On the pattern recognition of Verhulst-logistic Itô Processes in Market Price Data.

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Abstract.

We introduce a highly error resistant method of extracting Itô processes as applied to market data. This method is inspired by an AI method known as Hough transforms (HT). The HT method has been used in extracting geometric shape patterns from noisy and corrupted image data. We use this method to extract simultaneously logistic geometric Brownian motion trends from simulated price histories data. It turns out that this approach is an effective method of extracting market processes for both simulated and real-world market price data.

Key words: Itô processes, logistic function, logistic geometric Brownian motion, artificial intelligence (AI), Hough transforms, price histories, market processes, and pattern recognition.

1.0 Introduction

Recognition of patterns in market prices is very important to the market players for decision making. This will enable the players to understand how the market moves with time. The characteristics of a successful recogniser of patterns corrupted by noise are robustness of the following different types:

- ● resistance to statistical errors in the capturing process ('Gaussian noise'),
- ● resistance to extreme errors ('impulsive noise'),
- ● resistance to loss of data ('pattern occlusion').

There is a growing literature on attempts to apply multi-layer artificial neural nets (ANNs) to financial market data [8]; [15]. In this paper, however, we apply a method that has been less publicised than the ANN: the Hough transform.

1.1 Pattern recognition in asset price history.

The primary aim of pattern recognition is to identify objects and images from their shapes, forms, outlines, colour, surface extraction, temperature or sound or any sensed attributes- usually by automatic means [14]. Pattern recognition has developed into a powerful tool in many disciplines, business and finance included. One of the virtues of pattern recognition is the ability to simplify complex environments by picking out pattern from noise. Another virtue is its reliance on algorithms that are executable with tremendous simplicity, clarity, speed, and power. Pattern recognition supports effective and rapid decision-making in any business environment, however time sensitive it may be. In an increasingly changing, chaotic, and complex business environment, understanding and internalising market patterns is vital and crucial, for helping management and other market players to make safe, rapid, and effectively supported executive decisions.

In financial markets, it is an established notion that prices of underlying assets move in patterns and that these patterns can be used to forecast asset prices statistically. Every financial analyst needs to define and search out meaningful patterns from historical time series quickly and efficiently. Such discovery of

patterns can be time consuming and technically challenging, especially in the absence of high-level pattern recognition techniques.

In this paper, our preferred technique of pattern recognition is inspired by the classical Hough transform (HT). We give a summary of the Hough transform in the next section and adapt it to identifying patterns in logistic stochastic dynamics of changing asset prices.

2. 0 The Classical Hough Transforms

The Hough transform (HT) (Hough 1962, Duda and Hart 1972; Leavers 1992; Princen and others 1992; Toft 1996) is a standard tool for extraction of geometrical primitives such as line-segments and arcs from noisy digital images. Hough transforms change the mode of presentation of data set in order to ease detection of a specific geometric form being looked fort.

The characteristic relation (1) below, of the looked-for feature is back-projected in the space of pattern parameters, $(\alpha_1, \alpha_2, ., \alpha_n)$. A pixel (x, y) that lies on a parametric curve of the characteristic relation that is inconsistent may be represented in parametric space, if $f((x, y), (\alpha_s)) = 0$ (1)

holds, then idea of the Hough transform is to convert a pixel position (x, y) into a relation between pattern parameters by fixing (x, y) in (1) above.

One of the main advantages of the Hough transforms is its robustness to noise and occlusion. This is due to the fact that each image point is considered independently of the other points. The occlusion of valid pixels only alters peak intensities in parameter space and not shape or pattern. Similarly, the addition of a few noise pixels only adds low-level peaks in parameter space. In financial markets, a direct consequence is the capability of transforms to detect broken line patterns as may be reflected in stock prices during nights, holidays, and strikes by workers or weekends when trading is suspended for some day(s) [10]. Hough transforms are also capable of detecting several different features in a given image in one go. That is, it can simultaneously accumulate 'votes' for several peaks in the accumulator array, if the given pattern consists of several primitives, as in the case of several processes [10].

2.1 The (ρ-θ) Hough transforms

The slope-intercept form of a line, (y = mx + c), described above has a problem with the vertical lines, both m (gradient) and c (y-intercept) are infinite i.e. they are unbounded. This problem is eliminated by expressing a line passing through point (x, y) in standard polar (ρ-θ) form instead of the Cartesian (m, c) form. That is each point (x, y) can be transformed to

$$x\cos\theta + y\sin\theta = \rho,$$ (2)

where ρ is the perpendicular distance to the line AB from the origin and θ is the angle between this perpendicular or 'normal' and the x-axis. By convention $0^0 \leq \theta < 180^0$.

If the point (x, y) is fixed, equation (2) describes a sinusoidal curve in (θ, ρ) parametric space. A pixel at point (x_i, y_i) in image <u>votes</u> into cells through which the sinusoidal curve (2) passes.

