Robustness of Load and Resistance Design Factors for RC Columns with Wind Dominated Combination Considering Random Eccentricity

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Abstract: Based on the capacity models in the 2008 edition of the American Concrete Institute (ACI) Standard Building Code Requirements for Structural Concrete, a more realistic limit state function is built for reinforced concrete (RC) columns with random eccentricity. Then, we discuss the applicability of the load and resistance factors in the code, which is mainly based on reliability calibrations with a fixed eccentricity criterion. Taking the wind dominated combination as an example, representative cases are established by selecting typical values of four related design parameters. Using the load and resistance models in previous reliability calibration of the code, the probabilistic distribution of eccentricity and the statistics of column resistance are analyzed for each case. The analysis indicates that the possible eccentricity shows a scatter over a large range, and the probabilistic model of column resistance varies from case to case, which is largely different from the constant resistance model assumed in previous reliability calibration. With Monte Carlo simulation (MCS), the column reliability is calculated and obtained for different cases. The results

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show that the fixed eccentricity criterion underestimates the reliability differences among cases and overestimates the column reliability in some tension failure cases. To attain a robust design, an improved measure is recommended by selecting optimum wind load factors varying with cases. The new calibration results prove that the recommended measures can achieve the goal better.

Key words: RC columns; wind dominated combination; random eccentricity; reliability evaluation; sensitivity analysis; robustness

Introduction

Wind hazard (e.g. hurricane) causes great losses every year all over the world. For example, Li and Ellingwood (2006) investigated the severe damage to residential construction and the social disruption caused by hurricanes in the past two decades in the United States; Li (2008) reported that some RC columns used to support aqueduct bridges collapsed under a strong wind action in China.

To reduce the loss, a better performance of the structures during strong wind action is desired. As one of the effective measures, a higher target reliability is prescribed for structural members in design codes (e.g. ACI 318-08 code) for the wind dominated case. Generally, with an application of the design methods in codes, the target safety level can often be achieved.

However, some unfavorable outcomes have been found recently for the load and resistance factors of the column design methods in codes, which are mainly based on a reliability calculation following the fixed eccentricity criterion. For example, the load and resistance design factors in codes cannot achieve a uniform reliability under different cases (Mohamed et al. 2001), and they can cause a possible unsafe design (i.e. reliability much lower than target value) in some cases of tension failure (Frangopol et al. 1996; Jiang and Yang 2013).

For a column only subjected to the vertical loads (e.g. dead load and live load), the fixed eccentricity criterion can usually be applied, because the random properties of eccentricity are not noticeable in this case, and the column reliability is high (Hong and Zhou 1999; Mirza 1996; Stewart and Attard 1999; Breccolotti and Materazzi 2010). However, for a column subjected to a horizontal load and a vertical load together (e.g. wind dominated combination), the load eccentricity is noticeably random under this combination, because the vertical load and the horizontal load follow different distributions and both have random properties. On this condition, the fixed eccentricity criterion is not applicable in general since it neglects the random properties of eccentricity, and, thus, unfavorable outcomes may possibly occur (Jiang et.al 2015; Jiang and Yang 2013).

To investigate the reasons for unfavorable outcomes in detail, the differences in the reliability results for fixed versus random eccentricity need to be investigated in detail. Generally, two primary capacity models are often used to obtain column reliability results. One model follows the analytical formulas in codes (e.g. code-based models used by Szerszen et.al 2005; Hong and Zhou 1999; Mirza 1996; Jiang et.al 2015; Jiang and Yang 2013), and another model works with finite elements (e.g. fiber section model used by Milner et.al 2001; ABAQUS model used by Mirza and Lacroix 2002).

Recently, robustness-based design methods have attracted increasing attention. Generally, there are two concepts of robust design mainly involved in the current studies. One concept is that robustness refers to structural redundancy and integral capacity under accidental actions, e.g., earthquake, impacts (Anitori et al. 2013; Masoero et al. 2010). Another concept is that robustness refers to invariable performance under normal variation of parameters (Sandgren and Cameron

2002; Oh et al. 2010). With the studies based on these concepts, effective measures have been proposed to improve the current design methods in codes. For example, Ching et al. (2013) discussed how to achieve a uniform reliability level for a wide range of stratum scenarios with constant or non-constant load and resistance factor design (LRFD). In this context, further studies are also required on how to achieve a robust design for columns with random eccentricity.

This study tries to build a more realistic failure function and resistance function for RC columns with random eccentricity, and to investigate the differences between the results obtained by the fixed and random eccentricity criterion for different cases. The built functions are based on the widely accepted column capacity model in the code (ACI Committee 318 2008). Considering random properties of column eccentricity, a set of improved wind load factors are recommended for design practice. They can be used to achieve a robust design with uniform reliability.

Design Method in the Code

Capacity model of RC column

For an RC column with an eccentricity *e* along a fixed principal direction (*e=M/N*, *M* and *N* are the moment and the compressive force, respectively) and a typical symmetrical rectangular section, its model for capacity calculation often adopts an equivalent rectangular stress block assumption in the code (ACI Committee 318 2008), as shown in Fig.1.

The column capacity formulas are specified as

$$M = \alpha_1 f_c bx(\frac{h}{2} - \frac{x}{2}) + f_s' A_s'(\frac{h}{2} - a_s') + f_s A_s(\frac{h}{2} - a_s) \tag{1}$$

$$N = \alpha_1 f_c bx + f_s' A_s' - f_s A_s \tag{2}$$

$$-f_y \le f_s = E_s \varepsilon_{cu}(h_0 / x_c - 1) \le f_y \tag{3}$$

$$-f_y \le f_s^{'} = E_s \varepsilon_{cu}(1 - a_s^{'}/x_c) \le f_y \tag{4}$$

$$x = \beta_1 x_c \tag{5}$$

where $\alpha_1 f_c$ is the concrete stress assumed uniformly distributed over an equivalent rectangular block, $\alpha_1=0.85$, and f_c is the concrete compressive strength; $f_s^{'}$ and f_s are the stress of steel for compression and tension, respectively; $f_y^{'}$ and f_y are the yield strength of steel for compression and tension, respectively, whereby $f_y^{'} = f_y$; $A_s^{'}$ and A_s are the area of compressive and tensile steel, respectively, whereby $A_s = A_s^{'}$ (assumed true in the whole paper); h and h_0 are the geometrical depth and effective depth, respectively; b is the section width; a_s ($a_s^{'}$) is the distance from the center of gravity of the tensile (compressive) steel to the extreme tensile (compressive) fiber, whereby $a_s = a_s^{'}$; x_c and x are the depth of the real compression zone and the equivalent rectangular stress block, respectively, $\beta_1=0.85$ for f_c between 17.2 and 27.6 MPa; $E_s=200$GPa is the elastic modulus of steel; $\varepsilon_{cu}=0.003$ is the assumed ultimate strain of concrete.

It is known that there are three basic failure modes: tension failure, balanced failure and compression failure for RC columns. Based on the capacity model given in Eqs.(1-5), the *N-M* diagram of RC columns can be obtained easily for all three failure modes, as shown in Fig.2.

Design factors in the code

The basic requirement for strength design in the code is expressed by

$$R_d = \varphi R_n \ge U_d \tag{6}$$

where R_d and R_n are design strength and nominal strength, respectively; U_d is the required strength which is related axial force and moment (M_d and N_d) and also expressed in terms of factored load effects; φ is the strength reduction factor.

For designing an RC column, the governing load combination is often determined as the

combination with the maximum moment. Generally, the axial force produced by vertical action is compressive. However, the axial force produced by horizontal action is either compressive or tensile due to its uncertain direction. If the axial forces produced by horizontal action is tensile, a negative value should be added to calculate the total axial force in load combination. Thus, the total required strength are the sum of factored items, no matter the axial force produced by horizontal action is compressive or tensile.

