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Theoretical Derivation of a Brie-like Fluid Mixing Law

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Abstract

Prediction of the velocity of acoustic waves in partially saturated rocks is very important in geophysical applications. The need to accurately predict acoustic velocities has resulted in a widespread popularity of Brie's effective fluid mixing law. This empirical model together with Gassmann's formula are used routinely in fluid substitution problems in petroleum geophysics and seismic monitoring of carbon capture and storage. Most attempts to justify Brie's model have been focused on interpretation in terms of patchy saturation models and attaching meaning to the Brie parameter in terms of the patch size. In this paper, using a microstructural description of the rock and a parameter relating to capillary pressure, we calculate an effective fluid modulus that is very similar to Brie's law. The fluid mixing law we propose is independent of frequency and has a solid theoretical foundation. This proposed law produces analytically harmonic and arithmetic averaging at the endpoints. Our results indicate Brie-like behaviour may not necessarily be related to frequency and patch-size dependent phenomena.

1 Introduction

In rock physics, it is both theoretically and experimentally shown that bulk elastic properties are strongly affected by partial gas saturation. Practical approaches to the problem of calculating partially saturated bulk moduli are easy to implement hence very useful for practical purposes. Almost exclusively the Gassmann (1951) formula is used in these approaches and often several of its input parameters are adjusted to match experimental measurements of S- and P-wave velocities from logs (see Simm, 2007).

From a physics perspective, the impact of partial saturation on the bulk modulus can be attributed to an effective fluid modulus. Accurately describing this effective fluid modulus as a function of the saturating moduli is a key component of fluid substitution. In Domenico (1974) it is argued that the weighted harmonic average of the two saturating fluid moduli describes this effective fluid in a pore space uniformly saturated by two or more fluids.

The harmonic average of Domenico (1974) predicts that a small amount of gas would result in a radical decrease of the effective fluid modulus. This effect was observed in seismic but not sonic frequencies in Murphy et al. (1993). Therefore, it became accepted that the description of the effective modulus of Domenico (1974) would have to be amended to account for a behaviour closer to an arithmetic averaging law as observed in Murphy et al. (1993).

A practical description of the effective fluid modulus was missing from the literature and in Brie et al. (1995) an empirical model was suggested.

Despite the lack of a theoretical foundation, this model has been used with success in both exploration (e.g. Lee, 2004) and carbon capture and storage applications (e.g. Queißer and Singh, 2013; Carcione et al., 2006; Grude et al., 2014).

In its original form, Brie et al. (1995) calculate this effective fluid modulus experimentally for a variety of water saturations in three unconsolidated shaly sands and fit it to an empirical model:

$$\mathcal{K}_f = (\mathcal{K}_\mathrm{w} - \mathcal{K}_\mathrm{g}) S_\mathrm{w}^e + \mathcal{K}_\mathrm{g}, \qquad 1 < e \lesssim 40 \tag{1}$$

where $\mathcal{K}_\mathrm{w}, \mathcal{K}_\mathrm{g}$ are the water and gas fluid moduli respectively whereas S_w is the water saturation. Its lower bound of 1 corresponds to an arithmetic averaging law and the upper bound of about 40 approximates harmonic averaging. In the well data fitted the dimensionless parameter e lies in the range $2 < e < 5$. This effective fluid modulus is plotted in Figure 1 for various values of its free parameter e.

This effect is commonly attributed to mesoscopic dispersion owing to wave induced fluid flow in a patchy saturated rock and there is direct experimental evidence (see Lebedev et al., 2009) to suggest this may be the case. The extent to which saturation can be considered uniform, determines the exact form of the effective fluid modulus (see Murphy et al., 1993; Mavko and Mukerji, 1998; Toms-Stewart et al., 2009). This is the main idea behind Hill (1963), White (1975) but see also Smith et al. (2003) and Carcione et al.