We illustrate the Hough transforms in parametric (ρ-θ) by considering a set of 7 collinear image points in Fig. 1. The corresponding sinusoidal voting curves intersect at a single point in parameter space as depicted in Fig. 2. Fig. 3 depicts a 3-dimensional accumulator array with the peak clearly labelled. The three dimensional accumulator is used to indicate the pattern in the dynamical system.
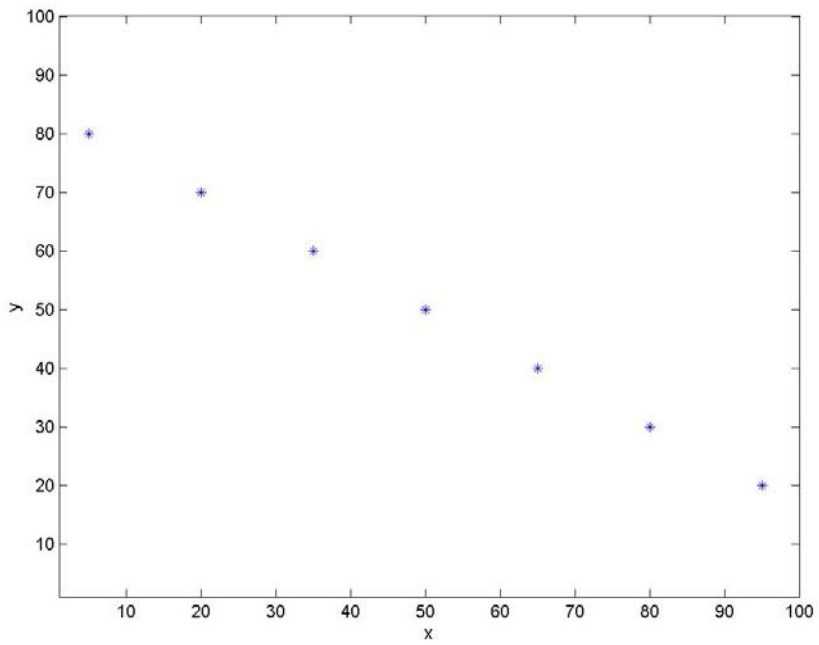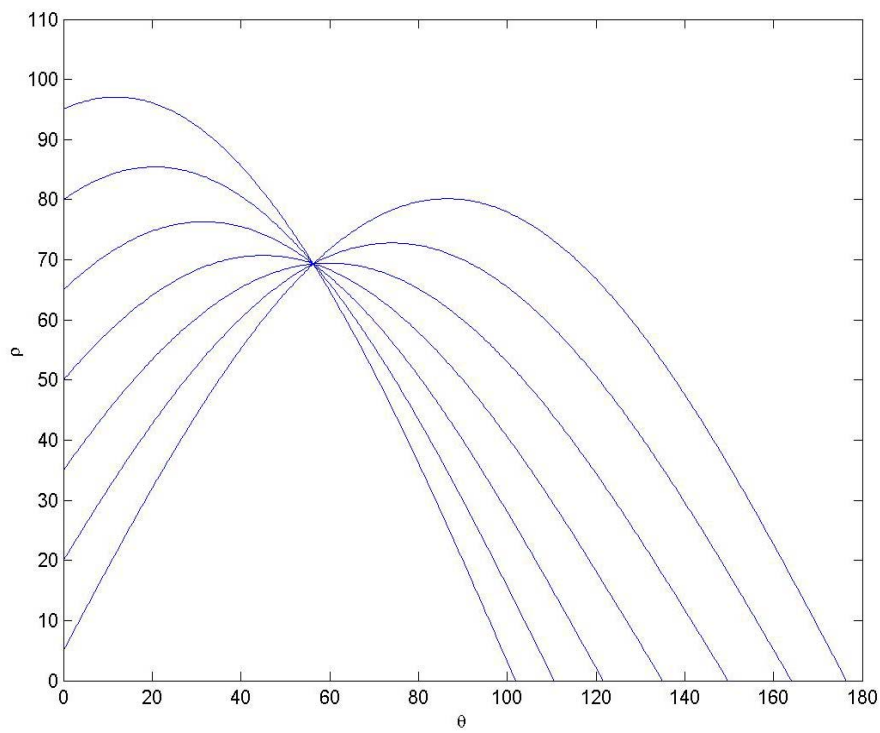
Figure 1: An image of collinear points.

Figure2: The corresponding Hough
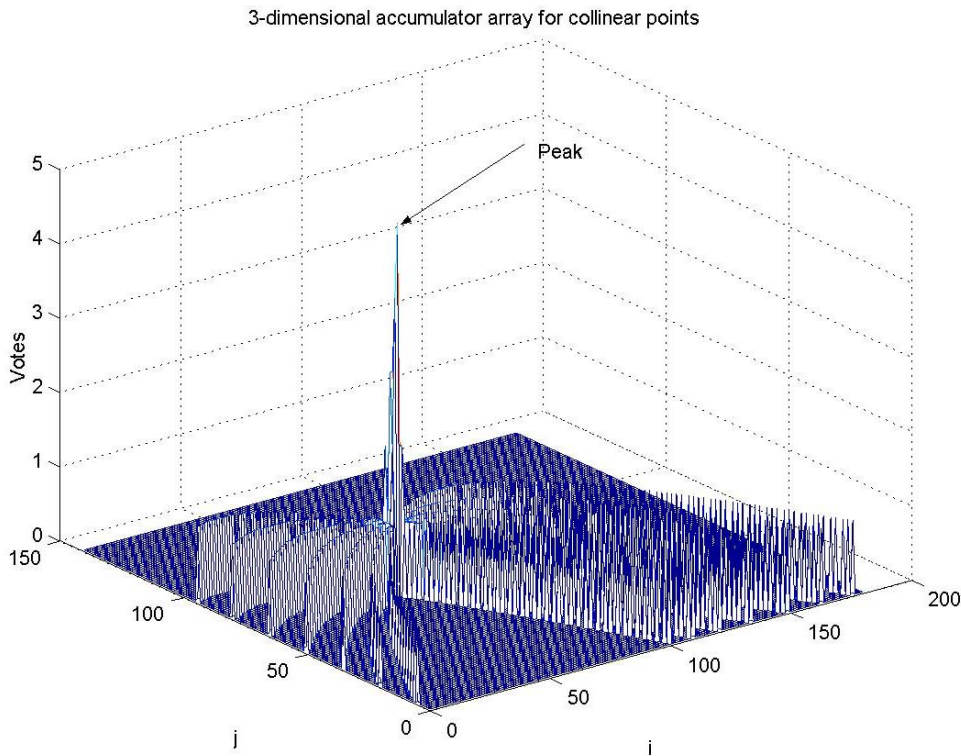
Transforms for collinear points.

Figure 3: A 3-dimensional accumulator array (butterfly shape) for the collinear points of Fig. 1.

2.2 Detection of two processes

Hough transform can be used to detect simultaneously two or more patterns (or processes in a given dynamical system). To illustrate this property, two lines (non-parallel) of collinear points are used (Figure 4) and the corresponding Hough transform is shown in Figure 5 which shows clearly two points of concentrated intersection of the sinusoidal curves. Figure 6 depicts a 3-dimensional accumulator array for the two processes of Figure 4. There are two peaks clearly labelled which depict the two processes in the dynamical system.

