Optimal battery charge/discharge strategies for prosumers and suppliers

Journal Item

Noname manuscript No.
(will be inserted by the editor)

Optimal battery charge/discharge strategies for prosumers and suppliers

B. D. Mestel

the date of receipt and acceptance should be inserted later

Abstract We discuss the application of classical variational methods to optimal charging/discharging strategies for a prosumer or storage supplier, where the price of electrical power is known in advance. We outline how a classical calculus of variations approach can be applied to two related problems: (i) how can a prosumer minimise the cost of charging/discharging a battery, when the price of electrical power is known throughout the charging/discharging period? and (ii) how can an electricity supplier incentivise desired prosumer/storage supplier behaviour by adjusting the price?

Keywords variational methods, prosumer, battery, storage, smart grid

1 Introduction

To meet the challenges of a low-carbon future incorporating significant renewable energy, there has been growing interest in the role of energy/electricity storage in the management of power systems, both (i) as an aid to short-term stabilization of the power system; and (ii) as means to handle mismatches between supply and demand. For example, in a recent report [9] from a UK perspective, the Institution of Mechanical Engineers calls for greater R&D investment in energy storage (in the heat, transport and electricity sectors) to meet the problem of intermittency and seasonal variability of energy obtained from wind, sun, tides and waves. Similarly, in 2013 the EU identified energy storage as a key area for development [5].

Moreover, two key challenges in energy research are (i) to understand the interplay between the electricity market and electricity generation; and (ii) to understand the dynamics of smart grids consisting of a network of prosumers (producers/consumers) interacting in the physical, cyber and social layers. (See, in particular, the contribution by Etorre Bompard in [2].)

In line with these trends, there has also been mathematical research in the field of energy storage. Cruise *et al* use stochastic control methods to understand how a storage supplier might optimise the use of storage for arbitrage purposes with particular reference to hydroelectric storage [4], and Li *et al* have studied the possibility of electric vehicles facilitating renewable energy integration in the power system [8]. The general impact of electric vehicles on the grid has been investigated by Lukszo and co-workers [7, 13, 14].

Whilst there are many possible approaches to energy storage (compressed air, hydroelectric storage, flywheels, hydrogen etc.), we restrict this paper to electricity storage in fuel cells/batteries, which are charged from the grid and discharged to the grid, by, for example, a commercial storage supplier or a prosumer with, say, a vehicle battery. There are many types of batteries currently in use or being researched and developed for the future, e.g., Lead-acid, Li Ion, etc., and in this article we do not distinguish between the battery types.

A key claim made by the Institution of Mechanical Engineers report is that "energy storage cannot be incentivised by conventional market mechanisms". It is therefore useful to investigate to what extent it might

Department of Mathematics and Statistics, The Open University, Milton Keynes MK7 6AA, UK
E-mail: Ben.Mestel@open.ac.uk

be possible to incentivise energy storage through the conventional market mechanism of price, and this is one motivation for the study of battery charging/discharging presented in this paper.

The modelling of batteries is a well-established and growing field. One approach is electrochemical and thermal modelling of the electrolyte, electrodes and their interaction (sometimes at nano-scales [12]); another is phenomenological or dynamical macroscale modelling of battery behaviour, often based on equivalent circuit diagrams [11]. Battery models can be elaborate (see for example, [3] for a successful but mathematically complex example), and only the very simplest (and largely unrealistic) models are amenable to analysis. Hence most studies use numerical simulation (often using software packages such as Matlab and Maple). However, simple battery models are successfully used to model charging of electric vehicles [1, 6, 11].

In this paper we consider the following problem. Given an agent (e.g., a prosumer or a storage supplier) with a storage battery (perhaps in an electric/hybrid vehicle) and an exogenous price $p(t)$, what is the optimal charging/discharging function to minimise cost and how should the price be chosen to induce a given behaviour in the prosumer?

To be specific, we consider battery models with state variable S, the State of Charge (SOC) of the battery. The variable S varies continuously between $S = 0$, when the battery is fully discharged, and $S = 1$, when the battery is fully charged. We ignore other state variables used in some battery models, in particular the so-called Depth of Charge and the battery temperature. (In some models battery temperature is a dynamic variable determined by the ambient temperature and other model variables; in any case it is clearly an important variable to control to prevent gassing, damage to the battery and fire.) We assume that the permitted charging regimes conform to the manufacturer's specification to avoid hazards and to minimize battery degradation. We consider a fixed time interval $[t_s, t_e]$ over which there is an applied current $I(t)$ and voltage $V(t)$. We adopt the convention that $I > 0$ when the battery is charging and $I < 0$ when the battery is discharging. Note that I and V cannot necessarily be determined independently. Certainly the power relation $W(t) = V(t)I(t)$ holds, but the precise relation between I, V and W will be dependent on the battery model chosen.