(2011) for an overview. Central to these models is the concept of the fluid patch size which is somewhat elusive and a difficult parameter to determine (for a discussion see Castagna, 2001).

In Amalokwu et al. (2015b) and Amalokwu et al. (2015a), measurements of a partially saturated bulk modulus are fitted to two dispersive models: one to account for the patchiness of the saturation, and one to account for the dispersive nature of squirt flow effects in full saturation. These two mechanisms are considered independently of each other so it would be appealing to fit both of them under a unified description which is the motivation behind this work.

2 Theory

In recent work, Papageorgiou and Chapman (2015) argued that the spatial fluid distribution in the pore space is not merely of patchy or uniform type. Their justification was that the characteristic patch size is not an applicable quantity when the pore space consists of narrow cracks and wider pores and using a model that is suited to describe such a pore space, they showed how the seismic properties of the saturated matrix are affected by different fluid distributions across different inclusion types.

In Papageorgiou and Chapman (2015) one of the fundamental assumptions is that the saturating fluids – water and gas – are held at equal pressures in the pore space and the seismic properties of the matrix are affected by variations in water distribution along cracks and pores. Strictly speaking, spatially inhomogeneous saturation and the assumption for equal pressure are inconsistent with each other. A difference in crack and pore saturation can only be driven by capillary forces. Capillary forces exist exactly because of the imbalance in surface energy between different fluids which results in a pressure difference between the two fluids if the matrix is assumed to be wetted by one of them.

In this work we use an Eshelby-based inclusion model where the inclusions are saturated by multiple fluids. The relationship of such models to Gassmann's theory is discussed in Chapman et al. (2002). Taking capillary pressure into account, we show that this theory admits an effective fluid modulus resembling the Brie et al. (1995) mixing law. The corresponding free parameter has a straightforward interpretation in terms of the capillary pressure.

We use the same notation as Papageorgiou and Chapman (2015) to express a pore network comprised of coin-like ellipsoids and spherical pores. Quantities referring to cracks are denoted by $^{\ominus}$ and to pores by $^{\odot}$. At the low frequency limit there is no dynamic fluid exchange between cracks and pores. As a consequence, even though the saturating fluids may have different pressures, there is no pressure imbalance between inclusions for either fluid and there is no dispersion due to fluid flow between pores:

$$P_{\mathrm{w}}^{\ominus} = P_{\mathrm{w}}^{\odot}\ (= P_{\mathrm{w}}) \qquad P_{\mathrm{g}}^{\ominus} = P_{\mathrm{g}}^{\odot}\ (= P_{\mathrm{g}}). \tag{2}$$

Note that dispersion occurs when the pressure difference in neighbouring pores induces fluid mass exchange as noted in (Papageorgiou and Chapman, 2015).

As shown in Papageorgiou and Chapman (2015), a choice of equivalent fluid pressure $\widetilde{P}$ needs to be made in order to balance externally applied stress σ in each inclusion.This stems from considerations in Zatsepin and Crampin (1997) where the specific compliance of the inclusions is calculated. Specifically the stress σ induces an infinitesimal volume change in each type of inclusion given by:

$$\begin{aligned} \phi^{\ominus} =& \phi_0^{\ominus}\left(1 - \frac{\sigma_n}{\sigma_c} + \frac{\widetilde{P}}{\sigma_c}\right) \\ \phi^{\odot} =& \phi_0^{\odot}\left(1 - \frac{3}{4\mu}\frac{(1-\nu)}{(1+\nu)}\sigma_{ll} + \frac{3}{4\mu}\widetilde{P}\right). \end{aligned} \tag{3}$$

where $\phi_0^{\odot}, \phi_0^{\ominus}$ are the unstressed porosities of each inclusion type. Here the symbols σ_n, σ_{ll} denote respectively the normal and the trace of the tensor σ and μ, ν are the shear modulus and Poisson ratio of the mineral grains. The parameter σ_c denotes a compressibility relating to the ellipsoid geometry defined in Chapman et al. (2002) as

$$\sigma_c = \frac{\pi \mu r}{2(1-\nu)} \tag{4}$$

where r is the aspect ratio of the ellipsoid.