. In finance markets it is usually assumed during analysis that there is only one stochastic process, the linear geometric Brownian motion, driving the stock prices and hence the prices of derivatives. This might not be true in some markets. There might be more than one process within the time period of trade. It may also not necessarily be the linear Brownian motion in play; the asset price may be following a logistic Brownian motion [10]; [11]. Nevertheless, a generalisation of the Hough transforms is capable of detecting such stochastic processes in a financial market.
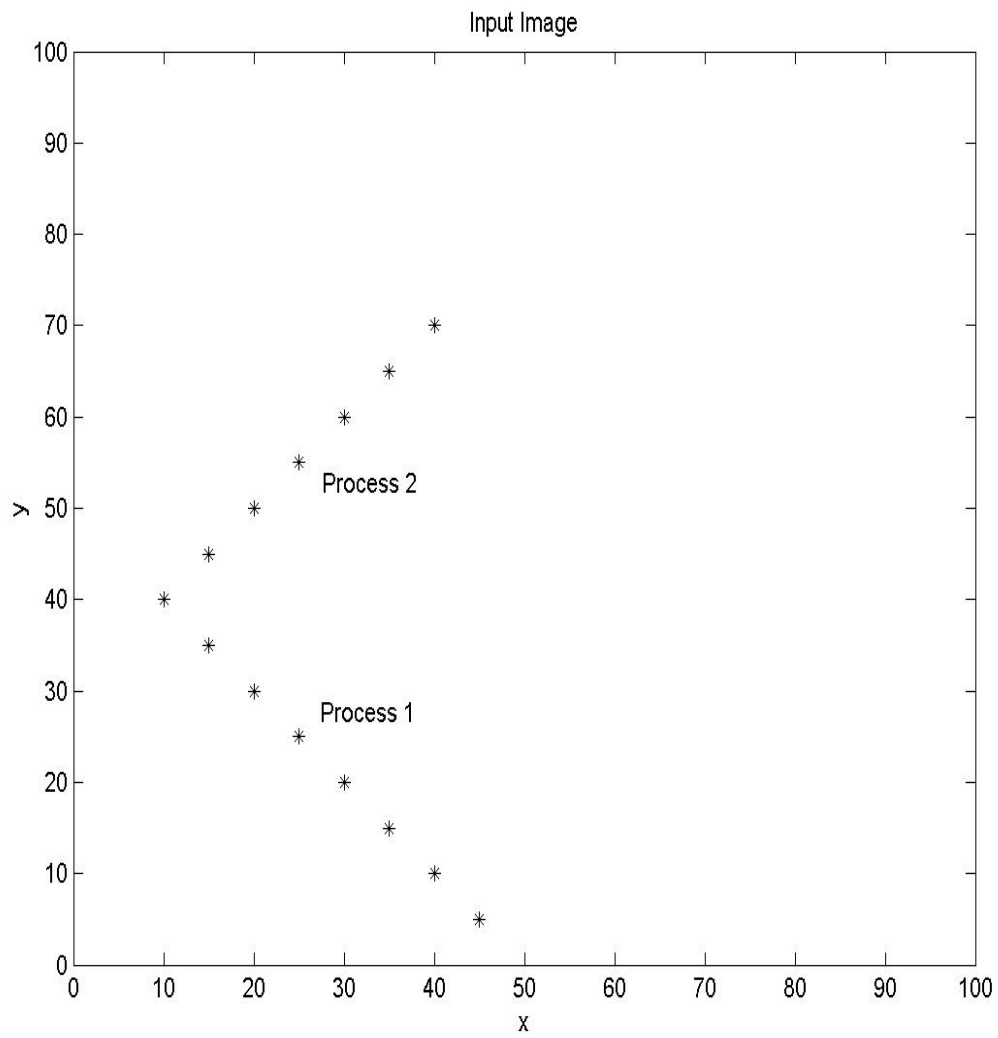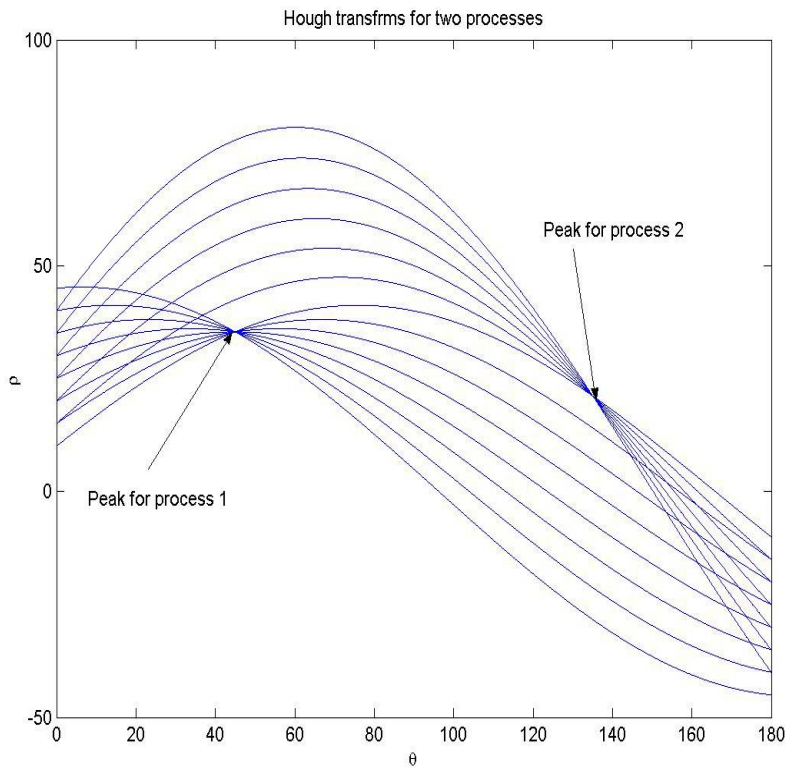
Figure 4: Images of two simple processes

Hough transfrms for two processes

Peak for process 2

Peak for process 1

ρ

θ

Figure 5: The corresponding Hough   Transforms of two processes.

3-dimensional accumulator array for two processes
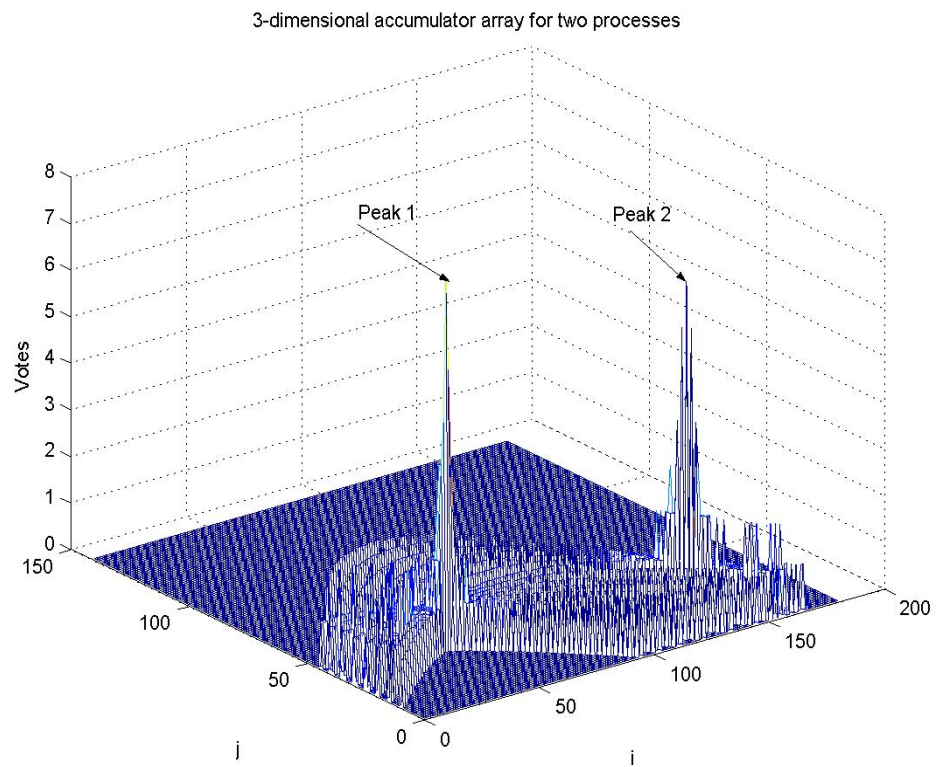
Peak 1