2 Cost functional and fundamental questions

Let us consider a model problem of a single agent in a fixed market. The agent may be a prosumer or a storage supplier, since we envisage the agent buying and selling electricity from and to the grid. We assume the agent is a price taker, and the price, which may be regarded as an exogenous variable, is set by the electricity supplier (which might be a transmission, distribution or retail company). In general we might consider two prices, given in, say, £ per KWh: $p_o(t)$, the offer price, i.e., the price the agent buys electricity, and $p_b(t)$, the bid price, i.e., the price the agent sells electricity. In general $p_b(t) \leq p_o(t)$, but in this paper we consider the friction-free case $p_o(t) = p_b(t) = p(t)$. We also assume forward pricing, so that $p(t)$ is a known function on the whole of the interval $[t_s, t_e]$, and that the agent has sufficient prior knowledge to optimise charging/discharging during the period $[t_s, t_e]$.

The total cost of charging/discharging a battery for the price function $p(t)$ and power $W(t)$ is given by the functional

$$C = \int_{t_s}^{t_e} p(t)W(t)\, dt\,, \tag{1}$$

where $W(t) = V(t)I(t)$. The goal is to minimize C subject to any constraints such as maximum/minimum current, voltage, and power that are imposed by the battery manufacturer, the electricity supplier etc. Note that it possible to include a discount factor e^{-rt} in the integrand in equation (1), where r is the discount rate, but we have not done so because of the short timescales envisaged in applications of the model.

The following are key questions that any mathematical approach would attempt to answer.

1. For a given price $p(t)$, what is the optimal charging function $S(t)$ within a given class of approved charging functions, e.g., satisfying given constraints? Is the optimal charging function unique?
2. For a given charging function $S(t)$, what is the price function $p(t)$ for which the given $S(t)$ is the optimal charging function for $p(t)$? And is $p(t)$ unique?
3. For a given power function $W(t)$, what is the charging function $S(t)$ inducing $W(t)$, and is it unique and in a given class of approved charging functions?

4. For a given power function $W(t)$, what is the price function $p(t)$ that has optimal power function $W(t)$?
5. What is the minimum cost in each of these cases?

As we shall see in the next section, we may use a classical variational approach to answer some of these questions.

3 A classical calculus of variations approach

Let us outline how the questions above may be answered using variational methods. We assume that the voltage $V(t)$ may be expressed in terms of the State of Charge $S(t)$ and its derivative $\dot{S}(t)$, and that all functions are sufficiently smooth for the relevant equations to be well defined and to have solutions of the required degree of smoothness. We note that $I = Q_{max}\dot{S}$ where Q_{max} is the maximum charge that battery can hold. Our approach does not exclude other state variables (such as temperature), but here we assume that any such variables are completely determined at time t by additional parameters and by the State of Charge $S(t)$ and its derivative $\dot{S}(t)$. Thus a charging regime is determined completely by the State of Charge as a function of time and we may write $W(t) = G(S, \dot{S})$, for some function G. Then equation (1) becomes

$$C = \int_{t_s}^{t_e} p(t')G(S(t'), \dot{S}(t'))\, dt' \,, \tag{2}$$

where $S(t_s) = S_s$ and $S(t_e) = S_e$, and where $0 \le S_s \le S_e \le 1$ are the States of Charge (considered known parameters) at the start and end of the charging time-interval $[t_s, t_e]$.

In this context Question 1 is a classical problem in the calculus of variations, which may be solved (at least in principle) by standard techniques. For a given price function $p(t)$, the optimal charging regime is a stationary path of (2) given by a solution of the Euler-Lagrange equation,

$$\frac{d}{dt}\left[p(t)G_{\dot{S}}\right] - p(t)G_S = 0 \,, \tag{3}$$

with boundary conditions $S(t_s) = S_s$ and $S(t_e) = S_e$. In this equation, and what follows, we use subscripts to denote partial derivatives, so that $G_S = \partial G/\partial S$ etc. As is standard for the calculus of variations, $G_{\dot{S}} = \partial G/\partial \dot{S}$ denotes the partial derivative with respect to the second argument of G. Existence of a solution of the second-order boundary value problem (3) is guaranteed under reasonably wide conditions. However, solutions of (3) may not be unique and may not be minimisers/maximisers in general, even when the solution is unique. Convexity of $pG(S, \dot{S})$ with respect to $\dot{S}$ is sufficient to guarantee a unique minimizer. In general, determining whether or not a solution is a minimizer requires analysis of the second variation (and higher) of (2), which is not, in general, an easy task. Constraints may be tackled using Lagrange multipliers and/or slack variables, but again the analysis can be quite involved.