For example, for a basic load combination: vertical load (including dead load D and live load L) and horizontal wind load W, the total design moment and axial force are given by

$$M_d = \gamma_D M_{Dn} + \gamma_L M_{Ln} + \gamma_W M_{Wn} \tag{7}$$

$$N_d = \gamma_D N_{Dn} + \gamma_L N_{Ln} + \gamma_W N_{Wn} \tag{8}$$

where γ_D, γ_L and γ_W are 1.2, 1.0, and 1.6 in the code (ACI Committee 318 2008), respectively; D_n, L_n and W_n are nominal dead load, live load and wind load, respectively.

The values of strength reduction factor vary largely in different cases. They are 0.65 and 0.90 for compression-controlled sections (i.e. x_c/d_t larger than 0.6) and tension-controlled sections (i.e. x_c/d_t less than 0.375), respectively; and it can be determined by a linear interpolation for transition sections, which is expressed as

$$\varphi = 0.65 + 0.25(d_t / x_c - 5/3) \tag{9}$$

where d_t is the distance from the extreme tension steel to the extreme compression fiber ($d_t = h_0$ for case of one layer tension steel).

Design and nominal strength of column

To illustrate the differences between design strength and nominal strength of RC columns, two columns with different steel areas are considered, as shown in Table 1. Herein, the materials:

f_c=20.7 MPa concrete and Grade 60 steel (f_y=413.8 MPa) are selected based on the code. The obtained results is given in Fig.3.

It is seen that the design strength and nominal strength are closer to each other for tension-controlled section case of a column, while the differences between them are much larger for compression-controlled and transition sections case, due to the effects of strength reduction factor.

Applicability of the Design Factors in Code

Limit state functions under random eccentricity

Let Z be the performance function value, which is dependent of all random variables. Then, based on Eqs(1,2), a more realistic limit state function for columns with random eccentricity can be expressed by

$$Z = (N - f_s^{'} A_s^{'} + f_s A_s)(\frac{h}{2} - \frac{N - f_s^{'} A_s^{'} + f_s A_s}{2\alpha_1 f_c b}) + f_s^{'} A_s^{'}(\frac{h}{2} - a_s^{'}) + f_s A_s(\frac{h}{2} - a_s) - M = 0 \qquad (10)$$

Following the fixed eccentricity criterion, the assumed limit state function can be expressed by

$$Z = (R|e = e_d) - M = 0 \qquad (11)$$

where e_d is a fixed eccentricity determined by the design moment and axial force ($e_d = M_d / N_d$).

It is seen that Eq(10) is a nonlinear expression of resistance and load effects term. Li and Melchers (1995) also pointed out that the limit state surface is nonlinear for RC columns under the combined actions of moment and axial force. However, Eq(11) is a linear expression of moment M and resistance with a fixed eccentricity. Thus, the differences are large between Eq(10) and Eq(11).

Applicability of the design factors in code

For a wind dominated case, as mentioned earlier, the column eccentricity has many possible values, which also documented as possible load path by Milner et.al (2001). Therefore, the fixed

eccentricity criterion, which only involves the design load path, as shown in Fig.4, neglects the uncertainties of load path, and the resultant reliability results will be less accurate. The probability density function of eccentricity can be obtained with an improved approach proposed by Jiang and Yang (2013).

Note that the design factors (i.e. load factors and strength reduction factor) in the code are mainly determined by the reliability calibration results. However, such reliability calibrations are performed with a fixed eccentricity criterion rather than a random eccentricity criterion. Thus, if the random properties of eccentricity is not so important, the design factors in the code are applicable well and a robust reliability can be achieved. Otherwise, they are less applicable.

Probabilistic Analysis of Resistance

Related design parameters

Generally, design moment M_d, design axial force N_d, strength reduction factor φ, concrete nominal strength f_{cn} and steel nominal strength f_{yn} are adopted in Eq(10) for check when considering safety margin. Therefore, the design equation is given by

$$Z(M_d, N_d, \varphi, f_{cn}, f_{yn}, A_s, \cdots) = 0 \tag{12}$$

where only terms of interest are highlighted in the equation for simplification.

For an RC column with a certain dimension of section and material configurations (i.e. concrete and steel), its moment capacity is mainly determined by the reinforcement and axial force. Herein, two normalized ratios: reinforcement ratio and axial compression ratio are defined as

$$\rho_s = A_s / (bh) \tag{13}$$

$$\lambda_N = N_d / N_{cr} \tag{14}$$

where N_{cr} is the design axial force for an RC column under balanced failure. Thus, if these two

ratios are specified, the design moment M_d can be solved by Eq(12).

To distinguish the differences between different load effect cases, another two ratios of

moment and axial force are often introduced in reliability analysis. They are given by

$$\rho_M = M_{Wn}/(M_{Dn} + M_{Ln}) \tag{15}$$

$$\rho_N = N_{Wn}/(N_{Dn} + N_{Ln}) \tag{16}$$

Then, the nominal values of moment and axial force for each load are expressed as:

$$M_{Dn} = M_d / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_M (1 + \frac{L_n}{D_n})] \tag{17}$$

$$N_{Dn} = N_d / [\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_N (1 + \frac{L_n}{D_n})] \tag{18}$$

$$M_{Ln} = \frac{M_d}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_M (1 + \frac{L_n}{D_n})} \frac{L_n}{D_n} \tag{19}$$

$$N_{Ln} = \frac{N_d}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_N (1 + \frac{L_n}{D_n})} \frac{L_n}{D_n} \tag{20}$$

$$M_{Wn} = \frac{M_d \rho_M}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_M (1 + \frac{L_n}{D_n})} (1 + \frac{L_n}{D_n}) \tag{21}$$

$$N_{Wn} = \frac{N_d \rho_N}{\gamma_D + \gamma_L \frac{L_n}{D_n} + \gamma_W \rho_N (1 + \frac{L_n}{D_n})} (1 + \frac{L_n}{D_n}) \tag{22}$$

For engineering structures, it is often assumed that the random properties of the load effects

(e.g. moment and axial force) result from the random properties of loads. Thus, for the wind

dominated combination, a random value of the total moment and total axial force are given by

$$M = M_{Dn} \frac{D}{D_n} + M_{Ln} \frac{L}{L_n} + M_{Wn} \frac{W}{W_n} \tag{23}$$

$$N = N_{Dn} \frac{D}{D_n} + N_{Ln} \frac{L}{L_n} + N_{Wn} \frac{W}{W_n} \tag{24}$$

The random normalized eccentricity e' is calculated as

$$e'=\frac{e}{e_d}=\frac{M/N}{M_d/N_d}=\frac{[\frac{D}{D_n}+\frac{L_n}{D_n}\frac{L}{L_n}+\rho_M(1+\frac{L_n}{D_n})\frac{W}{W_n}][\gamma_D+\gamma_L\frac{L_n}{D_n}+\gamma_W\rho_N(1+\frac{L_n}{D_n})]}{[\frac{D}{D_n}+\frac{L_n}{D_n}\frac{L}{L_n}+\rho_N(1+\frac{L_n}{D_n})\frac{W}{W_n}][\gamma_D+\gamma_L\frac{L_n}{D_n}+\gamma_W\rho_M(1+\frac{L_n}{D_n})]} \tag{25}$$

For concrete structures, a typical value of L_n/D_n is 1.0 (Ellingwood et al. 1980). For a simplification, $L_n/D_n=1.0$ is assumed in the following analysis. It is known that the reliability can be largely determined by the values of ρ_s, λ_N, ρ_M and ρ_N for RC columns, when the random properties of resistance and loads are all given. Based on the reported W_n/D_n ranges (Ellingwood et al. 1980), the reported analysis results for three typical structural schemes (Jiang et al. 2015) and requirements in design practice, the common ranges of these normalized design parameters are determined tentatively and given in Table 2.

Herein, 3, 2 and 4 representative values are selected for λ_N, ρ_s and ρ_N, respectively, which are distributed uniformly in the ranges of interest for No.1-No.24 case, as shown in Table 3. Besides, 3 representative ρ_M values ($\rho_M=1.0, 2.5\ 4.0$) are also considered. Thus, 72 cases are included totally.

Probabilistic models of random variables

The randomness of five variables: D, L, W, f_c and f_y is considered in column reliability analysis. These variables show a considerable coefficient of variation (COV). The other remaining variables (e.g. dimensions of column section) are considered as deterministic since their COV is much smaller and no significant sensitivities are present so that the effects on the reliability can be neglected.