Peak 2

Votes

j

i

Figure 6: 3-dimensional accumulator array for two processes indicated by two peaks.

We recall that Figure 1 shows that a single line defines a well-delineated peak as shown in Figure 2 and Figure 3. Several lines will show several peaks in the accumulator. In particular, two lines of collinear points, representing two processes (see Figure 4), will show two peaks in the accumulator array as depicted in Figure 6. If the number of processes is not known in advance, the number of peaks in the accumulator array will represent the number of processes in the system.

3.0 Determinist price logistic function

The deterministic price logistic function is given by

$$\frac{dP(t)}{dt} = rP(t)(P* - P(t)) \qquad (3)$$

[10]; [11].

This is a deterministic logistic (first- order) ordinary differential equation in asset price, P(t). Thus the fractional growth of P(t) is linear in P(t). This contrasts with exponential growth, where the fractional growth is constant (independent of P(t)).

From equation (3), the logistic equation models price increase as a quadratic function of security price P(t), with a peak value $\frac{rP*^2}{4}$. Several trajectories of the growth plotted against P(t) are depicted in Fig. 3. It should be noted that the growth

The solution set of equation (3) is given by

$$P(t) = \frac{P* P(0)}{P(0) + (P* - P(0))e^{-rP*t}} \qquad (4)$$

where P(0) is a parameter interpreted as the initial price.

From equation (4) we observe as $t \to \infty$, the term e^{-rP*t} in the denominator will be negligibly small, and so $P(t) \to \frac{P* P(0)}{P(0)} = P*$. The security price thus settles into a constant level, called a steady state or equilibrium, at which no further change will occur.

A member of this set corresponding to P(0) = 2 is shown in graphically in Fig. 7 and similar graphs to different growth rates for P(0) = 10 are shown in Fig. 8.
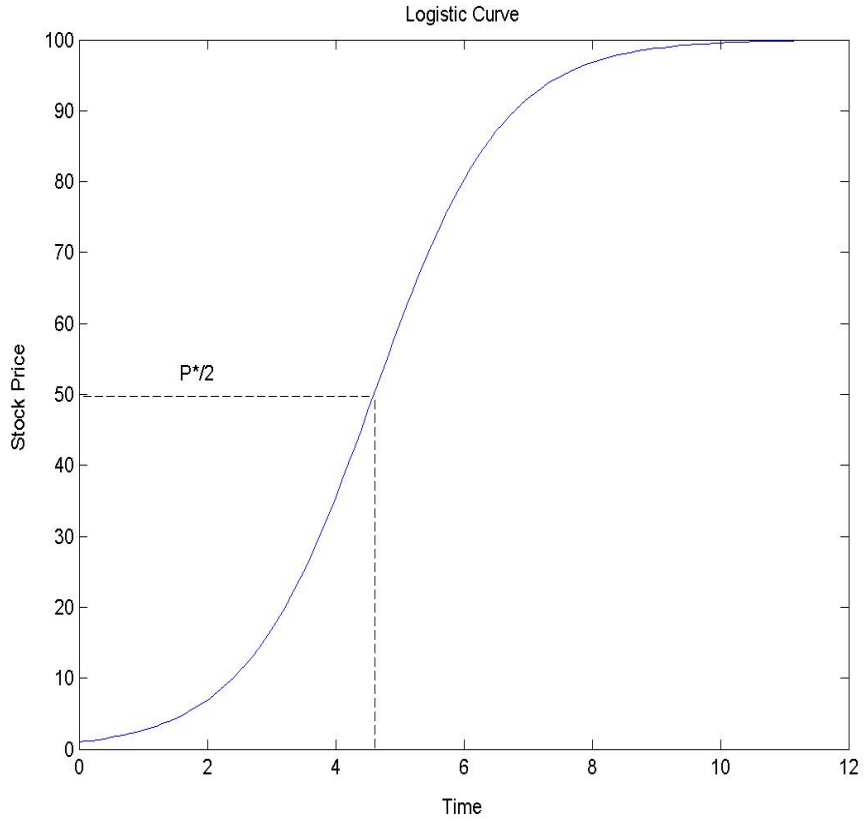
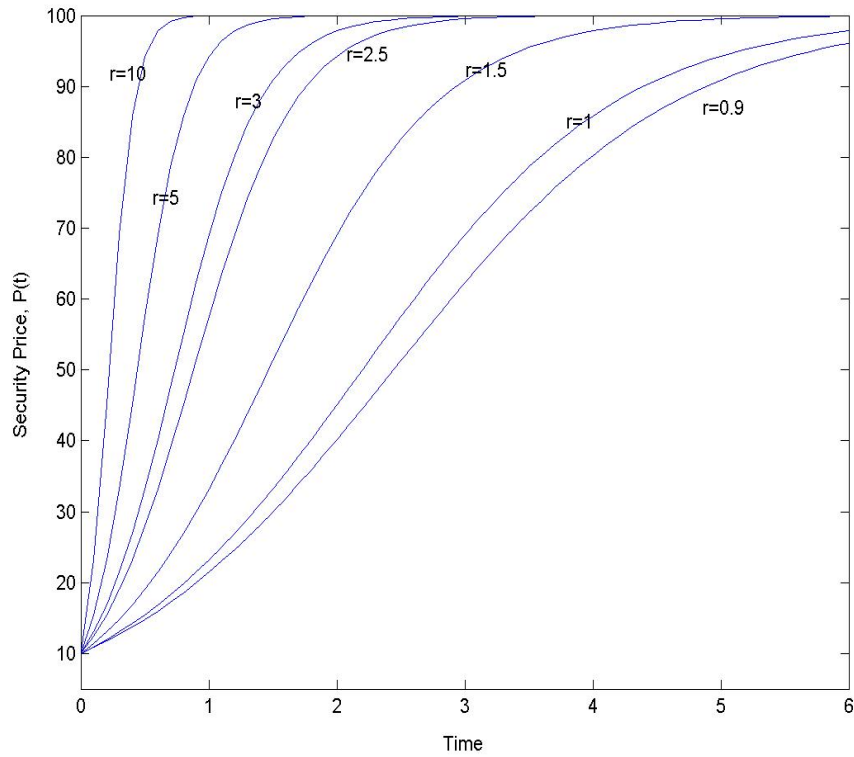**Fig. 7** A typical Stock Price Logistic (sigmoid) curve (P* = 100, P(0) = 2, r = 0.025).

