Formalising flexible multi-material surfaces as weighted shapes

Conference or Workshop Item

Formalising Flexible Multi-material Surfaces as Weighted Shapes

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Abstract

The introduction of multi-material additive manufacturing makes it possible to fabricate objects with varying material properties, leading to new types of designs that exhibit interesting and complicated behaviours. But, computational design methods typically focus on the structure and geometry of designed objects, and do not incorporate material properties or behaviour. This paper explores how material properties can be included in computational design, by formally modelling them as weights in shape computations. Shape computations, such as shape grammars, formalise the description and manipulations of pictorial representation in creative design processes. The paper explores different ways that material properties can be formally modelled as weights, and presents examples in which multi-material surfaces are modelled as weighted planes, giving rise to flexible behaviours.

CR Categories: I.3.5 [Computational Geometry and Object Modeling]: Curve, surface, solid, and object representations; J.6 [Computer-Aided Engineering]: Computer-aided design (CAD)

Keywords: additive manufacturing, shape computation, design

1. Introduction

Additive manufacturing is rapidly becoming an essential and ubiquitous process in creative design, and has introduced new possibilities with respect to the types of shapes that can be realised. The technology continues to evolve, and over recent years has introduced variability in material properties alongside variability in form. This is achieved either by colouring material as it is extruded or by combining different materials within one fabricated object. The result is a greater range in the types of objects that can be fabricated, but these advances are not reflected in computational methods used in design. As discussed by Oxman and Rosenberg (2007), computational methods are typically restricted to defining and exploring the structure and geometry of a design models, and do not incorporate material properties or behaviour. This research explores how these can be incorporated into shape computations, which have been shown to formalise creative design processes (Prats et al., 2008; Paterson and Earl, 2010), and support generative design (Stiny, 2006). In this paper, the focus is on identifying how the material properties of a surface can be formally modelled. The paper explores mechanisms necessary to support computation with weighted shapes, building on theoretical developments presented Stiny (1992). Ultimately, the aim of the research is to enable the generation of design models, with reference to material properties and expected behaviour.

2. Multi-Material Additive Manufacturing

In additive manufacturing, multi-material fabrication is made possible via technologies such as the *Objet Connex*[1], which combine different materials in a single fabricated object. Materials with various transparencies, colours and material properties, are combined in layers, or are mixed as composites that simulate the properties of common materials such as plastic or rubber. For example, using the *Objet Connex* a hard white plastic material called *VeroWhitePlus*, can be mixed in different proportions with a soft rubber-like black material called *TangoBlackPlus* to produce a range of composite materials, as illustrated in Figure 1. These composite materials vary in colour, from opaque white through to opaque black. They also vary in material properties[2]: as the proportion of *TangoBlackPlus* increases the shore rating (a measure of resistance to permanent indentation) decreases, the tensile strength decreases, and the elongation at break increases. Consequently, composite materials become softer and more flexible as the proportion of *TangoBlackPlus* increases.

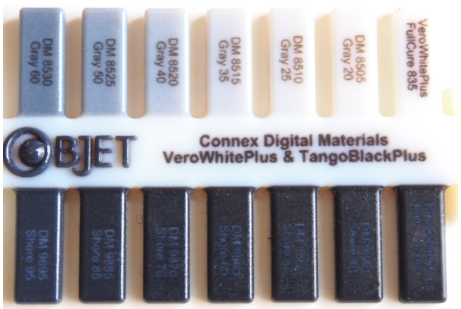

Figure 1. *Sample .material for the Objet Connex*

Fabrication processes that combine materials in layers or mix them in composite materials make it possible to produce objects that have variable material properties. The result is objects which exhibit different physical behaviours. For example, Figure 2 illustrates a flat surface composed of composite materials that are a mix *VeroWhitePlus* and *TangoBlackPlus*. In the surface the composite materials are arranged in stripes, where the darkness of a stripe reflects the amount of *TangoBlackPlus* included in the mix: the darker the stripe, the higher the proportion of *TangoBlackPlus* and the higher the flexibility of that segment of the surface. The result is a flat surface that has stripes of varying flexibility that are arranged to give a gradient of flexibility, starting from a very flexible stripe and ending with a very stiff stripe. This gives rise to a natural curvature so that the flat surface can be deformed into a curved surface, as illustrated.

