EMPIRICAL TESTING FOR BUBBLES DURING THE INTER-WAR EUROPEAN HYPERINFLATIONS

by

KAI-YIN WOO

University of Stirling
May, 2004

Thesis submitted for the degree of Doctor of Philosophy in Economics

Table of Contents

Abstract

In this thesis, I undertake an empirical search for the existence of price and exchange rate bubbles during the inter-war European hyperinflations of Germany, Hungary and Poland. Since the choice of an appropriate policy to control inflation depends upon the true nature of the underlying process generating the inflation, the existence or non-existence of inflationary bubbles has important policy implications. If bubbles do exist, positive action will be required to counter the public's self-fulfilling expectation of a price surge. Hyperinflationary episodes have been chosen as my case study because of the dominant role that such expectations play in price determination. In the literature, there are frequently expressed concerns about empirical research into bubbles. The existence of model misspecification and the nonlinear dynamics in the fundamentals under conditions of regime switching may lead to spurious conclusions concerning the existence of bubbles. Furthermore, some stochastic bubbles may display different collapsing properties and consequently appear to be linearly stationary. Thus, the evidence against the existence of bubbles may not be reliable. In my thesis, I attempt to tackle the above empirical problems of testing for the existence of bubbles using advances in testing procedures and methodologies. Since the number of bubble solutions is infinite in the rational expectations framework, I adopt indirect tests,

rather than direct tests, for the empirical study. From the findings of my empirical research, the evidence for stationary specification errors and the nonlinearity of the data series cannot be rejected, but the evidence for the existence of price and exchange rate bubbles is rejected for all the countries under study. It leads to the conclusion that the control of the inter-war European hyperinflations was attributable to control of the fundamental processes, since the dynamics of prices and exchange rates for these countries might not be driven by self-fulfilling expectations.

Acknowledgement

For the completion of this thesis, I owe a great deal to many people. I am most indebted to my supervisors, Dr. Yue Ma and Prof. Bob Hart, who gave me invaluable comments, encouragement and advice on the organization of this thesis. I have also greatly appreciated the comments and advice on how to improve the quality of my thesis from my internal and external examiners, Dr. Ron Shone, and Prof. Martin Sola, respectively. I am also grateful to Dr. Hing-Lin Chan who provided me with data sets, econometric packages and all other materials necessary for my research work. His critical evaluations and suggestions on some chapters of my thesis enlightened me very much.

In addition, I would like to express my sincere thanks to Chulsoo Kim and Tom Engsted for their opinions, which were essential for an understanding of the orthogonality tests and of the cointegration tests, when they were invited to attend seminars in Hong Kong. I also appreciate Kyung-So Im, Jeremy Piger and In Choi, who sincerely sent me their own programs for my research works. Moreover, I express my gratitude to Bruce Hansen, Chang-Jin Kim and Neil Haldrup, who let practitioners download their Gauss codes for research uses. I am grateful for Hans-Martin Krolzig, who let Ox users to download his Ox package, MSVAR, for estimation of regime-switching models. Moreover, I benefited from many members of the mailing lists of GAUSS, RATS and Ox, who had provided me with codes, and solutions to the problems of econometric programming. Without their help, many works of econometric estimation and simulation in my thesis cannot be finished.

Besides, I feel thankful to Miss Susan Sprengeler and Ms. Gillian Gaston for proofreading my thesis. Finally, my gratitude is extended to the Hong Kong Shue Yan College for granting me financial support for my studies.

Needless to say, all the errors and omissions that may remain are my sole responsibility.

CHAPTER ONE INTRODUCTION

1.1 Purposes

Price bubbles are defined as explosive processes of asset prices generated by self-fulfilling expectations independently of market fundamentals. The existence of bubbles represents a possible explanation for the deviation of asset prices from the underlying fundamentals. There are many historical examples of incidents that could be considered from the evidence as being self-fulfilling bubbles. Famous classic cases include the tulip-mania in the Netherlands from 1634 to 1637, 'the Mississippi bubble' in France in 1719-1720 and the contemporaneous and related 'South Sea bubbles' in Britain (Garber, 1989 and 1990). In addition, the US stock market crashes of 1929, 1987 and 2000, the Asian stock market slump of 1997, as well as the Japanese property market crash in the 1990s, are usually deemed as recent examples of bubble bursts. Keynes (1936) considered that the stock prices in the 1920s might not be governed by an objective view of fundamentals but by "what average opinion expects average opinion to be". The study of bubbles has attracted much research interest because bubble bursts will normally have negative wealth effects and create economic confusion (Blanchard and Watson, 1982).

According to Kindleberger (1987), a bubble is defined loosely as a sharp rise in price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers, who are generally speculators interested in profits from trading in the asset rather than its use of earning capacity; the rise is usually followed by a reversal of expectations and a subsequent sharp decline in price often resulting in financial crisis.

In the literature, general equilibrium arguments can be found about the theoretical restrictions concerning the existence of bubbles and the effects of bubbles on the economy. For instance, Tirole (1982) considers that rational bubbles are ruled out when there exists a finite number of agents in the market. If the number of agents is infinite, bubble existence will become possible (Tirole, 1985, Weil, 1989). On the other hand, within a monetary framework, Obsteld and Rogoff (1983) assert that price bubbles can be ruled out during hyperinflationary episodes if the government guarantees a probable minimal redemption value for the currency in units of capital. Nevertheless, the analysis in this thesis focuses on the empirical examination of the existence of a bubble. This is because, while the existence of bubbles cannot always be proven theoretically, it may be reasonable to rely upon econometric methods to detect them.

Although the empirical search for evidence of bubbles has largely focused on capital markets, I contend that the empirical investigation of inflationary bubbles is equally important. Since the choice of an appropriate policy to reduce the inflation rate may very much depend on the true nature of the underlying process generating the inflation, the existence of inflationary bubbles has far-reaching policy implications. If inflationary bubbles are not present in the observed price series, then it is only necessary to take control of the market fundamentals, by such means as the restrictive control of money supply growth and the reduction of fiscal deficits. If, however, this inflation has a stubborn self-sustaining momentum and is thus being driven by a bubble phenomenon, then positive action will be required to work on the expectation mechanism to shock expectations off the speculative bubble path (Funke *et al.* 1994). For instance, it would require the government to commit itself to a change in its policies for controlling fiscal deficits and money growth in a way that is sufficiently binding and convincing for them to be widely believed. Further, since bubbles are associated with self-fulfilling prophecies, it is reasonable to deduce that if bubbles do actually occur in the data, they are more likely to be observed when the expected future market price is an important factor determining the current market price level. During hyperinflation, expectation plays a dominant role in the determination of the asset price. Hence, it is believed that hyperinflationary episodes provide fascinating environments for the empirical study of bubbles (Flood and Garber, 1980b). The classic examples include the inter-war European hyperinflations of Germany, Hungary and Poland. Sargent (1982) provides a detailed description of how the hyperinflation in these countries was stopped. It has been found that the government authorities stopped inflation by announcing a binding and credible policy regime change and at the same time taking control of market fundamentals. Thus, the resulting control of inflation cannot explain fully the true nature of the hyperinflation that occurred. It is suggested that econometric methods could be used to test for the presence of inflationary bubbles during these classic hyperinflationary episodes.

It is also to be noted that a floating exchange rate system was first implemented in European countries during the 1920s following World War I. According to Okina (1984), if price bubbles occur, and the purchasing power parity is not violated, bubbles in the nominal exchange rate will also appear and are reflected in the form of the price bubbles. The country's external competitiveness, therefore, would not be adversely affected. On the other hand, when price bubbles are not present but exchange rate bubbles do exist, the nominal exchange rate bubbles are represented by an explosive deviation from the purchasing power parity, and real exchange rate bubbles will appear as well. With the ups and pops of real exchange rate fluctuations, the export sectors will suffer serious consequences and will not recover quickly even when the bubbles finally burst. It is important, therefore, to check for the presence of both price and exchange rate bubbles over the same estimation periods.

The purpose of this thesis is to undertake empirical research into both the price and exchange rate bubbles. I have chosen the inter-war European hyperinflations of Germany, Hungary and Poland for my case study, because, the data series for both prices and the free market exchange rates are available, and they have been widely discussed in the literature.

1.2 Outline of the Thesis

The thesis is divided into seven chapters, which are structured as follows:

Chapter Two introduces specifications and solutions of the Cagan hyperinflation models. Both the fundamental and bubble solutions of the Cagan models under rational expectations will be derived. In the rational expectations framework, the specification of a bubble process is related to an arbitrary martingale. For any value of a bubble coefficient, there exists an infinite set of bubble processes because there also exists an infinity of possible martingales with respect to a given sequence of information sets. Some examples of theoretical bubble specifications will be explored. In addition, several bursting bubble specifications will be illustrated. Owing to the problems of multiple solutions, indirect testing methodologies that do not require the specification of particular forms of bubble are more appropriate and have been employed for identification of bubbles.

Chapter Three provides a brief description of the data series for the inter-war European hyperinflations of Germany, Hungary and Poland. In addition, since the stochastic properties of the data series will affect the econometric methods to be adopted and the economic interpretations of the empirical results in subsequent chapters, I also investigate the stochastic properties of the observed variables. Using structural time series modeling techniques, I extract the unobserved structural components of the observed data variables and examine the integration orders of the data on the basis of the specifications of the structural time series components.

In the existing literature, three main concerns have been aired about the empirical investigation of bubbles. First of all, most of the previous empirical studies of bubbles have assumed at the outset that the models they use are correctly specified. This means that if the models are in fact misspecified, this may be falsely interpreted as evidence for the existence of bubbles. The bubble test is, however, a joint test for both bubble existence and correct model specification. The evidence for no bubbles implies that no bubbles are present and that the model under study is correctly specified. Hence, the appropriate testing procedures and econometric methods should be effective enough to separate model misspecification from the evidence of bubbles. Secondly, there is a problem of observational equivalence between expected future changes in economic fundamentals and bubbles. When the Governments attempted to bring runaway inflation under control by enforcing monetary reforms during the hyperinflationary episodes in the 1920s, the nonlinear movements of price or exchange rate series that are caused by the possible regime shifts in underlying fundamentals may often be misunderstood as representing a bubble path. Thirdly, it is found that the stochastic bubble process, as illustrated in Chapter Two, exhibits an explosive dynamic path over the expanding phase of the bubble process only, but not over the whole sample period. Consequently, standard econometric methods will be biased towards the rejection of bubble existence. In subsequent chapters, I apply advances in econometric procedures and methodologies to handle the above empirical issues of bubble detection in different ways.

In Chapter Four, I design a set of orthogonality testing procedures for empirical study, which can help separate tests on model specification from tests for bubbles in a more rigorous manner. The orthogonality testing procedure is expected to detect any kinds of bubble process that are not orthogonal to information sets. I employ the fully modified econometric methodologies to conduct the orthogonality tests, which are developed under the assumption of the linear data generation process. Hence, I restrict the empirical analysis on pre-reform samples as has been done in the previous literature. This chapter develops the ideas contained in my work published in Progress in Economic Research (Chapter Two) and the International Review of Economics and Finance.

In order to extend the empirical analysis to cover the excluded observations of monetary reforms and to permit a comparison with the evidence for bubbles contained in Chapter Four, in Chapter Five I employ the threshold cointegration method for bubble detection. Since the threshold cointegration methodology can be used to test simultaneously for the existence of nonstationary roots and for threshold nonlinearity in two regimes, it is expected to be robust to the presence of both a nonlinear switching process and a stochastic bubble. I also choose traditional linear cointegration tests and the cointegrating RALS-ADF test for carrying out the comparison study. Moreover, by conducting cointegration analyses between the real money balances and price changes, and subsequently between the real money balances and money growth rate, the existence of a bubble can be separated from the model misspecification. In addition, a comparison of the orthogonality tests and the cointegration tests in detecting bubbles is made.

Further, while the regime-switching behaviour of market fundamentals discussed in Chapter Five is restricted because it depends on an observed threshold value, the switching process described in Chapter Six is specified to depend on unobservable Markov-switching states generated by a first-order Markov chain. The probability law that governs the Markov-switching states is more flexible in that it allows the observed data to determine the specific form of the nonlinearities, which are consistent with the sample information. Following the same cointegration-testing procedure as in Chapter Five, I adopt the Markov-switching cointegrating ADF method in Chapter Six, in order to simultaneously model the Markovian regime shifts in underlying fundamentals and to test for bubble existence. This method is considered to be effective in identifying nonstationary dynamics from the stochastic bubbles.

Finally, Chapter Seven summarizes the major findings of the empirical research, assesses the suitability of the econometric methods for carrying out tests for the existence of bubbles, and contains the concluding remarks.

CHAPTER TWO SPECIFICATIONS AND SOLUTIONS OF THE CAGAN MODEL

2.1 Introduction

In this chapter, I will briefly describe the specifications of the Cagan model under rational expectation in which the price and exchange rate series are expressed in first-order linear difference equations. The particular and the homogenous solutions to the Cagan model can then be derived. The particular or fundamental solution characterizes a unique dynamic movement of an underlying fundamental process. Several explicit representations of the fundamental solution will be explored. The homogenous or bubble solution is non-unique in a rational expectations framework. I attempt to specify some examples of bubble solution with different dynamic properties. Also, several bursting bubble specifications will be illustrated. It is concluded that the problems of multiple solutions make indirect tests more attractive than direct tests for bubble detection. In addition, the general solution, which is just the sum of particular and homogenous solutions, will be discussed. Hence, the bubble paths are characterized as any deviations of the general solution from the fundamental solution when the model is specified correctly. The remainder of the chapter proceeds as follows: The specifications of the Cagan's hyperinflation models will be explored in Section 2. Sections 3 and 4 explore the particular and homogenous solutions respectively. The general solution is discussed in Section 5. A summary is offered in the final section. Proofs of some equations are shown in the appendices.

2.2 Specifications of the Cagan Model

Money balances are held as a reserve of purchasing power for contingencies. The desired real money balances depend upon several variables including real wealth, real income, and the expected opportunity cost of holding money. The expected cost of holding money refers to the difference between the expected monetary return on holding cash balance and on substitutes of local currency. The money return on cash balance is negligible and is usually assumed to be zero. Therefore, to the extent that money is held as a substitute for financial assets, the expected cost of holding money includes the expected interest rate and capital gain yield of holding those financial assets. To the extent that money is held as substitutes for non-perishable consumers' goods, the expected cost of holding money is the expected rate of depreciation in the real value of money, or equivalently, the rate of inflation. According to Cagan (1956), hyperinflation refers to the rise in prices at a rate at least equal to 50% per month and only the expected inflation rate accounts for the drastic fluctuations in real cash balances during hyperinflation, with all other variables being considered to have minor effects on desired cash balance. Cagan (1956) assumes the expectation mechanism to be adaptive. Sargent and Wallace (1973), Sargent (1977) and Salemi and Sargent (1979), however, introduce the rational expectation hypothesis of Muth (1961) to the Cagan model. Mathematically, the linear form of the Cagan model under rational expectations and instantaneous clearing in the money market is given as:

$$\boldsymbol{M}_t - \pi_{1,t} = \alpha_1 + \beta_1 \boldsymbol{E}_t(\Delta\pi_{t+1}) + \boldsymbol{u}_{1,t} \qquad (2.1)$$

where $\boldsymbol{M}_t$ is the natural logarithm of the money stock at time t, $\pi_{1,t}$ is the natural logarithm of the price level, $\boldsymbol{E}_t(.)$ denotes the mathematical expectations operator conditional on information set Ω_t, α_1 is a constant, β_1 is the semi-elasticity of real money demand with respect to the expected inflation rate and $\boldsymbol{u}_{1,t}$ refers to a money demand disturbance term representing all deviations from the exact Cagan model under rational expectations such as demand velocity shocks and all other omitted real variables. Theoretically, the value of β_1 should be negative because money holders will substitute consumers' goods for money when the real value of money is expected to fall or the expected inflation rate rises.

Since local currency loses its value very rapidly during hyperinflation, foreign currency balances are often held in order to perform the functions of a medium of exchange and a store of value. Even if foreign currencies are held merely as a store of value, they are often converted back into domestic money and then goods at a later time. Hence, the substitution between domestic money and goods can occur, directly or indirectly, via foreign currencies (Moosa, 1999). Such phenomenon of currency substitution is documented in Sargent (1982) for the inter-war European hyperinflations. In light of this, it is appropriate to replace the future inflation rate in Eq.(1) with the expected depreciation rate of domestic currency to represent the cost of holding domestic money balance.[1] If I further assume that the purchasing power parity (PPP) relationship holds and that all the foreign money demand determinants, for example, foreign interest rates and income levels, are assumed to be constant, the real money balance represented by Eq.(2.1) can be alternatively expressed as:

$$M_t - \pi_{2,t} = \alpha_2 + \beta_2 E_t(\Delta\pi_{2,t+1}) + u_{2,t} \qquad (2.2)$$

[1] Frenkel (1977 and 1979) estimated the Cagan money demand using forward premium and expected depreciation rate as a measure of expected cost of holding local currency during the German hyperinflation.

where $\pi_{2,t}$ is the natural logarithm of the exchange rate measured as the value of domestic currency per unit of foreign currency, $\Delta\pi_{2,t+1}$ represents the exchange rate change at time $t+1$, α_2 is an intercept, β_2 is the semi-elasticity of currency substitution between domestic and foreign currency and $u_{2,t}$ is a measure of model noise from the linear exact Cagan model under rational expectations that include all the domestic and foreign money demand shocks and omitted real-side determinants.

Re-arranging Eq. (2.1) and Eq. (2.2) in terms of $\pi_{j,t}$ ($j = 1, 2$) gives:

$$\pi_{j,t} = \frac{\alpha_j}{\beta_j - 1} - \frac{M_t}{\beta_j - 1} + \frac{\beta_j}{\beta_j - 1} E_t(\pi_{j,t+1}) + \frac{u_{j,t}}{\beta_j - 1} \qquad j = 1, 2. \qquad (2.3)$$

Eq.(2.3) is expressed as a first-order dynamic linear difference equation with rational expectations. The future expectation and the current variables are determined simultaneously. The general solution of (2.3) is the sum of a particular solution and a homogenous solution.

2.3 Particular Solution

For sake of notional simplicity, I eliminate the subscript j in subsequent equations. By recursively substituting forward for $E_t\,(\pi_{t+1+i})$ and using the law of iterated expectations, I obtain:

$$\pi_t = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i} - u_{t+i}) + \lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i})\ (2.4).$$

When $|\frac{\beta}{\beta-1}|< 1$, the transversality condition:

$$\lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) = 0, \qquad (2.5)$$

is then satisfied. Under this circumstance, the solution of π_t is given by:

$$\pi_t^f = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i} - u_{t+i}) \qquad (2.6)$$

The expression of π_t^f represents a forward-looking particular solution or the fundamental solution to the Cagan models (2.1) and (2.2) under rational expectations, which is determined by the present discounted value of expected levels of the market fundamentals, $(M_{t+i} - u_{t+i})$, for all i $\geqq$ 0. If the expectation of $(M_t - u_t)$ grows at a constant rate g, the infinite sum, π_t^f, will converge when

$(1+g) < \frac{\beta - 1}{\beta}$ or $g < |\frac{1}{\beta}|$.[2]

By assuming special stochastic processes for the sequence of M_t and u_t, π_t^f can be written in explicit manners. Let's define X_t as $\frac{1}{1-\beta}M_t$ or $\frac{-1}{1-\beta}u_t$. Gourieroux *et al.* (1982) considers the ARMA solutions of the Cagan model. Assume that $|\frac{\beta}{\beta-1}|<1$ and X_t admits an ARMA (p,q) representation, that is, $\Psi(L)X_t = \theta(L)\eta_t$, where L is a lag operator, $\Psi(L) = 1 - \Psi_1 L - \ldots - \Psi_p L^p$, $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ and η_t is a white noise. Then, the present discounted value of X_t, $\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(X_{t+i})$ can be explicitly written as a unique stationary ARMA solution:

$$\{\frac{1}{(L-b)}[L - \frac{b\theta(b)\Psi(L)}{\Psi(b)\theta(L)}]\}X_t, \tag{2.7}$$

where b is defined as $\frac{\beta}{\beta-1}$.

However, many economic variables exhibit nonstationarity. If X_t follows random walk with drift and linear time trend, that is, $\Delta X_t = \mu + \omega t + \eta_t$, then the present discounted value of X_t is written as:[3]

---

[2] In fact, it is similar to the case of the constant dividend growth model in which the growth rate of dividend is no larger than the discount rate.

[3] The steps of proof are shown in Appendix 2.1.

$$(1-\beta)X_t + \beta(\beta-1)\{\mu + \omega[(1-\beta)+t]\}, \tag{2.8}$$

During hyperinflation, the economic variables are likely to contain double unit roots (Haldrup, 1998). I then consider the case of double unit roots with drift and polynomial time trend. Assume that $\Delta^2 X_t = \mu + \omega_1 t + \omega_2 t^2 + \eta_t$. The infinite sum of X_t will be represented as:[4]

$$(1-\beta)X_t + (1-\beta)^2 \Delta X_t - \beta(1-\beta)^2\{\mu + \omega_1(1-\beta) + \omega_2[(1-\beta)^2 - \beta(1-\beta) + t^2] + t[\omega_1 + 2\omega_2(1-\beta)]\} \tag{2.9}$$

Since the fundamentals, M_t and u_t, may be represented by different stochastic processes, for instance, M_t is usually I(2) and u_t is either I(1) or I(0), the explicit representation of the fundamental solution, π_t^f, will be written as a combination of Eqs. (2.7), (2.8) and (2.9).[5]

2.4. Homogenous Solution

The homogenous solution of (2.3) denoted by π_t^h is equal to the general solution of the homogenous counterpart as follows:

---

[4] The work of proof is illustrated in Appendix 2.2.

[5] The conditions for the particular solutions of the Cagan model to be unique are documented in Broze and Szafarz (1991) and Broze *et al.* (1995).

$$\pi_t^h = b^{1+i} E_t(\pi_{t+1+i}^h), \qquad i \geqq 0 \tag{2.10}$$

Multiplying both sides of Eq.(2.10) by b^t obtains:

$$b^t \pi_t^h = b^{t+1+i} E_t(\pi_{t+1+i}^h), \tag{2.11}$$

Gourieroux *et al.* (1982) derive the homogenous solution, π_t^h, by using the martingale process. Let's define m_t as $b^t \pi_t^h$, and the stochastic process of m_t satisfies the martingale property such that $E_t(m_t) = m_t$, and $E_t(m_{t+i}) = m_t$, for all i > 0. The homogenous solution, π_t^h, is represented in terms of the martingale process:

$$\pi_t^h = \frac{m_t}{b^t} \tag{2.12}$$

Any arbitrary martingale process, m_t, can be considered as a component of π_t^h. It implies the existence of multiple solutions for the Cagan models under rational expectations. Since $E_t(\pi_{t+1}^h) = E_t(\frac{m_{t+1}}{b^{t+1}}) = \frac{1}{b}(\frac{m_t}{b^t})$ where $|b| < 1$, the stochastic process of π_t^h follows a submartingale such that:

$$E_t(\pi_{t+1}^h) = \frac{\pi_t^h}{b} > \pi_t^h. \tag{2.13}$$

Therefore, π_t^h satisfies a bubble process that explodes in expected value and it can be interpreted as a bubble solution, B_t. Also, when Eq. (2.11) and (2.12) are substituted into the transversality condition of (2.5), it implies $E_t(m_{t+1+i}) = m_t = 0$, for $i \to \infty$. Consequently, the transversality condition of (2.5) implies the nonexistence of B_t.

The stochastic unit root process suggested by Granger and Swanson (1997) can be generalized to the martingale process, m_t:

$$m_t = q_t m_{t-1} + \omega_t, \qquad (2.14)$$

where $E_t(q_{t+1}) = E_{t-1}(q_t) = 1,\ E_t(\omega_{t+i}) = 0,$ for all $i > 0$.

Suppose that $x_t \sim N(\mu_x, \sigma_x^2)$. For an arbitrary λ, the moment-generating function of a normally distributed variable, x_t, is given by $E(\exp(\lambda x_t))$ = $\exp(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)$. Hence, q_t is represented by $\exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)]$. Dividing Eq.(2.14) by b^t yields the following general bubble specification:

$$B_t = \frac{(q_t B_{t-1})}{b} + \frac{\omega_t}{b^t} \qquad (2.15a)$$

$$= \exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\, B_{t-1} + \frac{\omega_t}{b^t} \qquad (2.15b)$$

By restricting underlying parameters of bubble process given by (2.15b) such as λ and μ_x, there are different theoretical bubble specifications with particular stochastic properties to be derived (Salga, 1997). I illustrate them with further modifications and refinements.

2.4.1. $\lambda = 0$

Let's first assume that $\lambda = 0$, the resulting bubble process will be obtained as follows:

$$\begin{aligned} B_t &= \frac{B_{t-1}}{b} + \frac{\omega_t}{b^t} \\ &= \frac{B_0}{b^t} + \frac{\sum_{i=1}\omega_{t=i}}{b^t} \end{aligned} \qquad (2.16)$$

Since the above bubble process is driven by time only, it is known as pure time-driven bubble process. As $t \rightarrow \infty$, the time-driven bubble must converge toward infinity with $|b| < 1$ and its dynamics must then be asymptotically unstable. In particular, if m_t is a constant, the sequence of ω_t in Eq.(2.14) will become zero. Consequently, B_t is represented by $\frac{B_0}{b^t}$ only, which is known as the deterministic bubble.

2.4.2. $\lambda \neq 0$ and $\mu_x + \frac{1}{2}\lambda\,\sigma_x^2 = 0$

If $\lambda \neq 0$ and $\mu_x + \frac{1}{2}\lambda\,\sigma_x^2 = 0$, it then implies $\mu_x \neq 0$ because $\lambda = \frac{-2\mu_x}{\sigma_x^2} \neq 0$. The bubble process can be specified as:

$$B_t = \exp\left(\frac{-2\mu_x}{\sigma_x^2} x_t - \ln b\right) B_{t-1} + \frac{\omega_t}{b^t} \qquad (2.17a)$$

Suppose that $x_t = w_t - w_{t-1} = \mu_x + \varepsilon_{xt}$, where $\varepsilon_{xt} \sim N(0,\sigma_x^2)$. Replacing x_t by $w_t - w_{t-1}$, substituting one period forward for B_{t+1} and re-arranging yield:[6]

$$B_t = \exp\left[\frac{-2\mu_x}{\sigma_x^2} w_t - (\ln b)t\right] \qquad (2.17b)$$

where $\ln b < 0$ since $b < 1$ or $\beta < 0$.

Assume that w_t represents a vector of underlying I(1) fundamental variables in the model. The stochastic bubble process of (2.17b) thus depends upon both time and the underlying fundamental process. Further, by recursively forward substitution, $w_t = w_0 + \mu_x t + \sum_{i=1}^{t} \varepsilon_{xt-i}$, the bubble process given by (2.17b) can be alternatively written as:

---

[6] Appendix 2.3 proves the general specification of bubble when $x_t = w_t - w_{t-1}$.

$$B_t = \exp\{\frac{-2\mu_x}{\sigma_x^2} w_0 - [\frac{2\mu_x}{\sigma_x^2}(\mu_x + \frac{\sum_{i=1}^{t}\varepsilon_{xt-i}}{t}) + \ln b]t\} \qquad (2.17c)$$

Given that $\frac{\sum_{i=1}^{t}\varepsilon_{xt-i}}{t} \rightarrow 0$ as $t \rightarrow \infty$, B_t will converge toward zero when $\frac{-2\mu_x}{\sigma_x^2}\mu_x < \ln b < 0$. As a result, the dynamics of B_t is asymptotically stable. The divergent bubble process driven by the time component, $\exp[-(\ln b)t]$, would be somehow offset by the fundamental component, $\exp[\frac{-2\mu_x}{\sigma_x^2}\mu_x]$, to a certain degree. Hence, the inclusion of the fundamental-dependent component may help stabilize bubble dynamics and exhibit more dynamic properties of the bubble process (Ikeda and Shibata, 1992 and 1995).[7]

One special case is that $\mu_x + \frac{1}{2}\lambda\,\sigma_x^2 = 0$ but λ is restricted to be 1, $\mu_x = -\frac{\sigma_x^2}{2} < 0$, then the bubble process of (2.17a) and (2.17c) will be simplified to be:

$$B_t = \exp(x_t - \ln b)\, B_{t-1} + \frac{\omega_t}{b^t} \qquad (2.18a)$$

$$= \exp\{w_0 + (\mu_x + \frac{\sum_{i=1}^{t}\varepsilon_{xt-i}}{t} - \ln b)t\} \qquad (2.18b)$$

[7] Ikeda and Shibata (1992 and 1995) however derived the specifications of fundamental-dependent bubbles in a continuous-time framework.

Similarly, the asymptotic dynamics of bubble process given by (2.18b) depend on the sign of $(\mu_x - \ln \boldsymbol{b})$. If $\mu_x < \ln \boldsymbol{b} < 0$ or $\exp(\mu_x) < \boldsymbol{b} < 1$, $\boldsymbol{B}_t$ will converge toward zero and is then asymptotically stable.

2.4.3. $\lambda \neq 0$ and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K) = 0$

Let $\ln \boldsymbol{b} = (K + H) < 0$, where K and H are arbitrary constants. Assume that $\lambda \neq 0$ and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K) = 0$ with λ_1 and λ_2 being the two characteristic roots. Hence,

$$\boldsymbol{B}_t = \exp(\lambda_1 \boldsymbol{w}_t - H\boldsymbol{t}) \tag{2.19a}$$

or $$\boldsymbol{B}_t = \exp(\lambda_2 \boldsymbol{w}_t - H\boldsymbol{t}) \tag{2.19b}$$

where $$\lambda_1 = \frac{-\mu_x + [\mu_x^2 - 4(\frac{1}{2})\sigma_x^2 K]^{1/2}}{2(\frac{1}{2})\sigma_x^2} = \frac{-\mu_x + (\mu_x^2 - 2\sigma_x^2 K)^{1/2}}{\sigma_x^2} \tag{2.20a}$$

$$\lambda_2 = \frac{-\mu_x - (\mu_x^2 - 2\sigma_x^2 K)^{1/2}}{\sigma_x^2} \tag{2.20b}$$

I first consider the case of $\mu_x \neq 0$. While $\mu_x \neq 0$ and $(\mu_x^2 - 2\sigma_x^2 K) = 0$, then, $K = \frac{\mu_x^2}{2\sigma_x^2} > 0$, H must be negative. From (2.20a) and (2.20b), $\lambda_1 = \lambda_2 = -\frac{\mu_x}{\sigma_x^2}$. The bubble process of (2.19a) and (2.19b) will be written as:

$$B_t = \exp(-\frac{\mu_x}{\sigma_x^2} w_t - Ht) \quad (2.21a)$$

$$= \exp\{-\frac{\mu_x}{\sigma_x^2} w_0 - [\frac{\mu_x}{\sigma_x^2}(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t}) + H]t\}, \quad (2.21b)$$

When $-\frac{\mu_x^2}{\sigma_x^2} < H$, which implies $\exp(-\frac{\mu_x^2}{2\sigma_x^2}) < b$, the bubble process will converge towards zero asymptotically.

On the other hand, when $(\mu_x^2 - 2\sigma_x^2 K) \neq 0$, then it can be seen that $\lambda_1 \neq \lambda_2$. The bubble process of (2.19a) or (2.19b) or any linear combination of them still satisfies the submartingale process of (2.13). Let's define A_1 and A_2 as two arbitrary constants. The linear combination of bubble process (2.19a) and (2.19b) is given as:

$$B_t = A_1 \exp(\lambda_1 w_t - Ht) + A_2 \exp(\lambda_2 w_t - Ht) \quad (2.22a)$$ [8]

$$= A_1 \exp(\lambda_1 w_0 + [\lambda_1 (\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t}) - H]t) +$$

$$A_2 \exp(\lambda_2 w_0 + [\lambda_2 (\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t}) - H]t) \quad (2.22b)$$

[8] Appendix 2.4 provides the proof that the bubble solution (2.22a) can satisfy the submartingale property.

In particular, while $(\mu_x^2 - 2\sigma_x^2 K) > 0$, or $K < \frac{\mu_x^2}{2\sigma_x^2}$, then $\lambda_1 > \lambda_2$ and the λ values are real numbers. The stochastic stability of bubbles specified by (2.22b) depends upon whether $\lambda_i \mu_x < H$ or $\exp(\lambda_i \mu_x + K) < b$ for all i = 1,2.

Moreover, if $(\mu_x^2 - 2\sigma_x^2 K) < 0$, or $K > \frac{\mu_x^2}{2\sigma_x^2} > 0$, then H must be negative and the λ values contain imaginary numbers:

$$\lambda_1 = \frac{-\mu_x + [(2\sigma_x^2 K - \mu_x)^{1/2}]i}{\sigma_x^2}, \tag{2.23a}$$

$$\lambda_2 = \frac{-\mu_x - [(2\sigma_x^2 K - \mu_x)^{1/2}]i}{\sigma_x^2}, \tag{2.23b}$$

where i is an imaginary number, $\sqrt{-1}$. Let's define $h_1 = \frac{-\mu_x}{\sigma_x^2}$, and $h_2 = \frac{[(2\sigma_x^2 K - \mu_x)^{1/2}]}{\sigma_x^2}$, so that $\lambda_1, \lambda_2 = h_1 \pm h_2 i$. The bubble process is specified as:[9]

$$B_t = \exp(h_1 w_t - Ht)[A_3 \cos(h_2 w_t) + A_4 \sin(h_2 w_t)] \tag{2.24a}$$

$$= \exp\{h_1 w_0 + h_1 (\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})t - Ht\}[A_3 \cos(h_2 w_t) + A_4 \sin(h_2 w_t)] \tag{2.24b}$$

Under this circumstance, the bubble process of (2.24a) and (2.24b) can exhibit cyclical patterns. While $\frac{-\mu_x^2}{\sigma_x^2} < H < 0$, or $\exp(\frac{-\mu_x^2}{\sigma_x^2} + K) < b$, the cyclical

[9] Appendix 2.5 offers the detailed steps of proof.

dynamics of $\boldsymbol{B}_t$ is asymptotically damped.

Now, I consider the case of $\mu_x = 0$. When $\mu_x = 0$, then K must be negative since $(\frac{1}{2}\lambda^2\sigma_x^2 + \mathrm{K}) = 0$. Also, $\lambda_1 = \frac{(-2\sigma_x^2\mathrm{K})^{1/2}}{\sigma_x^2} = \frac{(-2\mathrm{K})^{1/2}}{\sigma_x} > 0$, $\lambda_2 = -\frac{(-2\mathrm{K})^{1/2}}{\sigma_x} < 0$. The bubble process will be specified as:

$$\boldsymbol{B}_t = \mathrm{A}_1 \exp[\frac{(-2\mathrm{K})^{1/2}}{\sigma_x}\boldsymbol{w}_t - \mathrm{H}\boldsymbol{t}] + \mathrm{A}_2 \exp[-\frac{(-2\mathrm{K})^{1/2}}{\sigma_x}\boldsymbol{w}_t - \mathrm{H}\boldsymbol{t}] \tag{2.25a}$$

$$= \mathrm{A}_1 \exp(\lambda_1 \boldsymbol{w}_0 + [\lambda_1(\frac{\sum_{i=1}^{t}\varepsilon_{\mathrm{xt-i}}}{\boldsymbol{t}}) - \mathrm{H}]\boldsymbol{t}) + \mathrm{A}_2 \exp(\lambda_2 \boldsymbol{w}_0 + [\lambda_2(\frac{\sum_{i=1}^{t}\varepsilon_{\mathrm{xt-i}}}{\boldsymbol{t}}) - \mathrm{H}]\boldsymbol{t}) \tag{2.25b}$$

On condition that H > 0, the bubble process (2.25b) will converge towards zero as $\boldsymbol{t} \rightarrow \infty$.

2.4.4. $\lambda \neq 0$ and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln \boldsymbol{b}) = 0$

Suppose that $\lambda \neq 0$ and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln \boldsymbol{b}) = 0$, the bubble process will be purely driven by a fundamental process:

$$\boldsymbol{B}_t = \exp(\lambda_1 \boldsymbol{w}_t) \tag{2.26a}$$

or
$$\boldsymbol{B}_t = \exp(\lambda_2 \boldsymbol{w}_t) \tag{2.26b}$$

where $\lambda_1 = \dfrac{-\mu_x + (\mu_x^2 - 2\sigma_x^2 \ln \boldsymbol{b})^{1/2}}{\sigma_x^2}$ (2.27a)

$\lambda_2 = \dfrac{-\mu_x - (\mu_x^2 - 2\sigma_x^2 \ln \boldsymbol{b})^{1/2}}{\sigma_x^2}$ (2.27b)

Given the fact that ln $\boldsymbol{b}$ < 0, when $\mu_x \neq 0$, it is impossible for $(\mu_x^2 - 2\sigma_x^2 \ln \boldsymbol{b})$ = 0 and $(\mu_x^2 - 2\sigma_x^2 \ln \boldsymbol{b})$ < 0, which imply that ln $\boldsymbol{b}$ = $\dfrac{\mu_x^2}{2\sigma_x^2} > 0$ and ln $\boldsymbol{b}$ > $\dfrac{\mu_x^2}{2\sigma_x^2} > 0$ respectively. The only possible case is given by $(\mu_x^2 - 2\sigma_x^2 \ln \boldsymbol{b}) > 0$, or ln $\boldsymbol{b}$ < 0 $< \dfrac{\mu_x^2}{2\sigma_x^2}$. Then, $\lambda_1 > \lambda_2$ and the λ values are real numbers. The specification of the bubble process will be written as:

$$\boldsymbol{B}_t = \mathrm{A}_1 \exp(\lambda_1 \boldsymbol{w}_t) + \mathrm{A}_2 \exp(\lambda_2 \boldsymbol{w}_t) \quad (2.28a)$$

$$= \mathrm{A}_1 \exp(\lambda_1 \boldsymbol{w}_0 + [\lambda_1(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{\boldsymbol{t}})]\boldsymbol{t}) + \mathrm{A}_2 \exp(\lambda_2 \boldsymbol{w}_0 + [\lambda_2(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{\boldsymbol{t}})]\boldsymbol{t})$$

(2.28b)

The stochastic stability of bubbles specified by (2.28b) depends upon whether $\lambda_i \mu_x < 0$ for all i = 1,2.

In case of $\mu_x = 0$, then $\lambda_1 = \dfrac{(-2\sigma_x^2 \ln \boldsymbol{b})^{1/2}}{\sigma_x^2} = \dfrac{(-2 \ln \boldsymbol{b})^{1/2}}{\sigma_x} > 0$ and $\lambda_2 = \dfrac{-(-2 \ln \boldsymbol{b})^{1/2}}{\sigma_x} < 0$ since ln $\boldsymbol{b}$ must be negative. The bubble process is shown as:

$$B_t = A_1 \exp[\frac{(-2\ln b)^{1/2}}{\sigma_x} w_t] + A_2 \exp[-\frac{(-2\ln b)^{1/2}}{\sigma_x} w_t] \quad (2.29a)$$

$$= A_1 \exp\{\frac{(-2\ln b)^{1/2}}{\sigma_x}[w_0 + (\frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})t]\} +$$

$$A_2 \exp\{-\frac{(-2\ln b)^{1/2}}{\sigma_x}[w_0 + (\frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})t]\} \quad (2.29b)$$

The bubble process of (2.29b) must exhibit stable dynamics as $t \rightarrow \infty$.

From the above, although all bubble processes are derived to explode in expected values, they may converge towards zero as $t \rightarrow \infty$ under certain restrictions on parameters. The different examples of bubble specifications are summarized in Table 2.1. The asymptotic stability of bubble process leads to difficulties in bubble testing. Nevertheless, the numbers of observations are usually not large during hyperinflationary episodes and consequently, such difficulties may not be so serious in my subsequent empirical studies.[10]

Other than the asymptotic dynamics of bubble, the bursting properties are the main issues about the theoretical specifications of bubble solution. The submartingale property of bubble process (2.13) can be further modified by the inclusion of a probability that a bubble continues to grow ($0 \leqq \Pi \leqq 1$):

---

[10] However, it may create serious problems of bubble detection in financial markets with long data horizons.

$$E_t(B_{t+1}) = \Pi\ E_t(B_{t+1} \mid G) + (1-\Pi)E_t(B_{t+1} \mid C)$$

$$= \frac{B_t}{b} \tag{2.30}$$

where $E_t(B_{t+1} \mid G)$ and $E_t(B_{t+1} \mid C)$ refer to the expected values of B_{t+1} given the regimes of bubble growth (G) and bubble collapse (C) respectively.[11]

One particular example of a bubble process that is satisfied with the above bursting bubble specification (2.30) is given as:

$$B_{t+1} = \exp\{\lambda x_{t+1} - [\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln(b\Pi)]\}B_t + \frac{\omega_{t+1}}{b^{t+1}}; \tag{2.31a}$$

$$B_{t+1} = \frac{\omega_{t+1}}{b^{t+1}} \tag{2.31b}$$

where $E_t(\omega_{t+1}) = 0$.

The bubble process of (2.31a) and (2.31b) would occur with the probability of Π in regime G and with the probability of $(1-\Pi)$ in regime C respectively. They represent a general version of the bursting bubbles suggested by Blanchard and Watson (1982) who restricted the value of λ in (2.31a, b) to be zero. It is noted that the expected value of the bubble in regime G, $E_t(B_{t+1} \mid G) = (b\Pi)^{-1} B_t$, where $(b\Pi)^{-1} > b^{-1}$, and the bubble will collapse to zero expected value as it bursts, $E_t(B_{t+1} \mid C) = 0$.

---

[11] The probability, Π, can be a variable as a function of the size of bubble (Norden, 1996).

In addition, Evans (1991) suggests a periodically collapsing bubble specification:

$$B_{t+1} = \exp[\lambda x_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\, B_t + \frac{\omega_{t+1}}{b^{t+1}} \quad \text{for } B_t \leqq \kappa \text{, and}$$

$$= \exp[\lambda x_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2)][\delta_o + \theta_{t+1}\Pi^{-1}b^{-1}(B_t - \delta_o b)] + \frac{\omega_{t+1}}{b^{t+1}}$$

$$\text{for } B_t > \kappa . \qquad (2.32)$$

where both κ and $\delta_o > 0$, θ_t is an exogenous independently and identically distributed Bernoulli process that takes the value of 1 with probability of Π in regime G and 0 with a probability of $(1-\Pi)$ in regime C. Since $(B_{t-1} - \delta_o b)$ is restricted to be positive, δ_o must be smaller than $(\kappa\, b^{-1})$.[12]

For $B_t \leqq \kappa$, it implies that $\Pi = 1$ and $E_t(B_{t+1}) = \frac{B_t}{b}$. For $B_t > \kappa$, $E_t(B_{t+1} \mid G) = [\delta_o - \delta_o\Pi^{-1} + \Pi^{-1}b^{-1}(B_t)]$ and $E_t(B_{t+1} \mid C) = \delta_o > 0$. Hence, $E_t(B_{t+1})$ is equal to $\frac{B_t}{b}$ for any value of B_t and the bubble process of Evans (1991) can satisfy the submartingale property. It is found that the collapsing bubble is strictly positive and never vanishes. Moreover, the size of bubble collapse or explosion and the probability Π are dependent upon the sizes of the bubble

[12] The collapsing bubble is specified to be positive since if a bubble collapses to zero, it cannot re-start (Diba and Grossman, 1988b).

compared to the value of κ . Also, the bubble bursts partially in contrast to the total bubble collapse of the bursting bubble of (2.31).

2.5. General Solution

The general solution to the difference equation (2.3), denoted by π_t^g , is equal to the sum of particular and homogenous solutions, i.e. $\pi_t^f + B_t$. The stochastic process of π_t^f characterizes the long-run equilibrium path of π_t^g ; on the other hand, the movement of B_t characterizes the deviation of π_t^g from π_t^f . If the model under study is correctly specified, the task of bubble testing consists in detecting whether any movements of asset price deviate from the paths predicted by the market fundamental solution.

2.6. Summary

I have specified two versions of the Cagan model under rational expectations, in which the opportunity costs of holding money are measured by the expected inflation rate and the expected depreciation rate of local currency respectively. Hence, they will be used for the subsequent study of price and exchange rate bubbles in this thesis. The general solution of the Cagan model is simply the sum of fundamental and bubble solutions. The fundamental solutions can be expressed in explicit representations dependent upon the assumed generating processes of the underlying fundamentals. There exist arbitrary martingales in the bubble solution, which is therefore non-unique in the rational expectations model. By restricting parameters of the bubble solution, several examples of different theoretical bubble specifications can be explored. Some exhibit asymptotic stability and some display different switching behaviours under alternate regimes of explosion and collapse. It makes the indirect testing methodologies more attractive for bubble detection. In subsequent chapters, I will conduct econometric studies to examine whether the price or exchange rate series deviate from the particular solutions. Before doing so, the first step is to examine the statistical properties of the relevant economic variables in the next chapter so that I can adopt appropriate econometric procedures and methods on the data set.