Two groups of probabilistic models for load and resistance variables: MA1 and MA2, are considered in the following reliability analysis, which is performed in correspondence with the code (ACI Committee 318 2008). Note that the load models are the same for MA1 and MA2, and their resistance models are documented as the old and new model by Szerszen and Nowak (2003),

respectively. The statistics are shown in Table 4, where the statistics of column resistance R/R_n are also given for reliability calibration with the fixed eccentricity criterion.

Herein, the statistics of the live load adopts an arbitrary-point-in-time model, because the wind load is considered to dominate the load combination, as mentioned earlier.

Distributions of random eccentricity

From Eq.(25), it is known that the random properties of e' depend on the random properties of load variables and two normalized parameters: ρ_M, ρ_N. When ρ_M and ρ_N are given, MA1 and MA2 have the same probability distributions of eccentricity, because their probabilistic models of loads are the same. For typical cases, the probability distributions of normalized eccentricity are shown in Fig.5.

It is seen that the random values of normalized eccentricity are distributed over a large range (e.g. [0.5, 1.75] range for e' in most cases), even a larger range for the case of larger ρ_M (e.g. [0.25 2.5] range for $\rho_M=4.0$ and $\rho_N=-0.15$). The probability of the event $e<e_d$ is the same for all cases, about 0.73 with the probabilistic models of load variables given in Table 4. This is because Eq.(25) has a special feature regarding ρ_M and ρ_N, and the probability can be simplified and calculated as

$$P(e < e_d) = P(\frac{\gamma_D + \gamma_L \dfrac{L_n}{D_n}}{\dfrac{D}{D_n} + \dfrac{L_n}{D_n}\dfrac{L}{L_n}} \frac{W}{\gamma_W W_n} < 1.0) \tag{26}$$

where ρ_M and ρ_N are not involved.

Statistics of resistance with random eccentricity

Based on the *N-M* diagram shown in Fig.2, it is known that the statistics of column strength depends not only on the resistance variables (e.g. concrete strength, steel strength), but also on the random properties of eccentricity. Let M_u denote the bending strength of a column. Thus, M_u is a

function of multiple variables: eccentricity e, concrete strength f_c, steel strength f_y, and so on. Herein, a normalized resistance factor R' is introduced and given by

$$R' = \frac{R}{R_n} = \frac{M_u(e, f_c, f_y)}{M_u(e_d, f_{cn}, f_{yn})} \tag{27}$$

If the fixed eccentricity criterion is assumed for a column, the statistics of such normalized resistance factor is only dependent on the resistance variables. For simplification, constant values for mean and COV of R' are usually used in previous reliability calibration of design codes, and the corresponding data are shown in Table 4. However, for a column with a random eccentricity, the mean and COV of R' are different from case to case.

With Monte Carlo simulation (MCS) and statistics of each resistance variables, Mirza (1996) obtained the statistics of resistance for columns with fixed eccentricity based on the capacity formulas in codes as well as an associated reliability result. Herein, in a similar manner, the statistics of resistance is obtained for columns with random eccentricity through MCS (run 5×10^5 times). It is found that the statistics of resistance vary largely within No.1-No.24 for columns with random eccentricity, but are nearly independent of ρ_M. Thus the results are only given for $\rho_M=1.0$, as shown in Fig.6.

It is seen that the mean value varies largely from 0.89 to 1.53 for MA1 and from 0.91 to 1.59 for MA2 across the cases. For COV, the differences between the cases are much smaller, varying from 0.071 to 0.087 for MA1 and from 0.042 to 0.057 for MA2. They are both different from the constant values assumed in the previous reliability calibrations.

Moreover, the mean values in tension failure cases (No.1- No.16) are much smaller than those in compression failure cases (No.17- No.24). Note that the value of the strength reduction factor for

compression cases is 0.65, which is much smaller than for tension failure cases (0.9 for tension-controlled sections, and 0.65-0.9 for transition sections). Thus, the design reliability in tension failure cases can be much lower than that in compression failure cases.

Resistance distributions in eccentricity intervals

Assume that the eccentricity range of interest can be divided into n intervals with the step size τ. To quantify the contribution of terms in different eccentricity intervals to the mean value of resistance, a ratio r is introduced and calculated as

$$r_i = \frac{\int_{e_i-0.5\tau}^{e_i+0.5\tau}\int_{\Theta} M_u(e, f_c, f_y)F(e)F(f_c, f_y)d\Theta de}{\int_{-\infty}^{+\infty}\int_{\Theta} M_u(e, f_c, f_y)F(e)F(f_c, f_y)d\Theta de} \tag{28}$$

where Θ is the integral domain of resistance variables f_c and f_y; $F(e)$ is the probability density function of eccentricity e; $F(f_c, f_y)$ is the joint probability density function of f_c and f_y (assumed two independent variables), and defined as the product of the two probability density functions for both f_c and f_y.

Similarly, values of ratio r are obtained for all 72 cases with $\tau=0.1e_d$ and the MCS (run 5×10^5 times). It indicates that the r values of MA1 are very close to those of MA2, though these two models have different probabilistic models of resistance variables as shown in Table 4. The distributions of r vary more largely within eccentricity intervals when ρ_M and ρ_N vary. For typical cases, r values are shown in Table 5.

Reliability Evaluation of the Design Method

Reliability analysis strategies

After the design parameters have been assigned, the reliability of RC columns can be

calculated based on the statistics given in Table 4. Due to the complex characteristics of the limit state function, as shown in Eq.(10), MCS is used for reliability calculation. Herein, the main purpose of MCS application is for searching the design point (needed in sensitivity analysis) rather than recording failure frequency.

Let $\boldsymbol{Y}^* = [y^*_1, y^*_2, \cdots, y^*_m]$ denote the design point in standard normal space, where m is the number of random variables. Then, the reliability index can be given by

$$\beta = \sqrt{\boldsymbol{Y}^* \boldsymbol{Y}^{*\mathrm{T}}} \tag{29}$$

The main steps are shown in Fig.7. To obtain an accurate reliability result, the sampling number is selected as large as enough for each case. The MCS results are also checked with another method, which searches the design point by selecting 50 nodes uniformly distributed within the ranges of interest for each one of m-1 random variables, obtaining 50^{m-1} points on the failure surface, calculating distances from the origin for each point, and recording the point with the minimum distance. The reliability results given by these two methods agree well with each other.

Analysis results and discussions

Considering a short RC column with a typical symmetrical section, its configuration (e.g. section dimension and materials) is shown in Table 1. Characterization of the parameters that are required to define the short column is also shown in Table 2 and Table 3.

Using the flowchart in Fig.7 and the statistics of random variables in Table 4, the reliability index is calculated for different cases of columns with random eccentricity, including MA1 and MA2 cases. For comparison, the corresponding reliability indices are also calculated for the fixed eccentricity cases. Finally, all the obtained results are shown in Fig.8.

For the code-based design method, if a fixed eccentricity criterion is used, the reliability only

varies with different values of ρ_M and λ_N (φ varies with λ_N) for both MA1 and MA2 cases. Compared with MA2 cases, MA1 cases have a lower reliability value. This is because MA1 has a lower mean and a larger COV of resistance than MA2, as shown in Fig.6.

However, if random eccentricity is considered, the reliability indices vary much strongly from case to case for both MA1 and MA2. For example, the maximum and minimum value is 5.51 and 2.47 for MA1, respectively; the maximum and minimum value is 6.71 and 2.59 for MA2, respectively.

In some tension failure cases (No.1-No.16), a lower reliability may possibly be found, especially with a larger ρ_M. Actually, even for the results with fixed eccentricity criterion, the lower reliability cases can also be reported for load combinations involving wind load (Ellingwood 1980).

