ANALYTICAL MODELS OF DISEQUILIBRIUM GROWTH AND MACRODYNAMICS

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CONTENTS.

Part B.

TO MY PARENTS

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The thesis presents a number of analytical models on disequilibrium growth as well as medium term dynamics. It began some time back under the supervision of Professor Avinash Dixit. He read the first drafts of my work and I am extremely grateful to him for suggestions, advice and encouragement. My work later on continued under the guidance of Professor Marcus Miller. I am deeply indebted to him for all the help and academic suggestions that he has given me freely, over the years, and which have helped me tremendously. In fact this thesis would never have been completed without his co-operation, as well as his recommendations for meeting the deadline to finish the work. However, if the human capital formation, as evidenced in the thesis, still shows signs of disequilbrium maladjustment, I must bear all responsibility for them.

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SUMMARY

Disequilibrium analysis, particularly in the context of explicit dynamic economic models, is an area of considerable interest. Disequilibrium is important when markets fail to clear and dynamic adjustments are required. Three essential strands of the literature seem the most important: non market-clearing temporary equilibria; long term growth theory which allows for the possibility of unemployment of labour and underutilisation of capital stock; medium term dynamics where aggregate demand fails to match up to potential output. This thesis presents a number of theoretical and analytical models which analyse various aspects of the last two issues. Even though we use some concepts from short-run rationing models of temporary equilibria, the central focus is exclusively on long run growth and more shorter-term dynamic systems, where capital stock is exogenous. The work is also emphatically macroeconomic in nature, emphasising aggregative structures which conform to stylised facts and have interesting policy conclusions.

The first part of the thesis discusses growth models. Given the lack of an unified theoretical structure in the area itself, we concentrate on specific issues: income-expenditure models with independent investment functions leading on to capital formation and (possible) movement towards steady states; unemployment of labour, and capital; monetary growth and asset structure; open economy considerations when markets may fail to clear. The second part analyses macrodynamics, assuming fixed capital, and is concerned with medium term adjustments of variables such as output, price and exchange rate under disequilibrium and rigidities.

The purpose of the research is to present a diversity of concepts and conclusions. The objective is not to present a comprehensive 'general' or 'meta' theory; it is not clear whether encompassing concepts will necessarily be more insightful; in any case the current state of the arts preclude such a schema. The chapters that follow deal with a wide range of possible topics; model specifications are adapted to tackle the specific problem at hand.

The conclusions clearly demonstrate that specification of regime, Keynesian or Classical, is vital to the understanding of how the economy will behave under disequilibrium. Even if the steady state depends on exogenous parameters (such as the natural rate or potential output) the paths that approach it are essentially different in characteristics, depending on what sort of disequilibrium regime the economy is in. This, of course, has important policy relevance. Discretionary policies, as well as policy rules, must carefully study the underlying structural features of the economy if they are to have significance.

Chapter I

Introduction

Section 1: Disequilibrium economics.

Disequilibrium models have a long and respected tradition in economics. Keynesian short run macro theory, analysing unemployment and the inability of investment demand to match up to full employment saving, emphasises the notion that markets may not clear and that disequilibrium is a more general state of the economy than equilibrium. Harrod's discussion of the knife-edge instability of growth paths in a capitalist economy, where independent decision makers determine saving and investment which therefore may not be equal ex ante, once again emphasise deviations from dynamic equilibrium.

Formal analysis of short run disequilibrium models in macroeconomics has burgeoned recently with explicit treatments of non market-clearing temporary equilibria, optimum choice under rationing, spill-over effects consequent to agents failing to buy (sell) their desired demands (supplies) as well as sluggish prices which tend to perpetuate the initial cause of disequilibrium. These models are analytically powerful, but are essentially short run , static, single period (or sometimes two-period) systems.

Growth theory, particularly in the neoclassical tradition, has generally eschewed disequilibrium methods and the vast literature in the field is usually concerned with equilibrium states. This is in spite of the early lead from the Harrodian model. However, the emphasis, on equilibrium, is understandable since even the simplest disequilibrium formulations lead to considerable complications and problems of interpretation.

A third strand, following dynamic extensions of the IS/LM models, emphasise the discrepancy between aggregate demand and full employment aggregate supply and therefore take on disequilibrium features right from the start. But capital stock behaviour is not often analysed nor is the interdependence of markets under disequilibrium (which is radically different from general equilibrium because of rationing constraints and spill-overs) given importance.

The purpose of this thesis is to present several analytical models of growth and short/medium term dynamic theory, when markets do not clear and general disequilibrium prevails. It therefore concentrates on the second and third issues discussed in the preceeding paragraphs. Though it borrows concepts from temporary equilibrium rationing models, the thesis concentrates exclusively on long run growth theory, with capital accumulation considered endogenous, as well as more short run dynamic systems, where capital stock is taken to be exogenous.

At this point it is probably counterproductive to go into a long and pedagogical discussion as to the exact definition of disequilibrium that we intend to use. The choice of a specific scheme will depend upon the particular application that will be analysed within the general framework. Therefore, what is necessary here is a brief description of some of the concepts we shall be using.

In a sense equilibrium, as a concept, can be totally pervasive if properly defined. For example, the three criteria used by Malinvaud (1977) to delineate the characteristics of equilbrium are so broad that they can include almost any economic states. For the macroeconomy, in aggregate, these are: (i) trades balance or the amount (value) of purchase equals sales for each good; (ii) trades are voluntary, thus total purchase (sales) cannot exceed demand (supply); (iii) a rationed buyer (seller) in a market implies that there does not exist a rationed seller (buyer). These are essentially consistency conditions and will hold under many possible situations. However, they do not rule out the discrepancy between ex ante demand and supply, the failure of prices to adjust quickly to clear markets, adjustments in response to rationing, unemployment with labour demand not being on the marginal productivity schedule, capital accumulation or decumulation consequent to capital shortage or excess capacity ----- all important features of disequilibrium.

The essence of disequilibrium is dynamic adjustments. Malinvaud (1977) puts it clearly: "If one objects to thinking with equilibria, one must use a dynamic formulation in which the relevant variables will simultaneously move according to some properly specified rules". These adjustments are of two pure types; hybrids are of course possible. In the tatonnement models prices adjust rapidly to clear markets and make notional (desired) demands equal supply; transactions take place at market clearing temporary equilibrium prices. Further adjustments may take place with changes in state variables which are assumed to be given during the actual tatonnement process. Interest lies in the attainment of full equilibrium where even the underlying state variables are constant; this is the domain of stationary or steady state. Disequilibrium analysis can be meaningful here since the sequence of temporary equilibria and its time path may be related with the way prices actually adjusted during tatonnement. In non-tatonnement models, prices are rigid during the period of operation of temporary equilibrium; quantitites adjust to attain consistency; markets may 'clear' at the minimum of ex ante supply and demand or alternative, and more complicated, rationing rules can be proposed. But the seeds of disequilibrium remain in the failure of some agents to achieve their optimum (unrationed) positions. Thus prices adjust and the sequence gravitates, if stable, towards full equilibrium.

Specifications of adjustment rules and values of adjustment parameters are crucial. Some of these are 'natural': excess demand raises price, though in a competitive market with atomistic price-taking agents it is not always easy to answer the question as to who actually changes price. Others are more controversial. Major criticism, from the new classical equilibrium theorists, centers around the value of the specific speed of adjustment parameter of relevant variables in non-clearing markets. In particular, any finite positive value of say the speed of adjustment of prices in response to a discrepancy between demand and supply, seems to be unacceptable. Yet it is not at all clear why speed of adjustment (of price and expectations) should be infinite (markets always clear) in every case; in a sense postulating infinite speed seems to be as ad hoc as to give finite values, including zero (where prices do not change).

It is true that postulating dynamic adjustments of any variable, and giving arbitrary values to the speed of adjustment parameter, without explicit optimisation is ad hoc. Tobin (1981) puts it strongly: "-- the adjustment process itself has not in general been successfully described as optimizing behaviour, the only paradigm that carries theoretical conviction in our profession. This failure, neither surprising nor discreditable in view of the intrinsic difficulties of the task, is the root of the chronic crisis in macroeconomics." However, the current state of the arts makes it imperative to use these concepts without too fine a discussion of the theoretical foundation. As mentioned earlier, these difficulties are not necessarily related to macro models; competitive general equilibrium theory also fails to answer the questions, why do price adjustments take place 'instantaneouly' to clear markets and who changes prices?

Macroeconomic models can analyse disequilibrium in various stages. First, aggregate demand may not be equal to aggregate supply. This will manifest itself in ex ante or notional saving being unequal to ex ante or desired investment. Some form of price/quantity adjustments are required to equalise these two variables. Second, the level of actual output supplied, now equal to aggregate demand, may not be equal to the potential, full employment, full capacity output. A second level dynamic adjustment is now required. Third, actual saving or investment will determine capital accumulation. This in turn will have implications for long run growth. Full equilibrium will be reached, and all disequilibrium eliminated, when the capital stock reaches the stationary state level. Additional complications, at all these stages, are provided by the existence of labour market disequilibrium.

The foregoing issues will be dealt with in a sequence of models that constitute Chapters II to X divided into two parts; Part A deals with growth theory while Part B analyses issues in short run dynamic modelling. At the current stage of our knowledge it is not possible to give a general model which can handle all the issues concurrently. The very nature of disequilibrium modelling forces us to analyse each problem at a time. Further, modelling techniques and assumptions made need to be adapted to the specific problem that we wish to emphasise on. Model specifications change depending on the issue on which we focus. The analysis as a whole can, therefore, deal with a wide range of topics. Thus, the chapters that follow are therefore relatively independent. Each also has its own introduction and conclusion as well as a brief literature survey.

For the purposes of integration we will discuss some of the major issues that need investigation in the area. The next Section deals briefly with short run rationing models. Though these are not elaborated in the thesis they do provide implicit microeconomic justification to our subsequent analysis; hence a review is essential. Section 3 analyses the sorts of questions that disequilibrium growth theory should answer; this overview sets the stage for much of the specifics in the main body of this work. Section 4 does the same but for medium run dynamic models with capital as exogenous. Since adjustment processes are so vital in what we do, Section 5 gives a succint review of methods. The final Section is important since it gives the structure of the thesis, how the chapters unfold, the links between them as well as their implications.

It should be mentioned, right at the outset, that the work presented here is unflaggingly macroeconomic in nature. Even though many of the relations and concepts have implicit microfoundations their use is within aggregative structures which conform to stylised facts and have interesting policy implications. Analysing the concept of a 'metalanguage' Fitoussi (1983) defends the use of macro theory in the following way: (it is believed that) "macroeconomic relations must have microeconomic foundations. This proposition establishes from the outset the subordination of the macro to the micro approach, and at the same time it ranks economic arguments in implicitly acknowledging that microeconomics is itself well founded. Yet it is not clear that macroeconomic relations can be derived in this fashion ------ macroeconomic theory should set itself up as an autonomous discipline and seek also other foundations". This is not to say that macro and micro theory cannot cohabit; however they do have independent raison de´tre; one does not have to necessarily rely on the other.

Section 2: Short run models of rationing.

The rationing models, where quantity constraints are binding and agents cannot transact their desired amounts, started life formally in Patinkin´s (1965) discussion of Keynesian unemployment. All the basic arguments had already been made in the General Theory and a major theoretical foundation had been established in the Hicksian fix price flex price distinction. But it lies to the credit of Patinkin to point out the issues so clearly.

He considers the aggregate labour market alone where if the firm could sell all its output then labour demand would be on the marginal productivity schedule and wage rate would be equal to marginal product of labour. If now the firm faces a binding sales constraint (and there is no inventory) then this ration will change the firms behaviour. In particular it will produce the amount it can sell and hire the corresponding amount of labour, independent of the wage rate. The demand curve for labour implicitly becomes kinked and vertical at the point of the sales constraint. Compared to its initial position the firm will now employ less labour and unemployment will appear. Given the kink, the marginal product of labour will be generally greater than wage at this level of labour demand. This unemployment is Keynesian since it occurs due to the firm's inability to sell its optimum output; hence there is deficient aggregate demand. A reduction in wage does not help since it is not the cost of labour that precludes the firm from hiring more people. Rather, lower wage may mean a further reduction in the demand for the output. On the other hand if the firm is actually on the unconstrained labour demand curve (and maximising profits with wage equals marginal product), and still there is unemployment then the wage rate must be too high to clear the labour market. This is the case of Classical unemployment where there is no problem of demand deficiency and the firm can sell whatever it wishes.

Clower (1965) laid even stronger and more formal micro-foundations to the Keynesian short run model within this framework. He concentrates on the aggregate household which, in the absence of any other constraints, derives its labour supply and output demand from an optimisation process given the wage price configuration. Now if there is unemployment, and the household fails to sell its desired quantity of labour, it is rationed in its sales and has to take this binding constraint into account. This leads to the 'dual decision hypothesis' which shows that a second level decision needs to be taken by the agent in one market subject to a binding constraint in another market. There will also be a fundamental difference between notional demands (supplies) and effective demands (supplies); the latter takes into account all the binding or effective constraints that the agent may be facing.

Clearly, there will be interdependence among markets as these dual decisions are taken and consistent positions sought to be reached. In the firm's case the initial reduction in employment leads to a fall in wage income; hence there may be a further fall in effective demand emanating from the household. Similarly, the household faced with rationing in employment will reduce its demand for the output which in turn may lead to further cuts in labour demand.

In an aggregate two good economy ---- commodity and labour --- there are four possible configurations of rationed equilibria when the markets are brought together. We can have excess demand(supply) for commodity as well as for labour. These four situations have been termed: Keynesian Unemployment (excess supply of commodity, excess supply of labour); Classical Unemployment (excess demand for commodity, excess supply of labour); Repressed Inflation (excess demand for commodity; excess demand for labour); Underconsumption (excess supply of commodity, excess demand for labour).

The implicit adjustment at the market level, that is presupposed to bring the economy into rationed equilibrium from an initial situation of disequilibrium, is the so called "min" condition. Here markets clear on the short side so that actual transactions is the minimum of demands and supplies. Thus price adjustment is zero while speed of adjustment for quantity is infinite in the very short run. The "min" condition may also give a criterion, beyond temporary equilibrium, for price adjustments to take place in response to excess effective demands.

There have been significant extensions to the basic structure fully formalised by Barro and Grossman (1976). Open economy considerations have been extensively studied in Dixit (1978) and Neary (1980). A major innovation in terms of our understanding of the microfoundations of these systems is provided in Neary and Roberts (1980) through the concept of "virtual prices"; an example of this is discussed a bit later. Another highly significant extension is to incorporate expectations and a sequence of temporary equilibria in a very rigorous fashion. This is done in a paper by Neary and Stiglitz (1983); though the analysis contains two periods and full equilibrium is not discussed, the paper incorporates all the complications that arise when microeconomic considerations are fully explored.

Since the major purpose of the thesis is not to analyse these types of models per se, rather to borrow ´stylised results´ for use, we do not go into much further details. It is probably better to give a simple illustrative example which will clarify the basic issues. Consider the aggregative household whose utility depends on consumption (C) and leisure (R). The total time available to the household is T, so that labour supply is L = T-R. It faces a real wage rate w in the labour market; thus its wage income is wL = w(T-R). In this one period static optimisation model there are no savings. The household is also constrained by the maximum amount of labour time it can sell; this is given by $\bar{L}$.

The constrained optimisation problem can be written as

$$\text{Max} \quad U = U(C, R) \tag{1}$$

subject to

$$C \leq w(T-R) \tag{2}$$

$$(T-R) \leq \bar{L} \quad (3)$$

We assume that all variables, C, R, L, T, $\bar{L}$ are non-negative

The Lagrangian is:

$$Z = U(C,R) - l[C - w(T-R)] - m[(T-R) - \bar{L}] \quad (4)$$

where l and m are the relevant Lagrange multipliers.

For positive values of consumption and leisure, the Kuhn-Tucker (necessary) conditions for maximisation imply:

$$U_C - l = 0 \quad (5)$$

$$U_R - l[w - (m/l)] = 0 \quad (6)$$

$$l[C - w(T- R)] = 0 \quad (7)$$

$$m[(T - R) - \bar{L}] = 0 \quad (8)$$

$$l \geq 0, \quad m \geq 0 \quad (9)$$

When the constraints are binding then we have

$$C = w(T -R), \quad l > 0 \quad (10)$$

$$(T - R) = \bar{L}, \quad m > 0 \quad (11)$$

From (5), (6), (10), (11) we get

$$(U_R/U_C) = w - (m/l) \quad (12)$$

$$C = w\bar{L} \quad (13)$$

The implications of (12) and (13) are clear. If the household is rationed in its labour supply, such that it is forced to sell less labour (utilise more leisure) than it would actually do so under an unconstrained regime, the constrained optimum labour sold is at the upper limit and the corresponding level of consumption is given by (13). Condition (12) tells us that if the real wage was actually w - (m/l), rather than w, then the household would optimally choose $\bar{L}$ as its labour supply independent of the constraint given by (3). The shadow wage-price configuration w-(m/l) implies that the household does not perceive the ration on its supply of labour. Its "as if" optimum choice would be (C, R) = ($w\bar{L}$, T - $\bar{L}$). We note from Neary and Roberts (1980) that [w - (m/l)] is the "virtual price" of leisure time.

For subsequent analysis we denote the optimal choice in the absence of rationing (on labour supply) as

$$(C, R, L) = (C^*, R^*, L^*) \tag{14}$$

and the rationed levels as

$$(C, R, L) = (W\bar{L}, T-\bar{L}, \bar{L}) \tag{15}$$

To understand how (labour) rationing occurs we must turn to te behaviour of the aggregate firm. Assume initially that the firm faces no binding sales constraints; in other words it can sell whatever is produced (there are no inventories, for simplicity). Profit (Π), in real terms, is given by

$$\Pi = F(L^d) - wL^d \tag{16}$$

where $Y = F(L^d)$ is the output of the firm and wL^d is the real wage cost. Profit maximisation implies

$$F'(L^d) = w \tag{17}$$

$$L^d > 0 \tag{18}$$

Let us assume henceforth that the firm can always buy the labour services it wants, thus the firm faces no rationing in its demand for labour.

If the real wage rate is fixed at w, and transactions can take place at disequilibrium (demand not equal to supply), then one possibility might be that

$$L^d = \bar{L} < L^* \tag{19}$$

Here the firms' demand for labour is less than the households optimum unconstrained supply, L^*. Thus the household is faced with a ration in its labour sales to the firm. This necessitates a "dual decision" i.e. a second round optimisation, incorporating the upper limit that labour sales is $L = L^d$. The "minimum" condition

$$L^a = \min (L^d, L^*) \tag{20}$$

(L^a is actual labour services transacted) holds and the market for labour "clears" at $L^a = L^d = \bar{L}$. The household's rationed optimisation is as discussed earlier and given by conditions (17), (13). This case assumes implicitly that the firm can sell all its output. Thus

$$F(L^d) = F(\bar{L}) = C = w\bar{L} \tag{21}$$

We are in the region of classical unemployment where the household is forced to supply labour services less than its (unconstrained) optimum level. The firm however maximises profits (see (17)) without additional constraints.

The second possibility occurs where the firm faces a sales ration such that its desired output (given by L^d from (17) and the production function) is greater than the amount that the market (or household) will demand. Let there be an effective sales ration for the firm such that output must be less than or equal to an exogenous limit $\bar{Y}$.

Then the firm's problem is,

$$\text{Max } \Pi = F(L^d) - wL^d \qquad (22)$$

$$Y = F(L^d) \leq \bar{Y} \qquad (23)$$

The Lagrangeian is

$$Z = [F(L^d) - wL^d] - n[F(L^d) - \bar{Y}] \qquad (24)$$

Optimality implies the following:

$$(1 - n)\ F'(L^d) - w = 0 \qquad (25)$$

$$L^d \geq 0 \qquad (26)$$

$$F(L^d) - \bar{Y} \leq 0 \qquad (27)$$

$$n[F(L^d) - \bar{Y}] = 0 \qquad (28)$$

$$n \geq 0 \qquad (29)$$

Note that <u>here</u> L^d represents the firm's demand for labour, when the sales constraint is operative.

Given that the sales ration is binding, so that the firm would have liked to produce more than $\bar{Y}$ at the unconstrained profit maximisation level, we get

$$F'(L^d) = (w/1-n) \quad (30)$$

$$L^d > 0 \quad (31)$$

$$F(L^d) = \bar{Y} \quad (32)$$

$$1 > n > 0 \quad (33)$$

Figure 1.1 (adapted from Patinkin (1965)) shows the constrained (rationed) optimisation of the firm. The current wage rate is given at w_1. The unconstrained supply of output at the optimum is Y_1. Faced with a binding sales ration, $\bar{Y} < Y_1$ the firm chooses its labour demand such that $F(L^d) = \bar{Y}$. Once again, the firm obeys the dual decision hypothesis, and re-calculates its optimum with the additional constraint. Two points should be noted. First, that the marginal product of labour is greater than the real wage rate at the production level $\bar{Y}$. Second, that the shadow real wage rate (w/1-n) is again the 'virtual price' of labour such that if the actual real wage was set at this "as if" level, the firm would have no sales constraint, de facto.

It is clear how the household may be rationed in its sale of labour. If

$$L = \bar{L} = F^{-1}(\bar{Y}) \quad (34)$$

$$\bar{L} < L^* \quad (35)$$

then the household cannot sell its unconstrained level of labour supply and is forced to accept the ration of $\bar{L}$.

This is a situation of Keynesian unemployment where effective demand for output is <u>less</u> than what the firm desires to produce at the going real wage. It is characterised by the fact that the virtual price of labour is greater than the actual real wage rate.

If the aggregate household and firm are brought together we get a consistent characterisation of the general disequilibrium model. The following equations will give the solution to the relevant variables.

$$F(\bar{L}) = w\bar{L} \tag{36}$$

$$F'(\bar{L}) = w/1-n \tag{37}$$

$$C = w\bar{L} \tag{38}$$

$$U_C\ (C,\ T - \bar{L}) = 1 \tag{39}$$

$$U_R\ (C,\ T - \bar{L}) = l[w - m/l] \tag{40}$$

$$(n,\ l,\ m) > 0$$

Equation (36) simply states that supply of output is equal to demand, though not at the level required for unconstrained optimisation. If $\bar{L}$ is a binding labour supply constraint for the houehold, then we have unemployment since desired L^* is greater than the actual L. Equations (37) to (40) we have met earlier, respectively equations (30), (13), (5) and (6) with actual labour transacted $(\bar{L})$ as the argument for the various functions. The 5 equations solve for the endogenous variables L, C, n, l, m, given the exogenous level of w.

The shadow (Lagrange) multipliers, showing whether the constraints are effective or not, have special implications. When m, the multiplier associated with the labour supply ration of the household, is positive we have unemployment. For m = 0, the household is on its desired labour supply function. Similiarly, given unemployment and $m > 0$, when $n = 0$ we get classical unemployment where the firm's sales ration is not binding. Alternaively, for $n > 0$ and an effective sales constraint, we are in a situation of Keynesian unemployment.

The general disequilibrium model postulated above, and depicted diagramatically in Figure 1.1, says nothing about dynamic adjustment per se. However, implicit in the discussion there lies a mechanism through which a dynamic story, over time, can be told. If the objective of the economy is to reach Walrasian equilibrium (WE), then it is important to see which form of adjustment can actually take the system towards WE. The real rate is exogenously fixed in the model specification. So it is natural to postulate a wage adjustment equation which will enable the economy, over time, to move towards WE. In Classical unemployment the wage rate is too high relative to WE; thus wage adjustment is a function of excess demand for labour i.e. the difference between actual demand and notional supply. This requires wages to fall under Classical unemployment. On the other hand, if the unemployment is Keynesian, the reduction in the wage rate will cause effective demand to fall reducing the firm's sales ration even further; this in turn will aggravate unemployment. The best way to reach WE in a state of Keynesian unemployment is to <u>increase</u> the wage rate. Thus, the adjustment equations tend to be fundamentally different depending on the type of regime we are in. This feature is crucial and will figure later on with regularity. Specifications of dynamic adjustments depend essentially on the system we have.

Section 3: Disequilibrium growth theory.

Compared to the unified theoretical structure of equilibrium growth theory, as evidenced say in the neoclassical models, disequilibrium growth presents a bewildering variety of possibilities. At this stage of our knowledge and research potential it is impossible to analyse the most general theoretical framework and then see how the specific features fit in. The methodology cannot be from the general to the particular; rather it is expected that analyses of special problems will indicate how fundamental issues can be potentially modelled for a general theory. Nor is it possible, in the narrow confines of a thesis, to deal with all the major outstanding dificulties and we have to confine ourselves to some of the important concerns in the field.

Dixit (1976) emphasises the difficulties clearly: "To set up a model of disequilibrium, we have to specify which markets fail to clear, how the actual transactions in these markets occur when this happens, and how the various plans, expectations and then the realizations of various prices and quantitites alter in response to it. There is a bewildering range of possibilities to choose from., and often a seemingly slight difference in assumptions can lead to a major diference in results. --- With such variety of approaches and lack of consensus, it is not possible to set down simple representative models." Given these limitations, we can in the thesis discuss specific models exemplifying issues which are either considered important or tractable; even here, given the size limitations of a thesis, the analysis can encompass some and not all the relevant features.

The approach we adopt overall is to link up disequilibrium behaviour with the underlying equilibrium growth process. In this we concur with Dixit: "On this question of compatibility with an underlying equilibrium model, there are two approaches evident in disequilibrium models. Some simply ignore the question --- I think that this is a mistaken argument of practicality ---- Other models attempt to study the outcome under hypothetical stationary circumstances, some finding compatibality and others, not." We follow, in the analytical models discussed later on, the second method which seems to be the most fruitful. For example, suppose a neoclassical (Solow) steady state full equilibrium is disturbed by a parametric change. The interest rate is not fully flexible and does not equal the marginal product of capital. This implies that at the given capital stock profit is not being maximised and provides a rationale for capital accumulation (or decumulation); we can then postulate an independent investment function which can motivate capital formation, rather than relying exclusively on saving as in the standard growth model. If investment is not sufficient to absorb full employment saving then clearly firms will be rationed in their sale of potential, full employment output. Hence unemployment may occur. The question is whether this disequilibrium state will actually converge to the new steady state or if there is a possibilty of a vicous cycle developing. The analysis of stability, with full equilibrium as the final objective of convergence, will have overriding consideration in our analysis. In particular, we shall be careful to watch for saddle path (in)stabilty since this will clearly demonstrate the need for regulation and intervention. We will need, at least implicitly, an exogenous authority, to put the economy on the stable manifold if it is to avoid cumulative divergence.

In a masterly survey of the state of the arts in disequilibrium growth theory Dixit (1976) identifies three major areas, within the noe-classical/Keynesian growth paradigm, in which analysis should proceed. These are (a) Income-expenditure models leading on to the more formal temporary equilibrium theories of sequence economies; here the work of Bliss (1975, 1983) stands out in terms of rigour; (b) Monetary models and the explicit treatment of asset structures as well as expectations; this follows the seminal work by Tobin (1965); (c) returns to scale and monopolistic markets. He does not deal with open economy macro growth models, though this is an important area of investigation.

Given the excellent and comprehensive survey already available in the last chapter of Dixit (1976) there seems to be little necessity in discussing the material once again. We take as a starting point the research agenda laid down in that book and continue from there. We emphasise income expenditure models with independent investment functions which can be built on profit maximising principles or be of the Keynesian marginal efficiency type. Our growth models here stress unemployment and underutilisaton of (capital) capacity. In addition some income distributional considerations, affecting saving, consumption and effective demand, are also analysed. One model is exclusively devoted to open economy considerations. This follows research topic (a) above. We then move on to a large and comprehensive chapter on money and growth (topic (b)) which focusses on the neutrality proposition and builds a relatively general discussion on policy, asset structure, transitional paths and disequilibrium. Unfortunately, space and time precludes the proper analysis of the third issue (c) mentioned above. The analysis of monetary models seems a natural place to leave growth theory and move on to short run dynamics (fixed capital) where the asset structure and concomitant expectations on rates of returns have more crucial influence.

Section 4: Dynamics with exogenous capital.

A third type of macroeconomic dynamic models, following from the IS/LM tradition, looks at short to medium term adjustment processes when markets do not clear. Models of this type do not consider the evolution of capital stock over time; thus capital accumulation is either assumed to be zero or exogenous. Commodity market balance is assured in the sense that saving and investment (desired and/or actual), based on realised income, are equalised; but this saving/investment decision does not carry over onto long run considerations of changes in capital stock. Disequilibrium is almost always introduced through the notion of an aggregate supply function; when aggregate demand emanating from the IS/LM equilibrium is not equal to supply of the national product we have disequilibrium.

The analysis of the supply side can be done in alternative ways. Sachs (1980) starts with the labour market and derives the supply function as dependent on the real wage rate. However, wage/price movements are rapid enough to equate aggregate supply and demand; thus there is no rationale for disequilibrium. The Dornbusch type models treat supply as fixed (at a full employment level independent of nominal wage and price) but analyse fully adjustments under disequilibrium. Both approaches are partial. As we shall see later (in Chapter IX) a more fruitful method of analysis is to integrate the two major desirable properties --- a proper supply function and slow price adjustment to allow for disequilibrium.

Another problem with this class of models is that, even though supply and demand are not equal in the commodity market, spill over effects are not always properly integrated. If aggregate demand, derived from the commodity and money markets, cannot be met by suppliers then a further round of decision making on the demand side of the market (IS/LM) is necessary. Again, this will be a major focus of our attention in a later chapter.

Many of the interesting theoretical developments in this area have come from open economy macroeconomics so we also emphasise this aspect (in the last two chapters). Money, exchange rates and expectations play quite important parts in the models and their implications are considered in some detail. The structure of the analysis can be considered briefly. For a given exchange rate and price level, the IS and LM functions determine aggregate demand and the interest rate. The supply function gives aggregate supply again dependent on the level of the 'state variables'. Demand, supply and the interest rate are used in turn in the adjustment equations for price and rate of exchange which then change to give the next period values. The transition path exhibits disequilibrium in the sense that aggregate demand and supply are not equal; the presence of this 'gap' gives rise to dynamics.

The Dornbusch model places exclusive reliance on (sluggish) price adjustments to clear markets. Another method, familiar with early Keynesian economics, is to use inventory adjustments with suitable (and major) extensions to take into account the open economy. Depending on the structural assumptions made, inventory dynamics can give some interesting results for the disequilibrium behaviour of the economy.

The models that we will analyse all assume rational expectations or perfect (myopic) foresight. Thus expectational variables adjust rapidly (instantaneously); expectations are realised quickly except for the influence of pure uncertainty or white noise. The rapid speed of adjustment of expectational variables, as contrasted with other alternative assumptions such as in adaptive expectations, creates problems for stability analysis. In a sense it may be necessary to have slow adjustments in some variables, and the formal presence of disequilibrium, to achieve at least saddle path stability; the alternative might mean that all characteristic roots are positive or lie outside the unit circle. In practice, we often have saddle point equilibrium. Only the stable manifold approaches long run equilibrium over time; all other paths diverge away from stationary (steady) state. In growth theory, this was appreciated early, in the model pioneered by Harrod where the equality of the actual and natural rates of growth defined the stable manifold; stability is therefore a 'knife-edge' problem since almost all paths exhibit instability.

If we wish to guarantee convergence it is necessary to identify the unstable roots with 'jump' variables while the stable roots are linked with predetermined variables, which cannot change in response to current information or shocks. This is akin to the distinction between control and state variables in dynamic optimisation. Heuristic descriptions akin to transversality conditions are used to motivate the economy to reach long run equilibrium. Though ad hoc, the models have the great virtue of policy realism and the ability to explain quite complicated empirical phenomena such as business cycles.

The choice of which variables should be predetermined or non-predetermined is usually a matter of empirical judgement based on stylised facts; there are no clear cut rules. For example, short run, dynamic, open economy, macro models often assume that the price level (or rate of inflation) is predetermined and backward-looking while the exchange rate can jump. Closed economy models may assume that output falls into the former class while price variables constitute the latter group. This thesis makes an innovation in the last chapter by assuming that inventories can be regarded as a 'jump' variable for the small open economy. Since this may be controversial we will discuss it in some detail in the general framework being presented here.

We are proposing the rather unusual assumption that an aggregate stock variable (level of inventory holding) can move discontinuously at the point of time when the economy experiences a shock. Yet a careful consideration will show why this may be possible. First, there are other areas, often discussed and considered plausible, where a stock variable is changed discontinuously. The obvious example is money 'stock'; policy discussions based on increased (decreased) money stock are common. The main point is that the agent handling the quantity transaction is able to change its level 'quickly'. It is thought that the authorities can do so for money; hence the acceptability of discontinuity in the time path of stocks. Second, a single and small firm in a large competitive market can change inventory stocks at will simply by ordering them without affecting prices (due to its 'smallness'). There are of course practical difficulties such as transportation constraints and transaction costs; but these are present even in 'instantaneous' price adjustments; the difference is one of degree not of kind. The third point to note is that a small open economy selling a homogeneous product in a large competitive world market can behave exactly like a single firm of the domestic economy. By virtue of its smallnes it can dispose of inventory stocks 'quickly' to the rest of the world exactly like a small firm; thus discontinuous movements of inventories are possible and this variable can in principle jump, even though in practice a distributed lag mechanism will work. Basically, it is a question of ranking speeds of adjustment. In the model of Chapter X it is assumed that the exchange rate and inventory stock have a much faster speed of adjustment ('infinite') compared to the price level or rate of inflation as the case may be.

Section 5: Adjustment Process

When markets do not clear and the system is in disequilibrium it is necessary to postulate adjustment equations which give the behaviour of the endogenous variables over time. Disequilibrium analysis, particularly with rationing and spill-overs, creates major problems for dynamic behaviour. Growth theory, where the time path of capital stock is crucial, adds additional difficulties. At the current state of the macroeconomist's art, it is only possible to be ad hoc and search for consistency, rather than trying to analyse a full blown optimising framework where adjustment equations and parameters come out as by-products of the analysis. We intend to give some structure, at this stage, to the myriad viewpoints and specifications that are potentially available.

One extreme point of view is the Walrasian tatonnement model where all transactions must take place at prices where notional or ex ante demands and supplies are equalised. Thus, dynamic adjusments, necessary to clear markets, take place before quantity transactions are actually made. In a sense the adjustments are hypothetical since they are done at "auction" prices; only when equilibrium is actually reached can trades occur. The opposite view point, given non-tatonnement, assumes that transactions occur at the short side of the market so that in disequilibrium one of the agents (demander/supplier) is satisfied while the other is rationed. The spill over effects emanating from disequilibrium in various markets are then taken into account; agents reallocate their effective demands and supplies; price adjustments follow in response to excess effective demand.

Both these paradigms have micro economic foundations and the results are derived from optimising models. The analysis gains from preciseness but is not always robust or subject to empirical evaluation or appropriate for institutional policy discussions. Macroeconomics tends to take the middle path, borrowing elements from both and using relatively ad hoc assumptions to construct dynamic models.