Table 2.1 Summary for different theoretical bubble specifications

Parameter restrictions	Equations	Conditions for dynamic stability
$\lambda = 0$	2.16	None
$\mu_x + \frac{1}{2}\lambda\,\sigma_x^2 = 0$		
$\mu_x \neq 0$ and $\lambda \neq 0$	2.17a, b, c	$\frac{-2\mu_x}{\sigma_x^2}\mu_x < \ln \boldsymbol{b} < 0.$
$\mu_x = -\frac{\sigma_x^2}{2} < 0$ and $\lambda = 1$	2.18a, b	$\mu_x < \ln \boldsymbol{b} < 0$
$\lambda \neq 0$, and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K) = 0$, where $\ln \boldsymbol{b} = (K + H) < 0$		
$\mu_x \neq 0$ and $\lambda_1 = \lambda_2$	2.21a, b	$-\frac{\mu_x^2}{\sigma_x^2} < H$
$\mu_x \neq 0$ and $\lambda_1 \neq \lambda_2$	2.22a,b	$\lambda_i\mu_x < H$
$\mu_x \neq 0$ and $\lambda_1, \lambda_2 = h_1 \pm h_2 i$	2.24a,b	$\frac{-\mu_x^2}{\sigma_x^2} < H < 0$
$\mu_x = 0$ and $\lambda_1 \neq \lambda_2$	2.25a,b	$H > 0$
$\lambda \neq 0$ and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln \boldsymbol{b}) = 0$		
$\mu_x \neq 0$ and $\lambda_1 \neq \lambda_2$	2.28a,b	$\lambda_i\mu_x < 0$
$\mu_x = 0$ and $\lambda_1 \neq \lambda_2$	2.29a,b	Must be asymptotically stable

Appendix 2.1

Given that $|b| < 1$, the presented discounted value of X_t can be expressed as follows:

$$\sum_{i=0}^{\infty} b^i E_t(X_{t+i}) = X_t + bE_t(X_{t+1}) + b^2 E_t(X_{t+2}) + b^3 E_t(X_{t+3}) + \dots$$

$$= X_t + bE_t(\Delta X_{t+1}) + b^2 E_t(\Delta X_{t+2}) + b^3 E_t(\Delta X_{t+3}) + \dots$$

$$+ bX_t + b^2 E_t(\Delta X_{t+1}) + b^3 E_t(\Delta X_{t+2}) + \dots$$

$$= X_t + \sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i}) + b \sum_{i=0}^{\infty} b^i E_t(X_{t+1+i})$$

$$= \frac{X_t}{1-b} + \frac{1}{1-b} \sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i}) \qquad \text{(A.2.1.1)}$$

Suppose that $\Delta X_{t+j} = \mu + \omega(t + j) + \eta_{t+j}$, the values of $b^i E_t(\Delta X_{t+i})$ are given as:

$$bE_t(\Delta X_{t+1}) = b\mu + b\omega t + b\omega \qquad b^2 E_t(\Delta X_{t+2}) = b^2\mu + b^2\omega t + 2b^2\omega$$

$$b^3 E_t(\Delta X_{t+3}) = b^3\mu + b^3\omega t + 3b^3\omega \qquad b^n E_t(\Delta X_{t+n}) = b^n\mu + b^n\omega t + nb^n\omega$$

as $n \rightarrow \infty$

Hence, $\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i})$ is equal to the sum of the following three components:

$$\sum_{i=1}^{\infty} b^i\mu + \sum_{i=1}^{\infty} b^i\omega t + \sum_{i=1}^{\infty} ib^i\omega .$$

The value of each component is calculated as follows:

$$\sum_{i=1}^{\infty} b^i \mu = \frac{b\mu}{1-b}, \sum_{i=1}^{\infty} b^i \omega t = \frac{b\omega t}{1-b}, \quad \sum_{i=1}^{\infty} i b^i \omega = \frac{1}{(1-b)} \sum_{i=1}^{\infty} b^i \omega = \frac{b\omega}{(1-b)^2}$$

(A.2.1.2)

It is known that $\frac{1}{1-b} = 1-\beta$, and $\frac{b}{1-b} = -\beta$, then, from (A.2.1.1) and (A.2.1.2), I obtain:

$$\begin{aligned}\sum_{i=0}^{\infty} b^i E_t(X_{t+1+i}) &= \frac{X_t}{1-b} + \frac{1}{1-b}\left[\frac{b\mu}{1-b} + \frac{b\omega t}{1-b} + \frac{b\omega}{(1-b)^2}\right] \\ &= (1-\beta)X_t + (1-\beta)[-\beta\mu - \beta\omega t - \beta(1-\beta)\omega] \\ &= (1-\beta)X_t - \beta(1-\beta)[\mu + \omega t + (1-\beta)\omega] \\ &= (1-\beta)X_t + \beta(\beta-1)\{\mu + \omega[(1-\beta) + t]\}\end{aligned}$$

Appendix 2.2

Given that $\Delta^2 X_{t+j} = \mu + \omega_1(t+j) + \omega_2(t+j)^2 + \eta_{t+j}$, the values of $b^i E_t(\Delta X_{t+i})$ are shown as:

$$bE_t(\Delta X_{t+1}) \quad = b\Delta X_t + b\mu + b\omega_1(t+1) + b\omega_2(t+1)^2$$

$$b^2 E_t(\Delta X_{t+2}) \quad = b^2 \Delta X_{t+1} + b^2\mu + b^2\omega_1(t+2) + b^2\omega_2(t+2)^2$$

$$= b^2\Delta X_t + 2b^2\mu + b^2\omega_1(t+1) + b^2\omega_2(t+1)^2 + b^2\omega_1(t+2) + b^2\omega_2(t+2)^2$$

$$b^3 E_t(\Delta X_{t+3}) \quad = b^3\Delta X_t + 3b^3\mu + b^3\omega_1(t+1) + b^3\omega_2(t+1)^2 + b^3\omega_1(t+2) + b^3\omega_2(t+2)^2 + b^3\omega_1(t+3) + b^3\omega_2(t+3)^2$$

$$b^n E_t(\Delta X_{t+n}) \quad = b^n\Delta X_t + nb^n\mu + b^n\omega_1(t+1) + b^n\omega_2(t+1)^2 + b^n\omega_1(t+2) + b^n\omega_2(t+2)^2 + b^n\omega_1(t+3) + b^n\omega_2(t+3)^2 + \ldots + b^n\omega_1(t+n) + b^n\omega_2(t+n)^2 \quad \text{as } n \to \infty$$

From the above, $\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i})$ is equal to the sum of the following four components:

$$\sum_{i=1}^{\infty} b^i \Delta X_t + \sum_{i=1}^{\infty} ib^i\mu + \sum_{i=1}^{\infty} b^i\omega_1[\Pi_{j=1}^{i}(t+j)] + \sum_{i=1}^{\infty} b^i\omega_2[\Pi_{j=1}^{i}(t+j)^2]$$

The finite values of the above four components of $\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i})$ are derived as follows:

$$\sum_{i=1}^{\infty} b^i \Delta X_t = \frac{\Delta X_t}{1-b}, \qquad \sum_{i=1}^{\infty} ib^i \mu = \frac{1}{1-b}\sum_{i=1}^{\infty} b^i \mu = \frac{b\mu}{(1-b)^2},$$

$$\begin{aligned}
&\sum_{i=1}^{\infty} b^i \omega_1 [\Pi_{j=1}^{i}(t+j)] \\
= \;& \sum_{i=1}^{\infty} t(ib^i\omega_1) + \sum_{i=1}^{\infty} b^i \omega_1 (\sum_{j=1}^{i} j) \\
= \;& \frac{1}{1-b}\sum_{i=1}^{\infty} t(b^i\omega_1) + (b\omega_1 + (1+2)b^2\omega_1 + (1+2+3)b^3\omega_1 + \ldots) \\
= \;& \frac{1}{(1-b)^2} b\omega_1 t + \frac{1}{1-b}\sum_{i=1}^{\infty} ib^i\omega_1 \\
= \;& \frac{1}{(1-b)^2} b\omega_1 t + \frac{1}{(1-b)^2}\sum_{i=1}^{\infty} b^i\omega_1 \\
= \;& \frac{1}{(1-b)^2} b\omega_1 t + \frac{b\omega_1}{(1-b)^3},
\end{aligned}$$

$$\begin{aligned}
&\sum_{i=1}^{\infty} b^i \omega_2 [\Pi_{j=1}^{i}(t+j)^2] \\
= \;& \sum_{i=1}^{\infty} t^2(ib^i\omega_2) + \sum_{i=1}^{\infty} tb^i\omega_2(\sum_{j=1}^{i} 2j) + \sum_{i=1}^{\infty} b^i\omega_2(\sum_{j=1}^{i} j^2) \\
= \;& \frac{1}{1-b}\sum_{i=1}^{\infty} t^2(b^i\omega_2) + \frac{1}{1-b}\sum_{i=1}^{\infty} t(2ib^i)\omega_2 + \frac{1}{(1-b)}\sum_{i=1}^{\infty}(i^2)b^i\omega_2 \\
= \;& \frac{t^2 b\omega_2}{(1-b)^2} + \frac{1}{(1-b)^2}\sum_{i=1}^{\infty} t(2b^i)\omega_2 + \frac{1}{(1-b)^2}\sum_{i=1}^{\infty}(2i-1)b^i\omega_2 \\
= \;& \frac{t^2 b\omega_2}{(1-b)^2} + \frac{t(2b)\omega_2}{(1-b)^3} + \frac{b\omega_2}{(1-b)^3}\sum_{i=1}^{\infty} 2b^i\omega_2 \\
= \;& \frac{t^2 b\omega_2}{(1-b)^2} + \frac{t(2b)\omega_2}{(1-b)^3} + \frac{b\omega_2 + b^2\omega_2}{(1-b)^4}. \qquad \text{(A.2.2.1)}
\end{aligned}$$

From (A.2.1.1) and (A.2.2.1), the value of $\sum_{i=0}^{\infty} b^i E_t(X_{t+i})$ is equal to:

$$\frac{X_t}{1-b} + \frac{1}{1-b}[\frac{\Delta X_t}{1-b} + \frac{b\mu}{(1-b)^2} + \frac{b\omega_1 t}{(1-b)^2} + \frac{b\omega_1}{(1-b)^3} + \frac{t^2 b\omega_2}{(1-b)^2} + \frac{t(2b)\omega_2}{(1-b)^3} +$$
$$\frac{b\omega_2 + b^2\omega_2}{(1-b)^4}]$$

$$= \frac{X_t}{1-b} + \frac{\Delta X_t}{(1-b)^2} + \frac{b}{(1-b)^3}[\mu + \frac{\omega_1}{(1-b)} + \frac{\omega_2(1+b)}{(1-b)^2}] + \frac{b}{(1-b)^3}[\omega_1 + \frac{2\omega_2}{(1-b)}]t$$
$$\frac{b\omega_2}{(1-b)^3}t^2$$

$$= (1-\beta)X_t + (1-\beta)^2\Delta X_t - \beta(1-\beta)^2\{\mu + \omega_1(1-\beta) + \omega_2[(1-\beta)^2 - \beta(1-\beta) + t^2] +$$
$$t[\omega_1 + 2\omega_2(1-\beta)]\}$$

Appendix 2.3

Let $x_t = w_t - w_{t-1}$. Substituting one period forward for B_{t+1} from the general bubble specification (2.15), I obtain:

$$B_{t+1} = \exp[\lambda x_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\,B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

$$= \exp[\lambda w_{t+1} - \lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\,B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

$$= \exp[\lambda w_{t+1} - \lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1) +$$

$$(\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1) - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\,B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

$$= \exp\{\lambda w_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1) -$$

$$[\lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]t\}B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

Assume that $\{\omega_t\} = 0$, then:

$$\frac{B_{t+1}}{B_t} = \frac{\exp[\lambda w_{t+1} - \lambda(\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1)]}{\exp[\lambda w_t - \lambda(\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t)]},$$

Hence, $B_t = \exp[\lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)t]$. (A.2.3.1)

By imposing different parameter restrictions upon (A.2.3.1), I can obtain different bubble specifications summarized in Table 2.1.

Appendix 2.4

Let's examine whether the linear combination of bubble process (2.19a) and (2.19b) or the bubble process (2.22a) can still satisfy the submartingale property such that:

$$bE_{t-1}(B_t) = E_{t-1}(q_t B_{t-1}).$$

It is known that $bE_{t-1}(B_t) = E_{t-1}[\exp(\ln b)B_t]$. By substituting the bubble process (2.22a) into $E_{t-1}[\exp(\ln b)B_t]$, I obtain:

$$\begin{aligned} bE_{t-1}(B_t) &= E_{t-1}\{\exp(\ln b)[\mathrm{A}_1 \exp(\lambda_1 w_t - \mathrm{H}t) + \mathrm{A}_2 \exp(\lambda_2 w_t - \mathrm{H}t)]\} \\ &= E_{t-1}\{\mathrm{A}_1 \exp[\lambda_1 w_t - \mathrm{H}t + \ln b] + \mathrm{A}_2 \exp[\lambda_2 w_t - \mathrm{H}t + \ln b]\} \end{aligned}$$

Also, it is known that $E_{t-1}(q_t B_{t-1}) = E_{t-1}(\exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)]B_{t-1})$. By substituting the bubble process (2.22a) into $E_{t-1}(\exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)]B_{t-1})$, I find that $E_{t-1}(q_t B_{t-1})$ is equal to:

$$\begin{aligned} &E_{t-1}\{\exp[\lambda_1 w_t - \lambda_1 w_{t-1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2)]\mathrm{A}_1 \exp[\lambda_1 w_{t-1} - \mathrm{H}(t-1)] + \\ &\quad \exp[\lambda_2 w_t - \lambda_2 w_{t-1} - (\lambda_2\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2)]\mathrm{A}_2 \exp[\lambda_2 w_{t-1} - \mathrm{H}(t-1)]\} \end{aligned}$$

Given the assumption that $\ln \boldsymbol{b} = \mathrm{K} + \mathrm{H}$, and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \mathrm{K}) = 0$, then:

$$\boldsymbol{E}_{t-1}(\boldsymbol{q}_t\boldsymbol{B}_{t-1}) = \boldsymbol{E}_{t-1}\{\mathrm{A}_1 \exp[\lambda_1\boldsymbol{w}_t - (\lambda_1\mu_x + \tfrac{1}{2}\lambda_1^2\sigma_x^2 + \mathrm{K}) - \mathrm{H}\boldsymbol{t} + \ln\boldsymbol{b}] +$$

$$\mathrm{A}_2 \exp[\lambda_2\boldsymbol{w}_t - (\lambda_2\mu_x + \tfrac{1}{2}\lambda_2{}^2\sigma_x^2 + \mathrm{K}) - \mathrm{H}\boldsymbol{t} + \ln\boldsymbol{b}]\}$$

$$= \boldsymbol{E}_{t-1}\{\mathrm{A}_1 \exp[\lambda_1\boldsymbol{w}_t - \mathrm{H}\boldsymbol{t} + \ln\boldsymbol{b}] + \mathrm{A}_2 \exp[\lambda_2\boldsymbol{w}_t - \mathrm{H}\boldsymbol{t} + \ln\boldsymbol{b}]\}$$

$$= \boldsymbol{b}\boldsymbol{E}_{t-1}(\boldsymbol{B}_t)$$

Appendix 2.5

Given the fact that $\exp(\pm i h_2 w_t) = [\cos(h_2 w_t) \pm i \sin(h_2 w_t)]$, after substituting

$\lambda_1 = h_1 + h_2 i$, and $\lambda_2 = h_1 - h_2 i$ into (2.22a), I obtain:

$$B_t = \exp(-Ht)[A_1 \exp(h_1 + h_2 i)w_t + A_2 \exp(h_1 - h_2 i)w_t]$$

$$= \exp(h_1 w_t - Ht)[A_1 \exp(i h_2 w_t) + A_2 \exp(-i h_2 w_t)]$$

$$= \exp(h_1 w_t - Ht)\{A_1[\cos(h_2 w_t) + i \sin(h_2 w_t)] + A_2[\cos(h_2 w_t) - i \sin(h_2 w_t)]\}$$

$$= \exp(h_1 w_t - Ht)[(A_1 + A_2)\cos(h_2 w_t) + (A_1 - A_2) i \sin(h_2 w_t)]$$

$$= \exp(h_1 w_t - Ht)[A_3 \cos(h_2 w_t) + A_4 \sin(h_2 w_t)].$$

where $A_3 = A_1 + A_2$, $A_4 = (A_1 - A_2)i$

CHATPER THREE STRUCTURAL TIME SERIES ANALYSIS OF DATA

3.1 Introduction

In this chapter, I will provide a brief description of the data series for the three inter-war European hyperinflations of Germany, Hungary and Poland. Also, since the stochastic properties of economic variables affect the econometric methods adopted for subsequent studies, I attempt a structural time series analysis to identify the unobserved stochastic components of the observed data series. From the reduced forms of the trend component, the integration orders of the data series can be found. In addition, some evidence of regime changes in data generation can be detected from the trend component or the slope of the trend. Such findings are important for empirical studies conducted in the subsequent chapters. This chapter is structured as follows. Section 2 introduces the statistical specifications of the unobserved components in a structural time series model. Section 3 discusses the data sources, sample lengths and definitions of the variables under study. The statistical and graphical analysis of the unobserved components will also be presented. Section 4 summarizes the findings.

3.2. Statistical Specifications of Structural Time Series Model

Before taking the empirical testing in subsequent chapters, I try to examine the stochastic properties of the economic variables under study. Using the structural time series modeling techniques (Harvey, 1989), I attempt to decompose and analyze the unobserved components of the observed data series. A univariate structural time series model is formulated as:

$$(1-\Psi(L))y_{ot} = \mu_t + \gamma_t + \psi_t + \varepsilon_t, \qquad \varepsilon_t \sim \text{NID}(0,\ \sigma_\varepsilon^2), \qquad t = 1, \ldots, T. \tag{3.1}$$

where y_{ot} is an observed time series variable, $\Psi(L) = 1 - \Psi_1 L - \ldots - \Psi_p L^p$ where Ψ_i is a parameter of a lagged value of y_{ot}; the elements μ_t, γ_t, ψ_t and ε_t represent the unobserved trend, seasonal, cyclical and irregular components respectively.[13]

The trend is the long-run component in the series, which indicates the general moving direction of the observed series under study. There are two parts to the trend specified as:

---

[13] A first-order autoregressive component and a vector of exogenous variables should also be included in the univariate structural time series model (3.1), but they are not found in my empirical results and thus are excluded here for simplicity.

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \qquad \eta_t \sim \text{NID}(0,\ \sigma_\eta^2), \tag{3.2a}$$

$$\beta_t = \beta_{t-1} + \zeta_t, \qquad \zeta_t \sim \text{NID}(0,\ \sigma_\zeta^2), \tag{3.2b}$$

where μ_t is the level, which is the actual value of the trend, β_t is the slope of the trend. If σ_η^2 and σ_ζ^2 are zero, μ_t and β_t will be fixed respectively.

Different properties of the level, slope and irregular component would result in different specifications of the trend model. Let's illustrate them briefly.[14] When both σ_ε^2 and σ_η^2 are zero, then the trend specification given by:

$$\mu_t = \mu_{t-1} + \beta_{t-1}, \tag{3.3a}$$

$$\beta_t = \beta_{t-1} + \zeta_t, \qquad \zeta_t \sim \text{NID}(0,\ \sigma_\zeta^2), \tag{3.3b}$$

is known as a second differencing model. When σ_ε^2 is not zero; the trend is known as a smooth trend model. Also, in case where both σ_ε^2 and σ_ζ^2 are zero, the trend model specified as:

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \qquad \eta_t \sim \text{NID}(0,\ \sigma_\eta^2), \tag{3.4a}$$

$$\beta_t = \beta_{t-1}, \tag{3.4b}$$

[14] The details of the trend specifications are documented in Koopman *et al.* (2000).

is called random walk with a drift, or random walk if $\beta_t = \beta_{t-1} = 0$. While σ_ε^2 is not zero, the trend component is known as the local level with a drift or the local level, dependent upon whether $\beta_t = \beta_{t-1}$ is different from zero.

The seasonal component may be based on the dummy variable form, or the trigonometric formulation. Given that s refers to the number of seasonal frequencies, the seasonal dummy is given by:

$$\gamma_t = \sum_{j=1}^{s-1} -\gamma_{t-j} + \omega_t, \qquad \omega_t \sim \text{NID}(0,\ \sigma_\omega^2). \qquad (3.5)$$

Moreover, the trigonometric seasonal formulation is:

$$\gamma_t = \sum_{j=1}^{s/2} (\cos\lambda_j \gamma_{j,t-1} + \sin\lambda_j \gamma_{j,t-1}^*) + \omega_{j,t},\ \text{j} = 1,\ldots,\ \frac{s}{2}, \quad \omega_{j,t} \sim \text{NID}(0,\ \sigma_{\omega_j}^2). \qquad (3.6)$$

where $\lambda_j = 2j\pi / s$ refers to the frequency in radians, $\gamma_{j,t}^*$ is constructed to estimate γ_t.

On the other hand, the stochastic cycle is specified as follows:

$$\psi_t = \rho(\cos\lambda_c \psi_{t-1} + \sin\lambda_c \psi_{t-1}^* + \kappa_t), \qquad \kappa_t \sim \text{NID}(0,\ \sigma_\kappa^2). \qquad (3.7)$$

where λ_c is the frequency in radians, in the range $0 \le \lambda_c \le \pi$, ρ is a damping factor in the range $0 \le \rho \le 1$ and as in the trigonometric seasonal form (3.6), ψ^*_{t-1} is constructed to generate ψ_t. The period of the cycle is equal to $2\pi / \lambda_c$.

All the disturbance terms of the structural components, and the irregular component, $\{\eta_t, \zeta_t, \omega_t, \omega_{j,t}, \kappa_t, \varepsilon_t\}$ are independent of one another. The inclusion of the disturbance terms produces stochastic properties of the corresponding unobserved components. The q-ratio is the ratio of the standard deviation of each disturbance term to the standard deviation associated with the largest variance. The q-ratio corresponding to a particular component is zero when that component is deterministic or nonexistent.

3.3. Data Description and Structural Time Series Analysis of Data

The data from Germany, Hungary and Poland include money supply, price index and exchange rate series. The money supply series are month-end data, whereas the other series are monthly averages; I therefore follow Abel *et al.* (1979) in applying the geometric averaging method to make the money supply series conform to the rest of the data. Also, all of the exchange rate series that are originally quoted as the number of US cents per unit of local currency are transformed in terms of the values of domestic currency per US dollar. The German exchange rate series and all the data for Hungary and Poland are taken from Young (1925), while the German money supply and price index are collected from Tinbergen (1934). All series are transformed in logarithm.

The statistical treatment of the univariate structural time series model (3.1) is based on the state space form. The values of the parameters and the unobserved components are estimated using the maximum likelihood (ML) method with the Kalman filter algorithms. Since the unobserved components are in general stochastic, they can only be assessed by examining their behaviours throughout the whole samples, not just at the end. The filtered and smoothed estimates of the components will then be plotted to provide a guide as to whether the model is best decomposed by the estimated components. The model can also be evaluated through goodness-of-fit measures and diagnostic statistics.

For each country under study, the log level and the log difference of money supply, price index and exchange rate series, as well as the log level of real money balances in terms of both price and exchange rate series will be decomposed into unobserved components for analysis. They are denoted by M_t, $\pi_{1,t}$, $\pi_{2,t}$, ΔM_t, $\Delta\pi_{1,t}$, $\Delta\pi_{2,t}$, $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$ respectively.

3.3.1 Germany

The German data are collected from January 1920 to December 1923. Money in circulation is employed to represent the money supply series, and the cost of living index is used as a price index. The exchange rate figures are transformed from US cents per German mark.

Table 3.1 reports the empirical results and Figures 3.1 to 3.8 show the graphical components of the economic variables under study. From Table 3.1, all observed series do not contain any lagged dependent variables and irregular components, so that all of the Ψ_i and ε_t are equal to zero. The trend component for the series of M_t, $\pi_{1,t}$ and $\pi_{2,t}$ follows a second differencing specification. From the q-ratio and the seasonal test, both $\pi_{1,t}$ and $\pi_{2,t}$ contain significant stochastic dummy seasonal components. Further, M_t contains a fixed seasonal and $\pi_{1,t}$ contains a nonzero stochastic cycle.

Furthermore, the trend component is a random walk with a fixed drift for the series of $\Delta\pi_{1,t}$, $\Delta\pi_{2,t}$, $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$ as well as a random walk for the series of ΔM_t. The seasonals in M_t, $\pi_{1,t}$ and $\pi_{2,t}$ remain in the structural components of $\Delta\pi_{1,t}$, $\Delta\pi_{2,t}$ $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$. In addition, the stochastic cycle contained in $\pi_{1,t}$ is carried forward to $\Delta\pi_{1,t}$.

[Table 3.1 to be inserted here]

From the figures of the structural components, the slopes of the trend for the level series, $M_t, \pi_{1,t}$ and $\pi_{2,t}$, as well as the trend for $\Delta\pi_{1,t}$, $\Delta\pi_{2,t}$ $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$ exhibit changes in moving direction toward the end of 1923. It signifies the possible regime shifts in data generation.

[Figures 3.1 to 3.8 to be inserted here]

3.3.2. <u>Hungary</u>

The Hungarian data sets starts from July 1921 to March 1925. The money supply series includes notes in circulation and deposits. The price index numbers from July 1921 through December 1923 represent retail prices based on 60 commodities. From December 1923 through March 1925, the figures of the price index represent wholesale prices based on 52 commodities. The exchange rate data are originally quoted as US cents per Hungarian crown.

The empirical results of the structural time series models are presented in Table 3.2 with the components graphics plotted from Figures 3.9 to 3.16. All series under study do not contain any seasonal components. The trend for M_t and $\pi_{1,t}$ is found to follow a second differencing specification, but given that the irregular component is not zero, the trend for $\pi_{2,t}$ is known as a smooth trend specification. Moreover, there is one cyclical component found in M_t, $\pi_{1,t}$ and $\pi_{2,t}$. The model for the series of M_t, $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$ include lagged values of the corresponding dependent variables.

The trend follows a random walk for the series of ΔM_t, $\Delta\pi_{1,t}$, and $M_t - \pi_{1,t}$. When the irregular components are nonzero, the trend models for the series of $M_t - \pi_{2,t}$ and $\Delta\pi_{2,t}$ are known as the local level and the local level with a fixed slope respectively, dependent upon whether the fixed slope of the trend is existent or not. Also, the stochastic cycles are carried forward to the series of ΔM_t, $\Delta\pi_{1,t}$ and $M_t - \pi_{1,t}$ from the series of M_t and $\pi_{1,t}$, but no cycle is found in the series of $\Delta\pi_{2,t}$ and $M_t - \pi_{2,t}$.

[Table 3.2 to be inserted here]

The figures of the structural components indicate that the slopes of the trend and the trend or the economic variables start to shift in the second half of 1923. They all display some evidence of regime-switching behaviour in the observed data series.

[Figures 3.19 to 3.16 to be inserted here]

3.3.3. Poland

The Polish data are collected from January 1921 to March 1924. The money supply includes notes in circulation and the wholesale price index is chosen to represent the price level. The exchange rate series is transformed from US cents per Polish mark.

The empirical results of the structural models are shown in Table 3.3. As in the case of Hungary, all observed series under study do not contain any seasonal components. Only the model for the series of M_t and ΔM_t include corresponding lagged dependent variables.

The trend components for the series of M_t and $\pi_{2,t}$ follow a second differencing specification but the trend for $\pi_{1,t}$ is a smooth trend in which the irregular component exists. Moreover, $\pi_{1,t}$ and $\pi_{2,t}$ contain a stochastic cycle, which however cannot be found in M_t.

[Table 3.3 to be inserted here]

For the series of ΔM_t, $\Delta\pi_{1,t}$, $\Delta\pi_{2,t}$, $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$, the fixed slope of the trend cannot be found. Also, the irregular components exist for the series of $\Delta\pi_{1,t}$ and $\Delta\pi_{2,t}$ only. Hence, the trend model for the series of ΔM_t, $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$ follows a random walk but it follows a local level for the series of $\Delta\pi_{1,t}$ and $\Delta\pi_{2,t}$. Furthermore, the stochastic cycles remain in series of $\Delta\pi_{1,t}$, $M_t - \pi_{1,t}$ and $M_t - \pi_{2,t}$ but not in $\Delta\pi_{2,t}$.

From the movements of the trend as well as the slope of trend for the time series variables under study shown in Figures 3.17 to 3.24, some evidence of structural changes is found in the data generation occurred in the late 1923.

[Figures 3.17 to 3.24 to be inserted here]

3.4. Summary

From the analysis of structural time series components, the cycles and seasonals are found in some data series. More importantly, the trend for the levels of money supply, price and exchange rate is composed of a fixed level with a stochastic slope. From the reduced form of the structural time series models (Harvey, 1989), the integration order of these level series is two; in other words, they contain double unit roots. It is consistent with Haldrup (1998) that the economic variables are likely to be I(2) during hyperinflation. Also, the real money balances, the first-differenced price and exchange rate series have a stochastic trend with a fixed or zero slope, implying that these series contain a unit root. From the figures of the structural components, it indicates possible regime changes in data generation, resulting from monetary regime changes that will be further described in Chapter Five and Chapter Six.[15] Such findings of the stochastic properties play an important role in the econometric methods adopted in subsequent chapters.

---

[15] Due to the possible existence of structural breaks and nonlinearity in the raw data series, I do not formally conduct the unit root tests in this chapter. Nevertheless, I will conduct unit root tests in the residuals of the Cagan money demand functions using nonlinear cointegration methodologies in Chapter Five and Chapter Six.

Table 3.1 ML Estimation Results of the Structural Time Series Model for Germany

Variables	M_t	$\pi_{1,t}$	$\pi_{2,t}$	ΔM_t	$\Delta\pi_{1,t}$	$\Delta\pi_{2,t}$	$M_t - \pi_{1,t}$	$M_t - \pi_{2,t}$
Estimated standard deviation of disturbances [q-ratio]								
σ_η	——	——	——	0.5004 [1.0000]	0.0460 [0.0979]	0.4112 [0.8001]	0.1837 [1.0000]	0.2151 [1.0000]
σ_ζ	0.5024 [1.0000]	0.2895 [0.7189]	0.4506 [1.0000]	——	——	——	——	——
σ_κ	——	0.4026 [1.0000]	——	——	0.4703 [1.0000]	——	0.1139 [0.6198]	——
σ_ω	——	0.0711 [0.1767]	0.2958 [0.6564]	——	0.1557 [0.3311]	0.5139 [1.0000]	0.1328 [0.7228]	0.2022 [0.9401]
Filtered estimates of final state vector at time T with the corresponding root mean square error (RMSE) in the brackets								
μ_T	33.2690* (0.4544)	23.186* (1.0809)	29.373* (0.5543)	2.6041* (0.2694)	1.1880* (0.3342)	3.4429* (0.4157)	6.0249* (0.2683)	4.5411* (0.1974)
β_T	2.5691* (0.5701)	1.7910* (0.5297)	3.5183* (0.6051)	0.0566 (0.0789)	0.0377** (0.0158)	0.0861 (0.0659)	-0.0731** (0.0301)	-0.0691** (0.0343)
φ_T	——	4.0166* (2.7505)	——	——	1.5518 (2.2428)	——	-0.8133 (0.4415)	——
$\gamma_{1,T}$	-0.8713*** (0.4544)	0.6487 (2.6230)	-0.2587 (0.5543)	-0.2419 (0.2694)	-2.0990 (2.2129)	-2.8041* (0.4157)	0.8982** (0.4033)	0.3066 (0.1974)
$\gamma_{2,T}$	-0.8839** (0.4325)	3.4688 (2.6711)	2.6207* (0.4200)	-0.0318 (0.2617)	0.4432 (2.2157)	3.6239* (0.3267)	-0.6341 (0.4062)	-1.6454* (0.1577)
$\gamma_{3,T}$	-0.5708 (0.4245)	4.2237 (2.7078)	-0.8030*** (0.4075)	0.2899 (0.2578)	1.2357 (2.2030)	0.0011 (0.3035)	-0.6560 (0.4047)	0.6287* (0.1505)
$\gamma_{4,T}$	-0.0511 (0.4245)	4.0646 (2.7080)	-0.6008 (0.4127)	0.4926*** (0.2578)	1.2808 (2.2020)	0.1735 (0.3000)	-0.2270 (0.4037)	-0.1109 (0.1502)
$\gamma_{5,T}$	0.6133 (0.4325)	3.3656 (2.6712)	-0.7189 (0.4228)	0.6334** (0.2617)	2.2227 (2.2121)	0.1296 (0.2994)	-0.8710** (0.4064)	-0.3126** (0.1503)
$\gamma_{6,T}$	0.6925 (0.4544)	1.5230 (2.6257)	-0.8964** (0.4307)	0.0442 (0.2694)	2.0706 (2.2224)	-0.2761 (0.2993)	-0.5816 (0.4104)	0.0287 (0.1501)
$\gamma_{7,T}$	0.6790 (0.4557)	-0.7016 (2.6248)	-0.6769 (0.4309)	-0.0350 (0.2773)	1.4104 (2.2268)	-0.3992 (0.3001)	-0.1708 (0.4124)	0.2725*** (0.1507)
$\gamma_{8,T}$	0.5831 (0.4495)	-2.7581 (2.6712)	-0.2818 (0.4260)	-0.1037 (0.2825)	0.4452 (2.2149)	-0.1219 (0.3018)	0.0684 (0.4094)	0.2427 (0.1518)
$\gamma_{9,T}$	0.4370 (0.4443)	-4.1619 (2.7171)	-0.1315 (0.4222)	-0.1403 (0.2850)	-0.7557 (2.1988)	-0.3371 (0.3033)	0.4284 (0.4046)	0.4940* (0.1527)
$\gamma_{10,T}$	0.1600 (0.4443)	-4.4652 (2.7170)	0.2009 (0.4226)	-0.2576 (0.2850)	-1.8780 (2.1988)	-0.6201** (0.3038)	0.6170 (0.4045)	0.3923** (0.1530)
$\gamma_{11,T}$	-0.2185 (0.4495)	-3.4875 (2.6713)	0.7738*** (0.4270)	-0.3456 (0.2825)	-2.1109 (2.2151)	0.1295 (0.3031)	0.5048 (0.4093)	-0.1621 (0.1526)
Estimated parameters of cycle								
Variance	——	18.3227	——	——	8.0315	——	0.5105	——
ρ	——	0.9956	——	——	0.9861	——	0.9872	——
Period (yr.)	——	0.9604	——	——	0.9684	——	0.9218	——
λ_c	——	0.5452	——	——	0.5407	——	0.5680	——
Seasonal test (at time T)								
$\chi^2(11)$	——	21.5743**	421.345*	——	34.9284*	514.924*	95.5476*	361.616*

Goodness-of-fit measures and diagnostic checking								
SE	0.4175	0.7133	0.8542	0.4168	0.6535	0.8706	0.3613	0.3846
R(1)	0.2022	0.1393	-0.0865	0.1710	0.0991	-0.1347	-0.0388	-0.0414
R(6)	-0.0756	-0.0790	-0.0099	-0.0797	-0.0456	-0.0134	-0.0793	0.1283
Q(4)	6.3655	3.8261	1.3000	6.6425	5.3202	1.4800	8.6160	3.5528
Q(7)	6.8320	4.6983	3.1197	7.2309	5.6424	3.2953	9.0970	5.8950
PEV	0.1743	0.5087	0.7297	0.1737	0.4271	0.7579	0.1305	0.1479
R_d^2	0.8634	0.6813	0.5699	0.2581	0.4060	0.5412	0.3166	0.5676
AIC	-1.1281	0.1337	0.3515	-1.1314	-0.0412	0.3894	-1.2267	-1.2443
BIC	-0.5903	0.8370	0.9307	-0.5936	0.6621	0.9687	-0.5233	-0.6651
Max ln L	7.1769	-4.6230	-7.8919	7.8000	-2.4095	-7.8991	16.6318	15.6629

Notes:

1. A cycle component is not persistent throughout the series; a t-value is therefore not appropriate.
2. SE is the standard error of the residuals of the estimated equations.
3. r(k) is the residual autocorrelation coefficient at lags (k).
4. Q(k) is the Box-Ljung Q statistics with degrees of freedom = k.
5. PEV is the prediction error variance.
6. R_d^2 is a modified coefficient of determination based on the first difference of the dependent variable.
7. AIC and BIC refer to the Akaike information criterion and Bayes information criterion respectively.
8. Max ln L is the maximum log-likelihood function.
9. */**/*** Denotes the significance at the 1%, 5%, and 10% level.
10. All computations are produced using the STAMP package written by Koopman, *et al.* (2000).

Figure 3.1 Structural Time Series Components of the German Money Supply Level (M_t)

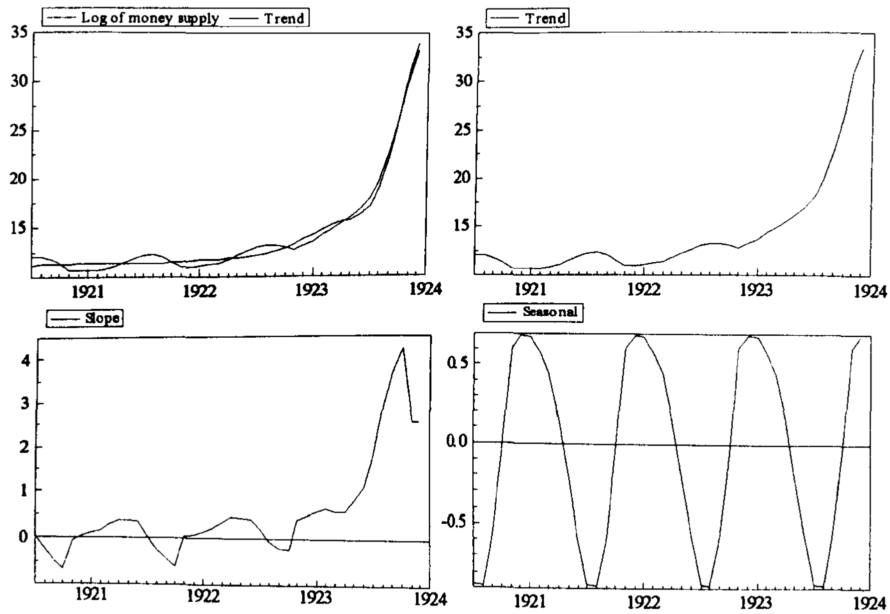


Figure 3.2 Structural Time Series Components of the German Consumer Price Level (π_{1t})

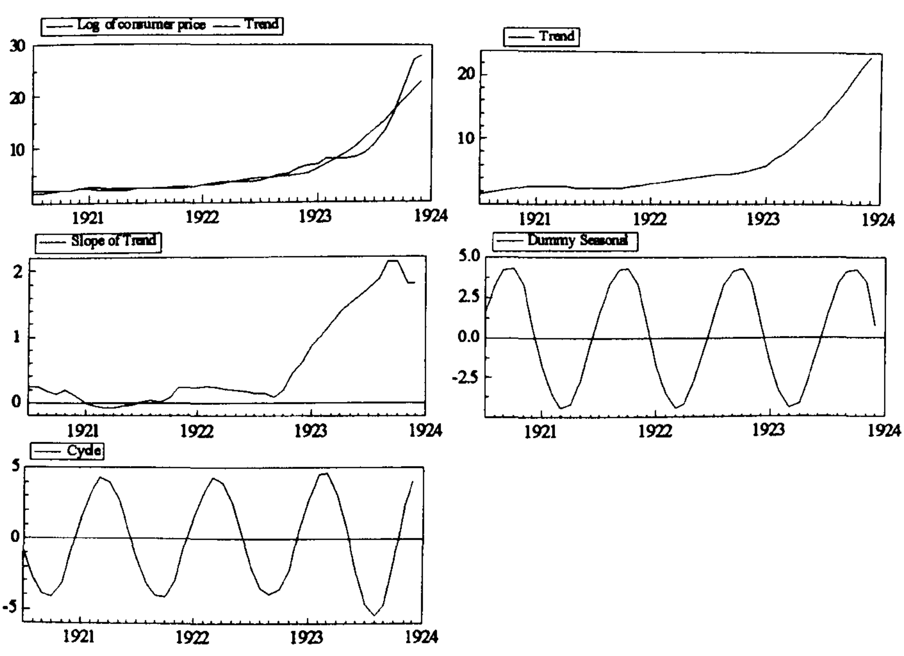


Figure 3.3 Structural Time Series Components of the German Exchange Rate (π_{2t})

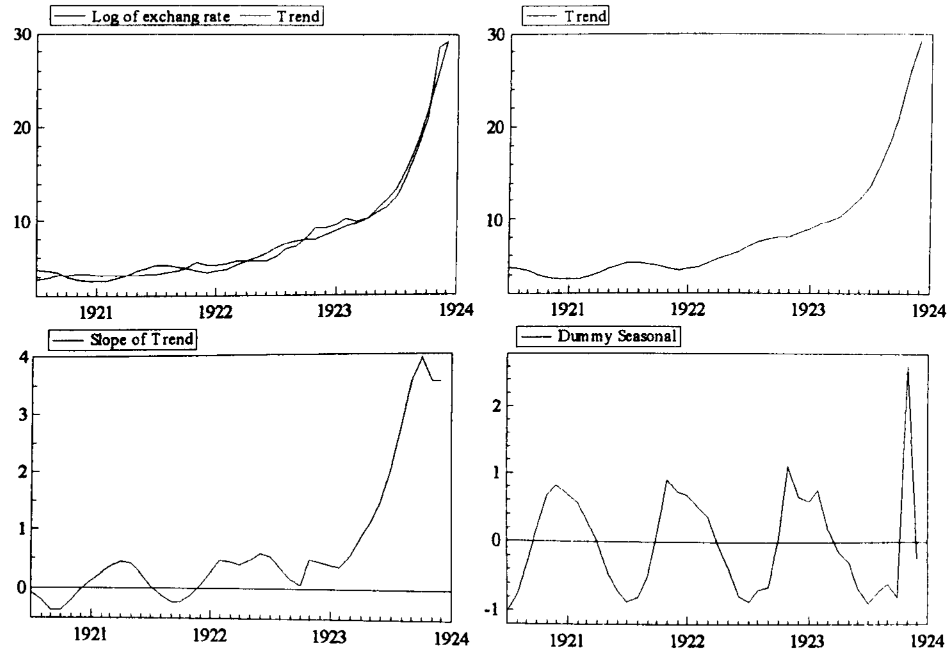


Figure 3.4. Structural Time Series Components of the Money Supply Growth (ΔM_t) for Germany

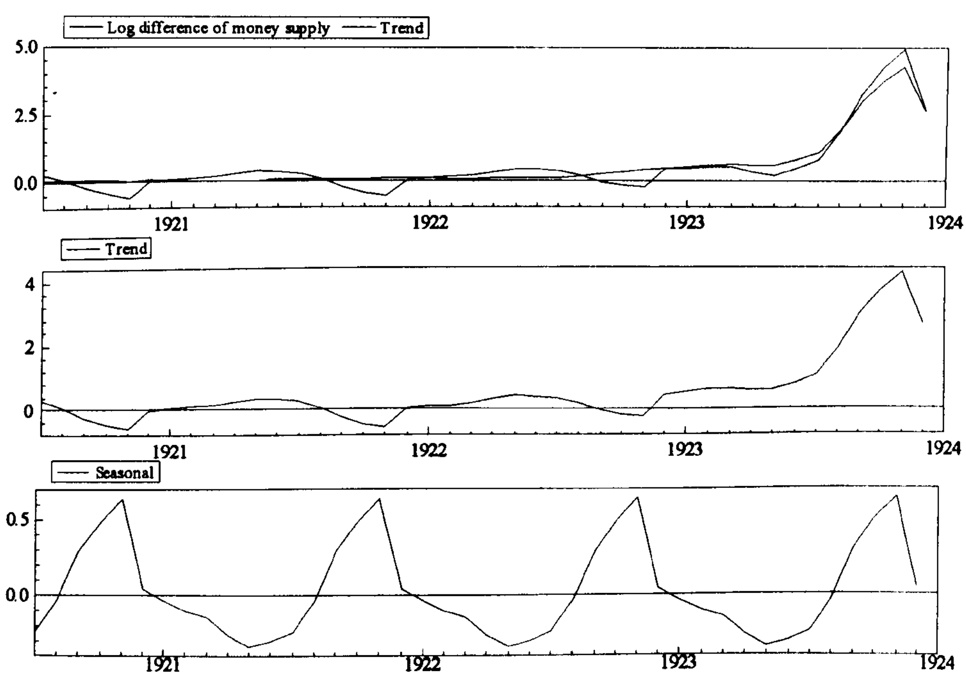


Figure 3.5 Structural Time Series Components of the Price Change ($\Delta\pi_{1t}$) for Germany

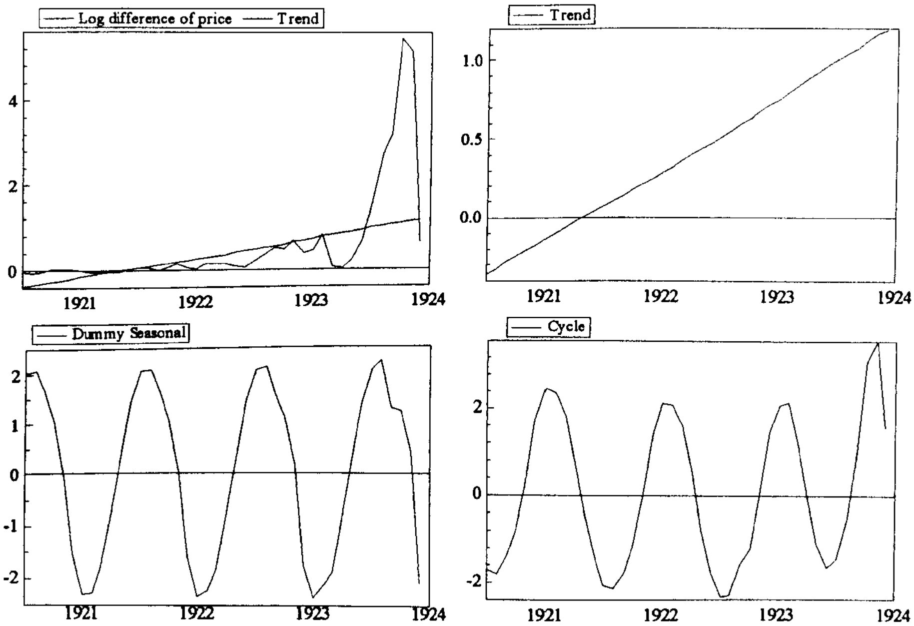


Figure 3.6 Structural Time Series Components of the Exchange Rate Change ($\Delta\pi_{2t}$) for Germany