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[1] http://www.sys-uk.com/Connex

[2] http://www.stratasys.com/materials/polyjet/rubber-like

The curved surface in Figure 2 was generated using the Kangaroo Physics tools for Grasshopper[3] by modelling the stripes as springs, in a method similar to that described in Oxman and Rosenberg (2007). A bending resistance force is applied to each stripe, which corresponds to the weight applied to the stripe; as the proportion of *TangoBlackPlus* increases the resistance force decreases and the stripe allows a greater flexibility. Each corner point is modelled as an 'anchor' point, initially 'anchored' to the *xy* plane. Kangaroo allows anchor points to be moved in real time during the simulation, allowing the user to interact and explore the kinetic properties of a spring system, and so the potential flexibility of a weighted surface. The images shown are snapshots of this process, which provides a simulation of the flexible behaviour of the multi-material plane, and is determined based on the weights applied to the plane in combination with the geometry of the plane. This presents an interactive approach to designing material properties and behaviour: material properties are incorporated in representations used in shape computation, so that they, and the resulting behaviour, can be defined and explored during design generation.

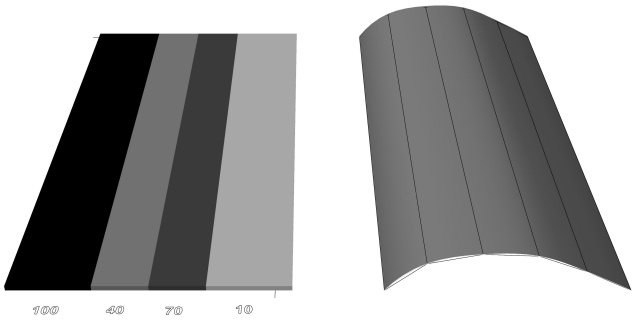

Figure 2. *The behaviour of a multi-material surface*

3. Shape Computation and Weighted Shapes

In shape computation (Stiny, 2006), design representations are formalised as shapes composed of finite numbers of geometric elements (points, lines, planes, etc.) of finite extent. Shapes are described according to their parts and these are ordered by a part relation, are combined by shape operations of sum, product and difference, and manipulated according to Euclidean transformations. Shapes have no inherent parts and do not have a unique decomposition. Instead, decompositions arise through use, through processes of enquiry and description, such as analysis or communication. Shapes present a more design-orientated approach to design representation and generation than the point-set formalism commonly used in computer-aided design (Earl, 1997). This is because they can be freely interpreted, even according to parts which are not be apparent in the initial construction of a shape.

The ordering of the parts of a shape by the part relation gives rise to a formal structure corresponding to a Boolean algebra (Stiny, 2006). This algebra is partially ordered by the part relation, closed over operations of sum and product, with the complete shape as unit, the empty shape as zero, and complements defined accordingly. The algebra enumerates *all* potential decompositions of a given shape according to *all* possible parts. For example, Figure 3 presents a lattice that is equivalent to a sub-algebra of a shape, defined according to triangular parts. In the lattice, the sum of parts is given by the supemum (join) and the product by the infimum (meet). The grey lines represent missing lines, and are included for legibility.

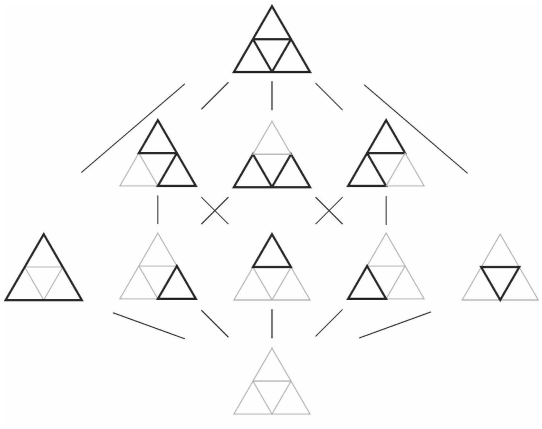

Figure 3. *A lattice of triangular parts.*

Similarly, shapes in general define generalised Boolean algebras ordered by the part relation, closed over operations of sum and product, with the empty shape as zero, but lacking a unit, because an infinite shape is not defined. These are equivalent to Boolean rings (Stiny, 2006), and denoted U_{ij}, where i is the dimension of the geometric element, and j is the dimension of the embedding space. For example, designs represented in 2D sketches are in the algebra U_{12}, 3D wire frame models are in U_{13}, and solid models are in U_{33}. More interesting design representations, composed of combinations of different types of shapes give rise to composite algebras defined by the Cartesian products of these shape algebras. Shape computations formally define manipulations and transformations of shapes within shape algebras. Shape grammars exemplify such computations, and formalise creative design processes (Prats et al., 2008; Paterson and Earl, 2010).