In a recent paper, Heal (1986) has discussed some of the stability problems that are associated with macrodynamics. He postulates the following model which "could be thought of as an aggregative one-sector macroeconomic model, in the spirit of neoclassical growth models and many subsequent macrodynamic models" (Heal (1986)).

We have

$$\dot{q} = a[p - c(q)] \tag{41}$$

$$\dot{p} = b[D(P) - q] \tag{42}$$

where q is output, p is its price, D is demand and c the average cost of production. A dot `.´ over a variable always denotes a time derivative, $\dot{x} = dx/dt$. If the price is above average cost then firms increase output; when demand is greater than supply price rises.

I will now take this basic system and adapt it to a growth framework with capital accumulation. Ignore labour and the wage rate for the moment. Then we can have the following:

$$\dot{K} = a\,[F'(K) - r] \tag{43}$$

$$\dot{r} = b[I(K,r) - s(r)F(K)] \tag{44}$$

where K is capital stock, r is the rate of interest, F is the production function (with standard properties) I is desired or ex ante investment so that I(K,r) is the investment function, s is the saving propensity and sF is total saving and (a, b) are speeds of adjustment, both positive.

Equations (43) and (44) are essentially similiar to (41) and (42) except that we are now within the realm of a growth model. With the wage cost fixed, if the marginal product of capital is greater than the interest rate, then as capital stock increases so will profit. Thus, with $F' > r$, the firm increases capital and output over time to reap extra profitability and this rationalises equation (43). In the words of Walras, this can be described as ´le loi de la revient'. On the other hand, equation (44) can be a description of the market for loanable funds, where capital is transacted. Here, excess demand for funds, given by ex ante investment greater than ex ante saving, generates a rise in its price i.e the rate of interest. The Walrasian statement would be 'le loi de l'offre et de la demande' (see Heal (1986)). Excess demand (supply) raises (lowers) price.

The model, of course, needs to be supplemented by other information. For example, the investment function needs to be specified. Equation (43) is one specification; there may be others. For example, a Harrodian investment function of the type

$$\dot{K} = z[I(K,r) - S(K,r)] \tag{45}$$

may be appropriate. If desired investment (I) is greater than the amount of saving, investors try to increase capital stock since they perceive a capital "shortage". On the other hand when ex ante investment is low, specifically less than saving, then there exists excess capacity which leads to decumulation. (z is the speed of adjustment).

It is useful to have a classifactory scheme for the various types of adjustments that can take place when markets do not clear and capital accumulation needs to be analysed at a macroeconomic level. A major distinction is between Classical and Keynesian Regimes (henceforth termed in this chapter CR and KR). These two regimes will generally have different equations for capital accumulation, as exemplified earlier by equation (43) and (45). Another distinction between them occurs when planned investment and saving are not equal and actual capital formation is assumed to be the lower of the two;

then, CR is the case where the saving constraint is binding and actual capital formation is equal to saving; KR would have binding investment and changes in capital stock are given by the investment function. Mixed features are also possible; for example, a behavioural function for investment like (43) which also happens to be binding and the main determinant of accumulation. Chapter II deals specifically with such a model. Chapter IV gives lot more attention to the Harrod-Keynes investment function (45).

The other distinction is between nontatonnement and tatonnement type adjustments. If the minimum condition holds so that

$$\dot{K}_a = \min[I, S] \tag{46}$$

where $\dot{K}_a$ is actual capital investment, then we call it the non tatonnement dynamics of the capital stock. The alternative, where actual investment is set at a level depending on desired saving and investment (for example, averaging them), as well as taking account of price adjustment, may be termed a tatonnement type process (see Rose (1966) for an example).

We now have four 'pure' cases. However, as mentioned earlier, some hybrid examples can also be constructed. The following table gives a clearer picture.

	Tatonnement	Non-tatonnement
Classical Regime	Case I	Case II
Keynesian Regime	Case III	Case IV

Models which analyse these cases will generally be tested for stability; some comparative dynamics excercises will also be carried out. When the system obeys the Routhian conditions, and is at least locally stable, we have no major problems in analysing growth paths. Starting from any initial conditions, the variables will converge over time to long run equilibrium, either steady or stationary states. Difficulties arise when the equilibrium exhibits saddle point properties. Then almost all paths will gravitate towards the unstable manifold unless specific restrictions are placed on them. As discussed earlier, systems with explicit expectational modelling, such as the short/medium term dynamics systems (with capital stock exogenous), usually impose restrictions on the nature of variables - pre determined and "jump". Given that long run equilibrium, with positive values of all variables, is a desirable objective for all agents, the "jump" variables move discontinuously to attain the stable mainfold. We will make similiar assumptions for the growth models, in this thesis, when they exhibit saddle paths. The implicit assumption is that there is an infinite horizon optimal control model "supporting" these descriptive systems that we analyse. Suitable transversality conditions can be imposed to make the system move towards long run equilibrium. Jump variables are then identified with controls which the "invisible" optimising agent can choose at will. Predetermined variables are state variables of control theory which cannot change at the time of shock.

The following chapters will have examples of these various cases (I to IV) with dynamic paths and concomitant growth behaviour when markets may not clear. The examples are selected for their economic interest and ability to exemplify how the growing economy behaves in disequilibrium. The models deal not only with capital stock but also labour dynamics, including unemployment. Together, they make up a relatively comprehensive picture of the disequilibrium process with capital formation.

Section 6: Structure of the thesis.

The papers collected in the thesis follow the traditions of macro growth and dynamic models, with and without endogenous capital formation, as discussed earlier in Sections 3 and 4. The thesis does not deal with short run rationing models per se. Thus, the behavioural relations are not always derived explicitly from optimising foundations incorporating quantity constraints. However, the implications of rationing, particularly the fact that actual transactions take place on the short side so that quantity transacted is the minimum of notional supply and demand, are carefully noted and integrated in the analysis. We also stress adjustment processes, both of the tatonnement and the non-tatonnement types, clearly distinguishing Classical and Keynesian regimes. It will be seen, in the models that follow, that the latter distinction is of crucial importance in delineating specific growth paths for the stylised economy.

Disequilibrium in the labour market is usually discussed in terms of unemployment; this is because unemployment is potentially the most important problem for an economy trying to achieve greater capital accumulation; the opposite case of over (full) employment, or even a dynamic version of repressed inflation, is given less importance though it is discussed occasionally in some of the following models. In addition, a great deal of emphasis is placed on the possibility that desired saving and investment may not be equalised; the consequences of such a divergence and their implications are major issues in the analysis.

The thesis naturally divides into two parts. The first part is concerned with long run capital formation and growth models in general. Here the behaviour of capital stock is endogenous, and an important area of investigation is to find out how investment and capital evolves over time when some markets fail to clear. This is the preserve of disequilibrium growth theory. The Second part shifts emphasis and implicitly assumes that capital stock is fixed exogenously. Our interest is primarily in the medium term where price adjustments and aggregate demand behaviour take on central roles, as in standard Keynesian short run macroeconomic models, while the supply side is given less prominence. Labour is also assumed to be fixed (from outside the model); thus the analysis of steady states and the convergence of actual to natural growth rate are unimportant here. The non-equalisation of aggregate demand and supply, which was interpreted as the divergence of investment and saving in the first part of the thesis, now causes price or inventory changes rather than influencing capital formation.

Part A, on growth theory, starts off with the one sector (Solow) neoclassical model of growth. This is as expected, given the seminal importance of that work in the theory of equilibrium growth. The Solow model assumes continuous equality of saving and investment and this is the major mechanism by which markets are equilibrated. We show that, even in an one sector model, it is not easy to defend the assumption that desired saving must be equal to desired investment. Thus, using Bliss's terminology (1970), investment must be 'accomodating'; in effect it adjusts to whatever level of saving (derived via the inter-temporal optimising decisions of the aggregate household) is available. Alternatively, saving always determine the short side of the market so that it is equal to capital formation and hence actual investment. We criticise this lack of an investment 'function', relating ex ante investment to other variables in the system, which can in principle be binding and hence determine capital accumulation. Chapter II then goes on to explore a model which implicitly analyses investment behaviour and shows how disequilibrium (in the sense of ex ante saving and investment not being equal) can be incorporated in the basic neoclassical model of growth.

The one sector disequilibrium growth model with neoclassical features ignores the problem of unemployment; this omission needs to be rectified. Chapter III concentrates exclusively on unemployment and discusses the stability properties of a system with capital accumulation but where labour markets fail to clear. The analysis is set in an optimising framework using the well known overlapping generations model. The 'younger' generation earn wage income alone and save part of it. The 'older' generation consume all their profit income. This allows us to consider saving and investment decisions as synonymous; thus the previous chapter's major concern has no significance here. However, the explicit focus on labour market disequilibrium has interesting implications.

An important missing feature of these two models, as well as others in the neoclassical vein, is the lack of an explicit investment function (though Chapter II does have some discussion of this). One of the earliest, and seminal, analysis in the context of growth theory was due to Harrod and his ideas are used formally in the next chapter to analyse various regimes of dynamic disequilibrium. The Harrodian investment function assumes that intended investment (as a proportion of current capital stock) rises when there is capital shortage i.e. desired investment can not be fulfilled due to lack of saving. Alternatively, ex ante investment (as a ratio of capital) falls when there is excess capacity; which implies potential full employment saving is greater than investment demand. In Chapter IV, armed with an explicit behavioural equation for investment, we find that it is possible not only to derive dynamic properties of the growth system but also to identify and study various regimes of unemployment, such as Classical and Keynesian, as well as repressed inflation with general excess demand.

Chapter V takes on board some distributional considerations through a two-class model of 'workers' and 'capitalists' who have different saving propensities. Here we are careful to take into account the implications of the dual decision hypothesis and the fact that unemployment will have implications for saving behaviour (through workers rationed income); hence capital formation, determined by the minimum of desired investment and saving, will also be affected. This chapter emphasises unemployment, again because of its overall macoeconomic importance. Classical and Keynesian (unemployment) regimes are distinguished, and dynamic behaviour on the long run growth path are analysed, separately for the two alternative systems. The implicit assumption is that if the economy finds itself in one of the two unemployment regimes then it will continue to do so in the long run. A more general disequilibrium model with regime switching and growth is beyond the scope of this thesis.

Uptil now the analysis is strictly concentrated on the closed economy. Chapter VI emphasises the open economy where a tradeables/nontradeables distinction is made, to characterise the major property of a small economy trading with the rest of the world. Once again we stress unemployment and analyse the disequilibrium of the labour market in the context of capital formation and trade. All models discussed thus far are 'real' in the sense that monetary assets play no explicit role. This follows the tradition of equilibrium growth theory where monetary models were latecomers in the overall analysis. As emphasised earlier, in the context of the Dixit research agenda, the very presence of money even in a world of certainty-equivalents create conditions for disequilibrium modelling. Chapter VII analyses certain issues in monetary growth within the framework of rational expectations, which in the absence of uncertainty implies perfect foresight.

We are interested in two questions, in the context of money and growth, both of which have received a lot of attention in the recent literature. First, the conditions under which 'superneutrality' holds; would the steady (stationary) state value of real variables be invariant to changes in the rate of growth of money supply; what conditions would guarantee a positive role to discretionary monetary policy; alternatively, can the monetary authorities influence the long run values of real capital stock and output. These questions are analysed in some detail, using a survey of the literature as well as a model, in this chapter.

The second set of issues are more intimately connected with disequilibrium behaviour; specifically, the motion of the economy in its transition path (to the new long run equilibrium) consequent to a policy shock. To focus attention on disequilibrium dynamics we postulate superneutrality so that initial and final capital stock, corresponding to two levels of the exogenously specified money growth rate, are the same. This allows us to concentrate on the behaviour of real variables when the economy is out of equilibrium without bothering about stationary states. It is demonstrated that activist monetary policy, both anticipated and unanticipated, does indeed have real effects on the transition path when the economy has not reached its long run equilibrium. This is independent of any subsidiary assumptions regarding superneutrality.

Chapter VIII begins the second part of the thesis where we eschew capital theory and focus on short and medium term issues. In a sense, this chapter and the preceeding one are linked for they both deal with (super)neutrality of money under rational expectations. Thus they form a formal bridge between the two parts of the work.

In the postulated short run framework Chapter VIII analyses monetary neutrality given that markets may not clear. Lucasian models usually assume equilibrium and then demonstrate that money is neutral under rational expectations. Neutrality, under uncertainty, is defined as the invariance of real variables to changes in the anticipated component of money supply; only unanticipated shocks have real effects. We investigate these propositions under disequilibrium.

The short run monetary framework continues into Chapter IX. A Dornbusch type open economy is considered with price and exchange rate dynamics; price movements depend on the difference between aggregate demand and supply; exchange rates obey covered interest parity under perfect foresight. Our innovation here is to model disequilibrium (giving rise to price movements over time) much more formally, taking into account, carefully, the difference between notional and effective demand (supply) as well as the concomitant dual decision effects. It is demonstrated that if the implications of non market clearing behaviour are properly taken into account then 'overshooting' type responses are exacerbated. This is not surprising since the pressure on the jump variable is enhanced by both the inertia of sluggish prices and the consequences of disequilibrium.

The standard way to justify and tackle disequilibrium, aggregate demand not equal to supply, is to assume 'slow' price adjustments. Another way would be to use inventory fluctuations as an adjustment mechanism; thus flow disequilibrium is met by stock changes as well as by some price movements. The crucial question is to model inventory behaviour. This is done in Chapter X in a Buiter-Miller framework by analysing a short to medium term macro model of the open economy. It is demonstrated that the dynamic movements of inventory in the small open economy can exhibit some surprisingly different properties (from the standard textbook Keynesian models), depending, of course, on the assumptions made. In particular, if inventory is assumed to be a 'jump' variable, as it can rightly be in a small open economy which can buy or sell any amount of the given aggregate commodity in the world market (acting like a 'small' competitive firm in a 'large' industry), then inventory holding can actually fall consequent to a contractionary monetary policy induced reduction in aggregate demand. This in turn may aggravate the business cycle as discussed fully in Chapter X.

The following chapters are relatively self-contained; each have their own short introduction, brief literature survey and succint concluding remarks. The linkages between them lie principally in methodology and the type of issues discussed rather than in the evolution of a specific model. It is difficult, given the current state of the literature, to construct a general disequilibrium growth model which will take care of all the major features involved. Thus specific models emphasise and analyse specific or individual problems, with the integrating link being provided by the framework and the type of questions asked, rather than the answers given or the solutions provided.

In general, the models discussed later stress the crucial importance of various adjustment processes, these in turn being dependent on the type of disequilibrium regime one has or one proposes to emphasise. The crucial distinction is that between 'Classical' and 'Keynesian' regimes. In the former, the real wage rate is usually equal to the marginal product of labour; thus unemployment is a product of too high a real wage rate and its sluggish adjustment. In addition, the capital (commodity) market clears only with a binding saving constraint; thus investment behaviour need not be analysed. The 'Keynesian' world, on the other hand, has the real wage rate below the marginal product of labour as well as accomodating the important role of the investment function. Depending on the regimes postulated, the adjustment equations are appropriately specified, as mentioned earlier in Section 5, and this gives rise to the contrasting dynamics and growth stories of the various models we discuss later in the thesis.

Finally it can be mentioned, given the brief discussions of the chapters, how the evolution of the first part of this work fits in with the research agenda set forth in Section 3. These six chapters (II to VII) monitor the major issues in disequilibrium growth theory identified in Dixit (1976). Together, they are expected to give a reasonably comprehensive picture of the field. However, as mentioned earlier, given the current literature, it is not possible to give an integrated and perfectly general model. But this may not be undesirable either, since a metatheory might not be sufficiently encompassing as to incorporate all the complications of the problem. At this stage it may even be preferable to analyse issues as they arise, with a loose interconnection (through the method and objectives) binding them informally together.

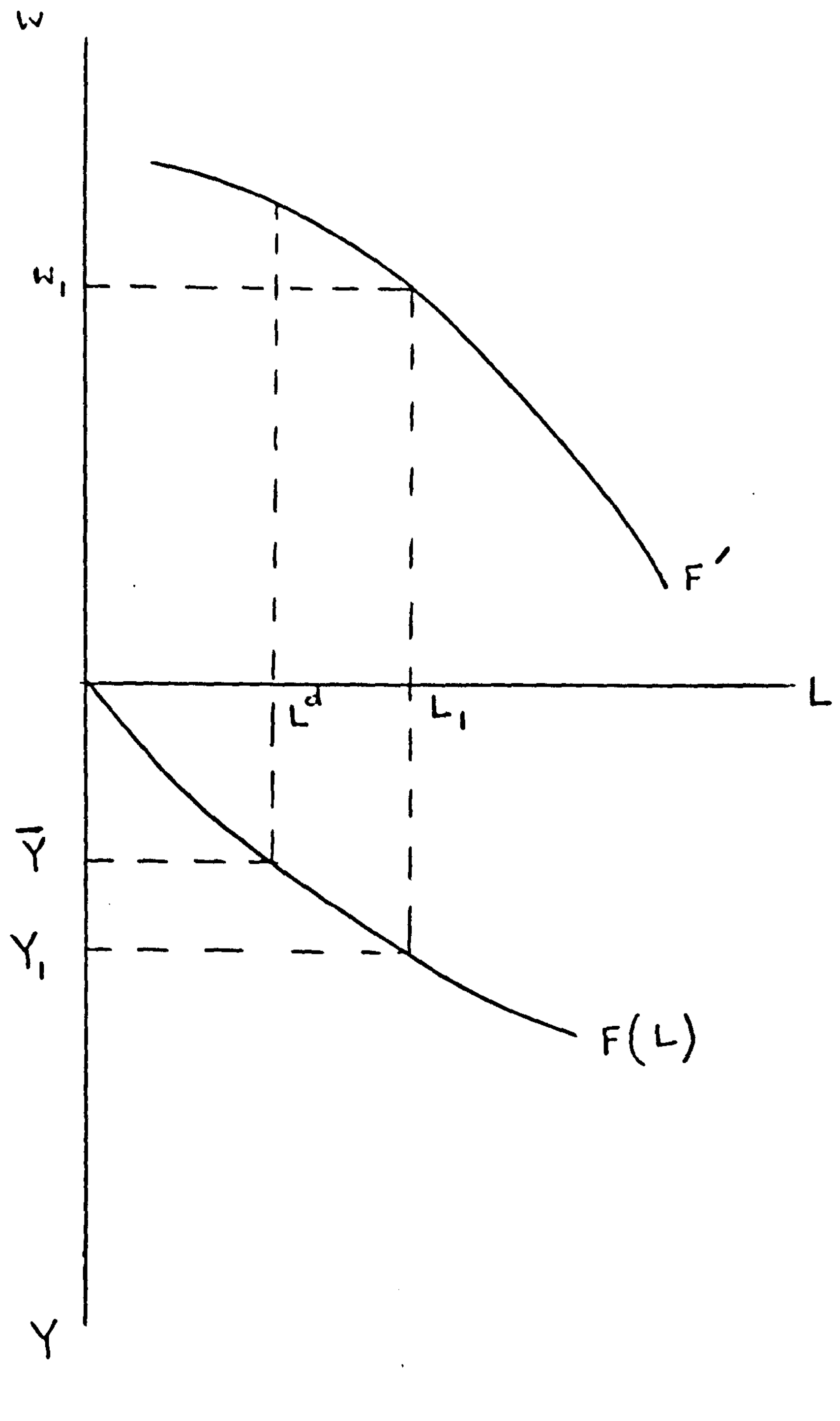

Figure 1·1

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Chapter II

One Sector Neoclassical Growth Model with Disequilibrium

Section 1: Introduction

Modern growth theory was pioneered by the work of Harrod (1939). Harrod was interested in two types of equilibrium (and therefore of disequilibrium), which were essentially different, though with interconnections. The first deals with the equality of actual and warranted rates of growth. The second investigates the equality of warranted (equal to actual) and natural growth rates, i.e. properties of steady states. In spite of the fact that Harrod stressed the crucial importance of the former problem[1], the neoclassical theory that followed concentrated exclusively on the latter issue.

Actual and warranted rates of growth are not equal to each other when desired saving is not equal to desired investment. In an atomistic environment when savers and investors are usually different groups of people, saving investment equality ex ante is not a truism. Rather, we should expect ex ante saving not to be equal to ex ante investment and investsigate the consequences of the inequality.

Unfortunately, this problem is completely ignored in the neoclassical theory of growth. It quickly passes on to the second problem concerning the equality of warranted and natural rates of growth. In other words the analysis of steady states become crucial in this theory. It also analyses the behaviour of the system when warranted (necessarily equal to actual) and natural growth rates are not equal and finds that the former converges to the latter by flexible capital output and capital labour ratios which in turn depend on the real prices of factors of production, labour and capital. The marginal productivity theory is an essential adjunct to this analysis.

The theory has been criticised on various points. The one asset structure of the model, the problem of valuation of capital in a more general model, the possibility of badly behaved production functions, indeed the very notion of analysing steady state equilibrium and stability have been attacked. We are not concerned with these. The essential problem from our point of view is that the possibility of desired investment saving inequality is ignored.

Harrod's knife edge growth equilibrium and related instability is based on an explicit investment demand function. However, neoclassical theory has generally ignored the analysis of investment behaviour in growth models. As Bliss (1975) states: "It is a most remarkable fact (that) ... since Solow's pioneering paper, there has been no subsequent discussion of this particular process of adjustment to equilibrium" (p. 312). An excellent paper by Nikaido (1975), highlights the issue of the knife edge problems as caused by investment behaviour of capitalists rather than rigid factor prices and inflexible factor substitution. Our paper tries to analyse these issues in a more neoclassical framework but with a greater emphasis on independent investment behaviour and the possibility that desired (ex ante) saving and investment may be unequal.

Though not always explicitly stated, three reasons can be found for such a neglect.[2] Firstly, there may be a government which bridges the deficit or withdraws the surplus. Secondly, savings plans are always realised and investment is an "accommodating" variable which adjusts automatically to the given level of saving. Thirdly, there exists a price mechanism in the capital market whereby the rate of interest adjusts to equate the desired demand for and supply of new capital (desired investment and saving respectively). The next section deals with these questions in more detail. It is shown that none of them are satisfactory.

To sum up, we try to investigate within an explicit neoclassical growth framework, the effects of investment behaviour, the divergence between desired investment and saving and the possible convergence of the model to a steady state equilibrium. The analysis emphasises disequilibrium behaviour as opposed to continuous saving investment equality, more appropriate to equilibrium theory. The latter view makes investment passive and accommodating, and neglects the consideration of "animal spirits" of the Keynesian investors. The alternative disequilibrium model, therefore, seems to be more interesting.

Section 2 deals with the reasons for the apparent neglect of investment behaviour in neoclassical growth models. Section 3 sets out the basic disequilibrium model. Section 4 analyses local and global stability of the system. Section 5 extends the model to include a variable saving income ratio. The last section concludes the chapter.

Section 2: Investment Saving Equality

The basic one sector neoclassical growth model (Solow (1956)) may be briefly summarised. All variables used here have aggregate values. There exists a neoclassical production function (whose properties are well known) linking output Y, capital stock K, and employment L (= total labour force available).

$$Y = F(K,L) \qquad (1)$$

By constant returns to scale we can write (1) in per capita form

$$y = f(k) \qquad f' > 0,\ f'' < 0 \qquad (2)$$

where $y = Y/L$, $k = K/L$.

The intertemporal utility function of the aggregate household determines a constant saving propensity (s) such that total saving (S) is:

$$S = sY$$

and $$\frac{S}{L} = sf(k) \qquad (3)$$

Per capita investment is by definition

$$i = \frac{I}{L} = \frac{\dot{K}}{L} \qquad (4)$$

The growth of full employment labour force is given by the natural rate of growth

$$n = \frac{\dot{L}}{L} \qquad (5)$$

Since $k = K/L$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \qquad (6)$$

and $$i = \frac{I}{L} = \dot{k} + nk \qquad (7)$$

Assuming continuous equality between saving and investment $I/L = S/L$ we have

$$\dot{k} + nk = sf(k) \qquad (8)$$

or $$\dot{k} = sf(k) - nk \qquad (9)$$

This is the basic differential equation that defines the path of capital stock per capita. The steady state values of $k = k^*$ is derived by putting $\dot{k} = 0$. Under the Solow condition of stability $sf(k^*) < n$, steady state equilibrium can be proved globally stable. The real factor prices of the model are derived from marginal productivity theory

$$r = P_K = \partial Y/\partial K = f'(k) \qquad (10)$$

$$w = P_L = \partial Y/\partial L = f(k) - kf'(k) \qquad (11)$$

(r is rate of interest, P_K is price of capital service, w is wage rate and P_L the price of labour).

In this equilibrium growth model, investment is <u>assumed</u> equal to saving. The possibility of disequilibrium between investment and saving may be rectified by three alternative methods, as suggested in Section 1. Consider each one in turn. In the given one asset model, the government can intervene to equalise investment and saving only through taxes (or transfers) of the given commodity. Suppose the rate of tax (or subsidy) is t_o so that the per capita amount of tax is $t = t_o f(k)$ and net private saving is $s(1-t_o)f(k)$. The condition for equilibrium is

$$i + g = s(1-t_o)f(k) \tag{12}$$

where $i = \dot{k} + nk$ is net investment per capita and $g = t_o f(k)$ is government expenditure or withdrawal per capita. Therefore

$$t_o(f(k) + sf(k)) = sf(k) - \dot{k} - nk$$

and

$$t_o = \frac{sf(k) - \dot{k} - nk}{f(k) + sf(k)} \tag{13}$$

Thus, the tax or subsidy rate must be continuously adjusted such that this equality holds. Unless the government is omniscient and omnipotent, such a continuous adjustment of tax (or subsidy) rates is virtually impossible and even as a "stylised fact" goes beyond credibility. Yet Burmeister and Dobell (1970) writes simply: "It may, of course, require some fiscal or monetary policy in the background to bring saving desires into line with investment intentions".

An alternative neoclassical procedure of tackling the problems arising out of an inequality of saving and investment is to make investment an "accommodating variable".[3] An equilibrium system consisting of a set of relations, gives some value (or values, if the solution is not unique) to the variables with which it deals. A variable may have a particular value either because of "uniquely determined decisions by the individual micro units of the model or it may be a consequence of aggregate balancing relations in the model" (see Bliss (1970)). The latter class of variables may change without violating the equilibrium conditions related to the individual decision makers. Its main purpose is to "accommodate" to a value which satisfies an aggregate balancing (equilibrium) condition, for example demand and supply have to be equal in a particular market. In the neoclassical growth model saving is a variable of the former type depending on the "decisions by the individual micro units of the model". On the other hand, investment is of the latter type because its value depends on savings, "a consequence of aggregate balancing relations", i.e. of the condition I = S. A variable like "investment" in a neoclassical model is called an "accommodating variable".

There are two ways in which the choice of investment as an "accommodating variable" can be defended. One can conceive the long run dynamic process as a sequence of short run temporary equilibrium, similar to a Hicksian "week". In any such short run "week", saving is determined by output (functionally dependent on given capital stock and labour force), and the saving propensity (determined by households intertemporal optimisation plan). Desired investment is equal to desired capital stock minus the initial capital stock. If at the given price set, desired capital stock is a <u>correspondence</u> rather than a <u>function</u>, the aggregate firm would willingly demand exactly the amount of capital stock supplied by household (savers). Thus desired investment passively accommodates to the given level of saving. If per chance, level of saving was different, investors would have been equally willing to invest that new (different) amount. The level of investment demand is not determined by any optimising behaviour of entrepreneurs, given the price vector. Whatever is saved is invested.

This is equilibrium economics par excellence. The essence of disequilibrium where markets do not clear, transactions take place at non-market clearing prices and price adjusts with non-tatonnement, are ignored completely. The balancing relations hold automatically, without any microeconomic incentive for change.

An alternative justification for an accommodating variable is more interesting. Consider any short run Hicksian "week". Suppose the aggregate firm wishes to maximise output subject to resource constraints, i.e. Max $Y = f(K^d, L^d)$ (14)

subject to $K^d < \overline{K} + S$

$L^d < (1 + n)\overline{L}$

where K^d, L^d are demand for capital and labour, $\overline{K}$ is initial capital stock, S is the flow of saving, $\overline{K} + S$ is the total capital stock available and $\overline{L}$ is initial labour.

Forming the Lagrangeian

$$Z = F(K^d, L^d) + r(\overline{K} + S - K^d) + w(\overline{L}(1 + n)\ L^d) \qquad (15)$$

where r and w are Lagrange multipliers and interpreted as shadow price of capital and labour resource.

By Kuhn-Tucker condition we have

$$\begin{aligned} &F_1 - r < 0 \\ &F_2 - w < 0 \qquad\qquad (16) \\ &\overline{K} + S - K^d > 0 \\ &\overline{L}(1 + n) - L^d > 0 \end{aligned}$$

(where $F_1 = \partial Y/\partial K^d$, $F_2 = \partial Y/\partial L^d$). If we assume further that $(K^d, L^d, w, r) > 0$ then all the relations (16) hold as equalities.

Specifically we have the marginal productivity theory of factor pricing $r = F_1$, $w = F_2$, full employment $L^d = \overline{L}(1 + n)$ and $\overline{K} + S = K^d$. If therefore investment is defined as $K^d - \overline{K}$, the desired change in capital stock, then we get $I = K^d - \overline{K} = S$.

If every single "week" obeys this equality, then the growth path conceived as a sequence of such weeks will always have investment savings equality.

Once again this stylised optimisation is well suited to equilibrium economics. However, as investors attempt to acquire more capital goods, various adjustments are taking place in the economy even within the Hicksian "week". In an attempt to reach the equilibrium position from an initial one, prices will change, and without an adjustment process in disequilibrium, we do not have an interesting analysis. A single firm under perfect competition can behave in the stylistic way set out, without causing changes in the underlying price vector (w,r). The aggregate firm in a macro model cannot do so and we must analyse the path of movement from the initial to the equilibrium points and any consequent disequilibrium adjustment.

Finally, we consider the price mechanism which gives the most interesting method by which disequilibrium may be corrected and investment saving equality brought about. By far the most complete analysis of this method is due to Solow (1956). He claims that in every period, a stock of capital (equal to saving plus existing stock) is thrown on the capital market inelastically and the real rental r(t) adjusts <u>instantaneously</u> to clear the market. In addition the marginal productivity condition must hold, i.e. real rental must equal marginal product of capital services. But prices never adjust <u>instantaneously</u>, disequilibrium can continue and Solow is aware of this problem. He says: "If saving and investment decisions are made independently, however, some additional marginal-efficiency-of-capital conditions have to be satisfied". But this enquiry is not followed up. There is a similar problem for the labour market too.

What does Solow's price mechanism, look like? Bliss (1975) has given an excellent formal summary of such a system. It consists of the following differential equations which are self-explanatory.

$$\dot{K} = \alpha[F_1(K,L) - r] \quad (17)$$

$$\dot{L} = \beta[F_2(K,L) - w] \quad (18)$$

$$\dot{r} = \delta[K - \overline{K}] \quad (19)$$

$$\dot{w} = \gamma[L - \overline{L}] \quad (20)$$

$$(\alpha, \beta, \delta, \gamma) > 0$$

(where $\overline{L}$, $\overline{K}$ are supplies of capital and labour). Even though general conditions of stability for such a system are difficult to derive, Bliss gives examples to show that stability in general cannot be established.

Neoclassicals may object to the characterisation given by equation (17) to (20). After all, the equilibrium model of Solow works on the basis of capital labour ratio and wage rental ratio, rather than absolute value of K, L, w, r as in system (17) to (20). We can re-interpret the equilibrating mechanisms and the working of the price system in a Solow model in the following way. Once again at the start of the Hicksian "week", the stock of capital to labour ($k=K/L$) is given inelastically on the market. Let it be $k=\overline{k}$. Similarly, the initial wage rental ratio ($\Omega=w/r$) is given. It the demand tor k exceeds the supply $\overline{k}$, wage rental ratio falls. A relative excess demand for capital w.r.t. labour raises the price of capital (r) w.r.t. price of labour (w). Further, if the wage rental ratio is greater than the ratio of marginal product of labour to capital, the capital labour ratio rises. The model is summarised as

$$\dot{k} = \alpha\left[\Omega - \left(\frac{f-kf'}{f'}\right)\right] \qquad (21)$$

$$\dot{\Omega} = \beta[\bar{k} - k] \qquad (22)$$

Forming the matrix

$$A = \begin{bmatrix} \partial\dot{k}/\partial k & \partial\dot{k}/\partial\Omega \\ \partial\dot{\Omega}/\partial k & \partial\dot{\Omega}/\partial\Omega \end{bmatrix}$$

$$= \begin{bmatrix} (\alpha f f'')/(f')^2 & \alpha \\ -\beta & 0 \end{bmatrix}$$

We can easily verify that the system is locally stable, since trace A < 0, det A > 0. The phase diagram for the model is given by Figure 2.1. and it is possible to have global stability.

The "week" is in equilibrium when $k=\bar{k}$, $\Omega=f(\bar{k}) - \bar{k}f'(\bar{k})/f'(\bar{k})$. When we attain such a temporary equilibrium in any "week", $\bar{k}$ is used to product output $y = f(\bar{k})$, saving is generated and a new capital stock for the next "week" is determined. The story continues until steady state.

The particular process described is silent about a number of important issues. Firstly, adjustments taking place are obviously one of tatonnement. No transactions take place until and unless capital stock, labour and factor prices have reached their temporary equilibrium values. This is not very satisfactory. Secondly, the

Hicksian "week" must be short enough for all adjustments to be completed and temporary equilibrium reached. Alternatively, speed of adjustment of k and Ω to equilibrium must be fast. Thirdly, and most important, even the attainment of the right equilibrium value for the <u>ratio</u> $k=\bar{k}$, does not guarantee full employment for the individual values of capital and labour. It is possible for the capital labour demand ratio to equal the given supply $\bar{k}$, but this may co-exist with unemployment and underutilisation of capital. Obviously, in such a situation, investment saving equality is not guaranteed.

Enough has been said, we hope, to justify the statement that saving investment equality (or inequality) is a fundamental economic problem. The model that we propose tries to analyse these factors in an explicit neoclassical framework.

Section 3: A Disequilibrium Model

This model concentrates exclusively on disequilibrium in the commodity market. Unemployment is not analysed explicitly but suitable extensions can be made to take this into account. It is similar to Bliss (1970), but is analysed in a much wider context. The following equations summarise the working of the system:

$$\dot{k}_p = \alpha[f'(k) - r] \qquad (23)$$

$$\dot{r} = \alpha[\dot{k}_p + nk - sf(k)] \qquad (24)$$

$$i_a = \text{minimum } [\dot{k}_p + nk,\ sf(k)] \qquad (25)$$

$(\alpha, \beta > 0)$

(where subscripts 'p' and 'a' denote planned and actual magnitudes and 'i' is investment per capita).