Sensitivity Analysis and Improved Measures

Parametric sensitivity analysis

As identified, the reliability index varies largely from case to case. To explore which random variable among all random variables (i.e. f_y, f_c, D, L and W for the random eccentricity method; R, D, L and W for the fixed eccentricity method) has stronger effects on column reliability, a vector of sensitivity indexes is introduced and calculated as

$$u_i = \frac{\partial \beta}{\partial y_i} = \frac{1}{\left\| \nabla Z(\boldsymbol{Y}^*) \right\|} \frac{\partial Z(\boldsymbol{Y}^*)}{\partial y_i} \tag{30}$$

No.1-No.16 cases have a lower reliability, as previously shown in Fig.8. Therefore, more attention is paid to these cases, and a sensitivity analysis is performed for these cases with the obtained design points by the flowchart in Fig.7. It is found that all the sensitivity results are similar. A typical case is shown in Fig.9.

For the fixed eccentricity criterion, randomness of W and R have the strongest effects on the reliability for both MA1 and MA2 cases. For example, the sensitivity indices for W and R are about 0.81 and 0.57 for MA1 cases in Fig.9, respectively. For MA2 cases, those values for W and R are about 0.87 and 0.48, respectively.

However, for the random eccentricity criterion, the sensitivity indices for W increase dramatically to about 0.95 for MA1 cases and about 0.99 for MA2 cases, and the sensitivity indices for the resistance variable f_y decrease dramatically to about 0.27 for MA1 cases and about 0.13 for MA2 cases. The sensitivity index for the resistance variable f_c is very small for all these cases. It indicates that the randomness of the wind load dominates the effects on reliability among all uncertainties of the variables for both MA1 and MA2 cases, when the random eccentricity criterion is used.

Comparing the two criteria, it is found that the sensitivity indices for wind and resistance are more comparable and the differences between them are smaller for the fixed eccentricity criterion.

Improved design measures and results

For an RC member with tension failure (e.g. RC beam), the target reliability is usually 3.5 (Szerszen and Nowak 2003). If the same target reliability is also assumed as $\beta_T=3.5$ for columns with tension failure (e.g. No.1-No.16 cases), then the design factors (e.g. load factors, strength reduction factor) used in codes are required to be improved to achieve this goal. To be consistent with the code and conveniently applied, only the wind load factor γ_W is improved and other design factors are still kept fixed, because the randomness of the wind load dominates the effects on reliability.

A tentative range from 0.8 to 2.5 with step size 0.05 is selected for γ_W to perform the reliability

calculation again. Generally, the optimum γ_W is the one that corresponds closest to the reliability index target value 3.5. The optimum values of γ_W are obtained for different cases (No.1-No.16 cases only), as shown in Fig.10.

It is seen that the optimum γ_W is not constant and increases as ρ_M increases, which varies from 1.1 to 2.45 for MA1 cases and from 0.95 to 2.25 for MA2 cases. However, a constant value 1.6 is adopted in the code (ACI Committee 318 2008) for column design. For comparison, the robustness evaluation of these two measures (i.e. non-constant and constant γ_W factor) is performed for total of 48 cases (i.e. No.1-No.16 and 3 ρ_M values) and the results are given in Table 6.

It is seen that the design method with the recommended values can achieve a robust design within 48 cases, for it has a smaller COV and a closer value to the target reliability 3.5.

Conclusions

A more realistic limit state function is built for RC columns with random eccentricity based on the capacity model in codes. The statistics of column resistance and its reliability are calculated for different cases. The major conclusions are drawn as follows.

(1) For wind dominated combinations, the column eccentricity varies over a large range, and the probabilistic model of resistance is largely different from the constant resistance model assumed in previous reliability calibration of the code.

(2) The fixed eccentricity criterion used in previous reliability calibration can underestimate the column reliability differences among cases and overestimate the reliability in some tension failure cases.

(3) For columns designed by the code-based factors, the reliability in tension failure cases is much

lower than that in compression failure cases, and it is even lower with a larger ratio of the moment produced by wind load to the moment produced by vertical load, when random properties of eccentricity are considered.

(4) Based on the sensitivity analysis results with the random eccentricity criterion, the randomness of wind load dominates the effects on reliability among all uncertainties of variables for all cases.

(5) The recommended wind load factors varying with cases can keep a mean reliability index closer to the assumed target reliability index 3.5 and a smaller coefficient of variation, thus a robust design can be achieved better.

Further studies are needed on how to achieve a uniform reliability design for the RC columns with random eccentricity for other load combinations.

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Table 1 Parameters of different columns

No.	b/mm	h/mm	a_s/mm	A_s/mm^2	f_{cn}/MPa	f_{yn}/MPa
C-A1	450	450	50	2040	20.7	413.8
C-A2	450	450	50	3060	20.7	413.8

Table 2 Ranges of normalized design parameters

No.1-No.24	
Normalized design parameters	Value ranges
ρ_M	[1.0, 4.0]
λ_N	[0.5, 2.0]
ρ_s	[1%, 2%]
ρ_N	[-0.15, 0.15]

Table 3 Values of design parameters for No.1-No.24 cases

No.	λ_N	ρ_s	ρ_N	φ	No.	λ_N	ρ_s	ρ_N	φ
1	0.5	1%	-0.15	0.9	13	1.0	2%	-0.15	0.66
2	0.5	1%	-0.05	0.9	14	1.0	2%	-0.05	0.66
3	0.5	1%	0.05	0.9	15	1.0	2%	0.05	0.66
4	0.5	1%	0.15	0.9	16	1.0	2%	0.15	0.66
5	0.5	2%	-0.15	0.9	17	2.0	1%	-0.15	0.65
6	0.5	2%	-0.05	0.9	18	2.0	1%	-0.05	0.65
7	0.5	2%	0.05	0.9	19	2.0	1%	0.05	0.65
8	0.5	2%	0.15	0.9	20	2.0	1%	0.15	0.65
9	1.0	1%	-0.15	0.66	21	2.0	2%	-0.15	0.65
10	1.0	1%	-0.05	0.66	22	2.0	2%	-0.05	0.65
11	1.0	1%	0.05	0.66	23	2.0	2%	0.05	0.65
12	1.0	1%	0.15	0.66	24	2.0	2%	0.15	0.65

Table 4 Statistics of random variables

Variable	Distribution	Mean	COV	Model
D/D_n	Normal	1.05	0.10	MA1/MA2
L/L_n	Gamma	0.24	0.65	MA1/MA2
W/W_n	Type-I-Largest	0.78	0.37	MA1/MA2
f_c/f_{cn}	Normal	1.18	0.18	MA1
f_c/f_{cn}	Normal	1.35	0.10	MA2
f_y/f_{yn}	Normal	1.125	0.10	MA1
f_y/f_{yn}	Normal	1.145	0.05	MA2
R/R_n	Normal	1.107	0.136	MA1
R/R_n	Normal	1.260	0.107	MA2

Table 5 Ratios in different eccentricity intervals for typical cases

e'	ρ_M=1.0		ρ_M=4.0	
	MA1,No.3	MA2,No.3	MA1,No.19	MA2,No.19
[-∞, 0.4]	0.0%	0.0%	0.5%	0.5%
[0.4, 0.5]	0.0%	0.0%	3.2%	3.2%
[0.5, 0.6]	0.5%	0.5%	9.1%	9.1%
[0.6, 0.7]	7.1%	6.9%	14.9%	14.9%
[0.7, 0.8]	20.3%	20.3%	17.2%	17.1%
[0.8, 0.9]	25.2%	25.4%	15.7%	15.9%
[0.9, 1.0]	20.3%	20.2%	12.7%	12.7%
[1.0, 1.1]	12.8%	12.7%	9.2%	9.2%
[1.1, 1.2]	7.0%	7.0%	6.3%	6.3%
[1.2, 1.3]	3.6%	3.6%	4.2%	4.1%
[1.3, 1.4]	1.7%	1.8%	2.7%	2.6%
[1.4, 1.5]	0.9%	0.9%	1.6%	1.7%
[1.5, 1.6]	0.4%	0.4%	1.0%	1.0%
[1.6, 1.7]	0.2%	0.2%	0.6%	0.6%
[1.7, 1.8]	0.1%	0.1%	0.4%	0.4%
[1.8, +∞]	0.1%	0.1%	0.5%	0.5%