In addition to the spatial information captured in shapes, shape computation takes into account non-spatial information, such as colour and function, and these are formalised in algebras of labelled shapes, V_{ij} and weighted shapes, W_{ij} (Stiny, 1992). In a shape computation, labels are defined according to a given vocabulary and serve to distinguish shapes from each other: overlapping geometric elements with different labels are distinct, and cannot be merged, as illustrated in Figure 4.

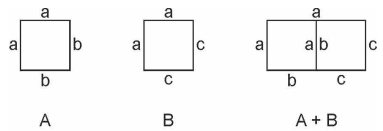

Figure 4. *The union of labelled shapes in V_{12} (Stiny, 1992).*

Weights represent properties of a shape, e.g. in W_{12} weights may represent thickness of line elements, in W_{23} weights may represent the texture of plane elements, and in W_{33} weights may represent physical properties of a solid. Weights also change the way that geometric elements interact, but this is not as straight-forward as with labels. This is because weights, like shapes, can be embedded as parts of each other, whereas labels are always distinct. In weighted shapes, overlapping geometric elements may or may not merge; it depends how a weight is defined.

[3] http://www.grasshopper3d.com/group/kangaroo

In order to include a weight in a shape computation, such as a shape grammar, it is necessary to define a relation between different weight-values, i.e. an order over the weights, which specifies how they are embedded in each other as parts. It is also necessary to define operations of sum, product and difference, which specify how weights combine in shape computations. The relation and operations should complement the equivalent shape relation and operations, and should reflect the properties being modelled. For example, in Stiny (1992), a weight in W_{12} is defined which models line thickness and reflects the use of lines in drawing. In this example, a part relation is defined where thin lines are embedded in, and subsumed, by thick lines; the sum of two weights is defined to be the maximum of the two; the product of two weights is defined to be the minimum of the two; and the difference between two weights is the arithmetic difference of the two, with a minimum value of zero. Note that geometric elements with zero weights are not defined, so when a weight of zero is applied to a geometric element, the result is its removal from the shape. This is illustrated in Figure 5, where examples of sum, product and difference of two weighted lines are presented. Details of shape computations with weighted shapes in algebras W_{ij} are presented in Stiny (1992). As illustrated in Figure 5, these incorporate both shape and weight operations, with weights interacting only on overlapping geometric elements. The relation between weights informs the part relation over weighted shapes, and the operations over weights inform the shape operations on weighted shapes.

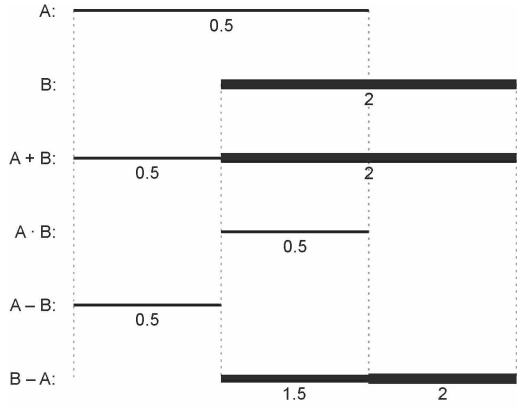

Figure 5. *Shape operations on weighted lines in W_{12} (Stiny, 1992).*

4. Modelling Material Properties with Weights

Weights can be defined in many ways, e.g. as values, vectors or matrices, and can be used to model any properties of a shape, e.g. physical properties such as mass, or intentional properties such as function. In this research, the aim is to model the material properties of multi-material objects, which are fabricated from composite materials. How weights can be used to do this will be illustrated by considering the combination of materials in the *Objet Connex*. *VeroWhitePlus* and *TangoBlackPlus* are combined in composite materials which vary in shade, hardness and flexibility: as the proportion of *TangoBlackPlus* increases the composite materials become darker, softer and more flexible. In practice, the composite materials that are produced by combining *VeroWhitePlus* and *TangoBlackPlus* are limited to the fourteen discrete examples illustrated in Figure 1. But, in this paper a more theoretical approach is followed, where any proportion of the base materials can be used. It is anticipated that this approach can be generalised to model other material properties defined according to combinations of materials.