Fig. 8 Several trajectories of Stock Price Logistic equations (P* = 100, P(0) = 10)

Fig. 9 depicts a case where the market equilibrium, P*, is equal 20, intrinsic growth rate, r = 0.025 and initial security prices are P(1) = 10 and P(2) = 30. In both cases the graph of asset price, P(t) is asymptotic to the line P(t) = 20. That is, the stock price approaches the market equilibrium with time.
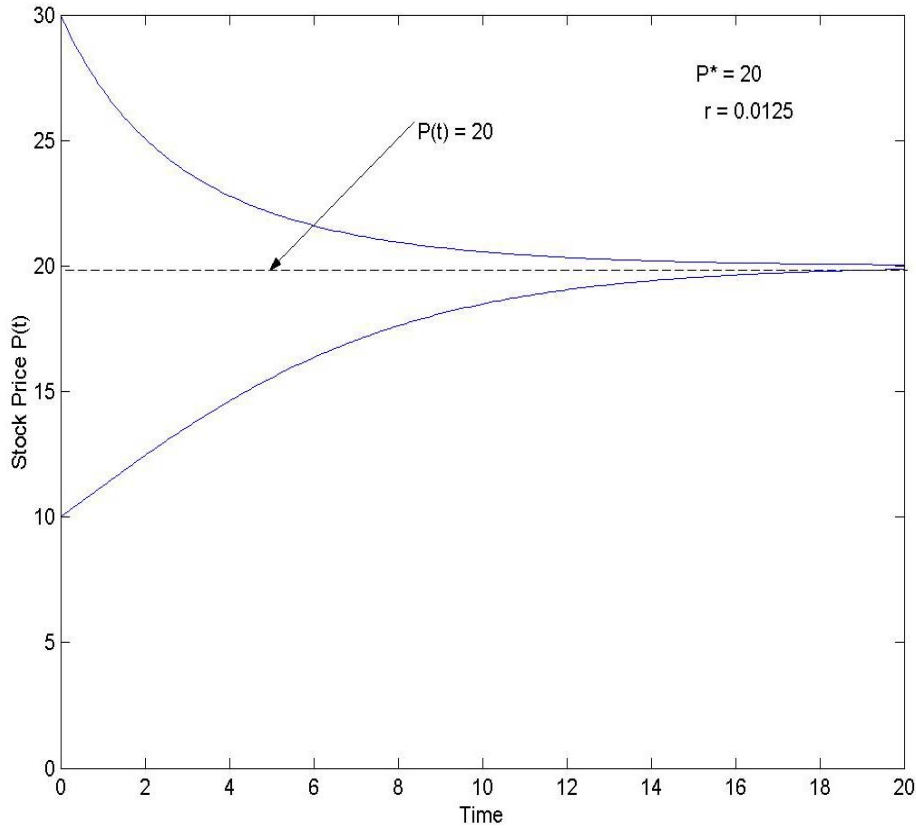
Fig. 9. Plots of increasing and decreasing Verhulst-type Stock Price logistic curves (P* = 20, $P_1(0)$ = 10, $P_2(0)$ = 30, r = 0.025)

3.1 The logistic Brownian motion

One of the relaxation of assumptions of Black-Scholes-Merton model [10]; [11], we have that the price of an underlying asset follows the logistic geometric Brownian motion given as

$$dP(t) = \mu P(t)(P^* - P(t))dt + \sigma P(t)(P^* - P(t))dZ(t) \qquad (5)$$

where P(t) is the price of the underlying at any time t, P^* is the market equilibrium price, μ is the rate of increase of the asset price, σ is the volatility of the underlying and dZ(t) ~N(0,dt). The solution of equation (5) using Itô's lemma is given as

$$\ln\left(\frac{P(t)}{|P^*-P(t)|}\right) = \ln\left(\frac{P(0)}{|P^*-P(0)|}\right) + \mu P^*(t-t_0) + \sigma P^* Z(t) \qquad (6)$$

Re-arranging and simplifying equation (6) and solving for P(t), we get

$$P(t) = \frac{P^* P(0)}{P(0) + (P^* - P(0))e^{-(\mu P^*(t-t_0)+\sigma P^* Z(t))}} \qquad (7)$$