Equation (23) states that planned changes in capital labour ratios are made when marginal product of capital is different from the rate of interest. Thus the deterministic marginal productivity theory does not hold in general. On the other hand, the rate of interest equilibrates between desired investment and saving as given by equation (24). This is in true neoclassical tradition where price variable (rate of interest) adjusts to clear a market (that of loanable funds). The third equation (25) follows the tradition of current disequilibrium literature, where actual investment is the minimum of desired investment and saving, i.e. market is cleared on the short side.

In a Solow type model, $i_a = sf(k)$ and investment accommodates. Let us make the opposite assumption in Keynesian fashion, that $i_a = \dot{k}_p + nk$ and saving accommodates. Thus investment behaviour given by $\dot{k}_p$ is crucial in the model. There are two ways in which the working of the model can be explained. Firstly, we have the forced consumption case. Given an initial vector (k,r) output $y = f(k)$ is actually produced. Desired saving $sf(k)$ is determined and happens to be greater than desired investment $\dot{k}_p + nk$. Thus actual investment (and actual saving) is $\dot{k}_p + nk$. Desired consumption was $f(k) - sf(k)$ while actual consumption is $f(k) - \dot{k}_p - nk$. Thus there is forced consumption of the order of $(sf(k) - \dot{k}_p - nk)$. The ex post saving propensity $(\hat{s})$ is no longer the ex ante value 's'. From the multiplier we have

$$y = \frac{i}{\hat{s}}$$

and

$$\hat{s} = \frac{\dot{k}_p + nk}{f(k)}$$

An alternative version of the working of the model may be called the underutilisation case. Given the vector (k,r), planned investment i_p is determined first. Savings accommodates and actual investment = $i_p = \dot{k}_p + nk$. Output is now determined through the multiplier

$$y = \frac{\dot{k}_p + nk}{s} < \frac{sf(k)}{s} = f(k)$$

Thus the actual output produced y is less than the full capacity output producible $f(k)$. We have underutilisation of capital stock. Figure 2.2 shows the process.

The ex ante saving function for full capacity output generates the curve OG $(sf(k))$. The full capacity production function gives curve OH (or $f(k)$). Desired investment function $\dot{k}_p + nk$, dependent on (k,r) is the dotted line ABC and it lies below the saving curve OG. Thus investment decisions are realised. Given the saving propensity 's', actual output produced is $\dot{k}_p + nk/s$ and is drawn as DEF. Full capacity output is never produced in disequilibrium. An index of excess capacity is given by $\hat{kk_1}$. The steady state values of the model are similar to the neoclassical one with $r = f'(k)$ and $sf(k) = nk$, $\dot{k} = 0 = \dot{r}$.

The growth process may be briefly discussed. Consider an initial steady state point with $s = s_o$, $k = k_o$ and $i_o = nk_o = s_o f(k_o)$. Now suppose the desired saving propensity rises to $s > s_o$ such that desired investment $i_o < sf(k_o)$. From (24), rate of interest will fall to equilibrate the capital market. This will increase $\dot{k}_D$ from equation (23) and planned (actual) investment will rise. The actual capital labour ratio will rise too and we may have a movement towards a new steady state $sf(k^*) = nk^*$. Whether the economy will actually attain the new steady state equilibrium will have to be answered by stability analysis - the subject matter of the next section. To discuss the problems of stability we shall assume that investment is always realised $\dot{k}_p + nk = i = \dot{k} + nk$, therefore actual and planned capital stock are equal. This is done to focus attention on investment behaviour and does not alter any conclusions. The opposite assumption that saving is realised brings us back firmly into the neoclassical fold.

Section 4: Stability

Given the disequilibrium model

$$\dot{k} = \alpha[f'(k) - r] \qquad \alpha > 0 \qquad (26)$$

$$\dot{r} = \beta[\dot{k} + nk - sf(k)] \qquad \beta > 0 \qquad (27)$$

we can now analyse the stability properties. To test for local stability we need to form the following matrix:

$$A_1 = \begin{bmatrix} \partial\dot{k}/\partial k & \partial\dot{k}/\partial r \\ \partial\dot{r}/\partial k & \partial\dot{r}/\partial r \end{bmatrix}$$

$$= \begin{bmatrix} \alpha f'' & -\alpha \\ \alpha\beta f''+\beta n-\beta sf' & -\alpha\beta \end{bmatrix}$$

The matrix A_1 is evaluated at $k=\hat{k}$ and $r=\hat{r}$, steady state values. The necessary and sufficient conditions for local stability are

(i) trace $A_1 < 0$

(ii) det $A_1 > 0$

In this case (i) trace $A_1 = \alpha f'' - \alpha\beta < 0$

(given $f'' < 0$, $\alpha > 0$, $\beta > 0$)

(ii) det $A_1 = \alpha\beta n - \alpha\beta(sf')$

Thus for local stability we need $n-sf'(k) > 0$ which is identical to the Solow (1956) condition. The disequilibrium model is locally stable under similar condition as the equilibrium one.

We can now derive a phase diagram to point out certain issues in global stability. Given the differential equation (26) for $\dot{k}$ we have

$$\left.\frac{dr}{dk}\right|_{\dot{k}=0} = \frac{-\dfrac{\partial\dot{k}}{\partial k}}{\dfrac{\partial\dot{k}}{\partial r}} = -\frac{\alpha f''}{-\alpha} = f''(k) < 0 \qquad (28)$$

This gives us the slope for $\dot{k}=0$. Since $\partial\dot{k}/\partial k < 0$, the phase lines are as indicated in Figure 2.3.

For $\dot{k}=0$ the capital labour ratio is constant, real rental of capital (r) is equal to marginal product of capital (f') and an increase in k will decrease r by diminishing marginal productivity theory. Thus $dr/dk < 0$ for $\dot{k} = 0$. The sign of $\partial\dot{k}/\partial k$ negative, can be explained intuitively. An increase in capital labour ratio (k), given the nature of the production function, implies a decline in the marginal product of capital as well as a decline in the output capital ratio. Both these factors tend to make capital accumulation take place at a lower rate, i.e. make $\dot{k}$ fall. A lower marginal product makes it less profitable to invest in capital goods while a lower output/capital ratio lowers savings per unit of capital, thus dampening the supply of new capital.

The fact that $\partial\dot{k}/\partial r$ is negative can be explained since in this model capital accumulation depends on the difference between the return to factor capital ($f'(k)$) and the rate of interest (interpreted as cost of hiring capital). A rise in r increases the cost of acquiring capital and thus has an inverse effect on $\dot{k}$.

The second equation (27) is given by

$$\dot{r} = \beta(\dot{k} + nk - sf(k))$$

$$\frac{\partial\dot{r}}{\partial k} = \beta\frac{\partial\dot{k}}{\partial k} + \beta n - \beta sf'(k)$$

$$= \beta\alpha f'' + \beta(n - sf')$$

and $$\frac{\partial\dot{r}}{\partial r} = \beta\frac{\partial\dot{k}}{\partial r} = -\alpha\beta$$

$$\left.\frac{dr}{dk}\right|_{\dot{r}=0} = \frac{-\frac{\partial\dot{r}}{\partial k}}{\frac{\partial\dot{r}}{\partial r}} = f'' + \frac{n - st'}{\alpha} \qquad (29)$$

The sign of dr/dk for $\dot{r} = 0$ is not apparent. In the neighbourhood of $k = 0$, $n - sf' < 0$ (by Inada condition), f'' is always < 0, therefore $dr/dk < 0$. But as k increases $n - sf'$ changes sign and it is possible that dr/dk for $\dot{r} = 0$ may change sign.

By reasoning similar to previous case, $\partial\dot{r}/\partial r < 0$, therefore the phase diagrams are as shown in Figure 2.3. The sign of $\partial\dot{r}/\partial k$ is unknown since n-sf' changes sign (see equation (29)). Intuitively note that a rise in capital labour ratio causing $\dot{k}$ to fall reduces the rate of demand for capital and so reduces the rate of change of the price for capital, i.e. $\dot{r}$. On the other hand, the existing labour force has to be supplied with additional capital and this greater demand has an upward effect on $\dot{r}$. Unless the strengths of the two effects are known, the final nature of sign of $\partial\dot{r}/\partial r$ is indeterminate since a rise in the rate of interest causes $\dot{k}$ to fall and as explained before, this will cause $\dot{r}$ to decline.

The important point from stability point of view is the relative slopes of

$$\left.\frac{dr}{dk}\right|_{\dot{k}=0} \quad \text{and} \quad \left.\frac{dr}{dk}\right|_{\dot{r}=0}$$

in the neighbourhood of steady state equilibrium. In that neighbourhood

$$\left.\frac{dr}{dk}\right|_{\dot{r}=0} = \left.\frac{dr}{dk}\right|_{\dot{k}=0} + \frac{n - sf'}{\alpha}$$

If $\left.\frac{dr}{dk}\right|_{\dot{r}=0}$ is > 0 we have the case as depicted in Figure 2.3a.

On the other hand, if $\left.\frac{dr}{dk}\right|_{\dot{r}=0} < 0$ then the absolute value of $\left.\frac{dr}{dk}\right|_{\dot{r}=0}$ is less than $\left.\frac{dr}{dk}\right|_{\dot{k}=0}$ so that the curve $\dot{k}=0$ cuts the curve $\dot{r}=0$ from above. This is shown in Figure 2.3b.

The phase diagrams give us an indication of the stability of steady state equilibrium.

The analysis until now has not proved conclusively that the system we are considering is globally stable. Phase diagrams can only give an indication of the movements towards equilibrium starting from any arbitrary starting point. We turn to this point now.

The steady state equilibrium of the foregoing model can be proved to be globally asymptotically stable (GAS), provided the movements towards equilibrium (if any) from any arbitrary point, is non-oscillatory. If the initial point is (k,r) and equilibrium is $(\hat{k}, \hat{r})$, then by assumption of non-oscillation, we have the following condition:

$$\text{if } \hat{k} - k > 0 \text{ (or } < 0) \text{ always}$$

$$\text{then } \hat{r} - r < 0 \text{ (or } > 0) \text{ always}$$

Define

$$\Omega_1 = k - \hat{k}$$

$$(\forall\ k, r)$$

$$\Omega_2 = \hat{r} - r$$

Next define a "distance" function

$$V = (\Omega_1 + \Omega_2)^2$$

We have (i) $V \in R^+$

(ii) $V > 0$

(iii) $V = 0$, iff $\Omega_1 = 0 = \Omega_2$ since Ω_1 and Ω_2 are of the same sign always.

(iv) $$\frac{1}{2}\frac{dV}{dt} = \frac{1}{2}\dot{V} = (\Omega_1+\Omega_2)(\dot{\Omega}_1+\dot{\Omega}_2)$$

$$= (\Omega_1+\Omega_2)\dot{k} - (\Omega_1+\Omega_2)\dot{r}$$

$$= (\Omega_1+\Omega_2)\dot{k} - (\Omega_1+\Omega_2)\dot{k} + (\Omega_1+\Omega_2)(sf-nk)$$

$$= (\Omega_1+\Omega_2)[sf(k) - nk]$$

$$= (\Omega_1+\Omega_2)[sf(\Omega_1+\hat{k}) - n\Omega_1 - n\hat{k}]$$

$$< (\Omega_1+\Omega_2)(sf(\hat{k}) + \Omega_1 sf'(\hat{k}) - n\Omega_1 - n\hat{k})$$

(by concavity of f(k))

$$= (\Omega_1+\Omega_2)(\Omega_1 sf'(\hat{k}) - n\Omega_1)$$

$$= \Omega_1{}^2[sf'(\hat{k}) - n] + \Omega_1\Omega_2[sf'(\hat{k}) - n]$$

$$< 0$$

$$\because \quad sf'(\hat{k}) - n < 0$$

and $\Omega_1\Omega_2 > 0$ always since

$$\Omega_1 > 0 => \Omega_2 < 0$$

$$\Omega_1 > 0 => \Omega_2 > 0$$

Hence by the standard theory of Liapunov, the equilibrium $(\hat{k}, \hat{r})$ is globally stable.

Section 5: Saving Function

The usual literature on growth theory assumes the saving-income ratio to be constant, independent of the rate of interest and the level of income. It has been shown that under certain assumptions (see Uzawa (1968)), the average propensity to save (S/Y) does not depend on income. But the propensity to save may still depend on the expected and actual rate of interest. Even though there may be a secular constancy of the saving income ratio, it does fluctuate over the cycle and this fluctuation may be related to the rate of interest. The predominant concern of growth theory is long run behaviour but disequilibrium analysis cannot consistently neglect short run fluctuations.

In what follows we assume that the saving income ratio (s=s/Y) is a function of the current rate of interest (r). Really, 's' should be a function of a whole complex of interest rates, i.e. present-value prices into the indefinite future. But this line of reasoning will take us away too far from the central theme, so we assume

$$s = s(r) \qquad \{s'(r) > 0\} \tag{31}$$

The basic equations governing the time path of k and r are now

$$\dot{k} = \alpha[f'(k) - r] \tag{32}$$

$$\dot{r} = \beta[\dot{k} + nk - s(r)f(k)] \tag{33}$$

The equilibrium as steady state is similar to before and we have $k = \hat{k}$ and $r = \hat{r}$ when $\dot{k} = 0 = \dot{r}$.

To analyse local stability, form the matrix

$$A_2 = \begin{bmatrix} \partial\dot{k}/\partial k & \partial\dot{k}/\partial r \\ \partial\dot{r}/\partial k & \partial\dot{r}/\partial r \end{bmatrix}$$

$$= \begin{bmatrix} \alpha f'' & -\alpha \\ \alpha\beta f'' + \beta(n - sf') & -\alpha\beta - \beta f s' \end{bmatrix}$$

Note trace $A_2 = \alpha f'' = \alpha\beta - \beta f s' < 0$

$$\det A_2 = \alpha\beta(n - sf') - \alpha\beta f s' f'' > 0$$

Given the Solow stability condition $n - sf' > 0$. Thus steady state equilibrium $(\hat{k}, \hat{r})$ is locally stable.

We can derive phase diagrams (Figure 2.4) for this system as before.

$$\left.\frac{dr}{dk}\right|_{\dot{k}=0} = -\frac{\frac{\partial \dot{k}}{\partial k}}{\frac{\partial \dot{k}}{\partial r}} = f'' < 0 \qquad (34)$$

$$\left.\frac{dr}{dk}\right|_{\dot{r}=0} = -\frac{\frac{\partial \dot{r}}{\partial k}}{\frac{\partial \dot{r}}{\partial r}} = -\frac{\beta\frac{\partial \dot{k}}{\partial k} + \beta[n-s(r)f'(k)]}{\beta\frac{\partial \dot{k}}{\partial r} - \beta f(k)s'(r)}$$

$$= \frac{\alpha f''(k) + n-s(r)f'(k)}{\alpha + f(k)s'r}$$

$$= \frac{f''(k) + [n-s(r)f'(k)]/\alpha}{1 + [f(k)s'(r)]/\alpha} \qquad (35)$$

In the neighbourhood of equilibrium $n-s(\hat{r})f'(\hat{k}) > 0$. But f'' always < 0 and therefore $\left.\frac{dr}{dk}\right|_{\dot{r}=0}$ may be positive or negative. But even if it is negative its absolute value will be less than $\left.\frac{dr}{dk}\right|_{\dot{k}=0}$ and thus the phase diagrams show a tendency towards stability.

The stability conditions are

$$n-s(\hat{r})f'(\hat{k}) > 0 \qquad (36)$$

$$s'(r) > 0$$

Section 6: Conclusion

The analysis of investment behaviour and possible disequilibrium between saving and investment decisions can be fruitfully incorporated in a neoclassical model of growth. The disequilibrium model is robust enough to be locally stable, the phase diagrams show that global stability is possible and formally global stability is proved under certain conditions. Thus it seems that neoclassical dynamic theory can be extended to include disequilibrium analysis.

Footnotes

1) The major part of Harrod's paper (1939) deals with this problem while the other issue takes up only the last few sections of the article.

2) An advanced textbook (Burmeister and Dobell (1970)) deals with the issue in just one page though it mentions all the three following reasons.

3) For reasons of space, the exposition here is brief. But see Bliss (1975) for an excellent discussion of "accommodating variables" in economic theory. See also Sen (1976) for further analysis.

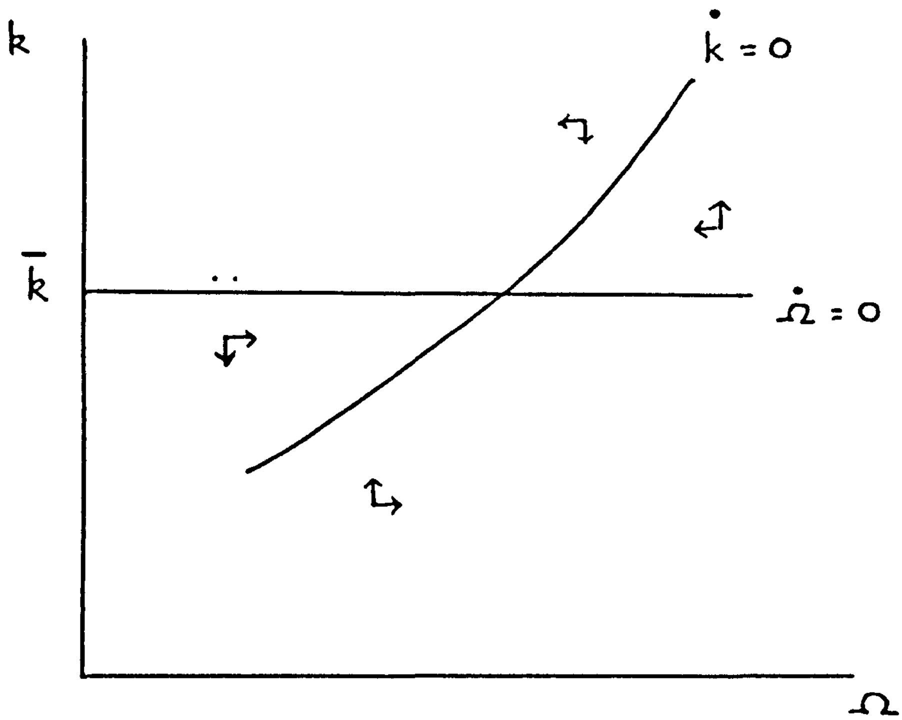


Figure 2.1

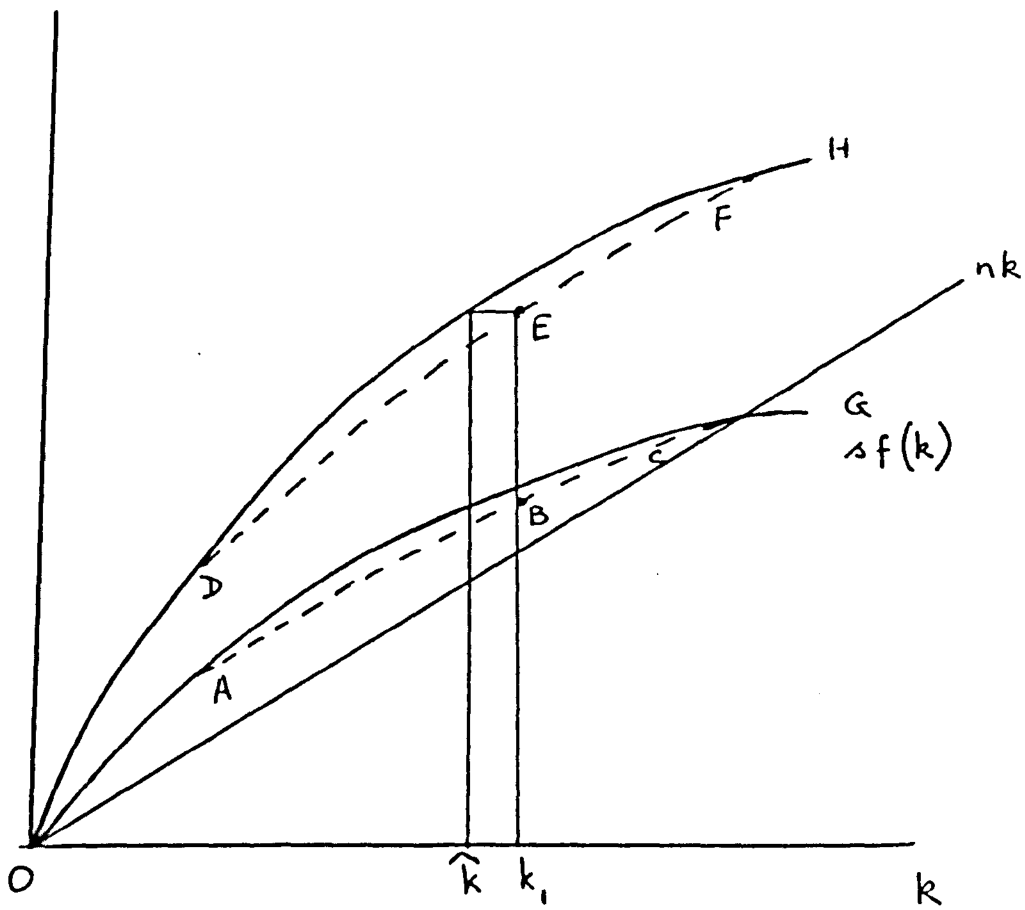
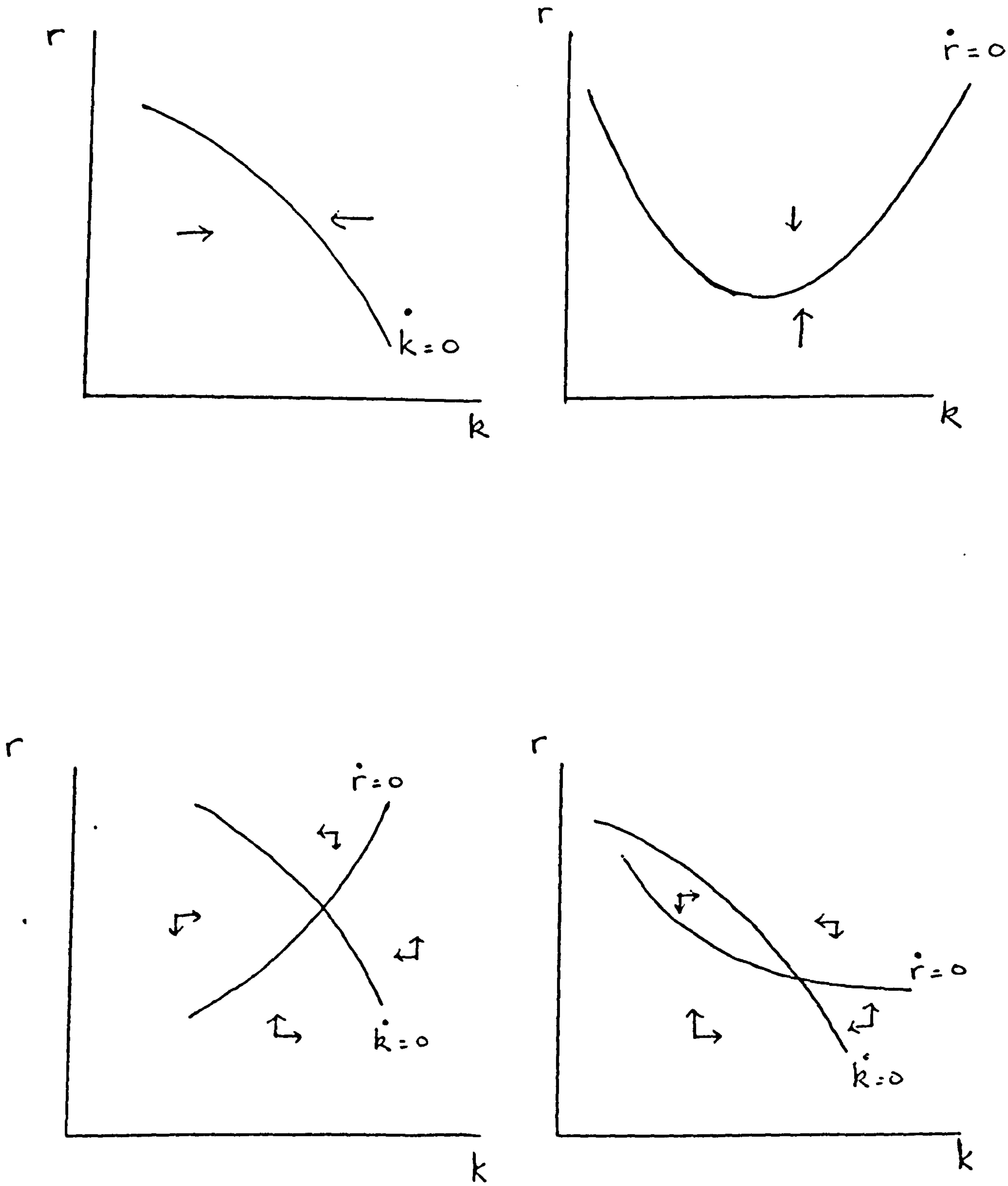


Figure 2.2



2·3a 2·3b

Figure 2·3

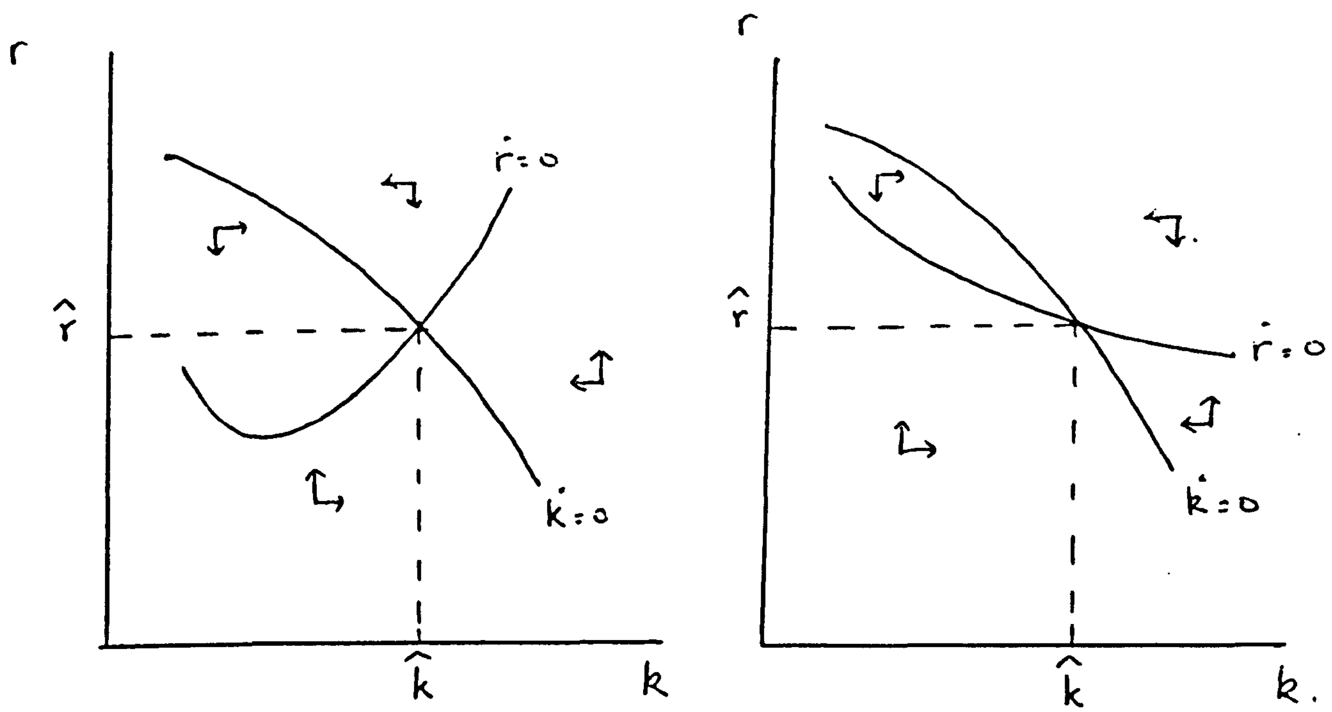


Figure 2·4.

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Chapter III

Intergenerational Model of Neoclassical Growth with Unemployment

Models of temporary equilibrium with quantity rationing have genrally emphasised transactions taking place at non-equilibrium prices. Thus in the labour market, when there is excess supply and the wage rate is sticky, actual labour transacted is the minimum of labour demand and supply. This results in unemployment and there may not exist an instantaneous adjustment in the wage rate to clear the market and consequently get full employment. Growth models on the other hand have generally assumed instantaneous price/wage adjustments and market clearing. We would like to present a model which embodies essential properties of neoclassical growth but which may have short run rigidities in the wage rate causing unemployment. We do not argue for any long lasting stickiness in the wage rate, rather a wage adjustment equation is formally integrated in the model. Our interest is in temporary equilibrium when markets transact on the short side thus leading to unemployment (or overemployment). We want to study how these sequences of temporary equilibrium tend to a full equilibrium steady state.

The formal model follows the structure of Diamond (1965) and Ito (1978), with an overlapping generation model, thus each period has two generations. The younger generation in each period offer employment, earn wage income, consume a part of their income and save the rest. The older generation are entrepreneurs or employers, earn profit income alone and do not save at all. For each period the wage rate 'w' is 'called' by the auctioneer (or determined by the bargaining process) and a temporary equilibrium has to be determined on the basis of given 'w'. The wage rate does not adjust within this period to clear the market so we may have disequilibrium in the sense that labour demand is not equal to labour supply. But the auctioneer (or bargaining process) is sensitive to the market mechanism and will vary the wage rate in subsequent periods according to whether there is excess demand or excess supply of labour.

Let the number of people in the younger generation be N. A convention to be followed throughout is that per capita value of variables X will mean $x = X/N$, i.e. in terms of the number in the younger generation rather than total population. Since only the younger generation work and supply labour we may say that 'x' is the value of X per worker. On the basis of utility maximisation, the workers or younger generation determine their labour supply $L^s = L^s(w)$. We assume that $\frac{L^s}{N} = l^s = f(w)$, which gives us the per worker labour supply function.

Labour demand is determined by capitalist employers. Given the capital stock K or $k = K/N$, employers maximise profits and through marginal productivity conditions determine $L^d = L^d(w,K)$. We assume that $l^d = L^d/N = g(w, k)$. This may not be true in general, but for some production functions like Cobb-Douglas, it will hold.

Full employment occurs when $l^d = l^s = l$. Otherwise transactions take place at the short side of the market such that $l = \min(l^d, l^s)$. When $l = l^d < l^s$ we have unemployment and $l = l^s < l^d$ implies overemployment.

Consider now the case of full employment; we have

$$f(w) = g(w,k) \qquad (1)$$

$$(f_1 > 0, \quad g_1 < 0, \quad g_2 > 0)$$

Solving for equilibrium in $w = w^*$ in terms of k we have

$$w^* = h(k) \tag{2}$$

From (1) $f_1 dw^* = g_1 dw^* + g_2\ dk$

or $\frac{dw^*}{dk} = \frac{g_2}{f_1 - g_1} > 0$

Thus $h' > 0$.

Further

$$l = f(w^*) = g(w^*, k)$$

Thus $l = l(k),\ l' > 0$ since w^* is a function of k.

Since investment and saving are done through wage income of workers alone we have

$$I = S = sw^*L \tag{3}$$

or $v = \frac{I}{N} = sw^*l = sh(k)l(k)$ (4)

Since $k = \frac{K}{N}$, $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{N}}{N}$

$$= \frac{I/N}{K/N} - n$$

$$= \frac{v}{k} - n$$

(where n is the natural rate of growth).

$$\dot{k} = v - nk$$

or $\dot{k} = sh(k)l(k) - nk$ (5)

Steady state under full employment is given by $\dot{k} = 0$, i.e. $sh(k)l(k) - nk = 0$. This equation gives us the steady state values $k = \hat{k}$ and w =

$\hat{w} = h(\hat{k})$. Assume also the Solow stability condition,

$$\left.\frac{\partial \dot{k}}{\partial k}\right|_{\dot{k}=0} = shl' + slh' - n < 0$$

$$\text{i.e. } shl' + slh' < n \qquad (6)$$

We thus know that full employment steady state is defined and is stable.

Unemployment

Let us now analyse the unemployment phase. We have $w > w^*$, $L^s > L^d$, $l^s > l^d$.(1) The important point is that actual employment $l = l^d = g(w,k)$. Investment per worker is defined by

$$v = I/N = S/N = swg(w,k) \qquad (7)$$

and $$\dot{k} = v - nk = swg(w,k) - nk \qquad (8)$$

The wage adjustment equation is the usual one,

$$\dot{w} = \alpha[l^d - l^s]$$

or $$\dot{w} = \alpha[g(w, k) - f(w)] \qquad (9)$$

$$(\alpha > 0)$$

Note that when $w = \hat{w}$ and correspondingly $k = \hat{k}$, $\dot{k} = 0$ in equation (8) since $g(\hat{w}, \hat{k}) = l(\hat{k})$ and $\hat{w} = h(\hat{k})$. Thus $\dot{k} = 0$ passes through $\hat{w}$, $\hat{k}$. Now consider the phases when $w > \hat{w}$ and $w < \hat{w}$ and let us analyse the signs of $\dot{k}$. We intend to show that under certain conditions $w \gtrless \hat{w} \rightarrow \dot{k} \gtrless 0$. Alternatively, it can be shown that $w \gtrless \hat{w} \rightarrow \dot{k} \lessgtr 0$.

---

(1) The equality signs are attached for analytical convenience. Strictly unemployment $\rightarrow w > w^*$, $l^s > l^d$. But we do not contradict the basic tenet of unemployment, i.e. supplies of labour are rationed and transactions in the labour market take place on the short (demand) side.

Compare the following equations for two points $(w, \hat{k})$ and $(\hat{w}, \hat{k})$.

$$s\hat{w}g(\hat{w},\hat{k}) - n\hat{k} = 0 = \dot{k} \quad \text{(steady state full employment)} \quad (10)$$

$$swg(w,\hat{k}) - n\hat{k} = \dot{k} \quad \text{(unemployment)} \quad (11)$$

For $w > \hat{w}$, $g(w, \hat{k}) < g(\hat{w}, \hat{k})$. If now the percentage change in 'w' is greater that the percentage change in $g(\ )$, then $swg(w,\hat{k}) > s\hat{w}g(\hat{w},\hat{k})$. This implies $\dot{k} > 0$ in equation (11). The condition which determines $\dot{k} > 0$ for $w > \hat{w}$ (when we are in the unemployment phase) is that the elasticity of per capita labour demand with respect to the wage rate is less than unity. If this condition is satisfied then $w \gtrless \hat{w} \rightarrow \dot{k} \gtrless 0$. Alternatively, if elasticity of l^d with respect to 'w' is greater than one (as in the Cobb Douglas case) we have $w \gtrless \hat{w} \rightarrow \dot{k} \lessgtr 0$.