The proportion of *VeroWhitePlus* and *TangoBlackPlus* included in a composite material can be modelled with weights in different ways, and three different definitions of a weight w have been identified, with an aim to illustrate the possibilities. These are subtly different, modelling the same composite materials but with emphasis on different properties of the materials.

1. $\{w \in \mathbb{N} : 0 < w \leq 100\}$
   w models the flexibility of a composite material. Here, w is assigned a value from 1 to 100 to reflect the percentage of *TangoBlackPlus* in the composite.
   e.g.
   $w = 100$, the material is black, very soft and very flexible
   $w = 50$, the material is grey, semi-soft and semi-flexible
   $w = 1$, the material is near-white, very hard and very rigid

2. $\{w \in \mathbb{N} : 0 < w \leq 100\}$
   w models the hardness of a composite material. Here, w is assigned a value from 1 to 100 to reflect the percentage of *VeroWhitePlus* in the composite.
   e.g.
   $w = 100$, the material is white, very hard and very rigid
   $w = 50$, the material is grey, semi-soft and semi-flexible
   $w = 1$, the material is near-black, very soft and very flexible

3. $\{w = (w_1, w_2) : w_1, w_2 \in \mathbb{N}_0\}$
   w models the mixture of *VeroWhitePlus* and *TangoBlackPlus* in a composite material. Here, w is a vector (w_1, w_2) with w_1 and w_2, representing independent values for *VeroWhitePlus* and *TangoBlackPlus*, respectively. The proportion of these is given by the ratio $w_1 : w_2$.
   e.g.
   $w = (x, 0)$, the material is white, very hard and very rigid
   $w = (x, x)$, the material is grey, semi-soft and semi-flexible
   $w = (0, x)$, the material is black, very soft and very flexible

For each of these three weights, it is necessary to define a relation between different weight-values (<), as well as operations of sum (+), product (□), and difference (–). There is currently little guidance on how these should be defined, nor any limitations on what is acceptable. But, definitions should reflect the material properties that are being modelled, and intuition about the context-specific meaning of a relation and operations. Generally, the operations of sum, product and difference can be defined similarly to the Boolean operations of union, intersection and complementation, as illustrated in Figure 6.

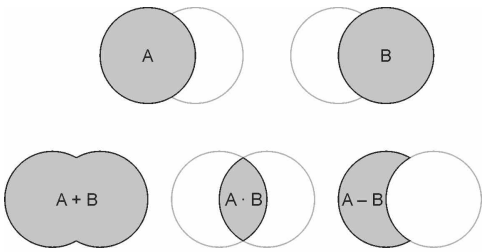

Figure 6. *Weight operations as Boolean operations.*

The result of a sum operation, $A + B$ is something that subsumes both A and B; the result of a product operation $A \square B$ is what A and B have in common; and the result of a difference operation $A - B$ is what remains of A after B is removed. Here, a variety of shape operations have been defined to illustrate the scope of what is possible.

Weight 1

w models the flexibility of a composite material, and the relation and operations defined over the weight should reflect this. Figure 7 illustrates definitions for these, with the relation and operations applied to weighted planes in a W_{22} algebra.

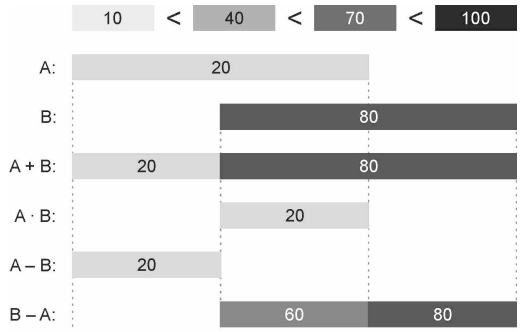

Figure 7. *Shape operations on weighted planes in W_{22} (Weight 1).*

The relation is defined as a linear total order, so that stiffer materials are embedded in, and subsumed by, flexible materials. Given two weights w and u,