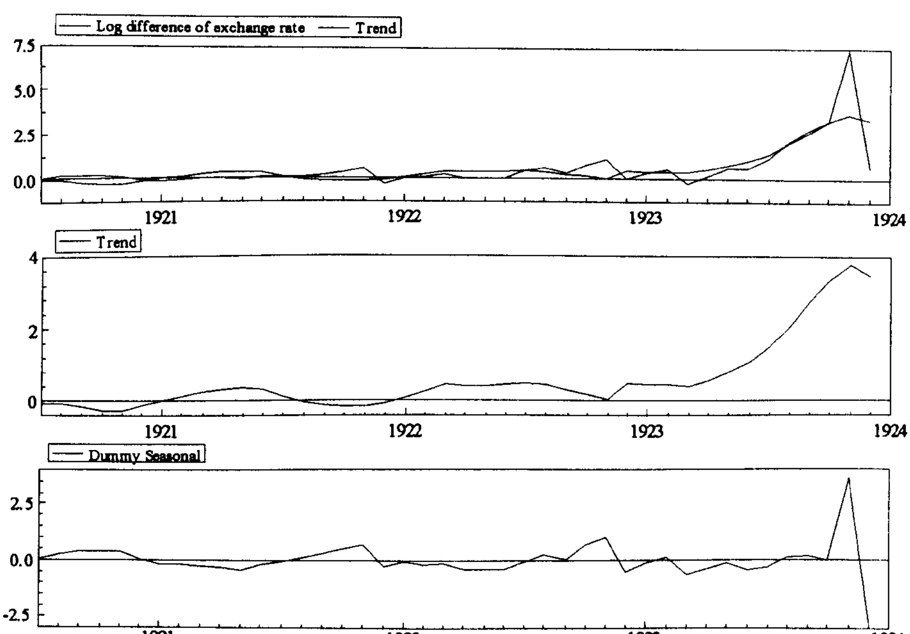


Figure 3.7. Structural Time Series Components of the Real Money Balance in terms of Consumer Price ($M_t - \pi_{1t}$) for Germany

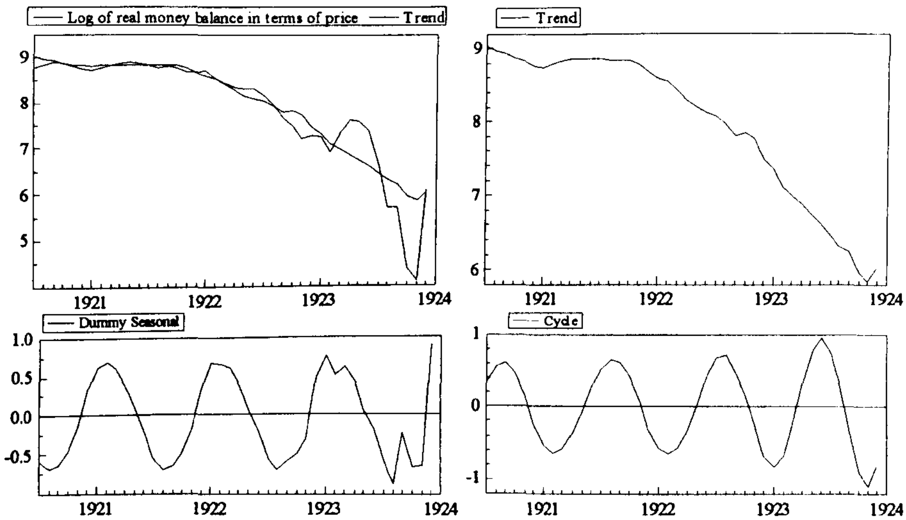


Figure 3.8. Structural Time Series Components of the Real Money Balance in terms of Exchange Rate ($M_t - \pi_{2t}$) for Germany

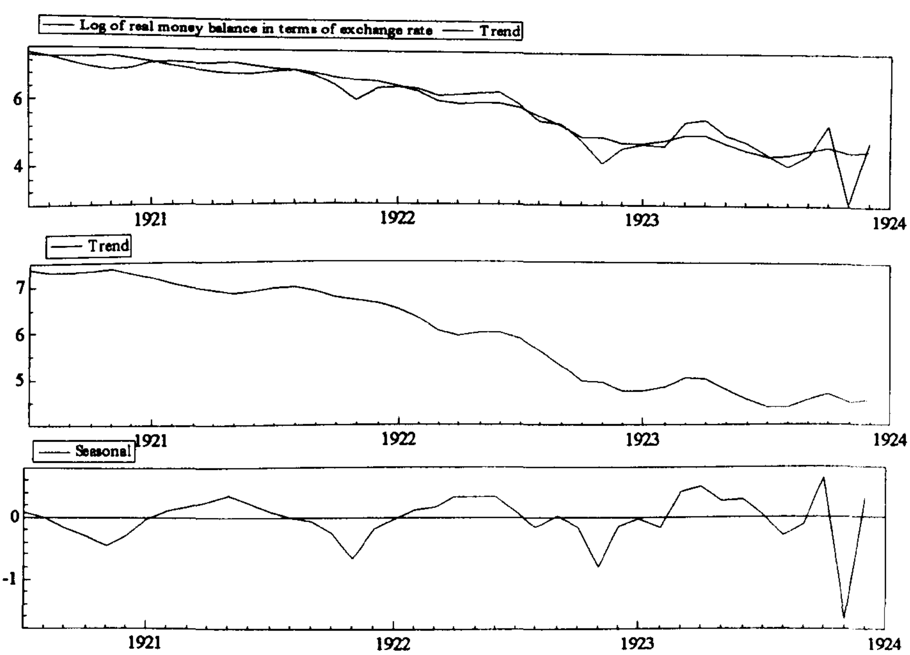


Table 3.2 ML Estimation Results of the Structural Time Series Model For Hungary

Variables	M_t	$\pi_{1,t}$	$\pi_{2,t}$	ΔM_t	$\Delta\pi_{1,t}$	$\Delta\pi_{2,t}$	$M_t - \pi_{1,t}$	$M_t - \pi_{2,t}$
Estimated standard deviation of disturbances [q-ratio]								
σ_η	—	—	—	0.0111 [0.7274]	0.0473 [0.4701]	0.0761 [0.4713]	0.0905 [1.0000]	0.1420 [1.0000]
σ_ζ	0.0150 [1.0000]	0.0531 [0.6997]	0.0393 [0.4932]	—	—	—	—	—
$\sigma_{\kappa 1}$	0.0132 [0.8826]	0.0759 [1.0000]	0.0796 [1.0000]	0.1520 [1.0000]	0.1006 [1.0000]	—	0.0716 [0.7917]	—
$\sigma_{\kappa 2}$	—	—	—	0.0109 [0.7142]	—	—	—	—
σ_ε	—	—	0.0667 [0.8372]	—	—	0.1615 [1.0000]	—	0.0813 [0.5724]
Filtered estimates of final state vector at time T and the estimated coefficient of lagged dependent variable with the corresponding RMSE in the brackets								
μ_T	5.5685* (0.9141)	14.571* (0.1138)	6.5584* (0.1476)	-0.0045 (0.0209)	-0.0153 (0.0737)	-0.0197 (0.1007)	0.5336* (0.1604)	5.2435* (1.2454)
β_T	0.0086 (0.0206)	-0.0227 (0.0826)	-0.0424 (0.0726)	—	—	-0.0057 (0.0121)	—	—
φ_{1T}	0.0025 (0.0178)	-0.0049 (0.1138)	0.0133 (0.1379)	0.0005 (0.02106)	-0.0313 (0.0737)	—	0.0254 (0.0868)	—
φ_{2T}	—	—	—	0.0101 (0.0201)	—	—	—	—
Ψ_1	1.4445* (0.0928)	—	—	0.7691* (0.1049)	—	—	0.5512* (0.1249)	0.4283* (0.1361)
Ψ_2	-0.7987* (0.0922)	—	—	—	—	—	—	—
Estimated parameters of the first cycle [second cycle]								
Variance	0.0006	0.0200	0.0411	0.0012 [0.0012]	0.0181	—	0.0130	—
ρ	0.8429	0.8438	0.9195	0.8979 [0.9513]	0.6645	—	0.7783	—
Period (yr.)	0.4424	0.6942	0.8341	0.4674 [0.8786]	0.5405	—	0.6661	—
λ_c	1.1834	0.7542	0.6278	1.1202 [0.5960]	0.9687	—	0.7861	—
Goodness-of-fit measures and diagnostic checking								
SE	0.0321	0.1403	0.1767	0.0325	0.1382	0.1997	0.1322	0.1748
R(1)	0.0203	0.1292	-0.0437	0.1149	0.0300	0.0886	0.0746	-0.0347
R(6)	-0.0389	0.0606	0.0126	-0.0821	0.0500	-0.1706	0.0518	-0.2389
Q(4)	4.5138	3.7516	4.4654	6.3933	2.8203	7.1317	5.2419	7.3353
Q(7)	6.7565	8.4173	6.2431	6.5268	5.6989	7.5821	8.3902	9.2621
PEV	0.0010	0.0197	0.0312	0.0011	0.0191	0.0399	0.0175	0.0306
R_d^2	0.9191	0.3233	0.1630	0.6929	0.2383	0.2479	0.3192	0.0893
AIC	-6.5683	-3.7064	-3.1996	-6.4889	-3.7769	-3.0852	-3.8200	-3.3518
BIC	-6.2873	-3.5057	-2.9587	-6.1645	-3.6147	-2.9636	-3.6173	-3.2302
Max ln L	140.084	83.0573	72.6282	142.888	84.2527	63.6983	83.8088	71.9792

Notes:

1. $\sigma_{\kappa 1}$ and $\sigma_{\kappa 2}$ are the standard deviation of the disturbance terms of the first and the second cycle respectively.
2. φ_{1T} and φ_{2T} are the final state of the first and the second cycle respectively.
3. Since money supply level contains a stochastic trend component, the standard statistical inference of Ψ_1 and Ψ_2 is interpreted with care.

Figure 3.9 Structural Time Series Components of the Hungarian Money Supply Level (M_t)

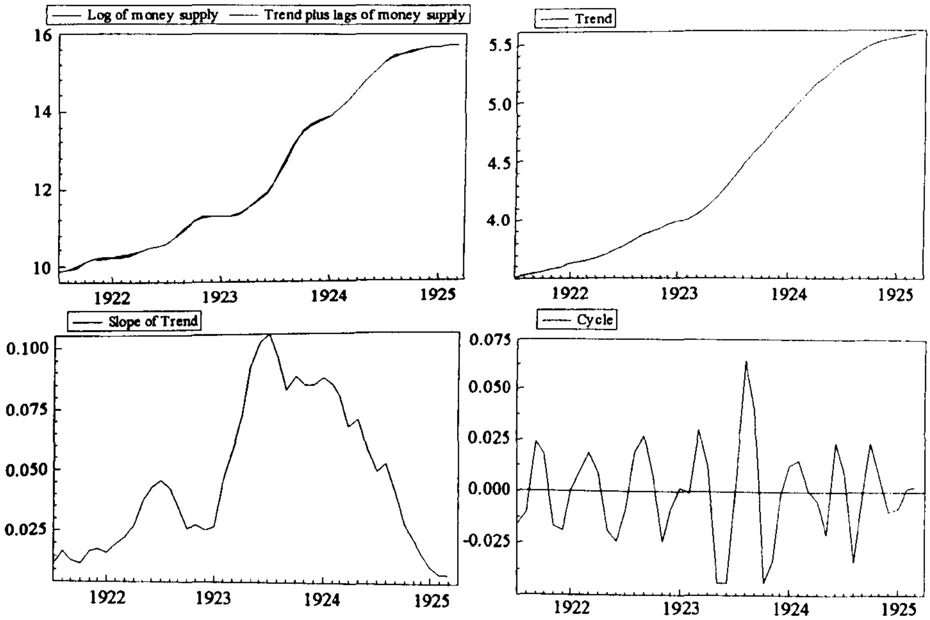


Figure 3.10 Structural Time Series Components of the Hungarian Composite Price Level (π_{1t})

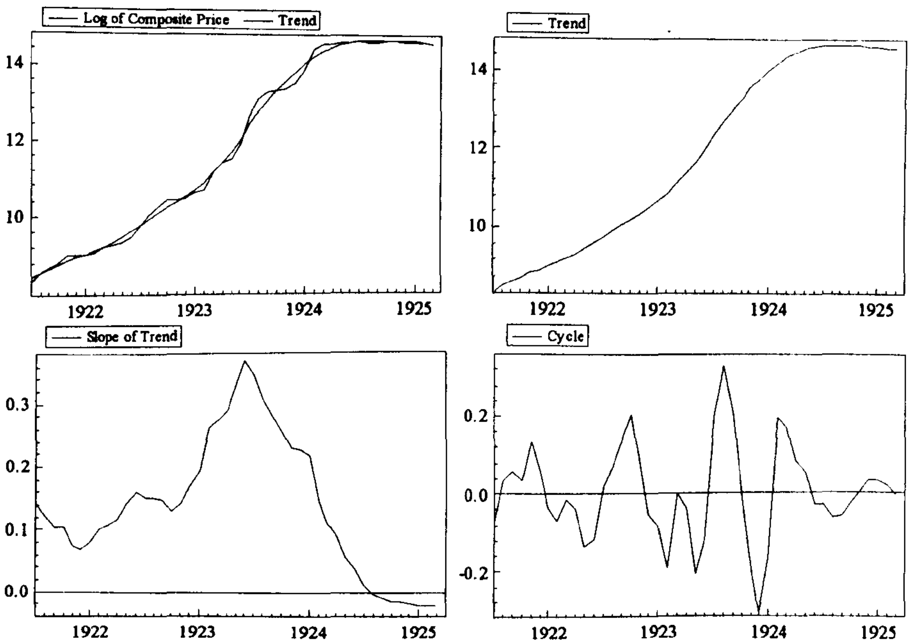


Figure 3.11. Structural Time Series Components of the Hungarian Exchange Rate (π_{2t})

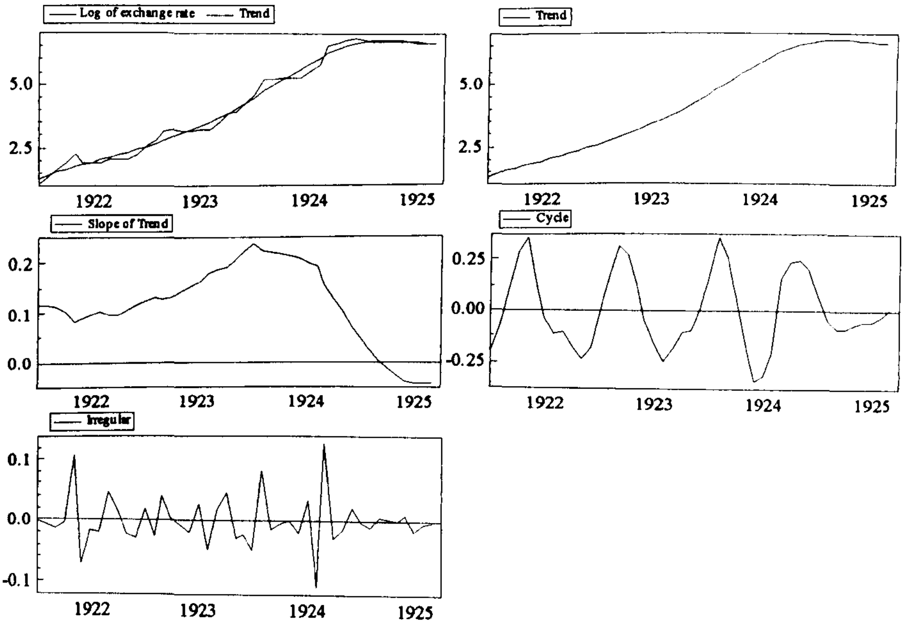


Figure 3.12. Structural Time Series Components of the Money Supply Change (ΔM_t) for Hungary

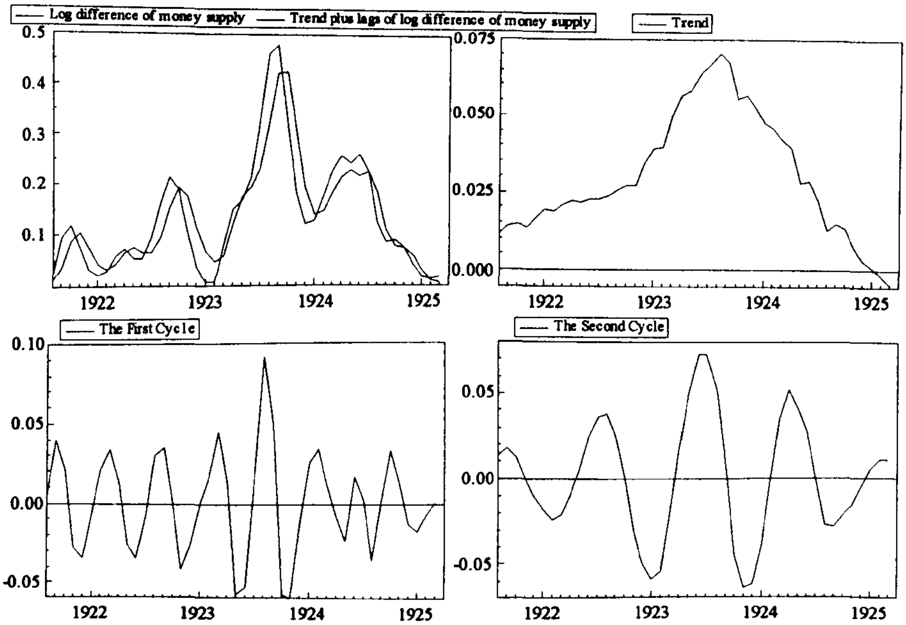


Figure 3.13. Structural Time Series Components of the Price Change ($\Delta\pi_{1t}$) for Hungary

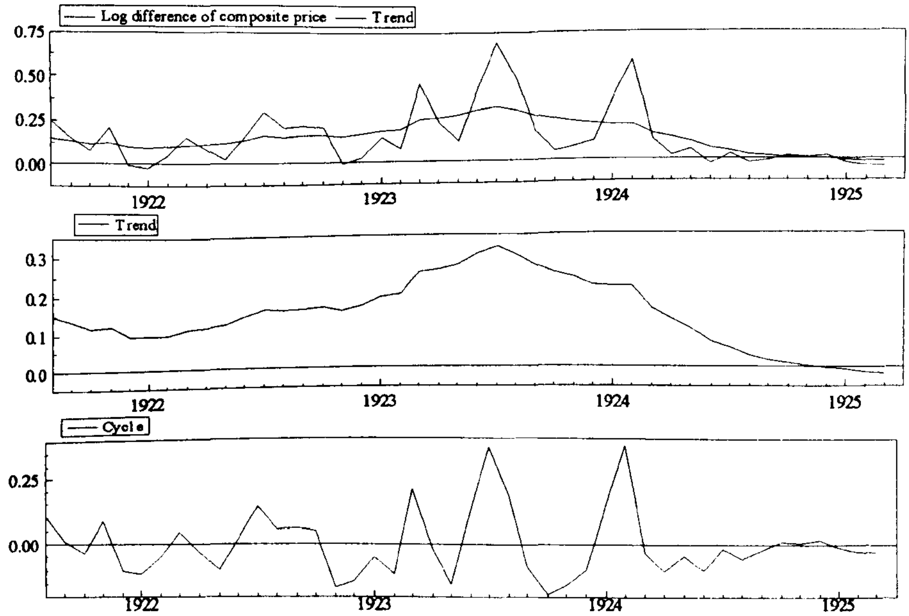


Figure 3.14. Structural Time Series Components of the Exchange Rate Change ($\Delta\pi_{2t}$) for Hungary

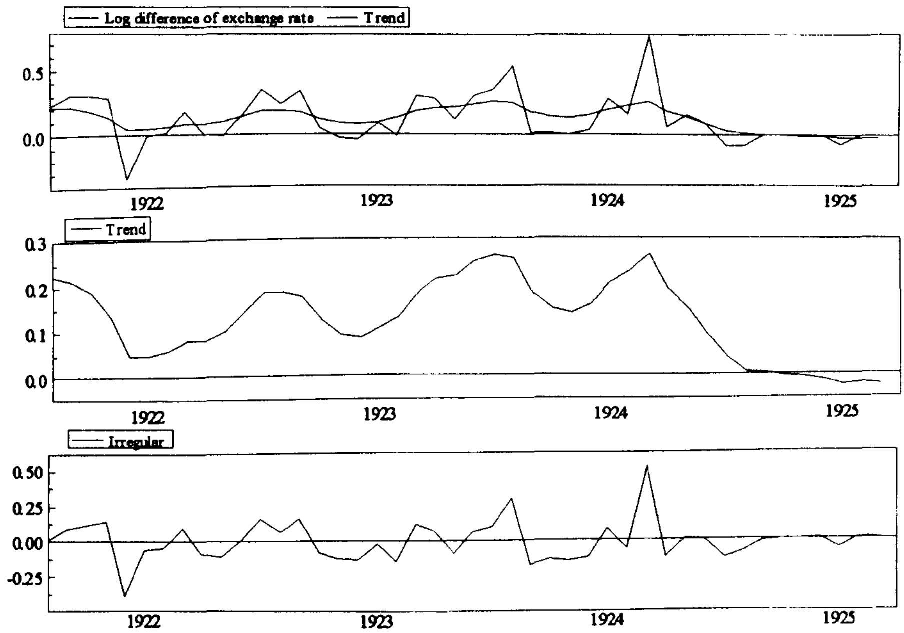


Figure 3.15. Structural Time Series Components of the Real Money Balance in terms of Composite Price ($M_t - \pi_{1t}$) for Hungary

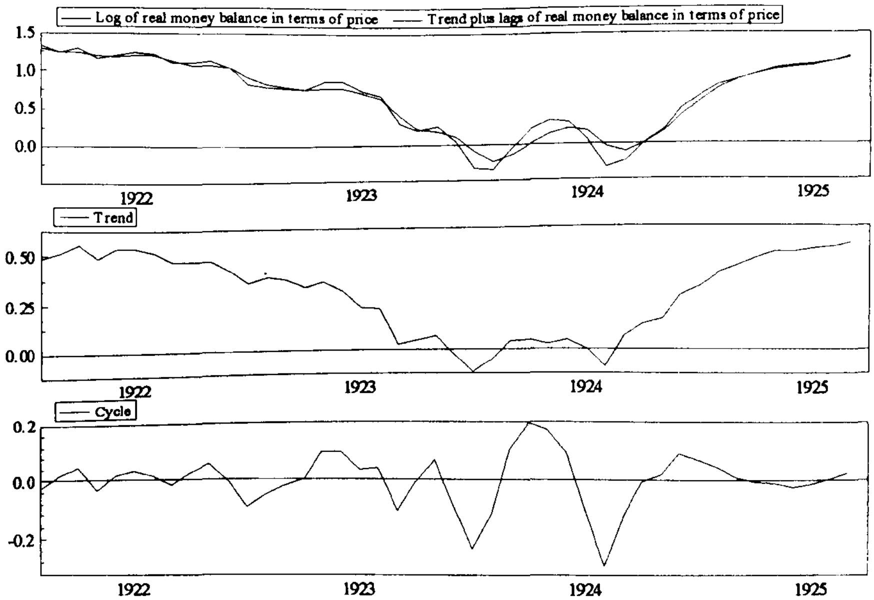


Figure 3.16. Structural Time Series Components of the Real Money Balance in terms of Exchange Rate ($M_t - \pi_{2t}$) for Hungary

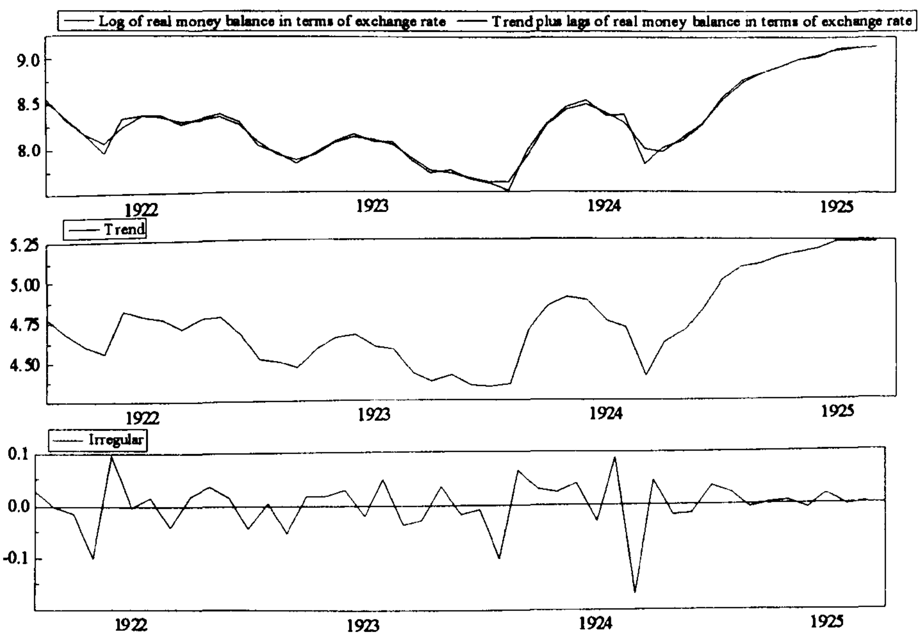


Table 3.3 ML Estimation Results of the Structural Time Series Model for Poland

Variables	M_t	$\pi_{1,t}$	$\pi_{2,t}$	ΔM_t	$\Delta\pi_{1,t}$	$\Delta\pi_{2,t}$	$M_t - \pi_{1,t}$	$M_t - \pi_{2,t}$
Estimated standard deviation of disturbances [q-ratio]								
σ_η	——	——	——	0.07877 [1.0000]	0.0608 [0.4229]	0.1068 [0.4595]	0.1468 [1.0000]	0.1159 [0.7036]
σ_ζ	0.0788 [1.0000]	0.0629 [0.4925]	0.0617 [0.3292]	——	——	——	——	——
σ_κ	——	0.1278 [1.0000]	0.1875 [1.0000]	——	0.0675 [0.4694]	——	0.0861 [0.5867]	0.1648 [1.0000]
σ_ε	——	0.0219 [0.1712]	——	——	0.1438 [1.0000]	0.2324 [1.0000]	——	——
Filtered estimates of final state vector at time T and the estimated coefficient of lagged dependent variable with the corresponding RMSE in the brackets								
μ_T	3.5694 (3.7453)	19.385* (0.2447)	11.350* (0.3109)	-0.2807** (0.1360)	0.4218* (0.1122)	0.2877** (0.1406)	0.2424 (0.1538)	8.1719* (0.1899)
β_T	-0.2807*** (0.1571)	0.4572* (0.1156)	0.4357* (0.1218)	——	——	——	——	——
φ_T	——	-0.1282 (0.2191)	0.0411 (0.3109)	——	-0.3867 (0.1127)	——	0.5860 (0.1538)	0.5837 (0.1899)
Ψ_i	0.8362* (0.1889)	——	——	0.8362* (0.1889)	——	——	——	——
Estimated parameters of cycle								
Variance	——	0.1384	0.1528	——	0.0392	——	0.0905	0.1114
ρ	——	0.9392	0.8774	——	0.9400	——	0.9581	0.8696
Period (yr.)	——	0.9567	1.0921	——	0.9142	——	0.7999	0.9153
λ_c	——	0.5473	0.4794	——	0.5727	——	0.6546	0.5720
Goodness-of-fit measures and diagnostic checking								
SE	0.0755	0.2104	0.2689	0.0766	0.2150	0.2880	0.1994	0.2399
R(1)	0.2588	-0.0226	-0.0386	0.2588	-0.0017	0.1144	0.1531	-0.0374
R(6)	-0.1492	-0.0100	0.0763	-0.1492	-0.0156	-0.0038	0.0433	0.0508
Q(4)	4.4356	1.4354	4.8692	4.4356	1.9140	3.0416	4.3929	6.2058
Q(7)	5.9300	2.6137	6.5065	5.9300	3.0594	6.6827	6.2526	8.1936
PEV	0.0057	0.0443	0.0723	0.0059	0.0462	0.0829	0.0397	0.0576
R^2_d	0.8936	0.4586	0.2024	0.3819	0.2135	0.2010	0.2749	0.1781
AIC	-5.0049	-2.8100	-2.3703	-5.0286	-2.8113	-2.3845	-3.0149	-2.6442
BIC	-4.8743	-2.5540	-2.1570	-4.9406	-2.5958	-2.2983	-2.8425	-2.4718
Max ln L	87.2775	55.8956	46.9873	87.2775	55.6872	45.3099	58.4529	51.9073

See Notes to Table 3.1 and Table 3.2.

Figure 3.17. Structural Time Series Components of the Polish Money Supply Level (M_t)

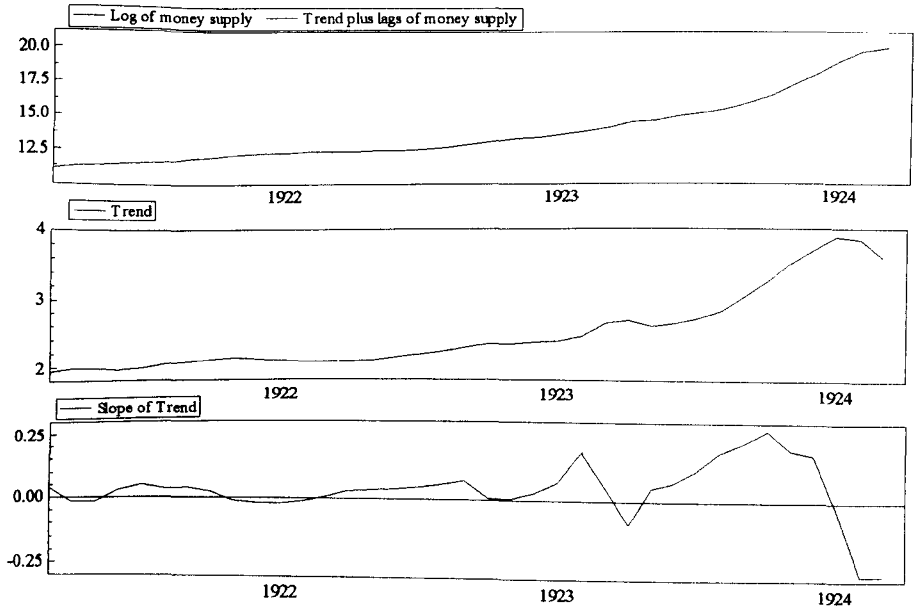


Figure 3.18. Structural Time Series Components of the Polish Wholesale Price Level (π_{1t})

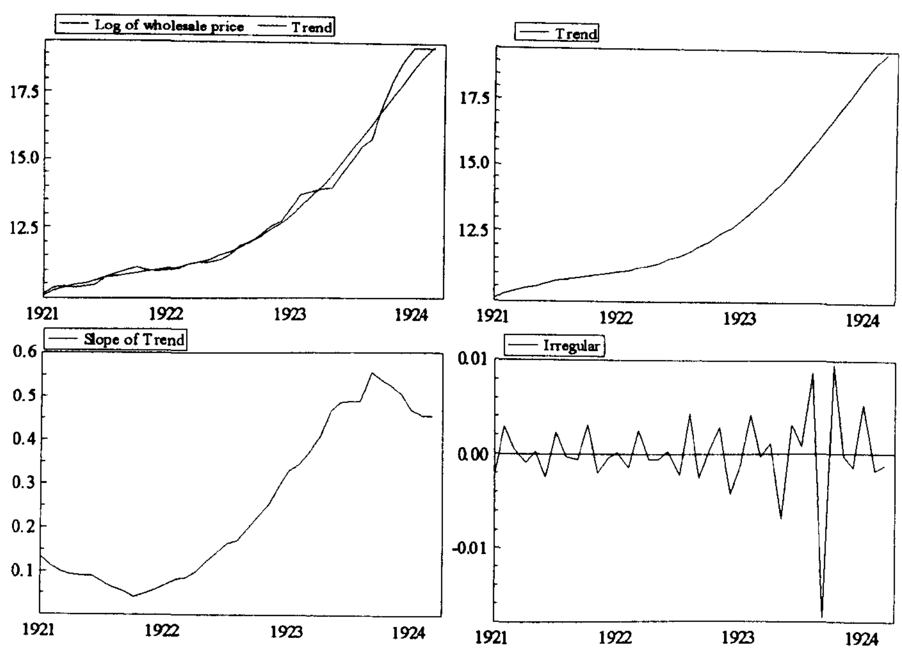


Figure 3.19. Structural Time Series Components of the Polish Exchange Rate (π_{2t})

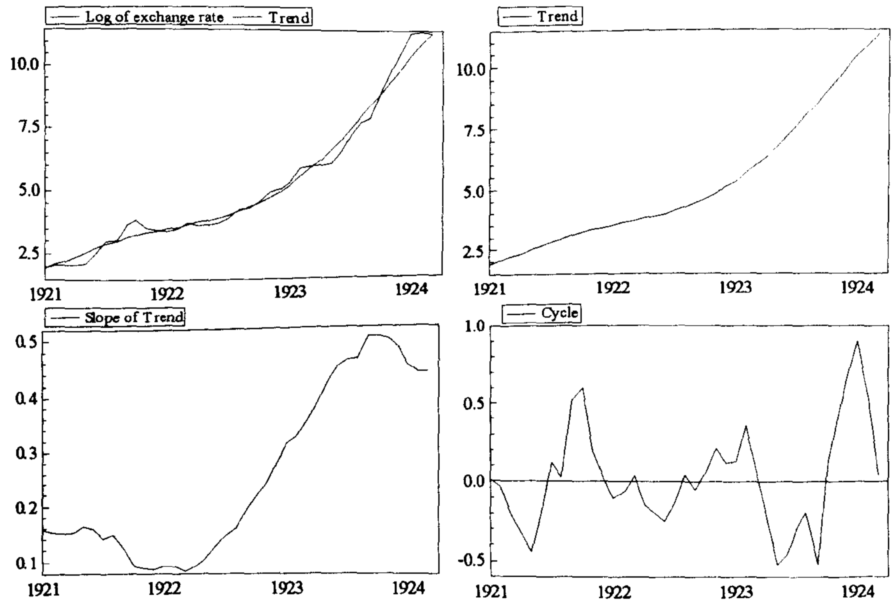


Figure 3.20. Structural Time Series Components of the Money Supply Growth (ΔM_t) for Poland

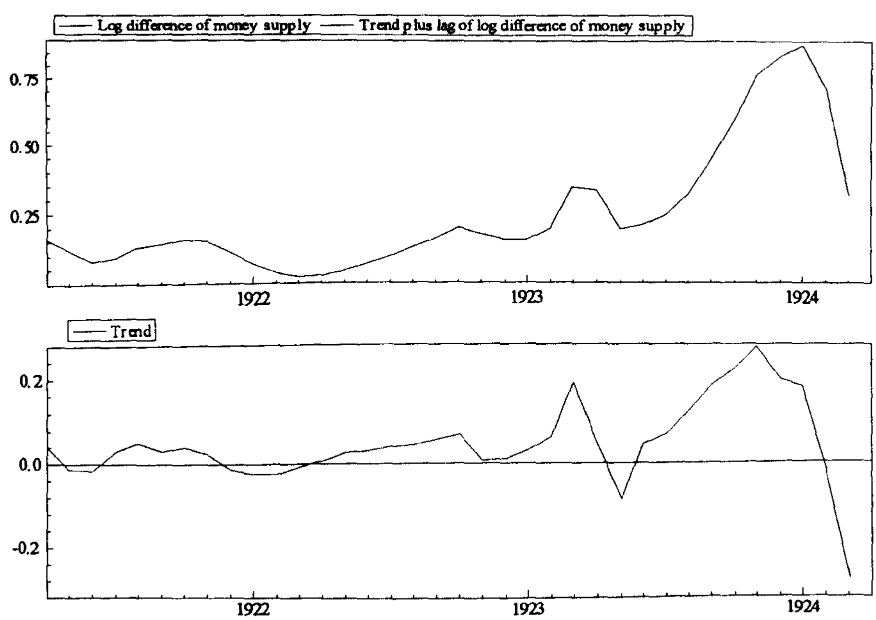


Figure 3.21. Structural Time Series Components of the Price Change ($\Delta\pi_{1t}$) for Poland

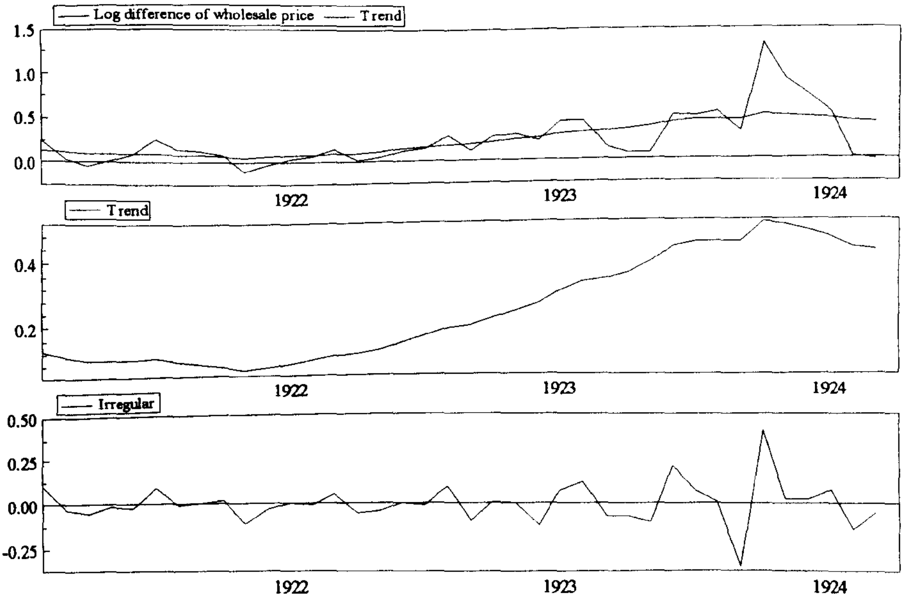


Figure 3.22. Structural Time Series Components of the Exchange Rate Change ($\Delta\pi_{2t}$) for Poland

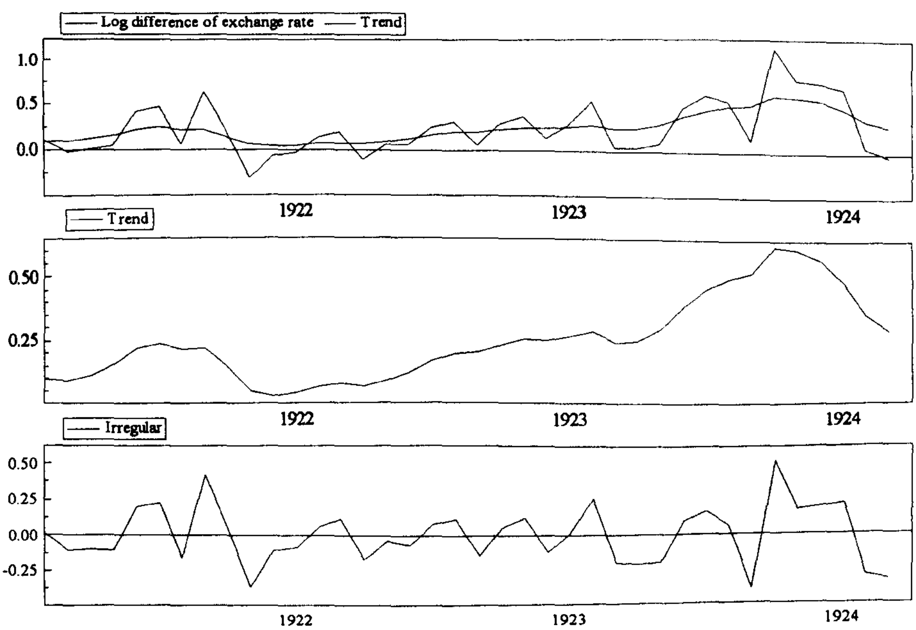


Figure 3.23. Structural Time Series Components of the Real Money Balance in terms of Wholesale Price ($M_t - \pi_{1t}$) for Poland

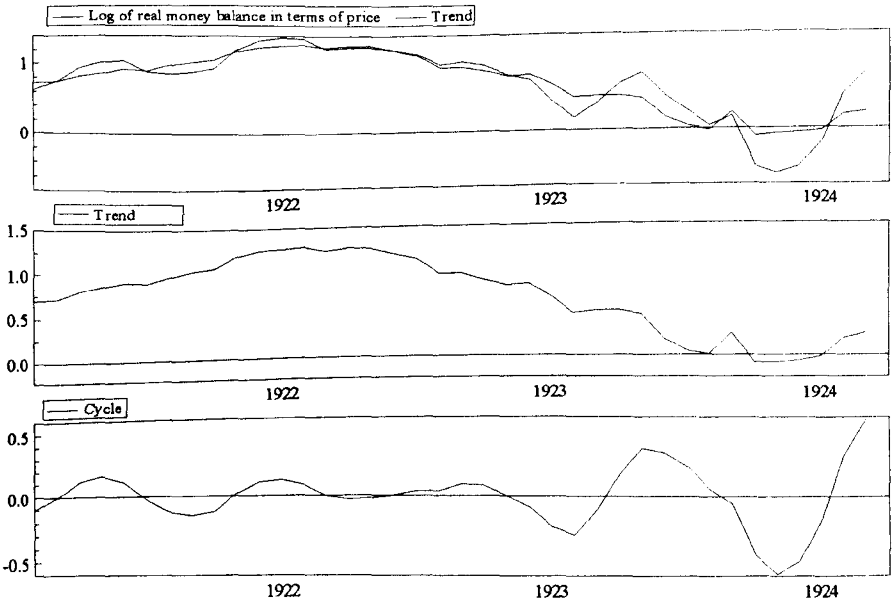
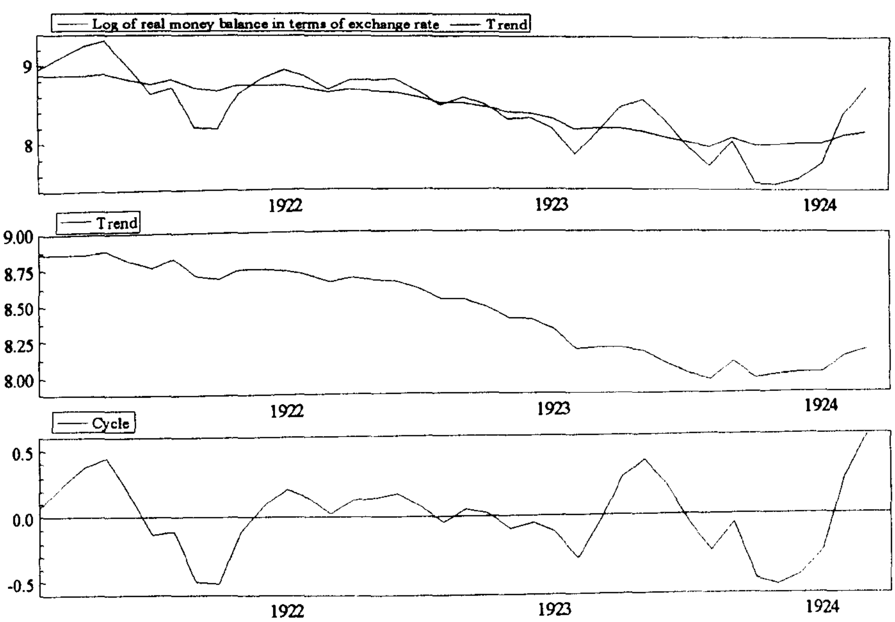


Figure 3.24. Structural Time Series Components of the Real Money Balance in terms of Exchange Rate ($M_t - \pi_{2t}$) for Poland



CHAPTER FOUR ORTHOGONALITY TESTS AND BUBBLES[16]

4.1 Introduction

This chapter starts the empirical testing of the presence of both price and exchange rate bubbles during the three hyperinflations of Germany, Hungary and Poland. Many previous studies have used the Cagan model to detect bubbles during hyperinflation but most of them assume that the model they use is correctly specified. The presence of model misspecification may be consequently interpreted as evidence of bubbles. This chapter shows that it is possible to test model misspecification separately from the presence of bubbles. I have designed a set of orthogonality tests for conducting an empirical study of bubbles, which can be used to identify specification errors and explosive bubbles. The set of orthogonality tests is an indirect testing procedure that does not require specifying a particular form of bubble process. Any bubbles not orthogonal to particular information sets will be detected. The problems of multiple bubble solutions in the framework of rational expectations illustrated in Chapter Two make indirect tests look more attractive. Moreover, based on the graphics of the structural time series components shown in Chapter Three, there are possible regime changes in data generation for the three countries under study. In order to avoid the problems of statistical inference caused by regime changes, the final few months of hyperinflations are all truncated in the present empirical work. The empirical results indicate the presence of misspecification in exact Cagan's hyperinflation model under rational expectations but do not support the evidence of price and exchange rate bubbles. The plan of this chapter is organized as

[16] This chapter extends the ideas and empirical results in Woo and Chan (2001) and Woo, *et al* (2003).

follows: Section 2 reviews the related literature with critical evaluation. Section 3 discusses the procedures of orthogonality tests and the econometric methodology. Sample periods of data and empirical results are reported in Section 4. The final section concludes.

4.2 Literature Review

In the bubble-testing literature, many econometric methods have been developed to carry out the analysis. I take a brief review of some of them, especially those of currency bubble testing.[17] I mainly focus on the indirect tests here, which rely on testing for the formulated null hypothesis of bubbles or no bubbles without specifying a particular bubble process.