The wage adjustment equation can be studied more analytically. For $\dot{w} = 0$, $g(w,k) = f(w)$ and we have already solved for $w^* = h(k)$. This gives us the phase line for $\dot{w} = 0$. We also know that $\left.\frac{\partial \dot{w}}{\partial w}\right|_{\dot{w}=0} = \alpha(g_1 - f_1) < 0$. We get Figure 3.1.

We can now use the phase line for $\dot{w} = 0$ and coupled with our previous analysis on $\dot{k}$ we get Figure 3.2.

Let us study the movements of $\dot{k}$ and $\dot{w}$ in some detail. Suppose initially the economy is at $k < \hat{k}$. If the initial wage is lower than the steady state $(w < \hat{w})$ and a state of unemployment exists, the income of wage earners is low. This is because of both low wages and the quantity constraint (rationing) set by unemployment. Thus saving (investment) will be low and the growth of capital will be less than the natural rate of growth. Per capita capital $k = K/N$ will fall. This leads to further unemployment, further reduction in wage and a vicious cycle results with falling 'k' and 'w'. If on the other hand the initial wage set by the auctioneer is $w > \hat{w}$, even though there is unemployment, the level of wage is higher enough to offset the quantity contraints of low unemployment. Thus income is high enough to have investment at such levels which increases k. Consequently, we have decreasing w (due to unemployment) and increasing k (due to high investment), and a movement towards steady state (for $k < \hat{k}$). This accords with our intuitive ideas of Keynesian economies where unemployment persists and growth is hampered when wages are too low while a high enough wage is beneficial to growth and employment.

The alternative case of unemployment but with $\varepsilon = \frac{w}{l^d} \frac{\partial l^d}{\partial w} > 1$ is given by the diagram, Figure 3.3. We can again derive a lot of conclusions from this figure, but I will not go into details.

<u>Overemployment</u>

Finally, let us study the phase of overemployment when $L^s < L^d$, $l^s < l^d$, $w < w^*$. We now have $v = I/N = S/N = swf(w)$ and $\dot{k} = swf(w) - nk$. Note that for $w = \hat{w}$, $k = \hat{k}$, $\dot{k} = 0$. Thus the phase line for $\dot{k} = 0$ passes through $(\hat{w}, \hat{k})$. Further taking

$$\dot{k} = 0 = swf(w) - nk$$

we get $$\left.\frac{dw}{dk}\right|_{\dot{k}=0} = \frac{n}{sf + swf_1} > 0$$

and $$\left.\frac{\partial \dot{k}}{\partial k}\right|_{\dot{k}=0} = - n < 0.$$

This information helps us to draw the phase diagrams (see Figure 3.4) for the overemployment case. We can distinguish two cases, one in which slope of $\dot{w} = 0$ is higher than slope of $\dot{k} = 0$ (for overemployment) and the other where it is lower. The movement of w and k can be analysed in the neighbourhood of steady state as before. A wide variety of cases are possible if we combine the various alternatives in the unemployment and overemployment regimes. A stable possibility (using $\varepsilon > 1$) is shown in Figure 3.5.

Footnotes.

1. The equality signs are attached for analytical convenience. Strictly speaking, unemployment --> $w > w^*$, $l^s > l^d$. But we do not contradict the basic tenets of unemployment, i.e. suppliers of labour are rationed and transactions in the labour market take place on the short (demand) side.

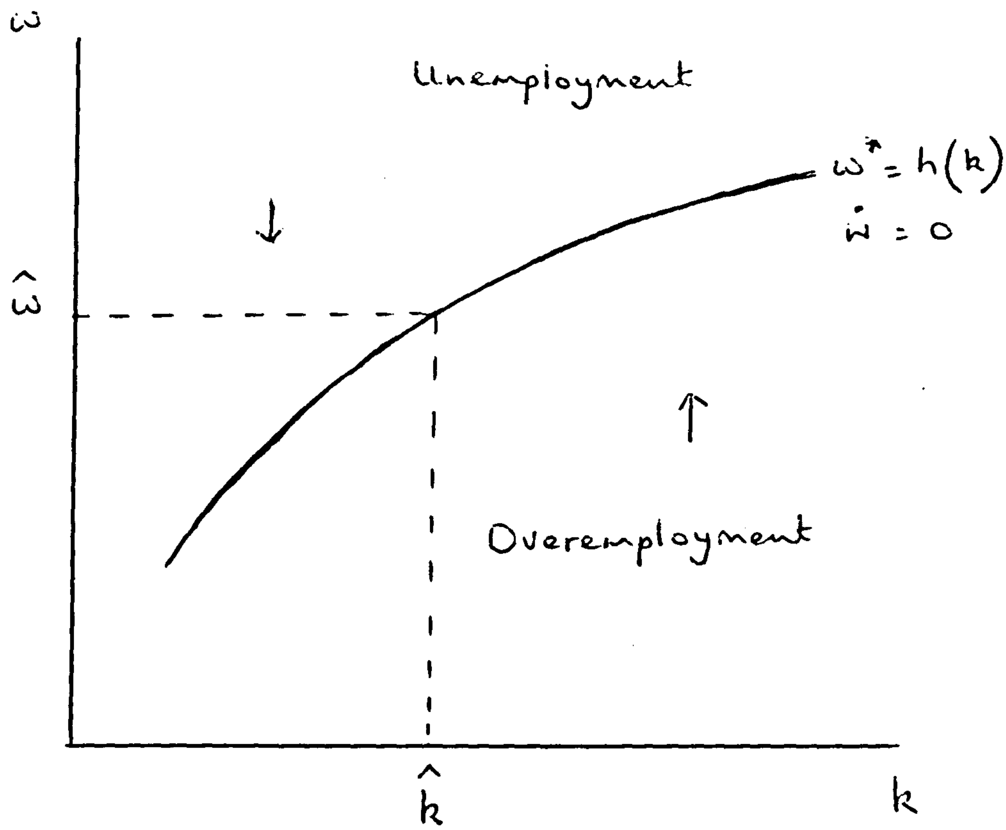
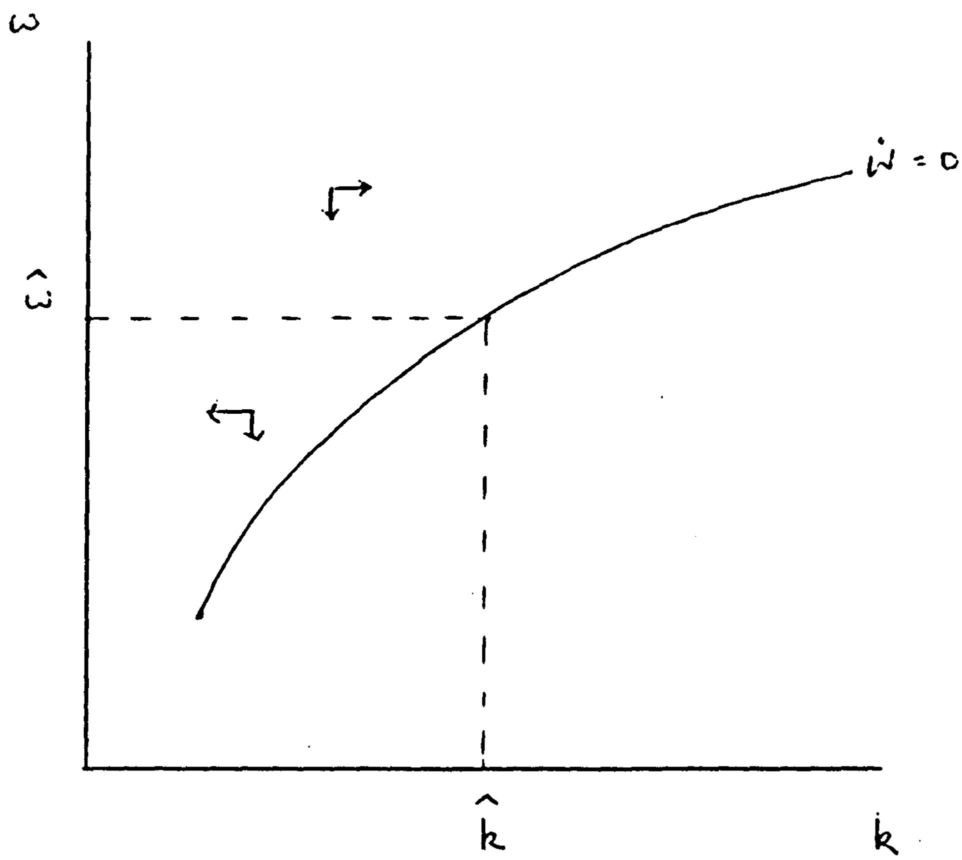

ω
Unemployment
$\omega^* = h(k)$
$\dot{w} = 0$
$\hat{\omega}$
Overemployment
$\hat{k}$
k

Figure 3.1



(Relevant elasticity $\epsilon = \frac{\omega}{L^d}\frac{\partial L^d}{\partial \omega} < 1$)

Figure 3·2

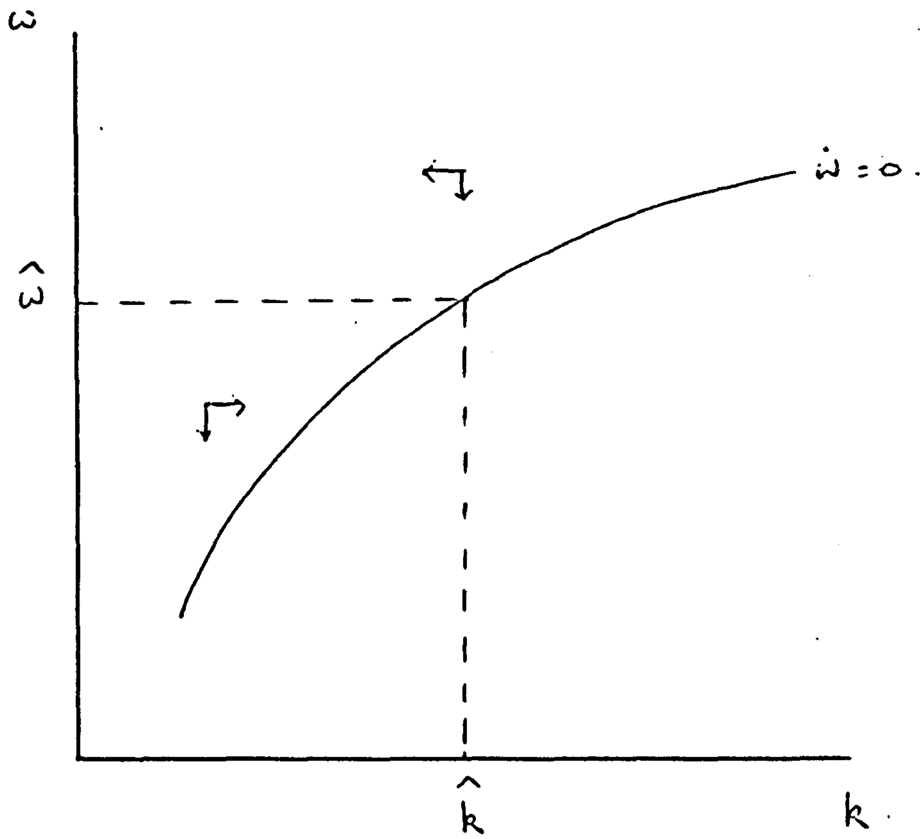


Figure 3.3

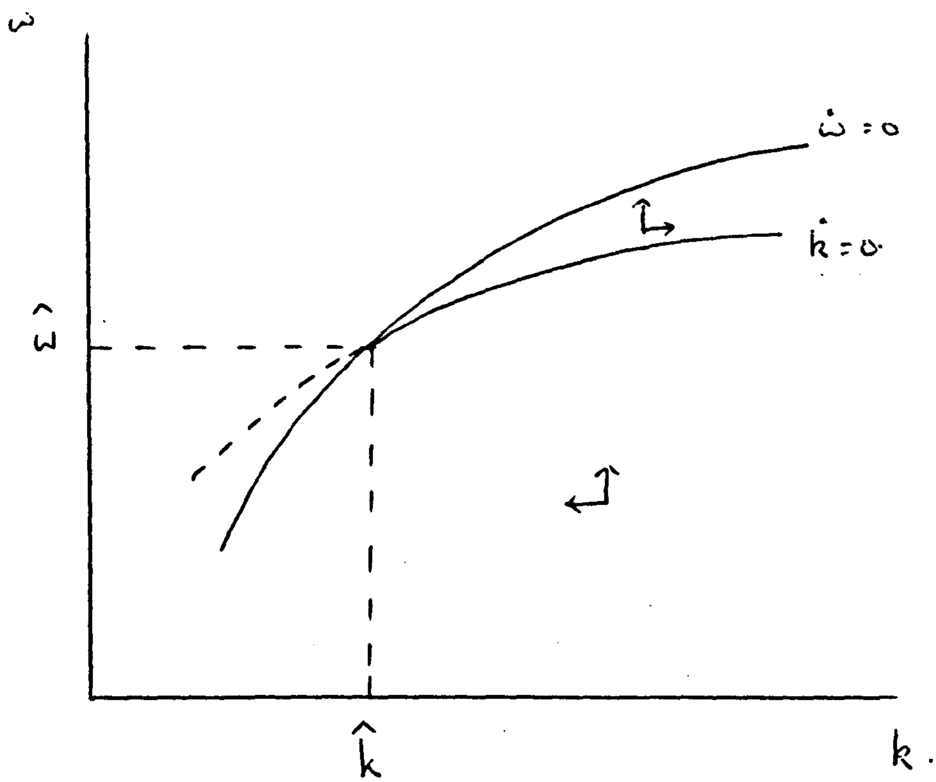


Figure 3·4

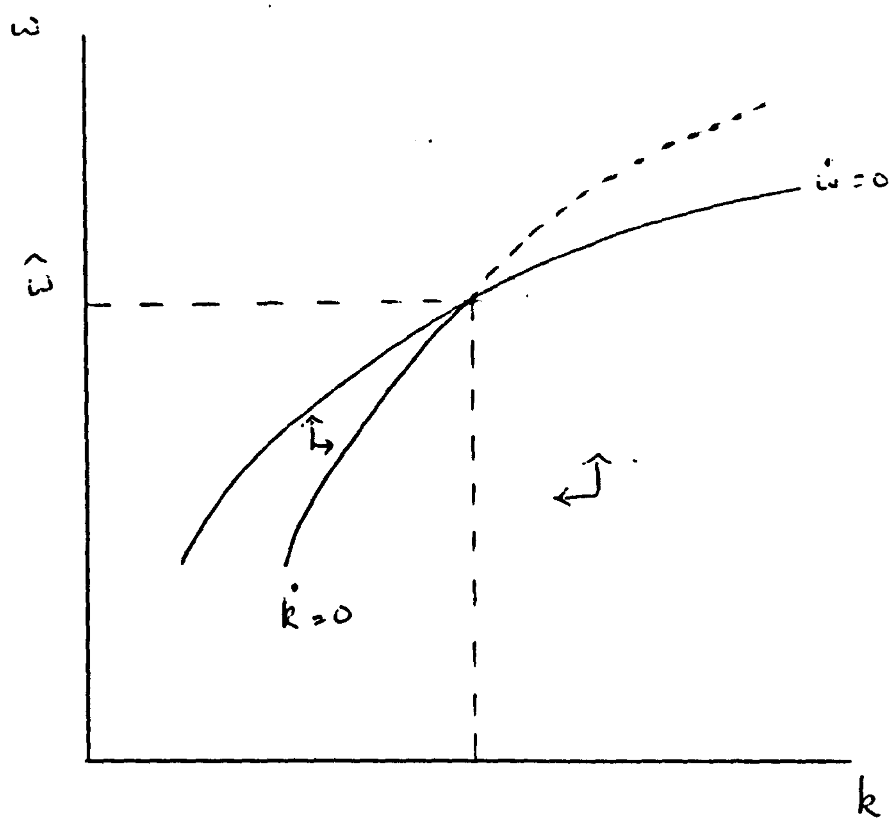


Figure 3.5

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Chapter IV

Harrodian Investment Function with Non-Clearing Markets

Section 1: Introduction

The following model tries to investigate the nature of growth equilibrium when desired investment and desired saving are not equal. It assumes a Harrodian investment behaviour within the broad framework of a neoclassical growth model and tries to investigate stability properties of the system. An important conclusion which follows is that Harrodian knife edge instability is just one of alternative possibilities for the model. Further, analysis of investment behaviour helps us to incorporate some Keynesian properties in a growth model. Introducing the labour market and coupled with the commodity market, we can get Malinvaud type classification of Keynesian unemployment, classical unemployment and repressed inflation. The last section deals with these regimes in some detail.

Section 2: The Model

We use the following notations:

$I \rightarrow$ intended investment

$s \rightarrow$ propensity to save

$F(K,L) \rightarrow$ potential full employment output

$Y \rightarrow$ actual national output (income)

$\dot{K} \rightarrow$ actual increase in capital stock

Following the usual practice in macromodels with rationing, markets are cleared on the short side so that actual increase in capital stock is the minimum of desired saving and investment. This gives us equation (1).

$$\dot{K} = \min(I,\ sF(K,L)) \tag{1}$$

$$Y = \frac{\dot{K}}{s} = \min\left(\frac{I}{s},\ F(K,L)\right) \tag{2}$$

$$\dot{K} = sY \tag{3}$$

$$\frac{I}{s} = \theta\ F(K,L) = F(\theta K,\ \theta L) \tag{4}$$

Equation (2) is the familiar multiplier mechanism and (3) follows from it. In (4) θ indicates the current level of capital shortage or excess capacity. If $\theta > 1$, desired investment (I) is greater than actual investment (= full employment investment), and there is capital shortage. If $\theta < 1$, desired investment is less than potential full employment investment and there is excess capacity. The former situation may be called the Solow-neoclassical case where full employment is maintained, actual capital formation is determined by full employment saving and investment is accommodating. The latter situation may be termed the Harrod-Keynes case where desired investment does not match up to the potential levels of a full employment economy.

As stated before investment behaviour is Harrodian, i.e.

$$\frac{d}{dt}\ (I/K) = \phi(\theta-1) \tag{5}$$

(assume $\phi > 0$ constant, or $\phi'(0) > 0$, $\theta \gtrless 1 \rightarrow \phi(\ ) \gtrless 0$).

If $\theta > 1$, and there is capital shortage, investors increase desired investment as a proportion of current capital stock. If $\theta < 1$, there is excess capacity and investors reduce the ratio of investment to capital stock $\left(\frac{d}{dt}\ (I/K) < 0\right)$.

Finally for labour supply we assume

$$\frac{\dot{L}}{L} = n \tag{6}$$

From (2) we have

$$\dot{K} = s \min[\frac{I}{s}, F(K,L)] \tag{7}$$

From (5),

$$\frac{d}{dt}(I/K) = \frac{K\dot{I} - I\dot{K}}{K^2} = \frac{\dot{I}}{K} - \frac{I}{K}\frac{\dot{K}}{K} = \phi(\theta-1)$$

or $$\dot{I} = I\frac{\dot{K}}{K} + K\,\phi(\theta-1)$$

or $$\dot{I} = \frac{sI}{K}\min\left(\frac{I}{s}, F(K,L)\right) + K\phi(\theta-1) \tag{8}$$

Equations (6), (7), (8) can be analysed further after a few simplications. Define per capita investment: $v = I/L$.

Given $k = K/L$

$$\frac{\dot{k}}{K} = \frac{I}{K} - n$$

$$\dot{k} = v - nk \tag{9}$$

Consider now the case where $\dot{K} = I < sF(K,L)$; i.e. $\theta < 1$, the so-called Harrod-Keynes case.

Since $v = \frac{I}{L}$, $\frac{\dot{v}}{v} = \frac{\dot{I}}{I} - n$

From (8), for this case

$$\dot{I} = \frac{I^2}{K} + K\,\phi(\theta-1)$$

or $$\frac{\dot{I}}{I} = \frac{I}{K} + \frac{K}{I}\,\phi(\theta-1)$$

Substituting ϕ in expression for $\dot{v}/v$ we get

$$\frac{\dot{v}}{v} = \frac{I}{K} + \frac{K}{I}\,\phi(\theta-1) - n$$

$$= \frac{v}{k} + \frac{k}{v}\phi(\theta-1) - n$$

or $$\dot{v} = \frac{v^2}{k} + k\,\phi(\theta-1) - nv \qquad (10)$$

Alternatively we may consider the opposite when $\theta > 1$ and $\dot{K} = sF(K.L) < I$, the Solow-neoclassical case.

$$\dot{I} = s\frac{I}{K} f(K.L) + K\phi(\theta-1)$$

$$\frac{\dot{I}}{I} = \frac{sF(K,L)}{K} + \frac{K}{I}\phi(\theta-1)$$

and finally substituting in $\frac{\dot{v}}{v}$ expression we have

$$\frac{\dot{v}}{v} = \frac{sF(K,L)}{K} + \frac{K}{I}\phi(\theta-1) - n$$

$$= \frac{\frac{sF(K,L)}{L}}{K/L} + \frac{K}{I}\phi(\theta-1) - n$$

$$= \frac{sF(k,1)}{k} + \frac{k}{v}\phi(\theta-1) - n$$

or $$\dot{v} = v\,\frac{sF(k,1)}{k} + k\phi(\theta-1) - nv \qquad (11)$$

Equations (9) and (10) together describe the Keynes-Harrod type of model while equations (9) and (11) give us Solow-neoclassical type models. They can be now analysed for steady state equilibrium and stability. Steady state is defined as $\dot{k} = \dot{v} = 0$, $\theta = 1$, $v = nk$.

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{\dot{L}}{L} = n$$

The two models need further simplification since we have to derive θ in terms of k, v.

Since $\theta = I/\ sF(K.L)$, therefore

$$\theta = \frac{v}{sF(k,1)} \tag{12}$$

Substituting in (10) we get

$$\dot{v} = \frac{v^2}{k} + k\phi \left(\frac{v}{sF(k,1)} - 1 \right) - nv \tag{13}$$

and in (11) we get

$$\dot{v} = v\,\frac{sF(k,1)}{k} + k\phi \left(\frac{v}{sF(k,1)} - 1 \right) - nv \tag{14}$$

We will work with the two pairs equations (9), (13) <u>and</u> equations (9), (14).

For stability analysis, let us take first equations (9) and (13), i.e.

$$\dot{k} = v - nk$$

$$\dot{v} = \frac{v^2}{k} + k\phi \left(\frac{v}{sF(k,1)} - 1 \right) - nv$$

The following matrix (evaluated at steady state) gives essential properties.

$$A = \begin{bmatrix} -n & 1 \\ \frac{-v^2}{k^2} + \phi \left(\frac{v}{sF} - 1 \right) & \frac{-2v}{k} + \frac{k\phi'}{sF} - n \\ -k\phi' \frac{vF'}{sF^2} & \end{bmatrix}$$

Given that at steady state $\theta = 1$, $\frac{v}{sF(k,1)} = 1$, $\frac{v}{k} = n$

$$A = \begin{bmatrix} -n & 1 \\ \frac{-v^2}{k^2} - k\phi' \frac{vF'}{sF^2} & n + \frac{k\phi'}{sF} \end{bmatrix}$$

$$\text{trace } A = \frac{k\phi'}{sF} > 0$$

$$\det A = \left\{ -n^2 - \frac{nk\phi'}{sF} + \frac{v^2}{k^2} + \frac{k\phi' vF'}{sF^2} \right\}$$

$$= \frac{nk\phi'}{sF} \left(-1 + \frac{kF'}{sF(k,1)} \right)$$

$$= \frac{nk\phi'}{sF} \left(-1 + \frac{kF'}{sLF(\frac{K}{L},1)} \right)$$

$$= \frac{nk\phi'}{sF} \left(-1 + \frac{kF'}{sF(K,L)} \right)$$

$$= \frac{nk\phi'}{sF} \left(-1 + \frac{\pi}{I} \right)$$

(π is total profits in the economy $F'K$ and $sF(K,L) = I$ in steady state equilibrium).

Thus the sign of det A is ambiguous depending upon whether $(\pi/I) >$ or < 1.

Trace $A > 0$ makes us aware of saddle point equilibrium but unless we know that det $A < 0$, we cannot say that the model is unstable. Of course it is plausible to assume $\pi < I$ and thus get det $A < 0$ and Harrodian instability. Further this raises distributional questions and possibilities of working with a 2-class model.

To conclude, if total profit in the economy is less than total investment, then steady state equilibrium is a saddle point and Harrod type knife edge results may follow.

Let us now turn our attention to the more neoclassical model using equations (9) and (14)

$$\dot{k} = v - nk$$

$$\dot{v} = \frac{vsF}{k} + k\phi \left(\frac{v}{sF} - 1 \right) - nv$$

To investigate stability properties form matrix

$$A = \begin{bmatrix} -n & 1 \\ \frac{vs(kF'-F)}{k^2} + \phi \left(\frac{v}{sF(k,1)} - 1 \right) & \frac{sF(k,1)}{k} + \frac{k\phi'}{sF} - n \\ -k\phi' \frac{vF'}{sF^2} & \end{bmatrix}$$

Since at steady state $\theta = 1$, $\frac{v}{sF(k,1)} = 1$ and $\dot{v} = 0 \rightarrow \frac{sF(k,1)}{k} = n$

we get

$$A = \begin{bmatrix} -n & 1 \\ \frac{vs(kF'-F)}{k^2} - \frac{k\phi' vF'}{sF^2} & \frac{k\phi'}{sF} \end{bmatrix}$$

$$\text{trace } A = -n + \frac{k\phi'}{sF} = -n + \frac{\phi'}{n} = \frac{\phi' - n^2}{n}$$

Note that trace A < 0 can be guaranteed if ϕ' is small. Thus Harrodian investment adjustment process must be slow.

$$\det A = \frac{-nk\phi}{sF} + \frac{vs(F-kF')}{k^2} + \frac{k\phi' vF'}{sF^2}$$

$$= \frac{ns(F-kF')}{k} + \frac{nk\phi'}{sF} \left(-1 + \frac{kF'}{F} \right)$$

$$= \frac{ns(F(K,L)-KF')}{K} + \frac{nk\phi'}{sF(k,1)} \left(-1 + \frac{KF'}{F(K,L)} \right)$$

$$= \frac{nsW}{K} + \frac{nk\phi'}{sF(k,1)} \left(-1 + \frac{\pi}{F(K,L)} \right)$$

where W is total wage fund and $F(K,L) = W + \pi$.

Therefore det $A = \frac{nsW}{K} + \frac{nk\phi'}{sF(k,1)} \left(\frac{W}{F(K,L)} \right)$

$$= \frac{nsW}{K} - \phi' \left(\frac{W}{F(K,L)} \right)$$

(since $nk = sF(k,1)$)

or det $A = W \left[\frac{nsF(K,L) - \phi' K}{K\, F(K,L)} \right]$

$$= \frac{W}{kF(K,L)} \left[nsF(k,1) - \phi' k \right]$$

$$= (W/F) \left[n^2 - \phi' \right]$$

If $\phi' - n^2 < 0$ and guarantees trace $A < 0$, then $n^2 - \phi' > 0$ guarantees det $A > 0$. Once again the same condition and the speed of adjustment is crucial .

To sum up, the first model comprising of equations (9) and (13) may be called Keynesian because it reflects excess capacity and ex ante investment is less than full employment capacity investment. Knife edge instability may occur depending on total profits and investment in the economy. But even here the answer is not unambiguous and distributional considerations may be important.

The second model may be called neoclassical, where ex ante investment is above the capacity full employment level. Savers are satisfied at the short side of the market but investors are not. Even though the investment mechanism is Harrodian, stability can be guaranteed provided the adjustment process is slow and investors do not react in a volatile fashion.

Let us now study the adjustment paths with phase diagrams in the neoclassical case. From equation (9),

$$\left.\frac{dk}{dv}\right|_{\dot{k}=0} = \frac{-\frac{\partial \dot{k}}{\partial v}}{\frac{\partial \dot{k}}{\partial k}} = -\frac{1}{-n} > 0$$

and $\left.\frac{\partial \dot{k}}{\partial k}\right|_{\dot{k}=0} = -n < 0.$

To analyse the v stationary, consider $\dot{v} = 0$. From equation (14) we get

$$\left.\frac{dk}{dv}\right|_{\dot{v}=0} = -\left.\frac{\frac{\partial \dot{v}}{\partial v}}{\frac{\partial \dot{k}}{\partial k}}\right|_{\dot{v}=0}$$

$$\frac{\partial \dot{v}}{\partial v} = \frac{sF(k,1)}{k} + \frac{k\phi'}{sF(k,1)} - n$$

It is difficult to attach a sign to this expression. Let us assume for simplicity that ϕ is a constant so that $\phi' = \phi$. Now when $\dot{v} = 0$, we have

$$\frac{vsF(k,1)}{k} + k\phi\left\{\frac{v}{sF(k,1)} - 1\right\} - nv = 0$$

or $\frac{sF(k,1)}{k} + \frac{k\phi}{sF(k,1)} = n + \frac{k\phi}{v}$

or $\frac{sF(k,1)}{k} + \frac{k\phi}{sF(k,1)} - n = \frac{k\phi}{v}$

Therefore $\left.\frac{\partial \dot{v}}{\partial v}\right|_{\dot{v}=0} = \frac{k\phi}{v} > 0$

Now consider

$$\left.\frac{\partial \dot{v}}{\partial k}\right|_{\dot{v}=0} = \frac{vs(kF'-F(k,1)}{k^2} + \phi\left(\frac{v}{sF(k,1)} - 1\right) - \frac{k\phi vF'}{sF^2}$$

Take each term in turn.

$$\frac{vs(kF' - F(k,1))}{k^2} = vs\frac{\left(\frac{KF' - LF(k,1)}{L}\right)}{k^2}$$

$$= vs\frac{\left(\frac{kF' - F(K.L)}{L}\right)}{k^2} < 0$$

(Since $KF' = \pi < F(K,L) = Y$).

This neoclassical type model assumes $\theta > 1$, $\frac{v}{sF} - 1 > 0$ and second term is > 0. The third term is negative. The final sign of $\left.\frac{dk}{dv}\right|_{\dot{v}=0}$ is unknown.

Let us take a few possible cases. Suppose ϕ is large, the positive second term dominates the others, $\left.\frac{\partial \dot{v}}{\partial k}\right|_{\dot{v}=0} > 0$ and $\left.\frac{dk}{dv}\right|_{\dot{v}=0} < 0$.

We have the following diagram showing saddle point equilibrium (see Figure 4.1)

Alternatively, ϕ is small, $\left.\frac{\partial \dot{v}}{\partial k}\right|_{\dot{v}=0} < 0$ and $\left.\frac{dk}{dv}\right|_{\dot{v}=0} > 0$.

We then have the two following alternatives, given by Figures 4.2 and 4.3. Once again, there are saddle point equilibria.

Section 3: Disequilibrium Regimes

It is possible to extend this model and do some more formal analysis. But let us now discuss more intuitively the different disequilibrium phases in which the economy may find itself. Our analysis is a wider generalisation of the basic model due to Nikaido (1980).

Suppose initially the economy is in a steady state with $I = \dot{K} = sF(K,L)$ and $\frac{\dot{K}}{K} = \frac{\dot{L}}{L} = n$. Now suppose the capitalists or investors become more bearish (a change in marginal efficiency of investment perhaps). They decide to reduce their ex ante investment such that $I < sF(K,L)$ or $\frac{I}{s} < F(K,L)$. Thus $\theta < I$ and excess capacity appears in the economy. Suppose simultaneously that the natural rate of growth rises due to exogenous shocks and $\frac{\dot{K}}{K} < \frac{\dot{L}}{L} = n$. Thus capital accumulation is not sufficient to absorb all of the growing labour force immediately and unemployment occurs. We therefore have excess supply of commodities (excess capacity) and excess supply of labour (unemployment). This may correspond to the Keynesian case.

Consider now the difference between $\frac{I}{s}$ and F(K,L) or more formally $\frac{I}{sK}$ and $\frac{F(K,L)}{K}$.

Note:

$$\frac{d}{dt}\left(\frac{I}{sK} - \frac{F(K,L)}{K}\right) = \frac{1}{s}\frac{d}{dt}\left(\frac{I}{K}\right) - \frac{L}{K}\left(n - \frac{\dot{K}}{K}\right)\frac{\partial F}{\partial\left(\frac{L}{K}\right)}\left(1, \frac{L}{K}\right) \qquad (15)$$

Let us try to determine the sign of this expression. From Harrod type investment behaviour, since $\theta < 1$, $\frac{1}{s}\frac{d}{dt}(I/K) < 0$. Since $\dot{K} = I$, this implies that $\frac{\dot{K}}{K}$ is falling. If initially $\frac{\dot{K}}{K} < n$ then it continues to be so and $n - \frac{\dot{K}}{K} > 0$. Therefore the second term of (15) is negative too. Thus $\left(\frac{I}{sK} - \frac{F(K,L)}{K}\right)$ declines over time. Excess supply or excess capacity increases over time. Simultaneously, $n > \frac{\dot{K}}{K}$ implies that unemployment persists and its volume increases. We have a typical Keynesian case, where investors plan less and less investment due to increasing excess capacity and at the same time there is growing unemployment.

We now move on to the second case of general excess demand or so-called repressed inflation. Again start from an initial steady state with $I = \dot{K} = sF(K,L)$ and $\frac{\dot{K}}{K} = \frac{\dot{L}}{L} > n$. Suppose due to exogenous change in expectations and demography I increases and n falls so that we have $I > sF(K,L)$ and $\frac{\dot{K}}{K} > n$. Since $\dot{K} = sF(K,L)$ we have $\frac{sF(K,L)}{K} > n$ This initial movement away from equilibrium is therefore characterised by capital shortage in the goods market and labour shortage in the labour market.

Once again consider equation (15). By investors behavioural equation (5) we have $\frac{1}{s}\frac{d}{dt}(I/K) > 0$ and since by assumption $n < \frac{\dot{K}}{K}$ the second term is positive. Therefore the whole expression is positive. This means the gap between I and sF(K,L) is increasing and there may be repressed inflation which is sustained.

On the other hand,

$$\frac{d}{dt}\left(\frac{F(K,L)}{K}\right) = \frac{L}{K}\left(n-\frac{\dot{K}}{K}\right)\frac{\partial}{\partial\left(\frac{L}{K}\right)} F(1,L/K) < 0$$

or

$$\frac{d}{dt}\left(\frac{sF(K,L)}{K}\right) = \frac{d}{dt}\left(\frac{\dot{K}}{K}\right) < 0.$$

Therefore $\dot{K}/K$ is declining towards the given natural rate (n) and ultimately $\frac{\dot{K}}{K} = \frac{\dot{L}}{L} = n$ and $sF(K,L) = nK$, the Solow steady state equilibrium. This fits in with our ideas of repressed inflation. Even though the economy is apparently in Solow growth equilibrium, investors bullishness can cause repressed inflation and apparent capital shortage. Firms whish to invest more than the amount of savings forthcoming and this causes implicit price levels to go up.