This price dynamics is referred to as logistic Brownian motion of stock price, P(t). When σ = 0, then we get the deterministic logistic equation, (4). Fig. 10 depicts a graph of a deterministic logistic trend and a typical logistic Brownian motion.
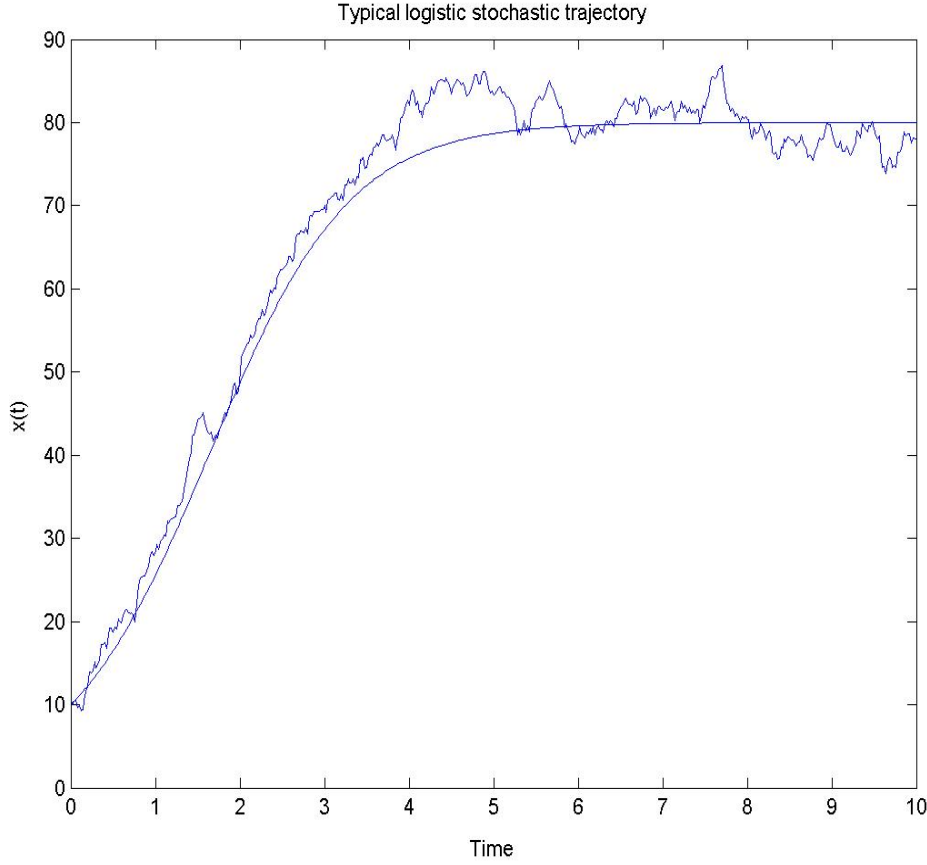
Fig. 10 A typical trajectory of simulated logistic Brownian motion (P(0) = 10, P* = 80, μ = 0.015, σ = 0.5).

3.2. Towards a computational model of the logistic Brownian motion

Equation (6) can be re-arranging and simplifying equation, to get

$$\sigma \varepsilon_i = \frac{\ln\left(\frac{P(t_i)(P^*-P(0))}{P(0)(P^*-P(t))}\right) - \mu P^*(t-t_i)}{P^*\sqrt{t-t_i}} \qquad (8)$$

Squaring equation (8), summing over i and simplifying we get

$$\sigma^2 = \frac{A - 2B\mu + \mu^2 C}{\chi^2} \qquad (9)$$

where

$$A = \sum_{i=1}^{n} \frac{1}{P^*} \frac{\log^2\left(\frac{P(t_i)(P^*-P(0))}{P(0)(P^*-P(t_i))}\right)}{t_i - t_0},$$

$$B = \sum_{i=1}^{n} \frac{1}{P^*} \log\left(\frac{P(t_i)(P^*-P(0))}{P(0)(P^*-P(t_i))}\right)\sqrt{t_i - t_0}, \qquad (10)$$

$$C = \sum_{i=1}^{n} (t_i - t_0)$$

Equation (10) links in a self-consistent way the LBM parameters $P(0)$, P^*, μ and σ^2 to the complex moments A, B, and C in a moving window $<P(t_0), P(t_1), P(t_2),....., P(t_N)>$. The measure σ^2 of volatility during price adjustment varies as the reciprocal of statistical variate χ^2, which has standard Pearson's distribution with most probable value N-1.

The analogous equation for the linear Brownian motion of $\log(P(t))$ has a simpler set of moments

$$A = \sum_{i=1}^{n} \left(\frac{\log(P(t_i)/P(0))}{\sqrt{t_i - t_0}} \right), \quad B = \sum_{i=1}^{n} \log(P(t_i)/P(0)),$$

$$C = \sum_{i=1}^{n} (t_i - t_0)$$

and these have can be used to extract processes from dynamical systems[10], [11]. The type of statistical pattern recognition used is highly resistant to gross errors or blips in the historical data.

4.0 Adapting Hough transform to logistic Brownian motion

In this section we adapt the Hough transforms to simulated logistic Brownian motion. We introduce and use the grey-scale voting techniques to calculate the area of cell in the accumulator array.

4.1. Detecting deterministic logistic curves in an image

In this subsection we consider values of simulated deterministic logistic function with carrying capacity equal to 90. We use the linear Hough transform to detect the logistic curve in the image. Fig. 11 depicts a typical deterministic logistic curve and Fig. 12 depicts the corresponding mesh 3-dimensional accumulator array. It is noticed that the accumulator array does not display the butterfly peak as in the case of collinear points (see Fig. 3)- it shows a ridge.
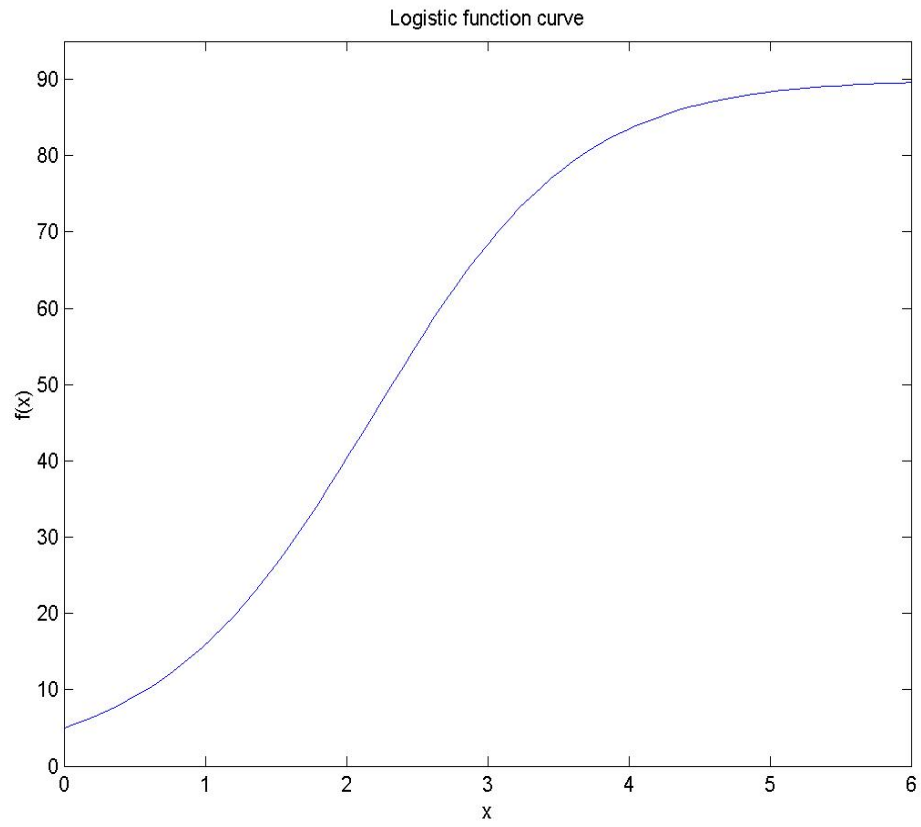
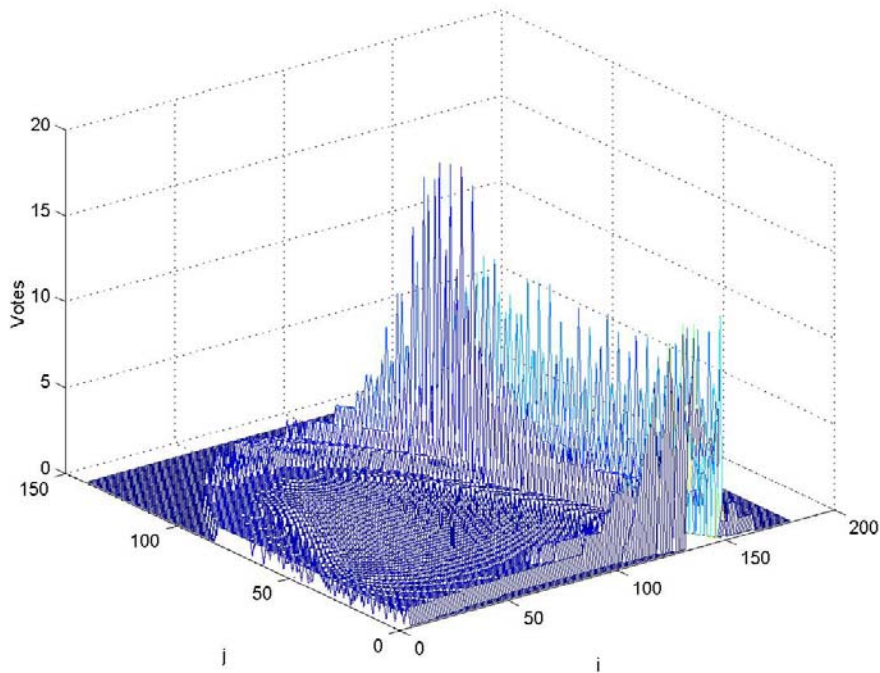**Fig. 11.** Typical deterministic logistic function curve with carrying capacity 90 and initial value 5.