Table 6 Robustness evaluation of the methods with different γ_W factors for 48 cases

γ_W	MA1				MA2			
	β_{max}	β_{mean}	β_{min}	COV	β_{max}	β_{mean}	β_{min}	COV
In the code	4.33	3.21	2.47	0.15	4.57	3.36	2.59	0.15
Recommended	3.57	3.50	3.45	0.008	3.56	3.50	3.44	0.009
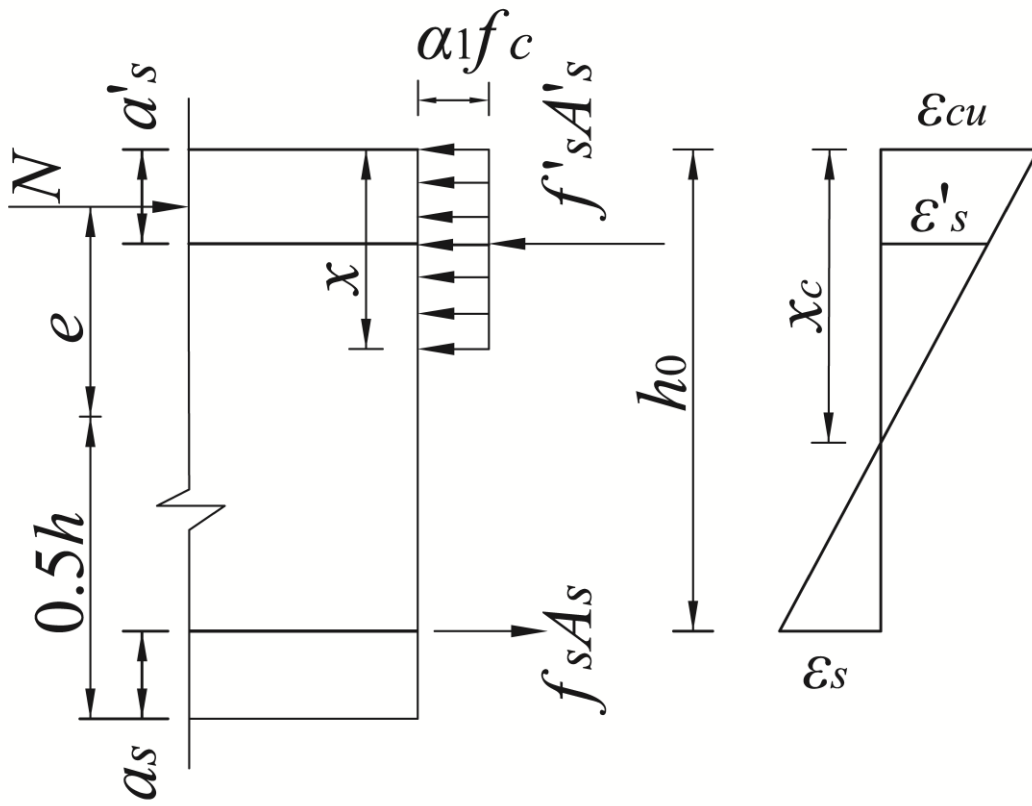
Fig.1 Capacity model of RC columns
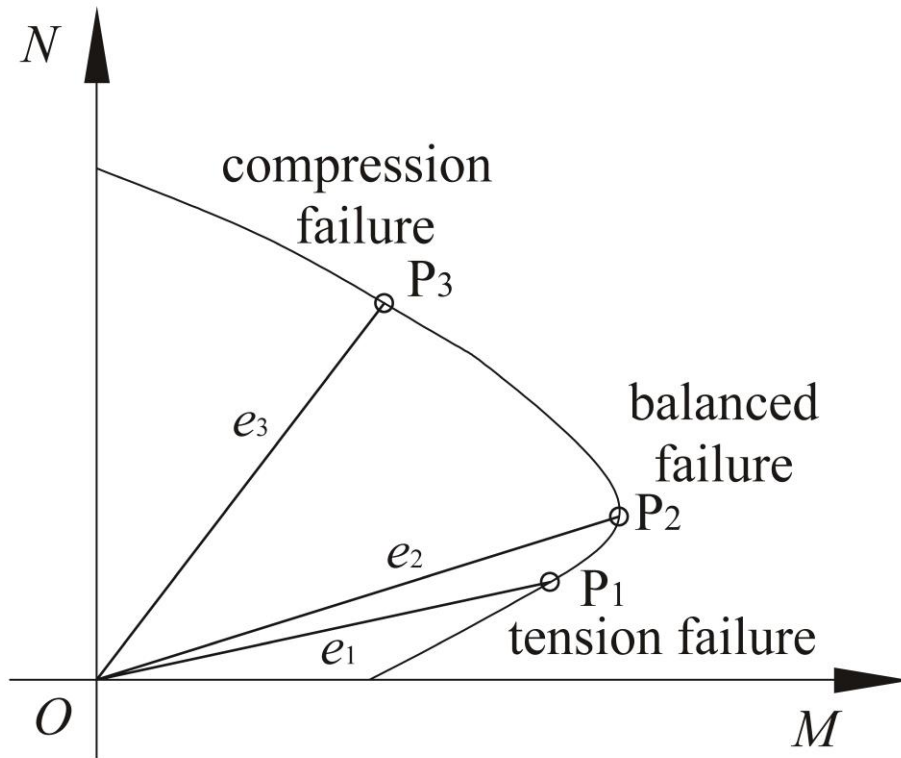