The final case we consider is the case of classical unemployment characterised by excess demand for commodities, but with excess supply of labour. We have $I > sF(K,L)$ or $\frac{I}{s} > F(K,L)$, but simultaneously $\frac{\dot{K}}{K} < \frac{\dot{L}}{L} = n$. Actual capital formation is $\dot{K} = sF(K,L)$. Since capital is growing slower than labour, either the capital labour ratio is declining or there is unemployment. But this system will not persist in the long run. We know that

$$\frac{d}{dt}\left(\frac{sF(K,L)}{K}\right) = s\left\{\frac{L}{K}\left(n-\frac{\dot{K}}{K}\right)\left(\frac{\partial}{\partial(L/K)}F(1, L/K)\right)\right\}.$$

Since $n-\frac{\dot{K}}{K} > 0$ this expression is > 0. Thus

$$\frac{d}{dt}\left(\frac{\dot{K}}{K}\right) = \frac{d}{dt}\left(\frac{sF(K,L)}{K}\right) > 0.$$

So $\frac{\dot{K}}{K}$ will rise until it catches up with the natural rate n and we have $\frac{\dot{K}}{K} = \frac{\dot{L}}{L} = n$ in long run equilibrium. We also have $\dot{K} = sF(K,L)$ and thus the two conditions give us the Solow steady state.

The final question is the difference between $\frac{I}{sK}$ and $\frac{F(K,L)}{K}$, alternatively the difference between I and sF(K,L) intended and actual investment. The outcome is difficult to predict since the two expressions in equation (15) have opposite signs. By (5) we have $\frac{1}{s}\frac{d}{dt}(I/K) > 0$. On the other hand the second expression contains $(n-\frac{\dot{K}}{K}) > 0$ and is thus negative. The final movement will depend on the strength of the two expressions. If Harrodian adjustment speed ϕ' (or ϕ if constant) is very low then the quantitative effect of $\frac{1}{s}\frac{d}{dt}\left(\frac{I}{K}\right)$ is small and thus $\frac{d}{dt}\left(\frac{I}{sK} - \frac{F(K,L)}{K}\right)$ is negative. We started with excess demand $\left(\frac{I}{s} - F(K,L)\right)$, but it will decline. Ultimately, $\frac{I}{sK} \rightarrow \frac{F(K,L)}{K}$ and we have a full equilibrium where intended capital formation is equal to the actual. Thus we have a Solow equilibrium coupled with an expectational equilibrium.

The moral of the story seems to be that the neoclassical model of growth is rather robust with respect to convergence towards steady state equilibrium. In spite of Harrod type investment functions and non-market clearing assumptions, stable equilibrium can be found under different types of assumptions.

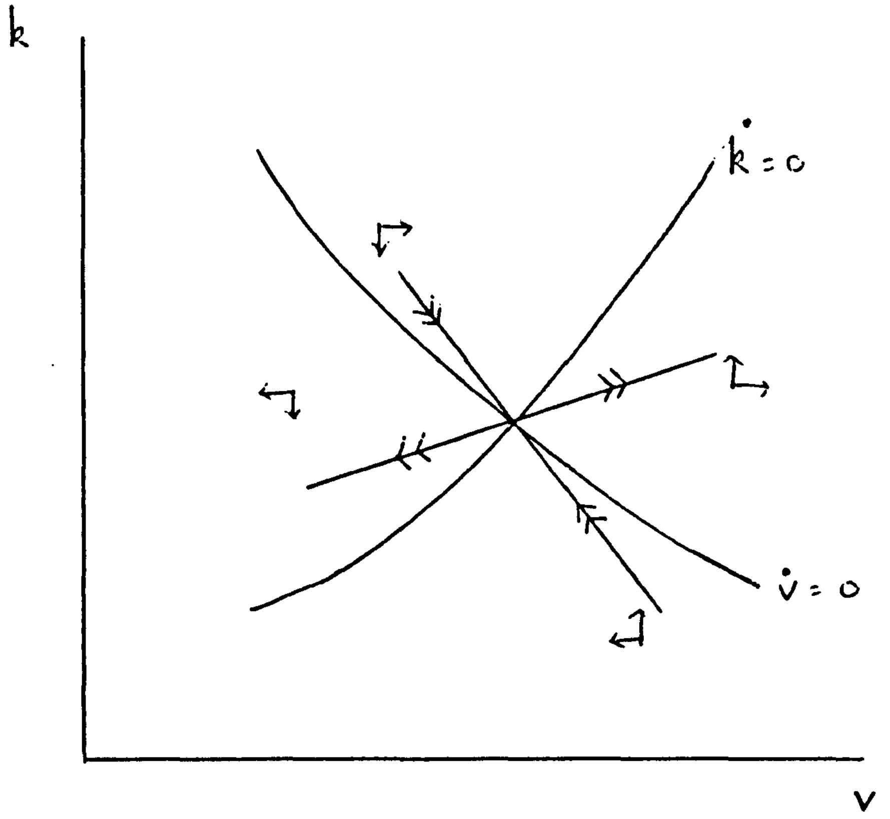

Figure 4.1

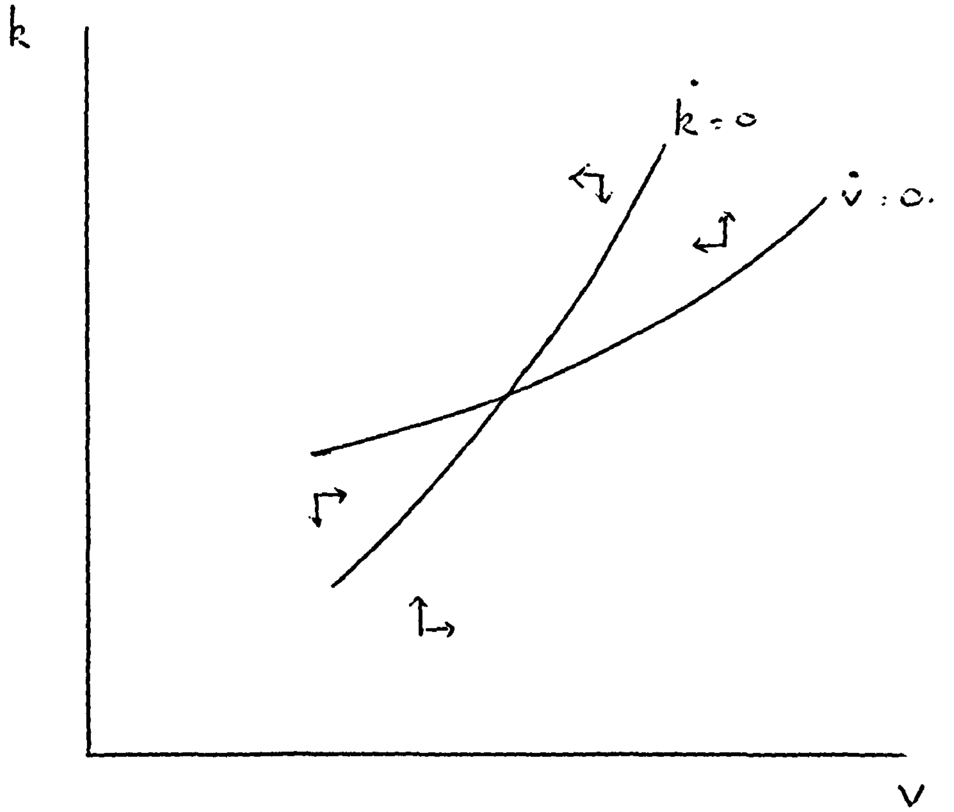

Figure 4.2

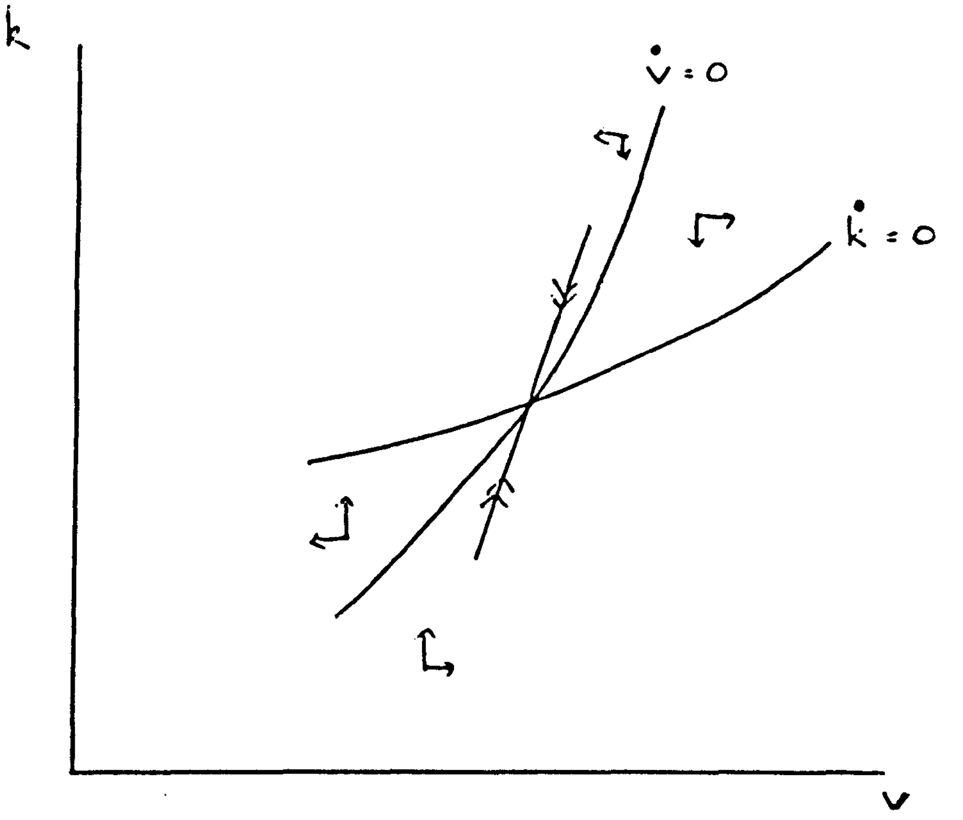

Figure 4.3.

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Chapter V

Unemployment, Income Distribution and Disequilibrium Growth

Section 1: Introduction

The purpose of this chapter is to study certain aspects of unemployment in a disequilibrium neoclassical growth model. Fix price models with non-clearing markets have been extensively studied in the recent literature (see Clower (1969), Barro and Grossman (1976) and Malinvaud (1977)). We propose to deal with similar issues connected with a disequilibrium labour market (specifically with unemployment) in an explicit dynamic framework.

As shown by Solow and Stiglitz (1968) income distribution parameters become important in specifying adjustment processes when markets do not clear. Our work follows in that tradition. The basic tenets of the neoclassical paradigm - marginal productivity theory and its role in the functional distribution of income are well known. Even though the theory in its basic form deals with market clearing and full employment, it can be extended to accommodate unemployment and quantity rationed temporary equilibrium, without losing its essential significance. But most of these analyses, though sophisticated, rest on a static framework. An elegant static model has been given by Bruno (1979), which may be used as a starting point. We propose to extend the scope of such models and put them in the explicit dynamic framework of neoclassical <u>growth</u> theory.

As is well known (Malinvaud (1977)), in a macro model with two goods (labour and commodity) and two transactors (households and firms), involuntary unemployment can be either classical or Keynesian. Further, the qualitative characteristics and the policy conclusions of these two types of unemployment are essentially different. One of our major aims is to highlight the important distinction between different forms of unemployment and to see its effects on the long run growth path of the economy.

In the next section, the case of neoclassical full employment is considered. First we look at the static single period representation and then consider the dynamic growth model. The same structure is repeated for the subsequent two sections, where we consider classical unemployment and Keynesian unemployment. Our main objective is to study the dynamic behaviour of the stylised growth economy when there is unemployment and when income distribution rules are fully specified.

Section 2: Neoclassical Full Employment

Let us first analyse the static case. Given the neoclassical production function $Y = F(K,L)$ (where Y,K,L are aggregate output, capital stock and labour), per capita income (output), $y = Y/L$ is distributed as

$$y = f(k) = w + rk \qquad (1)$$

(w is wage rate, r is rate of interest and $k = K/L$ is capital labour ratio). In a one sector model (we assume away capital-theoretic problems), given k, (1) can be represented as a straight line on the w-r space, with slope $-k$ and intercept $\bar{w} = f(k)$ when $r=0$. It is also easy for any point on this curve to read off w/rk which is the ratio of relative shares of labour and capital in national product.

Let the savings (S) function be of the Kaldor-classical type with two different propensities to save of two "classes", i.e. wage earners (s_w) and profit earners (s_r). Thus

$$S = s_w wL + s_r rK$$

or

$$\frac{S}{L} = s_w w + s_r rk$$

Denoting $i = I/L$ we have for saving equal to investment

$$i = s_w w + s_r rk \qquad (2)$$

Once again for a given i and k, this equation can be depicted as a straight line on w-r space with slope $-ks_r/s_w$ and intercept i/s_w.

We make the plausible assumption that $s_r > s_w$ and therefore given (1) and (2)

$$s_r > \frac{i}{y} > s_w \qquad (3)$$

Given this condition, the intersection of the two straight lines (1) and (2) is guaranteed.

In Figure 5.1, AB represents equation (1) for a given capital labour ratio k_0 and CD represents equation (2) for given investment per capita i_0 and k_0. To determine all absolute magnitudes we need an additional scale variable like size of labour force L_0, or capital stock K_0.

The static model has two degrees of freedom and alternative specifications are needed to close them. Within the neoclassical paradigm, one can use the marginal productivity theory of distribution as a first step towards closing these degrees of freedom.

It is well known (see Garegnani (1970), Burmeister and Dobell (1970) and Samuelson (1962)) that corresponding to the neoclassical production function (or a set of Samuelsonian blueprints), there exists a relationship between the marginal products of labour and capital. Given competitive assumptions and the marginal productivity theory of distribution, this relation becomes the factor price frontier (fpf in Figure 5.1). At every point on the fpf, the tangent line is represented by equation (1). Thus we have fpf given by $w = \phi(r)$ and $dw/dr = -k$.

We can now analyse the case of full employment in this short run model. Given the stock of capital (K_0) and the labour force (L_0), for full employment we have $k_0 = K_0/L_0$. With all factor markets operating under perfect competition, the appropriate wage rate (w_0) and profit rate (r_0) are determined by marginal productivity conditions,

$$r_0 = f'(k_0)$$

$$w_0 = f(k_0) - k_0 f'(k_0)$$

In Figure 5.1, this is represented by point Q_f. Of all parallel lines representing equation (2), the one passing through Q_f (in Figure 5.1 it is CD) will determine full employment investment per capita $i = i_0$. The ratio Q_fB/Q_fA gives the factor share of wage earnings to profit earnings. Thus, we have neoclassical full employment and all variables are determined. Many other comparative statics results can be derived, but let us turn our attention to the central issue of this paper, viz. growth models.

The standard neoclassical growth model (Solow (1956)) can be suitably adapted to fit the foregoing structure. Since labour grows at its natural rate (n) and is fully employed, $\dot{L}/L = n$.

Therefore,

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

$$= \frac{I}{K} - n$$

$$= \frac{s_w w}{k} + s_r r - n$$

$$\dot{k} = s_w w + s_r rk - nk$$

$$= s_w(f(k)-rk) + s_r rk - nk \qquad \text{(given (1))}$$

or

$$\dot{k} = s_w f(k) + (s_r - s_w) f'(k) k - nk \qquad (4)$$

For steady state $\dot{k} = 0$ and

$$s_w f(k) + (s_r - s_w) f'(k) k = nk \qquad (5)$$

This can be solved for steady state value $k = k^*$.

To analyse stability, we first deal with local stability properties of the system. We need $\partial \dot{k}/\partial k < 0$ in the neighbourhood of $k = k^*$,

$$\frac{\partial \dot{k}}{\partial k} = s_w f' + (s_r - s_w) f' + (s_r - s_w) f'' k - n$$

$$= s_r f' + (s_r - s_w) f'' k - n < 0$$

or

$$s_r f' < n - (s_r - s_w) f'' k \qquad (6)$$

Simplifying (4) we get for $k = k^*$

$$s_r f' k + [f - kf'] s_w = nk$$

or $$s_r f' = n - \frac{w s_w}{k} \qquad (7)$$

From (6) and (7), the stability condition reduces to

$$n - \frac{w s_w}{k} < n - (s_r - s_w) f'' k$$

or

$$(s_r - s_w) f'' k - \frac{w s_w}{k} < 0 \qquad (8)$$

which is always satisfied since $(s_r - s_w) > 0$, $f'' < 0$.

We need stronger assumptions to prove global asymptotic stability (GAS). Neoclassical production functions assume that $f(k)$ is concave but makes no assumptions regarding the curvature of $f'(k)$, we only know that its slope $f''(k) < 0$. Assume that f' is also a concave function. Let:

$$\Omega = k - k^*$$

Form the Liapunov function

$$V = \tfrac{1}{2}\Omega^2$$

Then

$$V \in R_1^+$$

$$V > 0$$

$$V = 0 \text{ iff } \Omega = 0$$

Finally,

$$\dot{V} = \Omega\dot{\Omega} = \Omega\dot{k}$$

$$= \Omega[s_w f(k) + (s_r - s_w) f'(k)k - nk]$$

$$= \Omega[s_w f(k^* + \Omega) + (s_r - s_w) f'(k^* + \Omega) \cdot (k^* + \Omega) - n(k^* + \Omega)]$$

$$\leq \Omega[s_w f(k^*) + s_w \Omega f'(k^*) + (s_r - s_w) \cdot (k^* + \Omega)\{f'(k^*) + f''(k^*)\Omega\} - nk^* - n\Omega]$$

$$= \Omega[s_w f(k^*) + (s_r - s_w) f'(k^*)k^* - nk^* + s_w \Omega f'(k^*) + (s_r - s_w)\Omega f'(k^*) + (s_r - s_w) k f''(k^*)\Omega - n\Omega]$$

$$= \Omega^2[s_w f'(k^*) + (s_r - s_w) f'(k^*) + (s_r - s_w) f''(k^*)k - n]$$

$$= \Omega^2[s_r f'(k^*) - n + (s_r - s_w) f''(k^*)k]$$

$$< 0 \qquad (9)$$

The last expression is negative for the following reasons:

(i) $\Omega^2 > 0 \; \forall \; \Omega \neq 0$

(ii) $(s_r - s_w) f''(k^*)k < 0$ for $f'' < 0$, $k > 0$

(iii) $s_r f'(k^*) - n < 0$

This is because $f' = r$, thus $s_r rK$ is the savings from profit income alone and must be less than total saving which in steady state is $S = \dot{K} = nK$.

$$\therefore \quad s_r rK < nK$$

$$\rightarrow \quad s_r f' < n$$

We have proved that $\dot{V} < 0$. Thus by a standard theorem of Liapunov, the system is globally asymptotically stable.

Section 3: Classical Unemployment

Unemployment in the short run model takes place when the equilibrium level of labour employed (L_1) is less than the given labour force, i.e. $L_1 < L_0$. Unemployment is termed "classical" when it is caused by the real wage rate being higher than the full employment level w_0. Once again, in temporary equilibrium, investment is equal to savings ex post, but the labour market does not clear.

In Figure 5.2, the initial full employment equilibrium is denoted by Q_f. Suppose the wage rate rises to w_1. At the given capital labour ratio k_0, distributional parameters change and the rate of interest becomes r_2. But point $(w_1 r_2)$ is off the fpf and producers are not maximising profits. They will thus reduce k and move to Q_c. The equilibrium wage rental values will be $(w_1 r_1)$. The new capital labour ratio k_1 is given by slope of $A_1 B_1$ where $k_1 > k_0$. This new $k_1 = K_0/L_1$ and thus $L_1 < L_0$. Unemployment of the order of $(L_0 - L_1)$ appears in the model.

Investment, in the classical unemployment model, has no important role to play. Either, we assume that investment is "accommodating" or that some sort of Says' Law operates. Therefore, given (w,r,k) saving per capita is determined and this in turn determines investment. In figure 5.2, investment per capita i remains the same at both Q_c and Q_f, but this is just a coincidence, it need not be the same. The major point is that both equations (1) and (2) are satisfied, and there is unemployment.

We now intend to analyse the long run dynamics of the system. From the factor price function or the unit cost function, given w we find r. Thus

$$c(r,w) + r = g(w) \qquad (10)$$

The function g is decreasing and convex; $g_1 = (-c_w/c_r)$ $= (- L/K) = - \phi$ Further the labour capital ratio is given by

$$L/K = \Phi(w/r) \qquad (11)$$

where Φ is decreasing and its elasticity is $(-\sigma)$. Note that L is the cost minimising optimum level of output that the firm wishes to employ, given capital stock.

The wage adjustment equation depends on labour demand (L) and labour supply (N). Thus

$$\dot{w} = \alpha \ (L/N) \qquad (12)$$

where α () is a function, $\alpha(1) = 0$, $\alpha_1 > 0$. L is determined by cost minimisation via (10) and (11), N is given by the supply of labour. Actual employment is the min of labour demand (L) and supply (N). $\dot{w} = 0$ means full employment.

From (12), (11)

$$\dot{w} = \alpha[K\Phi(w/g)/N]$$

$$= \alpha[k\Phi(w/g)] \qquad (13)$$

where k is capital per head of labour force. The $\dot{w} = 0$ or w stationary is given by

$$\alpha(k\Phi(w/g)) = 0$$

Taking total differentials,

$$\alpha_1[\Phi dk + kd\Phi] = 0$$

$$\Phi k\alpha_1[dk/k + d\Phi/\Phi] = 0$$

or

$$\hat{k} + d\Phi/\Phi = 0$$

(where

$\hat{k} = dk/k$, etc.)

$$\hat{k} + [d\Phi/d(w/g)].\ [d(w/g)/\Phi]\ [(w/g)/(w/g)] = 0$$

$$\hat{k} - \sigma[d(w/g)/w/g] = 0$$

$$\hat{k} - \sigma[dw/w - dg/g] = 0$$

$$\hat{k} - \sigma[\hat{w} - g_1 dw/g] = 0$$

$$\hat{k} - \sigma[\hat{w} - (wg_1 dw/g \; w)] = 0$$

$$\hat{k} - \sigma[\hat{w} + (w\Phi/g)\hat{w}] = 0$$

$$\hat{k} - \sigma[\hat{w} + (wL/rK)\hat{w}] = 0$$

$$\hat{k} - \sigma[\hat{w} + (\Theta/1-\Theta)\hat{w}] = 0$$

$$\hat{k} - \sigma[\hat{w}/1-\Theta] = 0$$

$$\hat{k} = \sigma/(1-\Theta)\hat{w}$$

(where Θ is labour's share in income, $\Theta = wL/Y$, $(1-\Theta) = rK/Y$)

Thus the slope of $\dot{w}=0$ is given by

$$dk/dw = (\sigma/1-\Theta)\ (k/w) \qquad (14)$$

Note for phase diagram,

$$\partial\dot{w}/\partial k = \alpha_1\ \Phi\ > 0 \qquad (15)$$

All this information is depicted in Figure 5.3

Consider the region where there is unemployment such that L < N. Thus employment is L. Then investment

$$I = \dot{K} = S = (s_w\ wL + s_r\ rK) - \delta K$$

$$= K\left[s_w\ w\Phi(w/g(w))) + s_r\ g(w)\right] - \delta K$$

(where δ is the rate of depreciation)

$$\dot{K}/K = s_w\ w\Phi(w/g)\ + s_r g - \delta$$

$$\dot{k}/k = \dot{K}/K - \dot{N}/N = s_w\ w\Phi(w/g)\ + s_r g - \delta - n \qquad (16)$$

Thus when $(\dot{k}/k)$:0 and $s_w w\Phi + s_r g - \delta - n = 0$, then we get

$$w = w^* \qquad (17)$$

We also note, from (16) that

$$\partial(\dot{k}/k)/\partial w = (s_w\Phi + s_r g' + s_w w\Phi')\ \left[\partial(w/g(w))/\partial w\right] =$$

$$= -(s_r - s_w)\Phi + s_w w\Phi' \left[\partial(w/g(w))/\partial w\right] < 0 \qquad (18)$$

Thus will help us to draw the phase line for the unemployment region.

Consider now the region of over full employment where labour demand is greater than supply. Thus L>N and employment must be N. The cost functions are not appropriate and we get

$$\dot{K} = s_w wN + s_r\left[F(K, N) - wN\right] - \delta K$$

$$\dot{K}/K = s_w w(N/K) + s_r\left[F(K,N)/K - w(N/K)\right] - \delta$$

$$= s_w w/k + s_r\left[f(k)/k - e/k\right] - \delta$$

$$\dot{k}/k = s_w w/k + s_r\left[f(k)/k - w/k\right] - \delta - n$$

$$\dot{k} = s_w w + s_r\left[f(k) - w\right] - (\delta+n)k \qquad (19)$$

We also note that the locus of $\dot{k}=0$, can be calculated by

$$(s_w - s_r)dw + \left[s_r f' - (\delta+n)\right]dk = 0 \qquad (20)$$

$$dw/dk = [(\delta+n) - s_r f'] / (s_w - s_r) \quad (21)$$

Assuming, for stability, that $(\delta+n) > s_r f'$

$$dw/dk \Big|_{\dot{k}=0} < 0 \qquad (22)$$

$$\text{Also } \partial\dot{k}/\partial k = s_r f' - (\delta+n) < 0 \qquad (23)$$

Equation (17), (18), (20) to (23) give the phase diagrams as depicted in figure 5.4.

We observe that when unemployment is classical, there is a strong possibility of real bus-iness cycles with capital labour ratio and wage rate fluctuating around steady state.

Section 4: Keynesian Unemployment

A model of classical unemployment emphasises the value of the wage rate as the main determinant of the model. In a static framework, the real wage rate is "too high" and this causes unemployment as producers reduce labour in an attempt to maximise profits. In a dynamic framework, wage rate adjustments take place to clear the market for labour and this has consequential effects on labour demand growth and capital labour ratio.

On the other hand, Keynesian unemployment comes about through a lack of effective demand, portrayed in this model by a change in investment. Thus, unlike full employment and classical unemployment, investment is not "accommodating". Investment behaviour must be postulated first, and it is the lack of this investment which starts off the chain of unemployment. The other important point to note is that wage rate now changes to clear the commodity market. A short run temporary equilibrium is reached through wages adjusting to bring the exogenously given (and non-accommodating) investment and the endogenously determined savings in line. This will be clear with the following analysis.

Suppose the economy is at full employment equilibrium in the short run model. Given the values of $K = K_0$ and $L = L_0$, we have determined levels of all variables. Now consider investment I to be determined by "animal spirits" of entrepreneurs and it falls below the original level. Thus current investment $I_1 < I_0$.

In Figure 5.5 Q_t once again represents full employment equilibrium AB is given by income distribution and value of $k = k_0 = \frac{K_0}{L_0}$ and CD gives the investment line with slope $\frac{-k_0 s_r}{s_w}$.

$$OC = \frac{i_0}{s_w} = \frac{I_0}{L_0 s_w} \quad \text{and} \quad OD = \frac{i_0}{k_0 s_r} = \frac{I_0}{K_0 s_r}.$$

Now suppose given k, aggregate investment demand falls to I_1 and we have a new line represented by C_1D_1 (parallel shift of CD leftwards). For the given k_0, consumption per head must rise and savings fall to equilibrate the commodity market (since investment is now less then the previous level of savings). Wage rate w will rise and r fall to increase consumption per head (given $s_r > s_w$). This change in income distribution consistent with equation (1) will move the economy to $(w_1 r_2)$, the intersection of AB and C_1D_1. But the distributional variables $(w_1 r_2)$ are off the factor price frontier and profits are not maximised. Capital labour ratio k will rise and this is done through a reduction in employment. A new equilibrium is reached at Q_k, with the investment saving line shifting to C_2D_1. Note that $OC_1 = \frac{I_1}{L_0 s_w} < OC_2 = \frac{I_1}{L_1 s_w}$ (new employment $L_1 <$ full employment level L_0).

Keynes assigned a similar role to distributional parameters and savings propensities in his analysis of the "inflationary gap" in his book "How to Pay for the War". In his model, the opposite sequence is studied when an excess of aggregate demand over supply (investment greater than saving ex ante) causes an inflationary gap. Distributional changes take place such that real wage falls, rental rate rises and extra saving is forthcoming to match up to surplus investment demand. As is clear the process in our model is just the opposite and we have a deflationary gap with consequential unemployment.

The dynamics of the temporary equilibrium analysed until now can be represented by the following differential equations:

$$\dot{w} = \alpha[(s_w - s_r)w + s_r y - i] \qquad \alpha > 0 \qquad (24)$$

$$\dot{k} = \beta[w - f(k) + kf'(k)] \qquad \beta > 0 \qquad (25)$$

Equation (24) gives the change in wage rate to clear the commodity market. This is the essence of Keynesian unemployment and is quite different from the classical one where wage rate adjusts to clear the labour market (see, Solow and Stiglitz (1968)). Equation (25) is derived from standard marginal productivity assumptions. Note once again that $k = k/L^d$ is capital employment ratio and not capital labour ratio even though the same notation is used. Note also, $y = Y/L^d$, $i = I/L^d$.

To analyse stability, form the following matrix evaluated at $\dot{w} = 0 = \dot{k}$

$$A = \begin{bmatrix} \frac{\partial \dot{w}}{\partial w} & \frac{\partial \dot{w}}{\partial k} \\ \frac{\partial \dot{k}}{\partial w} & \frac{\partial \dot{k}}{\partial k} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha(s_w - s_r) & \alpha s_r f' \\ \beta & \beta k f'' \end{bmatrix}$$

We need:

$$\text{trace } A = \alpha(s_w - s_r) + \beta k f'' < 0 \qquad (26)$$

$$\det A = \alpha(s_w - s_r)\beta k f'' - \alpha\beta s_r f' > 0 \qquad (27)$$

trace A is always negative, but det A may not be positive. Local stability condition becomes

$$\alpha(s_w - s_r)\beta k f'' - \alpha\beta s_r f' > 0 \qquad (28)$$

Phase diagrams give a clearer picture of movements of variables out of equilibrium. Noting

$$\left.\frac{dw}{dk}\right|_{\dot{w}=0} = -\frac{\frac{\partial \dot{w}}{\partial k}}{\frac{\partial \dot{w}}{\partial w}} = \frac{-s_r f'}{(s_w - s_r)} > 0$$

and

$$\left.\frac{dw}{dk}\right|_{\dot{k}=0} = -\frac{\frac{\partial \dot{k}}{\partial k}}{\frac{\partial \dot{k}}{\partial w}} = -kf'' > 0$$

and also $\frac{\partial \dot{w}}{\partial w} = \alpha(s_w - s_r) < 0$, $\frac{\partial \dot{k}}{\partial k} = \beta k f'' < 0$ we get

Fig. 5.6

$$\left.\frac{dw}{dk}\right|_{\dot{w}=0} > \left.\frac{dw}{dk}\right|_{\dot{k}=0}$$

Fig. 5.7

$$\left.\frac{dw}{dk}\right|_{\dot{w}=0} < \left.\frac{dw}{dk}\right|_{\dot{k}=0}$$

Figure 5.7 denotes the more stable case and follows from inequality (28). But as Figure 5.6 shows there is a distinct possibility of saddle point equilibrium in the short run case. In the absence of planning, a competitive Keynesian unemployment economy might not even reach temporary equilibrium in the short run.

We can extend the analysis of short run temporary equilibrium in the Keynesian unemployment case a bit further. Up till now, the focus has been on unemployment of labour alone; we have assumed that capital stock is fully utilised. But to be true to the spirit of Keynesianism, one must also consider the case of underutilisation of capital stock too. Keynesian unemployment should be characterised by unemployed labour and unutilised capital stock.

This can be incorporated by noting that changes in capital labour ratio can be brought about by changes in both capital and labour employed. Thus a rise in capital labour ratio can be caused by a fall in both labour and capital, with the proviso that the fall in labour employed is proportionately higher than the fall in capital used. This will be shown in Figure 5.8.

Once again, as in Figure 5.5, Q_f is full employment equilibrium point, with investment saving line given by CD, $OC = \frac{I_0}{L_0 s_w}$, $OD = \frac{I_0}{K_0 s_r}$, (it represents equation (2)).

A fall in investment to $I_1 < I_0$ shifts this line to C_1D_1. To restore effective demand, consumption must increase which leads to an increase in wage, and a fall in rate of profit given the capital labour ratio. This calls for an increase in capital/labour from its initial value $k_0 = \frac{K_0}{L_0}$ to be consistent with firms' optimisation.

Now suppose producers in addition to laying off workers also close down plants and use less capital stock. As mentioned earlier, this may still be consistent with an increase in capital labour ratio.

Starting from an initial saving investment equilibrium, a reduction of $K = K_1 < K_0$, will increase investment per unit of capital stock,

i.e. $\frac{i}{k} = \frac{I}{K} = \frac{I_1}{K_1} > \frac{I_1}{K_0}$. Faced with higher investment/capital ratio, saving/capital ratio should rise to preserve equilibrium. This calls for a rise in the rate of profit r. If this increase in r required to equilibrate the commodity market (note there is a corresponding fall in w) is high enough, it may exceed the marginal product of capital. Then the capital labour ratio k will have to <u>fall</u> to accommodate this very high rental rate r. We may finally end up at the point Q_k (Figure 5.8), the temporary equilibrium for Keynesian unemployment. It is characterised by lower investment, lower employment, lower capital utilised, lower wage rate and lower capital ratio, as compared to Q_f. As a representation of "stylised facts" this position seems quite reasonable.

The following equations summarise the analysis and are self-explanatory. Given

$$\frac{i}{k} = s_w \frac{w}{k} + s_r r \qquad \text{(from (2))}$$

$$\frac{i}{K} = \frac{s_w f}{k} + (s_r - s_w) r \qquad \text{(from (1))}$$

we have

$$\dot{r} = \alpha\left[\frac{i}{k} - \frac{s_w f}{k} - (s_r - s_w) r\right] \qquad (29)$$

and

$$\dot{k} = \beta\left[f'(k) - r\right] \qquad (30)$$

Equation (29) tells us how 'r' changes to equilibrate between investment and savings ex ante. Equation (30) states that capital labour ratio increases (falls) as marginal product of capital f' is greater (less) than a given r.

To analyse stability form the matrix of partial derivatives

$$A = \begin{bmatrix} -\alpha(s_r - s_w) & -\alpha\left[\dfrac{i}{\overline{K}^2} + \dfrac{s_w k f' - s_w f}{k^2}\right] \\ -\beta & \beta f'' \end{bmatrix}$$

We need

$$\text{trace } A = -\alpha(s_r - s_w) + \beta f'' < 0$$

$$\det A = (s_w - s_r)k^2 f'' - i + s_w(f - kf')$$

In the neighbourhood of equilibrium $f - kf' = w$, thus $-i + s_w w < 0$ (sum of last two terms in expression for det A). But $(s_w - s_r)k^2 f'' > 0$ and therefore sign is ambiguous.