Here, we assume that the effective pressure $\widetilde{P}$ balancing the stress at the inclusion level is the volumetric average of the water and gas fluid pressures. This choice is arbitrary but educated (see Santos et al., 1990b):

$$\widetilde{P} = S_{\mathrm{w}} P_{\mathrm{w}} + (1 - S_{\mathrm{w}}) P_{\mathrm{g}}, \tag{5}$$

where we have written S_{w} for the volume fraction of the water in the pore space. Note that this volumetric average refers to the effective fluid pressure in each inclusion. As the pore space consists solely of these inclusions this assumption translates to uniform fluid distribution within the pore space.

The expression for the pore-space volume from eq. (3) can also be written as a sum over fluid volumes:

$$\phi = \phi_0 \left(1 + \frac{S_{\mathrm{w}}}{\mathcal{K}_{\mathrm{w}}} P_{\mathrm{w}} + \frac{(1 - S_{\mathrm{w}})}{\mathcal{K}_{\mathrm{g}}} P_{\mathrm{g}} \right). \tag{6}$$

We note that for a water-wet medium, a capillary equation relates the wetting to non-wetting fluid pressure. A convenient way to parametrise this capillary pressure is by scaling it to the ratio of gas/water fluid moduli:

$$P_{\mathrm{g}} = \alpha \frac{\mathcal{K}_{\mathrm{g}}}{\mathcal{K}_{\mathrm{w}}} P_{\mathrm{w}}, \qquad 1 \leq \alpha \leq \frac{\mathcal{K}_{\mathrm{w}}}{\mathcal{K}_{\mathrm{g}}}. \tag{7}$$

The dependence of the coefficient α from fluid saturation may itself be complicated if flow is driven by capillary forces in the rock. We favour a quasi-static description in which the coefficient α is assumed constant and there is no capillary flow taking place. In Santos et al. (1990b) such ideas are extensively discussed and the bounds for the coefficient α are derived using a model of capillary pressure $P_{cap.} = P_{cap.}(S_{\mathrm{w}})$. The relation of α from this model is then calculated:

$$\alpha = \frac{\mathcal{K}_{\mathrm{w}} - S_{\mathrm{w}}(1 - S_{\mathrm{w}})P'_{cap.}(S_{\mathrm{w}})}{\mathcal{K}_{\mathrm{g}} - S_{\mathrm{w}}(1 - S_{\mathrm{w}})P'_{cap.}(S_{\mathrm{w}})}. \tag{8}$$

In this work we postpone the discussion of a pore scale model for capillary pressure $P_{cap.}(S_{\mathrm{w}})$ and instead we will let α be a free parameter. Since α depends on the *change* of capillary pressure from equilibrium which in turn depends on the specific fluid distribution in the pores, we feel justified in not fixing this parameter.

To calculate the bulk modulus in the context of the inclusion model (3), we use the Eshelby (1957) prescription where the elasticity of the bulk is calculated by a sum over inclusions. This summation involves the effective pressure in the inclusions:

$$\begin{aligned} \mathcal{K}_{\mathrm{eff}} &= \mathcal{K}_d + \\ \phi_0^{\ominus}\left(\frac{\mathcal{K}_m}{\sigma_c} + 1\right)\frac{\widetilde{P}}{\sigma} &\ + \ \phi_0^{\odot}\left(\frac{3\mathcal{K}_m}{4\mu} + 1\right)\frac{\widetilde{P}}{\sigma} \end{aligned} \tag{9}$$

where $\mathcal{K}_d$ is the dry and $\mathcal{K}_m$ the grain modulus. It was shown in Chapman et al. (2002) that the above is equivalent to Gassmann's formula in the zero frequency limit. But in order to calculate the ratio $\frac{\widetilde{P}}{\sigma}$ one needs to write eq.