Question 2 is also readily solved by variational methods. Let the SOC $S(t)$ be given. If S is optimal for a price function $p(t)$, then the Euler-Lagrange equation (3) must also be satisfied. Viewed as an equation for the unknown function $p(t)$, equation (3) is a first-order linear ordinary differential equation (o.d.e.), which may be solved via an elementary integrating factor method. Explicitly

$$p(t) = p(t_s)\frac{G_{\dot{S}}(t_s)}{G_{\dot{S}}(t)} \exp\left[\int_{t_s}^{t} \frac{G_S(t')}{G_{\dot{S}}(t')}\, dt'\right] \,, \tag{4}$$

provided $G_{\dot{S}} \neq 0$. Up to the (expected) arbitrary price normalization $p(t_s)$, the solution is unique.

We now turn to Questions 3 – 5. For a prescribed power function $W(t)$, the equation $G(S, \dot{S}) = W(t)$ is an implicit first-order ordinary differential equation for S which may be solved subject to the one of the boundary conditions, $S(t_s) = S_s$, say. As an initial value problem, it has a unique solution close to $t = t_s$, but may not necessarily extend to the whole of the charging interval $[t_s, t_e]$. Also it is not possible, in general, to specify S_e; instead it must be calculated from the solution of the o.d.e. This clearly imposes a constraint on $W(t)$ and/or S_e; too high a power function, for example, will lead to overcharging of the battery. Therefore determining which power functions produce admissible charging regimes is not always straightforward. Assuming that the value $S(t_e) = S_e$ has been determined, it is now possible, at least in principle, to use the above method to determine the price function $p(t)$ which gives $G(S,\dot{S}) = W(t)$ for the optimal SOC $S(t)$. Finally, having determined $p(t)$ and $G(S,\dot{S})$, it now straightforward, again in principle, to evaluate the cost functional (2) to obtain the minimum cost of charging.

The above discussion applies irrespective of whether the battery is being charged or discharged, or, indeed, a combination of the two. As we shall see in the model example below, there are circumstances when buying electricity and selling it back to the grid reduces the cost, even when an agent must end the charging period with a fully charged battery. Of course, an agent is quite severely constrained in the use of such a buy-low sell-high strategy, not just by the capacity of the battery, but also by other constraints on $I(t)$, $V(t)$ and $W(t)$, either imposed by the electricity supplier or by the characteristics of the storage battery. In the model example considered in the next section, constraints of this form are not imposed *a priori*, although any optimal solution must be checked to ensure no constraints are violated, and, if so, the optimal solution must be recalculated incorporating any binding constraints.

4 Model example

Let us illustrate this theory with a simple battery model, which has the twofold advantage (i) that the V-I constituent relation can be obtained without having to solve differential equations; and (ii) that the Euler-Lagrange equation may be readily solved. Furthermore, models of this type are used to model batteries in electric vehicles. See, for example, [1] and [6].

For this model, $V = V_{OC} + R_b(t)I$. Here $I(t)$ is the current flowing into the battery during charging. Recall that $I = Q_{max}\dot{S}$, where $S(t)$ is the State of Charge (SOC), and Q_{max} is maximum charge that can be stored in the battery. Furthermore, $V(t)$ is the voltage at the battery terminals, V_{OC} the open circuit voltage of the battery (i.e., the voltage presented in the absence of current), $R_b(t)$ is the internal resistance of the battery (which, for simplicity, we take as constant R_b, but which, in general, will be determined by $S(t)$, and will depend on whether the battery is charging or discharging.) For a price function $p(t)$, equation (1) becomes $C(S) = \int_{t_s}^{t_e} F(t, S(t), \dot{S}(t))\, dt$ where

$$F(t,S,\dot{S}) = p(t)W(t) = p(t)G(S(t),\dot{S}(t)) = p(t)\left(V_{OC} + R_b Q_{max}\dot{S}\right) Q_{max}\dot{S}\,. \tag{5}$$

We note that $G_S = 0$, so that, for general $p(t)$, the Euler-Lagrange equation (3) reduces to

$$Q_{max}\frac{d}{dt}\left[p_0(t)\left(V_{OC} + 2Q_{max}R_b\dot{S}\right)\right] = 0\,, \tag{6}$$

which may be integrated immediately to give

$$S(t) = S_s + c\int_{t_s}^{t}\frac{dt'}{p(t')} - \frac{V_{OC}}{2Q_{max}R_b}(t - t_s)\,, \tag{7}$$

where the constant c is determined by the condition $S(t_e) = S_e$. Since $F_{\dot{S}\dot{S}} = 2p(t)Q_{max}^2 R_b > 0$ for $R_b > 0$, F is strictly convex with respect to $\dot{S}$ and the solution is unique by standard results. We note that

$$I = Q_{max}\dot{S} = \frac{cQ_{max}}{p(t)} - \frac{V_{OC}}{2R_b}\,, \tag{8}$$

confirming that charging current is a decreasing function of price.