Fig.2 *N-M* diagram of RC columns with different failure modes
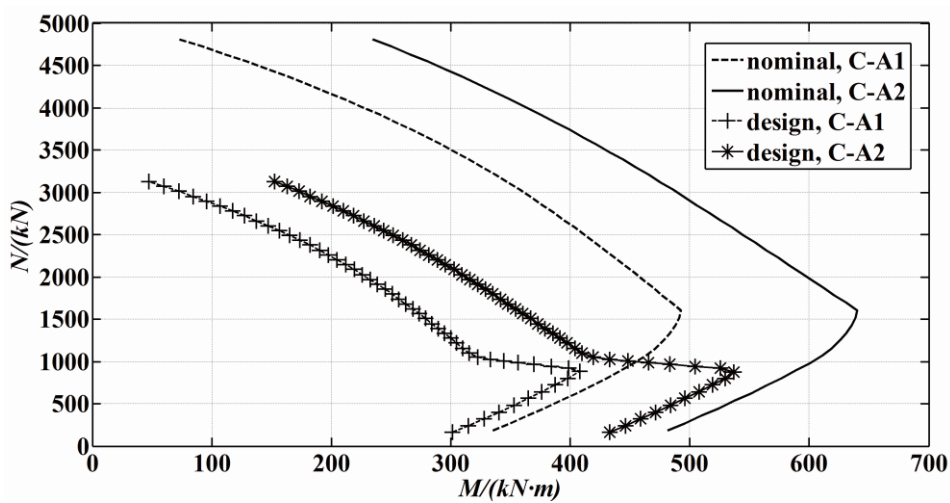
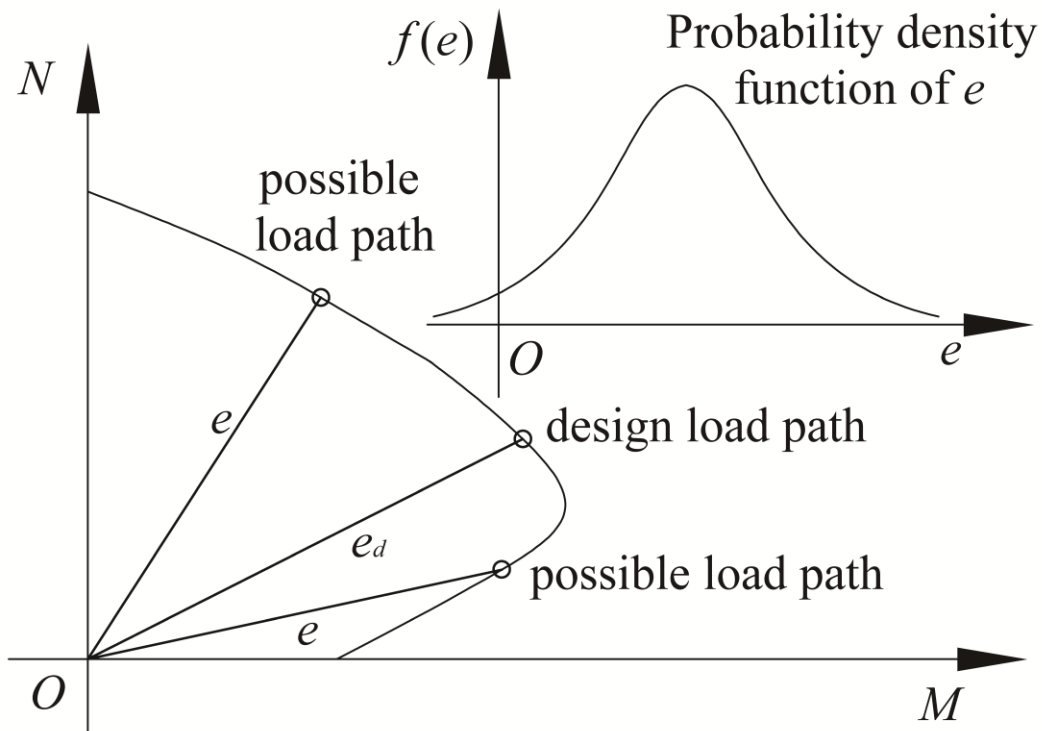Fig.3 Nominal and design strength for different columns based on code
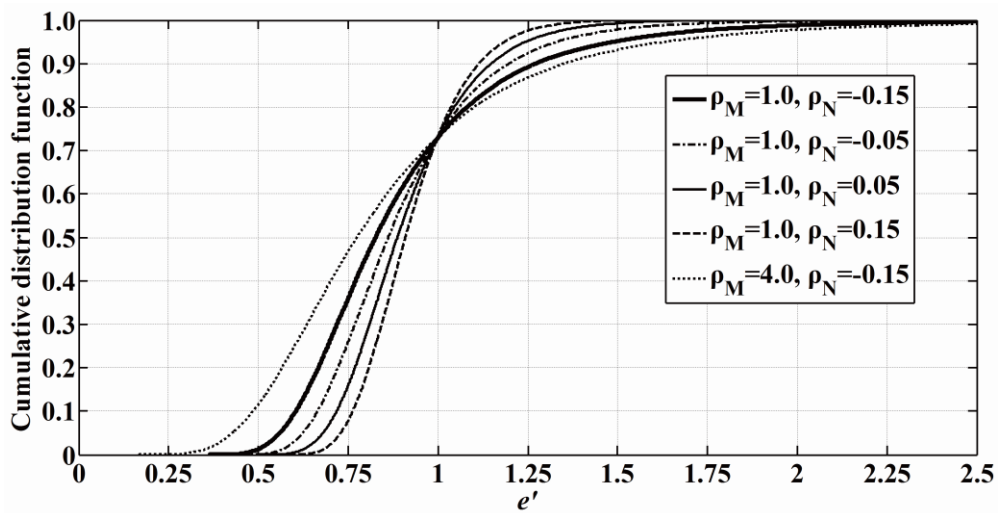
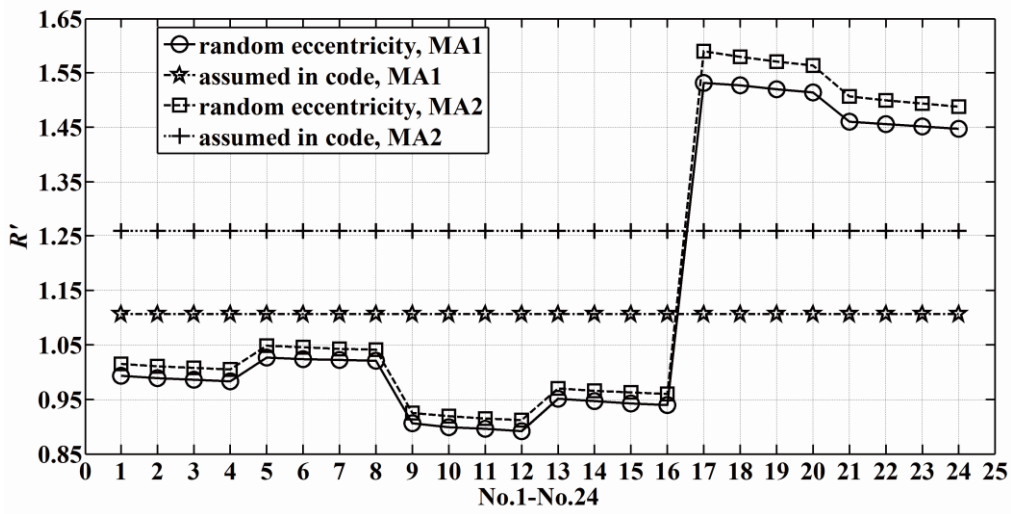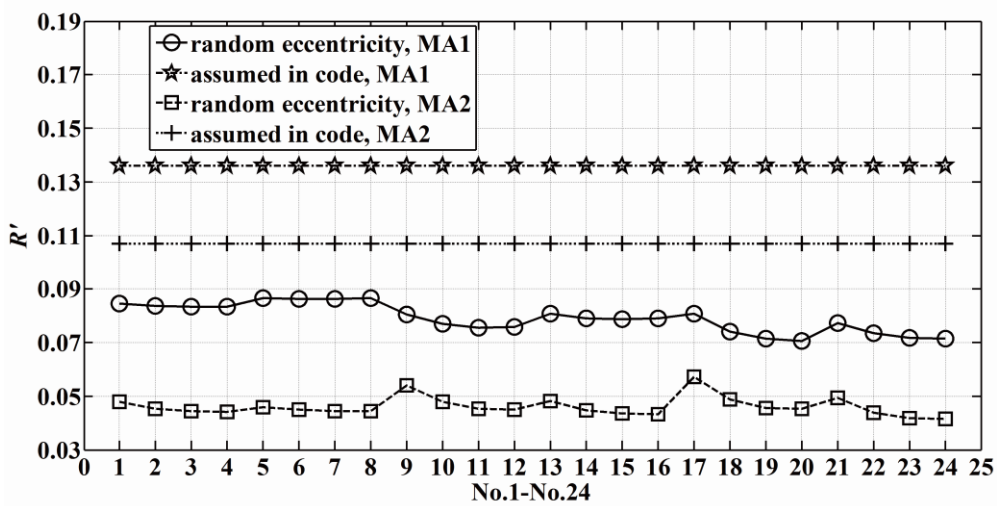Fig.4 Possible load paths for RC columns