Fig. 12. A 3-dimensional accumulator array for a deterministic logistic function with $\mu = 0.015$

In fact it is possible to design a special accumulator for logistic curves, which is useful when tracking the ground of faults in machines [6]. We are, however, more interested in accumulators for stochastic logistic curves or logistic Brownian motion

4.2. Detecting stochastic logistic curves in an image

We apply the adapted Hough transform to simulated logistic Brownian motion described in section 3. In this example, we take the carrying capacity or Walrasian equilibrium $P^* = 90$ and initial value $P(0) = 5$. Fig. 13 depicts the 3-dimensional Hough transform accumulator array of the stochastic logistic function. This accumulator takes the form of the 3-dimensional Hough transform accumulator array. The array does not show a butterfly shaped peak but a ridge of many peaks.
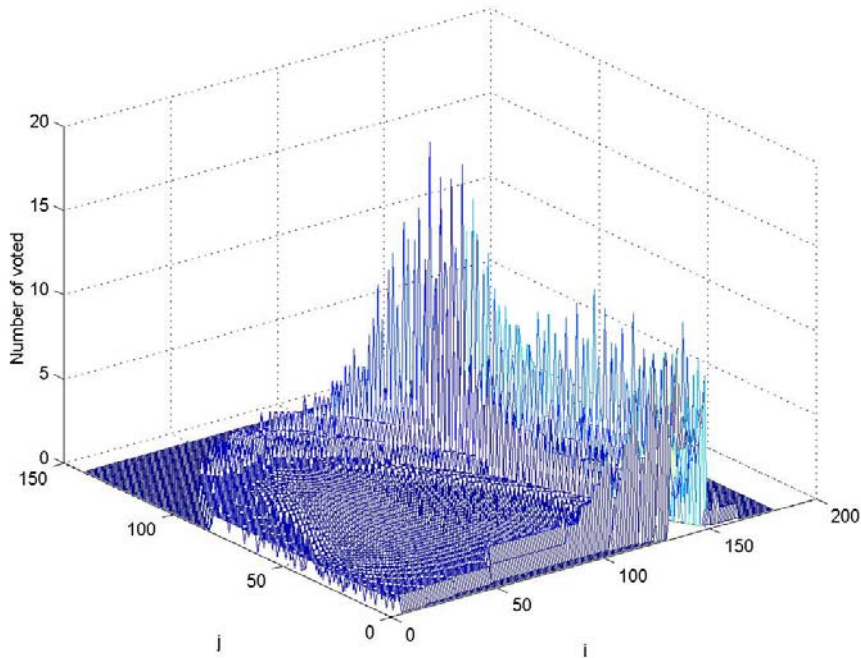
Fig. 13. A 3-dimensional accumulator array for a stochastic logistic function, with μ = 0.015 and σ = 0.5.