For constant price $p(t) \equiv p$, the solution reduces to constant current $I = Q_{max}(S_e - S_s)/(t_s - t_e)$. This is also a solution in the singular case $R_b = 0$, although, in that case, F is no longer strictly convex and the solution is not unique. For constant p, the total cost of charge is easily calculated to be $C = pQ_{max}\left(V_{OC} + R_b Q_{max}(S_e - S_s)/(t_e - t_s)\right)$ which is minimized when $t_e - t_s$ is maximized, i.e., for trickle charge. When $R_b = 0$, there is no solution except when p is constant and, in that highly degenerate case, the total cost is $pQ_{max}V_{OC}$, independently of the rate of charge. This answers Question 1.

Turning to Questions 2–5, we note that $G_S = 0$ and $G_{\dot{S}} = Q_{max}\left(V_{OC} + 2Q_{max}R_b\dot{S}\right)$, and equation (4) becomes

$$p(t) = p(t_s)\frac{V_{OC} + 2Q_{max}R_b\dot{S}(t_s)}{V_{OC} + 2Q_{max}R_b\dot{S}(t)}\,, \tag{9}$$

which again confirms the relationship between price and charging current.

For a specified voltage function $V(t)$ or power function $W(t)$, the equations

$$V_{OC} + R_b Q_{max} \dot{S} = V(t)\,, \quad Q_{max} \dot{S} \left(V_{OC} + R_b Q_{max} \dot{S} \right) = W(t) \tag{10}$$

may be solved to give the charging rate $\dot{S}$. For the latter equation,

$$S(t) = S(t_s) + \frac{1}{2 Q_{max} R_b} \int_{t_s}^{t} \left(V_{OC}^2 + 4 R_b W(t') \right)^{1/2} - V_{OC}\, dt' \tag{11}$$

with the condition $S(t_e) = S_e$ placing a constraint on the choice of $W(t)$. The price function that induces this power function is

$$p(t) = p(t_s) \left(\frac{V_{OC}^2 + 4 R_b W(t_s)}{V_{OC}^2 + 4 R_b W(t)} \right)^{1/2} \,, \quad \left(V_{OC}^2 + 4 R_b W(t) > 0 \right), \tag{12}$$

with a total cost

$$C = p(t_s) \int_{t_s}^{t_e} \left(\frac{V_{OC}^2 + 4 R_b W(t_s)}{V_{OC}^2 + 4 R_b W(t)} \right)^{1/2} W(t)\, dt\,. \tag{13}$$

For $R_b \geq 0$, the cases of constant power, constant voltage, constant current and constant pricing are all equivalent to constant $\dot{S}$, which has been considered above.

We now illustrate this theory with a couple of practical examples, of immediate relevance to power systems. We consider a storage battery with characteristics typical of an electric vehicle battery, with values of the constants taken for the Saft VL45E cell as given in Table I and Figure 1 in [1]. In particular we take $Q_{max} = 45Ah$, $V_{OC} = 3.5V$, and $R_b = 4m\Omega$. The results given below are for these parameter values and for an electricity price of £0.15 per kWh.

Our first example answers the question: how might an electricity supplier determine the price function $p(t)$ to help even out demand over a 24-hour period? Figure 1 illustrates a typical profile for UK electricity demand for a weekday (Monday – Thursday) (as published by the UK National Grid), with its characteristic mid-morning and mid-evening peaks. For an electricity supplier, incentivizing agent behaviour to help even out demand is a significant objective. Let us consider an agent with a storage battery who desires to charge the battery over a twenty-four hour period and let us assume that the electricity supplier wishes to incentivize a power profile $W(t)$ as illustrated in Figure 2. (The profile has been adjusted for the attributes of the cell and to ensure that S is 0 at the start of the 24-hour period and is 1 at the end.) The associated SOC that generates $W(t)$ is shown in Figure 3 and the price function $p(t)$ which induces this function $S(t)$ as the minimal-cost SOC is shown in Figure 2. Note that $\dot{S} < 0$ for part of the charging period, indicating that the optimal solution involves selling electricity back to the electricity supplier during a period of high price and buying back the electricity at a later time. That the price function $p(t)$ mimics the demand and complements the function $W(t)$ is not, of course, surprising. Perhaps more surprising is the low relative variation in the price. Indeed, in line with the Institution of Mechanical Engineers report, such a small price variation might not be sufficient to persuade an agent to depart from constant charging (also shown in Figure 3), since, the optimal solution offers a saving of only approximately 1% over constant charging.