Fig.5 Probability distribution of eccentricity for typical cases

(a) mean value for ρ_M=1.0

(b) COV for ρ_M=1.0

Fig.6 Statistics of resistance for columns with random eccentricity

```
┌─────────────────────────────────────────┐
│ Input values of all design parameters and │
│ statistics of random variables            │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│ Calculate required moment Md with given   │
│ parameters by Eq.(12)                     │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│ Calculate nominal values of load effects  │
│ with load factors and Eqs.(17-22)         │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│ Sampling each random variable according   │
│ to its probability distribution           │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│ Obtain a point on the limit state surface with │
│ Eq.(10) and sampling values of variables  │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│ Estimate the distance from the origin for │
│ each obtained point in standard normal space │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│ Select the closest point to the origin as the │
│ design point, and record it               │
└─────────────────────────────────────────┘
```
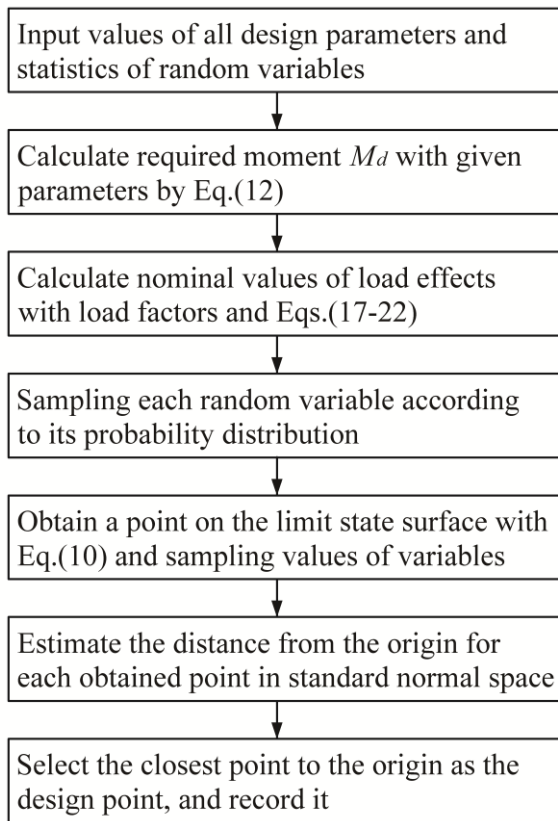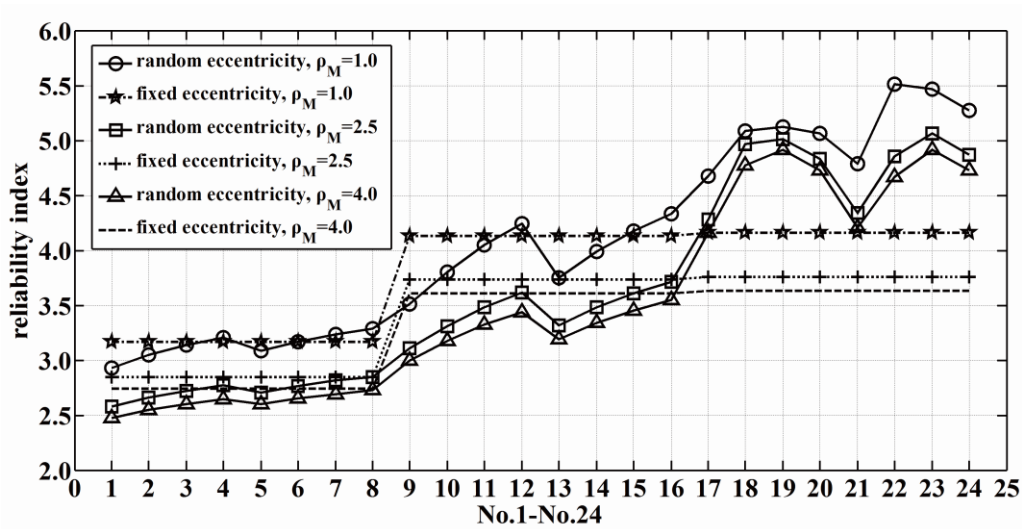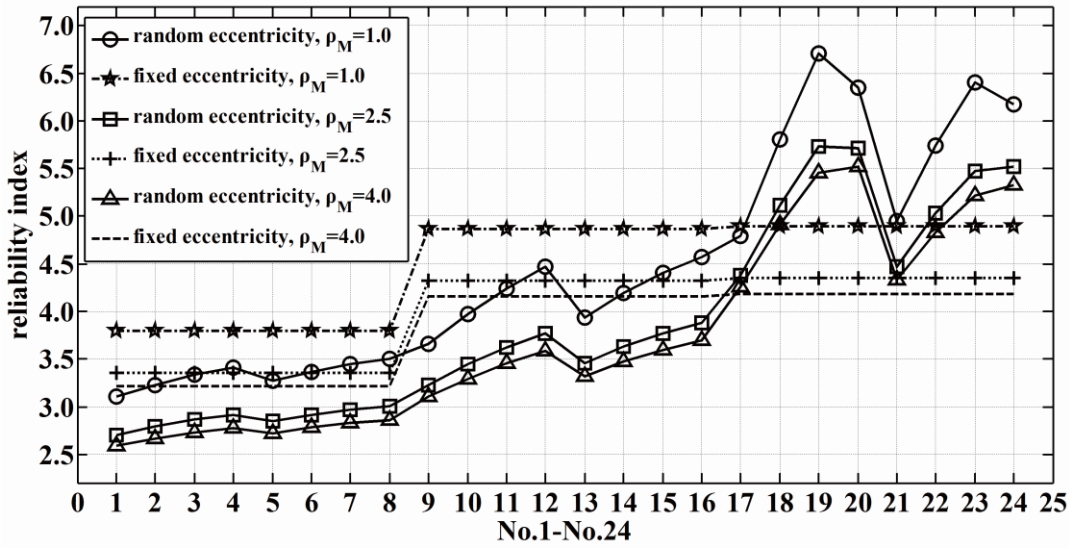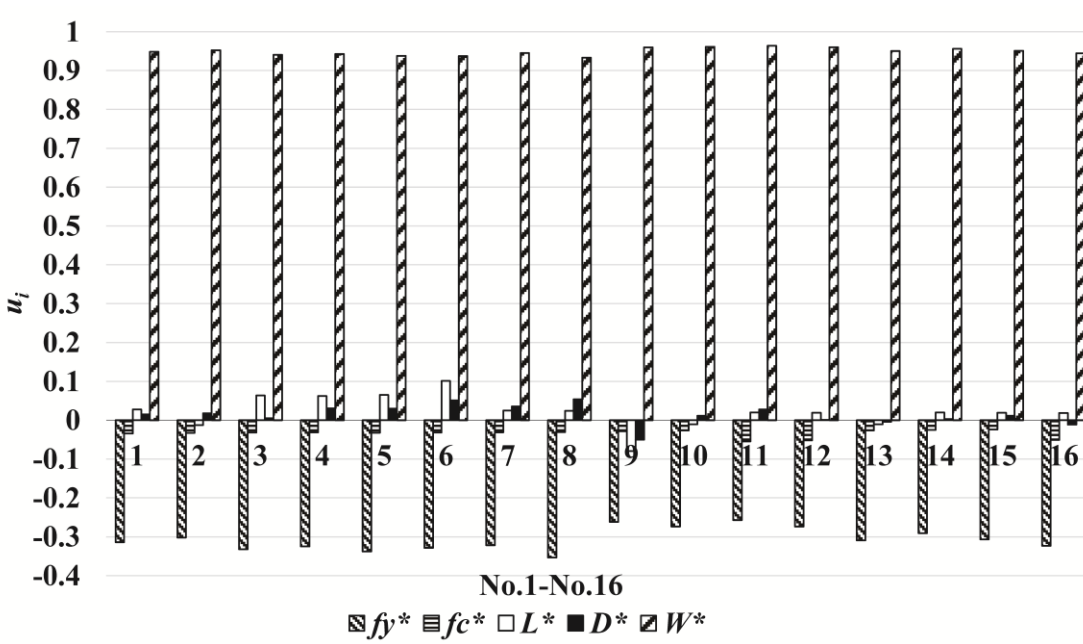
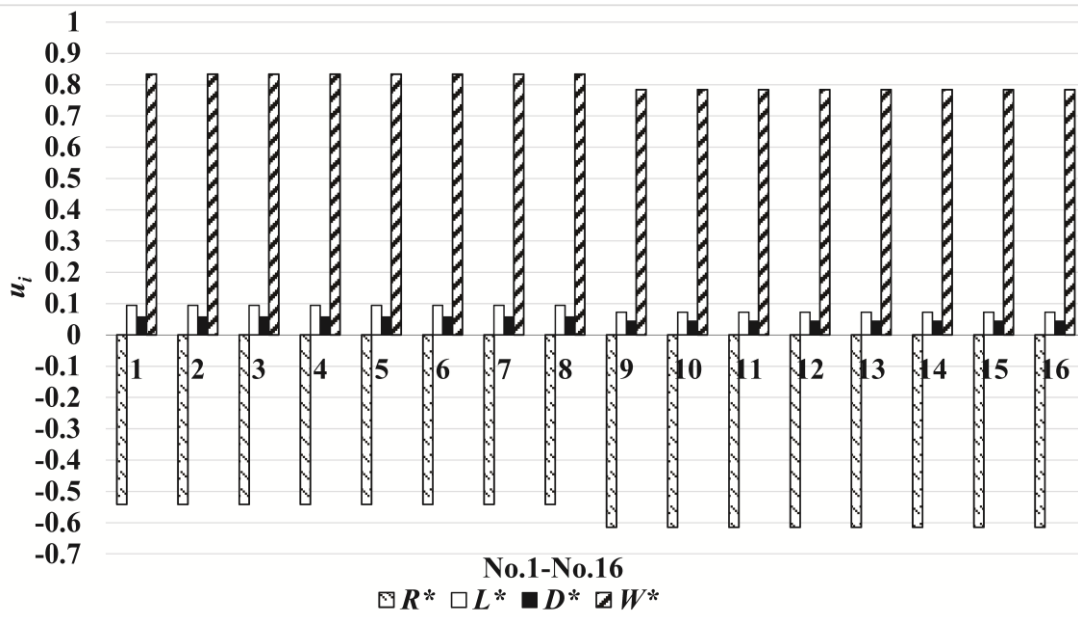Fig.7 Flowchart for reliability analysis with random eccentricity

