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Effects of diagrams showing relationships between variables in solutions to problems concerning relative values.

1. Introduction

"Relative values" taught in the fifth grade are said to be the most difficult among the contents of mathematics taught in Japanese elementary schools. Relative value problems can be classified into three types: (1) when the compared quantity and base quantity are known, and the relative value is required (First use), (2) when the base quantity and relative value are known, and the compared quantity is required (Second use), and (3) when the compared quantity and relative value are known, and the base quantity is required (Third use). It is known that the third type of problems are especially difficult for elementary school students.

In Japan, number lines are generally used when teaching relative values, because relationships between the actual quantity and corresponding relative value are visually indicated. On the other hand, Shindo and Shimizu (2015) examined whether number lines are useful for solving verbally presented relative value problems and obtained the following results:

(1) Solutions to verbal relative value problems are facilitated in elementary school students by using number lines.
(2) It is difficult for elementary school students to draw number lines by themselves when solving verbal problems.
(3) One reason for (2) was that they could not regard the base quantity as "once".

Furthermore, Shindo and Moriya (2015) conducted an experiment with university students that required solutions to relative value problems. The results indicated that many university students did not understand number lines, and that number lines are not effectively used in solving problems. Therefore, is necessary to develop tools for facilitating the understanding of relative values. This study presents a diagram method that clearly indicates relationships among the base quantity, compared quantity, the relative value of the base quantity (=1 (once)), and the relative value of the compared quantity. Relative values were taught to two fifth graders using such diagrams and the effects of the diagram on facilitating solutions to the problems were examined.

2. Experiment

Participants: M and N that were fifth graders participated in the experiment. They had already learned multiplication and division of decimal fractions, but had not learned relative values.

Procedures: A textbook that included the definition of relative values and three types of problems described above, which were developed by the authors, was used. One of the authors taught relative values following the textbook, and using a diagram called a "box diagram" (Figure 1).

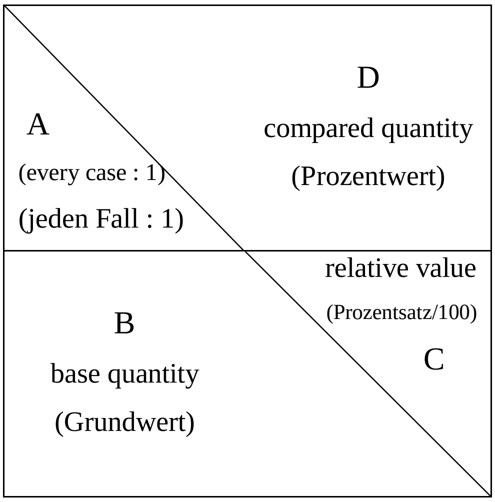

Figure 1 Box Diagram

Overview of the teaching process

Pre-tests was conducted to examine the effect of the box diagram. The problems used in the tests are shown below. These problems have been used in Shindo and Shimizu (2015). In their study, the mean percentage of correct answers among elementary school students (N=116), to whom relative values had already been taught using number lines, were 62%, 67%, and 58% respectively for the three problems (Table 1)

Problem 1. There is a 30 cm stick. If you cut 18 cm of the stick to use it for work. How many times the length of the original stick is the length of the stick that I cut? (First use)
Problem 2. A class consists of 40 students. The number of boys in the class is 0.6 times the total number of students in the class. How many boys are there in this class? (Second use)
Problem 3. A boy gave 15 marbles to his sister. The number of marbles that the boy gave to his sister was 0.6 times the marbles that he originally had. How many marbles did the boy originally have? (Third use)

Table 1 Sentence Problems imposed as Pre-test

In the pre-test, M developed a correct formula for Problem 1, whereas N responded "30-18=12, 12÷30=0.4". M also developed a correct formula for Problem 2, however N developed a wrong formula; "40÷0.6." M and N both developed a wrong formula for Problem 3; "15×0.6." This indicated that the third problem was the most difficult for both M and N. Teaching started after finishing the pre-test. First, a first use problem below was given: "at four basketball games the success rates for shoots by a player were as follows: 4/8, 4/10, 8/10, and 9/12. Compare the success rates for shoots between the four games." M calculated the difference between the number of shoots and failures, and responded that the player was most successful in the fourth game. On the other hand, N attended only to the third and fourth games and responded that although the player made 12 shoots (two more than 10) in the fourth game, he/she succeeded in 9 shoots (just one more than 8). Therefore, the success rate in the third game was higher. It can be seen from these responses that before teaching both students tended to make judgments based on the differences.

Following this, the definition of relative value (i.e. relative value =compared quantity÷base quantity) and the meaning of relative value were taught. Then, the box diagram was shown and the method of using it was explained (Table 2).

(1) Always enter "1" in the A division (see Figure1), which means "once" (2) Enter the actual quantity corresponding to "once" in the B division (3) Enter the actual quantity not corresponding to "once" in the D division (4) In the C division, enter how many times larger D is than B (5) Enter " ? " as the unknown value that is inquired in the problem

Table 2 Usage of Box Diagram

After the explanation about relative values using the box diagram, the two participants were required to solve the problem. They calculated the relative values of four games, and responded that the success rate in the fourth game was the highest.

Next, the participants were required to solve another three first use problems using the box diagram. Results indicated that they could respond correctly. At this point, percentage expressions of relative values were taught. Then, the second use problem below was shown: "There is 300 mL juice containing 30% fruit juice. How many milliliters is the fruit juice?" The two participants made box diagrams and gave the correct response. When given two other second use problems, they could also make box diagrams and give the correct response. Subsequently, the participants were required to solve the third use problem below: "The weight of a kitten is 168g, which is 160% of its birth weight. What was the birth weight of the kitten?" M and N made box diagrams by themselves and responded correctly. Because findings of previous studies have indicated that the third use problems are difficult to solve, another solution described below was was taught: using the unknown quantity X, make a formula of the second use, and converting it into the third use. That is, we taught that it would be easier to solve third use problems by first making the formula, “*X*×relative value=compared quantity,” and then converting it into “*X*=compared quantity÷relative value.” Two third use problems were given, which indicated that both participants could answer correctly without using an equation.

After going through the teaching process described above, identical problems to the pre-test were posed as the post-test. This indicated that M and N could make correct formulas and respond correctly to all the three problems. For examining the effects of box diagrams more objectively, three problems included in the national academic ability test that is given to all sixth graders in Japan was given to M and N, to which they responded correctly. The national average percentage of correct answers to these problems is less than 50%.

3. Discussion

The two participants did not have major problems during the teaching process. Furthermore, they could correctly answerer the problems in the post-test to problems that have a correct response percentage less than 50%. These results suggest that the introduction of box diagrams would facilitate understanding of relative values. This effect could be because the box diagrams indicates relationships among the compared quantity, base quantity, and relative value more clearly, compared to number lines. It is considered effective to enter “1 (once)” in the A division of the diagram at first, which indicates the relative value of the base quantity.

On the other hand, even when using box diagrams, it is necessary to identify the base quantity and the compared quantity in order to solve verbal problems, which is exactly the major difficulty in solving problems for many learners. In the future, it would be important to suggest a teaching method that accurately indicates the meaning of verbal problems. This study is a case study with just two participants. Further verification of the effect of box diagrams is required by increasing the number of participants.

4. References

Shindo, T. & Shimizu, M. (2015) Need of the number lines in the understanding of the ratio. *Proceeding of the 11th Annual Meeting of the Japanese Association of Teaching-Learning Psychology*, 14-15.

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