Our second example considers an electric vehicle that is to be charged overnight in the period 17:00 - 07:00, with the electricity supplier aiming to disincentivize charging in the high-demand period from 17:00 to 21:00. This example also illustrates how the theory can be readily applied to non-smooth charging functions. Indeed, in practice the price function is often piecewise constant, which leads to a piecewise-linear optimal charging function.

We assume that the vehicle has a battery pack consisting of 160 Saft VL45E cells so that the maximal storable energy is in the range 25–26 kWh. The particular series/parallel configuration is not important here provided there are no differences between individual cells. The SOC of the vehicle at 17:00 is denoted S_s, where, as before, $0 \leq S_s \leq 1$. The variable S_s might be modelled as a random variable, but here we take it as a parameter. For simplicity, the price function has two values only: one applicable before 21:00 and another afterwards. As an illustration, we set the function (i) so that the cost of a full steady charge over the 14-hour period is precisely £0.15 per kWh; and (ii) so that the price before 21:00 is 50% higher than the price after 21:00. This means that the price before 21:00 is c. £0.20 per kWh and after 21:00 is c. £0.13

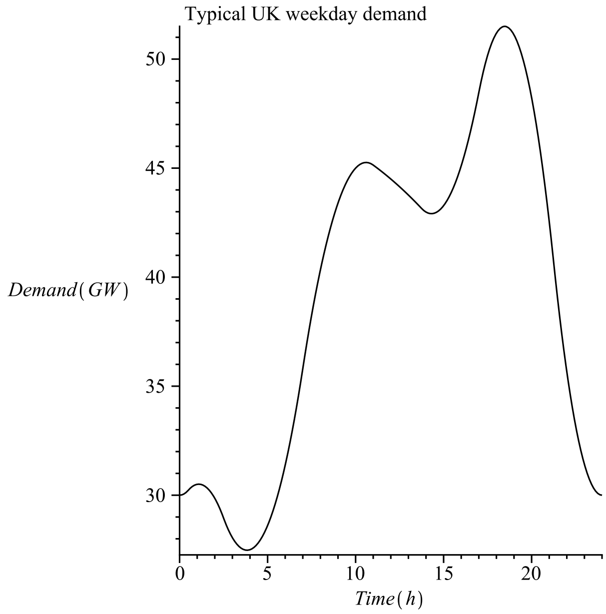

Fig. 1 A typical UK weekday demand profile

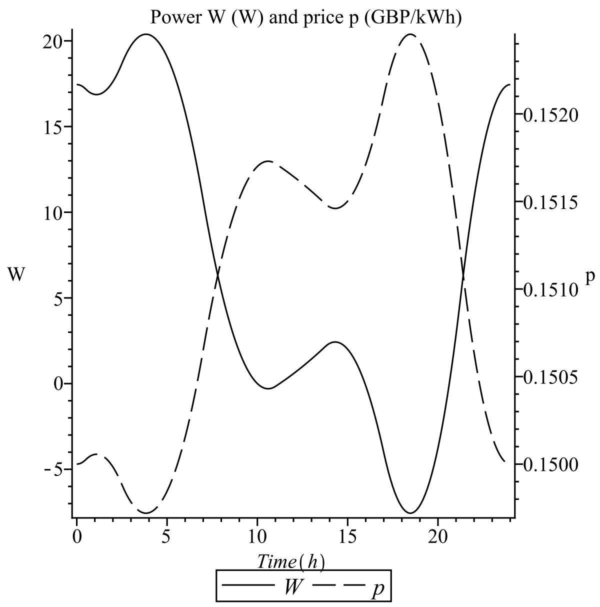

Fig. 2 Power and induced price functions

per kWh and the total cost of a full constant charge ($S_s = 1$) of the battery pack over the 14 hour period is £3.79. In Figure 4, we illustrate the total optimal cost of charging the battery pack as a function of S_s and in Figure 5 we show the percentage reduction obtained by changing to the optimal solution from constant charging from $S = S_s$ to $S = 1$.