One early attempt of indirect tests is the variance bounds or variance inequality test, which was first suggested by Shiller (1981), and LeRoy and Porter (1981) in order to detect whether the fundamental solution or the present value relation alone is valid to adequately characterize the actual asset prices. The variance bounds test is based upon the mathematical property that the conditional expectation of any random variable is less volatile than the variable itself. Hence, if the actual price (π_t) is an unbiased forecast of the corresponding perfect foresight analogue (π_t^*) represented by the present discounted value of future underlying fundamentals up to the m[th] period as $m \to \infty$, the variance of the perfect foresight price will constitute the upper bound for the variance of actual price such that $\mathrm{Var}(\pi_t^*) \geqq \mathrm{Var}(\pi_t)$. Violation of the variance bounds implies the rejection of the fundamental solution. Since the violation

[17] Camerer (1989) documents the detailed survey of bubble testing.

is attributable to the presence of an unobserved component with large variance, which is usually considered as a bubble, rejection of the fundamental solution is equivalent to the rejection of the no bubble hypothesis. However, there are some alternative methodological problems that cause violation of the variance bounds test, even if the fundamental solution is not rejected. According to Flavin (1983), the sample variance of π_t will be estimated with downward bias when π_t is positively autocorrelated and the sample size is smaller than m periods. The sample variance of π_t^* are however estimated with a greater downward bias because π_t^* is more strongly autocorrelated than π_t, and the sample size must be smaller than m periods which are infinite under the construction of π_t^*. Hence, in finite samples, the variance bounds estimated from sample variances may be reversed. Moreover, the population variances of π_t^* and π_t do not exist when both the underlying fundamentals and π_t follow an integrated process with infinite variance. Furthermore, if the sample's actual terminal price is used to construct a measurable counterpart to π_t^* as suggested by Shiller (1981), the presence of bubbles will not cause violation of the variance bounds (Flood and Hodrick, 1986).

West (1988b) derives another kind of variance inequality test to examine the presence of bubbles. The inequality states that the variance of the innovations (or forecast errors) to the expected present discounted value of fundamentals made with a limited information set is larger than that based on the full information set.[18] For $\boldsymbol{H}_t$ to denote a limited information set, it is noted that

[18] The West's (1988b) inequality on innovation variance is considered to be a direct implication of the LeRoy-Porter inequality (see Cuthbertson, 1996, chapter 6).

$E[(\pi_t^f - E(\pi_t^f \mid H_{t-1}))^2] > E[(\pi_t - E(\pi_t \mid \Omega_{t-1}))^2]$. The presence of bubbles will lead to violation of the inequality if the bubbles are positively correlated with fundamentals. Using West's (1988b) variance inequality test, West (1987b) examined whether the standard monetary models were consistent with the 1974-84 variability of the deutschemark exchange rate, and then provided evidence against speculative exchange rate bubbles. Instead of the variance of π_t and π_t^*, the variance bound test of West (1988b) just requires estimating the variance of innovations, which are stationary and assumed to be serially uncorrelated. Thus, West's (1988b) variance bounds test does not appear to be subject to the small sample bias, the problem of nonstationarity and the problems of the proxy for π_t^*, which are all found in the variance bounds of Shiller (1981) and LeRoy and Porter (1981). Nonetheless, all kinds of variance bounds tests involve a joint hypothesis of correct model specification and the absence of bubble solution. Violation of the variance bounds may be caused by several possible alternatives other than the presence of bubble solution, which include, for instance, the existence of time-varying discount rate, irrationality of expectations or omission of fundamentals in the present value relation under study. Then, the presence of model misspecification may be erroneously interpreted as evidence of bubbles.[19]

Moreover, it is expected that the bubble existence will cause some extremely large price increases as they explode, and even larger price drops when they collapse, so that the distribution of price changes is considered to have large kurtosis and negative skewness if stochastic bubbles exist. Okina (1985) found large kurtosis for

[19] Gilles and LeRoy (1991) and West (1988a) provide detailed surveys and critical evaluations of the variance bounds tests.

the short-term changes in exchange rates of the Canadian dollar, French franc, deutschemark, Japanese yen and British pound during 1973-1980, which was consistent with short-lived bubbles. Evans (1986) defined a bubble as a nonzero median in the distribution of the excess return to holding foreign currency. He found such a bubble in the British pound during 1981-1984. Furthermore, if bubbles grow for a while and then burst afterwards, the excess return to holding the assets or simply the asset price changes, will tend to be of the same sign while the bubble survives, and then of a reverse sign when the bubble collapses. The total number of runs for the existence of bubbles will be smaller than that for a random sequence of excess return on assets (or asset price changes). The results of run tests conducted by Okina (1985) were consistent with short-lived bubbles. Nevertheless, it is found that outliers and non-zero median may be equally present in the fundamental components. Also, run tests may not have high power against stochastic bubbles with alternate periods of explosion and crash. Hence, the above tests for bubble existence are still inconclusive.

Another approach for bubble testing is West's (1987a) specification test, which examines whether two sets of parameter estimates differ significantly using the Hausman's (1978) specification tests. The first set includes implied values of parameters which are constructed using the Hansen and Sargent's (1980) formula from the estimates of a pair of equations: one is the equation of an equilibrium valuation model that admits a general solution[20] and another is the ARIMA equation generating the fundamental process. The second set is derived by a distributed lag

[20] It is the arbitrage equation of stock price in West (1987a) that yields the discount rate, or the money demand equation in Casella (1989) that yields the elasticity of money demand with respect to expected inflation rate.

projection of asset price onto the underlying fundamental variables, which satisfies the fundamental solution only. The point estimated parameters from the second set would be asymptotically biased upwards when bubbles are present and result from an overreaction to news about fundamentals, leading to significance of the Hausman's (1978) test statistic. Apart from the correlation of the bubble with the fundamentals, the bias may also be created by correlation of the bubble innovation with innovations in any fundamental variables, or by the bubble's mean which may bias the estimate of the intercept term (Flood and Hodrick, 1990). Therefore, West's (1987a) approach can detect fundamental-dependent bubbles (Ikeda and Shibata, 1992 and 1995) or intrinsic bubbles (Froot and Obsteld, 1991). Using West's (1987a) approach, Meese (1986) found evidence of exchange rate bubbles for the deutschemark and British pound over the period 1973-82. Casella (1989) independently developed the same specification test as West (1987a), and examined the presence of price bubbles during the 1920's German hyperinflation. The evidence of bubbles was found under the restriction of exogenous money supply, whereas the presence of bubbles was rejected if money supply process was assumed to be endogenous.

However, the distribution of the Hausman's (1978) test statistic may not be consistent. That is, in the presence of bubbles, the asymptotic probability of rejecting the null hypothesis of no bubbles may not be unity, even if the two sets of parameter estimates will be asymptotically different with probability one (West, 1987a). More importantly, the actual size of the Hausman's (1978) test is likely to be larger than the designated nominal level in small samples, leading to an incorrect rejection of the no bubble hypothesis (Dezhbakhsh and Demirguc-Kunt, 1990). Furthermore, conventional econometric methods may lead to spurious empirical results (Phillips,

1986) while they were used in regressions with nonstationary variables. In addition, Casella (1989) imposed a random walk assumption on the money demand disturbances. The random walk assumption implies a misspecification of the Cagan model from the outset in sense of Engle and Granger (1987). If the assumption is incorrect, misleading conclusions of bubbles will be reached.

Flood and Hodrick (1990) emphasized that the bubbles tests involve the joint hypothesis of correct model specification and no bubbles. They provided a detailed description of the model misspecification in West (1987a).[21] The presence of model misspecification may be interpreted as evidence of bubbles. Durlauf and Hooker (1994) and Hooker (2000) focus on this issue in the context of price bubbles during the hyperinflationary episodes, and suggest a testing procedure that relies upon the orthogonality properties of the flow and stock variables. Such approach was originally designed to identify different sources of the violations of the fundamental solution so as to avoid erroneous interpretation of model misspecification as evidence of bubbles. Similar to West's (1987a) approach, the Durlauf-Hooker approach only requires the bubbles under study to be correlated with information sets. Chen (1995 and 1999) employed the Durlauf-Hooker approach to test for model misspecification and bubbles in the German, Canadian and the US stock markets from 1970 to 1993 and in the Taiwan's foreign exchange market from 1987 to 1995 respectively. The results supported the evidence of model misspecification but rejected the presence of bubbles in all cases. Woo, Chan and Lee (2001) employed this approach to test for

[21] Similarly, Flood, Hodrick and Kaplan (1986) found that the Euler equation of stock price and dividend forecasting equation employed in West (1987a) for stock market bubble testing were seriously misspecified, leading to the false evidence of price bubbles.

price bubbles in the Hong Kong residential property markets and found the evidence of bubbles in property market before 1997.

Despite the favourable ideas of the Durlauf-Hooker approach for detecting bubbles, there are several drawbacks of the practical testing procedures that warrant my attention. First, the works of Durlauf and Hooker (1994) and Hooker (2000) make restrictive parametric process assumptions on the unobservable money demand disturbance of the Cagan model. They impose zero, random walk and AR(1) assumptions on the money demand disturbance. This is undesirable because the assumptions about the nature of the disturbance term may be made erroneously. More seriously, specification errors may be caused by or enlarged by misspecification of the generating process for the unobserved demand disturbance. Second, for the case where both the flow and stock orthogonality conditions are rejected, it implies that model misspecification is likely to be present but it is not certain whether bubbles exist. As suggested by Durlauf and Hooker (1994) and Chen (1995 and 1999), unit root tests are applied to the flow and stock variables for further analysis.[22] Ironically, Durlauf and Hooker (1994) and Chen (1995) have ever critically evaluated the use of unit root econometric methods in bubble testing. In fact, the Durlauf-Hooker approach fails to determine whether the total model noise is caused by the existence of a bubble or model misspecification or both when the two orthogonality conditions are rejected, although the initiative of the Durlauf-Hooker approach is to separate tests of model specification from tests for the presence of bubbles. Third, the econometric analysis in literature was based on conventional regression methods

[22] The unit root methodologies for bubble testing will be discussed in Chapter Five and Chapter Six.

developed for analyzing stationary variables. It therefore cannot yield valid inferential statistics in regression models with nonstationary variables.

In view of the above drawbacks, my empirical work of bubble testing in this chapter differs from that of the conventional Durlauf-Hooker approach in the following aspects. In the first place, in contrast to Durlauf and Hooker (1994) and Hooker (2000), I have decided not to impose any restrictive and arbitrary assumptions on the parametric process of the money demand disturbance of the Cagan model on the grounds that these assumptions may not be made correctly. Instead, I propose to use the exact version of the Cagan model for conducting our analysis. The advantage of using the exact Cagan model is that any deviations from the model, including the nonzero demand disturbance of any parametric process, will be captured by a model misspecification term.[23] It, therefore, obviates the need to impose any arbitrary and restrictive assumptions on the unobservable demand disturbance.[24] Secondly, in the Durlauf-Hooker framework, the rejection of both the flow and stock orthogonality conditions fails to test for the evidence of bubbles, though it implies that model misspecification is likely to occur. I therefore suggest a testing methodology that is revised from the original Durlauf-Hooker approach with an aim to successfully detect the presence of bubbles when evidence of specification error is found. Thirdly, I adopt the fully modified (FM) econometric techniques to allow for nonstationary components of estimators. The purpose of using the FM methods is to make semiparametric correction for the endogeneity and serial

[23] Cagan model is a stochastic model so that a fundamental disturbance term is included in the model. Hence, model misspecification refers to the omission of the disturbance term in the exact Cagan model. Misspecification term, disturbance term and random error are the common terminologies and can be used interchangeably.

[24] The assumption of zero disturbance term in the Cagan money demand model is then less arbitrary than the assumption made by Durlauf and Hooker (1994).

correlation bias, caused by nonstationarity of variables in the limit distribution of the FM estimators (Phillips and Hansen, 1990).[25] On the other hand, the FM inferential statistics are employed for hypothesis testing. Since the sample sizes for all three hyperinflation episodes under study are not large, I use finite-sample rather than asymptotic critical values, generated from the Monte Carlo simulations for evaluating our empirical results. Finally, my empirical study detects the presence of price and exchange rate bubbles rather than price bubbles only.

4.3 Procedures of Orthogonality Test and Econometric Methodology

The models under study in this chapter are the exact version of the stochastic Cagan models (2.1) and (2.2) specified in Chapter Two. The linear form of the exact Cagan models under rational expectations for $\pi_{1,t}$ and $\pi_{2,t}$ are shown as follows:

$$\boldsymbol{M}_t - \pi_{1,t} = \alpha_1 + \beta_1 \boldsymbol{E}_t(\Delta\pi_{1t+1}) \tag{4.1}$$

$$\boldsymbol{M}_t - \pi_{2,t} = \alpha_2 + \beta_2 \boldsymbol{E}_t(\Delta\pi_{2,t+1}), \tag{4.2}$$

Re-arranging Esq.(4.1) and (4.2) in terms of $\pi_{j,t}$ ($j = 1, 2$) gives us the following linear difference equations:

$$\pi_{j,t} = \frac{\alpha_j}{\beta_j - 1} - \frac{\boldsymbol{M}_t}{\beta_j - 1} + \frac{\beta_j}{\beta_j - 1}\boldsymbol{E}_t(\pi_{j,t+1}) \; ; \qquad j = 1, 2. \tag{4.3}$$

Since Eq.(4.3) is applicable to both price and exchange rate bubbles, we are able to eliminate the subscript j in subsequent equations in order to make the presentation

[25] Appendix 4.1 provides a simulation study to compare the sampling performance of several FM estimators in a finite sample.

neater. By recursively substituting forward for $E_t(\pi_{t+1+i})$ and using the law of iterated expectations, I obtain:

$$\pi_t = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i}) + \lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) \qquad (4.4).$$

Imposing the transversality condition, $\lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) = 0$, yields the fundamental solution, π_t^f, in the following form:

$$\pi_t^f = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i}). \qquad (4.5)$$

Eq.(4.5) shows that the fundamental solution is determined by an infinite sum of current and expected levels of money supply.

If the transversality condition is violated, then π_t^f in Eq. (4.5) is only a particular solution. The homogenous or bubble solution, B_t , satisfies the submartingale property of Eq.(2.13) in Chapter Two:

$$\left(\frac{\beta-1}{\beta}\right) B_t = E_t(B_{t+1}), \qquad (4.6)$$

However, it is reasonable to expect that exact relationships do not generally hold during hyperinflation. This is partly because some important fundamental variables, such as income and real interest rate, have been omitted from the exact Cagan models given in (4.1) and (4.2) and partly because measurement errors, expectations errors and deviations from the instantaneous PPP may occur. Hence, it is useful to add a misspecification term, S_t, to the general solution, π_t^g, so that π_t is decomposed into the following unobserved components:

$$\pi_t \equiv \pi_t^g + S_t = \pi_t^f + B_t + S_t. \tag{4.7}$$

The misspecification term, S_t, represents that component of π_t which cannot be attributed to the exact Cagan models represented by Eqs.(4.1) and (4.2). The sum of B_t and S_t is defined as the total model noise.

The works of Durlauf and Hooker (1994) and Hooker (2000) employ flow and stock variables to help detect the presence of model noise. To understand how these variables are constructed, let me first introduce the perfect foresight solution, π_t^*, which is defined as:

$$\pi_t^* = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i M_{t+i}. \tag{4.8}$$

Since $E(\pi_t^* \mid \Omega_t) = \pi_t^f$, the relationship between the perfect foresight and fundamental solutions can be expressed as:

$$\pi_t^* = \pi_t^f + V_t, \tag{4.9}$$

where V_t denotes the rational expectations forecast error. By definition, V_t is orthogonal to Ω_t. Substituting V_t into Eq. (4.7) obtains:

$$\pi_t - \pi_t^* = -V_t + B_t + S_t. \tag{4.10}$$

The implementation of the flow and stock tests requires the generation of the flow and stock variables. By applying the forward quasi-difference operator, $\phi \equiv -(1 - \frac{\beta}{\beta - 1} L^{-1})$, to both sides of Eq. (4.10), I obtain the flow variable, R_{t+1} as follows:

$$R_{t+1} \equiv \phi(\pi_t - \pi_t^*) = \phi(-V_t) + \frac{\beta}{\beta - 1}\omega_{t+1} + \phi(S_t). \tag{4.11}$$

where ω_{t+1} denotes a bubble innovation in the dynamic process of B_{t+1} in Eq.(4.6), which implies $E_t(\omega_{t+1}) = 0$. The stock variable, on the other hand, is simply equal to $\pi_t - \pi_t^f$. The Durlauf-Hooker flow and stock tests are implemented by projecting R_{t+1} and $\pi_t - \pi_t^f$, respectively, onto time-t information sets. When applying these two tests, there are three possible outcomes. Firstly, if there is no bubble and the model is correctly specified, the correlations of R_{t+1} and $\pi_t - \pi_t^f$ with Ω_t will be statistically insignificant. Secondly, if a bubble is present and the model is correctly specified, the $\pi_t - \pi_t^f$ projection will be nonzero and the R_{t+1} projection will be zero. Finally, if specification error is present, both the flow and stock projections will be nonzero irrespective of whether a bubble exists or not. In this situation, it is no longer possible to use the flow and stock tests to extract bubble noise.[26] To solve this problem, I design a testing methodology to test for specification

[26] Woo, Chan and Lee (2001) used an alternative procedure to extract the bubble noise even when misspecification exists, and the results supported the evidence of model misspecification and price bubbles in the Hong Kong property markets during 1980s and 1990s.

errors and bubbles, which is revised from the original Durlauf-Hooker framework in such a way that the presence of bubbles can be detected in a more rigorous way, especially when misspecification is found in the model.

My procedure begins with the analysis of the flow variable, $\boldsymbol{R}_{t+1}$. As shown in Eq. (4.11), $\boldsymbol{R}_{t+1}$ contains three elements; two of which, $\phi(-\boldsymbol{V}_t)$ and $\frac{\beta}{\beta-1}\omega_{t+1}$, involve time t+1 white noises. This implies that if $\boldsymbol{S}_t$ or $\phi(\boldsymbol{S}_t)$ is equal to zero, $\boldsymbol{R}_{t+1}$ will be a white noise process because it is constituted merely of two white noise components, which are orthogonal to Ω_t. On the other hand, if $\phi(\boldsymbol{S}_t)$ is nonzero, $\boldsymbol{R}_{t+1}$ will not be a white noise and it can be represented by an ARIMA (p, d, q) structure as follows:[27]

$$\Psi(L)\Delta^d(\boldsymbol{R}_{t+1}) = \theta(L)\eta_{t+1}, \qquad (4.12)$$

where $\Psi(L) = 1-\Psi_1\boldsymbol{L}-\ldots-\Psi_p\boldsymbol{L}^p$, $\theta(L) = 1+\theta_1\boldsymbol{L}+\ldots+\theta_q\boldsymbol{L}^q$ and η_{t+1} is a white noise. Since $\boldsymbol{R}_{t+1}$ and $\phi(\boldsymbol{S}_t)$ share the same dynamic process, I can examine the model misspecification by investigating the ARIMA process of $\boldsymbol{R}_{t+1}$; notice that since $\theta(L)\eta_{t+1}$ is an MA(q) process, $\Psi(L)\Delta^d(\boldsymbol{R}_{t+1})$ is orthogonal to Ω_{t-q} provided that the values of p, d and q are specified correctly.

[27] For instance, if the nonzero $\boldsymbol{S}_t$ is a white noise; then $\phi(\boldsymbol{S}_t)$ follows an MA (1) process and so is $\boldsymbol{R}_{t+1}$.

Taking the d[th] difference of ($\pi_t - \pi_t^f$) in Eq. (4.7) and applying the autoregressive operator $\Psi(L)$ to the resulting equation give:

$$\Psi(L)\,\Delta^d(\pi_t - \pi_t^f) \equiv \Psi(L)\,\Delta^d B_t + \Psi(L)\Delta^d S_t. \qquad (4.13)$$

Since $|\frac{\beta}{\beta-1}| < 1$ and $E[\phi(\Delta^d S_t)\,|\,\Omega_t] = E[\Delta^d(R_{t+1})\,|\,\Omega_t]$, applying ϕ^{-1} to both sides of Eq.(4.13) yields, in the limit,

$$E_t[\Psi(L)\Delta^d S_t] = -E_t[\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i \theta(L)\eta_{t+1+i}]. \qquad (4.14)$$

It may be seen from Eq. (4.14) that $E_t[\Psi(L)\Delta^d S_t]$ is orthogonal to Ω_{t-q}. Hence, one can infer from Eq.(4.13) that, under the null hypothesis of no bubble, $\Psi(L)\Delta^d(\pi_t - \pi_t^f)$ is orthogonal to Ω_{t-q}. This implies that if the projection of $\Psi(L)\Delta^d(\pi_t - \pi_t^f)$ onto Ω_{t-q} is zero, I can reject the existence of a bubble. Clearly, even if the total model noise is nonzero, it is still possible for us to check if a bubble exists in the data.

The differences between the original Durlauf-Hooker approach and my approach can be summarized in Table 4.1 and Table 4.2. Moreover, the practical testing procedures of my orthogonality tests are shown in Fig.4.1.

[Table 4.1 and 4.2 to be inserted here]

[Figure 4.1 to be inserted here]

From Figure 4.1, my testing procedures involve several major steps. The first step is to estimate the values of the flow variable, R_{t+1}. By substituting Eq. (4.8) into Eq. (4.11), I can alternatively express R_{t+1} as:

$$M_{t+1} - \pi_{t+1} - \Delta M_{t+1} = \frac{\alpha}{\beta} + \frac{\beta - 1}{\beta}(M_t - \pi_t) - \frac{\beta - 1}{\beta} R_{t+1} \qquad (4.15)$$

Using estimates of α and β, I can construct R_{t+1} from the residuals of Eq. (4.15). To estimate Eq. (4.15), I propose to employ the FM version of the generalized method of moment (GMM) (Kitamura and Phillips, 1997) with time-t information sets used as instrumental variables. After obtaining the values of R_{t+1}, I can investigate the dynamic process of the specification errors by examining the ARIMA (p, d, q) structures of R_{t+1}. If R_{t+1} is white noise, i.e., $p = d = q = 0$, this implies that $\phi(S_t)$ or S_t is unlikely to exist. In this situation, I can proceed directly to extract the bubble noise. However, if the process of R_{t+1} is not white noise, nonzero specification error, S_t, is likely to exist. This implies that the orthogonality condition of the stationary components of the instruments and R_{t+1} may be violated and consequently, the estimation of Eq.(4.15) may not be consistent. In view of this, I propose to obtain the consistent estimators of the Cagan model. This can be achieved by applying the operator $\Psi(L)\Delta^d$ to both sides of Eq. (4.15), which allows us to obtain:

$$\Psi(L)\Delta^d(M_{t+1} - \pi_{t+1} - \Delta M_{t+1}) = \Psi(L)\frac{\alpha}{\beta} + \frac{\beta - 1}{\beta}\Psi(L)\Delta^d(M_t - \pi_t) - \frac{\beta - 1}{\beta}\Psi(L)\Delta^d R_{t+1} \qquad (4.16)$$

With time t-q information sets used as instruments, the estimated values of $\Psi(L)\Delta^d(R_{t+1})$ in Eq. (4.16) should be orthogonal to Ω_{t-q}. This orthogonality property implies that the FM-GMM estimators for α and β would have a limit theory that is normal for the **I**(0) components with $\sqrt{T}$ consistency and mixed normal for the **I**(1) components with T consistency, where T stands for the number of usable observations. The properties of asymptotic normality and consistency allow the derivation of the FM-GMM t-ratio statistics to conduct hypotheses testing of the parameters in Eq. (4.16). The orthogonality of $\Psi(L)\Delta^d(R_{t+1})$ can be examined by the FM-GMM instrument validity test statistics (FM-GMM IV) or equivalently by projecting $\Psi(L)\Delta^d(R_{t+1})$ onto time t-q information sets via the FM-OLS method (Phillips, 1995a). The FM-OLS Wald statistics, W_R, can be used to test for zero coefficients in the projections. If the projections are statistically insignificant, this implies that the values of $\Psi(L)\Delta^d(R_{t+1})$ and Ω_{t-q} are uncorrelated.

To carry out the analysis of bubble testing in the final step, I need to generate the stock variable, $\pi_t - \pi_t^f$, which is equivalent to $[(M_t - \pi_t^f) - (M_t - \pi_t)]$. In contrast to the method of Shiller (1981) adopted by Durlauf and Hooker (1994) and Hooker (2000) to estimate π_t^*, I employ the Hansen and Sargent's (1980) formula to approximate the value of $(M_t - \pi_t^f)$.[28] The value of $(M_t - \pi_t^f)$ is estimated from the following equation using the Hansen and Sargent's (1980) formula:

[28] Shiller's (1981) method requires the imposition of truncation and terminal values on the infinite sum to estimate perfect foresight price. It then implies that the closer an agent gets to the terminal value, the less forward-looking he is. Also, as pointed out by Flood and Hodrick (1986), if the terminal value in the Shiller approximation contains a bubble, it will be exactly cancelled out in the construction of the perfect foresight price and the stock variable will never reveal the bubble. Hence, I choose to employ Hansen and Sargent's (1980) formula to estimate fundamental price.

$$(M_t - \pi_t^f) = \alpha - [\sum_{i=0}^{\tau-1} a_i \Delta M_{t-i} + \sum_{i=1}^{\gamma} b_i (M_{t-i} - \pi_{t-i})] \qquad (4.17)$$

where the coefficients a_i and b_i in Eq. (4.11) are derivable from the consistent FM-GMM estimator of β as well as the parameters c_i, f_i, g_i and h_i of the following equations:[29]

$$\Delta M_t = \mu + \sum_{i=1}^{\tau} c_i \Delta M_{t-i} + \sum_{i=1}^{\kappa} f_i (M_{t-i} - \pi_{t-i}), \qquad (4.18a)$$

$$(M_t - \pi_t) = \eta + \sum_{i=1}^{\gamma} g_i (M_{t-i} - \pi_{t-i}) + \sum_{i=0}^{\tau-1} h_i \Delta M_{t-i}. \qquad (4.18b)$$

Eq.(4.18a) is a money growth forecasting equation and Eq.(4.18b) describes the autoregressive representation of real money balance in terms of lagged real money balance and money growth rates.[30]

As explained previously, I can check whether a bubble exists by projecting $\Psi(L)\Delta^d$ [$(M_t - \pi_t^f) - (M_t - \pi_t)$] onto time t-q information sets. I employ the FM-OLS method to conduct the projections with the FM-Wald statistics, W_Ψ, being used for hypothesis testing. If the null hypothesis is rejected, evidence of a bubble is found. If not, the existence of a bubble is ruled out.

[29] The estimates of fundamental real money balance may contain approximation errors when only a subset of full information is used to construct the fundamental variable. The approximation errors are considered as a kind of specification errors.
[30] Chen (1995) describes the cross equation restrictions of (4.18a) and (4.18b) in greater detail.

4.4 Estimation Periods and Empirical Results

One issue that arises in studies of hyperinflation is the choice of estimation sample, given that the European hyperinflations ended with a monetary reform. The possible regime changes in data generation caused by a monetary reform during the final months of the hyperinflations may result in coefficient instability and statistical problems of inference. The common practice in literature is to truncate the final few observations of the data. Hence, in the present study, the estimates and tests are conducted on the pre-reform samples (up to the months when the expectations of monetary reform became significant). Then, the German data used for estimation cover the periods from January 1920 up to June 1923, after which expectation of a monetary reform began nonnegligible (Flood and Garber, 1980b). For Hungary, the data sets include the periods from July 1921 to February 1924 with March 1924 identified as the reform date. The Polish data cover the period between January 1921 and December 1923 with the reform date taken as January 1924. The dates of monetary reforms are quoted from Sargent (1982).

4.4.1 Price bubbles

The results from the estimation of α_1 and β_1 as well as $\boldsymbol{R}_{1,t+1}$ in Eq. (4.15) are reported in Table 4.3. The time t information sets used as instruments consist of current and lagged $\Delta \boldsymbol{M}_t$ and $\Delta \pi_{1,t}$. As shown by the Phillips-Perron (1988) unit root tests, the values of $\boldsymbol{R}_{1,t+1}$ are likely to be stationary, i.e., $d = 0$, for all the countries under study. In addition, the FM-GMM instrument validity (FM-GMM IV) test statistics indicate that the null hypotheses of the overidentifying restrictions are all rejected, signifying the possible correlation of $\boldsymbol{R}_{1,t+1}$ and the stationary components of instruments, i.e., time-t information sets. Therefore, the parameters reported in Table 4.3 are not estimated consistently. This may be attributed to the presence of non-zero specification errors in the Cagan model (4.1).

[Table 4.3 to be inserted here]

Since the values of $\boldsymbol{R}_{1,t+1}$ are possibly correlated with the stationary components of the time t information sets, it is necessary to examine the ARMA (p, q) structures for $\boldsymbol{R}_{1,t+1}$, which are reported in Table 4.4. For Germany, and Poland, the dynamic processes of $\boldsymbol{R}_{1,t+1}$ with respect to all information sets are best represented by the ARMA (0,1) and ARMA (0,4) models, respectively. For Hungary, the ARMA (0,2) model fits the time series of $\boldsymbol{R}_{1,t+1}$ best, except in two cases where the ARMA (1,2) model is preferable. This implies that nonzero misspecification is likely to occur in the exact Cagan model (4.1) for all the three countries. This corroborates results reported by Engsted (1993) and Taylor (1991) who rejected the exact rational expectations Cagan model and found strong evidence for the presence of stationary specification errors. However, our result contradicts the random walk assumption on money demand disturbances made by, for example, Flood and Garber (1980b), Sargent and Wallace (1973), Sargent (1977), Salemi and Sargent (1979) as well as Casella (1989). It also contradicts the findings of Goodfriend (1982), which supported the exact rational expectations Cagan model for the same European countries as are examined here. Goodfriend's (1982) results were based upon the absence of serial correlation from the residuals of the Cagan model, but the possibility of an MA process in the residuals was ignored.

[Table 4.4 to be inserted here]

Since nonzero specification errors are found for all three countries, I proceed to step two in order to consistently estimate Eq.(4.16), using time t-q information sets as instruments. The empirical results of Eq. (4.16) are presented in Table 4.5. The consistent estimation allows us to derive the FM-GMM t-ratio statistics for testing the null hypotheses that $\frac{\alpha_1}{\beta_1} = 0$ and $\left(\frac{\beta_1 - 1}{\beta_1}\right) \leq 1$ in Eq. (4.16), The t-ratio statistics show that β_1's are negative for all countries and α_1's are nonzero in most cases.

[Table 4.5 to be inserted here]

The requirement for consistency relies upon the assumption that the valid instruments have been chosen in such a way that the estimated values of $\Psi(L)\ \boldsymbol{R}_{1,t+1}$ are orthogonal to Ω_{t-q}. From the FM-GMM IV statistics reported in Table 4.5, the null hypothesis of over-identifying restrictions cannot be rejected. However, as pointed out by Kitamura and Phillips (1997), the power properties of the FM-GMM IV statistics in finite samples are not fully known. Equivalently, an alternative way to confirm the evidence of orthogonality is to project $\Psi(L)\ \boldsymbol{R}_{1,t+1}$ onto Ω_{t-q}, with the FM-OLS Wald statistics, W_R, being used to test for zero projections under the null. As the W_R statistics in Table 4.6 show, I cannot reject the null of zero projections for all three countries. Therefore, the estimations of Eq. (4.16) are consistent. Based on these findings, I can proceed to the final step, which projects the values of $\Psi(L)\ (\pi_{1,t} - \pi_{1,t}^{f})$ onto Ω_{t-q}. From the FM-OLS Wald statistics, W_{Ψ}, reported in Table 4.6, the null hypothesis of zero coefficients in the projections cannot be rejected. I therefore conclude that evidence of price bubbles is not found in the data for Germany, Hungary and Poland. These no-bubble results concur with those of Engsted (1993, 1994). Hooker (2000) provided mixing evidence of flow and stock tests in exact case, AR(1) case and random walk case together. Hooker (2003) concluded that priors over the model cases could affect the interpretation of the results, and he was more inclined to place more weight on the random walk case and as contrary to the critical review from Engsted (2003), to view exact case results as reflecting type II errors. In my opinion, such arguments mainly arise from the puzzling evidence of flow and stock tests, produced by the Durlauf-Hooker testing procedure.

[Table 4.6 to be inserted here]

4.4.2 Exchange rate bubbles

The testing procedures in exchange rate bubbles testing are exactly the same as those for price bubbles. By estimating the values of α_2 and β_2, I construct $\boldsymbol{R}_{2,t+1}$ in Eq. (4.15) using time-t information sets as instruments. The instruments consist of current and lagged $\Delta \boldsymbol{M}_t$ and $\Delta \pi_{2,t}$. In Table 4.7, the Phillips unit root t statistics show that the values of $\boldsymbol{R}_{2,t+1}$ for all countries are stationary, i.e., d = 0. Also, the hypotheses of the overidentifying restrictions are rejected for all three countries and the estimations of α_2 and β_2 in Eq. (4.15) are therefore unlikely to be consistent. This indicates that specification errors in the Cagan model (4.2) may be non-zero.

[Table 4.7 to be inserted here]

As reported in Table 4.8, the parameters of the ARMA model for $\boldsymbol{R}_{2,t+1}$ show that the time series of $\boldsymbol{R}_{2,t+1}$ are best represented by ARMA (0,1), ARMA (0,2) and ARMA (0,4) for Germany, Hungary and Poland respectively. In other words, nonzero stationary specification errors are contained in $\boldsymbol{R}_{2,t+1}$. The results are different from Engsted (1996), which supported the exact model for Germany.

[Table 4.8 to be inserted here]

Since nonzero specification errors are found in the data for all three countries, it is necessary to initiate step two in order to obtain consistent estimates of α_2 and β_2. To do that, I estimate Eq.(4.16) with Ω_{t-q} used as instruments, where q is set equal to 1,2 and 4 for Germany, Hungary, and Poland respectively. The FM-GMM t-ratio statistics show that, for all three countries studied, the null hypothesis of $\beta_2 \geq 0$ can be generally rejected in favor of negative alternatives and the null hypothesis of $\alpha_2 = 0$ cannot be rejected in some cases.

[Table 4.9 to be inserted here]

To check if the parameters in Eq.(4.16) for all four countries are consistently estimated, I rely on two types of statistics. The first are FM-GMM IV statistics. As reported in Table 4.9, I cannot reject the overidentifying restrictions. The second are FM-OLS Wald statistics, W_R. As shown in Table 4.10, I cannot reject the zero projections of $\Psi(\boldsymbol{L})\,\boldsymbol{R}_{2,t+1}$ onto Ω_{t-q}. Hence, both of these test statistics indicate that the parameters in Table 4.9 are consistently estimated.

To check whether a bubble exists, I project the estimated values of $\Psi(\boldsymbol{L})(\pi_{2,t} - \pi_{2,t}^{f})$ onto Ω_{t-q}. The FM-OLS Wald statistics, W_Ψ, reported in Table 4.10, cannot reject the null hypothesis of no bubble. This leads us to conclude that evidence for exchange rate bubbles cannot be found for the four countries studied.

[Table 4.10 to be inserted here]

4.5 Concluding Remarks

In this chapter, I have proposed a testing procedure, which extends the original Durlauf-Hooker framework for detecting the presence of specification errors, and both price and exchange rate bubbles during the inter-war European hyperinflations of Germany, Hungary and Poland. I rely upon the FM-GMM instrument validity test statistics and the ARIMA structures of the flow variables to detect the presence of specification errors; stock variables can then be transformed in such a way that I can differentiate between the presence of bubbles and model misspecification. My empirical results show that evidence of nonzero stationary specification errors in the exact Cagan models have been found, but that neither exchange rate nor price bubbles are evident in the data of the countries under study. The results imply that the control of hyperinflation only requires the control of the fundamental process. The dynamics of price and exchange rate might not be driven by the self-fulfilling expectations for the three inter-war European hyperinflations over the pre-reform samples.

The monetary reforms that occurred in the last few months of hyperinflations might be the main reason for the possible nonlinear behaviours of time series, as shown from the graphics of structural components in Chapter Three. The distribution theory of the FM inferential statistics under the nonlinear process is however unknown. The evidence for the absence of bubbles is therefore limited to the pre-reform observations. In subsequent chapters, I attempt several recent advances in econometric techniques to study bubbles over the extended observations during which monetary reforms were expected and even implemented, so that I can compare the evidence for the existence of bubbles over different samples.

Table 4.1 Durlauf and Hooker's flow and stock tests

		Flow test (Checks whether $R_{t+1} \perp \Omega_{t-q}$)	
		Orthogonal	Not orthogonal
Stock test (Checks whether $\pi_t - \pi_t^f \perp \Omega_{t-q}$)	Orthogonal	No bubble and no specification error.	Not applicable
	Not orthogonal	Bubbles exist and no specification error.	Specification error exists; uncertain about whether bubbles exist.

Table 4.2 The orthogonality tests of model misspecification and bubbles.

		Misspecification test (Checks whether R_{t+1} is white noise or represented by an ARIMA (p, d, q) process)	
		White noise	ARIMA process
Bubble test (Checks whether $\Psi(L)\Delta^d(\pi_t - \pi_t^f) \perp \Omega_{t-q}$)	Orthogonal	No specification error; no bubble.	Specification error exists; no bubble.
	Not orthogonal	No specification error; bubbles exist.	Specification error exists; bubbles exist.

Figure 4.1 The Procedures of the Orthogonality Test

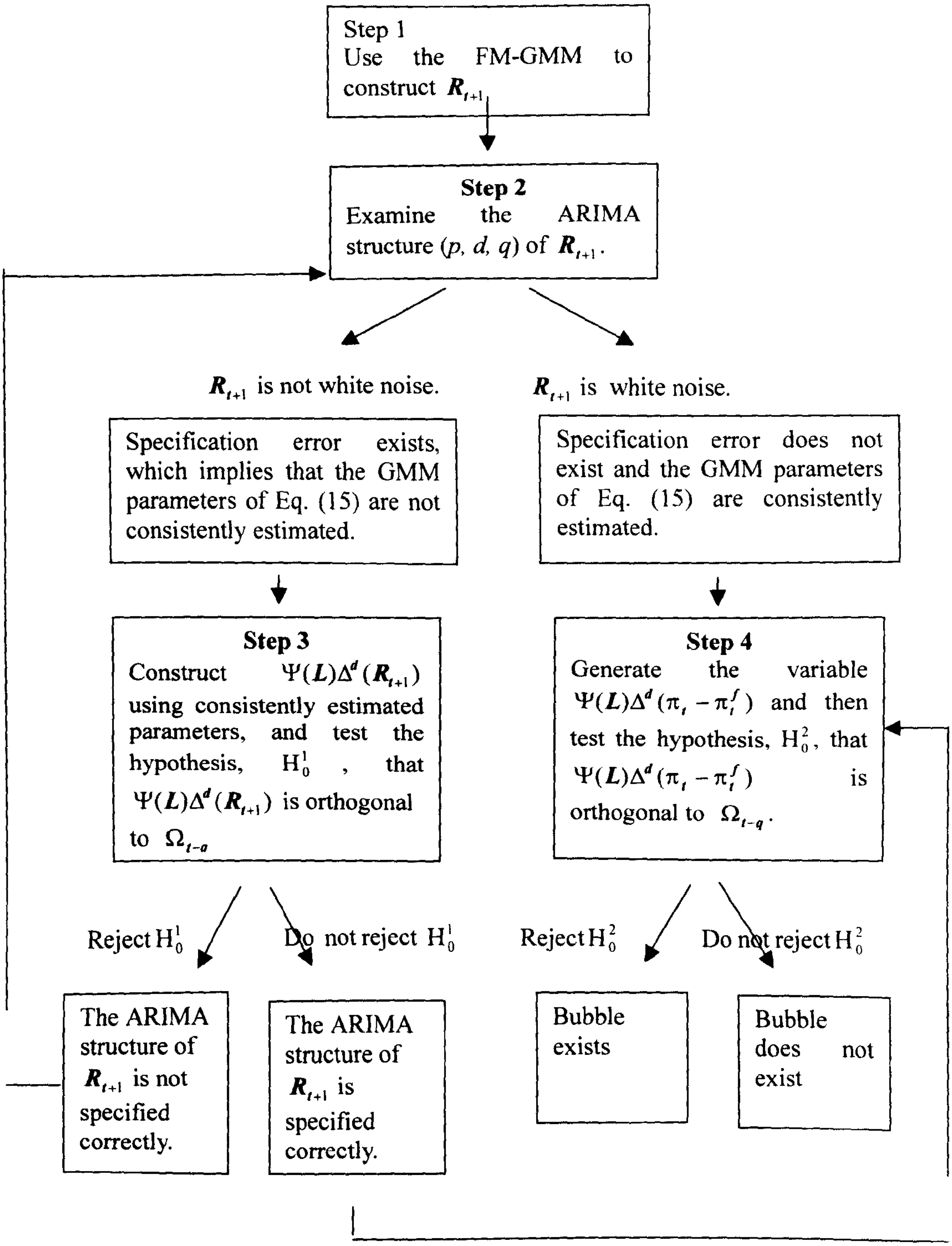


Table 4.3. Regression results of the Eq. (4.15):

Information set	α_1	β_1	FM-GMM IV	Phillips *t* (T)	Phillips *t* (T^2)
Germany					
Ω_t, Ω_{t-1}	9.3577	-3.4168	13.8468**	-4.3746**	-4.4592**
$\Omega_t,\ldots,\Omega_{t-2}$	8.8299	-2.0035	21.1945**	-4.0599**	-4.3837**
$\Omega_t,\ldots,\Omega_{t-3}$	8.9580	-2.8549	98.0218*	-4.3261**	-4.4827**
$\Omega_t,\ldots,\Omega_{t-4}$	8.8513	-2.6081	195.5892*	-4.2813**	-4.4802**
$\Omega_t,\ldots,\Omega_{t-5}$	9.1702	-4.0330	99.8418*	-4.3820**	-4.4170**
$\Omega_t,\ldots,\Omega_{t-6}$	9.1871	-3.8724	105.8213**	-4.3828**	-4.4285**
Hungary					
Ω_t, Ω_{t-1}	1.0238	-3.1244	23.0409*	-4.3910**	-4.3972**
$\Omega_t,\ldots,\Omega_{t-2}$	1.61172	-4.6958	93.5012*	-4.2998**	-4.3044***
$\Omega_t,\ldots,\Omega_{t-3}$	1.5087	-5.1365	29.8987**	-4.2638**	-4.2678***
$\Omega_t,\ldots,\Omega_{t-4}$	1.5744	-5.2658	50.2751**	-4.2686**	-4.2726***
$\Omega_t,\ldots,\Omega_{t-5}$	1.4569	-4.5001	158.8919*	-4.3113**	-4.3160**
$\Omega_t,\ldots,\Omega_{t-6}$	1.3913	-3.9659	140.4584*	-4.3437**	-4.3490*
Poland					
$\Omega_t,\ldots,\Omega_{t-1}$	1.5141	-2.7767	8.3806	-4.6156*	-4.6244**
$\Omega_t,\ldots,\Omega_{t-2}$	1.9062	-8.0407	34.1663*	-4.5614*	-4.5479**
$\Omega_t,\ldots,\Omega_{t-3}$	1.8195	-4.2307	35.5213**	-4.6549*	-4.6114**
$\Omega_t,\ldots,\Omega_{t-4}$	1.5976	-3.6010	86.9279*	-4.6572*	-4.6217**
$\Omega_t,\ldots,\Omega_{t-5}$	1.8037	-4.4056	473.4988*	-4.6520*	-4.6081**
$\Omega_t,\ldots,\Omega_{t-6}$	1.7964	-4.0199	608.1891*	-4.6573*	-4.6515**

Notes:

(1) The number of usable observations for Germany, Hungary and Poland are 32, 30 and 28 respectively.

(2) For Germany, the 5% simulated critical values of FM-GMM IV statistics are 10.4536, 20.0925, 27.4595, 39.7697, 55.6095, and 77.2497 with degrees of freedom = 2, 4, 6, 8, 10 and 12, respectively, while the corresponding 1% critical values are 15.8314, 29.5017, 38.3168, 55.8724, 77.5442 and 110.3933.

(3) For Hungary, the 5% simulated critical values of FM-GMM IV statistics are 11.1404, 21.4183, 28.9268, 42.6479, 60.1887 and 86.8796 with degrees of freedom = 2, 4, 6, 8, 10 and 12, respectively, while the corresponding 1% simulated critical values 16.8410, 31.8612, 41.6750, 61.0027, 88.6059 and 131.2547.

(4) For Poland, the 5% simulated critical values of FM-GMM IV statistics are 11.7754, 23.0888, 31.7244, 47.1863, 69.0410 and 102.2449 with degrees of freedom = 2, 4, 6, 8, 10 and 12, respectively, while the corresponding 1% critical values of FM-GMM IV statistics are 18.6293, 33.4206, 44.5272, 67.3659, 104.6666 and 155.2173.

(5) The finite-sample fractiles of the FM inferential statistics are based on 20,000 Monte Carlo simulations.

(6) Phillips t(T) and Phillips t(T^2) refer to the Phillips unit root t statistics with a linear trend and a quadratic trend respectively included in the Dickey-Fuller regression.

(7) The critical values for the Phillips' *t* statistics are obtained from Phillips and Ouliaris (1990).

(8) */** Denote significance at the 1%/5% levels, respectively.

Table 4.4. The ARMA model for the flow variable $R_{1,t+1}$ in Eq. (4.15).