(6) in terms of the effective pressure

$$\widetilde{P} = P_{\mathrm{w}} \left(S_{\mathrm{w}} + \alpha(1 - S_{\mathrm{w}}) \frac{\mathcal{K}_{\mathrm{g}}}{\mathcal{K}_{\mathrm{w}}} \right) \tag{10}$$

where we have expressed the effective pressure in terms of the definition in eq. (7). So, this theory reduces to Gassmann's theory with an effective fluid modulus $\widetilde{K}_f$ given by

$$\frac{S_{\mathrm{w}}}{\mathcal{K}_{\mathrm{w}}} P_{\mathrm{w}} + \frac{(1 - S_{\mathrm{w}})}{\mathcal{K}_{\mathrm{g}}} P_{\mathrm{g}} = \frac{\widetilde{P}}{\widetilde{K}_f}. \tag{11}$$

With the aid of the definition of capillary pressure in eq. (7) and having chosen an effective pressure in eq. (10), we arrive at an expression for the effective fluid modulus that is independent of water pressure. This expression is the central result of our paper:

$$\widetilde{K}_f = \frac{S_{\mathrm{w}} \mathcal{K}_{\mathrm{w}} + \alpha(1 - S_{\mathrm{w}}) \mathcal{K}_{\mathrm{g}}}{S_{\mathrm{w}} + \alpha(1 - S_{\mathrm{w}})}, \qquad 1 \leq \alpha \leq \frac{\mathcal{K}_{\mathrm{w}}}{\mathcal{K}_{\mathrm{g}}} \tag{12}$$

and it provides a fluid mixing law involving the free parameter α that is related to the capillary pressure via eq. (7). The effective modulus obtained in this way is depicted in Figure 2.

3 Equivalence to Brie's model and Discussion

For the limiting values of the parameter α, eq. (12) reduces analytically to an arithmetic ($\alpha = 1$) or harmonic ($\alpha = \frac{\mathcal{K}_{\mathrm{w}}}{\mathcal{K}_{\mathrm{g}}}$) mixing law much like Brie's empirical eq. (1). This means that the parameter α – a measure of capillary pressure – can be matched to the Brie parameter e by combining eq. (1) with eq. (12). The two parameters can be related by the saturation-dependent relationship:

$$e = \frac{\log\left(\frac{S_\mathrm{w}}{S_\mathrm{w}+\alpha(1-S_\mathrm{w})}\right)}{\log(S_\mathrm{w})}. \tag{13}$$

Taking Brie's law at face value eq. (13) shows how the Brie parameter relates to water saturation and the capillary pressure parameter. It should be noted that Brie's law does not analytically reduce to a harmonic averaging law for $e = 40$ but only approximates it. It may therefore be preferable to use eq. (12) over eq. (1) as it is based on a more solid theoretical foundation. In practice the simple relation $\alpha = e^{3/2}$ provides a good agreement between the two theories as can be seen in Figure 3. The comparison reveals that the effective modulus of eq. (12) is stiffer than that of eq. (1) at smaller water saturation and softer at greater water saturation. It remains to be seen if eq. (12) can be correlated to data relating to capillary pressure or even different fluid mixtures since the scaling can be appropriated to any wetting/non-wetting fluid mixture. In Monsen and Johnstad (2005) for instance, the saturating fluids are oil and nitrogen and the observed Brie exponent is larger than normally reported in the literature for brine/gas mixtures.

Since eq. (12) was derived as a static equations, and given its similarity to Brie's law, new interpretations of the Brie relation arise. The ad-hoc character of the Brie parameter was recognised in Dvorkin et al. (1999) and later in Johnson (2001). Resorting to the Hill average, where the saturation of the rock occurs in patches, is not the only plausible foundation for the stiffening of the effective fluid mixture.