The phase diagrams reflect the conditions needed for det A to be > 0 or < 0. We get Figure 5.9.

The first phase diagram in Fig. 5.9 is the stabler case and it can be generated if inequality det A > 0 holds. Otherwise we have saddle point equilibrium once again and in a competitive economy it will be difficult to achieve equilibrium.

To sum up thus far, even without any dynamic movements in a growth context, the basic temporary equilibrium model of Keynesian unemployment runs into problems. Independent of steady state analysis, even the short run equilibrium may not be reached, consequent to displacement. Thus Keynesian unemployment seems to create more problems than the other types of models.

The models discussed thus far are essentially static since the level of investment (per capita) is given; labour supply is constant; and finally, the adjustment of the capital labour ratio does not take into account the dynamics imparted by investment and labour growth. We now discuss the "proper" growth model emphasising the aforementioned adjustments. We choose the deal with the first model using wage adjustments; the second gives essentially similiar results.

Assume, for simplicity, that there is always unemployment. Thus we consider only the case where L<N and actual employment is equal to labour demand. All per capita variables are now defined in terms of labour supply. For example the capital labour ratio is $k = K/N$, the capital stock per head of total labour available. Once again $\dot{N}/N=n$

Desired saving is given by

$$S = s_w wL + s_r[F(K,L) - wL]$$

$$S/N = s_w w(L/N) + s_r[F(K,L)/N) - w(L/N)]$$

$$= (s_w - s_r)w(L/K)/(N/K) + s_r F(k, (L/K)/(N/K))$$

$$= (s_w - s_r)w\Phi k + s_r F(k, \Phi k) \qquad (31)$$

where we have used the fact that $(L/K) = \Phi(w/g(w))$ as in section 3.

The essence of the Keynesian model is the investment function. We assume that investment per capita, $i=I/N$, depends on capital stock per head (k) and the wage rate (w). Thus desired investment

$$i = i(k,w) \qquad (32)$$

It is expected that $i_w<0$; thus increase in the real wage, corresponding to a fall in the rate of profit r, would lower investment. We reserve judgement on the sign of i_k since a high capital stock may be an incentive to invest more; alternatively a diminishing marginal efficiency may lower investment. Therefore $i_k > 0$ (see Blinder and Solow (1973)).

Wage adjustment follows the equilibrium assumptions made earlier for the Keynesian static case. Wage rate rises (falls) when desired saving exceeds (is less than) desired investment. Firms wish to boost aggregate demand (consumption) which can be done here through an increase in wage rate since the consumption propensity of labour is higher than that of the capitalists. Thus using (31) and (32)

$$\dot{w} = \alpha[s_w - s_r)w\,\Phi k + s_r F(k,k\Phi) - i(k,w)] \qquad (33)$$

Noting, from the previous section, the expression

$$\hat{\Phi} = d\Phi/\Phi = (-\sigma/1-\Theta)dw/w$$

hence

$$d\Phi/dw = -(\sigma/1-\Theta)(\phi/w)$$

We get

$$\partial\dot{w}/\partial w = (s_w - s_r)[\Phi k - k\phi(\sigma/1-\theta)]$$

$$+ s_r F_2 k(-(\sigma/1-\Theta)(\Phi/w) - i_w$$

$$= (s_w - s_r)(\Phi k)(1 - \sigma/1-\Theta) - s_r F_2 k(\sigma/1-\Theta)(\Phi/w) - i_w$$

$$= (s_w - s_r)(\Phi k)(1-\Theta-\sigma/1-\Theta) - s_r F_2 k(\sigma/1-\Theta)(\Phi/w) - i_w$$

For Cobb-Douglas function, $\sigma = 1$ →

$$\partial\dot{w}/\partial w = -\Phi k(1/1-\Theta)[(s_w - s_r)\Theta + s_r F_2/w] - i_w$$

$$= -\Phi k(1/1-\Theta)\,[s_w\Theta + s_r(F_2/w-\Theta)] - i_w$$

$F_2 > w$ in Keynesian employment, $(F_2/w - \Theta) > 0$; $\Theta < 1$

$\partial\dot{w}/\partial w = < 0$; provided i_w is small, as expected.

Further,

$$\partial\dot{w}/\partial k = (s_w - s_r)w\phi + s_r F_1 + s_r F_2\phi - i_k$$

$$= s_w w\phi + s_r F_1 + s_r \phi(F_2 - w) - i_k > o$$

$$dw/dk = -(\partial\dot{w}/\partial k)/(\partial\dot{w}/\partial w) \quad > 0$$

The stationary $\dot{w}=0$, divides the (w,k) phase into two segments depending on whether S<I or I<S. We now need to determine the behaviour of capital stock. Again assume that the actual change in the capital stock is either desired investment or saving depending on which constraint is binding.

For the case S<I, we have

$$\dot{K} = S = s_w wL + s_r\left[F(K,L) - wL\right]$$

$$\dot{K}/K = s_w w(L/K) + s_r\left[F(K,L)/K) - (wL/K)\right]$$

$$= (s_w - s_r)w\phi + s_r F(1, \phi)$$

Thus,

$$\dot{k}/k = \dot{K}/K - \dot{N}/N$$

$$= (s_w - s_r)w\phi + s_r F(1,\phi) - n$$

when $\dot{k}=0$, this equation defines an unique real wage rate $w = w^*$. Further,

$$(\partial(\dot{k}/k/\partial w) = (s_w - s_r)\phi + s_w w(\partial\phi/\partial w) + s_r(F_2 - w)\partial\phi/\partial w$$

Noting, $\partial\phi/\partial w < 0$, $(F_2-w) > 0$, this expression is negative. The information helps us to draw the phase line for $(\dot{k}/k)=0$ in the region $\dot{K}=S$.

We now come to the case where the investment constraint is binding so that

$$\dot{K} = I$$

$$(\dot{k}/k) = (I/N)/(K/N) - n$$

$$= i/k - n$$

$$\dot{k} = i(k,w) - nk$$

$$\partial\dot{k}/\partial w = i_w < 0$$

$$\partial\dot{k}/\partial k = (i_k - n) \lesseqgtr 0$$

depending on the sign and magnitude of i_k.

Consider the case of 'passive' investor, where $i_k<0$. Thus $(\partial\dot{k}/\partial k)<0$. Then

$$dw/dk \Big|_{\dot{k}=0,\ \dot{K}=I} < 0$$

The phase diagram is essentially similar to the classical case (see figure 5.10).

The more interesting Keynesian model is that of active investors whose reaction to higher k is to increase investment significantly. Thus with $\dot{K}=I$, we may get $(\partial\dot{k}/\partial k) >0$. and

$$dw/dk\Big|_{\substack{\dot{k}=0\\ \dot{K}=I}} > 0$$

The phase diagram is rather complicated and relatively unstable. as shown in Figure 5.11.

Consider initial capital stock (per head of labour force), $k_0<k^*$. Then low initial wage is stabilising as $\dot{w}'_0$ path demonstrates. On the other hand, a high wage, at $w^2{}_0$, may lead to oscillatory wage fluctuations away from equilibrium. Within the region $\dot{K}=I<S$, the more realistic Keynesian case, wages either approach the steady state if starting from a low value; alternatively they oscillate and we have wages rising and falling as it crosses the boundary from $\dot{K}=S$.

Section 5: Conclusion

This chapter constitutes an early part of my research programme. In order to put the results in perspective, in this section, I am going to analyse a major paper by Ito (1980) and show how one of my models, in the chapter, relates to his analysis of disequilibrium growth theory. Since the paper has important links with the contents of this particular chapter, I propose to analyse it here in some detail and make a comparative evaluation. There are also some links with Chapter III which will be mentioned in due course.

Ito's model is "classical" in the sense that the two major characteristics of the classical unemployment regime (see the discussion in Section 3) are emphasised: desired saving is always realised so that no explicit role is given to the investment function; wage adjustment takes place according to whether there is excess supply or demand in the labour market. As regards the first feature, the possibility, of an independent investment function which can be binding, is ignored. There is no clear-cut discussion of why this is so. The implicit assumption being, as in a standard neo-classical growth model, that investment is accommodating; whatever is saved must be equal to capital formation. Regarding wage adjustment, the assumption is that real wage falls (rises) if there is unemployment (overemployment). Outside of full employment, the model therefore encompasses two regimes: unemployment and overemployment. Actual employment is the minimum of labour demand and supply. An interesting, though not essential, assumption is that the speed of wage adjustment is asymmetric such that it is different for the two regimes. A variant of the wage dynamics is to postulate that wage changes over time according to capital accumulation, in addition to the presence of labour market disequilibrium. Thus, even when there is full employment (labour supply equals demand), the real wage rate may change with capital stock. If the marginal productivity theory holds then the full employment case becomes identical with the standard one sector neoclassical (Solow) growth model. We will return to this point later.

The Ito model is therefore a perfect amalgamation of neoclassical growth theory with the basic ingredients of short run, quantity constrained, macroeconomic disequilibrium analysis. Rationing takes place on the short side of the market if desired demand and supply are not equal. Since the goods market is always in equilibrium (saving must equal investment, aggregate supply equal demand), the only quantity constraints are in the labour market which "clears" through the "min" condition. This in turn has a spill-over effect and effective or actual saving (investment) depends on wage income actually earned in employment.

Let us first briefly discuss the basic Ito model. Employment (L) is the minimum of labour demand (L^d) and supply (L^s). Thus

$$L = \min[L^d, L^s] \tag{34}$$

For a given real wage rate (w), the desired capital labour ratio is $k^d = K/L^d$, which is given by the marginal productivity relation:

$$w = f(k^d) - k^d f'(k^d) \tag{35}$$

Using standard notation, $y = f(.)$ is the output per capita. When employment equals labour demand, then equation (35) determines the capital labour ratio for a given w. This is the case of unemployment where $L = L^d < L^s$. Labour supply L^s is equal to the population, N. Thus when there is overemployment, $L = L^s < L^d$, the capital labour ratio is denoted by $k = K/N$.

As regards capital formation, the Ito model assumes, as we have done in Section 3, that in a two class economy savings (equal to investment) is given by

$$S = \dot{K} = s_w wL + s_r (Y - wL) \qquad (36)$$

Therefore

$$(\dot{k}/k) = (\dot{K}/K) - (\dot{N}/N)$$

$$= s_r(Y/K) + (s_w - s_r)w(L/K) - n \qquad (37)$$

Remember that n is the natural rate of growth; thus $n = \dot{N}/N$.

The two state variables of the model are w and k. Their dynamic behaviour gives us the long run growth story as well as the stability of the system. The dynamic equation for k is already given by (37). We therefore need to discuss the time path of wages.

As mentioned earlier, two possible wage adjustments are analysed in the Ito paper. We have either of the following:

$$\dot{w} = a[(L^d - L^s)/L^s] = a[(L^d/L^s) - 1] \qquad (38)$$

$$\dot{w} = -f''kk + a[(L^d/L^s) - 1] \qquad (39)$$

In equation (39), if there is full employment ($L^d = L^s$) then wages can rise with time when the capital labour ratio increases. This is the productivity factor emphasised in the full employment version of the neoclassical growth model. Thus, when $L^d = L^s$ and equation (35) holds we get the standard equilibrium Solow model as a special case of Ito's disequilibrium theory. Even though this is an useful formulation, the essential formal propeties of the model are very simillar independent of the use of (38) or (39), In what follows we will confine our attention to the dynamics given by equation (38).

The full employment part of the model is standard and needs no further discussion. Greater interest focuses on the two regimes where $L^d \neq L^s$.

Consider first the case of unemployment i.e. $L = L^d$. The firm is unrationed and thus can optimise; hence, marginal productivity given by (35) holds. Inverting this function we have

$$k^d = z(w) \qquad z' > 0 \tag{40}$$

Substituting (35) in (37), noting $L = L^d$ and $k^d = K/L^d$, we get

$$(\dot{k/k}) = [s_w \; f(k^d)/k^d] + [s_r - s_w]f'(k^d) - n \tag{41}$$

This gives us the time path of k. We note that, from (40), it depends wholly on the behaviour of w. Turning now to equation (38) we get

$$\dot{w} = a[(k/k^d) - 1]$$
$$\dot{w} = a[(k/z(w)) - 1] \tag{42}$$

Equation (41) and (42) define the dynamics of the unemployment regime.

For the overemployment regime, we have $L = L^s = N$. Using equation (37) and noting that $L = N$ and $k = K/N$, the dynamic equation for k is given by

$$\dot{k} = s_r f(k) + (s_w - s_r)w - nk \qquad (43)$$

In addition, wage dynamics is still being motivated by equation (41). Equations (43) and (42) define the time paths in the overemployment regime.

Ito makes two alternative assumptions for the relative values of the saving propensities: $s_w > s_r$ and $s_r > s_w$. The latter is the standard Kaldor assumption and we maintain it here; my own models in this chapter also make this assumption; it is quite easy to work out the dynamic properties of the first alternative. The analysis in Chapter III, and Ito (1978), use $s_w > s_r$ since it is a natural corollary of the overlapping generation model.

The phase diagrams of the composite system (both unemployment and overemployment) are given in Figures 1 and 2 of the Ito paper. The first diagram shows the case of $s_w > s_r$, an assumption culled from the overlapping generation model. The next case, used throughout Chapter V, is where $s_r > s_w$. The phase diagram now is exactly the same as my Figure 5.4 in the thesis. But this is not surprising since my classical unemployment regime is essentially similiar to a neoclassical orientated disequilibrium growth model (of the Ito type). The two fundamental assumptions that both share are: (a) wage adjusts when there is excess demand (supply) in the labour market and (b) capital accumulation is given by saving with no independent role for the investment function.

It is clear that my analysis of classical unemployment in Section 3 mirrors that of Ito (1980). This is not surprising since the basic features of the classical regime are also culled from neoclassical growth models and disequilibrium macroeconomics. The former implies that savings are always realised so that capital formation must be equal to desired saving. The latter emphasises the role of disequilibrium in the labour market and consequent spill-over via income on to aggregate consumption (demand) and hence saving and investment.

Inspite of the similarities, it is necessary to point out two relatively substantive issues whereby the model of section 3 differs from Ito (1980). Firstly, the Ito model analyses all price dynamics in terms of changes in the wage rate. We emphasise, on the other hand, both wage and the (real) interest rate, r (or the rental rate on capital stock). Given the factor price frontier (unit cost function), r is a function of w. Thus when the firm is unrationed and on the fpf (in the case of unemployment of labour) the dynamics of r mirrors that of w. Working with one factor price is sufficient to predict the behaviour of the other factor price. However in the case of over full employment the firm is rationed. Thus the economy is not on the (optimum) factor price frontier. Then

$$rK = F(K, N) - wN$$

$$r = F(1, 1/k) - [w/k]$$

$$(k = K/N)$$

Now it is important to analyse the behaviour of capital stock also to predict how r will change over time. The models in this chapter emphasise income distributional parameters precisely because these variables become crucial under disequilibrium and one can get asymmetric results depending on the nature of excess supply or demand in various markets.

The second difference in emphasis, within the (neo) classical framework), is that our model, in this chapter, can explain how the unemployed could receive a part of the national product without violating the basic macroeconomic identities of the system. I have shown in the Appendix that it is possible to evolve a tax subsidy mechanism, whereby we have the unemployed labour force getting a share of aggregate output, and yet maintaining the basic structure of the growth process. This is important because we do not need to worry about how the unemployed would survive in such a system. Ito's model does not have such a mechanism.

Global stability has been explicitly analysed in the Ito paper by taking the whole system together, with discontinuities at the switching points and at the border of adjacent regimes. In that way, by using the Filippov solutions, his analysis becomes more precise and allows him to get analytic solutions; whereas this chapter has relied on phase diagrams, for specific regimes, to demonstrate the general tendency towards long run equilibrium.

One further point should be noted. The analysis of Chapter III, similar to that of Ito (1978), was constructed with the aid of an overlapping generation model. This implies that the younger generation earn wages and have a high saving propensity. The older generation earn profit (dividend) income and have a high propensity to consume. Thus $s_r < s_w$. The Kaldorian two class assumption is, of course, just the opposite i.e. $s_r > s_w$. As pointed out earlier, we have used in Chapter V the latter assumption and derived our results. It would be a simple extension to accomodate the other case; but nothing substantive depends on this.

Uptil now the discussion has concentrated on what can be termed the classical regime. But I also have a model of Keynesian unemployment where capital accumulation is given by investment (rather than saving) and wage responds to disequilibrium in the commodity market (when ex ante investment and saving are not equal). These issues are not tackled in the Ito model since he is concerned with only the classical framework. Conceptually therefore we move away considerably from standard neoclassical (disequilibrium) growth theory by analysing Keynesian unemployment regimes. This is the main contribution of the chapter. Though the vehicle of analysis is the basic Solow growth model, the issues discussed, the adjustment machanism postulated and the nature of rationing (inadequate aggregate demand with investment being less than saving) are all fundamentally different.

It is instructive to quote the concluding remarks of Ito (1980) describing his model: "The model is philosophically classical (or anti-Keynesian) in the sense that the supply side determines output. The disequilibrium labour market determines the actual employment at the minimum of demand and supply. This in turn determines actual output, which is exactly absorbed either as consumption or capital accumulation. This story is clearly what Keynes and his "faithful" followers attack. This may seem paradoxical because disequilibrium macroeconomic models are praised mainly because they claim to be the restoration of "Keynes's economics" as opposed to "Keynesian economics". The present disequilibrium growth model is a hybrid of a neoclassical growth theory and a disequilibrium macroeconomic model, and the anti-Keynesian nature is inherited from neoclassical growth theory". However, my analysis presented in section 4 shows how to combine short run macroeconomic features of effective demand theory with the postulates of long run growth theory to produce a "Keynesian" disequilibrium growth model.

Finally, a few brief words summarising the general findings of this chapter. The full employment model of growth integrates income distributional parameters with a standard neoclassical framework. As expected, under fairly general conditions, both local and global stability can be proved. Thus its dynamic characteristics are similar to usual one sector growth models.

On the other hand models of unemployment have distinctly different characteristics depending on whether they are classical or Keynesian in nature. This accords with the view in the static literature (Malinvaud (1977)). The adjustment processes and consequently the conclusions are essentially different.

In a world of classical unemployment wage rate is higher than market clearing equilibrium and this <u>causes</u> excess supply of labour. The dynamic model therefore postulates that wage adjustments take place to clear the labour market. Stability conditions crucially depend on adjustment of price variables and rapid changes usually stabilise the system. As Dixit (1976) says, "In all these models .... rapid adjustment of prices in response to disequilibria is stabilising".

Keynesian unemployment is radically different. Even within the short run framework of temporary equilibrium, stability is not easily achievable. Rather stringent conditions are needed to get equilibrium in the short term. Given that temporary equilibrium is attained, the stability conditions of the growth model reveals interesting differences from classical unemployment depending on investor behaviour.

Given the structural differences, we believe that different types of unemploment have to be modelled separately. Both theoretical results and policy conclusions will be radically different depending on the regime the economy is on. When there is regime switching from one form of unemployment to another (from classical to Keynesian for example) a general disequilibrium theory becomes necessary. This must be a subject of future research.

APPENDIX

It may seem necessary to adjust our basic definitional equation (1) (2), determining distribution of income and saving investment equality (ex post), to take account of unemployment, when labour employed is less than labour force. But there is no need to change equations (1) (2) and we can readily demonstrate this.

Suppose output is distributed among wage earners (employees) and rental earners. Thus

$$Y = wL^d + rK = f(K,L^d) \tag{44}$$

$$y = \frac{Y}{L^d} = w + r\frac{K}{L^d} = f(k) \tag{1'}$$

where $k = \frac{K}{L^d}$

Let there be a lump sum tax on wage income and profit income denoted by T_w and T_r. Suppose a fixed social security (unemployment) benefit is paid out by the Government to unemployed workers. Let us call it C. We assume that this amount C is totally consumed. Then equilibrium in national accounts implies

$$Y = c_w(wL^d-T_w) + c_r(rK-T_r) + C + I + G \tag{45}$$

where $(c_w c_r)$ are the consumption propensities and (wL^d-T_w) and $(rK-T_r)$ are the disposable incomes of the two groups.

Equating (44) and (45) we get

$$I + G + C - c_wT_w - c_rT_r = s_wwL^d + s_rrK$$

where $(1-c_w) = s_w$ and $(1-c_r) = s_r$.

If now government expenditure is fixed such that

$$G = c_wR_w + c_rT_c - C$$

then we have

$$I = s_w wL + s_r rK$$

$$\frac{I}{L^d} = s_w w + s_r r \frac{K}{L^d} \tag{46}$$

$$i = s_w w + s_r rk \tag{2'}$$

for both unemployment models. Equations (1') (2') are exactly the same as (1) and (2).

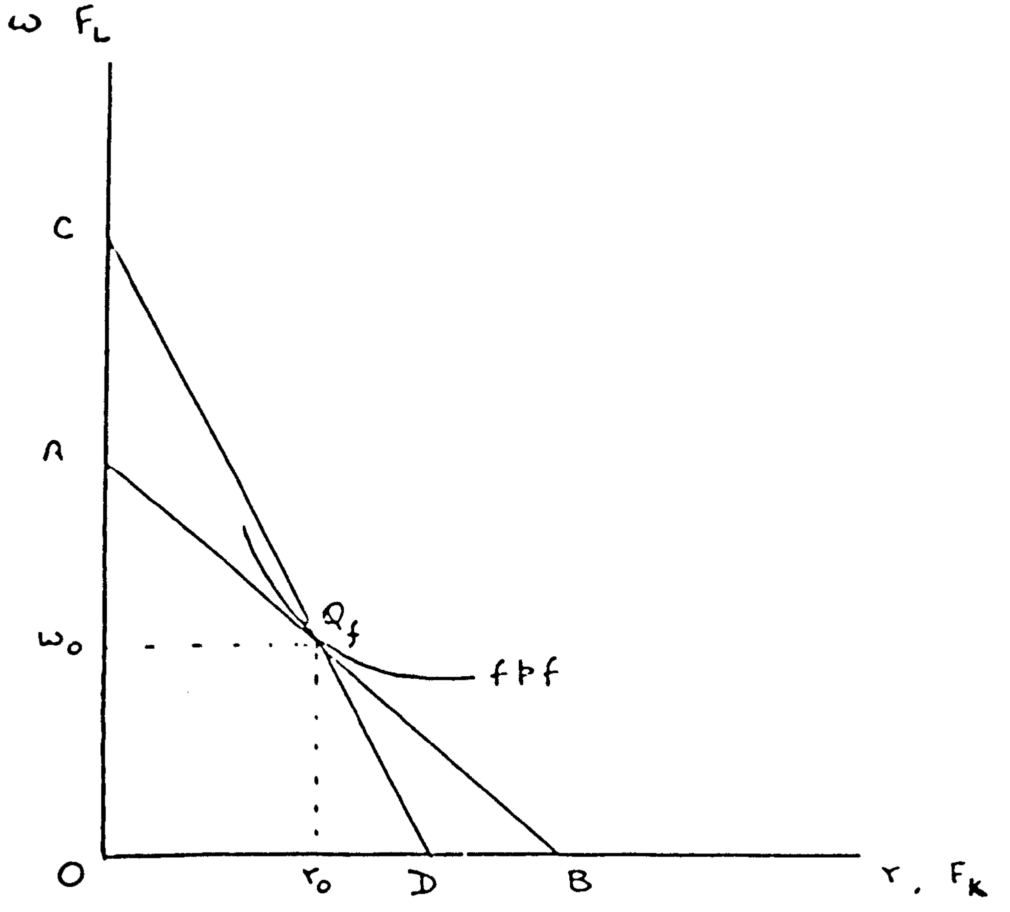


Figure 5.1

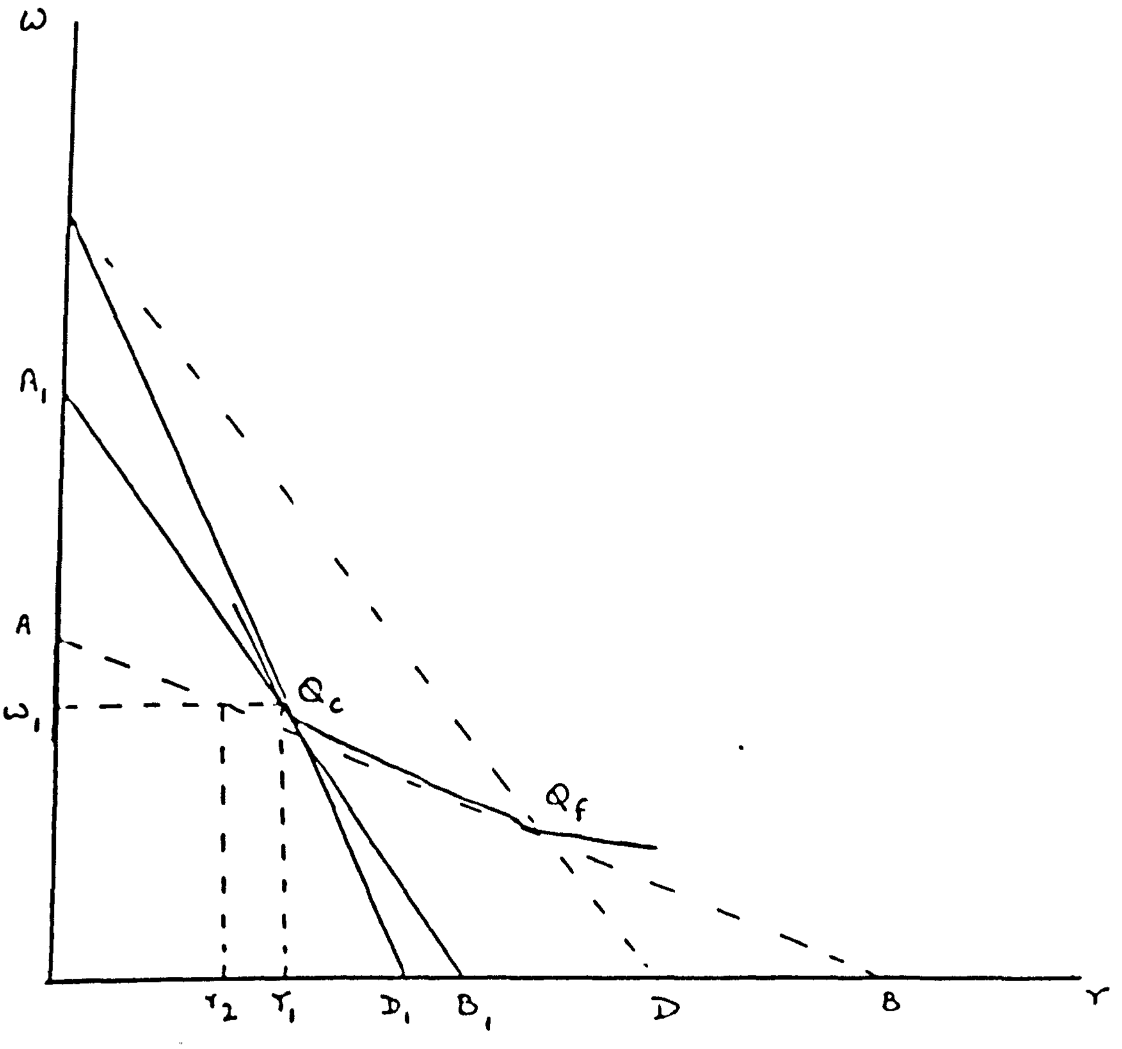


Figure 5.2

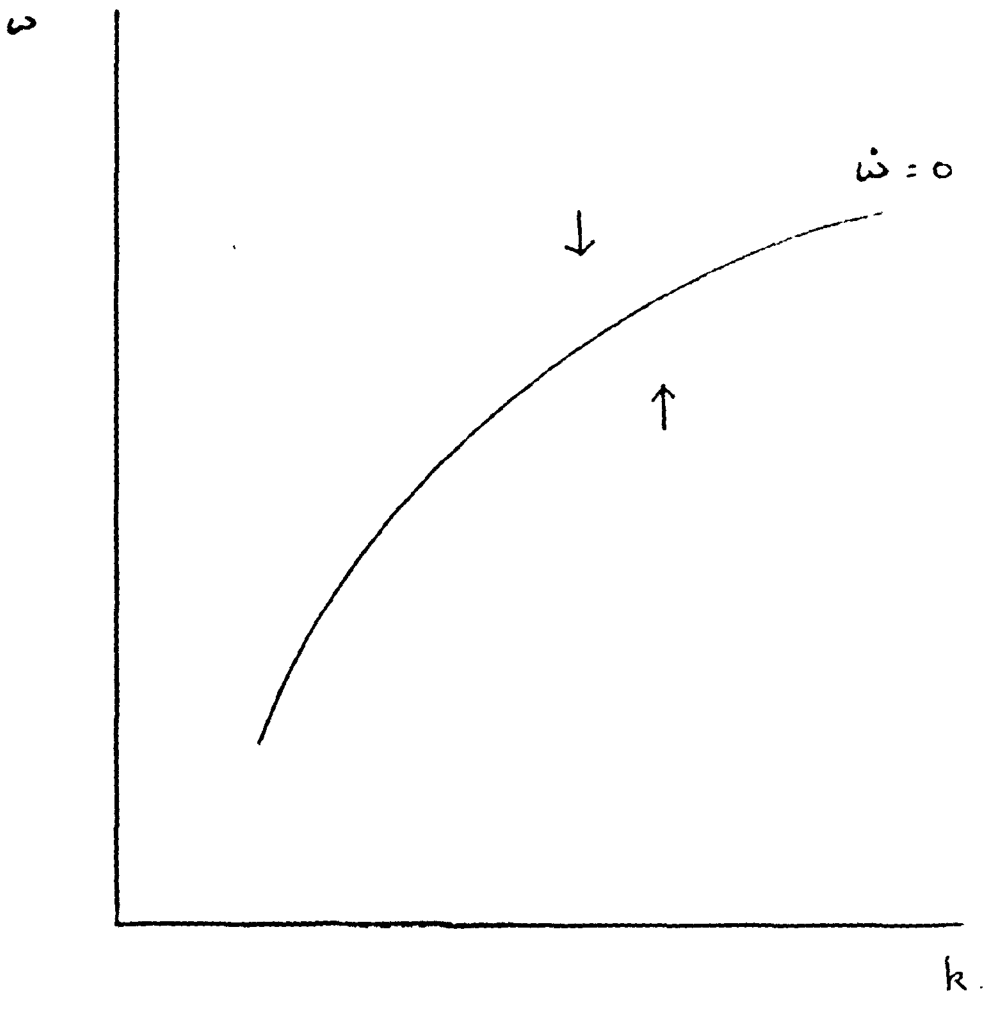


Figure 5.3

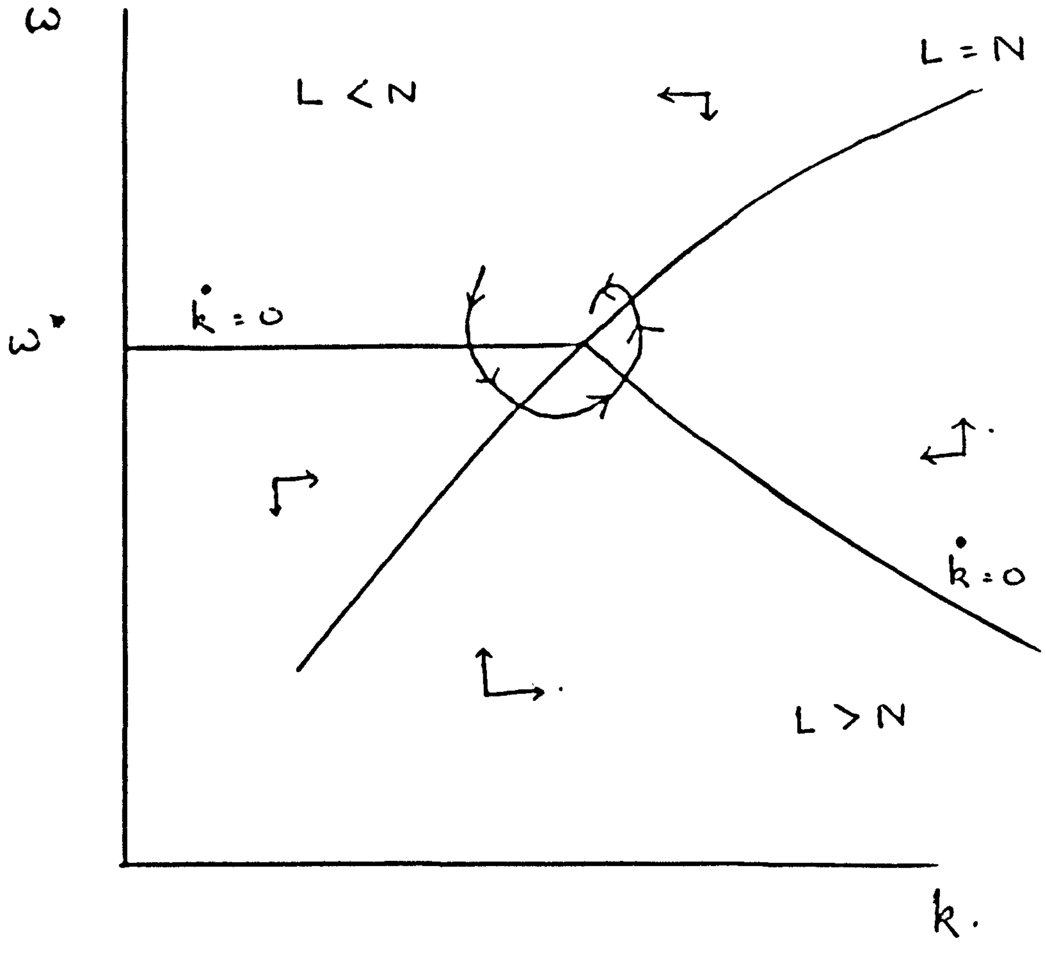


Figure 5.4.