$w < u$ if $|w| < |u|$

Applying the sum operation should give a material that is of a flexibility that subsumes w and u, and the result is the more flexible of the two

$w + u = \max(w, u)$

For product and difference, intuition gives little guidance, but the Boolean operations illustrated in Figure 6 suggest how these operations could be defined. The product operation should give a material that has a flexibility that is common in both w and u, and the result is the more rigid of the two

$w \square u = \min(w, u)$

The difference operation should give a material that has the flexibility in w after u is removed, so the result is a material more rigid than both w and u, given by their arithmetic difference with a minimum value of zero

$w - u = \max(|w| - |u|, 0)$

Weight 2

w models the hardness of a composite material, and the relation and operations defined over the weight should reflect this. The weight is essentially an inverse of Weight 1, and the relation and operations could be defined in a similar manner. But this is not necessary, and alternative definitions for the sum and product operations have been chosen. Figure 8 illustrates the relation and operations applied to weighted planes in a W_{22} algebra.

The relation is defined as a linear total order, so that soft materials are embedded in, and subsumed by, hard materials. Given two weights w and u,

$w < u$ if $|w| < |u|$

Applying the sum operation should give a material of hardness greater than w and u, and the result is their arithmetic sum, with a maximum value of 100

$w + u = \min(|w| + |u|, 100)$

The product operation results in a material that has a hardness that is between both w and u, and is defined as the arithmetic mean of the two

$w \square u = ½ (|w| + |u|)$

The difference operation should give a material that has the hardness of w after u is removed, and the result is a material softer than both w and u, given by their arithmetic difference with a minimum value of zero

$w - u = \max(|w| - |u|, 0)$

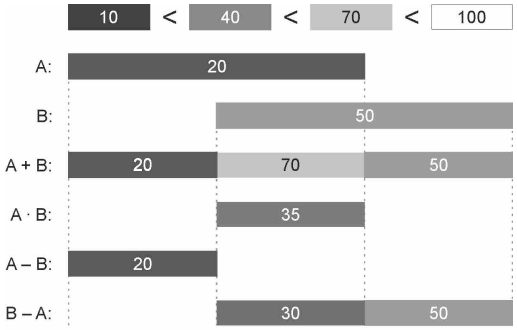

Figure 8. *Shape operations on weighted planes in W_{22} (Weight 2).*

Weight 3

w models the mixture of *VeroWhitePlus* and *TangoBlackPlus* in a composite material, and the relation and operations defined over the weight should reflect this. This third weight is different from the previous two, because it is defined as a vector instead of a single value. Specifically it is defined as (w_1, w_2), where w_1 and w_2, represent the amount of *VeroWhitePlus* and *TangoBlackPlus* in the mixture, respectively. A consequence of this is that the proportions of base material in a composite are not uniquely defined. For example, the vectors (10, 10), (30, 30) and (60, 60) all correspond to mixtures where the proportion of *VeroWhitePlus* to *TangoBlackPlus* is the same, so the three weights correspond to composite materials with the same properties. The relation and operations are defined by considering both co-ordinates of the vectors, and Figure 9 illustrates definitions for these, with the relation and operations applied to weighted planes in a W_{22} algebra.

The relation is defined as a partial order, so that both light and dark materials may be subsumed by grey materials. Given two weights w and u

$w < u$ if $|w_1| < |u_1|$ and $|w_2| < |u_2|$

Applying the sum operation is analogous to mixing paint of different shades of grey, giving a material with a shade that is a mixture of the w and u. The result is defined by adding the co-ordinates of the vectors, to a maximum value of 100

$w + u = (\min(w_1 + u_2, 100), \min(w_2 + u_2, 100))$

The product operation results in a material that has a shade that is common to both w and u, and is defined by the minimum co-ordinates of the two

$w \sqcap u = (\min(w_1, u_1), \min(w_2, u_2))$

The difference operation is analogous to removing paint of a given shade from a mixture, and the result is defined by the difference of the coordinates, with a minimum value of 0