Information set	AR(1)	MA(1)	MA(2)	MA(3)	AIC	Q(1)	Q(6)
Germany							
Ω_t, Ω_{t-1}	—	0.4744 (0.1234)	—	—	-1.0274	1.2503	4.1200
$\Omega_t,\ldots,\Omega_{t-2}$	—	0.5395 (0.1281)	—	—	-1.0909	0.6491	2.9166
$\Omega_t,\ldots,\Omega_{t-3}$	—	0.3592 (0.1567)	—	—	-1.2103	0.5281	2.4938
$\Omega_t,\ldots,\Omega_{t-4}$	—	0.3789 (0.1447)	—	—	-1.2111	0.9125	1.8512
$\Omega_t,\ldots,\Omega_{t-5}$	—	0.3288 (0.1516)	—	—	-1.1409	0.8885	2.7730
$\Omega_t,\ldots,\Omega_{t-6}$	—	0.3349 (0.1529)	—	—	-1.1493	0.9484	2.9381
Hungary							
Ω_t, Ω_{t-1}	0.8689 (0.0680)	-0.2367 (0.0870)	-0.6625 (0.1283)	—	-1.1993	1.4571	4.1370
$\Omega_t,\ldots,\Omega_{t-2}$	0.5020 (0.2141)	-0.1678 (0.1028)	-0.7255 (0.0860)	—	-1.2636	0.2005	1.8599
$\Omega_t,\ldots,\Omega_{t-3}$	—	0.2862 (0.1413)	-0.4351 (0.1822)	—	-1.2780	1.9328	4.3979
$\Omega_t,\ldots,\Omega_{t-4}$	—	0.1271 (0.1294)	-0.6113 (0.1448)	—	-1.3322	2.1137	5.7948
$\Omega_t,\ldots,\Omega_{t-5}$	—	0.2086 (0.1140)	-0.5204 (0.1330)	—	-1.3294	1.3074	3.0545
$\Omega_t,\ldots,\Omega_{t-6}$	—	0.3119 (0.1680)	-0.4058 (0.1834)	—	-1.3035	0.7092	1.8619
Poland							
	MA(1)	MA(2)	MA(3)	MA(4)	AIC	Q(1)	Q(6)
Ω_t, Ω_{t-1}	0.2325 (0.1292)	0.1297 (0.0542)	0.2854 (0.0649)	0.8359 (0.1440)	-0.7178	2.7172	6.3102
$\Omega_t,\ldots,\Omega_{t-2}$	0.3427 (0.1251)	0.3012 (0.0668)	0.4096 (0.0473)	0.79951 (0.2024)	-0.2460	1.2914	4.3136
$\Omega_t,\ldots,\Omega_{t-3}$	0.2134 (0.1253)	0.1117 (0.0555)	0.2596 (0.0705)	0.8113 (0.1808)	-0.5687	1.7832	5.1326
$\Omega_t,\ldots,\Omega_{t-4}$	0.2179 (0.1250)	0.1150 (0.0575)	0.2696 (0.0662)	0.8235 (0.1723)	-0.6253	2.1695	5.6755
$\Omega_t,\ldots,\Omega_{t-5}$	0.2154 (0.1263)	0.1122 (0.0570)	0.2579 (0.0714)	0.8109 (0.1850)	-0.5462	1.6943	4.9911
$\Omega_t,\ldots,\Omega_{t-6}$	0.2129 (0.1251)	0.1124 (0.0550)	0.2624 (0.0696)	0.8137 (0.1764)	-0.5916	1.9020	5.3051

Notes:

(1) The figures in parentheses are Newey-West heteroskedasticity-and-autocorrelation consistent standard errors.

(2) Q(k) refers to Ljung-Box Q-statistics with degrees of freedom = k.

(3) AIC refers to Akaike information criterion.

Table 4.5. Regression results of Eq. (4.16)

Information set	α_1	FM-GMM $t(\alpha_1)$	β_1	FM-GMM $t(\beta_1)$	FM-GMM IV	Phillips t (T)	Phillips t (T^2)
Germany							
Ω_{t-1}	8.9339**	-2.6573	-3.8664**	2.5370	____	-4.3828**	-4.4289**
$\Omega_{t-1},\Omega_{t-2}$	8.9892*	-3.8900	-3.4679*	3.6864	2.8954	-4.3766**	-4.4560**
$\Omega_{t-1},\ldots,\Omega_{t-3}$	8.9408*	-15.0462	-3.2367*	14.6351	19.3396	-4.3651**	-4.4694**
$\Omega_{t-1},\ldots,\Omega_{t-4}$	8.9395*	-13.3091	-3.3034*	13.0113	23.3083	-4.3692**	-4.4658**
$\Omega_{t-1},\ldots,\Omega_{t-5}$	9.0052*	-10.0640	-3.3701*	9.6698	17.1372	-4.3726**	-4.4620**
$\Omega_{t-1},\ldots,\Omega_{t-6}$	8.9654*	-8.1694	-3.9361*	7.8166	38.6320	-4.3827**	-4.4239**
Hungary							
Ω_{t-2}	1.4520	-1.5232	-6.2455*	6.3502	____	-4.5927*	-5.1002*
$\Omega_{t-2},\ldots,\Omega_{t-3}$	1.3861*	-5.7722	-4.0222**	2.8233	5.7219	-4.2213**	-4.2246**
$\Omega_{t-2},\ldots,\Omega_{t-4}$	1.5909*	-7.4402	-4.8989*	3.7227	12.2932	-4.2883**	-4.2927***
$\Omega_{t-2},\ldots,\Omega_{t-5}$	1.5853*	-4.9107	-5.5249**	2.7120	15.6025	-4.2555**	-4.2593***
$\Omega_{t-2},\ldots,\Omega_{t-6}$	1.5696*	-5.8921	-5.1498*	3.2270	23.0465	-4.2747**	-4.2788***
Poland							
Ω_{t-4}	1.4159*	-6.7685	-2.6879*	4.8145	____	-4.6053*	-4.6230**
$\Omega_{t-4},\Omega_{t-5}$	1.4163*	-7.8364	-2.5260*	5.7759	5.6631	-4.5817*	-4.6192**
$\Omega_{t-4},\ldots,\Omega_{t-6}$	1.4000*	-16.4952	-2.6003*	9.9810	9.3812	-4.5933*	-4.6212**

Notes:

(1) The numbers of usable observations are same as those of Table 4.3.

(2) For Germany, the 5% and 1% simulated critical values of the one-tailed FM-GMM t statistics are 1.9712 and 2.8954, respectively, while the corresponding two-tailed FM-GMM t statistics are 2.3852 and 3.3076, respectively. The 5% simulated critical values of FM-GMM IV statistics are 10.4536, 20.0925, 27.4595, 39.7697 and 55.6095 with degrees of freedom = 2, 4, 6, 8 and 10, respectively.

(3) For Hungary, the 5% and 1% simulated critical values of the one-tailed FM-GMM t statistics are 2.0052 and 2.9569, respectively, while the corresponding two-tailed FM-GMM t statistics are 2.4287 and 3.3575 respectively. The 5% simulated critical values of FM-GMM IV statistics are 11.1404, 21.4183, 28.9268 and 42.6479 with degrees of freedom = 2, 4, 6 and 8, respectively.

(4) For Poland, the 5% and 1% simulated critical values of the one-tailed FM-GMM t statistics are 1.9979 and 3.0044 respectively, while the corresponding two-tailed FM-GMM t statistics are 2.4504 and 3.3802, respectively. The 5% simulated critical values of FM-GMM IV statistics are 11.7754 and 23.0888 with degrees of freedom = 2 and 4 respectively.

(5) The finite-sample fractiles of the FM inferential statistics are based on 20,000 Monte Carlo simulations.

(6) Phillips t(T) and Phillips t(T^2) refer to the Phillips unit root t statistics with a linear trend and a quadratic trend respectively included in the Dickey-Fuller regression. The critical values are taken from Phillips and Ouliaris (1990).

(7) */** Denote significance at the 1%/5% levels, respectively.

Table 4.6. The FM-OLS Wald statistics for the projections of $\Psi(L)\,\boldsymbol{R}_{1,t+1}$ and $\Psi(L)$ $(\pi_{1,t} - \pi^f_{1,t})$

Information set	W_R	W_Ψ
Germany		
Ω_{t-1}	6.7109	6.8483
$\Omega_{t-1}, \Omega_{t-2}$	11.4442	11.5981
$\Omega_{t-1}, \Omega_{t-2}, \Omega_{t-3}$	22.3470	30.2699
$\Omega_{t-1}, \Omega_{t-2}, \Omega_{t-3}, \Omega_{t-4}$	38.6098	37.7832
$\Omega_{t-1},\ldots,\Omega_{t-5}$	40.2955	56.5292
$\Omega_{t-1},\ldots,\Omega_{t-6}$	43.0040	101.1938
Hungary		
Ω_{t-2}	5.9507	6.4725
$\Omega_{t-2}, \Omega_{t-3}$	20.9100	17.7286
$\Omega_{t-2}, \Omega_{t-3}, \Omega_{t-4}$	12.6699	20.3563
$\Omega_{t-2}, \Omega_{t-3}, \Omega_{t-4}, \Omega_{t-5}$	32.1923	34.6593
$\Omega_{t-2}, \ldots, \Omega_{t-6}$	47.8768	40.2012
Poland		
Ω_{t-4}	3.0237	3.2044
$\Omega_{t-4}, \Omega_{t-5}$	6.6163	18.7541
$\Omega_{t-4}, \Omega_{t-5}, \Omega_{t-6}$	22.5255	34.8514

Notes:

(1) The numbers of usable observations are same as those of Table 4.3.

(2) For Germany, the 5% simulated critical values of the FM-OLS Wald statistics are 10.5115, 21.4149, 35.9424, 55.7938, 86.2426 and 134.6337 with degrees of freedom = 2, 4, 6, 8, 10 and 12 respectively.

(3) For Hungary, the 5% simulated critical values of the FM-OLS Wald statistics are 10.8061, 22.1647, 38.0158, 60.7450 and 98.4709 with degrees of freedom = 2,4,6, 8 and 10 respectively.

(4) For Poland, the 5% simulated critical values of the FM-OLS Wald statistics are 11.5412, 23.2997 and 41.1154 with degrees of freedom = 2,4 and 6 respectively.

Table 4.7. The regression results of Eq. (4.15)

Information set	α_2	β_2	FM-GMM IV	Phillips *t* (T)	Phillips *t* (T^2)
Germany					
Ω_t, Ω_{t-1}	6.7069	-4.4022	15.5731*	-5.0227*	-5.0566*
$\Omega_t,\ldots,\Omega_{t-2}$	7.0440	-8.2336	61.6458*	-5.0559*	-5.0752*
$\Omega_t,\ldots,\Omega_{t-3}$	7.1005	-6.8321	46.9271*	-5.0599*	-5.0827*
$\Omega_t,\ldots,\Omega_{t-4}$	7.0357	-5.4984	47.2267**	-5.0524*	-5.0802*
$\Omega_t,\ldots,\Omega_{t-5}$	7.4699	-7.0597	63.0164**	-5.0598*	-5.0819*
$\Omega_t,\ldots,\Omega_{t-6}$	6.6139	-4.5453	236.4936*	-5.0288*	-5.0617*
Hungary					
Ω_t, Ω_{t-1}	8.9685	-5.5999	10.1694**	-3.9202**	-4.0676***
$\Omega_t,\ldots,\Omega_{t-2}$	8.8779	-4.1975	28.8040*	-3.8804***	-4.0993***
$\Omega_t,\ldots,\Omega_{t-3}$	8.9280	-4.0972	36.2334**	-3.8747***	-4.1009***
$\Omega_t,\ldots,\Omega_{t-4}$	8.7626	-3.8464	45.6930**	-3.8577***	-4.1042**
$\Omega_t,\ldots,\Omega_{t-5}$	8.7359	-4.1937	64.7463**	-3.8802***	-4.0993***
$\Omega_t,\ldots,\Omega_{t-6}$	8.5995	-4.7113	101.6567**	-3.9019***	-4.0888***
Poland					
Ω_t, Ω_{t-1}	9.0412	-3.0816	10.1344**	-5.5948*	-5.5951*
$\Omega_t,\ldots,\Omega_{t-2}$	9.0100	-3.0441	36.4123*	-5.5891*	-5.5906*
$\Omega_t,\ldots,\Omega_{t-3}$	9.1612	-2.7175	41.4412**	-5.5288*	-5.5453*
$\Omega_t,\ldots,\Omega_{t-4}$	9.0452	-2.4417	56.4205**	-5.4583*	-5.4950*
$\Omega_t,\ldots,\Omega_{t-5}$	9.0543	-2.8960	686.3372*	-5.5644*	-5.5717*
$\Omega_t,\ldots,\Omega_{t-6}$	8.7668	-2.6797	1080.582*	-5.5203*	-5.5391*

Notes:

(1) The numbers of usable observations are same as those of Table 4.3.

(2) For Germany, the 5% simulated critical values of FM-GMM IV statistics are 10.4536, 20.0925, 27.4595, 39.7697, 55.6095, and 77.2497 with degrees of freedom = 2, 4, 6, 8, 10 and 12, respectively, while the corresponding 1% critical values are 15.8314, 29.5017, 38.3168, 55.8724, 77.5442 and 110.3933.

(3) For Hungary, the 5% simulated critical values of FM-GMM IV statistics are 11.1404, 21.4183, 28.9268, 42.6479, 60.1887 and 86.8796 with degrees of freedom = 2, 4, 6, 8, 10 and 12, respectively, while the corresponding 1% simulated critical values 16.8410, 31.8612, 41.6750, 61.0027, 88.6059 and 131.2547.

(4) For Poland, the 5% simulated critical values of FM-GMM IV statistics are 11.7754, 23.0888, 31.7244, 47.1863, 69.0410 and 102.2449 with degrees of freedom = 2, 4, 6, 8, 10 and 12, respectively, while the corresponding 1% critical values of FM-GMM IV statistics are 18.6293, 33.4206, 44.5272, 67.3659, 104.6666 and 155.2173.

(5) The finite-sample fractiles of the FM inferential statistics are based on 20,000 Monte Carlo simulations.

(6) Phillips t(T) and Phillips t(T^2) refer to the Phillips unit root t statistics with a linear trend and a quadratic trend respectively included in the Dickey-Fuller regression. The critical values are obtained from Phillips and Ouliaris (1990).

(7) */** Denote significance at the 1%/5% levels, respectively.

Table 4.8. The ARMA models for the flow variables $\boldsymbol{R}_{2,t+1}$ in Eq. (4.15).

Information set	MA(1)	MA(2)	MA(3)	MA(4)	AIC	Q(1)	Q(6)
Germany							
Ω_t, Ω_{t-1}	0.3282 (0.1192)	——	——	——	0.1363	2.3002	8.9765
$\Omega_t, \ldots, \Omega_{t-2}$	0.3986 (0.1470)	——	——	——	0.2776	1.9510	6.7939
$\Omega_t, \ldots, \Omega_{t-3}$	0.3279 (0.1374)	——	——	——	0.1988	2.0933	7.7964
$\Omega_t, \ldots, \Omega_{t-4}$	0.2642 (0.1253)	——	——	——	0.1305	2.2416	8.8674
$\Omega_t, \ldots, \Omega_{t-5}$	0.2619 (0.1297)	——	——	——	0.1607	1.8602	7.7095
$\Omega_t, \ldots, \Omega_{t-6}$	0.3712 (0.1258)	——	——	——	0.1733	2.5162	8.8768
Hungary							
Ω_t, Ω_{t-1}	0.5458 (0.1128)	0.5675 (0.1429)	——	——	-0.5790	0.8610	4.1379
$\Omega_t, \ldots, \Omega_{t-2}$	0.5450 (0.1123)	0.5659 (0.1437)	——	——	-0.6372	0.9059	4.1627
$\Omega_t, \ldots, \Omega_{t-3}$	0.5469 (0.1132)	0.5665 (0.1448)	——	——	-0.6369	0.9190	4.1619
$\Omega_t, \ldots, \Omega_{t-4}$	0.5451 (0.1109)	0.5669 (0.1419)	——	——	-0.6598	0.9131	4.1731
$\Omega_t, \ldots, \Omega_{t-5}$	0.5458 (0.1105)	0.5686 (0.1407)	——	——	-0.6400	0.9073	4.1770
$\Omega_t, \ldots, \Omega_{t-6}$	0.5552 (0.1101)	0.5783 (0.1364)	——	——	-0.5981	0.9334	4.1996
Poland							
Ω_t, Ω_{t-1}	0.0037 (0.1352)	-0.1221 (0.0608)	0.4759 (0.0842	0.6071 (0.0633)	-0.4628	1.7421	4.7429
$\Omega_t, \ldots, \Omega_{t-2}$	-0.0035 (0.1669)	-0.1209 (0.1108)	0.4856 (0.0746)	0.5940 (0.0746)	-0.4589	1.8029	4.8382
$\Omega_t, \ldots, \Omega_{t-3}$	-0.0044 (0.1960)	-0.1065 (0.0843)	0.4248 (0.0572)	0.6421 (0.0572)	-0.5638	2.0156	4.9832
$\Omega_t, \ldots, \Omega_{t-4}$	0.0451 (0.2033)	-0.1947 (0.0744)	0.3842 (0.0456)	0.7149 (0.0456)	-0.6082	2.5710	5.6157
$\Omega_t, \ldots, \Omega_{t-5}$	-0.0026 (0.1731)	-0.1016 (0.0929)	0.4356 (0.0689)	0.6341 (0.0689)	-0.5066	1.8152	4.8106
$\Omega_t, \ldots, \Omega_{t-6}$	0.0974 (0.2232)	-0.1950 (0.0845)	0.4117 (0.1415)	0.7097 (0.1571)	-0.4199	2.7309	5.5394

Notes:

(1) The figures in parentheses are Newey-West heteroskedasticity-and-autocorrelation consistent standard errors.

(2) Q(k) refers to Ljung-Box Q-statistics with degrees of freedom = k.

(3) AIC refers to Akaike information criterion.

Table 4.9. The regression results of Eq. (4.16)

Information set	α_2	FM-GMM $t(\alpha_2)$	β_2	FM-GMM $t(\beta_2)$	FM-GMM IV	Phillips t(T)	Phillips $t(T^2)$
Germany							
Ω_{t-1}	7.2958**	-2.7145	-6.4436**	2.4202	____	-5.0595*	-5.0835*
$\Omega_{t-1}, \Omega_{t-2}$	7.4149*	-3.9739	-7.1355*	3.6500	5.2330	-5.0597*	-5.0816*
$\Omega_{t-1},\ldots,\Omega_{t-3}$	7.5052*	-3.3792	-5.9558*	2.9761	14.3855	-5.0571*	-5.0830*
$\Omega_{t-1},\ldots,\Omega_{t-4}$	7.7457*	-3.9792	-6.4084*	3.3432	13.4743	-5.0594*	-5.0835*
$\Omega_{t-1},\ldots,\Omega_{t-5}$	7.8523*	-4.6388	-7.5722*	3.7606	30.1904	-5.0586*	-5.0794*
$\Omega_{t-1},\ldots,\Omega_{t-6}$	8.2020*	-3.8777	-8.3767*	3.1127	33.4793	-5.0552*	-5.0742*
Hungary							
Ω_{t-2}	8.9671	-0.9857	-5.2457*	5.6265	____	-4.8517*	-4.8390**
$\Omega_{t-2},\ldots,\Omega_{t-3}$	9.5881	-0.6714	-9.8582*	6.2525	5.7191	-4.6606*	-4.6998**
$\Omega_{t-2},\ldots,\Omega_{t-4}$	9.9925	-0.9952	-10.7494*	9.7029	11.6432	-4.6363*	-4.6821**
$\Omega_{t-2},\ldots,\Omega_{t-5}$	9.7887	-1.3758	-8.3629*	10.8120	14.6191	-4.7096*	-4.7353**
$\Omega_{t-2},\ldots,\Omega_{t-6}$	10.1638	-1.0045	-11.1344*	9.7322	20.7000	-4.6268*	-4.6752**
Poland							
Ω_{t-4}	8.8654**	-2.8554	-1.9017**	2.7416	____	-5.2403*	-5.3467*
$\Omega_{t-4}, \Omega_{t-5}$	8.8084*	-5.2560	-1.6168*	5.0256	1.7582	-5.0619*	-5.2272*
$\Omega_{t-4},\ldots,\Omega_{t-6}$	8.7760*	-5.3281	-1.5409*	5.1112	5.8427	-5.0046*	-5.1886*

Notes:

(1) The numbers of usable observations are same as those of Table 4.3.

(2) For Germany, the 5% and 1% simulated critical values of the one-tailed FM-GMM t statistics are 1.9712 and 2.8954, respectively, while the corresponding two-tailed FM-GMM t statistics are 2.3852 and 3.3076, respectively. The 5% simulated critical values of FM-GMM IV statistics are 10.4536, 20.0925, 27.4595, 39.7697 and 55.6095 with degrees of freedom = 2, 4, 6, 8 and 10, respectively.

(3) For Hungary, the 5% and 1% simulated critical values of the one-tailed FM-GMM t statistics are 2.0052 and 2.9569, respectively, while the corresponding two-tailed FM-GMM t statistics are 2.4287 and 3.3575 respectively. The 5% simulated critical values of FM-GMM IV statistics are 11.1404, 21.4183, 28.9268 and 42.6479 with degrees of freedom = 2, 4, 6 and 8, respectively.

(4) For Poland, the 5% and 1% simulated critical values of the one-tailed FM-GMM t statistics are 1.9979 and 3.0044 respectively, while the corresponding two-tailed FM-GMM t statistics are 2.4504 and 3.3802, respectively. The 5% simulated critical values of FM-GMM IV statistics are 11.7754 and 23.0888 with degrees of freedom = 2 and 4 respectively.

(5) The finite-sample fractiles of the FM inferential statistics are based on 20,000 Monte Carlo simulations.

(6) Phillips t(T) and Phillips $t(T^2)$ refer to the Phillips unit root t statistics with a linear trend and a quadratic trend respectively included in the Dickey-Fuller regression. The critical values are taken from Phillips and Ouliaris (1990).

(7) */** Denote significance at the 1%/5% levels, respectively.

Table 4.10. The FM-OLS Wald statistics for the projections of $\Psi(L)\, \boldsymbol{R}_{2,t+1}$ and $\Psi(L)$ $(\pi_{2,t} - \pi^{f}_{2,t})$

Information set	W_R	W_Ψ
Germany		
Ω_{t-1}	0.7847	5.6989
Ω_{t-1}, Ω_{t-2}	8.1441	16.6208
Ω_{t-1}, Ω_{t-2}, Ω_{t-3}	28.2595	30.2699
Ω_{t-1}, Ω_{t-2}, Ω_{t-3}, Ω_{t-4}	38.5030	38.3649
$\Omega_{t-1}, \ldots, \Omega_{t-5}$	57.1144	28.8212
$\Omega_{t-1}, \ldots, \Omega_{t-6}$	47.5351	107.8442
Hungary		
Ω_{t-2}	6.6078	6.2457
Ω_{t-2}, Ω_{t-3}	12.0448	7.7946
Ω_{t-2}, Ω_{t-3}, Ω_{t-4}	18.6084	23.4400
Ω_{t-2}, Ω_{t-3}, Ω_{t-4}, Ω_{t-5}	44.4652	28.0724
$\Omega_{t-2}, \ldots, \Omega_{t-6}$	38.8802	60.7119
Poland		
Ω_{t-4}	2.0879	0.0937
Ω_{t-4}, Ω_{t-5}	8.8847	11.2366
Ω_{t-4}, Ω_{t-5}, Ω_{t-6}	17.4438	11.4788

Notes:

(1) The numbers of usable observations are same as those of Table 4.3.

(2) For Germany, the 5% simulated critical values of the FM-OLS Wald statistics are 10.5115, 21.4149, 35.9424, 55.7938, 86.2426 and 134.6337 with degrees of freedom = 2, 4, 6, 8, 10 and 12 respectively.

(3) For Hungary, the 5% simulated critical values of the FM-OLS Wald statistics are 10.8061, 22.1647, 38.0158, 60.7450 and 98.4709 with degrees of freedom = 2,4,6, 8 and 10 respectively.

(4) For Poland, the 5% simulated critical values of the FM-OLS Wald statistics are 11.5412, 23.2997 and 41.1154 with degrees of freedom = 2,4 and 6 respectively.

Appendix 4.1

In this appendix, I conduct a simulation study to examine the sampling performance of several FM estimators in a small sample. Other than the FM-OLS and FM-GMM that are used in the bubble tests, I include FM-LAD and FM-GIVE for comparison purposes. The FM-LAD is a FM version of the least absolute deviation (LAD) method (Phillips, 1995b), which is a robust estimation method in the presence of heavy-tailed errors in a nonstationary regression. FM-GIVE is a FM extension of the generalized instrumental variable estimation (GIVE) method (Kitamura and Phillips, 1997), which employs GLS transformation to data and instruments in a nonstationary regression under the assumption of strictly exogenous instruments.

The regression model used for data generation is shown as follows:

$$\boldsymbol{y}_t = \boldsymbol{a} + \boldsymbol{B}\boldsymbol{x}_t + \boldsymbol{u}_{ot} \qquad \boldsymbol{a} = 0, \boldsymbol{B} = 1. \qquad t = 1\ldots T. \qquad \text{(A.4.1)}$$

where $\boldsymbol{u}_{ot} = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \theta_4\varepsilon_{t-4}$ with $\theta_1 = 0.6, \theta_2 = 0.7, \theta_3 = 0.8$ and $\theta_4 = 0.9$. In other words, $\boldsymbol{u}_{ot}$ is an MA(4) process. $\boldsymbol{x}_t$ is a I(1) process and is a linear combination of two processes, $\boldsymbol{x}_{at}$, and $\boldsymbol{x}_{bt}$:

$$x_t = \alpha x_{at} + (1-\alpha)x_{bt} \quad \text{with } \alpha = 0.5 \text{ and} \qquad (A.4.2)$$

$$\Delta x_{at} = [\frac{1}{\sqrt{(1+c^2)}}]e_{1t} + [\frac{c}{\sqrt{(1+c^2)}}]\Delta z_t \qquad (A.4.3)$$

$$\Delta x_{bt} = [\frac{1}{\sqrt{(1+c^2)}}]e_{2t} + [\frac{c}{\sqrt{(1+c^2)}}]u_{ot} \qquad (A.4.4)$$

Δz_t is constructed by four first-differenced nonstationary instruments, Δz_{1t}, Δz_{2t}, Δz_{3t} and Δz_{4t}, which are generated in the following way:

$$\Delta z_t = 0.25 * \Delta z_{1t} + 0.25 * \Delta z_{2t} + 0.25 * \Delta z_{3t} + 0.25 * \Delta z_{4t} + e_{3t}, \qquad (A.4.5)$$

$(\Delta z_{1t}, \Delta z_{2t}, \Delta z_{3t}, \Delta z_{4t})'$ is a 4 x 1 matrix with:

$$\text{mean} = 0 \text{ and covariance } = \begin{bmatrix} 1 & 0.7 & 0.5 & -0.2 \\ 0.7 & 1 & 0.4 & -0.3 \\ 0.5 & 0.4 & 1 & -0.3 \\ -0.2 & -0.3 & 0.6 & 1 \end{bmatrix}$$

where $\Delta z_{it} = 0.8 * \Delta z_{it-1} + \varpi_{it}$ $\forall$ i = 1,2,3,4. It implies that the instruments z_{it} follow an ARIMA(1,1,0) process.

The residual terms, $e_t = \{ e_{1t}, e_{2t}, e_{3t} \}'$, $\varepsilon_t = \{ \varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t} \}'$ and $\varpi_t = \{ \varpi_{1t}, \varpi_{2t}, \varpi_{3t}, \varpi_{4t} \}'$are uncorrelated with one another. From Eq.(A.4.3) and (A.4.4), there is a feedback effect from Δz_t to Δx_t through Δx_{at}; and from u_{ot} to

Δx_t through Δx_{bt}, respectively. Hence, the instruments and the residual term are correlated with the regressors separately but they are uncorrelated with each other. Also, the parameter c controls the degree of association between Δx_{at} and Δz_t as well as between Δx_{bt} and u_{ot}. When $c \to \infty$, Δx_{at} and Δz_t as well as Δx_{bt} and u_{ot} become linearly dependent. When $|c| = 1$, the squared correlations between them are equally 0.5. In the simulation study, the innovation terms, $\{ e_{1t}, e_{2t}, e_{3t}, \varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t}, \varpi_{1t}, \varpi_{2t}, \varpi_{3t}, \varpi_{4t} \}$, are drawn from four distributions, which include standard normal distribution (N(0,1)), t distribution with 2 degrees of freedom (t_2), stable distribution with the characteristic exponent being equal to 1.5 ($S_{\alpha=1.5}$),[31] and standardized t_5 distribution with conditional variance from an IGARCH(0.5,0.5) model such that $\varepsilon_t = z_{ot}\sigma_t$ where $z_{0t} \sim t_5$ (IGARCH(0.5,0.5)-t_5). The values of c chosen for the simulation study are $c = 1$ and c=100. The sample size T is 40 and the number of Monte Carlo replications is 1000. The sampling performance of the FM estimators is indicated by the average bias ($BIAS_{ave}$), the root mean square error ($RMSE_{ave}$), the quantiles (Q_{25}, Q_{50} and Q_{75}), interquantile ranges (Q_{75}- Q_{25}) and the concentration probabilities ($\Pr(|\hat{b}-b| \le k)$) where k is a constant. In order to examine the performance of the

---

[31]When the characteristic exponent of a stable distribution is below 2, the variance will be infinite. The basic properties of stable distribution are described in McCulloch (1996).

FM estimators without the effect of the occasional outliers, I include the bias and RMSE based on the results excluding 1% in both tails, denoted by $BIAS_{98}$ and $RMSE_{98}$ respectively.

The results of simulation study are reported from Table A.4.1 to Table A.4.16 below. It is noted that with respect to the estimated intercept terms ($\boldsymbol{a}$), the mean bias of the FM-GMM estimator is the most serious in cases where c =1 and the distributions of innovations are non-normal, and where c=100 and the innovations are drawn from N(0,1) as well as IGARCH(0.5,0.5)-t_5. For the rest of cases, the mean bias of the FM-LAD estimator is however found to be the most serious. Also, the dispersion of the FM-GMM estimator for the intercept, measured by the RMSE and the interquantile range, is the largest in all cases and it is much larger than that of the FM-OLS and FM-LAD estimators. From the concentration probabilities, the FM-GMM estimator is unfavourably compared to other FM estimators, and the probability that the absolute deviation of the FM-GMM estimator for the intercept is smaller than 0.1 does not exceed 20%.

On the other hand, with respect to the I(1) coefficient ($\boldsymbol{B}$), the mean bias of the FM-GMM estimator is the lowest in most cases whereas the mean bias of the FM-GIVE estimator is the most serious in all cases. Based upon the RMSE, interquantile ranges and the concentration probabilities, the dispersion of the FM-GIVE estimator for the I(1) coefficient is the largest in all cases whereas the dispersion of the FM-LAD estimator is the lowest in almost all cases. The dispersion of the FM-GMM estimator for the I(1) coefficient is slightly lower than that of the FM-GIVE estimator under all circumstances and the dispersion of the FM-OLS estimator is found to be slightly lower than that of the FM-LAD in some cases.

In general, the overall performance of the FM-OLS and FM-LAD estimator for intercepts and I(1) coefficients are compared more favourably to that of the FM-GMM and FM-GIVE estimators in a small sample of the present simulation study whatever the values of c and the distributions of the innovations.

Table A.4.1. The simulation results of bias, RMSE, quantiles and interquantile range when $c=1$ and innovations are drawn from N(0,1)

Parameter	Estimator	$BIAS_{ave}$	$RMSE_{ave}$	$BIAS_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	$Q_{75}-Q_{25}$
a	FM-OLS	0.0267	0.4773	0.0256	0.4143	-0.2253	0.0166	0.2770	0.5023
	FM-LAD	0.0157	0.5436	0.0163	0.4774	-0.2857	0.0209	0.3161	0.6011
	FM-GMM	0.0108	2.5481	-0.0012	0.8632	-0.4309	0.0199	0.4199	0.8508
	FM-GIVE	0.0210	0.6547	0.0186	0.4942	-0.2372	0.0321	0.2679	0.5051
B	FM-OLS	-0.0270	0.1843	-0.0322	0.1390	-0.1164	-0.0302	0.0598	0.1762
	FM-LAD	0.0364	0.1412	0.0365	0.1303	-0.0499	0.0311	0.1232	0.1731
	FM-GMM	0.0186	0.4711	0.0099	0.2733	-0.1335	0.0099	0.1460	0.2795
	FM-GIVE	0.1468	0.7060	0.1226	0.4095	-0.0795	0.0867	0.3088	0.3883

Table A.4.2. The simulation results of bias, RMSE, quantiles and interquantile range when $c=1$ and innovations are drawn from t_2

Parameter	Estimator	$BIAS_{ave}$	$RMSE_{ave}$	$BIAS_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	Q_{75}- Q_{25}
a	FM-OLS	-0.0124	0.4929	-0.0133	0.4140	-0.2678	-0.0154	0.2422	0.5100
	FM-LAD	-0.1341	0.4870	-0.0160	0.3964	-0.2506	-0.0250	0.2239	0.4745
	FM-GMM	0.0008	1.8249	0.0122	0.7763	-0.3817	-0.0050	0.3696	0.7513
	FM-GIVE	0.0557	1.8284	-0.0014	0.4767	-0.2684	-0.0139	0.2434	0.5118
B	FM-OLS	-0.0277	0.1858	-0.0313	0.1383	-0.1165	-0.0295	0.0564	0.1729
	FM-LAD	0.0315	0.1336	0.0307	0.1196	-0.0415	0.0249	0.0991	0.1406
	FM-GMM	-0.0109	0.4085	-0.0042	0.2509	-0.1214	0.0044	0.1355	0.2569
	FM-GIVE	0.1122	0.8661	0.1297	0.4767	-0.0737	0.0983	0.2953	0.3690

Table A.4.3. The simulation results of bias, RMSE, quantiles and interquantile range when $c=1$ and innovations are drawn from $S_{\alpha=1.5}$

Parameter	Estimator	$BIAS_{ave}$	$RMSE_{ave}$	$BIAS_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	$Q_{75}-Q_{25}$
a	FM-OLS	0.0048	0.4712	0.0087	0.3910	-0.2344	0.0113	0.2614	0.4958
	FM-LAD	-0.0098	0.4923	-0.0042	0.3974	-0.2211	0.0163	0.2350	0.4561
	FM-GMM	0.0108	1.0886	0.0212	0.7398	-0.3646	0.0201	0.3991	0.7637
	FM-GIVE	0.0015	0.7217	0.0061	0.5196	-0.2650	-0.0022	0.2731	0.5381
B	FM-OLS	-0.0249	0.1589	-0.0275	0.1281	-0.1066	-0.0240	0.0530	0.1596
	FM-LAD	0.0314	0.1217	0.0302	0.1078	-0.0340	0.0198	0.0892	0.1232
	FM-GMM	0.0286	0.3337	0.0211	0.2216	-0.0952	0.0112	0.1457	0.2409
	FM-GIVE	0.0823	0.9616	0.1073	0.5410	-0.0793	0.0796	0.2914	0.3707

Table A.4.4. The simulation results of bias, RMSE, quantiles and interquantile range when c=1 and innovations are drawn from IGARCH(0.5,0.5)-t_5

Parameter	Estimator	$BIAS_{ave}$	$RMSE_{ave}$	$BIAS_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	Q_{75}- Q_{25}
a	FM-OLS	0.0111	0.5118	0.0065	0.4244	-0.2369	0.0036	0.2556	0.4925
	FM-LAD	0.0205	0.5107	0.0185	0.4273	-0.2274	0.0109	0.2443	0.4717
	FM-GMM	0.0983	2.4889	0.0199	0.7840	-0.3752	-0.0068	0.3783	0.7535
	FM-GIVE	0.0024	0.7321	0.0132	0.5024	-0.2635	-0.0028	0.2715	0.5350
B	FM-OLS	-0.0213	0.1712	-0.0247	0.1367	-0.1162	-0.0236	0.0559	0.1721
	FM-LAD	0.0351	0.1284	0.0340	0.1168	-0.0377	0.0235	0.1019	0.1396
	FM-GMM	0.0171	0.4679	0.0227	0.2305	-0.1027	0.0083	0.1432	0.2459
	FM-GIVE	0.1275	0.8040	0.1205	0.4485	-0.0627	0.0913	0.2947	0.3574

Table A.4.5. The simulation results of concentration probabilities when $c = 1$ and innovations are drawn from N(0,1)

		$(\Pr(\lvert\hat{b}-b\rvert \leq k))$				
Parameter	Estimator	$k \leq 0.1$	$k \leq 0.3$	$k \leq 0.5$	$k \leq 0.7$	$k \leq 0.9$
a	FM-OLS	0.2150	0.5790	0.7750	0.8790	0.9330
	FM-LAD	0.1720	0.4960	0.7220	0.8400	0.9090
	FM-GMM	0.1280	0.3700	0.5600	0.6820	0.7690
	FM-GIVE	0.2130	0.5590	0.7490	0.8460	0.9020
B	Estimator	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.05$	$k \leq 0.07$	$k \leq 0.09$
	FM-OLS	0.0540	0.1690	0.2870	0.3970	0.4990
	FM-LAD	0.0670	0.1940	0.2990	0.4060	0.5060
	FM-GMM	0.0350	0.1180	0.1910	0.2710	0.3370
	FM-GIVE	0.0320	0.0830	0.1450	0.2130	0.2710

Table A.4.6. The simulation results of concentration probabilities when $c = 1$ and innovations are drawn from t_2

		$(\Pr(\lvert\hat{b}-b\rvert \leq k))$				
Parameter	Estimator	$k \leq 0.1$	$k \leq 0.3$	$k \leq 0.5$	$k \leq 0.7$	$k \leq 0.9$
a	FM-OLS	0.2020	0.5850	0.7840	0.8860	0.9330
	FM-LAD	0.2180	0.5980	0.8140	0.9060	0.9430
	FM-GMM	0.1340	0.4030	0.6120	0.7280	0.8050
	FM-GIVE	0.2160	0.5700	0.7570	0.8560	0.9100
B	Estimator	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.05$	$k \leq 0.07$	$k \leq 0.09$
	FM-OLS	0.0710	0.1840	0.3070	0.4260	0.5210
	FM-LAD	0.0890	0.2300	0.3630	0.4900	0.5870
	FM-GMM	0.0450	0.1430	0.2210	0.3020	0.3770
	FM-GIVE	0.0290	0.0900	0.1490	0.2040	0.2600

Table A.4.7. The simulation results of concentration probabilities when $c = 1$ and innovations are drawn from $S_{\alpha=1.5}$

		$(\Pr(\lvert\hat{b}-b\rvert \leq k))$				
Parameter	Estimator	$k \leq 0.1$	$k \leq 0.3$	$k \leq 0.5$	$k \leq 0.7$	$k \leq 0.9$
a	FM-OLS	0.2020	0.5740	0.8060	0.9000	0.9480
	FM-LAD	0.2210	0.6280	0.8080	0.8990	0.9440
	FM-GMM	0.1590	0.4030	0.6170	0.7440	0.8210
	FM-GIVE	0.2040	0.5460	0.7430	0.8450	0.8940
B	Estimator	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.05$	$k \leq 0.07$	$k \leq 0.09$
	FM-OLS	0.0810	0.2160	0.3310	0.4560	0.5430
	FM-LAD	0.1080	0.2830	0.4210	0.5380	0.6460
	FM-GMM	0.0540	0.1710	0.2440	0.3480	0.4190
	FM-GIVE	0.0310	0.1010	0.1630	0.2260	0.2770

Table A.4.8. The simulation results of concentration probabilities when $c = 1$ and innovations are drawn from IGARCH(0.5,0.5)-t_5

		$(\Pr(\lvert\hat{b}-b\rvert \leq k))$				
Parameter	Estimator	$k \leq 0.1$	$k \leq 0.3$	$k \leq 0.5$	$k \leq 0.7$	$k \leq 0.9$
a	FM-OLS	0.2150	0.5730	0.7830	0.8790	0.9280
	FM-LAD	0.2460	0.5890	0.7970	0.8780	0.9190
	FM-GMM	0.1740	0.4140	0.5950	0.7230	0.8020
	FM-GIVE	0.2130	0.5690	0.7520	0.8420	0.9060
	Estimator	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.05$	$k \leq 0.07$	$k \leq 0.09$
B	FM-OLS	0.0830	0.1920	0.3230	0.4150	0.4980
	FM-LAD	0.0870	0.2490	0.3870	0.4840	0.5910
	FM-GMM	0.0470	0.1330	0.2340	0.3130	0.3830
	FM-GIVE	0.0260	0.0900	0.1570	0.2220	0.2930

Table A.4.9. The simulation results of bias, RMSE, quantiles and interquantile range when c=100 and innovations are drawn from N(0,1)

Parameter	Estimator	$BIAS_{ave}$	$RMSE_{ave}$	$BIAS_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	Q_{75}- Q_{25}
a	FM-OLS	-0.0215	0.4373	0.0199	0.3819	-0.1965	0.0181	0.2450	0.4415
	FM-LAD	0.0106	0.5662	0.0105	0.4985	-0.2710	-0.0017	0.2978	0.5688
	FM-GMM	-0.0960	3.0643	0.0229	0.8358	-0.4191	0.0181	0.4143	0.8334
	FM-GIVE	0.0320	0.6937	0.0200	0.5312	-0.2261	0.0199	0.2668	0.4929
B	FM-OLS	-0.0291	0.1199	-0.0301	0.1026	-0.0888	-0.0232	0.0345	0.1233
	FM-LAD	0.0346	0.1124	0.0346	0.10277	-0.0308	0.0281	0.0961	0.1269
	FM-GMM	0.0123	0.5044	0.0130	0.1938	-0.0909	0.01400	0.1208	0.2117
	FM-GIVE	0.1115	0.5705	0.1170	0.3534	-0.0426	0.0757	0.2605	0.3031

Table A.4.10. The simulation results of bias, RMSE, quantiles and interquantile range when $c=100$ and innovations are drawn from t_2

Parameter	Estimator	$MBias_{ave}$	$RMSE_{ave}$	$Mbias_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	Q_{75}- Q_{25}
a	FM-OLS	-0.0039	0.4383	-0.0016	0.3758	-0.2261	-0.0060	0.2323	0.4584
	FM-LAD	-0.0181	0.5045	-0.0151	0.4136	-0.2381	-0.0247	0.2173	0.4554
	FM-GMM	0.0123	0.9861	0.0035	0.7478	-0.3503	0.0047	0.3633	0.7136
	FM-GIVE	-0.0161	0.6357	-0.0186	0.5082	-0.2575	-0.0095	0.2412	0.4987
B	FM-OLS	-0.0314	0.1144	-0.0319	0.1035	-0.0946	-0.0300	0.0310	0.1256
	FM-LAD	0.0268	0.1033	0.0262	0.0931	-0.0317	0.0206	0.0778	0.1095
	FM-GMM	0.0020	0.2449	0.0009	0.1835	-0.0901	0.0035	0.0962	0.1863
	FM-GIVE	0.1191	0.5282	0.1152	0.3419	-0.0356	0.0806	0.2524	0.2880

Table A.4.11. The simulation results of bias, RMSE, quantiles and interquantile range when c=100 and innovations are drawn from $S_{\alpha=1.5}$

Parameter	Estimator	$MBias_{ave}$	$RMSE_{ave}$	$Mbias_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	Q_{75}- Q_{25}
a	FM-OLS	0.0040	0.4884	0.0300	0.3988	-0.2325	0.0037	0.2333	0.4658
	FM-LAD	-0.0261	0.5216	-0.0210	0.4142	-0.2361	0.0074	0.2190	0.4551
	FM-GMM	-0.0135	0.9843	-0.0080	0.7751	-0.3270	0.0126	0.3379	0.6649
	FM-GIVE	-0.0181	0.8706	0.0019	0.4894	-0.2290	0.0162	0.2573	0.4863
B	FM-OLS	-0.0283	0.1117	-0.0288	0.0975	-0.0871	-0.0251	0.0283	0.1154
	FM-LAD	0.0275	0.0985	0.0272	0.0870	-0.0241	0.0146	0.0725	0.0966
	FM-GMM	0.0088	0.2310	0.01334	0.1740	-0.0705	0.0079	0.0997	0.1702
	FM-GIVE	0.0884	1.0886	0.1124	0.3765	-0.0431	0.0618	0.2467	0.2898

Table A.4.12. The simulation results of bias, RMSE, quantiles and interquantile range when c=100 and innovations are drawn from IGARCH(0.5,0.5)-t_5

Parameter	Estimator	$MBias_{ave}$	$RMSE_{ave}$	$Mbias_{98}$	$RMSE_{98}$	Q_{25}	Q_{50}	Q_{75}	Q_{75}- Q_{25}
a	FM-OLS	0.0070	0.4934	0.0092	0.4111	-0.2066	0.0047	0.2234	0.4300
	FM-LAD	0.0237	0.5431	0.0260	0.4466	-0.2159	0.0148	0.2635	0.4794
	FM-GMM	0.0481	1.5786	0.0640	0.8350	-0.3154	0.0107	0.3923	0.7077
	FM-GIVE	-0.0230	1.2535	0.0130	0.5396	-0.2482	0.0104	0.2784	0.5266
B	FM-OLS	-0.0262	0.1172	-0.0278	0.1018	-0.0907	-0.0256	0.0361	0.1267
	FM-LAD	0.0297	0.1019	0.0285	0.0917	-0.0765	0.0201	0.0816	0.1581
	FM-GMM	0.0385	0.4052	0.0269	0.1931	-0.0798	0.0081	0.1058	0.1856
	FM-GIVE	0.0985	0.8456	0.1090	0.3546	-0.0479	0.0844	0.2360	0.2839

Table A.4.13. The simulation results of concentration probabilities when $c = 100$ and innovations are drawn from N(0,1)

		$(\Pr(\lvert \hat{b} - b \rvert \leq k))$				
Parameter	Estimator	$k \leq 0.1$	$k \leq 0.3$	$k \leq 0.5$	$k \leq 0.7$	$k \leq 0.9$
a	FM-OLS	0.2500	0.6050	0.7910	0.9050	0.9500
	FM-LAD	0.2000	0.5220	0.7280	0.8290	0.8870
	FM-GMM	0.1230	0.3850	0.5640	0.6950	0.7770
	FM-GIVE	0.2130	0.5590	0.7490	0.8460	0.9020
B	Estimator	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.05$	$k \leq 0.07$	$k \leq 0.09$
	FM-OLS	0.0870	0.2580	0.4020	0.5250	0.6500
	FM-LAD	0.0890	0.2550	0.3970	0.5200	0.6250
	FM-GMM	0.0520	0.1360	0.2440	0.3510	0.4380
	FM-GIVE	0.0430	0.1190	0.2040	0.2840	0.3490

Table A.4.14. The simulation results of concentration probabilities when $c = 100$ and innovations are drawn from t_2

		$(\Pr(\lvert \hat{b} - b \rvert \leq k))$				
Parameter	Estimator	$k \leq 0.1$	$k \leq 0.3$	$k \leq 0.5$	$k \leq 0.7$	$k \leq 0.9$
a	FM-OLS	0.2330	0.6290	0.8230	0.9020	0.9510
	FM-LAD	0.2250	0.6170	0.8040	0.8930	0.9370
	FM-GMM	0.1840	0.4430	0.6260	0.7480	0.8090
	FM-GIVE	0.2230	0.5780	0.7540	0.8420	0.8930
B	Estimator	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.05$	$k \leq 0.07$	$k \leq 0.09$
	FM-OLS	0.0750	0.2420	0.3880	0.5230	0.6100
	FM-LAD	0.2440	0.6200	0.8280	0.9250	0.9640
	FM-GMM	0.0700	0.1910	0.2970	0.3940	0.4850
	FM-GIVE	0.0380	0.1210	0.1970	0.2660	0.3390

Table A.4.15. The simulation results of concentration probabilities when $c = 100$ and innovations are drawn from $S_{\alpha=1.5}$

		$(\Pr(\lvert \hat{b} - b \rvert \le k))$				
Parameter	Estimator	$k \le 0.1$	$k \le 0.3$	$k \le 0.5$	$k \le 0.7$	$k \le 0.9$
a	FM-OLS	0.2260	0.6000	0.8070	0.9020	0.9440
	FM-LAD	0.2510	0.6130	0.8050	0.8900	0.9310
	FM-GMM	0.1560	0.4600	0.6530	0.7410	0.8040
	FM-GIVE	0.2370	0.5770	0.7670	0.8550	0.9110
B	Estimator	$k \le 0.01$	$k \le 0.03$	$k \le 0.05$	$k \le 0.07$	$k \le 0.09$
	FM-OLS	0.1020	0.2700	0.4110	0.5300	0.6570
	FM-LAD	0.1560	0.3520	0.5060	0.6380	0.7180
	FM-GMM	0.0740	0.2060	0.3380	0.4430	0.5230
	FM-GIVE	0.0520	0.1560	0.2460	0.3110	0.3750

Table A.4.16. The simulation results of concentration probabilities when $c = 100$ and innovations are drawn from IGARCH(0.5,0.5)-t_5

		$(\Pr(\lvert \hat{b} - b \rvert \le k))$				
Parameter	Estimator	$k \le 0.1$	$k \le 0.3$	$k \le 0.5$	$k \le 0.7$	$k \le 0.9$
a	FM-OLS	0.2520	0.6200	0.7980	0.8890	0.9360
	FM-LAD	0.2380	0.5870	0.7830	0.8710	0.9110
	FM-GMM	0.1740	0.4390	0.6190	0.7460	0.8200
	FM-GIVE	0.2220	0.5400	0.7270	0.8290	0.8870
B	Estimator	$k \le 0.01$	$k \le 0.03$	$k \le 0.05$	$k \le 0.07$	$k \le 0.09$
	FM-OLS	0.0890	0.2420	0.4080	0.5340	0.6380
	FM-LAD	0.0970	0.2890	0.4470	0.6040	0.7060
	FM-GMM	0.0680	0.1770	0.3130	0.4060	0.4980
	FM-GIVE	0.0410	0.1280	0.1930	0.2640	0.3350

CHAPTER FIVE REGIME SWITCHING AND BUBBLES

5.1 Introduction

The empirical study conducted in this chapter continues to examine the presence of bubbles during the three inter-war European hyperinflations. It nevertheless differs from the study in the last chapter in several aspects. First of all, the Cagan models under study are stochastic, and the model specification errors are captured by nonzero demand disturbances; moreover, the sample periods under study are extended to cover the periods of monetary reform expected and subsequently implemented, during which the data generating process and dynamic equilibrium adjustment process may be nonlinear; finally, I adopt recent advances in cointegration methodologies for the analysis and suggest that they are more powerful under nonlinear dynamics than the conventional tests so as to provide more reliable conclusions of bubbles. Evidence of nonlinear adjustment process is found in almost all cases, but evidence of bubbles still cannot be supported in any countries. The remainder of the chapter is set out as follows. Section 2 contains a discussion of related literature and empirical issues of unit root econometric methods in bubbles testing. Section 3 accounts for the cointegration methods adopted in this chapter. Section 4 presents Monte Carlo evidence on the power and size of the cointegration test statistics when the artificial data series exhibits threshold stationarity and when periodically collapsing bubbles are present in the data respectively. Data and empirical results are reported in section 5. The final section concludes the major findings.