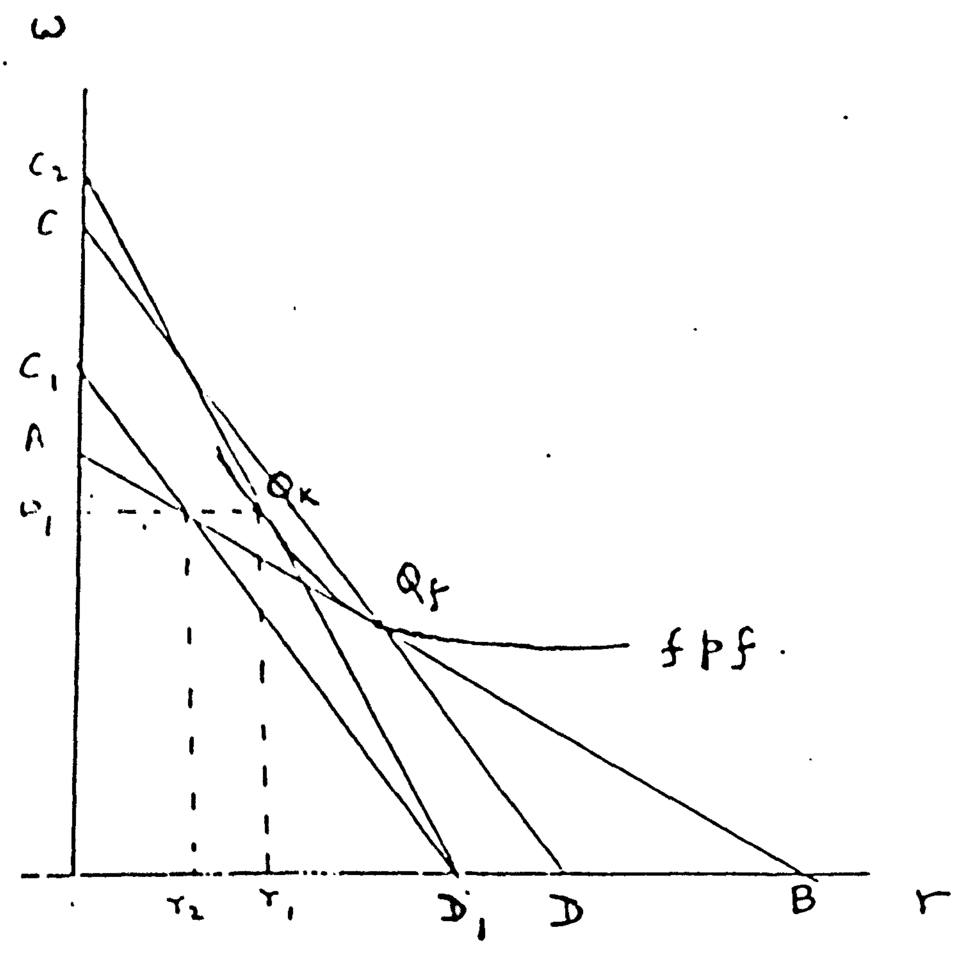
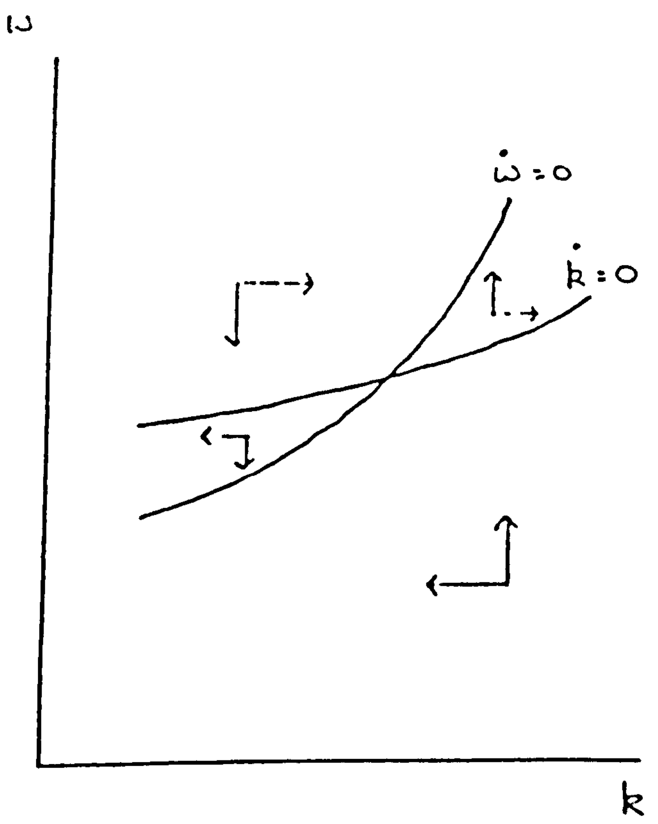


Figure 5·5



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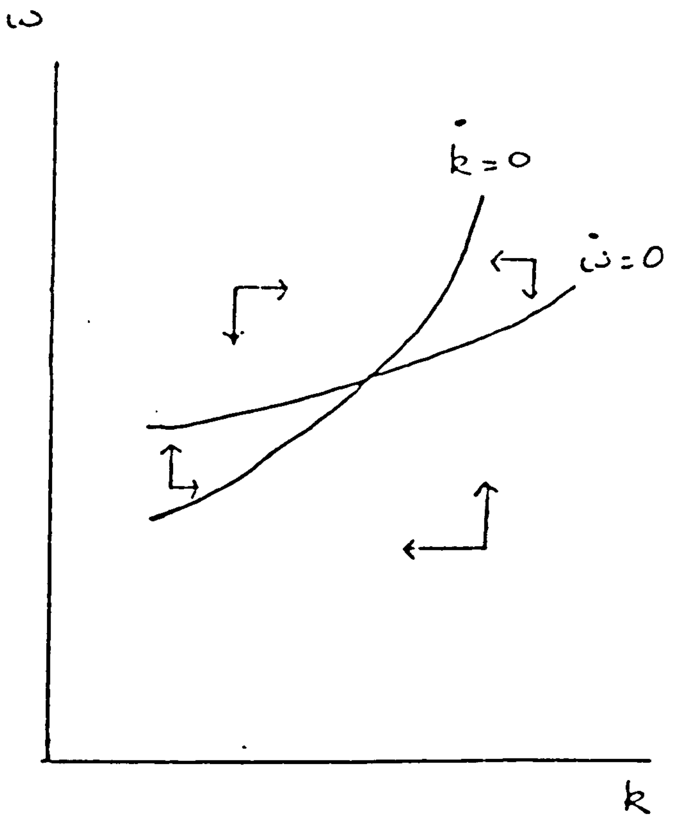

Figure 5.7

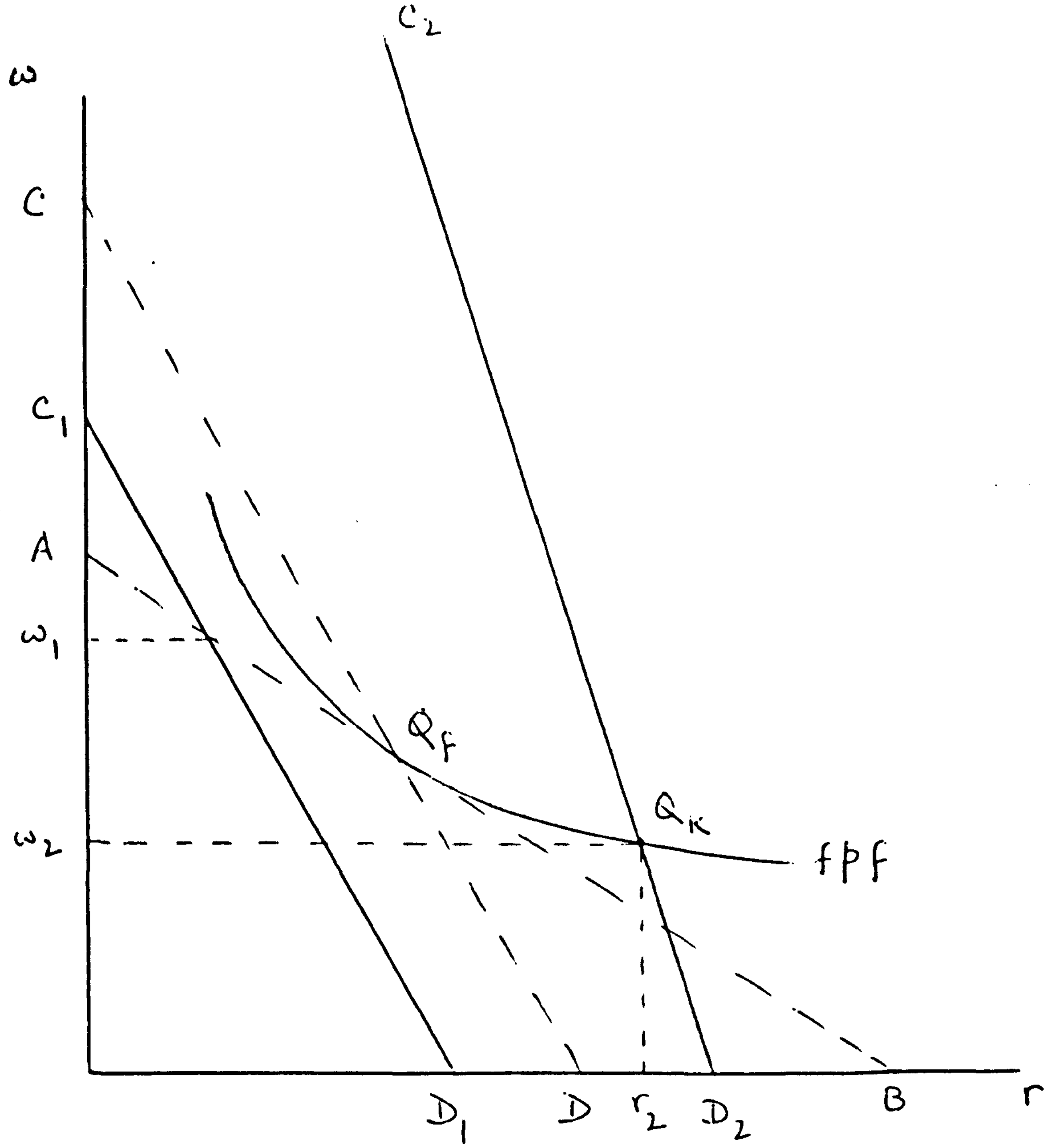

Figure 5. 8

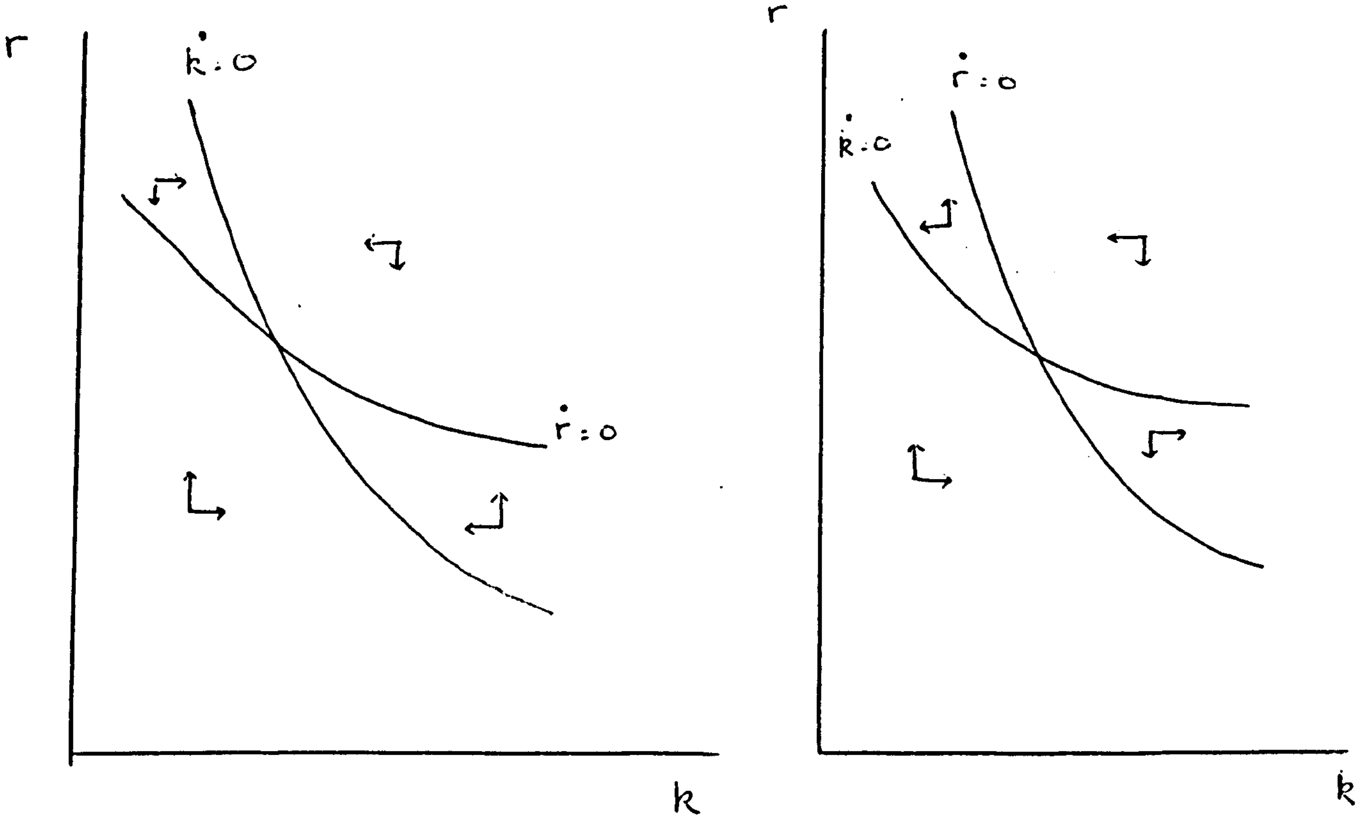

Figure 5.9

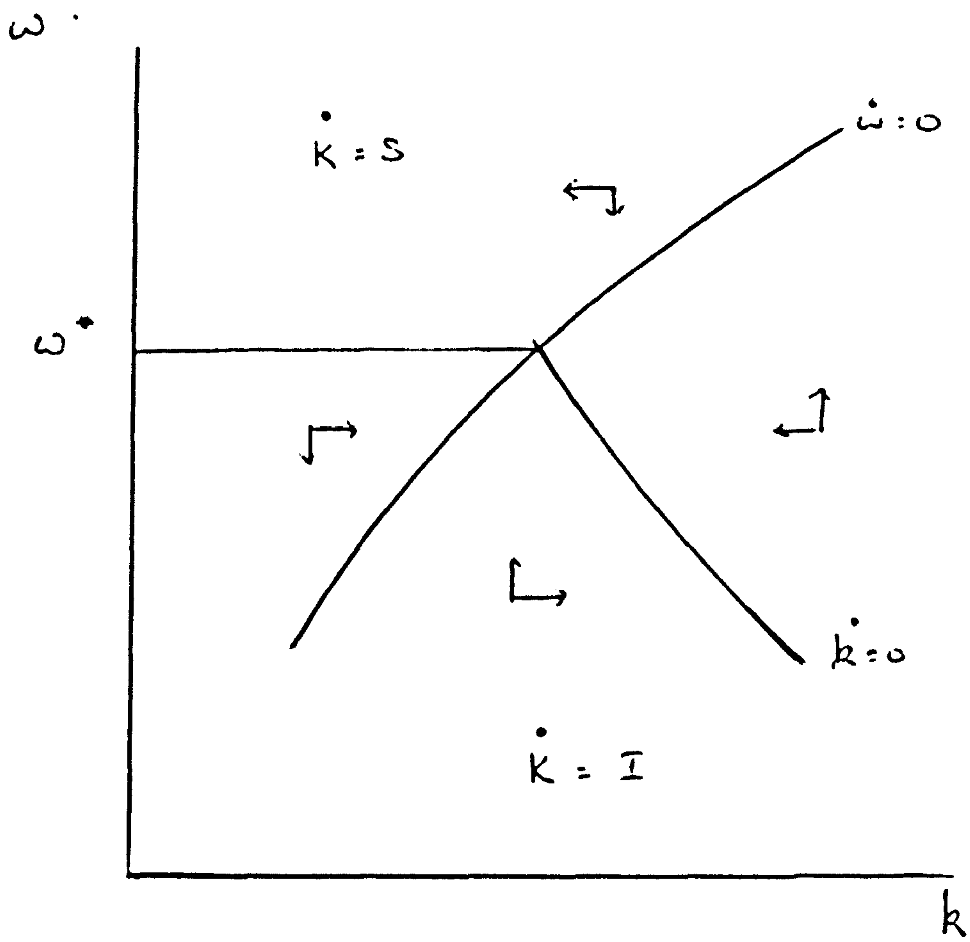

Figure 5.10

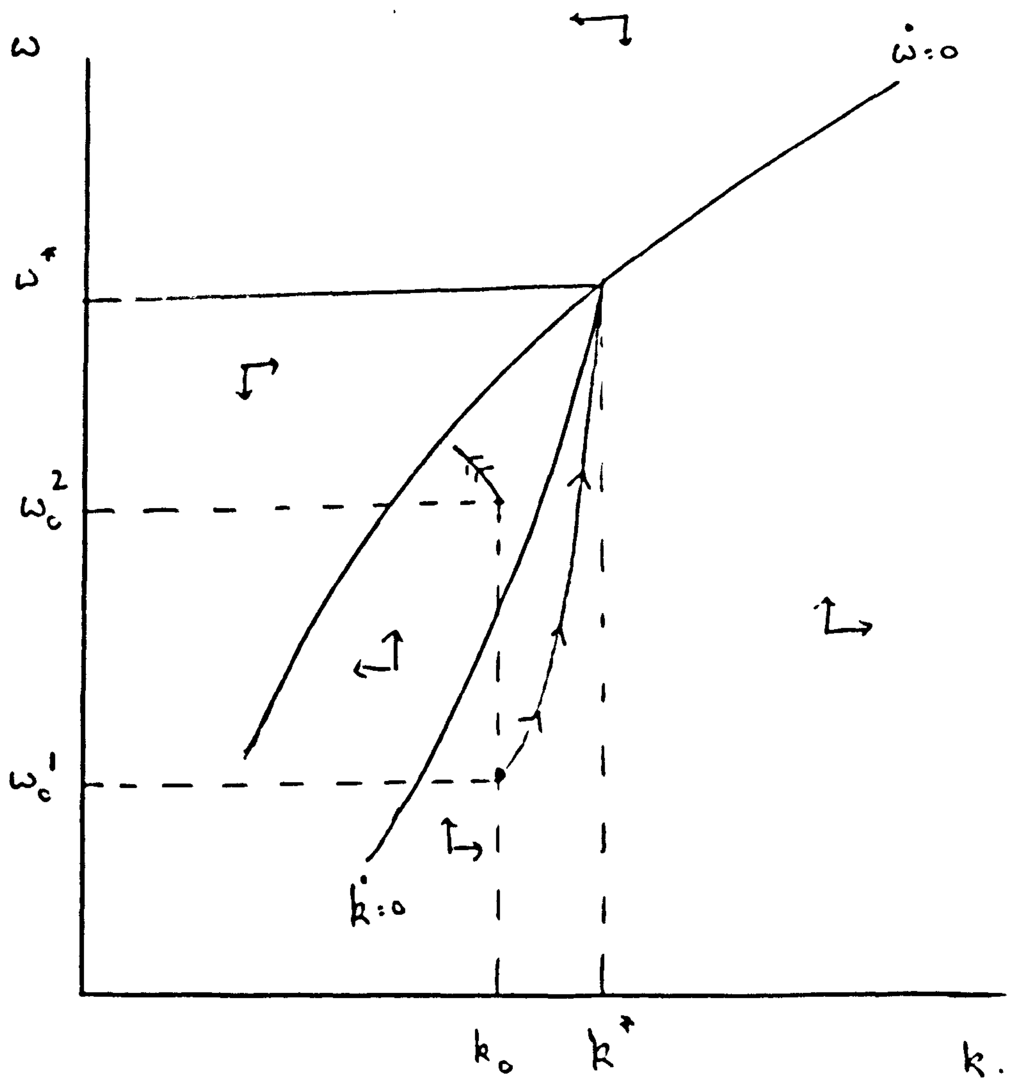

Figure 5.11

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Chapter VI

Growth in the Open Economy

Section 1: Introduction

The purpose of this chapter is to study the behaviour of the real exchange rate and endogenous capital formation in a macro-economic growth model of a small open economy. The analysis focuses on the dynamic movement of the real exchange rate as growth takes place in the open economy. We are also concerned with the effect of contractionary (or expansionary) monetary policy on steady state values of the exchange rate. Our major interest, however, is in the disequilibrium behaviour of the economy as it moves from one long run equilibrium to another in response to policy shocks. We therefore analyse in detail the transition paths of investment and exchange rates as they respond to new values of policy parameters.

In the standard "trade and growth" literature (Kemp (1969), Chacoliades (1978), the real exchange rate (identified with the terms of trade) is generally assumed fixed (given by international prices) and exogenous to the small economy. Its internal dynamics are thus neglected and problems related to its appreciation or depreciation are not fully studied. Further, full employment of both capital and labour always prevails and this seems to be a general weakness of the pure theories of international trade. However, capital growth is taken to be endogenous, and this characteristic is an useful one.

A second important development in the area comes from the macroeconomic models of the open economy recently studied in great depth by Dornbusch (1976a, 1976b), Branson (1979), Buiter and Miller (1981) and many others. These models are characterised by a rich analysis of the monetary asset structure of the economy (both internal and external), an emphasis on the balance of payments problem and a detailed study of the short run "overshooting" of the real exchange rate (with respect to its equilibrium value), in response to policy shocks. Given the complexity of these models, it is understandable that considerations of economic growth are left out in the interests of simplification.

A third important strand in the literature has been the "disequilibrium" models where non-market clearing behaviour, quantity rationing and spill-over effects are analysed carefully (see Dixit (1978) Neary (1980)). By their very nature, these models can study unemployment in great detail and notice the full implications for the real exchange rate when markets fail to clear. They also bring out to the forefront the important distinction between tradeables and non-tradeables. Emphasising the fix price approach, this method of analysis does not deal with dynamics in a rigorous way.

Kouri (1979)[1] has recently proposed a model which brings together some of the important features of the different developments noted earlier. In his analysis, the real exchange rate is endogenous and its dynamic movements have important bearing on capital formation, which itself is determined endogenously within a growth model.

Our analysis extends the Kouri model and establishes a more general framework to study the dynamics of real exchange rates and growth. It is characterised by three major features:

(i) capital stock is endogenously determined through the working of an investment function;

(ii) the effects of policy shocks on investment and exchange rate are analysed, in particular their behaviour on the transitional path prior to reaching long run equilibrium;

(iii) unemployment is explicitly introduced and the implication of quantity rationing in various markets is discussed.

Thus we bring together important characteristics of the three approaches to the analysis of a small open economy. In this respect we propose a partial synthesis of different aspects of the literature.

It must be noted, however, that our primary concern (as in Kouri (1979)) is to study the features of an open growth model. Thus relative price distortions, over-valuation (or under-valuation) of the exchange rate, etc., will be important insofar as it affects the capital stock (and the growth rate) of the economy. The balance of payments problem which has been extensively studied elsewhere, is relegated to a secondary issue in this paper.

In Section 2 we present the basic growth model and show the dynamic behaviour of capital stock and real exchange rate. A steady state equilibrium is defined and shown to be both locally and globally stable. In the next section adjustments to policy parameters are considered and comparative dynamics as well as transition paths of different variables are analysed. Section 4 concludes with ideas for further research.

Section 2: The Model

There are two aggregative sectors in the economy, one producing a tradeable good X_T and the other producing a non-tradeable good X_N. To focus attention on the real exchange rate, we assume that the price of tradeable good ($\hat{P}_T$) is fixed in the world market in terms of an international currency, say the dollar. Then the domestic currency price of tradeables is $\varepsilon \hat{P}_T$, where ε is the exchange rate (ε expressed as domestic currency per unit of foreign currency, e.g. £/\$). This is a reasonable assumption to make in the context of a small open economy, which does not have the power to influence world prices[2].

The following notations will be used from now on:

ϵ: the exchange rate £/$, the number of £'s per unit of foreign currency

X_T^s: supply of tradeable good

X_T^d: demand for tradeable good

$P_T = \epsilon \hat{P}_T$: price of tradeable in domestic currency (£)

W: wage rate in domestic currency

K_T: capital stock used in the production of tradeables

L_T^d: demand for labour in production of tradeables

X_N^s: supply of non-tradeable good

X_N^d: demand for non-tradeable good

P_N: price of non-tradeable good in domestic currency

L_N^d: demand for labour in non-tradeables sector

C: total consumption measured in units of non-tradeables

$L^s = L_0 e^{nt}$: total labour supply changing at the exogenously specified natural rate of growth, n

r_T: rate of profit in tradeable goods sector

Note: for quantity variables X_N X_T L_T L_N, a superscript 'd' implies demand, 's' implies supply and no superscript implies actual quantity transacted.

The tradeable goods sector has a constant returns to scale production function where capital and labour are both required to produce output. Thus

$$X_T^s = F_T(L_T^d, K_T) \qquad (1)$$

By constant returns to scale,

$$X_T^s = F_T \left(\frac{L_T^d/X_T^s}{K_T/X_T^s} \right) K_T \qquad (2)$$

Under competitive assumptions, the minimum unit cost function ϕ is given by

$$P_T = \phi(W, r_T P_T) \qquad (3)$$

By homogeneity

$$1 = \phi(\frac{W}{P_T}, r_T) \qquad (4)$$

Inverting this function we get the factor price frontier

$$r_T = \pi(P_T/W) \qquad (5)$$

$(\pi' > 0)$

Substituting in (4)

$$1 = \phi(\frac{W}{P_T}, \pi(\frac{P_T}{W})) \qquad (6)$$

By "Shephards Lemma", we can differentiate the cost function ϕ to get the optimal factor/output ratio. Thus

$$\frac{L_T^d}{X_T^s} = \phi_1(1/P_T/W, \pi(P_T/W)) \qquad (7)$$

and

$$\frac{K_T}{X_T^s} = \phi_2(1/P_T/W, \pi(P_T/W)) \qquad (8)$$

(where ϕ_i is the i^{th} partial derivative of ϕ).

Using (7), (8) in (2) we get

$$X_T^s = x(P_T/W)K_T \tag{9}$$

From (7), (9),

$$L_T^d = \ell(P_T/W)K_T \tag{10}$$

It is easy to derive the signs of

$$x' = \partial x/\partial(P_T/W) > 0$$
$$\ell' = \partial \ell/\partial(P_T/W) > 0$$

We assume that the economy is "small" in the sense that all its tradeable output can be sold (either at home or abroad). Further the tradeable sector has no constraint in getting labour from the homogenous labour force. Thus, in case of excess demand for labour (over full employment), it is the non-tradeables sector which has to be rationed in its requirement for labour. Armed with these two assumptions, we can now drop the superscripts (d,s) and note

$$X_T = x(P_T/W)K_T \qquad (9)'$$
$$L_T = \ell(P_T/W)K_T \qquad (10)'$$

Given the value of P_T (alternatively ϵ and $\hat{P}_T$) and W, the tradeable sector determines its output/capital and output/labour ratios through (9)' and (10)'.

We now turn our attention to the non-tradeable sector. Assume that capital is used <u>only</u> in the tradeable sector. Further, capital accumulation is achieved by using tradeable goods alone or importing from abroad. Given labour is the only factor of production for non-tradeables, we have a homogenous of degree 1 production function.

$$X_N^s = F_N(L_N^d); \; F_N' > 0 \qquad (11)$$

Labour productivity $X_N/L_N = F(1)$ is a constant. Inverting (11) we get

$$L_N^d = G_N(X_N^s); \quad G' > 0 \qquad (12)$$

Note that

$$F_N(1) = 1/G_N(1) \qquad (13)$$

The supply of output in the non-tradeable sector is determined by its effective demand. Thus we may have $X_N^s = X_N^d = X_N$, where X_N is the actual amount transactd. It should be clear that this is only consistent in an equilibrium situation. For a disequilibrium system, there may be excess effective demand or excess effective supply or goods. We would then need obviously a rationing rule, for example $X_N = \min(X_N^d, X_N^s)$. However, as we shall show later, the model will take care of disequilibrium in the goods market and we need not worry about the problem. The important disequilibrium that we shall face will be in the labour market where unemployment and over full employment will have to be analysed in detail.

The analysis of the model henceforth will be conducted in two stages. Firstly, we shall assume market clearing equilibrium and derive dynamic equations consistent with that equilibrium. At the second stage we relax this condition and study the implications of disequilibrium.

Assume a classical saving function such that all wage income is consumed and all profit income saved. Then

$$C = \frac{W}{P_N} L_N + \frac{W}{P_N} L_T \qquad (14)$$

Further, let us suppose that h is the proportion of total consumption C spent on non-tradeables. Obviously $h = h(P_T/P_N)$. Therefore,

$$X_N^d = h(P_T/P_N)C \qquad (h' > 0) \tag{15}$$

The system is completed by the unit cost function of the non-tradeable

$$P_N = \Omega(W) \tag{16}$$

Given the nominal wage W, the firm sets price at P_N.

The following results which follow from the properties of the production and cost functions will be useful later on.

$$\frac{W}{P_N} L_N = X_N \tag{17}$$

$$\frac{P_N}{W} = \Omega(1) \tag{18}$$

$$\frac{P_T}{W} = \frac{P_T}{P_N} \Omega(1) \tag{19}$$

$$\frac{W}{P_N} = F_N(1) = 1/G_N(1) \tag{20}$$

The implications of the relative price variables must be spelt out. By virtue of the simple cost function (16), the real wage in terms of non-tradeables W/P_N is a constant. However, real wage in terms of tradeables W/P_T is a crucial variable whose analysis is the focal point of this paper. For a given $\hat{P}_T$, the <u>real exchange rate</u> (in terms of units of non-tradeables) is given by $P_T/P_N = \varepsilon \hat{P}_T/P_N$. Alternatively, we can say that the real exchange rate is equal to ε/P_N (in terms of non-tradeables) or ε/W (in terms of labour). Note that by equation (19), the index of real exchange rate P_T/P_N is proportional to the real wage rate. Further, the real wage rate in terms of tradeables is itself inversely related to the real exchange rate. Thus a rise in P_T/W implies a decline in real wage as well as an appreciation of the real exchange.

Using (14), (15), (17), (18), (19), we get

$$X_N = \frac{\ell(P_T/W)K_T}{\Omega(1)[1-h(\frac{P_T}{W}\cdot\frac{1}{\Omega(1)})]}$$

$$= X(P_T/W)K_T \qquad (21)$$

(where $X' > 0$)

The total demand for labour is then

$$L^d = L_T^d + L_N^d$$

$$= (\ell(P_T/W)+G_N(X(P_T/W))K_T$$

$$= \psi(P_T/W)K_T \qquad (22)$$

$(\psi' > 0)$

Labour supply is assumed to be $L^s = L_0e^{nt}$. Therefore, for <u>equilibrium</u> in the labour market

$$L^s = \psi(P_T/W)K_T \qquad (23)$$

or

$$k_T = \frac{K_T}{L^s} = \frac{1}{\psi(P_T/W)} \qquad (24)$$

Since tradeables alone use capital, k_T is the total capital stock in the economy per head of labour force. To simplify notation, write $k = k_T$ and $p = P_T/W$. Then from (24), $k = 1/\psi(p)$, $(dk/dp < 0)$.

Inverting this function we can get

$$p = f(k) \qquad (f' < 0) \tag{25}$$

The f relation shows <u>full employment</u> of labour and is depicted in Figure 6.1.

Let us turn our attention now to disequilibrium behaviour. First, the labour market. It is easy to demonstrate that below f(k), the economy has excess supply of labour and above it has excess demand. The dynamic behaviour of p when the economy is not in full employment equilibrium is given by the following differential equation

$$\dot{p} = \lambda[f(k)-p] \tag{26}$$

When there is excess supply of labour, $f(k) > p$ and real wage rate W/P_T falls, thus $p = P_T/W$ rises ($\dot{p} > 0$). The opposite happens for excess demand.

Turning to the goods markets, we can show that the possibility of disequilibrium is ruled out by the structure of the model. Note that the tradeable sector faces no constraints since it can sell its output to the world market and has a first preference in the market for non-market clearing regimes and we have to look at the different alternatives.

First consider the case of excess supply of labour such that $L^d < L^s$. Then by a simple rationing rule (Malinvaud (1977)) actual labour in both sectors are given by labour demand, i.e.

$$L = \min (L^d, L^s) = L^d \text{ and } L_N = L_N^d.$$

Equation (21) determines the demand for non-tradeables. Since there is no rationing in firms requirement for labour and labour is the only factor of production, any demand for output can be met. Thus $X_N^s = X_N^d$ and from (11), (12) we can determine the output of non-tradeables. The relevant diagram is self-explanatory, see Figure 6.2.

The problem is more complicated when there is excess demand for labour. Since $L = \min(L^d, L^s) = L^s$ and the tradeable sector is not rationed, it is clear that the non-tradeable sector will face quantity constraints in its demand for labour. We have

$$L_N = \min(L_N^d, L_N^s)$$
$$= L_N^s = L^s - L_T \qquad (27)$$

Thus the maximum output that can be supplied is from (11),

$$X_N^s = F_N(L^s - L_T) \qquad (28)$$

By homogeneity

$$X_N^s = F_N(1)[L^s - L_T] \qquad (29)$$

Since employment is equal to labour supply, the amount of consumption

$$C = \frac{W}{P_N} L^s \qquad (30)$$

From (15),

$$X_N^d = h(P_T/P_N)C \qquad (31)$$

using (30),

$$X_N^d = h(P_T/P_N)\left(\frac{W}{P_N}\right)L^s \qquad (32)$$

From (14), (15), (17)

$$C = hC + \frac{W}{P_N} L_T \qquad (33)$$

or

$$(1-h)C = \frac{W}{P_N} L_T$$

$$(1-h)L^s \frac{W}{P_N} = \frac{W}{P_N} L_T$$

$$hL^s = (L^s - L_T) \qquad (34)$$

Since $F_N(1).G_N(1) = 1$ from (13),

$$\frac{hL^s}{G_N(1)} = (L^s - L_T)F(1) \qquad (35)$$

From (20), (29), (32), (35)

$$X_N^d = X_N^s \qquad (36)$$

Thus in the case of over full employment (excess demand for labour) the goods market is in equilibrium too.

We have demonstrated that only the labour market can be in disequilibrium. The non-tradeable goods sector will always have effective demand equal to effective supply. Thus "Keynesian" "Classical" cases of Malinvaud (1977) need not be distinguished in this model.

We now turn our attention to capital accumulation and growth. Following convention, labour supply is assumed to grow exogenously at rate n

$$L^s = L_0 e^{nt} \qquad (37)$$

The *per capita* investment function is given by

$$\frac{I_T}{L^s} = I(r_T, r^*) \tag{38}$$

$(I_1 > 0, I_2 < 0)$

Investment or capital accumulation in the economy[3] is dependent on the rate of profit r_T and the rate of *interest* r^*. The rate of profit represents the internal rate of return while rate of interest is a proxy for the *cost* of borrowing funds. Clearly the amount of investment is negatively related to the rate of interest.[4]

The major policy variable in our model is the rate of interest r^*. Government monetary policy determines r^* and the dynamics of the model are analysed on the basis of this given policy parameter. Obviously a contractionary monetary policy increases r^* and an expansionary monetary policy reduces it.