Three comments are in order. First, in this example, the optimal solution takes full advantage of the price differential to sell electricity and then buy it back later, so that the battery is fully discharged at 21:00. In fact, as S_s increases from 0 to 1, there are corresponding increases in both the absolute and percentage cost reduction, compared to steady charging, even though the vehicle requires less charge and so the cost of steady charging also reduces. As can be seen from Figure 4, for $S_s > 0.676$ the total cost is negative so that,

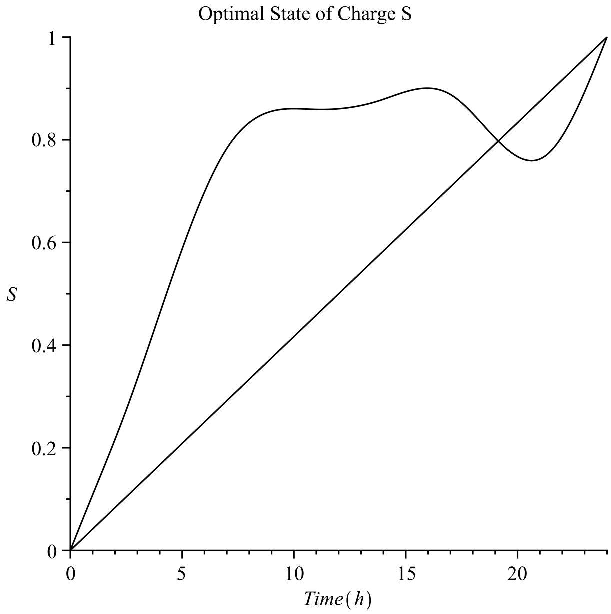

Fig. 3 The associated SOC (compared with constant charging)

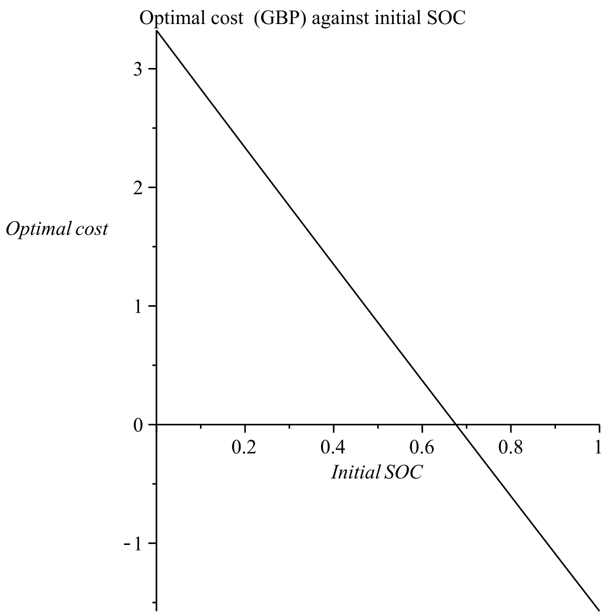

Fig. 4 The minimum cost of charge for the 14-hour period 17:00 to 07:00, with a 50% penalty for charging before 21:00.

ignoring battery degradation, an agent can make a profit by leaving a fully charged vehicle connected to the grid.

Second, the optimal solution changes if the constraints are binding, particularly for fast charging. For example, suppose, with the same pricing function, the vehicle is charged over a half-hour period from 20:45 to 21:15. The optimal charging function, taking into account the constraints, is shown for various initial S_s in Figure 6. For S_s close to 1, the optimal solution is unaffected by constraints. However, For $S_s < 0.884$ the solution is constrained in the period 21:00 to 21:15 by the maximum permitted steady current of 100 A for the Saft VL45E and it is not possible to take full advantage of the price differential between the two time periods. In this case the optimal solution consists of steady charge/discharge in the period before 21:00

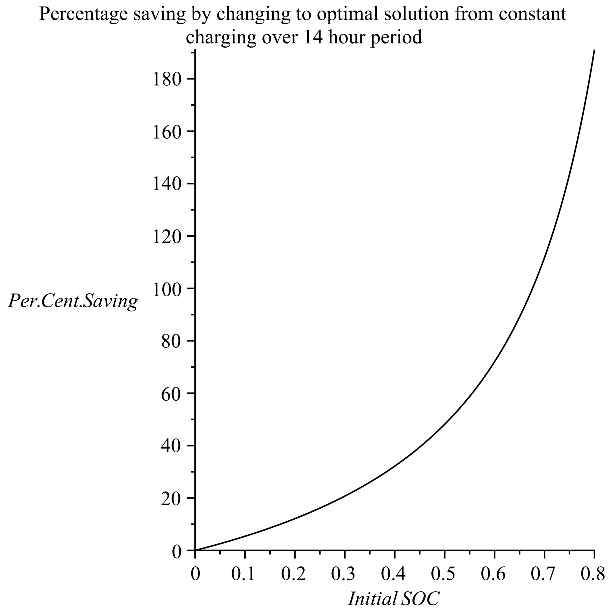

Fig. 5 The percentage saving of the minimum-cost charging over steady charge the 14-hour period 17:00 to 07:00, with a 50% penalty for charging before 21:00.