5.2 Related Literature and Empirical Issues of Unit Root Econometric Tests

The unit root econometric methods in the context of bubble testing were first proposed by Hamilton and Whiteman (1985), Campbell and Shiller (1987), and Diba and Grossman (1988a). They argue that if an asset price series contains bubbles, it will grow faster than an integrated series. The unit root test will therefore show that this kind of price series will remain nonstationary even though it is taken a finite number of differences. Hamilton and Whiteman (1985) noted that the presence of bubbles during the German hyperinflation in the 1920's would cause the price series to exhibit a higher order of integration than any of the underlying fundamentals in a finite sample. They found that both series for price and money contained two unit roots in the German hyperinflation, and rejected the existence of bubbles. For the same reason, the residuals obtained from the projection of the asset price onto their underlying fundamentals will also be nonstationary, which implies that it would not be possible to establish a cointegrated relationship between the asset price and its underlying fundamental process when a bubble exists. Meese (1986) found that over the period 1973-1982, the exchange rate of the deutschemark was less stationary than market fundamentals that included relative money supplies and relative real incomes. Then, the null hypothesis that the deutschemark, Japanese yen and British pound did not cointegrate with their corresponding fundamentals could not be rejected, signifying the possible existence of bubbles in currency markets.

There are, however, several caveats that one may find in applying unit root econometric methods to detect bubbles. First of all, taking a finite number of differencing may make the explosive series appear stationary in small samples (Durlauf and Hooker, 1994, Hall, *et al.* 1999). Furthermore, as emphasized by Hamilton and Whiteman (1985) and, Diba and Grossman (1988), the non-cointegration between asset prices and observable fundamentals may equally result from the omission of nonstationary unobservable fundamentals. Under such a circumstance, model misspecification may be erroneously interpreted as evidence of bubbles.

In addition, the simulation conducted by Meese (1986) shows that the nonlinear bursting bubbles of Blanchard and Watson (1982) may not exhibit nonstationary behaviour discernible from the autocorrelation function. Later, Evans (1991) finds that the conventional cointegration tests result in erroneous rejection in favour of linearly stable alternatives when the simulated data series contain periodically collapsing bubbles that display nonlinear explosive properties. The size distortion is serious since the collapsing bubbles may appear in the form of a stable linear autoregressive process especially when the probability of the bubble collapse is high.

Finally, there is a problem of observational equivalence between expected future changes in economic fundamentals and bubbles (Flood and Garber, 1980b, Hamilton, 1986). Assume that there are two regimes of money supply process, non-reform and reformed process. Expectation of reform refers to the possibility of a shifting from non-reform regime of money growth process to a reformed regime at some date. In periods prior to a given reform date, the expected inflation rate[32] is equal to the rate conditional on the regime of non-reform money supply process plus a fraction of the "reform effect", which is defined as the difference between reformed process and non-reformed process of money supply in the period immediately following the reform (LaHaye, 1985). The value of the reform effect

[32] From the Cagan model (2.2), the rate of inflation is equivalent in value to the rate of currency depreciation through the PPP.

on the expected inflation rate is negative. Hence, while the expectation of monetary reform occurred toward the end of hyperinflations, the expected inflation rate would decline and the actual real money balance would increase. Furthermore, after the reform was actually implemented, the real money balance and the expected inflation rate would shift in response to the regime of reformed money supply process. As a result, the expectation and the subsequent implementation of monetary reforms might bring about regime shifting of real money balances, money supply process and velocity of circulation (Evans, 1986, Flood and Garber, 1980a, Flood and Hodrick, 1986).[33] At the empirical level, the regime-switching behaviours of these time series variables may lead to nonlinear error-correcting dynamics in a cointegrated system.[34] To respond to this problem, the common practice in the literature as mentioned in the last chapter has been to truncate the final months of hyperinflations during which the expectations of monetary reform began to take effect (for instance, Casella, 1989, Flood and Garber, 1980b, Engsted, 1993, 1996, Hooker, 2000, Woo, *et al.* 2003). However, the existence of bubble

---

[33] In addition to the issue of monetary regime changes, asymmetric intervention rules, money financing of perpetual government deficits, and the presence of transaction costs are other possible economic justifications for non-linearity in a cointegrated system (Pippenger and Goering, 1993, Peel and Speight, 1994, and Balke and Fomby, 1997).

[34] The existence of nonlinear disequilibrium error will lead to nonlinear adjustment in the error-correcting system (Balke and Fomby, 1997 and Granger and Siklos, 2001).

during the truncated observations cannot be precluded at a prior.

This chapter attempts to circumvent the above problems of bubble tests by employing an alternative bubble testing strategy. In particular, I propose to apply a sequential cointegration-testing procedure, as suggested by Engsted (1993), to handle the problem of model misspecification. By conducting cointegration analyses between the real money balance and expected price change, and subsequently between the real money balance and the money growth rate, Engsted (1993) argues that the existence of a bubble can be separated from the model misspecification arising from omitted non-stationary fundamentals. In applying this testing procedure, however, the econometric methodology chosen must be robust to the presence of both a nonlinear dynamic process and a stochastic bubble in order to avoid producing misleading cointegration results. In light of this, I therefore suggest adopting the threshold autoregressive (TAR) unit root method of Caner and Hansen (2001), which is designed to simultaneously test for unit root and threshold nonlinearity so as to make it possible to distinguish threshold stationarity from nonstationarity. The threshold cointegration methodology is expected to have superior power to reject the null hypothesis of noncointegration when the series under study are subject to nonlinear regime-switching behaviors.[35] Thus, the existence of a nonlinear process in the residuals of the money demand models, which results from the agents' expectation of changes in market fundamentals that are not observed by econometricians, will probably not be identified as a nonstationary bubble path. Furthermore, since a stochastic bubble can exhibit both nonstationary and stationary patterns during different phases of its data generation process (Funke, *et al.*, 1994), the ability of the TAR method to separate the series under study into different regimes makes it more likely to identify the nonstationary dynamics of the collapsing bubbles in at least one regime. Thus, the use of the TAR method in bubble detection is expected to produce less serious size distortion than would the conventional tests. In order to investigate the suitability of the TAR method for bubble detection, I compare, by means of Monte Carlo experimentation, the relative empirical power and size performances of the TAR method with those of some other cointegration tests that do not allow for multiple regime shifts in the data generation process. I have chosen the

---

[35] When the nonlinearity of the data generating process is deterministic at known dates, it is suggested that the effects of deterministic shifts be captured by the inclusion of an appropriate set of intervention dummies in the DF regression or vector error correction model (VECM). For instance, Woo (1999) employed the bootstrapped algorithm of van Giersbergen (1996) to generate bootstrapped critical values of Johansen's (1995) likelihood ratio statistics when intercept dummies were added to the VAR model to capture the changes in parities of the ERM currencies during 1980s and 1990s.

conventional residual-based Augmented Dickey-Fuller (ADF) and the Phillips-Ouliaris (1990) Z statistics, as well as the Im's (1996, 2001) residuals-augmented least square augmented Dickey-Fuller (RALS-ADF) t statistic as comparison tests. The ADF and the class of Z statistics are among the most popular cointegration tests that have been used in the literature. Im's (1996, 2001) method was adopted by Taylor and Peel (1998) for price bubble tests in the US stock market and from the simulation study of Taylor and Peel (1998), the RALS-ADF test has favourable power and size performance in detecting price bubbles.

5.3 Econometric Testing Procedure and Methodology

I adopt the stochastic version of the Cagan models under rational expectations in this chapter that are the same as those described in Chapter Two. For expositional convenience, I re-write some of the equations in chapter two as necessary. Let's re-write the Cagan models under rational expectations for $\pi_{1,t}$ and $\pi_{2,t}$ as follows:

$$M_t - \pi_{1,t} = \alpha_1 + \beta_1 E_t(\Delta\pi_{1t+1}) + u_{1,t} \tag{5.1}$$

$$M_t - \pi_{2,t} = \alpha_2 + \beta_2 E_t(\Delta\pi_{2,t+1}) + u_{2,t} \tag{5.2}$$

where $u_{1,t}$ and $u_{2,t}$ capture the model misspecification components in the exact Cagan models (4.1) and (4.2) respectively.

By defining the rational expectation forecasting errors as $\eta_{j,t+1} = \Delta\pi_{j,t+1} - E_t(\Delta \pi_{j,t+1})$, which are assumed to be serially uncorrelated, Eqs.(5.1) and (5.2) can be re-written as:

$$M_t - \pi_{j,t} = \alpha_j + \beta_j \Delta\pi_{j,t+1} + \xi_{j,t};\quad j = 1, 2, \tag{5.3}$$

where $\xi_{j,t} = u_{j,t} - \beta_j \eta_{j,t+1}$

For notional convenience, I eliminate the subscript j in subsequent equations. Re-arranging Eq. (5.1) and (5.2) in terms of π_t and by recursively substituting forward for $E_t(\pi_{t+1+i})$ and using the law of iterated expectations, I obtain:

$$\pi_t = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i} - u_{t+i}) + \lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) \tag{5.4}$$

Imposing the non-bubble transversality condition, $\lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) = 0$, and re-arranging terms yield:

$$M_t - \pi_t = \alpha - \sum_{i=1}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(\Delta M_{t+i}) + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(u_{t+i}) \tag{5.5}$$

The further re-arrangement of terms in Eq.(5.5) gives:

$$(M_t - \pi_t) = \alpha + \beta \Delta M_t + (\beta - 1)\sum_{i=1}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(\Delta^2 M_{t+i}) + (\frac{1}{1-\beta})\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(u_{t+i}). \tag{5.6}$$

Engsted (1993) proposes a set of sequential testing procedures to detect the presence of B_t. The procedure is conducted by comparing the cointegrating relationship between $M_t - \pi_t$ and $\Delta\pi_{t+1}$ in Eq.(5.3) as well as between $M_t - \pi_t$ and ΔM_t in Eq.(5.6). The Cagan models given by Eq.(5.3) represent a money market equilibrium condition that admits a general solution. Therefore, provided that u_t is stationary, the Cagan models cannot be rejected even if B_t exists.[36] On the other hand, the non-bubble transversality condition is imposed upon Eq.(5.6) but not upon Eq.(5.3). Hence, Eq.(5.6) represents a fundamental solution only, which can be rejected if B_t is present in the price or exchange rate data.[37] Consequently, three possible outcomes may arise from the tests of these two cointegrating relationships. First of all, if $M_t - \pi_t$ cointegrates both $\Delta\pi_{t+1}$ and

[36] It should be noted that the rational expectation forecasting error, $\eta_{j,t+1}$, is a white noise by definition.

[37] The idea of Engsted (1993) is similar to that of West (1987a), which compares the differences of parameters obtained from two sets of equations rather than the differences in their cointegrating properties.

ΔM_t, it implies that u_t is I(0) and the presence of B_t is less likely to occur. Moreover, if $M_t - \pi_t$ only cointegrates with $\Delta\pi_{t+1}$ but not with ΔM_t, it indicates that u_t is I(0) and B_t may be present. However, if neither $\Delta\pi_{t+1}$ nor ΔM_t cointegrates with $M_t - \pi_t$, it indicates the fundamental failure of the Cagan models due to the presence of a nonstationary u_t. Therefore, this testing procedure can distinguish the existence of bubbles from the model misspecification caused by omitted nonstationary fundamentals. For further investigating whether B_t exists in the final case, the model must have to be re-specified so that the omitted nonstationary fundamental variables are included in the model.

One econometric method I choose to conduct the above cointegration-testing procedure in this chapter is the residuals-based threshold cointegration test. By applying the two-regime TAR unit root method of Caner and Hansen (2001) to the OLS estimated residuals of Eq.(5.3) and (5.6),[38] it can test for cointegration and threshold nonlinearity simultaneously, which is considered to be ideal for threshold cointegration analysis by Balke and Fomby (1997). The TAR regression can be represented by the following equation:

---

[38] As pointed out by Balke and Fomby (1997), the OLS estimates of a cointegration vector remain superconsistency under certain conditions even though the disequilibrium error exhibits threshold nonlinearity. Also, it is assumed to be constant in the threshold cointegration framework.

$$\Delta y_t = \Theta_1' S_{t-1} I(Z_{t-1} < \lambda_o) + \Theta_2' S_{t-1} I(Z_{t-1} \geq \lambda_o) + e_t \tag{5.7}$$

where y_t denotes the OLS residual of either Eq.(5.3) or (5.6); $S_{t-1} = (y_{t-1}, 1, t, \Delta y_{t-1}, \ldots, \Delta y_{t-k})'$ is a vector of regressors in which t denotes a linear time trend, $\Theta_i = (\rho_i, c_i, b_i, \Psi_{i,t-1}, \ldots, \Psi_{i,t-k})'$ represents the slope parameters associated with the corresponding regressors of S_{t-1}, $I(.)$ is an indicator function, Z_{t-1} is the observable threshold variable, λ_o is a threshold value and e_t is an i.i.d. error. In practice, there are two possible ways to specify Z_{t-1}, as either $y_{t-1} - y_{t-m-1}$ or $y_{t-m} - y_{t-m-1}$, which are known as long difference and lagged difference thresholds, respectively. Although the particular specification for the threshold variable is not essential, Z_{t-1} must be strictly stationary and ergodic. Hence, the long difference or lagged difference threshold specification is preferred to the lagged level of y_t because y_t is I(1) under the null.[39] The OLS estimate of λ_o is found by minimizing $\sigma^2(\lambda_o)$ in the unrestricted Eq.(5.7).

The sufficient conditions for the convergence of y_t in a two regime threshold model require $-2 < \rho_i < 0$ for i=1,2 (Enders and Granger, 1998).[40] Caner and Hansen (2001) propose using a class of R_T Wald statistics, which comprises the

[39] Eq. (5.7) is alternatively known as momentum TAR model, first introduced by Enders and Granger (1998), in which Δy_{t-1}, rather than y_{t-1}, is used as the threshold value.

[40] As given in Chan and Tong (1985), there are less stringent conditions for stability of a first-order threshold model.

one-sided (R_{1T}) and two-sided (R_{2T}) Wald statistics, which are expressed as $t_1^2\mathbf{I}(\rho_1 < 0) + t_2^2\mathbf{I}(\rho_2 < 0)$ and $t_1^2 + t_2^2$ respectively, to test for the null hypothesis that $\rho_1 = \rho_2 = 0$. The R_{1T} and R_{2T} Wald statistics can lead to a rejection of the null hypothesis when $\rho_i < 0$ and $\rho_i \neq 0$, respectively, for at least one i = 1,2. The distributions of the R_T statistics are nonstandard and have to be approximated by bootstrapping. In practice, there are two methods to carry out the bootstrapping. The first one is the unidentified bootstrap that assumes no threshold effects on parameters in Θ_i under the null and the second is the identified bootstrap that imposes the restriction of threshold effects. Caner and Hansen (2001), however, recommends the use of the unidentified bootstrap to calculate the bootstrap p-values because the size distortions of unit root test obtained by simulation from the identified bootstrap method are substantially large. Other than the joint test statistics, there are two individual t-ratio statistics, t_1 and t_2, for ρ_1 and ρ_2 respectively. Nevertheless, as found in the simulation study of Caner and Hansen (2001), the power of the t-ratio statistics associated with the lower absolute value of ρ_i will substantially deteriorate with an increase in $|\rho_1 - \rho_2|$ and $|c_1 - c_2|$. It is also found that even if the series of y_t is stationary in two regimes, the powers of the individual t-ratio statistics are no larger than those of the R_T Wald statistics. Following the lines of argument by Granger and Siklos (2001), the R_T Wald statistics are still useful when the point estimates for ρ_i fall within the ranges of –2 and 0, ∀ i, and especially when there are threshold effects in coefficients.[41] The evidence for the existence of nonstationary roots in two regimes is rejected when the R_T Wald statistics are significant with all point estimates of ρ_1 and ρ_2 lying between –2 and 0.

On the other hand, the existence of threshold effects can be tested under the null hypothesis of no threshold effect, i.e., $\Theta_1 = \Theta_2$. The Wald statistics, W_T, for examining this restriction is given by T $(\sigma_0^2/\ \sigma^2(\lambda_o) - 1)$, where T refers to the number of usable observation and σ_0^2 is the residual variances from OLS estimation of the null linear model. The asymptotic distribution of W_T depends upon the presence of nonstationarity in y_t. Hence, Caner and Hansen (2001) suggests the unconstrained and constrained bootstrapping approximations to the asymptotic distribution of W_T; the former based upon the unrestrictive estimates of parameters in Eq. (5.7) and the latter enforcing the restriction of a unit root. Also, the Wald statistics can be implemented to test for the equality of individual coefficients of Θ across regimes.

---

[41] Enders and Siklos (2001) find that the F statistic for the null hypothesis of non-cointegration in two regimes of a M-TAR model has more power than the individual t-ratio (t-max and t-min) statistics in simulation experiments. Hence, they recommend the use of F statistic for conducting a threshold cointegration test in cases where the point estimates of ρ_i imply convergence for all i.

The other cointegration tests considered for comparing with the TAR R_T statistics include the conventional ADF and the Phillips-Ouliaris (1990) Z statistics, as well as the Im's (1996, 2001) RALS-ADF t statistic. While the ADF **t** statistic corrects for serial correlation by adding lagged terms in the Dickey-Fuller (DF) regression (Said and Dickey, 1984), the cointegrating Z_a and Z_t statistics of Phillips and Ouliaris (1990) employ the semi-parametric method to asymptotically eliminate the bias caused by the weakly dependent and heterogeneously distributed innovations of the DF regression. Consider the standard DF regression for the OLS residual y_t :

$$y_t = \delta y_{t-1} + e_{ot}, \qquad \{e_{ot}\} \sim \text{i.i.d. } (0,\ \sigma^2_{e_o}). \tag{5.8}$$

The Z_a and Z_t statistics are given as:

$$Z_a = T(\hat{\delta}-1) - (s^2 - s_e^2)(2T^{-2}\sum_{t=1}^{T} y_{t-1}^2)^{-1} \tag{5.9}$$

$$Z_t = (\frac{s_u}{s})t_\alpha - (\frac{1}{2})(s^2 - s_e^2)(s^2 T^{-2}\sum_{t=1}^{T} y_{t-1}^2)^{-1/2} \tag{5.10}$$

where $t_\alpha = \dfrac{(\hat{\delta}-1)}{s_e(\sum_{t=1}^{T} y_{t-1}^2)^{-1/2}}$, $s_e^2 = T^{-1}\sum_{t=1}^{T} \hat{e}_{ot}^2$ .Given that $\sigma^2_{e_o} = \lim_{T\to\infty} T^{-1}E[\sum_{t=1}^{T} e_t^2]$, $\sigma^2 = \lim_{T\to\infty} T^{-1}E[\sum_{t=1}^{T} e_t]^2$. s_e^2 and s^2 are consistent estimates of $\sigma^2_{e_o}$ and σ^2

respectively. The term $(s^2 - s_u^2)(T^{-2}\sum_{t=1}^{T} y_{t-1}^2)^{-1}$ is referred to as the serial correlation correction factor. As argued by Perron and Ng (1996), the size distortion will be serious in the semiparametric procedure of the unit root tests when there are innovations of the DF regression with a moving average root close to -1.[42]

On the other hand, the RALS-ADF t statistic introduced by Im (1996, 2001), is a variant of the Hansen's (1995) covariate ADF (CADF) statistic, which is developed by including correlated stationary covariates into the ADF regression with an aim to increase the power of the ADF unit root test. To illustrate the working of CADF, let's suppose that there exists an m-vector x_t, which is related to the series, y_t, that is, the OLS residual of Eq.(5.3) or (5.6). Further suppose that x_t, is I(1) and its differenced series, Δx_t, is therefore I(0). Δx_t is known as stationary covariates of the following CADF regression:

$$\Psi(L)\Delta y_t = c + bt + \delta y_{t-1} + \theta(L)'(\Delta x_t - u_x) + e_t, \qquad (5.11)$$

where $\Psi(L) = 1 - \Psi_1 L - \ldots - \Psi_p L^p$, $u_x = E(\Delta x_t)$, $\theta(L) = \theta_{q_1} L^{-q_1} + \ldots + \theta_{q_2} L^{q_2}$ is a

---

[42] Haldrup (1994 and 1998) recognize that the size distortion for the semiparametric double unit root tests is serious in the presence of large negative MA errors in the DF regression. Appendix 5.1 presents a work of simulation study to examine the size distortion of the semiparametric double unit root tests.

polynomial allowing for both leads and lags of the deviation of the covariates from their expected means. The values of p, q_1 and q_2 are chosen to make e_t white noise. For conducting the analysis, it is useful to define a variable, v_t, which equals $\theta(L)'(\Delta x_t - u_x) + e_t$. Using this variable, I can further define the long-run squared correlation between v_t and e_t, denoted ρ^2, as $\sigma_{ve}^2/(\sigma_e^2\,\sigma_v^2)$, where $\sigma_{ve}^2 = \sum_{k=0}^{\infty}(v_t e_{t-k})$, $\sigma_e^2 = \sum_{k=0}^{\infty} e_t e_{t-k}$ and $\sigma_v^2 = \sum_{k=0}^{\infty} v_t v_{t-k}$, and the variance ratio, denoted R^2, as σ_e^2/σ_v^2. The value of ρ^2 lies between 0 and 1, which measures the relative contribution of Δx_t to v_t. As ρ^2 approaches zero (unity), Δx_t tends to explain all (none of) the movement in v_t. Hence, the inclusion of Δx_t in Eq.(5.11) leads to a smaller σ_e^2, which therefore increases the power of the ADF test statistic to reject the false null hypothesis that $\delta = 0$. The asymptotic critical values of the CADF statistic in the context of unit root testing are dependent upon the values of ρ^2 and are obtained in Hansen (1995) from simulation. Using the same number of Monte Carlo replications with the same length of sample as in Hansen (1995), the asymptotic critical values of the cointegrating CADF statistic are simulated and are reported in Appendix 5.2. The asymptotic critical values are obtained from applying the CADF to an OLS estimated residual of a regression. From Appendix 5.2, the critical values for $\rho^2 = 1$ are equal to those reported in Phillips and Ouliaris (1990). As the values of ρ^2 approach toward zero, the corresponding critical values (in absolute terms) become smaller.

When the distribution of v_t exhibits strong skew and excess kurtosis, the standardized third central moment of v_t will differ from zero, i.e. $E_t(v_t^3 - \sigma_v^3) = E_t[v_t(v_t^2 - \sigma_v^2)] \neq 0$, and the standardized fourth central moment will exceed three, i.e. $E_t(v_t^4 - 3\sigma_v^4) = E_t[v_t(v_t^3 - 3\sigma_v^3 v_t)] \neq 0$, respectively. Based on these observations, Im (1996 and 2001) suggest that the appropriate choice of Δx_t be $[(v_t^3 - 3\sigma_v^2 v_t)'\ (v_t^2 - \sigma_v^2)']'$. The t statistic for $\delta = 0$ is known as the RALS-ADF t-ratio statistic, which will the be more robust to the presence of skew and kurtosis in the distribution of the innovation term of the DF regression. The asymptotic critical values of the RALS-ADF and the CADF test statistics are the same with the same value of ρ^2.

5.4. Monte Carlo Experiments

In this section, I use Monte Carlo experiments to evaluate the power and size properties of the above cointegration methodologies in a finite sample.

5.4a. Power test

To implement the power tests, I first need to generate an I(0) cointegrating regression residual with a two-regime threshold nonlinearity, which can be obtained from the following data generating process of length 45:

$$P_t^1 = 0.5\, F_t + y_t, \tag{5.12a}$$

$$F_t = F_{t-1} + v_{1,t}, \qquad F_0 = 0 \text{ and } v_{1,t} \sim N(0,1), \tag{5.12b}$$

$$y_t = y_{t-1} + \Theta_1' S_{t-1} I(Z_{t-1} < \lambda_o) + \Theta_2' S_{t-1} I(Z_{t-1} \geq \lambda_o) + e_t, \tag{5.12c}$$

where $\Theta_i' = (\rho_i, c_i)'$, $S_{t-1} = (y_{t-1}, 1)'$, $Z_t = y_t - y_{t-1}$, λ_o is unknown and is set to be median of y_t and $e_t \sim N(0,1)$. Eq. (5.12a) represents a cointegrating regression and y_t is the residual of the Eq. (5.12a) with threshold effects. The empirical power of the cointegration tests is calculated by applying the test statistics to the estimated residuals, $\hat{y}_t$, which are obtained from regressing P_t^1 on F_t using the OLS method. By specifying the threshold effects on y_t, I denote $\Delta c = c_2 - c_1$ with Δc being varying among {0,1,3}; moreover, I vary ρ_2 among {-0.1, -0.3, -0.5, -0.7} when ρ_1 = -0.1, and vary ρ_2 among {-0.05, -0.3, -0.5, -0.7} when ρ_1 = -0.3. Notice that the threshold effect is absent when $\Delta c = 0$ and $\rho_1 = \rho_2$. The powers of the TAR R_T tests are evaluated using the corresponding 5% bootstrapped critical values. On the other hand, the corresponding finite-sample 5% critical values of the cointegrating ADF and Z statistics are obtained from Phillips and Ouliaris (1990), and the critical values of the cointegrating RALS-ADF test statistics are obtained from the simulated 20,000 draws of length 45 from two I(1) processes for U_t and F_t, and applying the test statistics to the estimated residuals, $\hat{U}_t$, from regressing P_t^2 on F_t, where P_t^2 is constructed as follows:

$$P_t^2 = F_t + U_t \tag{5.12d}$$

where $U_t = U_{t-1} + v_{2,t}$, $U_0 = 0$ and $v_{2,t} \sim N(0,1)$

In conducting the Monte Carlo study, the simulated series of P_t^1, P_t^2 and F_t are demeaned and detrended using the OLS method. For each cointegration test, 1000 Monte Carlo replications were performed. From the results reported in Table 5.1, the TAR R_T statistics have the greatest powers among other test statistics across all parameterizations of ρ_i and Δc. In addition, the power superiority of the TAR R_T statistics over other cointegration tests in the simulation study remains unchanged even when there are no identified threshold effects, i.e. $\Delta c = 0$ and $\rho_1 = \rho_2 = -0.1$ or $= -0.3$. By comparison, the power of the cointegrating RALS-ADF test generally shrinks with an increase in Δc. Nevertheless, the RALS-ADF test still performs much better than the classes of ADF and Z tests, though its power is inferior to that of the TAR R_T statistics especially when Δc is large. On the other hand, the class of ADF and Z statistics suffers noticeable power loss especially when Δc is different from zero.[43] The performance of the ADF and Z_t tests are somewhat better than that of the Z_a test. The rejection frequencies of Z_a fall to about the 5% nominal size in 13 of the total of 24 cases that I conducted in the experiment, and the rejection frequencies of the ADF and Z_t are lower than 10% in about 12 of the 24 cases.

[Table 5.1 to be inserted here]

5.4b. Size test

As pointed out by Evans (1991), the periodically collapsing bubbles may appear to be stationary as well as nonstationary (both integrated and explosive). This means that the statistical rejections of the noncointegration between asset prices and underlying fundamentals may be attributable to size distortion rather than the high power of the tests. Taylor and Peel (1998) thus argue that the cointegration methods chosen for bubble testing must have a relatively small size distortion in accepting the non-cointegration null when the stochastic bubbles

[43] Likewise, Pippenger and Goering (1993) produced unfavourable conclusions on the power of standard DF statistics under the threshold process.

actually exist in the data.[44] It is therefore necessary to assess the relative empirical sizes of the above cointegration test statistics while the Evans' (1991) periodically collapsing bubbles are present in the artificially generated data. The percentages of rejection frequencies for the size test are calculated in the same way as for the above power test, except the fact that $\boldsymbol{y_t}$ in Eq.(5.12c) is now replaced by the collapsing bubble, $\boldsymbol{B_t}$, with the same parameterization as given in Evans (1991) such that $\boldsymbol{P_t^1} = \boldsymbol{F_t} + \boldsymbol{B_t}$.[45]

Table 5.2 presents the findings for the empirical sizes of all the cointegration tests examined in the simulation experiment with different values of Π, which denote the probability that the collapsing bubble will continue, as illustrated in Chapter Two. In general, the size distortions of the cointegration tests tend to increase with the frequency of bubble collapses. In particular, when the bubble process is almost deterministic with the value of Π approaching zero, the empirical sizes would become much smaller. More specifically, when the value of Π does not exceed 0.7, the classes of ADF and $\boldsymbol{Z}$ tests reject over 20%, and even up to about

[44] It is different from Bohl (2003), which focuses upon the asymmetric adjustment of the M-TAR model proposed by Enders and Siklos (2001) in detecting periodically collapsing bubbles in the US stock market.

[45] The specification and the parameterisation of the Evans' (1991) periodically collapsing bubbles were illustrated as Eq. (2.32) in Chapter Two.

80%. Therefore, the collapsing property of the bubble process makes it difficult for these tests to identify a nonstationary root contained in the artificial series. When the value of Π rises to 0.9 or above, the empirical rejection frequencies of the ADF and the ***Z*** tests are found to fall below the 5% nominal size. As for the TAR R_T tests, they perform much better than the ADF and ***Z*** tests, especially when the simulated bubble process exhibits a high frequency of periodic collapses. The TAR R_T tests reject no more than 14.5% for all values of Π. Only in cases where the value of Π is equal to 0.9 or higher, are the rejection rates of the TAR R_T statistics higher than those of the ADF and ***Z*** statistics, but the empirical sizes of the TAR R_T statistics converges toward the 5% nominal level. As mentioned above, the threshold model that is designed to separate the artificial data series into two separate regimes makes it more capable of identifying the non-stationary dynamics of the collapsing bubbles in at least one regime. In the simulation experiment, the number of times that an explosive root is found in any one regime ranges from about 56% to about 72% (unreported here) when the value of Π rises from 0.2 to 0.95. Moreover, the cointegrating RALS-ADF test performs the best in the size test. The rejection frequencies of the RALS-ADF test are higher than those of the TAR R_T tests when Π is not greater than 0.5, they however fall sharply to be the lowest as Π exceed 0.5. The rejection rates of the RALS-ADF test is even under-sized, that is, lower than the 5% nominal size, while the value of Π is greater than 0.625. Such findings of favourable size performance for the RALS-ADF test are similar to those of Taylor and Peel (1998) with a larger sample size. As mentioned in Section 4.2, the existence of bursting bubbles may cause large kurtosis and excessive skewness in the distribution of price changes. The RALS-ADF test based upon an estimator originally designed to be robust to the presence of skewness and kurtosis, as argued by Taylor and Peel (1998), may be less biased toward erroneous rejection of noncointegration when the periodically bursting bubbles are present under the null.

To sum up, the power of the TAR R_T tests is the highest among all the tests for all parameterizations of ρ_i and Δc under the threshold stationary processes in the finite sample that I have simulated. The power of the RALS-ADF test is reasonably high enough to detect the nonlinear dynamics with the inclusion of stationary covariates, although the RALS-ADF statistic is a linear cointegration test. On the other hand, in the presence of periodically collapsing bubbles, all statistics under study may falsely reject the null of noncointegration when Π is low. As Π is lower than 0.5, the empirical sizes for the TAR R_T tests are the lowest. As Π increases, the RALS-ADF test is more likely to accept the null of noncointegration than other cointegration test statistics.

[Table 5.2 to be inserted here]

5.5 Estimation Periods and Empirical Results

My sampled data for the three European hyperinflations in this chapter are extended to cover the periods of monetary reforms that are truncated in literature such as Engsted (1993, 1994 and 1996) and Taylor (1991). In other words, the observation periods of the data in this chapter are the same as those used in Chapter Three for the structural time series analysis.

5.5.1 Price bubbles

I first use the OLS method to estimate β_1 by regressing $\boldsymbol{M}_t - \pi_{1,t}$ onto $\Delta\pi_{1,t+1}$ in Eq. (5.3), and by regressing $\boldsymbol{M}_t - \pi_{1,t}$ onto $\Delta\boldsymbol{M}_t$ in Eq. (5.6). I then implement the conventional ADF and $\boldsymbol{Z}$ tests of bubble existence and report all of the results in Table 5.3. From this I learn that all the values of β_1 of Eqs (5.3) and (5.6) are found to have the correct negative sign. In addition, the cointegrating ADF and $\boldsymbol{Z}$ tests could not identify any cointegrated relationships between $\boldsymbol{M}_t - \pi_{1,t}$ and $\Delta\pi_{1,t+1}$ or between $\boldsymbol{M}_t - \pi_{1,t}$ and $\Delta\boldsymbol{M}_t$ for the three countries under study. In other words, the classes of ADF and $\boldsymbol{Z}$ tests reject the validity of the Cagan model for the sampled countries.

[Table 5.3 to be inserted here]

However, from the results of the cointegrating RALS-ADF tests presented in Table 5.4, the RALS-ADF test statistics are all significant, thereby indicating that the residuals, u_{1t} of Eq.(5.3) and υ_{1t} of Eq.(5.6) are both stationary. The empirical findings are interpreted against the evidence of price bubbles in all countries under study. In addition, the values of ρ^2 vary from 0.11 to 0.77. Therefore, the power of the RALS-ADF test is likely to be increased by the inclusion of the stationary covariates in the ADF regressions.

[Table 5.4 to be inserted here]

To extend the analysis, I repeat the bubble testing exercise but this time using the residual-based threshold cointegration tests, which take into account a threshold process with multiple regime shifts in the data series. The empirical results of this test are reported in Table 5.5, which can be compared with those of the ADF, Z, and RALS-ADF tests. As shown in Table 5.5, the point estimates of ρ_1 and ρ_2 for the regression of $M_t - \pi_{1,t}$ on $\Delta\pi_{1,t+1}$, are all negative with significant R_T statistics. Hence, the existence of nonstationary roots across two regimes in the money demand equilibrium errors, u_{1t}, in Eq. (5.3) for the sample countries, are ruled out.

In addition, the significance of the W_T statistics indicates that the equilibrium errors, u_{1t}, exhibit threshold nonlinearity in all three countries, which are caused by the regime shifting of one or more elements in Θ_i across their corresponding thresholds. On the other hand, when the threshold cointegration tests are applied on the regression of $M_t - \pi_{1,t}$ on ΔM_t, I obtain negative estimates of ρ_1 and ρ_2 as well as significant R_T statistics, thereby implying that the residuals of Eq. (5.6) are stationary for all the sampled countries. Therefore, given the fact that the cointegrating relationships can be established for both Eqs (5.3) and (5.6), I do not find any evidence of price bubbles in the data for all of the sampled hyperinflations. It is also noted that the W_T statistics for the joint threshold effects in the residuals of Eq. (5.6) are significant for all the countries except Poland, where only W(*t*) is significant at the 10% level. Hence, the evidence of threshold nonlinearity cannot be rejected for the residuals of both Eqs (5.3) and (5.6).

[Table 5.5 to be inserted here]

From the above empirical results, I find that the TAR R_T tests and the RALS-ADF test consistently reject the null of non-cointegration, while the ADF and Z tests accept the null. Based upon the finite-sampled power properties of the cointegration tests considered in my simulation experiment, the frequent acceptance of the noncointegration null for the classes of the ADF and $\boldsymbol{Z}$ tests are likely to be caused by their low power characteristics in handling data series that possibly display threshold nonlinear dynamics with multiple regime shifts. It is important to note that when the final few observations of three hyperinflationary episodes are truncated, Engsted (1993,1994) and Taylor (1991) could find evidence of cointegrating relationships in Eqs.(5.3) or (5.6). Further, the Monte Carlo findings of my size test show that the TAR R_T tests and the RALS-ADF test are more likely to accept the true null of non-cointegration than the classes of ADF and $\boldsymbol{Z}$ tests when the periodically collapsing bubbles are actually present. Consequently, my empirical results of the tests do not point to the existence of collapsing bubbles.

5.5.2 Exchange rate bubble

I now turn to the exchange rate bubbles testing. The results of the ADF and $\boldsymbol{Z}$ tests are presented in Table 5.6 The OLS estimates of β_2 are all correctly signed. The ADF and $\boldsymbol{Z}$ test statistics provide evidence of cointegration between $\boldsymbol{M}_t - \pi_{2,t}$ and $\Delta\pi_{2,t+1}$ for Germany only. Moreover, the cointegration relationships between $\boldsymbol{M}_t - \pi_{2,t}$ and $\Delta\boldsymbol{M}_t$ for all three countries are rejected. Therefore, the classes of the ADF and $\boldsymbol{Z}$ statistics provide some empirical evidence of exchange rate bubbles in the German data.

[Table 5.6 to be inserted here]

However, it should be noted that the ADF and $\boldsymbol{Z}$ tests would be biased towards the null of noncointegration when the data are subject to regime changes. To investigate further, I apply the cointegrating RALS-ADF test statistics to the residuals of Eqs (5.3) and (5.6) again. I report the results of the test in Table 5.7, which cannot reject the validity of the Cagan model and can provide evidence against exchange rate bubbles in all sampled countries. Note that the values of ρ^2 vary from 0.17 to 0.67, indicating an increase in power of the test that is made possible by the inclusion of stationary covariates.

[Table 5.7 to be inserted here]

Finally, I apply the TAR unit root tests to the residuals of Eq. (5.3) and Eq. (5.6) with the results presented in Table 5.8. I find that the point estimates of ρ_1 and ρ_2 for the residuals of Eq. (5.3) are all negative and the R_T statistics are significant. In addition, the W_T statistics are significant except in the case of Poland, indicating that $\boldsymbol{u}_{2t}$ exhibits threshold nonlinearity only for Hungary and Germany. Based on the individual Wald statistics, I observe that the nonlinearity in the data for Hungary is caused by the asymmetric values of ρ_1 and ρ_2, whereas the nonlinear dynamics for Germany are produced by the asymmetric changes in all the elements in Θ_{i}. For the estimation of Eq. (5.6), the results suggest that $M_t - \pi_{2,t}$ and ΔM_t are cointegrated for all the sampled countries as indicated by the negative values of ρ_1 and ρ_2 as well as the significance of the R_T statistics. The dynamics of the residuals in the regression (5.6) are found to be nonlinear in the cases of Germany and Hungary, as shown by the asymmetric values of ρ_1 and ρ_2. In the case of Poland, however, the joint and individual hypotheses of threshold effects for the residuals of Eq. (5.6) are all rejected.[46] In conclusion, since the cointegrating relationships are established for Eqs (5.3) and (5.6), these results do not suggest any evidence of exchange rate bubbles in the data for the sampled hyperinflations. Also, the dynamics of the equilibrium errors u_{1t} of Eq. (5.3) and the residuals of Eq. (5.6), generally display threshold nonlinearity in the data for all the sampled countries.

[Table 5.8 to be inserted here]

The findings of threshold effects in the data series leads me to conclude that the frequent non-rejection of the ADF and Z tests is probably attributable to the low

[46] As shown in the above Monte Carlo findings, the powers of the TAR R_T statistics are still the highest in a small sample even under unidentified threshold effects.

power property of these tests under nonlinear processes with multiple shifts. Similar to the case of the price bubble tests, the results of the threshold cointegration tests corroborate those of the RALS-ADF test. It is found that when the final few observations of the German data were truncated, Engsted (1996) could reject the evidence of the exchange rate bubble in Germany. Moreover, the overall size distortion of the TAR R_T tests and the RALS-ADF test from the Monte Carlo simulation are found to be smaller than that of the class of the ADF and ***Z*** tests. Thus, the empirical results that the null of noncointegration is rejected by the TAR R_T tests and the RALS-ADF test while it is frequently accepted by the ADF and ***Z*** tests can be interpreted as evidence that contradicts the existence of collapsing bubbles in the data for all the three countries.

5.5 Concluding Remarks

In order to overcome the problems of identifying bubbles caused by anticipation of changes in fundamentals, the common practice is to concentrate the empirical tests on pre-reform samples. Using longer sample horizons, I attempt to employ a threshold cointegration method together with some linear cointegration test statistics for the further investigation of bubbles in this chapter. From the findings of the Monte Carlo study, the power of the TAR R_T tests is the highest in a finite sample no matter whether there are threshold processes or not. The RALS-ADF test gains substantial power improvement under the threshold processes by the inclusion of stationary covariates in the ADF regression in finite samples. The empirical results of the TAR R_T concur with those of the RALS-ADF test statistics and support the evidence of cointegrating relationships between real money balance and expected inflation rate (or expected depreciation rate) as well as real money balance and money growth rate for all countries of interest. Given the fact that the orthogonality tests in Chapter Four reject the evidence of bubbles on the pre-reform observations, the evidence against the existence of bubbles in Chapter Five conclude that bubbles cannot be found on periods after the monetary reforms as well. Furthermore, the evidence of threshold nonlinear processes cannot be rejected in almost all cases for the countries under study. Hence, it is found that the low power of the conventional cointegration tests in small samples under nonlinear process is likely to produce a spurious conclusion of bubbles. Moreover, the Monte Carlo findings show that in the presence of Evans' (1991) bubbles, the TAR R_T tests and the RALS-ADF test are more likely to accept the null of noncointegration than the classes of ADF and ***Z*** statistics. The above empirical results are obviously not consistent with the evidence for collapsing bubbles in the hyperinflationary data for Germany, Hungary and Poland in the inter-war period.