$w - u = (\max(w_1 - u_2, 0), \max(w_1 - u_2, 0))$

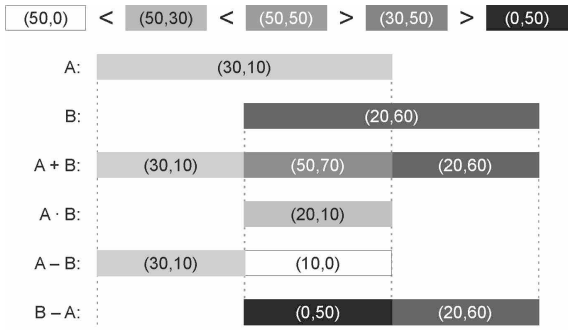

Figure 9. *Shape operations on weighted planes in W_{22} (Weight 3).*

5. Computing with Flexible Surfaces

Three different weights have been specified, and for each of these, the relation between weights, and operations on weights have been defined differently. These weights provide different models for the material properties that result from mixing the materials *VeroWhitePlus* and *TangoBlackPlus*, and each has different strengths and limitations. For example, Weight 1 intuitively models the flexibility of a composite material but cannot model material that is 100% *VeroWhitePlus*. Similarly, Weight 2 intuitively models the hardness of a composite material but cannot model material that is 100% *TangoBlackPlus*. Weight 3 intuitively models the mixture of materials and can model all variations, but does not provide a unique description of these.

Choosing which weight to use, and which definitions of relations and operations to use depends on what is appropriate for the task at hand. Experimentation with different shape computations will give some insight into how the different weights, relations and operations behave, and how they reflect different contexts. The examples presented in the previous section illustrated the relations and shape operations on plane segments with single weight values associated with them in a W_{22} algebra. In this section, more complicated arrangements of weights on surfaces are explored resulting in curved surfaces in a W_{23} algebra, such as the surfaces in Figure 2 and Figure 10. The width and height of the two planes are equal, so that they can be arranged with the weighted stripes running orthogonally.

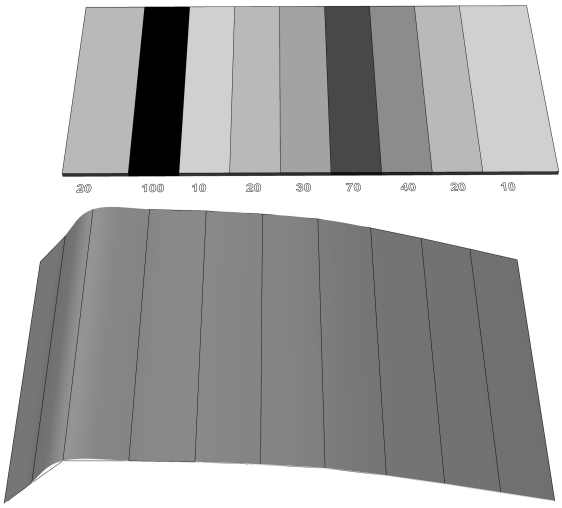

Figure 10. *Multi-material surface, modelled as a weighted shape.*

The properties of the materials in the planes are modelled according to Weight 1 (illustrated in Figure 7), and examples of shape computations with the surfaces are presented in Figure 11. In Figure 11a) the sum operation is applied, and the result is a more complicated arrangement of weights, with the more flexible stripes dominating the rigid stripes. In Figure 11b) the product operation is applied, and the result is a more complicated arrangement of weights, with the more rigid stripes dominating the flexible stripes. In Figure 11c) the difference operation is applied, and the result is a checkerboard of weights, but with segments missing where the result of arithmetic subtraction is zero or less. If the planes were modelled according to Weight 2 then the hard stripes would dominate the soft stripes, and if they were modelled according to Weight 3 the results would reflect the mixing of these properties.

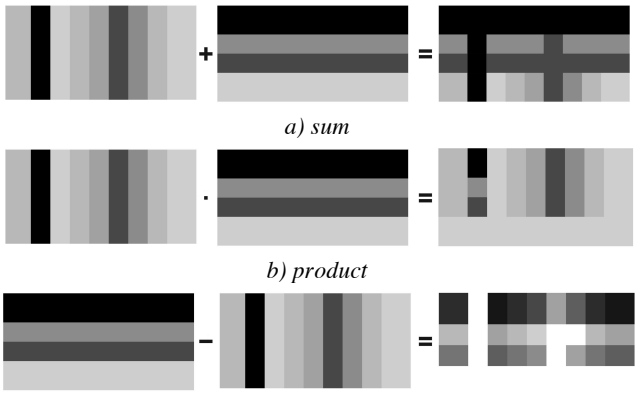

Figure 11. *Shape computations with multi-material planes*

Working from these shape computations the flexible behaviours of the resulting surfaces can be simulated, as described in Section 3. For example, Figure 12 presents a simulation of the result of the sum operation. The resulting W_{23} surface is doubly curved, combining the orthogonal curving of the two original surfaces in an interesting way.