(a) MA1

(b) MA2

Fig.8 Reliability indexes with random eccentricity or fixed eccentricity
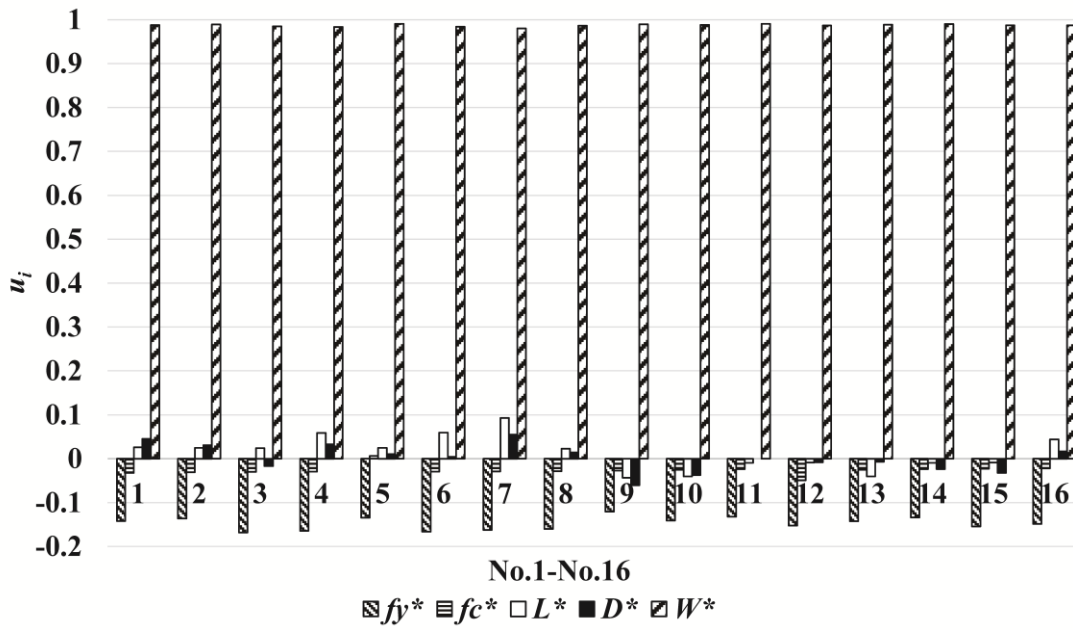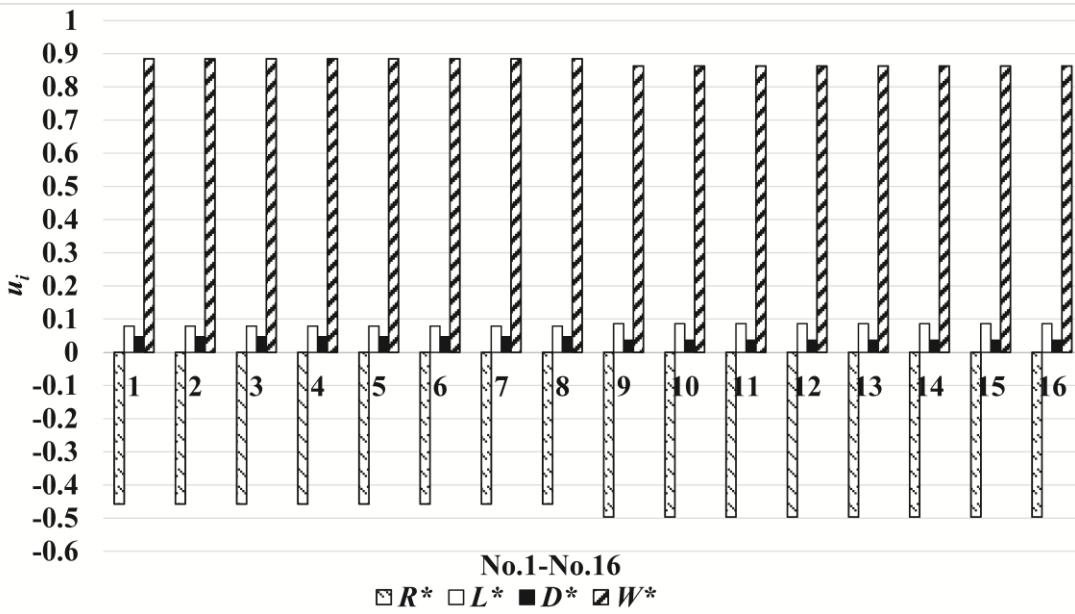

(a) ρ_M=1.0, MA1, random eccentricity

(b) ρ_M=1.0, MA1, fixed eccentricity

(c) ρ_M=1.0, MA2, random eccentricity

(d) ρ_M=1.0, MA2, fixed eccentricity
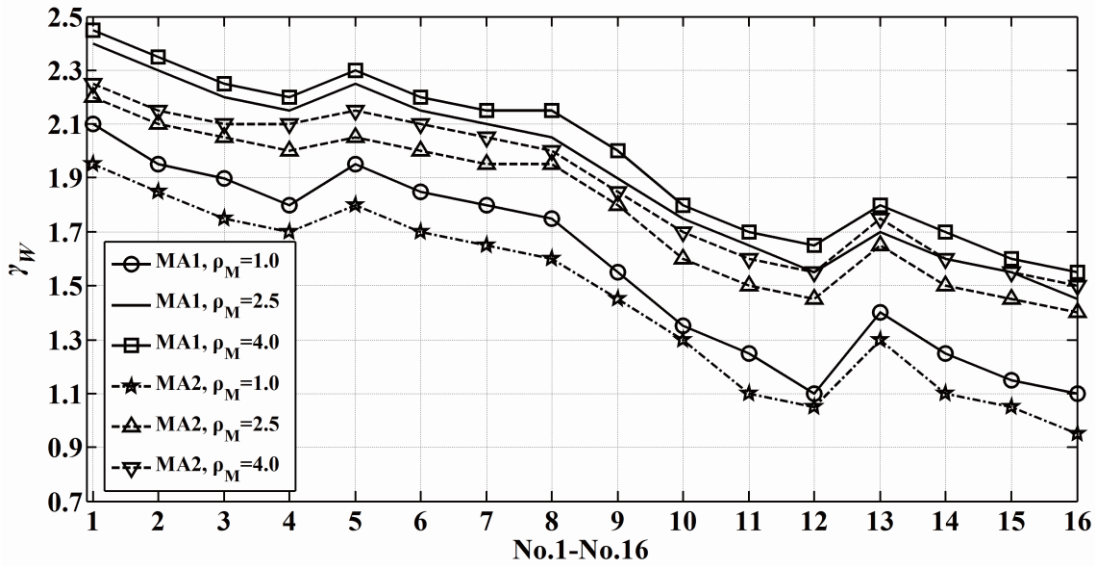
Fig.9 Sensitivity analysis results for typical cases

Fig.10 Recommended values of γ_W for different cases