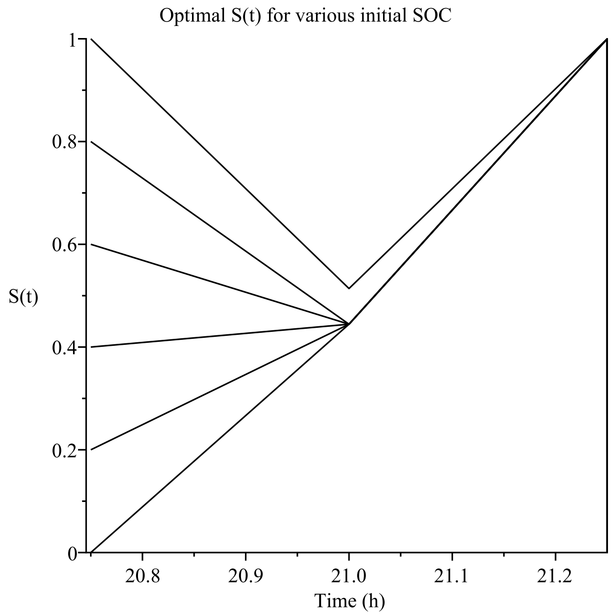

Fig. 6 The optimal charging function S(t) for the period 20:45 to 21:15 for initial SOC = 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0. Note that, for initial SOC < 0.884, the maximum permitted current is a binding constraint in the period between 21:00 and 21:15.

followed by steady charge between 21:00 and 21:15. For $S_s < 0.444$ the battery does not discharge at all, although charging takes place at a slower rate before 21:00. For higher values of S_s, the battery discharges before 21:00, but full discharge does not occur.

Third, it is important to understand the economic cost of continually charging and discharging the battery, especially if prosumers are to be persuaded to connect an electric vehicle power pack to the grid as in V2G storage. Although a full analysis involving realistic models of battery degradation is beyond the scope of the present study, an estimate of lifetime battery costs may be readily obtained. Let us assume that the effective battery lifetime consists of 1000 full charge-discharge cycles. Taking into account recent falls in battery prices, let us also assume a price of £150 per kWh, so that a 25 kWh battery costs £3750 to replace. Hence each full charge-discharge cycle costs £3.75 in battery degradation, in addition to the cost of the electricity required to charge the battery. For simplicity, we assume that a partial charge-discharge cycle (or vice versa) incurs costs proportional to the SOC charged & discharged, so that, for example, discharging from full SOC to 50% SOC, followed by recharging up to full SOC, costs £1.88 in battery degradation.

Returning to the second example above, in which a car is charged over a 14-hour period with a price ratio of 1.5:1 for the peak and off-peak charging periods, the additional cost of charging taking into account battery degradation increases linearly with initial SOC. For initial SOC $S_s = 0$ there is no additional cost so that the total cost of charging to full charge is unchanged. However, when $S_s = 1$, and the battery discharges fully before recharging, the additional cost of £3.75 exceeds the profit from price arbitrage. Indeed, for these choices of the parameters the optimal solution consists of constant charging between 21:00 and 07:00 for all values of S_s. However, if, as an illustration, we now assume a battery lifetime of 2000 charge-discharge cycles and a price ratio of 2:1 for the periods before and after 21:00, it becomes worthwhile to sell electricity to the grid and buy it back at a later time for all initial SOC $S_s > 0.120$. This illustrates further the sensitivity of the optimal solution to the parameter values and the difficulty of incentivising storage given current costs and technology.

5 Discussion

In this paper we have outlined a classical variational approach to optimising battery charge/discharge for a storage supplier or prosumer. In many circumstances, these classical variational techniques may be sufficient for the analysis of the problems considered in this paper. However, it may be argued that modern developments in variational methods offer a better framework. First, the modern theory provides a relaxation in the smoothness requirements of the functional and of the optimal solution to account for non-smoothness in price and battery models. Second, the modern theory provides for the incorporation of constraints and penalties, without the need for Lagrange multipliers or slack variables. There are, however, several disadvantages of the modern theory. It is possible that unphysical/unrealistic solutions might be introduced and also the modern theory is in many ways mathematically oversophisticated. Indeed, once theoretical questions have been decided, hands-on computations may require classical techniques in any case.