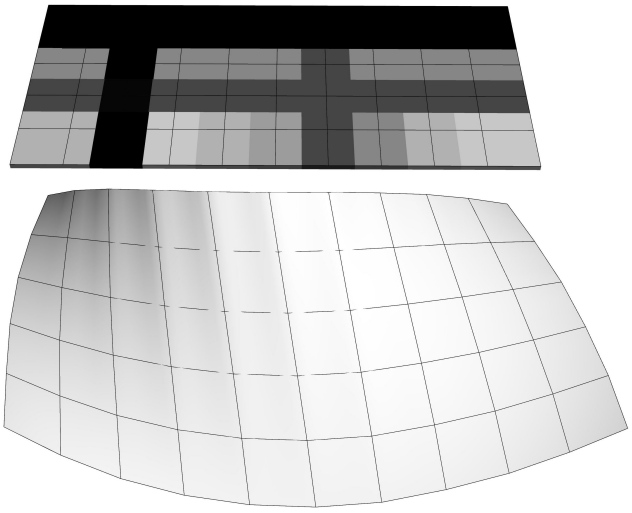

Figure 12. *Multi-material surface; result of the sum operation.*

Similarly, Figure 13 presents a simulation of the result of the product operation. The resulting W_{23} surface is again doubly curved, but it combines the curving of the two original surfaces in a very different way. These two surfaces are simple examples that illustrate the explorative potential of shape computation, with respect to material properties and resulting behaviour.

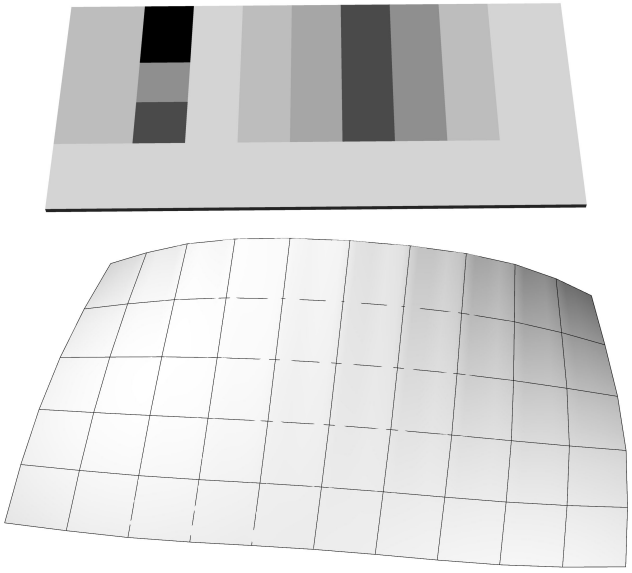

Figure 13. *Multi-material surface; result of the product operation.*

6. Discussion

In this paper, weights have been used to model the material properties of multi-material surfaces. The research has focussed on composite materials that are defined as combinations of two base materials, *VeroWhitePlus* and *TangoBlackPlus*. The surfaces are composed of stripes of different composites which are mixtures of the base materials. In different proportions, these mixtures give rise to different properties, in terms of shade, flexibility and hardness, and when combined in layers in multi-material surfaces in a W_{23} algebra, they result in flexible behaviours. Even for such a simple example, there is wide variability in how to model the materials, as exhibited by the three different definitions for weights and the range of definitions for the relations and operations over the weights. Currently, there is little theoretical guidance to suggest what makes a *good* definition of a weight, and other material properties may be defined in different ways, in different W_{ij} algebras. For example, combinations of three or more base materials might be used, such as weights that model colour which may be defined according to combinations of materials following specific colour-models, e.g. RGB or CMYK. Introducing more materials may increase the variability and complexity of the modelling process, but the same steps of defining a weight, and defining the relation between weight-values and the operations on weights, should be followed.