However, the cointegration-testing methodology has its own drawbacks for bubble testing. For example, if an explosive bubble exists and the money supply is endogenous, then the money supply and the price series may contain a common explosive component. Real money balance may be stationary but the inflation rate, the currency depreciation rate or the money growth is nonstationary. Consequently, it may lead to misleading results. Given the fact that every econometric methodology has its own limitations as well as advantages, it is then suggested to adopt more than one method for the empirical study of bubbles. The orthogonality methodology and the cointegration methodology should be complementary for bubble testing. In fact, the two methodologies have their own specialties. The former can exploit the high-frequency properties of the data whereas the latter exploits the low-frequency properties (Engsted, 2003).

On the other hand, the switching processes so far have been restricted to depend upon observable threshold values. Actually, threshold and Markov models can be observationally equivalent (Carrasco, 2002). Thus, it is required to further examine the evidence of bubbles when the regime-switching process of time series variables is assumed to depend on unobservable Markov states. In the next chapter, I carry out a Markov switching cointegration analysis to achieve the objective.

Table 5.1. Percentage of rejection frequencies against an alternative of threshold stationarity

ρ_1=-0.1	$\Delta c = 0$				$\Delta c = 1$				$\Delta c = 3$			
ρ_2 =	-0.10	-0.30	-0.50	-0.70	-0.10	-0.30	-0.50	-0.70	-0.10	-0.30	-0.50	-0.70
R_{1T}	30.20	45.90	72.80	91.00	30.00	35.50	52.70	75.60	29.00	29.60	35.00	41.30
R_{2T}	29.30	44.30	70.50	90.00	29.30	33.70	50.80	74.30	28.10	29.00	33.30	40.20
RALS-ADF	25.60	38.60	53.20	66.00	20.20	21.00	24.60	33.40	24.00	20.40	20.50	18.70
ADF	9.10	15.30	29.70	48.00	5.70	5.50	7.40	15.90	8.80	6.40	6.80	5.60
Z_a	5.80	10.40	21.60	40.30	2.50	2.20	4.50	11.20	4.80	3.20	3.20	3.20
Z_t	10.30	18.10	33.70	54.40	7.00	7.30	10.00	18.10	9.90	7.40	8.00	7.00
ρ_1=-0.3	$\Delta c = 0$				$\Delta c = 1$				$\Delta c = 3$			
ρ_2 =	-0.050	-0.30	-0.50	-0.70	-0.050	-0.30	-0.50	-0.70	-0.050	-0.30	-0.50	-0.70
R_{1T}	47.40	56.70	74.20	86.90	64.10	67.30	76.00	87.10	61.10	68.70	76.40	83.80
R_{2T}	44.10	54.90	72.30	86.00	62.80	66.10	74.00	85.50	59.30	67.60	75.30	82.20
RALS-ADF	37.90	52.70	70.70	83.00	23.40	27.40	36.00	49.10	35.80	28.30	26.60	25.20
ADF	12.20	30.00	50.00	72.40	5.80	7.10	12.70	26.40	17.40	4.80	3.90	4.80
Z_a	7.50	21.30	38.90	63.00	3.30	4.70	9.50	20.00	9.30	2.70	1.90	2.30
Z_t	14.50	34.00	55.40	76.20	7.10	9.50	17.20	32.30	19.80	6.10	5.90	6.90

Notes:

1. Each entry is the percentage of instances in which the null hypothesis was correctly rejected.
2. The Monte Carlo study is based on 1,000 replications for each case, with the finite sample of 45.

Table 5.2. Percentage of rejection frequencies of non-cointegration in the presence of periodically collapsing bubbles

Π	R_{1T}	R_{2T}	RALS-ADF	ADF	Z_a	Z_t
0.20	12.17	11.50	46.10	49.90	48.80	52.60
0.40	11.87	11.63	43.60	71.80	72.50	73.70
0.50	13.60	13.30	40.70	78.10	75.80	79.50
0.60	14.43	14.14	9.50	72.40	55.60	76.20
0.625	14.14	13.71	5.60	65.10	45.10	69.90
0.65	12.29	11.86	3.30	56.00	36.80	62.00
0.70	12.70	12.60	0.80	37.00	21.70	42.60
0.80	7.80	7.90	0.30	10.10	4.90	13.60
0.90	4.00	4.00	0.00	2.10	1.50	2.50
0.95	4.30	4.20	0.80	1.20	1.20	1.50

Notes:

1. Each entry is the percentage of instances in which the null hypothesis was incorrectly rejected.
2. The Monte Carlo study is based on 1000 replications for each case, with the finite sample of 45.

Table 5.3. Cointegrating ADF and Z tests for price bubbles

Country	α_1	β_1	ADF	Z_a	Z_t
Cointegration test for the regression of $M_t - \pi_{1,t}$ on $\Delta\pi_{1,t+1}$					
Germany	8.3271	-0.6871	-2.1934	-18.5794	-2.0782
Hungary	0.8043	-1.0091	-1.9475	-8.8528	-2.0928
Poland	0.9500	-1.0635	-3.5561	-17.6590	-3.6762
Cointegration test for the regression of $M_t - \pi_{1,t}$ on ΔM_t					
Germany	8.3731	-0.9652	-3.4910	-20.8454	-3.5395
Hungary	1.1051	-3.3883	-3.0770	-9.1919	-2.1644
Poland	1.1947	-2.003	-3.2723	-13.8429	-2.7591

Notes:

1. For conducting the ADF and Z tests the data are demeaned and detrended by using the OLS method.
2. The significance of the ADF and Z tests are based upon the critical values from Phillips and Ouliaris (1990).
3. */**/*** denote significance at the 1%/5%/10% levels, respectively.

Table 5.4. Cointegrating RALS-ADF test for price bubbles

Country	RALS-ADF[p]	$t(\Delta x_t)$	ρ^2	R^2	Q(1)	Q(6)
Cointegration test for the regression of $M_t - \pi_{1,t}$ on $\Delta\dot{\pi}_{1,t+1}$						
Germany	-3.6496[3]**	(2.2286**, 6.9937*)	0.2039	0.2153	0.7590	1.7591
Hungary	-3.5117[3]*	(-6.4991*, -3.5032*)	0.1087	0.1354	1.3321	5.1044
Poland	-4.8816[0]*	(6.0132*)	0.5861	0.5157	0.2790	2.0401
Cointegration test for the regression of $M_t - \pi_{1,t}$ on ΔM_t						
Germany	-3.8575[3]**	(-2.3619**, 4.7527*)	0.3467	0.4129	0.1467	2.7035
Hungary	-5.4648[3]*	(1.3867, -3.6878*)	0.5495	0.2449	0.3422	2.7865
Poland	-3.8267[1]**	(-0.0582, 1.5024)	0.7751	0.8883	0.0434	2.9459

Notes:

1. The significance of the RALS-ADF tests depends upon the simulated finite-sample critical values. The asymptotic critical values reported in appendix 4.1, however, lead to the same results of significance.
2. R^2 refers to the variance ratio defined in Hansen (1995).
3. $t(\Delta x_t)$ denotes **t** statistics for the coefficients of stationary covariates included in the ADF regression.
4. Q(k) refers to Ljung-Box Q-statistics with degrees of freedom = k.
5. */**/***Denote significance at the 1%/5%/10% levels, respectively.

Table 5.5. Threshold Cointegration test of price bubbles

Threshold cointegration test statistics for the regression of $M_t - \pi_{1,t}$ on $\Delta\pi_{1,t+1}$									
Country	λ[m]	R_{2T}	R_{1T}	ρ_1	ρ_2	W_T#	W(c)	W(t)	W(y_{t-1})
Germany	-0.061[4]	85.8* (0.0092)	85.8* (0.0000)	-1.0200	-1.6900	130* (0.0000) [0.1090]	2.8700 (0.8370)	18.2** (0.0464)	4.39* (0.0020)
Hungary	0.055[1]	22.0*** (0.0631)	22.0*** (0.0622)	-0.9290	-0.2710	25.3** (0.0226) [0.0319]	12.2*** (0.0541)	19.1* (0.0092)	6.5700 (0.1160)
Poland	-0.266[2]	21.8** (0.0502)	21.8** (0.0485)	-0.8300	-0.6540	22.4** (0.0339) [0.0339]	4.9800 (0.3490)	10.8** (0.0519)	0.2850 (0.7520)
Threshold cointegration test statistics for the regression of $M_t - \pi_{1,t}$ on ΔM_t									
Country	λ[m]	R_{2T}	R_{1T}	ρ_1	ρ_2	W_T#	W(c)	W(t)	W(y_{t-1})
Germany	0.087[1]	32.7** (0.0484)	32.7** (0.0482)	-0.1380	-1.2800	59.4** (0.0194) [0.0264]	28.9** (0.0374)	31.9** (0.0267)	16.4** (0.0486)
Hungary	0.162[4]	28.3** (0.0357)	28.3** (0.0356)	-0.1270	-0.9840	26.2** (0.0188) [0.0362]	0.0044 (0.9860)	0.9820 (0.6650)	16.00* (0.0039)
Poland	-0.046[2]	24.9** (0.0456)	24.9** (0.0452)	-0.9900	-0.7690	12.2000 (0.2940) [0.4360]	0.1320 (0.9090)	8.08*** (0.0736)	0.3750 (0.7330)

Notes:

1. The number of usable observations for Germany, Hungary and Poland are 37, 42, and 37, respectively.
2. λ[m] denotes the value of threshold with delay number, m.
3. W(c), W(t), W(y_{t-1}) and W(Δ y_{t-1}) refer to the Wald statistics to examine the hypothesis that the intercepts, the coefficients of linear time trend, y_{t-1} and Δ y_{t-1} respectively are equal between two regimes.
4. # the figure in the (.) denotes the unconstrained bootstrapped p value; whereas the figure in the [.] denotes the constrained bootstrapped p-value.
6. */**/***Denote significance at the 1%/5%/10% levels, respectively.
5. Since the empirical results reject the null of unit root, the */**/*** refer to the significance to the unconstrained bootstrapped p value.

Table 5.6. Cointegrating ADF and *Z* tests for exchange rate bubbles

Country	α_2	β_2	ADF	Z_a	Z_t
Cointegration test for the regression of $M_t - \pi_{2,t}$ on $\Delta\pi_{2,t+1}$					
Germany	6.0429	-0.3154	-4.9178*	-34.0155*	-5.1365*
Hungary	8.3513	-0.8426	-2.7531	-12.0403	-2.7210
Poland	8.5968	-0.5950	-2.9242	-18.3954	-3.1626
Cointegration test for the regression of $M_t - \pi_{2,t}$ on ΔM_t					
Germany	6.1433	-0.5996	-2.2389	-11.1121	-2.3384
Hungary	8.5766	-2.1281	-2.1803	-8.3965	-2.3195
Poland	8.8065	-1.4736	-2.2823	-11.9840	-2.4829

See notes to Table 5.3.

Table 5.7. Cointegrating RALS-ADF Test for exchange rate bubbles

Country	RALS-ADF[*p*]	t(Δx_t)	ρ^2	R^2	Q(1)	Q(6)
Cointegration test for the regression of $M_t - \pi_{2,t}$ on $\Delta\pi_{2,t+1}$						
Germany	-3.9711[0]**	(-4.8783*, 8.4394*)	0.4837	0.3788	0.6681	3.4756
Hungary	-4.1294[0]*	(-6.3331*, -3.7160*)	0.1739	0.1757	0.5769	6.8238
Poland	-4.7557[0]*	(-2.5401**, -1.2725)	0.6773	0.6667	0.0004	2.8828
Cointegration test for the regression of $M_t - \pi_{2,t}$ on ΔM_t						
Germany	-3.9074[3]*	(1.6479, 3.1490*)	0.2846	0.1606	0.0046	4.7146
Hungary	-3.7618[2]**	(-0.2391, 6.7937*)	0.5412	0.6136	0.5093	6.4561
Poland	-5.1258[3]*	(7.1830*)	0.1601	0.2777	0.5532	2.1798

See notes to Table 5.4.

Table 5.8 Threshold Cointegration test of exchange rate bubbles

Threshold cointegration test statistics for the regression of $M_t - \pi_{2,t}$ on $\Delta\pi_{2,t+1}$									
Country	λ[m]	R_{2T}	R_{1T}	ρ_1	ρ_2	W_T#	W(*c*)	W(*t*)	W(y_{t-1})
Germany	0.304[1]	48.4* (0.0039)	48.4* (0.0039)	-0.5100	-1.9300	31.0** (0.0108) [0.0254]	27.6* (0.0027)	25.2* (0.0032)	15.6* (0.0082)
Hungary	0.284[1]	25.6** (0.0337)	25.6** (0.0324)	-0.1700	-1.5200	19.3*** (0.0839) [0.1210]	2.5500 (0.5620)	2.8600 (0.4320)	17.0* (0.0068)
Poland	-0.350[2]	31.7* (0.0060)	31.7* (0.0058)	-1.7600	-0.7410	11.900 (0.2890) [0.3180]	1.3100 (0.6770)	0.0156 (0.9470)	5.25 (0.140)
Threshold cointegration test statistics for the regression of $M_t - \pi_{2,t}$ on ΔM_t									
Country	λ[m]	R_{2T}	R_{1T}	ρ_1	ρ_2	W_T#	W(*c*)	W(*t*)	W(y_{t-1})
Germany	0.166[2]	33.3** (0.0163)	33.3* (0.0158)	-0.4730	-1.7100	20.3000 (0.1060) [0.2110]	1.4500 (0.7490)	2.6800 (0.5250)	7.45*** (0.0698)
Hungary	0.325[3]	26.3*** (0.0699)	26.3*** (0.0692)	-0.3620	-1.4200	13.2000 (0.2060) [0.3440]	7.0700 (0.1980)	5.0500 (0.2630)	9.65** (0.0461)
Poland	0.42[2]	25.5*** (0.0504)	25.5*** (0.0501)	-1.2200	-1.2800	15.5000 (0.1650) [0.3100]	0.0061 (0.9820)	4.0800 (0.3160)	0.0106 (0.9480)

See notes to Table 5.5.

Appendix 5.1

Consider a regression of the generalized ADF form:

$$\Delta^2 y_t = (a_1 - 1)y_{t-1} + (a_2 - 1)\Delta y_{t-1} + \sum_{i=1}^{p-2} \phi_i \Delta^2 y_{t-i} + e_t \qquad \text{(A.5.1)}$$

where y_t is a variable of interest, p is the autoregressive order of y_t and e_t is an i.i.d innovation term. If y_t contains two unit roots, then $\hat{a}_1 = \hat{a}_2 = 1$. Hence, it is natural to test the joint hypothesis by a F-test. Haldrup (1994) documents the advantages of a joint test for a double unit root over a sequential testing procedure of Dickey and Pantula (1987).

Haldrup (1994) derives a semiparametric F test (H-F) to implement the joint tests for double unit roots, which is an extension to the parametric F test of Haza and Fuller (1979). Moreover, a symmetrized version of (A.5.1) suggested by Sen and Dickey (1987) is given as:

$$\Delta^2 y_t = (a_1 - 1)y_{t-1} + (a_2 - 1)\Delta y_{t-1} + \sum_{i=1}^{p-2} \phi_i \Delta^2 y_{t-i} + e_{1t} \qquad t = p+1,\ldots,T.$$

(A.5.2)

$$\Delta^2 y_t = (a_1 - 1)y_{t-1} - (a_2 - 1)\Delta y_t + \sum_{i=1}^{p-2} \phi_i \Delta^2 y_{t+i} + e_{2t} \qquad t = 3,\ldots,T-p+2.$$

(A.5.3)

where e_{1t} and e_{2t} refer to i.i.d. innovation terms. Likewise, a symmetrized joint test for double unit roots is conducted by a F-test for $\hat{a}_1 = \hat{a}_2 = 1$. Shin and Kim (1999) introduces a semiparametric F test (SK-F) based on the symmetric estimation of Sen and Dickey (1987). The detailed survey of the I(2) tests are documented in Haldrup (1998).

When the semiparametric procedure is adopted, no augmentation through lagged second differences, $\Delta^2 y_{t\pm i}$, in (A.5.1), (A.5.2) and (A.5.3) is required. Nevertheless, Perron and Ng (1996) explain how these size distortions relate to the kernel estimator of σ^2 in the semiparametric procedure of the Phillips and Perron (1988) when there are innovations of the DF regression with an MA root close to -1. Since the test statistics of Haldrup (1994) and Shin and Kim (1999) are derived with an application of the semiparametric procedure of Phillips and Perron (1988), they are likely to have a similar size distortion problem. In this appendix, I provide a size test of the above two I(2) tests when the data generating process is simulated as $\Delta^2 y_t = (1+\theta)\varepsilon_t$ with $\varepsilon_t \sim N(0,1)$ and θ = -0.1, -0.2 , ..., -0.9. The number of incorrect rejections is calculated using the finite-sample critical values obtained from the Monte Carlo simulation with $\theta = 0$.

The results of the empirical size are shown in Table A.5.1. The size distortion is quite serious for the large negative MA roots (θ) for both semiparametric I(2) test. In particular, when T=30, the size distortion of SK-F(C) is much lower than that of H-F(C); whereas the incorrect rejection rate of the SK-F(T^2) and SK-F(T) are generally larger than that of H-F(T^2) and H-F(T) respectively. When T increases to 50, the semiparametric tests of Shin and Kim (1999) generally suffer from less serious size distortions than that of Haldrup (1994) for any orders of the time polynomial in fitted regressions. Therefore, symmetric estimators are likely to result in smaller size distortion when T is large.

It is also noted that the size distortion of the SK-F(T^2) is larger than that of the SK-F(T), which is in turn more substantial than that of the SK-F(C). For the SK-F(C), the percentage of incorrect rejection frequencies is around the 5% nominal size when the absolute value of θ is no larger than 0.4. It however rises sharply from less than 30% to over 60% when the absolute value of θ increases from 0.8 to 0.9. On the other hand, the size distortion of the H-F(C) is larger than that of the H-F(T^2), which is in turn more serious than that of the H-F(T) for most values of θ. The percentage of incorrect rejection frequencies for the tests of Haldrup (1994) is larger than the 5% nominal size for all detrending specifications even when the absolute value of θ is

0.1 only.

From the above, semiparametric tests of double unit root suffer from size distortion in the presence of negative MA roots in the innovation terms of the DF regression. Hence, I feel doubtful about adopting semiparametric double unit root test methods to examine the stochastic properties of the relevant time series variables.

Table A.5.1. **Empirical sizes of the semiparametric double unit roots tests when the true data generating processes contain negative MA roots**

T	MA (θ)	SK-F(T²)	H-F(T²)	SK-F(T)	H-F(T)	SK-F(C)	H-F(C)
30	-0.1	0.0925	0.0610	0.0486	0.0613	0.0465	0.0853
	-0.2	0.1063	0.0588	0.0601	0.0526	0.0421	0.0979
	-0.3	0.1646	0.0697	0.0801	0.0553	0.0532	0.1308
	-0.4	0.2094	0.1006	0.1118	0.0806	0.0598	0.2001
	-0.5	0.3167	0.1583	0.1394	0.1381	0.0966	0.3106
	-0.6	0.4186	0.2597	0.2518	0.2384	0.1101	0.4627
	-0.7	0.5686	0.4030	0.3877	0.3998	0.2433	0.6420
	-0.8	0.7386	0.5715	0.5803	0.6189	0.2841	0.8150
	-0.9	0.7829	0.6991	0.8182	0.8322	0.7070	0.9196
50	-0.1	0.0551	0.0627	0.0386	0.0577	0.0331	0.0847
	-0.2	0.0691	0.0679	0.0386	0.0566	0.0331	0.1166
	-0.3	0.0979	0.0951	0.0562	0.0722	0.0360	0.1793
	-0.4	0.1436	0.1588	0.0562	0.1182	0.0457	0.2905
	-0.5	0.2193	0.2905	0.1136	0.2126	0.0623	0.4618
	-0.6	0.3373	0.4886	0.1136	0.3761	0.0926	0.6789
	-0.7	0.5238	0.7303	0.3045	0.6143	0.1553	0.8767
	-0.8	0.7517	0.9180	0.3045	0.8689	0.2966	0.9800
	-0.9	0.9272	0.9883	0.8607	0.9916	0.6017	0.9995

Notes:

1. SK-F(T²) and H-F(T²) are the semiparametric F tests of Shin and Kim (1999) and Haldrup (1994) respectively when y_t is detrended by a constant, a linear trend and a polynomial trend.
2. SK-F(T) and H-F(T) are the semiparametric F tests of Shin and Kim (1999) and Haldrup (1994) respectively when y_t is detrended by a constant and a linear trend.
3. SK-F(C) and H-F(C) are the semiparametric F tests of Shin and Kim (1999) and Haldrup (1994) respectively when y_t is detrended by a constant.
4. The number of the Monte Carlo replications is 10,000.

Appendix 5.2

Table A5.2. Asymptotic critical values for cointegrating CADF t-statistics

	Standard			Demeaned			Detrended		
ρ^2	1%	5%	10%	1%	5%	10%	1%	5%	10%
1.0	-2.4631	-2.7648	-3.3369	-3.9060	-3.3502	-3.0486	-4.3599	-3.7791	-3.4962
0.9	-3.3268	-2.7410	-2.4431	-3.8625	-3.3044	-2.9950	-4.2754	-3.7208	-3.4195
0.8	-3.3076	-2.7246	-2.4187	-3.8078	-3.2484	-2.9407	-4.2037	-3.6333	-3.3358
0.7	-3.2872	-2.7031	-2.3963	-3.7636	-3.1900	-2.8716	-4.1261	-3.5503	-3.2443
0.6	-3.2886	-2.6825	-2.3737	-3.7176	-3.1262	-2.8054	-4.0344	-3.4582	-3.1422
0.5	-3.2795	-2.6575	-2.3425	-3.6531	-3.0570	-2.7222	-3.9646	-3.3568	-3.0343
0.4	-3.2480	-2.6319	-2.3079	-3.6007	-2.9856	-2.6464	-3.8779	-3.2409	-2.9147
0.3	-3.2257	-2.6003	-2.2795	-2.5581	-2.9028	-2.5581	-3.7779	-3.1188	-2.7822
0.2	-3.1927	-2.5824	-2.2542	-3.4707	-2.8260	-2.4739	-3.6592	-2.9743	-2.6298
0.1	-3.1800	-2.5602	-2.2279	-3.3993	-2.7422	-2.3736	-3.5247	-2.8236	-2.4575

Notes:

1. The critical values were calculated from 60,000 draws generated from samples of sizes 1,000 with i.i.d. Gaussian innovations.
2. Parzen kernel is used and the bandwidth is calculated using the suggestion of Andrews (1991).

CHAPTER SIX MARKOV-SWITCHING COINTEGRATION TEST FOR BUBBLES

6.1 Introduction

It is noted that the expectations and implementation of monetary reforms during periods of hyperinflation might lead to regime changes in economic variables. Failure to model the regime-shifting behaviour of time series may lead to biased conclusions with respect to cointegration and the existence of bubbles. In the previous chapter, threshold cointegration analysis was employed to model the switching processes that are restricted to depend upon observable threshold values. However, threshold nonlinearity and Markovian regime shifts may be observationally equivalent. In this chapter, I will continue to use the same Cagan model specifications, and the testing procedure of Engsted (1993), as in the previous chapter, to conduct the empirical study of bubbles. The regime shifting behaviour of time series variables are, alternatively, assumed to depend on unobservable states generated by a first-order Markov chain, and I adopt the Markov-switching cointegration method for bubble testing. The probability law that governs the Markov-switching regimes is advantageous in that it is more flexible and allows the data to determine the specific form of nonlinearities that are consistent with the sample information. The empirical findings still do not support the evidence for bubbles but the Markovian regime-switching behaviour can be identified. Also, inferences about the probabilities of the unobservable states at each point in time can be made. The chapter is structured as follows: Section 2 contains a discussion of the econometric methodology; Section 3 reports the empirical results; concluding remarks are contained in Section 4.

6.2. Econometric Methodology

Krolzig (1996, 1997), and Yao and Attali (2000) argue that a linear cointegration method is asymptotically valid to test for the number of cointegrating vectors in a Markov error-correction model. However, Nelson *et al.* (2001), Psaradakis (2001) and Cavaliere (2003) consider that the conventional unit root tests will result in biased conclusions when the series under study exhibit Markov shifts. Hence, Cavaliere (2003) suggests that unit root or cointegration tests be carried out using a statistical method that allows for the Markov switching process.

In this chapter, I conduct the cointegration-testing procedure of Engsted (1993), as illustrated in Chapter 5, for the identification of bubbles by sequentially applying the MS-ADF unit root test of Hall *et al.* (1999) to the OLS residuals of Eqs (5.3) and

(5.6). The MS-ADF cointegration methodology can be used to simultaneously test for the existence of nonstationary roots and allow for the possibility of Markovian regime shifts in the structure of the disequilibrium errors.[47] The Markov shifts in the series under study can be detected by allowing the ADF parameters to switch values between different regimes generated by a Markov process. Further, the simulation study conducted by Hall *et al.* (1999) confirm that, compared to the standard ADF test, the MS-ADF **t** test statistics can effectively detect the periodically collapsing bubbles of Evans (1991) by identifying the existence of an explosive root at least in one regime. Using the MS-ADF unit root test for empirical studies, Funke *et al.* (1994) and Hall *et al.* (1999) found some evidence of inflationary bubbles in the data for Poland from 1991-1993 and for Argentina from 1983 to 1989 respectively.

I start with the general form of the Markov-switching ADF regression of order p with 2 regimes, which allows for different regime shifts in the parameters: [48]

$$\Delta y_t = c(s_t) + \rho(s_t) y_{t-1} + \sum_{j=1}^{p-1} \Psi_j(s_t) \Delta y_{t-i} + b(s_t) t + e_t \quad e_t \mid s_t \sim \text{NID}(0, \sigma_e(s_t)),$$

$$t = 1 \ldots T. \qquad (6.1)$$

[47] Similarly, the methodology of Caner and Hansen (2001) used in Chapter Five is designed to test for unit root or cointegration under threshold nonlinearity.

[48] Some argue that the inclusion of regime-dependent deterministic trend may capture the explosive dynamics of a bubble process.

where y_t is an OLS residual of Eq. (5.3) or (5.6),[49] is a white noise process with zero mean and regime-switching standard deviation $\sigma_e(s_t)$. All parameters of the autoregression, $c(s_t)$, $\rho(s_t)$, $\psi_j(s_t)$, and $\sigma_e(s_t)$ are conditioned on a finite number of stochastic unobservable Markov-switching state variable $s_t \in \{1,2\}$ such that: [50]

$$c(s_t) = c_1 s_{1t} + c_2 s_{2t},\ \rho(s_t) = \rho_1 s_{1t} + \rho_2 s_{2t},\ b(s_t) = b_1 s_{1t} + b_2 s_{2t},$$

$$\psi_j(s_t) = \psi_{j1} s_{1t} + \psi_{j2} s_{2t},\ \sigma_e(s_t) = \sigma_1 s_{1t} + \sigma_2 s_{2t} \tag{6.2}$$

where s_{it} takes on the value 1 when $s_t = i$, and 0 otherwise, for $i = 1, 2$.

The stochastic process generating the unobservable regimes is an ergodic Markov chain governed by the transition probabilities, $P_{ij} = \Pr[s_t = j \mid s_{t-1} = i]$ with $\sum_{j=1}^{2} P_{ij} = 1\ \forall \text{i, j} \in \{1,2\}$. For an ergodic Markov chain, regime shifts are persistent if $P_{ij} \neq P_{ii}$ for some i $\neq$j, but not permanent if $P_{ii} \neq 1\ \forall$ i. The filtered probability of $s_t = j$, denoted by $\Pr[s_t = j \mid \Omega_t]$, is equal to $\sum_{i=1}^{2} \Pr[s_t = j, s_{t-1} = i \mid \Omega_t]$, conditional on information up to time t, Ω_t. The smoothed probability of $s_t = j$, denoted by $\Pr[s_t = j \mid \Omega_T]$, conditional on all the information in the sample, Ω_T, and is

[49] As pointed out by Yao and Attali (2000), the OLS estimates of a cointegration vector remain superconsistent under regular conditions even though the disequilibrium error exhibits a Markov-switching process.

[50] In Hall *et al.* (1999), the Markov state s_t is set to 0 or 1, but in this chapter, s_t is set to be 1 or 2 instead.

calculated by $\sum_{i=1}^{2} \Pr[s_t = j, s_{t+1} = i \mid \Omega_T]$. The Hamilton's (1989) filtering and the Kim's (1994) smoothing algorithms are employed to make inferences about the filtered and smoothed probabilities of the unobservable Markov regimes respectively. An expectation-maximization (EM) algorithm for maximum likelihood estimation is used to yield estimated parameters of the ADF regression (6.1).

The existence of nonstationary roots is rejected when the two individual t-ratio statistics, t_1 and t_2, reject the null hypothesis that $\rho_i \geqq 0$ against the alternative of $\rho_i < 0$, for all i = 1 and 2.[51] In addition to the t-ratio statistics, I propose a Wald statistic to test for the joint hypothesis that $\rho_1 = \rho_2 = 0$ against the alternative of $\rho_i \neq 0$ for at least one i. When both the point estimates of ρ_1 and ρ_2 lie in the open interval of -2 and 0, the significance of the Wald statistic implies the rejection of the existence of nonstationary roots across two regimes in the data series of interest. The associated p-values of the Wald and t-ratio statistics are obtained via simulation.

On the other hand, the number of the Markov-switching regimes cannot be tested using conventional testing approaches due to the presence of unidentified nuisance parameters such as the transition probabilities under the null of linearity.

---

[51] The more rigorous stability conditions for a Markov-switching model are documented in Yao and Attali (2000).

Hansen (1992, 1996) derived formal tests of Markov-switching nonlinearity, which involve the approximation of the asymptotic distribution of the likelihood ratio (LR) via simulation and evaluation of the likelihood function across a grid of different values for the transition probabilities as well as for each state-dependent parameter. This is, however, computationally demanding and time consuming. In practice, Krolzig (2002) suggests the alternatives that include the upper bound of Davies (1977, 1987) for the significance level of the LR test statistics under nuisance parameters,[52] and information criteria such as AIC, SC and HQ (see for example, Krolzig, *et al.* 2002 and Clarida, *et al.* 2003).

6.3 Empirical Results

The data and the sample periods in this section are identical to those in Chapter Five. They include data series of price levels, money supply and exchange rates for the inter-war European hyperinflations of Germany, Hungary and Poland. In the subsequent empirical studies, the estimation is conducted using the maximum likelihood method with the expectation-maximization (EM) algorithm. The intercept

[52] The Wald statistic for the joint hypothesis that the intercept terms are equal across two regimes is valid (Krolzig, *et.al.* 2002). Hence, if singular matrix occurs when the LR test is implemented, the Wald statistic will be used instead.

terms $c(s_t)$, standard deviation of error $\sigma_e(s_t)$, and the coefficients $b(s_t)$, $\psi_j(s_t)$, and $\rho(s_t)$, of the MS-ADF regression (6.1), can be made regime-dependent if the corresponding upper bound LR test statistic of Davies (1977,1987) is significant. The number of lag length, p, is chosen to make e_t white noise. The p-values of the MS-ADF Wald and t-ratio statistics are obtained via simulation.[53]

6.3.1 Price bubbles

Table 6.1a contains the maximum likelihood estimates of the MS-ADF regressions for the OLS residuals of Eq.(5.3), which are obtained from regressing $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$. The estimation results show that all parameters of the MS-ADF regression are made regime-dependent for all of the hyperinflations under study, except the fact that the intercept term is regime-invariant for Germany. All the point estimates of ρ_1 and ρ_2 in two regimes are negative and the corresponding Wald and t-ratio statistics are significant from the associated p-values. Hence, the results favour the evidence of cointegrating relationships between $M_t - \pi_{1,t}$ and $\Delta\pi_{1,t+1}$. Table 6.1b presents the estimation results of the MS-ADF test for the OLS residuals obtained from regressing $(M_t - \pi_{1,t})$ on ΔM_t. As shown in Table 6.2b, the

[53] For some cases, I add a vector of regime-dependent stationary covariates into the MS-ADF regressions to increase power as suggested in Hansen (1995).

intercepts of the MS-ADF regression are allowed to be regime-dependent for Hungary only. The coefficients $b(s_t)$, $\psi_j(s_t)$, and $\rho(s_t)$ can vary across different regimes for Germany and Poland. The standard deviation of e_t can be regime-dependent for Germany and Hungary. All point estimates of ρ_1 and ρ_2 are negative, and both the Wald and t-ratio statistics are significant. This signifies the acceptance of the cointegrated relationships of $(M_t - \pi_{1,t})$ on ΔM_t. When $(M_t - \pi_{1,t})$ cointegrates with $\Delta\pi_{1,t+1}$ and ΔM_t the evidence for price bubbles in the data, as suggested by Engsted (1993), is rejected.

[Table 6.1a and 6.1b to be inserted here]

Figures 6.1 to 6.6 plot the filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ and of $(M_t - \pi_{1,t})$ on ΔM_t, from which the Markovian regime shifts are found in the residuals of Eqs (5.3) and (5.6) throughout the whole samples.

[Figure 6.1 and Figure 6.6 to be inserted here]

6.3.2 Exchange rate bubbles

The maximum likelihood estimates of the MS-ADF regressions for the OLS residuals of Eq.(5.3) are obtained from regressing $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$. The results are presented in Table 6.2a. The intercept term of the MS-ADF regression is made state-dependent for Germany and Hungary. The joint LR allows the coefficients $b(s_t)$, $\psi_j(s_t)$, and $\rho(s_t)$, of the MS-ADF regression to switch between regimes for Germany and Hungary. Also, the standard error of e_t can be state-dependent for Germany and Hungary. In other words, all the parameters of the MS-ADF regression for Poland are restricted to be regime-independent and the t-ratio value of ρ is the standard ADF **t** test statistic. For Germany and Hungary, the point estimates of ρ_1 and ρ_2 in two regimes are found to be negative and the MS-ADF Wald and t-ratio statistics are significant. From the above, it can be concluded that there exists a cointegrating relationship between $(M_t - \pi_{2,t})$ and $\Delta\pi_{2,t+1}$ for all of the countries under study. Moreover, the results of the cointegrating tests applied to the OLS residuals of Eq.(5.6) obtained from regressing $(M_t - \pi_{2,t})$ on ΔM_t are presented in Table 6.2b. For Germany, only the intercept of the MS-ADF regression is restricted to be state-invariant; whereas for Poland, only the intercept can be made regime-dependent. All the parameters of the MS-ADF regression are different across regimes for Hungary. From the negative point estimates of ρ_1 and ρ_2 with significant Wald and t-ratio statistics, the existence of a cointegrated relationship between $(M_t - \pi_{2,t})$ and ΔM_t cannot be rejected. While $(M_t - \pi_{2,t})$ cointegrates with $\Delta\pi_{2,t+1}$ and with ΔM_t, no nonstationary roots are found in the residuals of Eqs (5.3) and (5.6) respectively. Hence, the evidence for an exchange rate bubble is rejected in the data for all of the hyperinflations under study. .

[Table 6.2a and Table 6.2b to be inserted here]

The patterns of regime shifts can be seen from Figures 6.7 to 6.11 where the filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$ and of $(M_t - \pi_{2,t})$ on ΔM_t , are plotted. The regime-switching behaviours can be found throughout the whole estimation periods for the countries under study.

[Figure 6.7 and Figure 6.11 to be inserted here]

6.4. Concluding Remarks

In this chapter, I have continued the cointegration tests to examine the bubble existence using the cointegrating Markov-switching ADF tests. The regime shifts in the MS-ADF regression are allowed to depend on an unobservable state variable governed by the Markov chain rather than on an observable threshold value. The empirical results show that the evidence of Markovian regime shifts is found in the cointegration residuals from the regression of $M_t - \pi_t$ and $\Delta\pi_{t+1}$ as well as $M_t - \pi_t$ and ΔM_t. Also, the point estimates of ρ_i are all negative and the Wald and t-ratio statistics are all significant. Hence, the evidence favours the MS cointegrating relationship in both Eqs (5.3) and (5.6), and it rejects the presence of bubbles in any of the countries under study.

Table 6.1a. MS Cointegrating ADF test for the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$

Country	Germany		Hungary		Poland	
Parameters	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Intercept of the MS-ADF regression:						
$c(s_t)$	0.2028*** (0.1072)		1.0282* (0.2199)	0.0918 (0.0769)	1.0687* (0.0859)	0.1634 (0.1576)
$LR(c_1 = c_2)$	0.0025# [0.9599]		18.4070* [0.0004]		8.7839** [0.0323]	
Coefficients of the ADF regression:						
$b(s_t)$	-0.0046 (0.0037)	-0.0294* (0.0053)	-0.0508* (0.0078)	-0.0006 (0.0021)	-0.0495* (0.0026)	-0.0021 (0.0069)
$\rho(s_t)$	-0.2697	-1.5341	-0.8473	-0.2527	-0.7355	-0.6644
$t(\rho(s_t))$	-4.3517*	-6.7925*	-6.4622*	-3.8816**	-25.1221*	-3.8598**
$W(\rho(s_t))$	66.0191*		51.4814*		652.829*	
Joint LR	51.0888* [0.0000]		25.6972* [0.0001]		12.1681** [0.0300]	
Standard error of residuals						
$\sigma_e(s_t)$	0.0695	0.9889	0.1200	0.1153	0.0370	0.2731
$LR(\sigma_1 = \sigma_2)$	53.6782* [0.0000]		14.4253* [0.0024]		9.1883** [0.0269]	
Transition probability matrix:						
P11 P12	0.8310	0.1690	0.4837	0.5173	0.4110	0.5890
P21 P22	0.6773	0.3227	0.1729	0.8271	0.1816	0.8184
Diagnostic checking						
AIC	-0.0332		-0.3292		0.6719	
HQ	0.1663		-0.1782		0.8250	
SC	0.5386		0.0804		1.1208	
Q(12-p)	8.1515		7.9668		9.8592	

Notes:

1. $t(\rho(s_t))$ and $W(\rho(s_t))$ refer to the MS-ADF t and Wald statistics respectively.
2. Joint LR refers to the joint LR linearity test for the coefficients $\psi_1(s_t)$, $b(s_t)$ and $\rho(s_t)$, of the MS-ADF regression.
3. # Denotes the Wald statistic, rather than the LR statistic, for linearity tests.
4. The figures in (.) are standard errors. The figures in [.] are the p-values for the significance of the upper bound LR linearity tests.
5. AIC, HQ and SC refer to the Akaike, Schwarz, and Hannan and Quninn criterion respectively.
6. Q(k) refers to Ljung-Box Q-statistics with degrees of freedom = k.
7. */**/*** Denote significance at the 1%, 5% and 10% level.
8. For the case of Germany, the stationary covariates include $\Delta^2\pi_{2,t-1}$ and $\Delta^2 M_{t-1}$.

Table 6.1b. MS Cointegrating ADF test for the regression of $(M_t - \pi_{1,t})$ on ΔM_t

Country	Germany		Hungary		Poland	
Parameters	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Intercept of the ADF regression:						
$c(s_t)$	0.6770* (0.1069)		-0.1264** (0.0499)	0.1889* (0.0477)	0.1935* (0.0164)	
$\text{LR}(c_1 = c_2)$	0.0000# [0.9932]		11.2275** [0.0106]		0.0003# [0.9855]	
Coefficients of the ADF regression:						
$\psi_1(s_t)$	———		0.5581* (0.0873)		-0.0385 (0.0444)	0.6612* (0.1968)
$\psi_2(s_t)$	———		———		0.3992* (0.0452)	0.6127* (0.2025)
$b(s_t)$	-0.0258* (0.0036)	-0.0249* (0.0042)	-0.0027** (0.0013)		-0.0178* (0.0008)	-0.0044* (0.0014)
$\rho(s_t)$	-0.3968	-0.4378	-0.6259		-0.2293	-0.8422
$t(\rho(s_t))$	-5.0306*	-4.4110*	-8.5393*		-7.9263*	-5.6670*
$W(\rho(s_t))$	33.5772*		72.9204*		87.431*	
Joint LR	50.4305* [0.0000]		9.4629 [0.2284]		35.2855* [0.0000]	
Standard error of residuals						
$\sigma_e(s_t)$	0.0562	0.1384	0.09442		0.0175	0.1200
$\text{LR}(\sigma_1 = \sigma_2)$	32.5865* [0.0000]		0.0232 [0.8789]		19.4283* [0.0002]	
Transition probability matrix:						
P11 P12	0.5937	0.4063	0.8625	0.1375	0.4373	0.5627
P21 P22	0.5332	0.4668	0.0769	0.9231	0.2889	0.7111
Diagnostic checking						
AIC	-0.8563		-0.9086		-0.7081	
HQ	-0.6568		-0.7873		-0.4804	
SC	-0.2786		-0.5777		-0.0211	
Q(12-p)	13.2538		12.7802		10.8851	

Notes:

1. For the cases of Germany and Poland, the stationary covariates include $\Delta^2 M_{t-i}$, for i=1,2, and $\Delta^2 \pi_{2,t-1}$ respectively.
2. Other notes to Table 6.1a still apply.