Although the phenomenon of mesoscopic dispersion occurring in patchy-saturated rock is not refuted here, we shown that the low frequency limit of the inclusion theory of Chapman et al. (2002), leads to a similar type of fluid stiffening if the saturating fluids have different pressures. Furthermore, the mixing law of eq. (12) is independent of seismic frequency although frequency-dependent effects arising in such inclusion models have been described in Jakobsen and Chapman (2009).

In future work we intend to show that the characteristic frequency where squirt flow becomes important depends on this capillary pressure parameter. But at its low frequency limit, the theory presented here is essentially Gassmann's theory with the modified fluid modulus of eq. (12). This is important as the most popular interpretations of Brie's law have been to tie the parameter e to the characteristic patch size of White (1975) which is tuned to a characteristic frequency associated to the patch size. See Mavko et al. (1998), Pham et al. (2002), Carcione et al. (2003a) and Carcione et al. (2003b) for examples.

Of course, there is no need to exclude the dispersion due to patchy saturation as an explanation for the effective fluid stiffening, or indeed both phenomena combined. Some progress towards this work has been done in Qi et al. (2014) where the capillary stiffening occurs at the boundaries of the patches. But attributing this effect exclusively to the patchy dispersion mechanism is not necessary as demonstrated by this work.

4 Conclusions

We have derived from first principles a simple model describing an effective fluid mixing law similar to the empirical law described in Brie et al. (1995). Our model's contains a parameter with a straightforward interpretation in terms of the capillary pressure at the microscopic scale. Furthermore the model exactly produces both a serial and a parallel law. We have identified a matching condition between our parameter and the free parameter in Brie's model pointing out the difference between the two across the saturation range.

In the theoretical construction of this effective fluid mixing law, the stiffer effective fluid is not due to dispersive effects but it is a consequence of capillary pressure at the pore scale. Our work presents an alternative point of view in which Brie-like effects are not related to dispersion or patch size.

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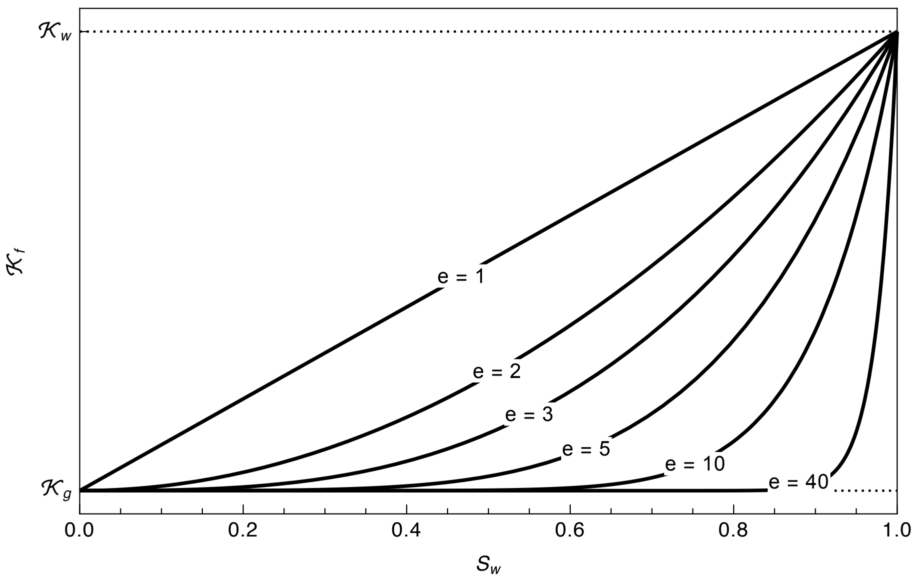

Figure 1: Brie's model for different values of the free parameter e. The relative scaling of the two moduli is $\frac{\mathcal{K}_w}{\mathcal{K}_g} = 100$.

