Optical eigenmode collapse

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The optical eigenmode representation of light fields defines a natural orthogonal basis of solutions of Maxwell's equations taking into account the geometry and interactions involved in a problem. Formally, the optical eigenmodes are similar to the wave functions in quantum mechanics. Here, I put forward that, in a linear interaction at the single photon level, the electromagnetic field collapses into an optical eigenmode of the interaction in question. This is to satisfy the principle that no linear optical system can distinguish between pure states and their superposition for single photons. To exemplify this statement, we will consider the case of angular momentum and linear momentum transfer in optical scattering.

In the following, we describe two typical scattering processes that correspond to a mechanical force or torque acting on the scattering object. We briefly introduce these effects in the classical regime and then discuss their implications, with respect to the optical eigenmodes, at the single photon level.

ANGULAR MOMENTUM TRANSFER

Firstly, we consider linearly polarised light incident on a quarter wave plate (QWP) that is transforming the 45^o polarisation into left circular polarisation and 135^o into right circular polarisation. This can be achieved by correctly aligning a birefringent crystal of suitable thickness. As the polarisation state of the light field changes, the QWP will experience a small torque of equal amplitude and different sign for each of the two polarisations considered. In this geometry, the polarisation state of horizontally and vertically polarised light is left unchanged as these polarisation states correspond to the fast and slow axis of the crystal. As their overall polarisation state does not change there is no torque associated with these two polarisations.

LINEAR MOMENTUM TRANSFER

The second thought experiment involves the superposition of two plane waves scattered by a spherical particle. The scattered electromagnetic vector fields can be calculated exactly using Mie theory. This allows us to define using Maxwell's stress tensor the flux of momentum of the optical fields. Integrating Maxwell's stress tensor on a sphere surrounding the particle determines the linear momentum transferred to the particle which experiences an optical scattering force. In this specific case, this total force will depend on the angle between the two incident plane waves. Due to interference effect, this total force is, in general, not the sum of the optical forces associated with each individual plane wave.

DISCUSSION

These two test cases can be used to discuss the need for the single photon to be described by an optical eigenmode[1, 2] and the collapse of the electromagnetic field onto this eigenmodes upon interaction.

At the single photon level, the QWP performs a weak polarisation measure. The interaction is described by the difference in spin angular momentum $i(\mathbf{E}^* \cdot \mathbf{H} - \mathbf{H}^* \cdot \mathbf{E})/(2c)$ of the beam before and after each QWP. There are two optical eigenmodes associated with this interaction, namely light polarised at 45^o and at 135^o. If the wave were not to collapse onto one of these eigenmodes, it would be possible to distinguish between the pure states of the diagonal polarisation basis and the horizontal/vertical polarisations which are superposition states. This would, for example, break the BB84 quantum key distribution protocol.

We have already noticed that the optical momentum transfer form the superposition of two plane waves is not equal to the sum of the momenta from each individual plane wave. Let us assume that single photons are described by plane waves and let us consider a 50%/50% superposition state of two photon each described by a different plane wave. In this case, statistically the average momentum transfer should be equal to the average force due to each individual plane wave. However, this will not deliver the correct total force due to interference effects. The only way to eliminate the interference is to consider only orthogonal optical eigenmodes as possible single photon interactions.

In conclusion, the electromagnetic field representation of single photons are the optical eigenmodes of the interaction considered. Upon interaction the fields collapse into one of the optical eigenmodes with a probability given by the coupling efficiency between the incident field and the optical eigenmodes.

[1] M. Mazilu. Spin and angular momentum operators and their conservation. *Journal of Optics A: Pure and Applied Optics*, 11(9), 2009.

[2] M. Mazilu. Optical eigenmodes; spin and angular momentum. *Journal of Optics*, 13(6), 2011.