Response to Comment on: "Matter-Wave Interferometry of a Levitated Thermal Nano-Oscillator Induced and Probed by a Spin"

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Our papers [1, 2] propose an experiment in which the observation of Ramsey fringes would be evidence for a spatial superposition. We analyzed this as a magnetic effect creating a Stern-Gerlach like spin dependent separation of the centre of mass (COM) states in conjunction with a gravitational effect imparting a relative phase between the states. The comment points out that this could be interpreted in a different way. It contends that the interference manifested in the spin states is not due to the spatial separation as the gravity effects can also be interpreted as a Zeeman effect. To support its contention, the comment splits the Hamitonian into parts H_1 and H_2 where only H_1 couples the COM with the spin states, while H_2 imparts the phase factor. However, the periodic factorizability of the COM and the spin states requires the action of H_1 as well. It is this factorizability which makes the phase detectable by a measurement on the spin alone. For instance, if the COM and spin states are not entangled at $T/2$, the evolution by H_1 alone for an additional time $T/2$ will not be able to factorise them. This will lead to the Ramsey interference pattern being supressed. Thus the very visibility of the phase due to H_2 hinges on the interference brought about by H_1. Both treatments (our's and the comment's) are valid and equivalent as they use the same Hamiltonian. In both cases there is a spatial superposition except for certain periodic moments in time (at integer multiples of the oscillator time period T). In both cases, the absence of coherence in the COM motion (which could be due to decoherence from air molecules for example) would remove these fringes.

In the absence of decoherence, an arbitrary initial coherent state $|\beta\rangle$ of the COM and an initial spin state $\frac{1}{\sqrt{2}}(|+1\rangle + |-1\rangle)$ evolves jointly as

$$\frac{1}{\sqrt{2}}(e^{-i\phi_+(t)}|\beta(t,+1)\rangle|+1\rangle + e^{-i\phi_-(t)}|\beta(t,-1)\rangle|-1\rangle), \tag{1}$$

where $|\beta(t,\pm1)\rangle$ are COM coherent states with the time-varying separation of $\Delta z(t) = \frac{8\lambda\delta_z}{\hbar\omega_z}(1-\cos\omega_z t)$ with $\delta_z = \sqrt{\frac{\hbar}{2m\omega_z}}$ being the ground state position spread of the oscillator. Despite the fact that $|\beta(t,\pm1)\rangle$ oscillate about centres $\frac{-g\cos\theta}{\omega_z^2} \pm \frac{4\lambda\delta_z}{\hbar\omega_z}$ where there are finite magnetic fields, in our approach, the entire inhomogeneous magnetic field term of the Hamitonian is "used up" to accompish the Stern-Gerlach like separation $\Delta z(t)$, and is thereby, not available any more to impart a Zeeman phase between the separated states. The integrated gravitational phase shift $\int_0^T \frac{mg\cos\theta\Delta z(t)dt}{\hbar}$ gives exactly the phase shift $\phi = \phi_+(T) - \phi_-(T) = \phi_{\text{grav}}$ of Refs.[1, 2].

Let us now clarify that even if the comment's interpretation that the measured signal results from "the common displacement of the COM position of both ±1 states" is adopted, the visibility of this signal is affected by the coherence between the superposed COM states. Consider a case where only the COM motion is decohered: the off diagonal terms $|\beta(t,+1)\rangle\langle\beta(t,-1)|$ are damped by a factor of $e^{-\gamma(t)}$. Then the evolved state at $t = NT$ is

$$\rho(NT) = |\beta\rangle\langle\beta| \frac{1}{2}\{|+1\rangle\langle+1| + |-1\rangle\langle-1| + e^{-\gamma(NT)}(e^{-iN\phi}|+1\rangle\langle-1| + e^{iN\phi}|-1\rangle\langle+1|)\}$$

Thus we see that the spin density matrix has also decohered (thereby lowering the visibility of ϕ as a relevant parameter, say θ, is varied) despite the fact that the decoherence was exclusively for the COM state [3, 4]. In particular if the COM state is completely decohered ($\gamma(NT) \to \infty$) the phase to be measured completely disappears from the density matrix. Thus the visibility of the phase is an evidence of the coherence (interference) between $|\beta(t,+1)\rangle$ and $|\beta(t,-1)\rangle$. That the Stern-Gerlach mechanism in an external inhomogeneous magnetic field does cause a spin dependent spatial splitting of the COM states hardly needs to be independently verified, as it is a long verified effect in quantum mechanics (it is the *coherence* between the split states which is the new challenging thing to be verified, which our papers aimed for). Nonetheless, the fact that indeed there was a time varying spatial separation between $|\beta(t,+1)\rangle$ and $|\beta(t,-1)\rangle$ can be concluded from a time modulation $e^{-(\frac{4\lambda}{\hbar\omega_z})^2(1-\cos\omega_z t)}$ of the visibility of $\phi_+(t) - \phi_-(t)$ from the spin state alone [5, 6]. Moreover, the spin dependent position splitting can be enhanced by a lower ω_z or free flight [7] if one feels that an independent verification of the Stern-Gerlach effect through spin-position correlation measurements is truly necessary.

The pitfalls of a purely Zeeman interpretation of the relative phase development between $|\beta(t,+1)\rangle|+1\rangle$ and $|\beta(t,-1)\rangle|-1\rangle$ can be highlighted by considering a case where we start with $\theta=\pi/2$ so that the gravitational term in the Hamiltonian vanishes and evolve till time $t=T/2$ to obtain a maximal spatial separation $\Delta z(T/2)$ between the superposed coherent states $|\beta(T/2,\pm1)\rangle$. At time $t=T/2$ we instantaneously switch off the magnetic field term (either directly, or for practical purposes by promoting electronic spin states $|\pm1\rangle$ to nuclear spin states) and then apply a gravitational (acceleration) pulse by changing θ from $\pi/2$ to 0 for a very short time $\delta t << T$. The off diagonal component of the spin part of the density matrix evolves as [8]:

$$\langle +1|\rho_S(T/2)|-1\rangle \rightarrow e^{-i\frac{mg\delta t\Delta z(T/2)}{\hbar}}\langle +1|\rho_S(T/2)|-1\rangle \quad (2)$$

The phase $\frac{mg\delta t\Delta z(T/2)}{\hbar}$ cannot be recast as a Zeeman phase.

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