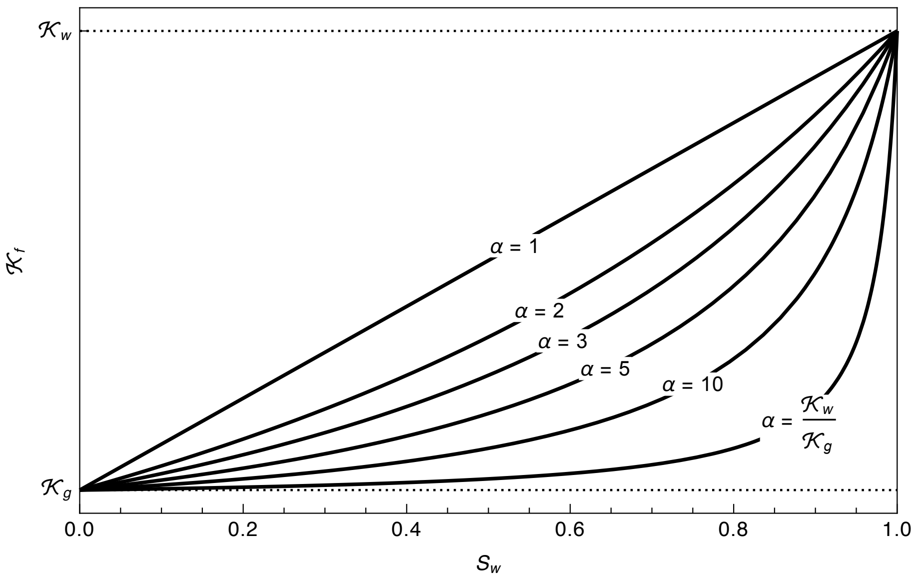

Figure 2: The model of eq. (12) for various values of the parameter α. The relative scaling of the two moduli is $\frac{\mathcal{K}_w}{\mathcal{K}_g} = 100$.

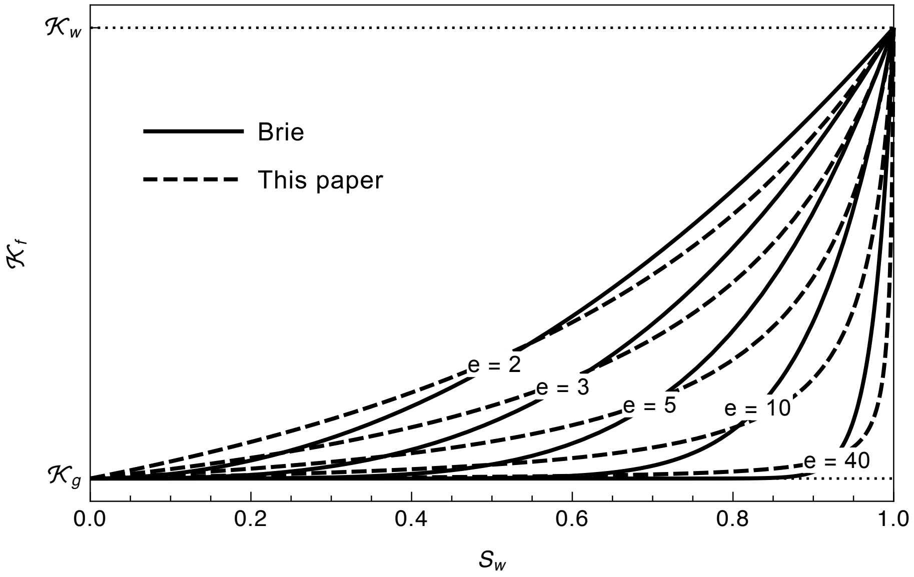

Figure 3: Comparison between Brie's effective fluid modulus of eq. (1) and the effective modulus presented in this paper in (12) for $\alpha = e^{\frac{3}{2}}$. The relative scaling of the two moduli is $\frac{\mathcal{K}_w}{\mathcal{K}_g} = 100$.