For example, let us consider the variational methods developed by Rockafellar and others as surveyed in [10]. In this approach the functional (1) is replaced by

$$C = \int_{t_s}^{t_e} L(t, S, \dot{S})\; dt + \ell(S_{t_s}, S_{t_e})\,, \tag{14}$$

where $L = p(t)G(S, \dot{S})$ and ℓ have relatively mild technical restrictions, are able to take infinite values (and so can naturally incorporate constraints).

A word of caution is needed. Convexity is a typical requirement to ensure uniqueness of the solution but may not always be appropriate in the context of battery charging/discharging; indeed it is not even evident that $L(t, S, \dot{S})$ should be defined on a convex set. Indeed, there are several distinct charging regimes commonly recommended/used in industrial applications. These include constant-current, constant-voltage, hybrid methods (such as constant-current followed by constant-voltage), as well as pulsed, trickle and quick charging. Each regime has its characteristic charging time and effect on battery temperature and battery lifetime and, of course, each battery has its own attributes and manufacturer-recommended charging regimes. Hence, it is possible that rapid and trickle charge (but not intermediate options) might both be acceptable for a given battery design, thereby violating the convexity requirement.

Let us conclude by listing several other directions for future research. It would clearly be helpful to develop the theory presented here to include, for example, generalized battery models; differential pricing for charging and discharging; random initial and final SOCs; a formal treatment of constraints; a ramp-up penalty (so that, for example, equation (1) is modified to $C = \int_{t_s}^{t_e} p(t)W(t) + \alpha(dI/dt)^2\; dt$ where $\alpha > 0$); more complicated pricing structures (where, for example, the price $p = p(W, t)$ is explicitly dependent on power); and ensembles of prosumers/battery models. These extensions will assist the design of pricing policies needed to control the power system and to incentivize the development of storage.

6 Acknowledgement

I would like to express my thanks to the referees for their helpful suggestions to improve the manuscript.

References

1. Binding C, Sundström O: A simulation environment for vehicle-to-grid integration studies. In: Proceedings of the 2011 Summer Computer Simulation Conference (SCSC '11), pp. 14–21 (2011)
2. Bompard E, Connors S, Full G, Han B, Masera M, Mengolini A, Nuttall WJ: Smart Energy Grids and Complexity Science. Joint Research Centre – Institute for Energy and Transport, European Union (2012)
3. Ceraolo M: New dynamical models of lead-acid batteries. IEEE Transactions on Power Systems, vol. 15, no. 4, pp. 1184–1190 (2000)
4. Cruise JR, Gibbens RJ, Zachary, S: Optimal control of storage for arbitrage, with applications to energy systems. In: 48th Annual Conference on Information Sciences and Systems (CISS 2014), pp. 1–6 (2014), doi: 10.1109/CISS.2014.6814090
5. The future role and challenges of energy storage. DG ENER Working Paper, European Union (2013)
6. Guzzella L, Sciarretta A: Vehicle propulsion systems : introduction to modeling and optimization. Springer, Berlin; New York 2005
7. Kleiwegt E, Lukszo Z: Grid impact analysis of electric mobility on a local electricity grid. In: 9th IEEE International Conference on Networking, Sensing and Control (ICNSC 2012), pp. 316–321 (2012)
8. Li Y, Lukszo Z, Weijnen MPC: The potential of electric vehicles to facilitate a high wind power penetration. In: 10th IEEE International Conference on Networking, Sensing and Control (ICNSC 2013), pp. 901–906 (2013)
9. Energy storage: The missing link in the UK's energy commitments. Institution of Mechanical Engineers (2014)
10. Rockafellar RT: Convex analysis in the calculus of variations. In: Advances in convex analysis and global optimization (Pythagorion, 2000), Nonconvex Optim. Appl., vol 54, Kluwer Acad. Publ., Dordrecht, pp. 135–151 (2001)
11. Seaman A, Dao TS, McPhee J: A survey of mathematics-based equivalent-circuit and electrochemical battery models for hybrid and electric vehicle simulation. Journal of Power Sources vol. 256, pp. 410–423 (2014)
12. Tippmann S, Walper D, Balboa L, Spier B, Bessler WG: Low-temperature charging of lithium-ion cells part i: Electrochemical modeling and experimental investigation of degradation behavior. Journal of Power Sources vol. 252, pp. 305 – 316 (2014)
13. Verzijlbergh RA, Lukszo Z: System impacts of electric vehicle charging in an evolving market environment. In: 8th IEEE International Conference on Networking, Sensing and Control (ICNSC 2011), pp. 20–25 (2011)
14. Verzijlbergh RA, Grond MOW, Lukszo Z, Slootweg JG, Ilic MD: Network impacts and cost savings of controlled EV charging. In: 9th IEEE International Conference on Networking, Sensing and Control (ICNSC 2012) pp. 1203–1212 (2012)