \\
&+ \mu(v+1) P_{x,y,v+1}(t)
\end{aligned}
$$

The Lagrange Partial Differential Equation

$$\frac{\partial G}{\partial t} = \{(z_1 - 1)\lambda + (z_3 - 1)\gamma\}G + (1 - z_1)\delta\frac{\partial G}{\partial z_1} + (Nz_3 - z_2)\kappa\frac{\partial G}{\partial z_2}$$

$$+ \quad (1 - z_3)\mu\frac{\partial G}{\partial z_3} + \beta(e^{-\rho\tau}z_2 - z_1 z_3)\frac{\partial^2 G}{\partial z_1 \partial z_3}$$

Moments of X(t), Y(t) and V(t) from the pgf

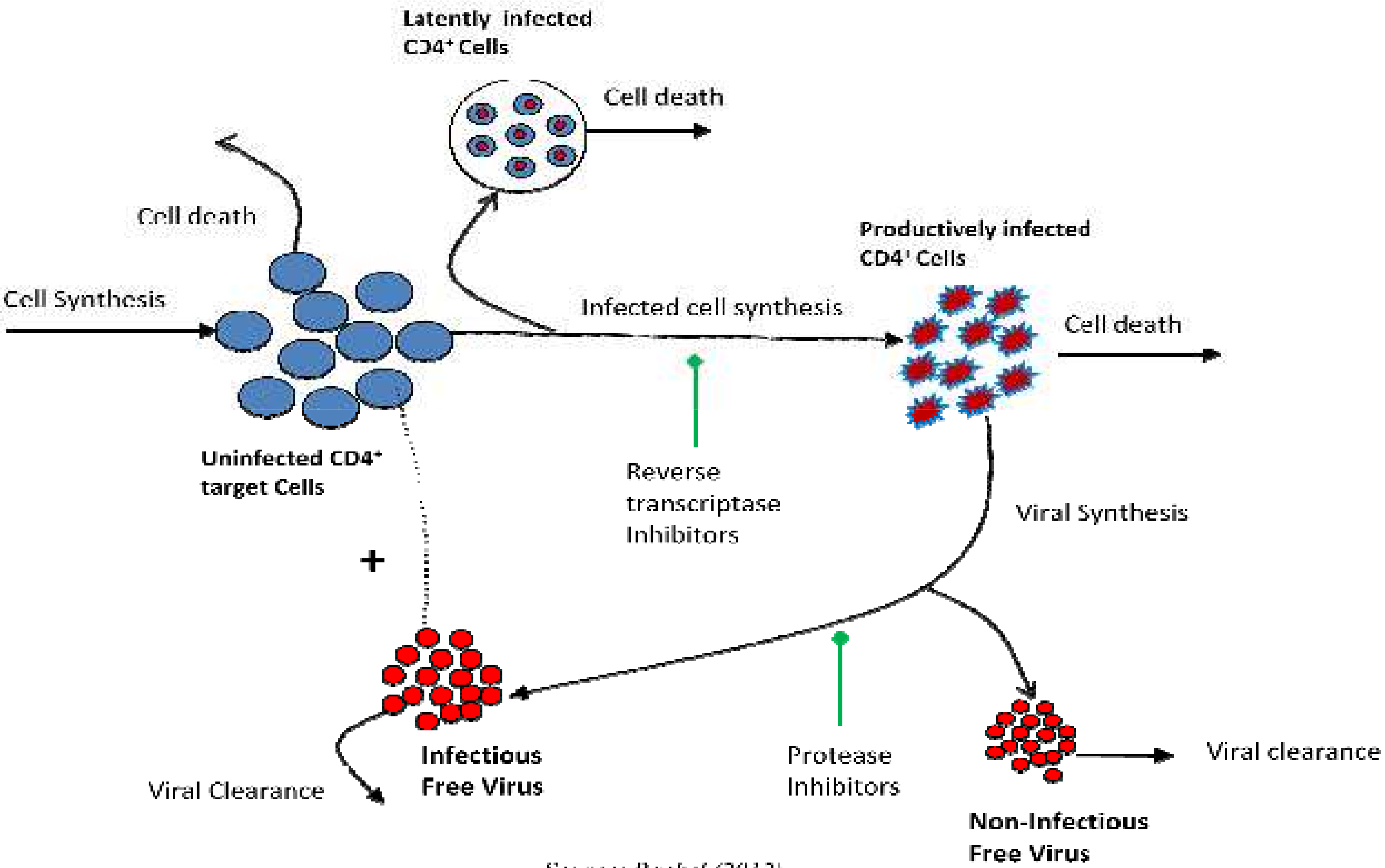
$$\frac{\partial}{\partial t}E[X(t)] = \lambda - \delta E[X(t)] - \beta E[X(t)V(t)]$$

$$\frac{\partial}{\partial t}E[Y(t)] = -\kappa E[Y(t)] + \beta e^{-\rho\tau} E[X(t)V(t)]$$

$$\frac{\partial}{\partial t}E[V(t)] = \gamma + N\kappa E[Y(t)] - \mu E[V(t)] - \beta E[X(t)V(t)]$$

With **x(t) = E(X(t))**, **y(t) = E(Y(t))** and **v(t) = E(V(t))**,
Where the variables X(t) and V(t) are independent

Effects of treatment therapy



Source: Rachel (2012)

Model incorporating treatment therapies

$$\frac{\partial G}{\partial t} = \{(z_1 - 1)\lambda + (z_3 - 1)\gamma\}G + (1 - z_1)\delta\frac{\partial G}{\partial z_1} + (\omega N z_3 - z_2)\kappa\frac{\partial G}{\partial z_2}$$

$$+ \quad (1 - z_3)\mu\frac{\partial G}{\partial z_3} + \alpha\beta e^{-\rho\tau}(z_2 - z_1 z_3)\frac{\partial^2 G}{\partial z_1 \partial z_3}$$

Probability of Virus clearance

$$\begin{aligned}\frac{\partial}{\partial t}P(V=0,t) &= \frac{\partial}{\partial t}\left(\frac{1}{0!}\frac{d^0 G}{dz_2^0}\right)\Big|_{z_1=z_2=1,z_3=0} \\ &= \frac{\partial G(1,1,0;t)}{\partial t} \\ &= \gamma - \omega N\kappa E[Y(t)] + \mu E[V(t)] + \beta E[X(t)V(t)]\end{aligned}$$

Expected outcome

- Probability distributions of the virus and cell reservoirs with combined treatment therapy.
- Moments of the random variables
- Probability of virus clearance

Thank you