When weights are fully defined they can be applied to shapes and incorporated in shape computations. Simple examples of such computations were illustrated according to weighted planes in a W_{23} algebra. The results of the computations were also planes, but with more complicated structures, as defined by the weights. The structure imposed on a shape by applying weights is more restrictive than the visual structure associated with un-weighted shapes in a U_{23} algebra, which is defined according to embedded parts, as illustrated in Figure 3. Weights force a decomposition of geometric elements into segments with different weights, as illustrated in Figure 5. This is analogous to the use of labels in Figure 4 where co-linear lines cannot be merged because their assigned labels keep them distinct. A consequence of this is that visually recognised parts may not be embedded in a given weighted shape. For example, the shape in Figure 14a) is weighted according to line thickness (as illustrated in Figure 5). It has a different structure to the un-weighted shape in Figure 3 and, because of this structure, the weighted triangle illustrated in Figure 14b) is not a part of the shape. The weighted triangle illustrated in Figure 14c) is part of the shape, but recognising the triangular parts does not decompose the shape according to the lattice in Figure 3. Applying the weights has fundamentally changed the shape according to its parts and its structure. This raises questions about the types of shape structures in W_{ij} algebras concerning different definitions of weights, and different definitions of the relation and operations. For example, the relation over Weights 1 and 2 define a linear order, whereas Weight 3 defines a partial order. As a consequence the algebraic structures defined by shapes assigned these different weights will be different. In Stiny (1992), it is suggested that weights can be freely chosen to meet the situation being modelled. But, there may be more formal constraints that need to be considered when selecting the definition of a weight.

a) *b)* *c)*

Figure 14. *Three weighted shapes in a W_{13} algebra.*

Operations in weighted shapes computations, in W_{ij} algebras, work within and retain the structure defined by weights. Indeed, often they impose more structure, because different weights interact to further decompose a shape into different-weighted segments, as illustrated in Figures 11. It is this structure, in combination with the material properties modelled by the weights that gives rise to the behaviours of the surfaces. And, potentially it is this structure that can inform the design of material properties

and behaviour in more complicated computations, for example using shape grammars, as described in Stiny (1992).

Building on this research, it is possible to define a shape grammar that supports the generative design of weighted surfaces, based on an exploration of shape and material properties. Weighted shape computations could be directed so that they decompose shapes into weighted parts with material properties that give rise to desired behaviours, and that recognise and explore emergent behaviours. The surfaces explored in this paper, are geometrically simple, but the complicated arrangements of weights give rise to complicated flexible behaviours. The computational nature of systems of weights allows them to be easily applied in physical simulation packages, as the initial examples in the paper have demonstrated. Weighted surfaces could be subjected to relevant forces according to the intended design objectives, such as pressure forces or draping, informing the design process. These have potentially interesting applications in design. For example, flat surfaces are cheaper to manufacture than curved surfaces but the introduction of flexible behaviours through multi-material fabrication makes it possible to manufacture flat surfaces that deform into desirable curved shapes. Also, it is possible to to fabricate one-piece objects which have rigid structural elements, embedded in flexible, malleable materials. These could be applied in a range of innovative contexts, for example they could be used to develop adjustable canopies, sporting clothing, or safety equipment. Fundamentally, weighted shapes allow computational methods to extend beyond the spatial aspects of designs, so that designers can also creatively explore and develop material properties and behaviour. This paper has explored the mechanisms to make such exploration possible, but further research is needed to investigate how these can be employed in computational methods for creative design.

References

EARL, C. F. 1997. Shape boundaries. Environment and Planning B: Planning and Design, 24(5): 669-687.

OXMAN, N., AND ROSENBERG, J. L. 2007. Material-based Design Computation: An Inquiry into Digital Simulation of Physical Material Properties as Design Generators. International Journal of Architectural Computing, 5(1): 26-44.

PATERSON, G., AND EARL, C. 2010. Line and Plane to Solid: Analysing their use in design practice through shape rules. In JS Gero (ed) Design Computing and Cognition 2010, (pp. 251-267). Springer.

PRATS, M., LIM, S., JOWERS, I., GARNER, S., AND CHASE, S. 2009. Transforming shape in design: Observations from studies of sketching. Design Studies, 30(5): 503-520.

STINY, G. 1992. Weights. Environment and Planning B: Planning and Design, 19(4), 413-430.

STINY, G. 2006. Shape: Talking about Seeing and Doing. MIT Press.