Figure 6.1. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ in Germany

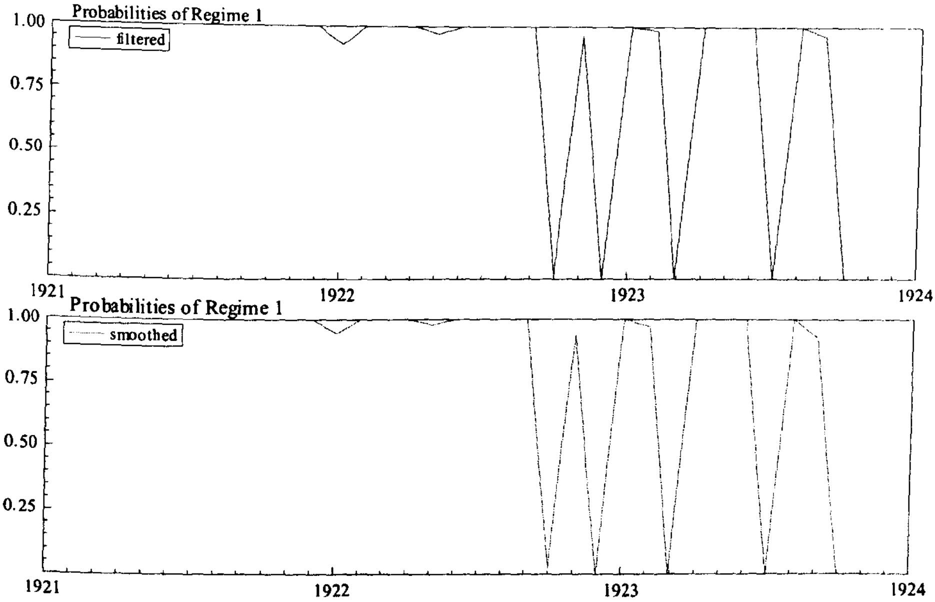


Figure 6.2. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ in Hungary

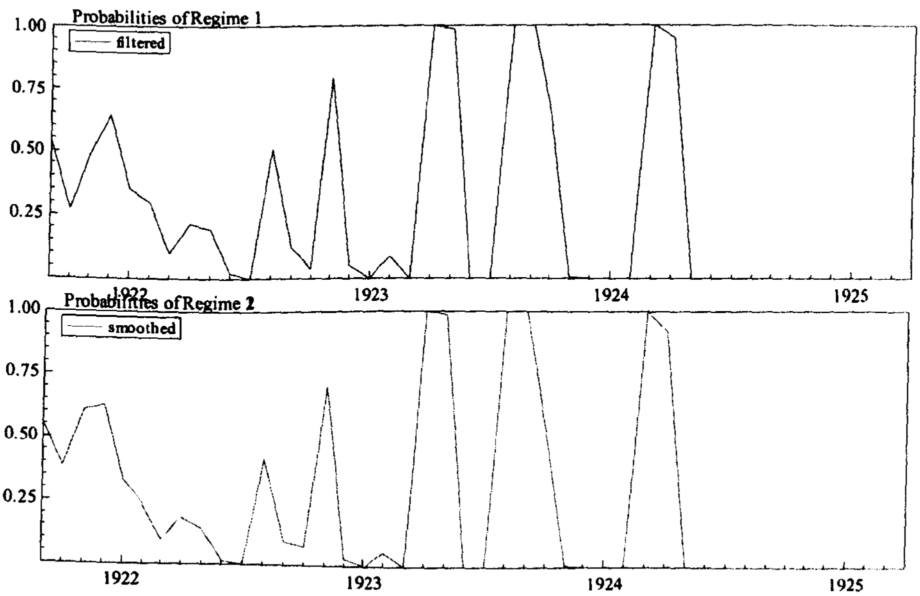


Figure 6.3. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ in Poland

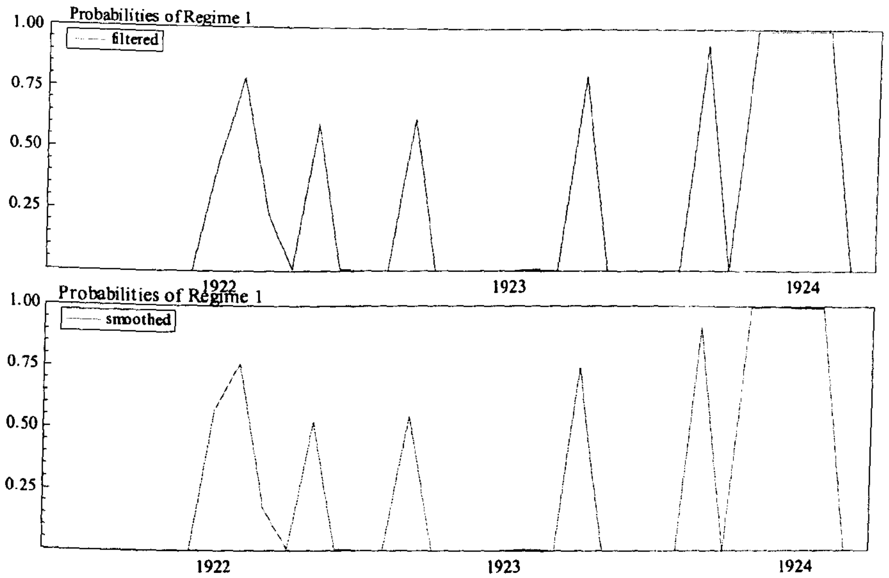


Figure 6.4. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on ΔM_t in Germany

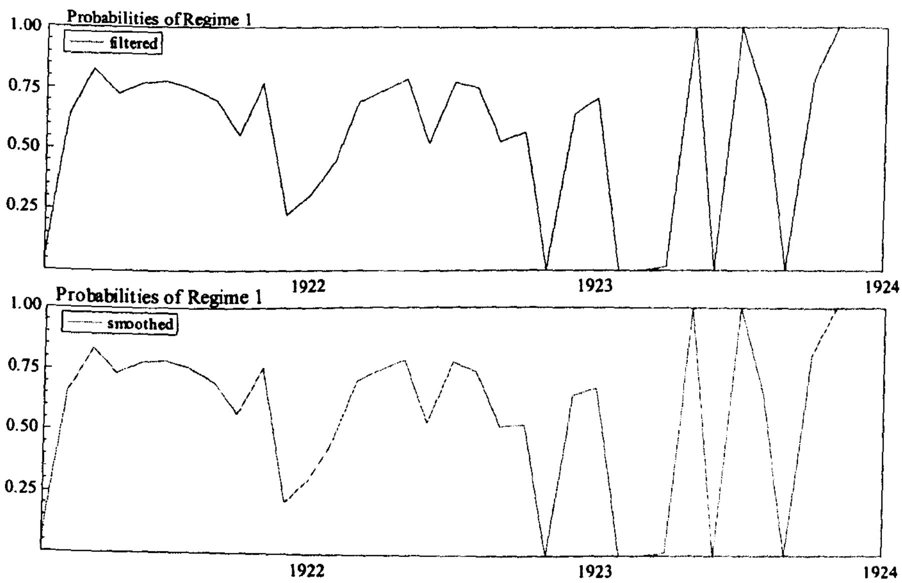


Figure 6.5. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on ΔM_t in Hungary

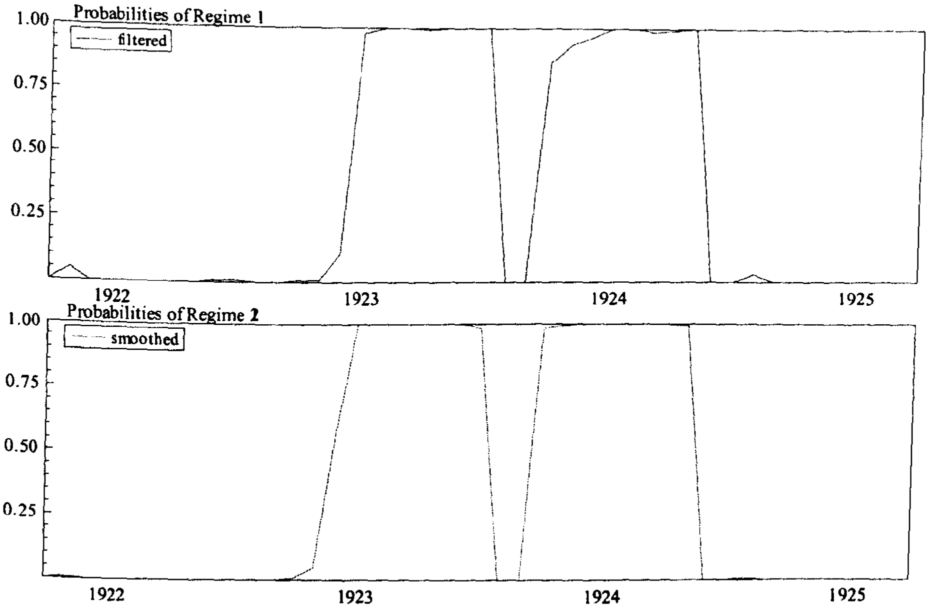


Figure 6.6. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on ΔM_t in Poland

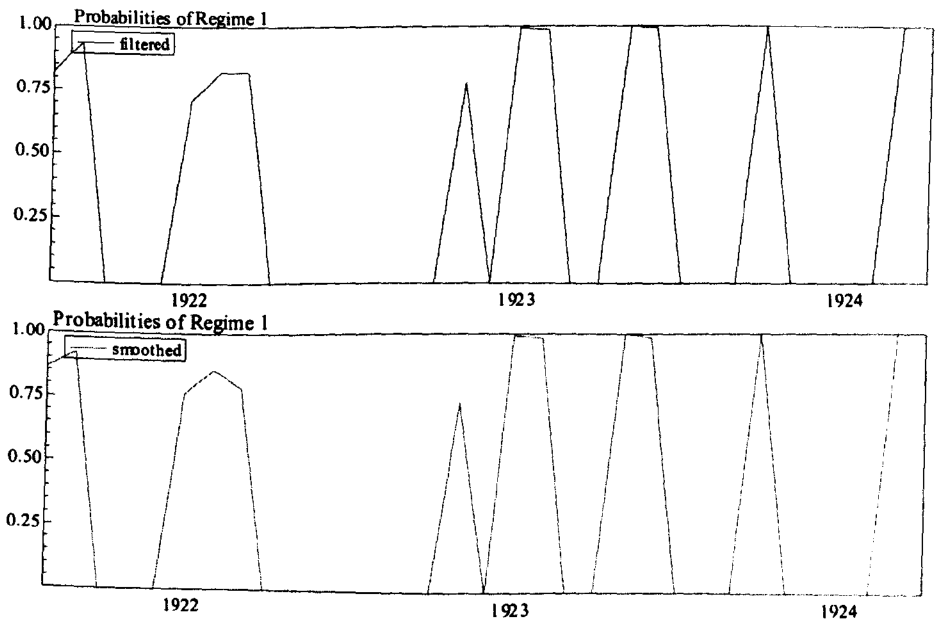


Table6.2a Cointegrating MS-ADF test for the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$

Country	Germany		Hungary		Poland	
Parameters	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Intercept of the ADF regression:						
$c(s_t)$	2.1098** (0.7046)	1.3878* (0.3523)	-0.8864* (0.0676)	-0.0460 (0.0791)	0.4547* (0.1595)	
$LR(c_1 = c_2)$	9.2272** [0.0264]		65.0991*# [0.0000]		3.6115 [0.3066]	
Coefficients of the ADF regression:						
$\psi_1(s_t)$	——		-0.7453* (0.0956)	0.1874** (0.0738)	——	
$\psi_2(s_t)$	——		0.2020** (0.0936)	0.2024* (0.0627)	——	
$b(s_t)$	-0.0909* (0.0191)	-0.0412* (0.0114)	0.0237* (0.0018)	0.0044*** (0.0023)	-0.0217* (0.0070)	
$\rho(s_t)$	-1.0375	-0.6391	-0.1768	-0.2710	-0.7360	
$t(\rho(s_t))$	-6.8660*	-5.1620*	-4.0354**	-3.8991**	-4.5379*	
$W(\rho(s_t))$	73.8228*		32.1445*		20.6209*	
Joint LR	33.0602* [0.0000]		44.9593* [0.0000]		6.6673 [0.5732]	
Standard error of residuals						
$\sigma_e(s_t)$	0.4678	0.1119	0.0344	0.0849	0.3107	
$LR(\sigma_1 = \sigma_2)$	22.7773* [0.0000]		6.7481*** [0.0804]		2.3391 [0.5051]	
Transition probability matrix:						
P11 P12	0.7322	0.2678	0.1909	0.8091	——	
P21 P22	0.1516	0.8484	0.3819	0.6181		
Diagnostic checking						
AIC	0.8359		-0.6609		0.7222	
HQ	1.0201		-0.3847		0.7836	
SC	1.3637		0.1390		0.8981	
Q(12-p)	9.0479		11.7997		7.3214	

Notes:

1. For the cases of Germany and Hungary, the stationary covariates are $\Delta^2\pi_{1,t-1}$, and $\Delta^2\pi_{1,t-i}$, i = 1,2, respectively.
2. Other notes to Table 6.1a still apply.

Table 6.2b. Cointegrating MS-ADF test for the regression of $(M_t - \pi_{2,t})$ on ΔM_t

Country	Germany		Hungary		Poland	
Parameters	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Intercept of the ADF regression:						
$c(s_t)$	0.8312* (0.1875)		0.1508* (0.0453)	-0.4197* (0.0879)	-0.3880* (0.0667)	0.0220 (0.0548)
LR$(c_1 = c_2)$	0.0022# [0.9625]		32.1756*# [0.0000]		9.1718** [0.0271]	
Coefficients of the ADF regression:						
$\psi_1(s_t)$	1.7130* (0.2334)	-0.4274* (0.0861)	-0.5732* (0.0925)	0.3166* (0.1085)	———	
$b(s_t)$	-0.0236* (0.0070)	-0.0310* (0.0059)	-0.0103* (0.0018)	0.0141* (0.0026)	0.0029 (0.0023)	
$\rho(s_t)$	-0.5922	-0.3631	-0.2790	-0.4094	-0.7084	
$t(\rho(s_t))$	-4.0692*	-5.0590*	-3.5969**	-4.9534*	-8.1681*	
$W(\rho(s_t))$	34.0088*		38.2115*		66.7179*	
Joint LR	34.8726* [0.0000]		23.1208* [0.0068]		10.5068 [0.3214]	
Standard error of residuals						
$\sigma_e(s_t)$	0.2600	0.2276	0.0463	0.0824	0.1170	
LR$(\sigma_1 = \sigma_2)$	25.9889* [0.0000]		12.0458* [0.0072]		0.0000# [0.9973]	
Transition probability matrix:						
P11 P12	0.2778	0.7222	0.6893	0.3107	0.5628	0.4372
P21 P22	0.3094	0.6906	0.1540	0.8460	0.1811	0.8189
Diagnostic checking						
AIC	1.1407		-1.0614		0.0253	
HQ	1.3095		-0.8179		0.1631	
SC	1.6295		-0.3927		0.4293	
Q(12-p)	11.5172		10.3732		11.1613	

Notes:

1. For the cases of Hungary and Poland, the stationary covariates are equally $\Delta^2\pi_{1,t-i}$, i = 1,2.
2. Other notes to Table 6.1a still apply.

Figure 6.7. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$ in Germany

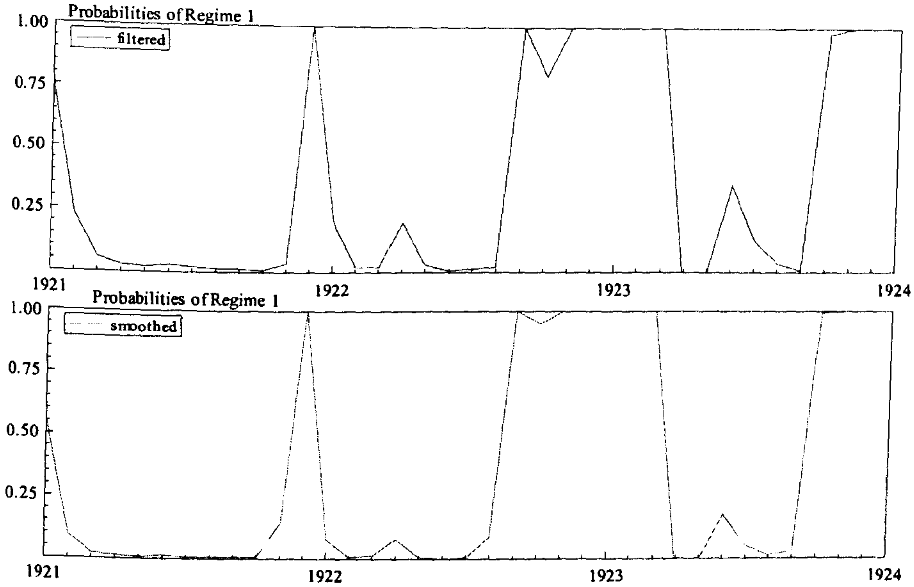


Figure 6.8. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$ in Hungary

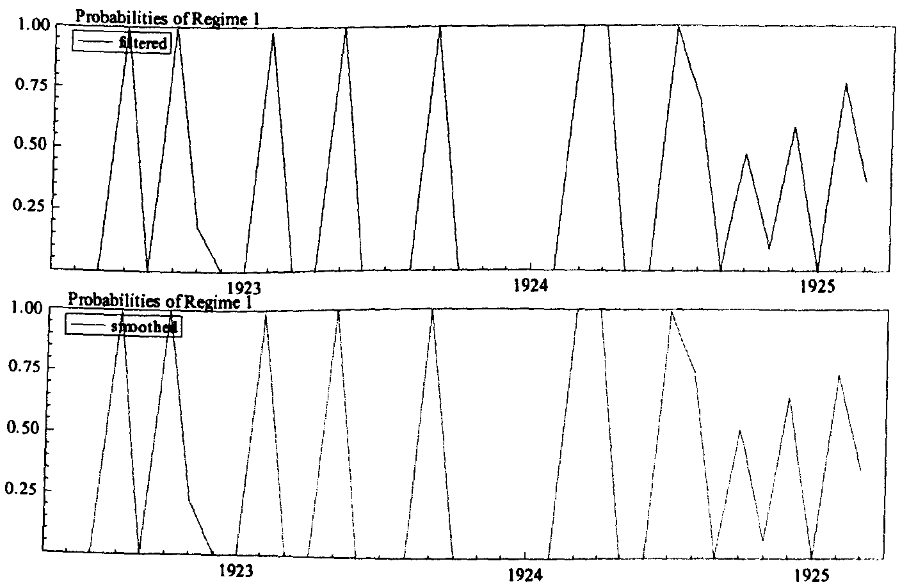


Figure 6.9. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on ΔM_t in Germany

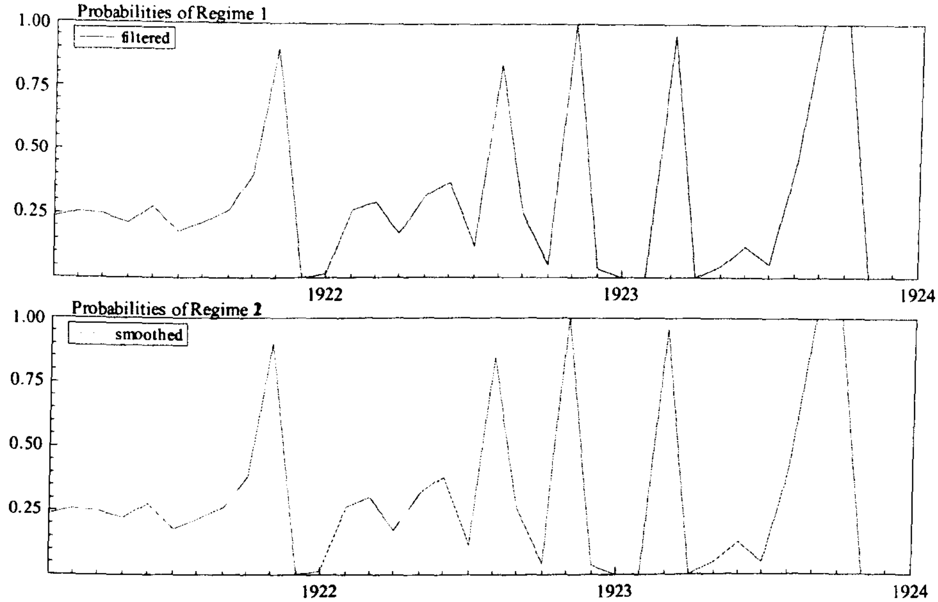


Figure 6.10. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on ΔM_t in Hungary

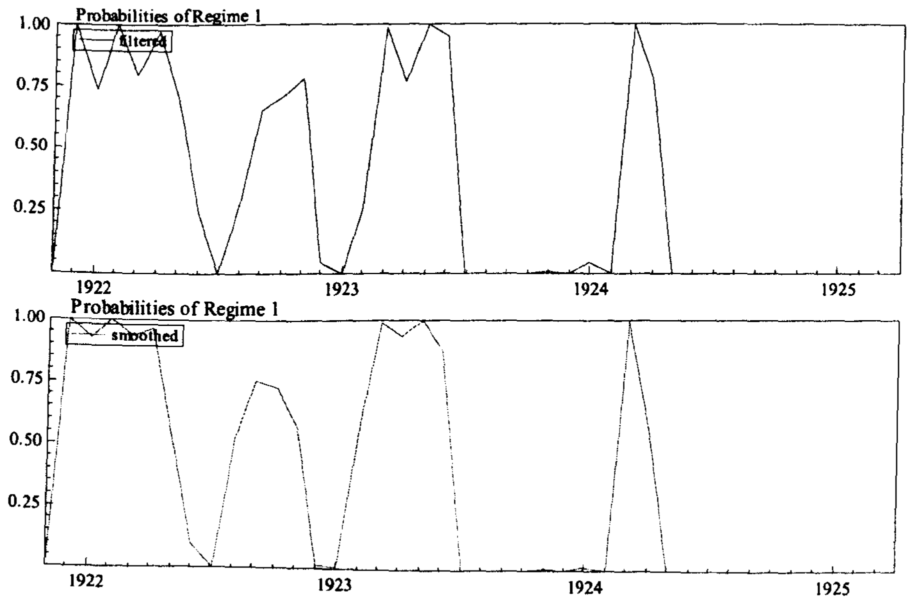
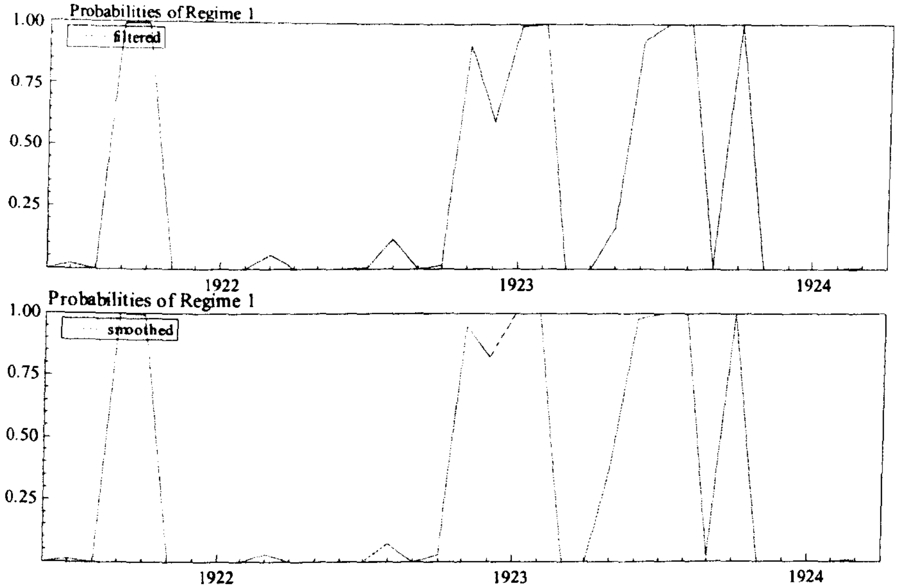


Figure 6.11. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on ΔM_t in Poland



CHAPTER SEVEN SUMMARY and CONCLUSION

In this thesis, I have adopted several econometric methods to investigate for the presence of price and exchange rate bubbles, using the inter-war European hyperinflations of Germany, Hungary and Poland as a case study. This chapter summarizes the major findings of the thesis and offers my concluding remarks.

In Chapter Two, two versions of the Cagan model under rational expectations are specified. The general solution to the Cagan models is the sum of the fundamental and bubble solutions. Dependent upon the assumed generating processes of the driving fundamentals, several explicit representations of the fundamental solutions are derived. In the rational expectations framework, the bubble solution contains an arbitrary martingale process. Consequently, there exists an infinite set of bubble processes for any value of a bubble coefficient. Several examples of theoretical bubble specifications are explored by imposing parametric restrictions. Some of these exhibit asymptotically dynamic stability, and some display different bursting properties. The problem of non-uniqueness in the bubble solutions makes it difficult to specify and test for every form of bubble. Hence, the general or indirect tests look more attractive for empirical research, because they focus upon the statistical properties of the data series used for bubble detection and do not require the specification of a particular form of bubble process.

In Chapter Three, I carry out a structural time series analysis to decompose the various structural time series components of the data over the full sample periods. This facilitates the examination of the statistical properties and integration orders of the relevant economic variables. It is found that the trend components for the data series on the level of money supply, prices and exchange rates follow a second differencing or a smooth trend specification. For the data series on real money balances, money supply growth, inflation rates and currency depreciation rates, the trend components follow a random walk or a local level representation. Some evidence for seasonal or cyclical components is found in the data series. On the basis of the reduced forms of the structural time series models, the series of the different levels are integrated at the order of two; and the other data series contain one unit root. Such findings are important for the selection of the econometric methods to be adopted for bubble testing in the thesis.

Some concern has been frequently expressed in the literature about the value of empirical research into the existence of bubbles. It has been argued that model misspecification and the process switching in fundamentals could lead to false conclusions concerning the existence of bubbles. Furthermore, as illustrated in Chapter Two, some stochastic bubbles may exhibit nonlinear explosive dynamics and consequently appear to be linearly stationary. Under these circumstances, conventional econometric methods may fail to test for the existence of bubbles even if they actually occur in the data series. In this thesis, I have employed new econometric procedures and methods to handle the above empirical problems of bubble identification.

In Chapter Four, I propose a new testing procedure using orthogonality tests, which is designed to effectively distinguish between bubbles and specification errors. I rely upon the FM-GMM instrument validity test statistics and the *ARIMA* structures of flow variables to detect the presence of specification errors. The stock variables are transformed in such a way that I can differentiate between the presence of bubbles and any model misspecification. Evidence for stationary specification errors in the exact Cagan models can be found. Moreover, while the transformed stock variables are orthogonal to the information sets in the FM regressions, both exchange rate and price bubbles are rejected in the data for those European hyperinflations under study. The results conclude that the control of inflation in those countries might only be required to gain control of the fundamental process. This is because the dynamics of prices and exchange rates might not be driven by the self-fulfilling expectations for these countries. It is argued that the expectation and implementation of monetary reforms that occurred in the final months of the sampled European hyperinflations might lead to nonlinear behaviour by the underlying fundamental variables. The distribution theory of the FM inferential statistics is developed, however, under the assumption of linear data generation. Hence, the nonlinear movements of fundamentals that display regime-switching behaviour may be misinterpreted as indicating a bubble path. The usual practice in such cases is to truncate the estimation periods under study, as was done in Chapter Four. The evidence for no bubbles given in Chapter Four is therefore limited to the pre-reform observations. The dilemma is that by doing this, the ability to detect bubbles may be diminished even when bubbles actually exist during the truncated observation periods. I suggest that advanced cointegration-testing methodologies be used to further identify bubbles over the full sample periods.

In Chapter Five, I employ a residual-based cointegrating TAR method to investigate the evidence for bubbles over the full sample periods. I also choose the conventional ADF and Z tests as well as the RALS-ADF test to carry out a comparison study. From the Monte Carlo findings, the power of the TAR tests is shown to be the highest no matter whether there are nonlinear threshold processes in the data generation or not. The RALS-ADF test gains substantially in power when using the threshold processes and it is made robust to the presence of non-normal errors by the inclusion of stationary covariates in the ADF regression. The results of the TAR and the RALS-ADF tests reject the evidence for the existence of bubbles, but some evidence of threshold nonlinear processes is nevertheless found in the data generation throughout the whole sample periods for all of the countries of interest. The low power of the conventional ADF and ***Z*** tests under the nonlinear process is likely to produce a spurious conclusion concerning the existence of bubbles. On the other hand, it is argued that rejection of non-cointegration may be attributable to a serious size distortion when nonlinearly explosive bubbles are present. The Monte Carlo findings, however, show that in the presence of periodically collapsing bubbles, the TAR tests and the RALS-ADF test are more likely to accept the null of non-cointegration than are cointegrating ADF and **Z** statistics. The empirical results reported in Chapter Five are not consistent with the presence of collapsing bubbles in the data.

The switching processes described in Chapter Five are assumed to depend upon observable threshold values. Actually, the threshold model and the Markov model can be observationally equivalent. Hence, in Chapter Six, I continue the cointegration tests to test for bubble existence using the Markov-switching cointegration ADF tests. The regime shifts in the Markov-switching ADF regression are allowed to depend on an unobservable Markov state variable rather than on an observable threshold value. The empirical results favour some evidence of Markov regime shifts in the data but reject the evidence of bubbles for any of the European hyperinflations under study.

The cointegration-testing methodology, however, has been criticized in the literature as having its own drawbacks for bubble testing. Given the fact that every econometric methodology has limitations as well as advantages, it is proposed that more than one method be adopted for the empirical study of bubbles. The orthogonality methodology and the cointegration methodology should be complementary for this purpose. They each have their own specialties. The orthogonality method can exploit the high-frequency properties of the data whereas the cointegration method exploits the low-frequency properties.

From the above, it is apparent that it is necessary to select the appropriate econometric procedures and methods for bubble testing, in order to handle the issues of model misspecification, the regime-switching behaviour of fundamentals and the nonlinear stochastic properties of bubbles. If any of these is ignored, misleading conclusions concerning the existence of bubbles are likely to result. Although the above econometric procedures and methods are useful for studying inflationary bubbles, they can be equally applied to the investigation of asset price bubbles in capital markets, and it is expected that this will be tackled in future research work.

References:

1. Abel, A., Dornbusch, R., Huizinga, J. & Marcus, A. (1979). Money Demand During Hyperinflation. *Journal of Monetary Economics*, 5, 97-104.

2. Andrews, D.W.K. (1991). Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica*, 59, 817-858.

3. Balke, N. and Fomby, T. (1997). Threshold Cointegration. *International Economic Review*, 38, 627-645.

4. Bhargava, A. (1986). On the Theory of Testing for Unit Roots in Observed Time Series. *Review of Economic Studies*, 53, 369-384.

5. Blanchard, O. J. and Watson, M. W. (1982), "Bubbles, Rational Expectations, and Financial Markets," in Paul Wachtel (eds.) *Crisis in the Economic Financial Structure*, MA: Lexington Books.

6. Bohl, M. T. (2003), Periodically Collapsing Bubbles in the US Stock Market? *International Review of Economics and Finance*, 12, 358-397.

7. Broze, L., Gourieroux, C. and Szafarz, A. (1995). Solutions of Multivariate Rational Expectations Models. *Econometric Theory*, 11, 229-257.

8. Broze, L. and A. Szafarz, (1991). *The Econometric Analysis of Non-Uniqueness in Rational Expectations Models*, Amsterdam: North Holland.

9. Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In M. Friedman (eds.), *Studies in the Quantity Theory of Money*. Chicago: University of Chicago Press.

10. Camerer, C. (1989). Bubbles and Fads in Asset Prices. *Journal of Economic Surveys*, 3, 3-41.

11. Campbell, J. Y. and Shiller, R. J. (1987). Cointegration and Tests of Present Value Models. *Journal of Political Economy*, 95, 1062-1088.

12. Caner, M. and Hansen B. E. (2001). Threshold Autoregression with a Unit Root. *Econometrica*, 69, 1555-1596.

13. Carrasco, M. (2002). Misspecified Structural Change, Threshold or Markov-Switching
Models. *Journal of Econometrics*, vol. 109, 239-273.

14. Casella, A. (1989). Testing for Rational Bubbles with Exogenous or Endogenous Fundamentals: The German Hyperinflation Once More. *Journal of Monetary Economics*, 24, 109-122.

15. Cavaliere, G. (2003). Asymptotics for Unit Root Tests under Markov Regime-switching. *Econometric Journal*, 6, 193-216.

16. Chan, K. S. and Tong, H. (1985). On the Use of the Deterministic Lyapunov Functions for the Egodicity of Stochastic Difference Equations. *Advances in Applied Probability*, 17, 666-678.

17. Chen, L. T. (1995). Essays on Testing for Speculative Bubbles in the Stock Market. *Monograph Series*, No.65, The Institute of Economics, Academia Sinica.

18. Chen, L. T. (1997). Misspecification Versus Bubbles in the Stock Market: The Case for Time-varying Discount Rates. *Academia Economic Papers*, 25, 427-461.

19. Chen, L. T. (1999). Model Misspecification Versus Speculative Bubbles in the Foreign Exchange Market: The Case of Taiwan's Effective Exchange Rate. *Taiwan Economic Review*, 27, 43-68.

20. Clarida, R. H., Sarno, L., Talyor, M. P. and Valente, G. (2003). The Out-of-Sample Success of Term structure Models as Exchange Rate Predicators: A Step Beyond. *Journal of International Economics*, 60, 61-83.

21. Cuthbertson, K. (1996). *Quantitative Financial Economics*. John Wiley & Sons.

22. Davies, R. B. (1977). Hypothesis Testing when a nuisance parameter is present only under the Alternative. *Biometrika*, 64, 247-254.

23. Davies, R. B. (1987). Hypothesis Testing when a nuisance parameter is present only under the Alternative. *Biometrika*, 74, 33-43.

24. Dezhbakhsh, H. and Demirguc-Kunt, A. (1990). On the Presence of Speculative Bubbles in Stock Prices. *Journal of Financial and Quantitative Analysis,* 25, 101-112.

25. Diba, B. T., and H. I. Grossman (1988a). Explosive Rational Bubbles in Stock Prices? *American Economic Review*, 78, 520-30.

26. Diba, B. T., and H. I. Grossman (1988b). Rational Inflationary Bubbles. *Journal of Monetary Economics,* 21, 35-46.

27. Dickey, D.A. and Pantula, S. G. (1987). Determining the Order of Differencing in Autoregressive Processes. *Journal of Business and Economic Statistics*, 5, 455-461.

28. Durlauf, S. N. & Hooker, M. A. (1994), "Misspecification versus Bubbles in the Cagan Hyperinflation Model," in C. Hargreaves (eds.), *Non-stationary Time Series Analysis and Cointegration*. Oxford: Oxford University Press.

29. Enders, W. and Granger, C. W. J. (1998). Unit Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates. *Journal of Business and Economic Statistics*, 16, 304-311.

30. Enders, W. and Siklos, P. (2001). Cointegration and Threshold Adjustment. *Journal of Business and Economic Statistics*, 19, 166-176.

31. Engle, R. F. and Granger, C. W. J. (1987). Cointegration and Error-correction: Representation, Estimation and Testing. *Econometrica*, 55, 251-276.

32. Engsted, T. (1993). Cointegration and Cagan's model of Hyperinflation under Rational Expectations. *Journal of Money, Credit and Banking*, 25, 350-360.

33. Engsted, T. (1994). The Classic European Hyperinflations Revisited: Testing the Cagan Model Using a Cointegrated VAR Approach. *Economica*, 61, 331-341.

34. Engsted, T. (1996). The Monetary model of the Exchange Rate under Hyperinflation: New Encouraging Evidence. *Economics Letters*, 51, 37-44.

35. Engsted, T. (2003). Misspecification versus Bubbles in Hyperinflation Data: Comment. *Journal of International Money and Finance*, 22, 441-451.

36. Evans, G. E. (1986). A Test for Speculative Bubbles in the Sterling-Dollar Exchange Rate: 1981-84. *American Economic Review*, 76, 621-636.

37. Evans, G. E. (1991). Pitfalls in Testing for Explosive Bubbles in Asset Prices. *American Economic Review*, 81, 922-930.

38. Flavin, M. (1983). Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence. *Journal of Political Economy*, 91, 929-956.

39. Flood, R. P. and Garber, P. M. (1980a). An Economic Theory of Monetary Reform. *Journal of Political Economy*, 88, 24-58.

40. Flood, R. P. and Garber, P. M. (1980b). Market Fundamentals versus Price-Level Bubbles: The First Tests. *Journal of Political Economy*, 88, 745-770.

41. Flood, R. P. and Hodrick, R. J. (1986). Asset Price Volatility, Bubbles and Process Switching. *Journal of Finance*, 41, 831-842.

42. Flood, R. P. and Hodrick, R. J. (1990). On Testing for Speculative Bubbles. *Journal of Economic Perspectives*, 4, 85-101.

43. Flood, R. P., Hodrick, R. J. and Kaplan, P. (1986). An Evaluation of Recent Evidence on Stock Market Bubbles. *NBER Working Paper*, No. 1971.

44. Frenkel, J. A. (1977). The Forward Exchange Rate, Expectations and the Demand for Money: The German Hyperinflation. *American Economic Review*, 67, 653-670.

45. Frenkel, J. A. (1979). Further Evidence on Expectations and the Demand for Money During German Hyperinflation. *Journal of Monetary Economics*, 5, 81-96.

46. Funke, M., Hall, S. and Sola, M. (1994). Rational Bubbles During Poland's Hyperinflation: Implications and Empirical Evidence. *European Economic Review*, 38, 1257-1276.

47. Garber, P. (1989). Tulipmania. *Journal of Political Economy*, 97, 535-560.

48. Garber, P. (1990). Famous First Bubbles. *Journal of Economic Perspective*, 4, 35-54.

49. Gilles, C. and LeRoy, S. F. (1991). Econometric Aspects of the Variance-Bounds Tests: A Survey. *Review of Financial Studies*, 4, 753-791.

50. Granger, C. W. J. and Swanson, N. R. (1997). An Introduction to Stochastic Unit-Root Process. *Journal of Econometrics*, 80, 35-62.

51. Goodfriend, M. S. (1982). An Alternate Method of Estimating the Cagan Money Demand Function in Hyperinflation under Rational Expectations. *Journal of Monetary Economics*, 9, 43-57.

52. Gourieroux, C., Laffont, J.J. and Monfort, A. (1982). Rational Expectations in Dynamic Linear Models: Analysis of the Solutions. *Econometrica*, 50, 409-425.

53. Haldrup, N. (1994). Semiparametric Tests for Double Unit Roots. *Journal of Business and Economic Statistics*, 12, 109-122.

54. Haldrup, N. (1998). An Econometric Analysis of I(2) Variables. *Journal of Economic Surveys*, 12, 595-650.

55. Hall, S. G., Psaradakis, Z. and Sola, M. (1999). Detecting Periodically Collapsing Bubbles: A Markov-Switching Unit Root Test. *Journal of Applied Econometrics*, 14, 143-154.

56. Hamilton, J. D. (1986). On Testing for Self-fulfilling Speculative Price Bubbles. *International Economic Review*, 27, 545-552.

57. Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica*, 57, 357-384.

58. Hamilton, J. D. and Whiteman, C. H. (1985). The Observable Implications of Self-Fulfilling Expectations. *Journal of Monetary Economics*, 16, 353-373.

59. Hansen, B. E. (1995). Rethinking the Univariate Approach to Unit Root Testing. *Econometric Theory*, 11, 1148-1171.

60. Hansen, L. P. & Sargent, T. J. (1980). Formulating and Estimating Dynamic Linear Rational Expectations Models. *Journal of Economic Dynamics and Control*, 2,7-46.

61. Harvey, A.C. (1989). *Forecasting, Structural Time Series Models and Kalman Filter*. Cambridge University Press.

62. Hausman, J. A. (1978). Specification Tests in Econometrics. *Econometrica*, 46, 1251-1271.

63. Haza, D. P. and Fuller, W. A. (1979). Estimation of Autoregressive Processes with Unit Roots. The Annuals of Statistics, 7, 1106-1120.

64. Hooker, M. A. (2000). Misspecification versus Bubbles in Hyperinflation Data: Monte Carlo and Interwar European Evidence. *Journal of International Money and Finance*, 19, 583-600.

65. Ikeda, S. and Shibata, A. (1992). Fundamentals-dependent Bubbles in Stock Prices. *Journal of Monetary Economics*, 30, 143-168.

66. Ikeda, S. and Shibata, A. (1995). Fundamentals Uncertainty, Bubbles and Exchange Rate Dynamics. *Journal of International Economics*, 38, 199-222.

67. Im, K. S. (1996). Least Squares Approaches to Non-Normal Errors. *DAE working paper*, No. 9603, University of Cambridge.

68. Im, K.S. (2001). Unit Root Tests Using More Moment Conditions Than Least Squares. *Working paper*, University of Central Florida.

69. Imrohoroglu, S. (1993). Testing for Sunspot Equilibria in the German Hyperinflation. *Journal of Economic Dynamics and Control*, 17, 289-317.

70. Johansen, S. (1995). *Likelihood Based Inference Cointegrated Vector Autoregressive Models*. Oxford University Press.

71. Keynes, J. M. (1936). *The General Theory of Employment, Interest, and Money*. Macmillan.

72. Kim, C. J. (1994). Dynamic Linear Models with Marko-Switching, *Journal of Econometrics* 60, 1-22.

73. Kindleberger, C. P. (1987). Bubbles. *New Palsgrave Dictationary*, Vol. 1, 20-22.

74. Kitamura, Y. & Phillips, P. C. B. (1995). Efficient IV Estimation in Nonstationary Regression: An Overview and Simulation Study. *Econometric Theory*, 11, 1095-1130.

75. Kitamura, Y. & Phillips, P. C. B. (1997). Fully Modified IV, GIVE and GMM Estimation with Possibly Non-stationary Regressors and Instruments. *Journal of Econometrics*, 80, 85-123.

76. Kleidon, A. W. (1986). Variance Bounds Tests and Stock Price Valuation models. *Journal of Political Economy*, 94, 953-1001.

77. Koopman, S. J., Harvey, A. C., Doornik, J. A. and Shephard, N. (2000). *STAMP: Structural Time Series Analyser, Modeller and Predictor*. Timberlake Consultants Ltd.

78. Krolzig, H. M. (1996). Statistical Analysis of Cointegrated VAR Processes with Markovian Regime Shifts. *SFB 373 Discussion Paper*, 25/1996, Humboldt Universitat zu Berlin.

79. Krolzig, H. M. (1997). *Markov Switching Vector Autoregression, Modelling, Statistical Inference and Application to Business Cycle Analysis*. Berlin: Springer.

80. Krolzig, H. M. (2002). *Regime-Switching Models*. Mimo, University of Oxford.

81. Krolzig, H. M., Marcellino, M. Mizon, G.. E. (2002). A Markov-switching Vector equilibrium correction model of the UK Labour Market. *Empirical Economics*, 27, 233-254.

82. LaHaye, L. (1985). Inflation and Currency Reform. *Journal of Political Economy*, 93, 537-560.

83. LeRoy, S. F. and Porter, R. D. (1981). The Present-Value Relation: Tests Based on Implied Variance Bounds. *Econometrica*, 49, 555-574.

84. McCulloch, J. H. (1996), "Financial Applications of Stable Distributions," in G. S. Maddala and C. R. Rao (eds.), *Handbook of Statistics*, Vol., 14, Elsevier, pp.393-425.

85. Meese, R. (1986). Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates? *Journal of Political Economy*, 94,

86. Moosa, I. A. (1999). Testing the Currency-Substitution Model under the German Hyperinflation. *Journal of Economics*, 70, 61-78.

87. Muth, J. F. (1961). Rational Expectations and the Theory of Price Movements. *Econometrica*, 29, 315-335.

88. Nelson, C., Piger, J. and Zivot, E. (2001), Markov Regime Switching and Unit-Root Tests. *Journal of Business and Economic Statistics*, 19, 404-415.

89. Okina, K. (1984). Rational Expectation, Bubbles and Foreign Exchange Market. *Bank of Japan Monetary and Economic Studies*, 2, 81-118.

90. Okina, K. (1985). Empirical Tests of Bubbles in the Foreign Exchange Market. *Bank of Japan Monetary and Economic Studies*, 3, 1-45.

91. Peel, D. A. and Speight, A. E. H. (1994). Testing for Non-Linear Dependence in Inter-War Exchange Rates. *Weltwirtschaftliches Archiv*, 130, 391-417.

92. Perron, P. and Ng, S. (1996). Useful Modifications to some Unit Root Tests with Dependent Errors and their Local Asymptotic Properties. *Review of Economic Studies*, 63, 435-463.

93. Phillips, P.C.B. (1986). Understanding Spurious Regressions in Econometrics. *Journal of Econometrics*, 33, 311-340.

94. Phillips, P. C. B. (1995a). Fully Modified Least Squares and Vector Autoregressions. *Econometrica,* 63, 1023-78.

95. Phillips, P. C. B. (1995b). Robust Nonstationary Regression. *Econometric Theory,* 11, 912-951.

96. Phillips, P. C. B. & Hansen, B. E. (1990). Statistical Inference in Instrumental Variables Regression with I(1) Processes. *Review of Economic Studies*, 57, 99-125.

97. Phillips, P. C. B. & Ouliaris, S. (1990). Asymptotic Properties of Residual Based Tests for Cointegration, *Econometrica*. 58, 165-193.

98. Phillips, P. C. B. and Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika*, 75, 335-346.

99. Pippenger, M. K. and Goering, G. E. (1993). A Note on the Empirical Power of Unit Root Tests under Threshold Processes. *Oxford Bulletin of Economics and Statistics*, 55, 473-481.

100. Psaradakis, Z. (2001). Markov Level Shift and the Unit Root Hypothesis. *Econometrics Journal*, 4, 226-42.

101. Salge, M. (1997). *Rational Bubbles*. Berlin: Springer.

102.Salemi, M.K. & Sargent, T.J., (1979). The Demand for Money during Hyperinflation under Rational Expectations: II. *International Economic Review*, 20, 741-758.

103.Sargent, T. J. (1977). The Demand for Money during Hyperinflations under Rational Expectations: I. *International Economic Review*, 18, 59-82.

104.Sargent, T. J. (1982), "The Ends of Four Big Inflations", in: Robert E. Hall (Ed.), *Inflation: Causes and Effects*. Chicago University Press.

105.Sargent, T. J. and Wallace, N. (1973). Rational Expectations and the Dynamics of Hyperinflation. *International Economic Review*, 14, 328-50.

106.Sen, D. L. and Dickey, D. A. (1987). Symmetric Test for Second Differencing in univariate time series. *Journal of Business and Economic statistics*, 5, 463-473.

107.Shiller, R. J. (1981). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review*, LXXI, 421-36.

108.Shin, D. W. and Kim, H. J. (1999). Semiparametric Tests for Double Unit Roots Based on Symmetric Estimators. *Journal of Business and Economic Statistics*, 17, 67-73.

109.Taylor, M. P. (1991). The Hyperinflation Model of Money Demand Revisited. *Journal of Money, Credit and Banking*, 23, 327-351.

110.Taylor, M. P. and Peel, D. (1998). Periodically Collapsing Stock Price Bubbles: A Robust Test. *Economics Letters*, 61, 221-228.

111.Tinbergen, J. (ed.) (1934). *International Abstract of Economic Statistics* 1910-30. London: International Conference of Economic Services.

112.Tirole, J. (1982). On the Possibility of Speculation under Rational expectations. Econometrica, 50, 1163-1181.

113.Tirole, J.(1985). Asset Bubbles and Overlapping Generations. *Econometrica*, 53, 1499-1528.

114.van Giersbergen, N.P.A. (1996). Bootstrapping the Trace Statistic in VAR models: Monte Carlo Results and Applications. *Oxford Bulletin of Economics and Statistics*, 58, 391-408.

115.Webb, S. B. (1985). Government Debt and Inflationary Expectations as Determinants of the Money Supply in Germany: 1919-23. *Journal of Money, Credit and Banking*, 17, 479-92.

116.Weil, P. (1989). Overlapping Families of Infinitely-lived Agents. *Journal of Public Economics*, 38, 183-198.

117.West, K. D. (1987a). A Specification Test for Speculative Bubbles. *Quarterly Journal of Economics*, 102, 553-580.

118.West, K. D. (1987b). A Standard Monetary Model and the Variability of the Deutschemark-Dollar Exchange Rate. *Journal of Monetary Economics*, 23, 57-76.

119.West, K. D. (1988a). Bubbles, Fads and Stock Price Volatility Tests: A Partial Evaluation. *Journal of Finance*, xx, 639-656.

120.West, K. D. (1988b). Dividend Innovations and Stock Price Volatility. *Econometrica*, 56, 37-61.

121.Woo, Kai-Yin (1999). Cointegration Analysis of the Intensity of the ERM Currencies under the European Monetary System. *Journal of International Financial Markets, Institutions and Money*, 9, 393-406.

122.Woo, Kai-Yin and Chan, H. L. (2001), "*Testing for Model Misspecification and Bubbles Under the Chinese Hyperinflation*", in Albert Travidze (Ed.), Progress in Economics Research, Vol. 4, NOVA, pp.13-28.

123.Woo, Kai-Yin, Chan, H.L. and Lee, S.K. (2001). Detecting Rational Bubbles in the Residential Housing Markets of Hong Kong. *Economic Modelling*, 18, 61-73.

124. Woo, Kai-Yin, Chan, H.L. and Lee, S.K. (2003). An Empirical Investigation of Price and Exchange Rate Bubbles during the Interwar European Hyperinflations. *International Review of Economics and Finance*, 12, 327-344.

125. Yao, J. F. and Attali, J. G. (2000). On Stability of Nonlinear Processes with Markov Switching. *Advances in Applied Probability*, 32, 394-407.

126. Young, J.P. (1925). *European Currency and Finance*, Vol. 2. Washington, D.C: U.S. Government Printing Office.