4.3 Application to real world market data: A case of Charter Plc, UK

In this subsection we test the Hough transform for logistic Brownian motion by using real world market data over periods where slow price adjustment was evident graphically. We use stock prices from Charter Plc from 22nd Apr.2003 to 17th Oct. Fig.14 depicts the curve of asset prices and Fig.15 depicts the 3-dimensional grey-scale accumulator array. In the accumulator it is clear that there are two distinct processes represented by peak 1 and peak 2. The processes are symmetrical about some middle point (turning point) of the logistic curve of logarithms of the prices.
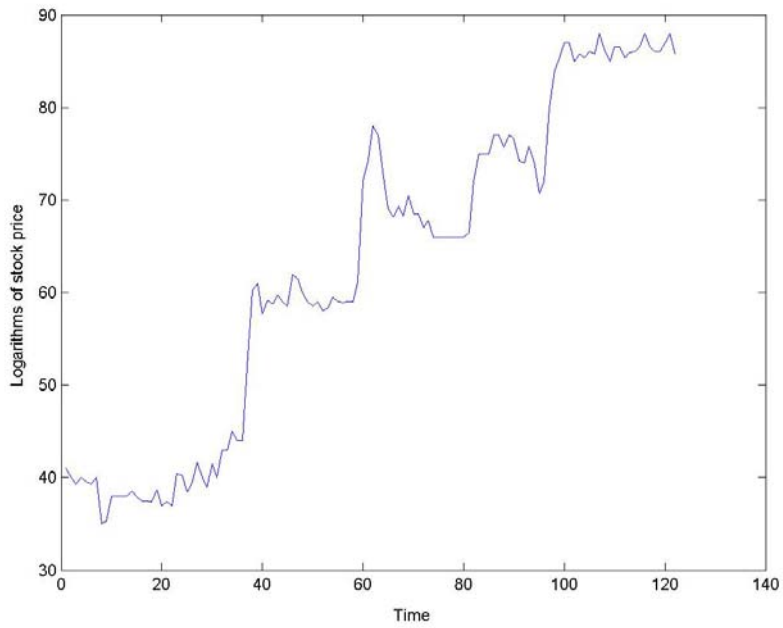
Fig. 14 Price trajectory for Charter Plc from 22nd April 2003 to 17th October 2003.
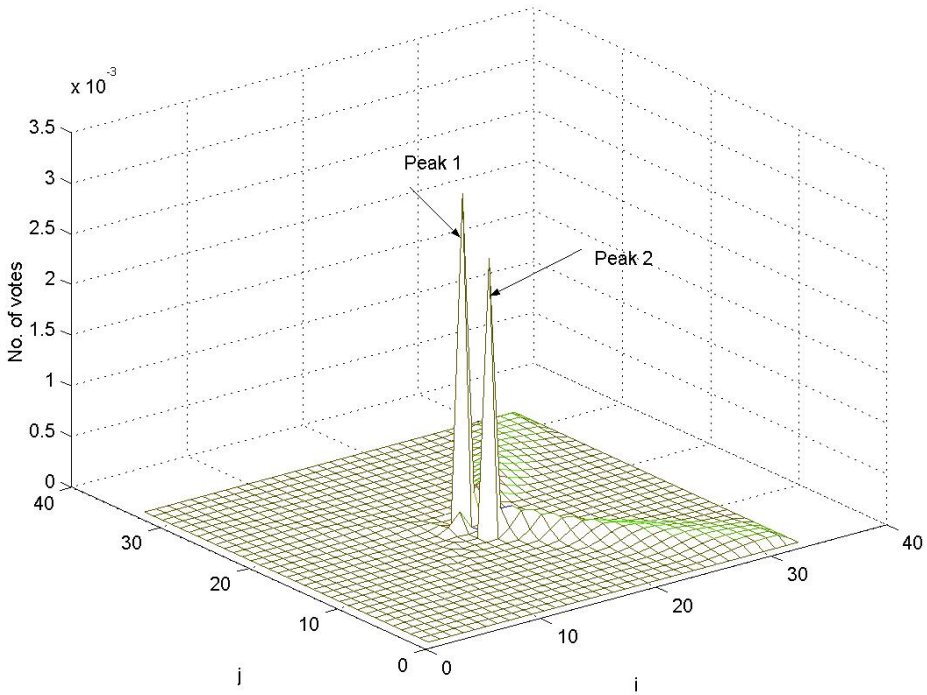

Fig. 15 A 3-dimensional grey-scale accumulator array for Charter Plc. Two peaks are evident, peak 1 and peak 2.

Fig.16 depicts a contour of Fig. 15 and the highest peak (peak 2) in the contour has the highest colour intensity as shown in the colour bar. The contour clearly confirms that there are two processes within the market.
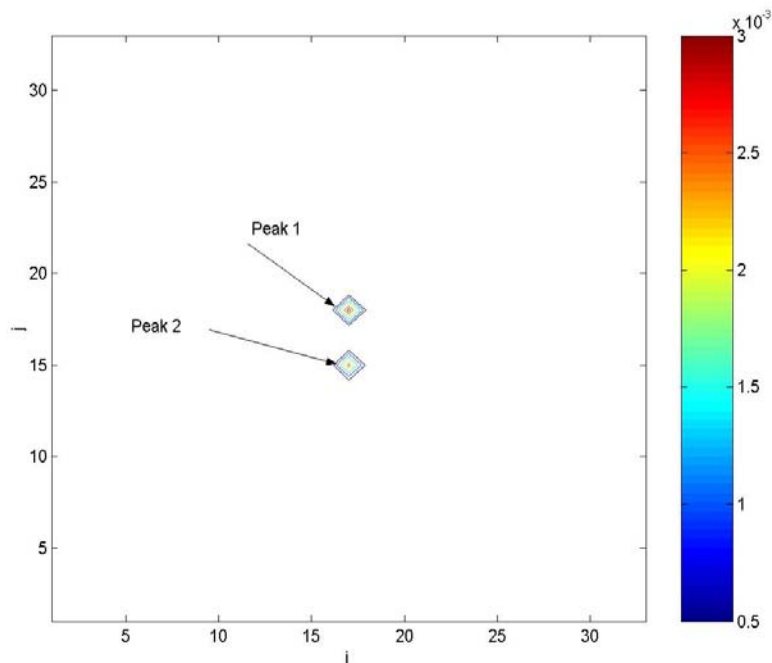

Fig. 16 A 2-dimensional contour of grey-scale image.

5.0 Summary

In this chapter, we have shown that AI can be adapted to detect stochastic process in real market data. This technique has been shown to be robust to market data occlusion, noise in data and is able to detect multiple processes in a market. An additional virtue of this technique is its ability to communicate complex ideas with tremendous simplicity, clarity, speed and power. It also supports effective or rapid decision making in the market. As is evident in the market, velocity of change is increasing, amidst greater complexity and chaos. So, possessing a deep understanding of the market patterns, can enable critical decision making whenever trading is to be undertaken an urgent issue-this helps the trader to decide when to trade and when not to trade. For financial management, knowledge of market patterns will also help in the quality of decision to be made.

6.0 Conclusion

In this paper, we have shown that AI can be adapted to detect logistic stochastic process in simulated data involving logistic geometric Brownian motion. This technique has been shown to be robust to market data

occlusion, noise in data and is able to detect multiple processes in a market. An additional virtue of this technique is its ability to communicate complex ideas with tremendous simplicity, clarity, speed and power. It also supports effective or rapid decision making in the market. As is evident in the market, velocity of change is increasing, amidst greater complexity and chaos. So, possessing a deep understanding of the market patterns can enable critical decision making whenever trading is to be undertaken an urgent issue-this helps the trader to decide when to trade and when not to trade. For financial management, knowledge of market patterns will also help in the quality of decision to be made.

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