Since $k = K_T/L^s$

$$\frac{\dot{k}}{k} = \frac{\dot{K}_T}{K_T} - \frac{\dot{L}^s}{L^s}$$

From (37), (38)

$$\dot{k} = I(r_T, r^*) - nk \tag{39}$$

Using (5)

$$\dot{k} = I(\pi(p), r^*) - nk \tag{40}$$

The other dynamic equation can be re-written now

$$\dot{p} = \lambda[f(k) - p] \tag{41}$$

Equations (40), (41) together give the behaviour of our growth model and the dynamic movements of capital stock (k) and the real exchange rate (p).

It can be easily verified that:

$$\frac{\partial \dot{k}}{\partial p} = I'\pi' > 0 \qquad \frac{\partial \dot{k}}{\partial k} = -n < 0$$

$$\frac{\partial \dot{p}}{\partial p} = -\lambda < 0 \qquad \frac{\partial \dot{p}}{\partial k} = \lambda f' < 0 \qquad (42)$$

$$\left.\frac{dp}{dk}\right|_{\dot{k}=0} = \frac{n}{I'\pi'} > 0 \qquad \left.\frac{dp}{dk}\right|_{\dot{p}=0} = f' < 0$$

Using information in (42), the phase diagram is given in Figure 6.3.

Local stability at steady state equilibrium (p* k*) is easily verified by forming the Jacobean

$$J = \begin{bmatrix} -n & I'\pi' \\ \lambda f' & -\lambda \end{bmatrix} \qquad (43)$$

and noting that

$$\text{trace } J = -\lambda - n < 0$$

$$\det J = \lambda n - \lambda f' I'\pi' > 0$$

(evaluated at steady state).

However, an even stronger stability result can be proved. Using Olech's Theorem we can show that our model is globally stable. Olech's Theorem states that for all points in the (p,k) plane, the following three conditions are sufficient for global stability:

(i) trace $J < 0$

(ii) det $J > 0$

(iii) either the product of diagonal or off diagonal terms must be non-zero.

(Note that these conditions must hold for <u>all</u> p, k and not only for p*, k*).

Looking at the Jacobean it is seen that for all values of the variables

$$\text{trace } J = -n-\lambda > 0$$

$$\det J = \lambda n - \lambda f'I'\pi' > 0$$

and product of diagonal terms $\lambda n > 0$.

Thus, by Olech, global stability is assured.

Section 3: Comparative Dynamics and Transitional Paths

In the previous section we have analysed a growth model of an open economy, which tries to explain the behaviour of capital stock and real exchange rate over time. The major policy variable in the model is the rate of interest r*. We would now like to consider the effects of changing r* by monetary authorities, and study comparative dynamics properties as well as transitional paths when the economy moves from one steady state equilibrium to another. The model is not suited for a proper analysis of demand management through fiscal policy since the goods markets always clear. However, government policy on the "supply side" which stimulates capital formation can easily be incorporated.

Firstly, consider the effect of a contractionary monetary policy which increases rate of interest r*. From (40), keeping p at any predetermined level and for $\dot{k} = 0$, taking total differentials,

$$I_2 dr^* - n dk = 0$$

or

$$\left.\frac{\partial k}{\partial r^*}\right|_{\dot{k}=0} = \frac{I_2}{n} < 0$$

Therefore, the curve $\dot{k} = 0$ shifts leftwards as in Figure 6.4.

Comparing the two steady state equilibria E_1 and E_2 it is clear that there is an increase in real exchange rate p, a decrease in the real wage rate W/P_T (in terms of tradeables), a decrease in the realtive price of non-tradeables P_H/P_T and a decrease in capital stock per capita in the tradeables sector and thus in the whole economy. Even though competitiveness rises in the long run, the economy is unable to take advantage of a cheaper domestic currency and general deflation. Thus its capital stock per head of labour declines. This may be likened to a case of de-industrialisation, where contractionary policy reduces capital and productivity in export goods in spite of an improvement in international competitiveness, signalled by a rising real exchange rate. One possible transitional path is given by the dotted line in the previous diagram, moving from E_1 to E_2.

Let us now turn to government parametric changes emanating from the supply side, which has the effect of encouraging capital formation in the tradeables sector. These may take various forms - creation of enterprise zones, help from the Export Credit Guarantee Department, reduction of export duties, opening up new markets, etc. If it affects the form of the investment function I(.), then KK will shift rightwards and steady state equilibrium value of capital stock k will rise while the exchange rate falls. It is more likely, however, that government policy will affect the structure of production per se so that capital intensity of output will rise. An increase in the capital output ratio will move the LL curve rightwards. For any given p, more capital intensive methods are encouraged and this increases k on the $\dot{k} = 0$ line, (see Figure 6.5).

It is clear that in the new steady state E_3, both the capital per capita and real exchange rate are high compared to E_1. Thus the economy has high profitability and can take full advantage of an improvement in international competitiveness to produce more tradeable output as well as have more investment.

We would now like to study the movement of variables along the transition path from one equilibrium to another more formally. To set the framework, consider a simultaneous operation of a contractionary monetary policy (increase in r*) and a supply side stimulus (encouraging capital formation). Suppose KK moves leftwards and LL moves rightwards so that p unambiguously rises. Let us assume that the equilibrium capital stock k* remains unchanged so that government policy is <u>neutral</u> in the long run (this is not strictly necessary but is a useful device to concentrate only on the transition path and not to worry about the position of equilibrium). The next diagram below (Figure 6.6) shows these changes. Note that the transitional path may follow the dotted line E_1DE_2. Specifically, it is important to see that along this path $k(t) < k^*$.

From the expression of Jacobean, (43) we derive the characteristic equation

$$\alpha^2 - \alpha[\lambda+n] + [\lambda n-\lambda f'I_1\Pi] = 0 \qquad (44)$$

where α is the characteristic root. We can write (44) as

$$\Lambda(\alpha,r^*) = \alpha^2-\alpha[\lambda+n] + [\lambda n-\lambda f'I_1\Pi'] = 0 \qquad (45)$$

From stability conditions we know that there are two values of α and both are negative. We consider the dominant characteristic root, i.e. the one with the <u>higher absolute value</u>.

The dynamic path of the variables will be determined by the behaviour of α. Thus if we wish to study the effects of r* on the transitional path of k(t) as it approaches equilibrium, we need to find the sign of $d\alpha/dr^*$. From (45)

$$\frac{d\alpha}{dr^*} = \frac{-\frac{\partial\Lambda}{\partial r^*}}{\frac{\partial\Lambda}{\partial\alpha}} \qquad (46)$$

Observe that when $\alpha = -\infty$, $\Lambda(-\infty, r^*) = +\infty$. Thus, for a given r*, Λ can be represented by the curve shown in Figure 6.7.

α_1 and α_2 are the values of characteristic root, i.e. the solution of $\Lambda(\alpha,r^*) = 0$. Note that at the dominant root $\alpha = \alpha_1$, $\partial\Lambda/\partial\alpha < 0$. Thus the sign of $d\alpha/dr^*$ is the same as the sign of $\partial\Lambda/\partial r^*$.

To get specific results at this point we have to give some additional structure to the most general form of the investment function used up till now. We assume that $I_T/L^s = I(r_T, r^*)$ (see equation (38)) can be written as

$$\frac{I_T}{L^s} = I(r_T - r^*) \tag{47}$$

where $\frac{\partial I}{\partial r_T} > 0$, $\frac{\partial I}{\partial r^*} < 0$, thus $I' > 0$. Further $I'' < 0$.

(This form has been rationalised in Kouri (1979)).

Then (45) becomes:

$$\Lambda(\alpha,r^*) = \alpha^2 - \alpha[\lambda+n] + [\lambda n - \lambda f''I'\Pi'] \tag{48}$$

and $$\frac{\partial\Lambda}{\partial r^*} = -\lambda f'I''\Pi' > 0$$

Therefore $$\frac{d\alpha}{dr^*} > 0 \tag{49}$$

Inequality (49) implies that as r^* rises, α rises, i.e. $(-\alpha)$ falls.

We know that

$$k(t) - k^* = (k_0 - k^*)e^{\alpha t}$$

Differentiating w.r.t. time

$$\begin{aligned} \dot{k} &= \alpha(k_0 - k^*)e^{\alpha t} \\ &= \alpha(k(t) - k^*) \\ &= (-\alpha)(k^* - k(t)) \end{aligned} \tag{50}$$

It has already been shown that $k^* > k(t)$. Therefore as r^* rises and $(-\alpha)$ falls, $\dot{k}$ falls. Thus investment falls along the time path of transition as (k,p) approach the new steady state equilibrium.

We have demonstrated a rather strong result. Contractionary monetary policy reduces the equilibrium capital labour ratio of a growing open economy. However, even if expansionary "real" policies are used to counteract the effects of contraction and ultimately there is long run neutrality (equilibrium k^* remains the same), the economy will still suffer. Along the whole transition path from one steady state to another, investment will be lower if the interest rate is raised.

Section 4: Conclusion

To conclude briefly, the foregoing analysis has allowed us to understand the dynamics of real exchange rate movements in a simple growth model with endogenous capital formation. The fundamental objectives of the paper have been threefold. Firstly, to integrate some of the important issues in a study of macro open economies, namely unemployment (or over full employment), growth and exchange rates. Secondly, we have shown explicitly the disequilibrium behaviour of the economy as it moves along the traverse from one steady state equilibrium to another. Thirdly, the effects of economic policy on the equilibrium values of variables as well as their transition paths have been analysed. In particular, we have seen the effect of "supply side" changes in the form of governments trying to increase the capital stock of the economy.

It may be emphasised that contractionary monetary policy has a uniformly harmful effect on investment along the whole transition path even though steady state capital stock may be higher due to authorities' efforts in capital formation. Thus the time period taken to move from one equilibrium to another is longer when the interest rate is pushed upwards. The model clearly demonstrates that a government dedicated to a contractionary monetary policy will reduce the growth rate of capital even though it may try to help directly to stimulate investment.

Footnotes

1 The reference for Kouri is

PENTTI KOURI (1979), "Profitability and growth in a small open economy" in A Lindbeck (ed), Inflation and employment in Open Economies, North-Holland

2 Alternatively one can assume an exogenously specified growth rate of $\hat{P}_T$ such that $\hat{P}_T = Pe^{\beta t}$; without loss of generality we assume $\beta = 0$.

3 Remember that only the tradeable sector uses capital in the production process.

4 For further justifications, see Kouri (1979).

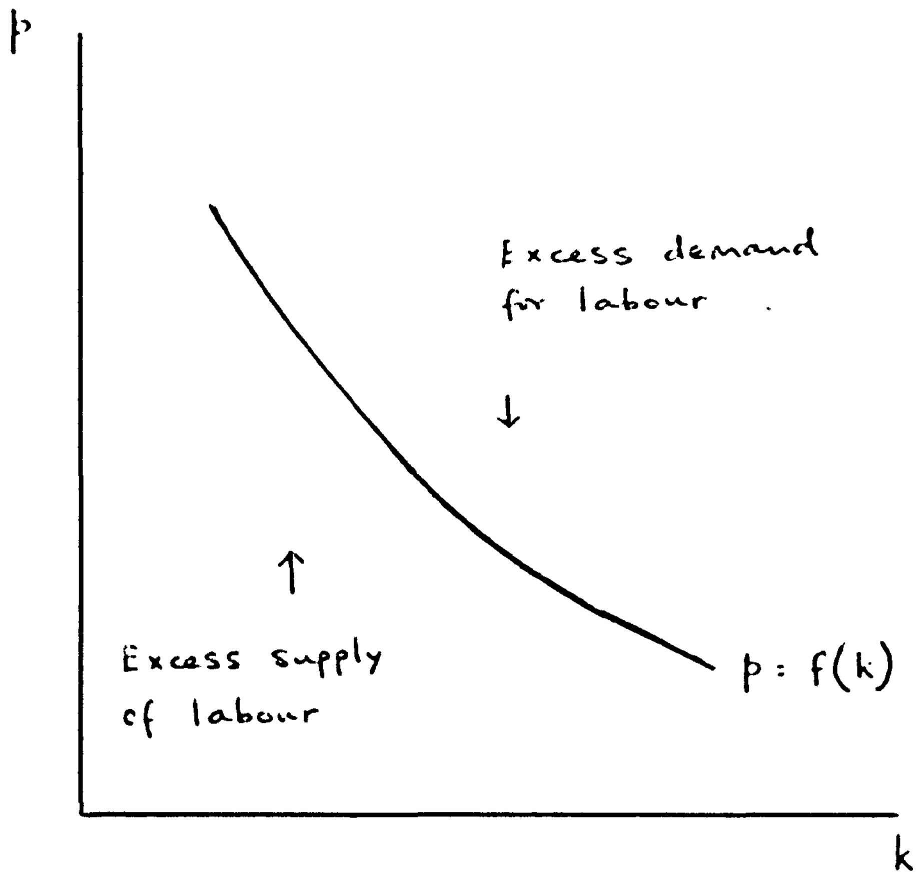

Figure 6.1

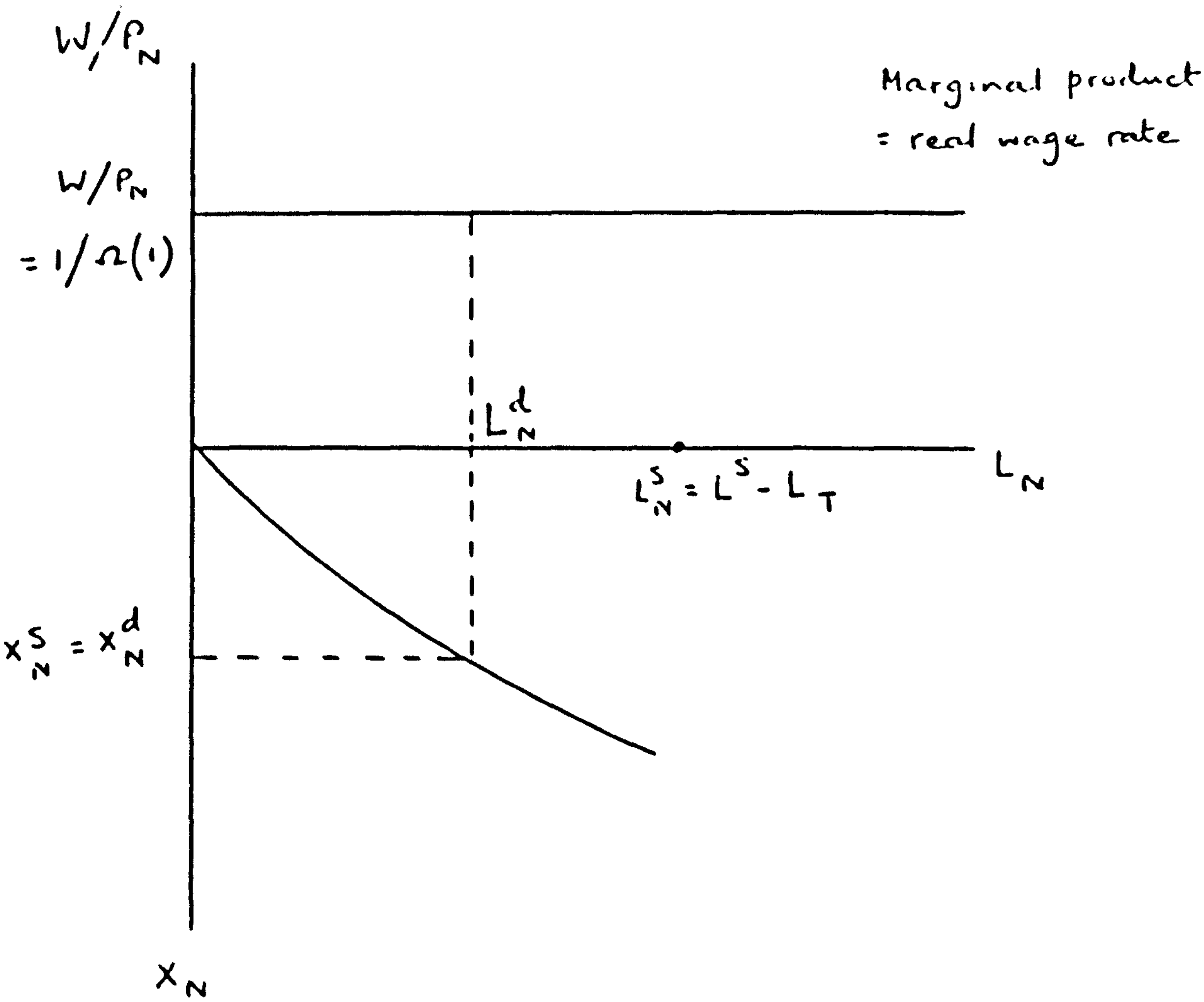
W/P_N
Marginal product
= real wage rate
W/P_N
= 1/Ω(1)
L^d_N
L_N
L^S_N = L^S - L_T
x^S_N = x^d_N
X_N

Figure 6.2

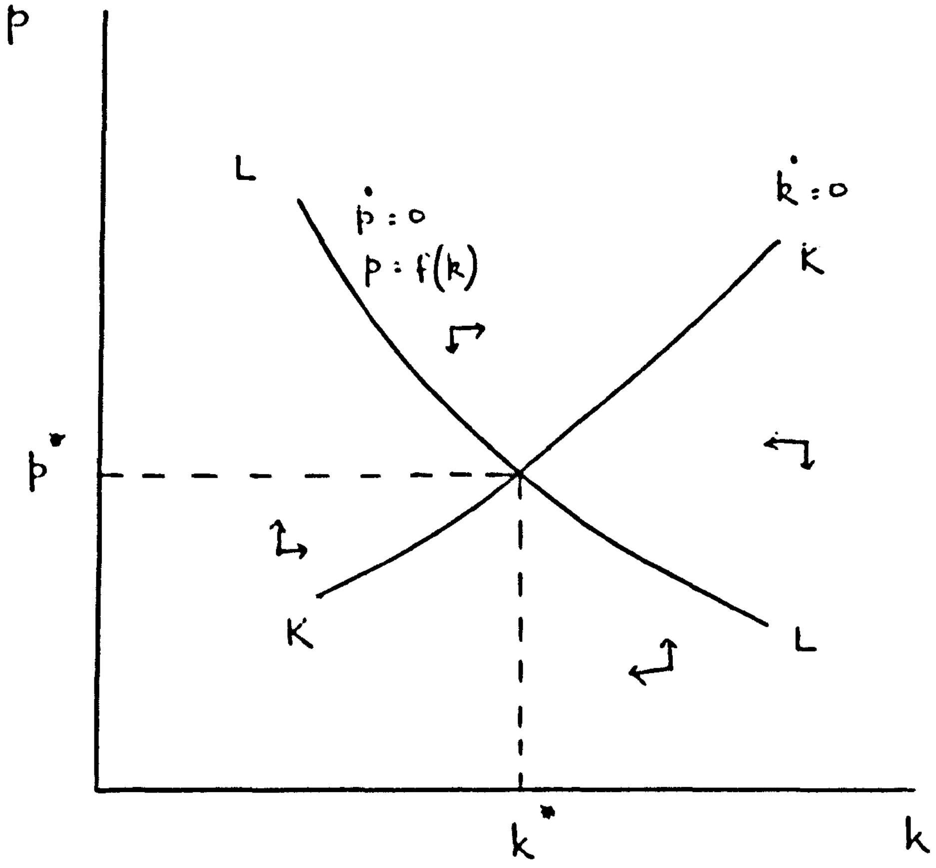

Figure 6.3

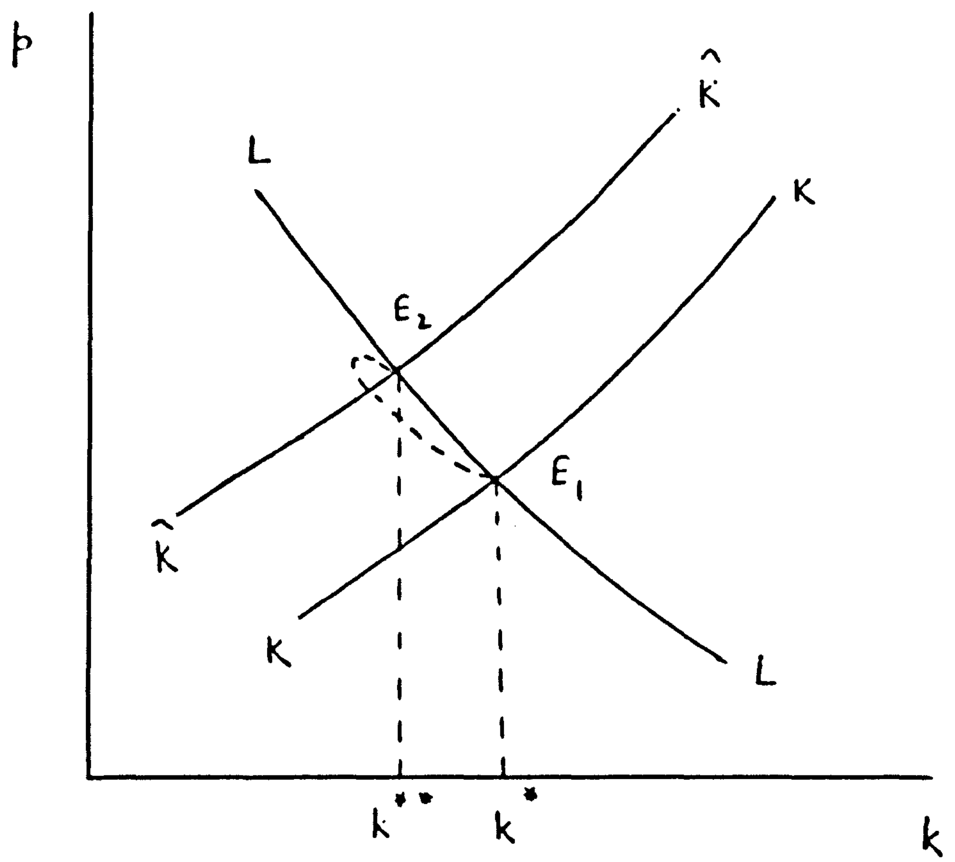

Figure 6·4

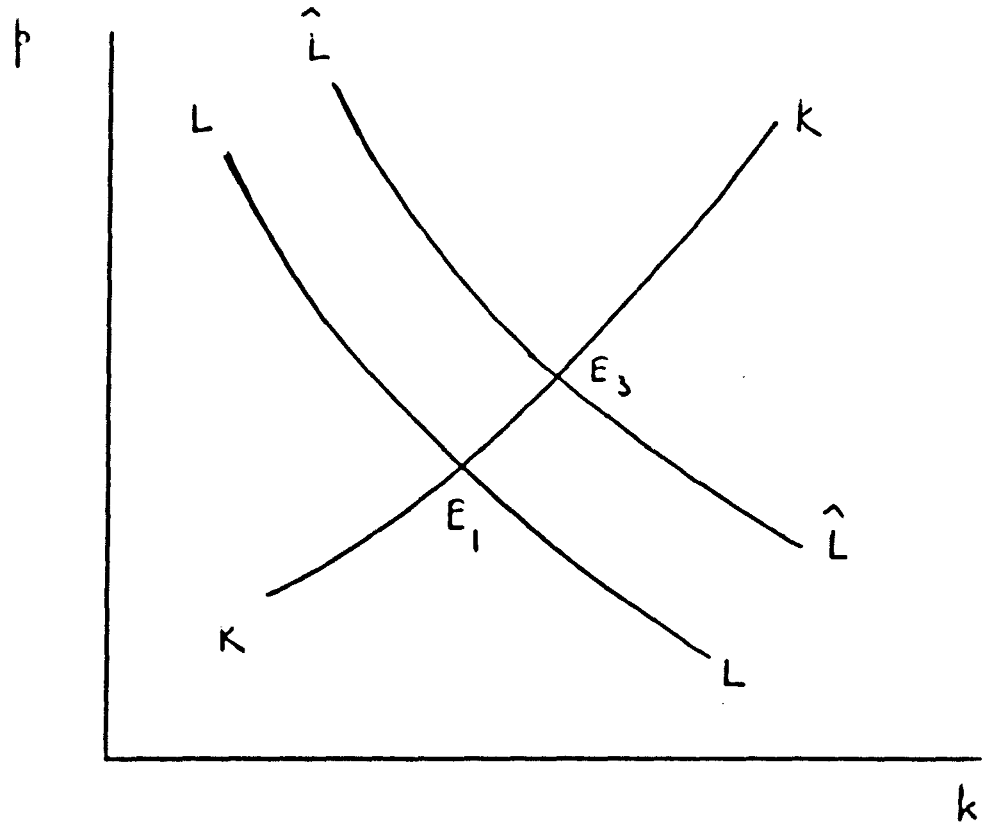

Figure 6.5.

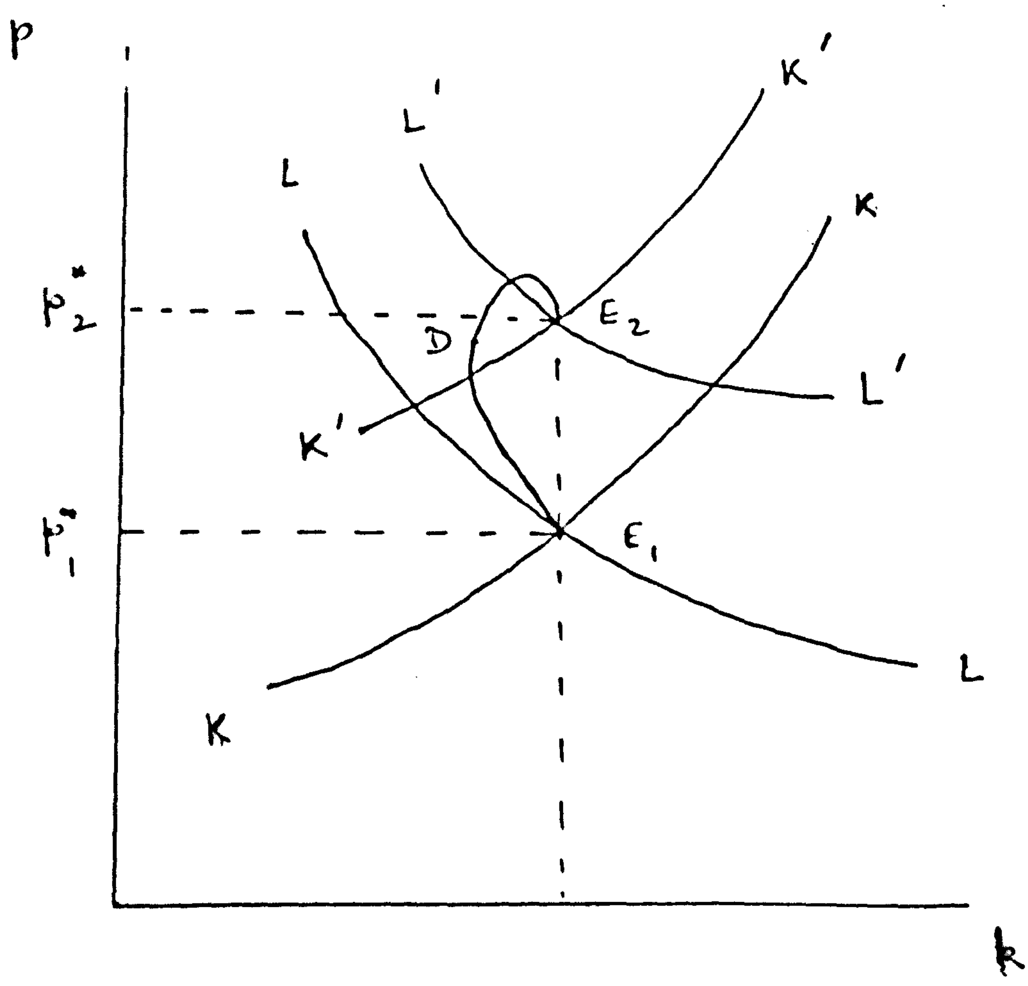

Figure 6.6

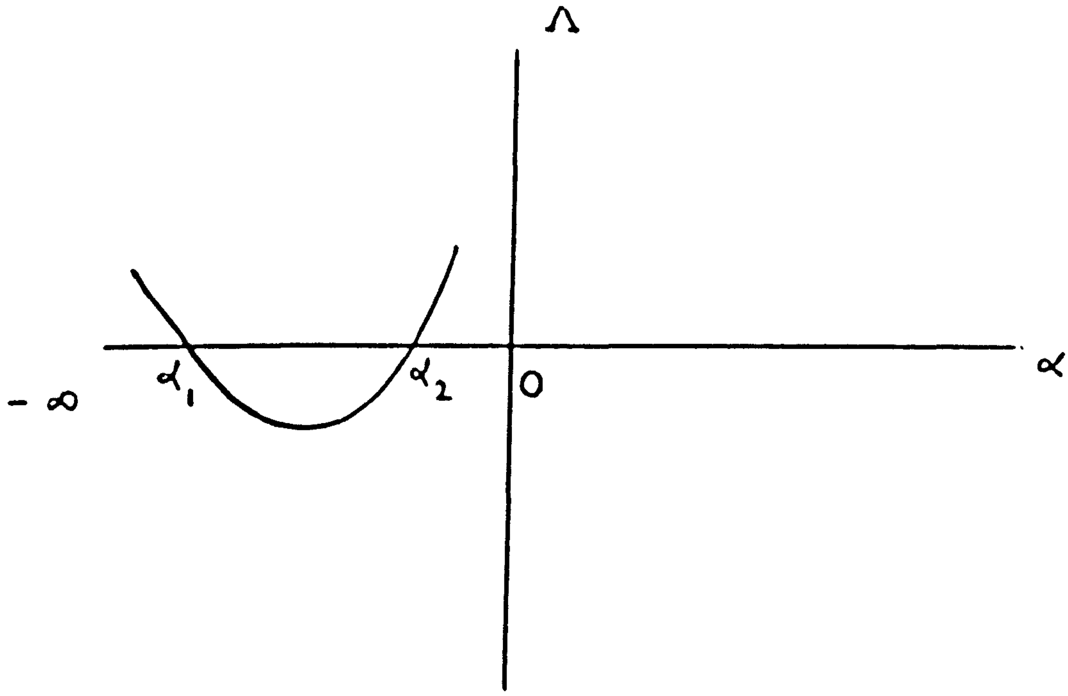

Figure 6·7

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Chapter VII

A Monetary Growth Model

Section 1: Introduction

The proposition that money is neutral has a long and respectable history in macroeconomics. Patinkin (1965) showed that monetary policy has no real effects in a static equilibrium model. Sidrauski (1967) proved, in the context of a perfect foresight growth model with money, that money is superneutral, in the sense that a change in the growth rate of money will not affect steady state values of real variables. The rational expectation literature (hereafter called RE) claims once again that systematic and perfectly anticipated monetary poicy has no effect on output provided expectations are held rationally.

However, monetary growth models (hereafter called MG) of Tobin (1965) and others, do not get strong superneutrality results, in general. Many of these models assume perfect foresight, which in the absence of uncertainty or in steady state, is equivalent to RE. It is a matter of considerable theoretical interest why RE and MG models make similar assumptions regarding the formation of expectations, yet come out with radically different conclusions regarding neutrality.

Hahn (1980) has recently pointed out that the fundamental difference between RE-structural neutrality propositions and others may be the nature of equilibrium analysed and the possible existence of multiple equilibria. Elsewhere I have shown (Sen (1980)) that neutrality propositions depend not on the assumptions of RE per se but on the type of non-Walrasian equilibrium one postulates. However, even within the limited scope of market clearing equilibrium models, that both RE and MG theories share in common, they have diametrically opposite conclusions regarding the effectiveness of perfectly anticipated monetary policy, while making similar assumptions regarding expectations.

In a recent paper Begg (1980) has raised this important issue by an interesting comparison of these two theories. He claims that the essential difference between RE and MG models and their neutrality conclusions lies in the absence (or presence) of real balance effects. Rational expectations models typically ignore the role of real money balance in aggregate demand and get neutrality. Growth models with money stress this role and get non-neutrality.

It is difficult to accept that a fundamental division in the policy literature is based on a relatively minor issue, such as the existence of real balance effects. Moreover, there are important models of neutrality which do assume real balance effects and perfect foresight - yet get neutral money. Consider Patinkin (1965) where real balance effects are crucial in analysing existence and stability of non-negative solutions. There is no explicit expectations generating mechanism but the dynamics of the model imply that agents have perfect foresight. In spite of the crucial role of real balances, money is neutral. In Sidrauski (1967), aggregate households maximise an intertemporal utility function

$$J = \int_0^\infty e^{-\rho t} u(c, \hat{m}) dt$$

where c is per capita consumption and $\hat{m}$ per capita real balance) subject to a budget constraint. It is assumed that $u_{12} \neq 0$, $u_{21} \neq 0$. Thus the stock of real balance does affect consumption decisions. Given perfect foresight and the existence of real balance effects, money is proved to be superneutral.

These two models in the neutrality literature alert us to the fact that existence of real balance in aggregate expenditure functions may not be as crucial as Begg suggests. We propose that the difference between rational expectations models and growth models with money are more fundamental. The first major difference is that models of neutrality use suitable assumptions to make equilibrium output independent of the expected rate of inflation. In the first instance, a change in the anticipated growth rate of money affects the expected inflation rate which in turn influences output. By severing the link in this chain, it is easy to derive neutrality propositions. On the other hand, models of non-neutrality stress the link in every single case. Secondly, RE models deny portfolio allocation by which changes in money cause inflation and thereby affect real capital formation through the wealth constraint. MG models on the other hand rely heavily on the role of inflation in determining optimal choice of real balance and capital in a given portfolio (see Dixit (1976)). In the next section we show, by taking a large number of examples from the literature, that these are the two crucial differences between alternative policy conclusions. Real balance effects play a minor role in the story.

In the third section we use a monetary growth model to get exact analytic solutions to the behaviour of the economy out of steady state, consequent to changes in money stock. It is shown that <u>even if</u> money is superneutral, rate of investment will be different for different values of growth rate of money. Thus even superneutral and fully anticipated money has real effects in disequilibrium. The exact nature of this change is seen to depend on real factors. Section 4 discusses a monetary disequilibrium model where markets may not clear. The final section concludes.

2. Rational Expectations and Monetary Growth Models Compared

A slightly modified version of the RE-neutrality model by Sargent and Wallace (1975) is given by

$$y^s = y = a_1 k_{-1} + a_2(p-p^e) + u_1 \qquad (1)$$

$$y^d = y = b_1 k_{-1} + b_2 x + b_3 Q + u_2 \qquad (2)$$

$$m = p + c_1 y + c_2(x + p^e_{+1} - p^e) + u_3 \qquad (3)$$

$$k = d_1 k_{-1} + d_2 x + d_3 Q + u_4 \qquad (4)$$

(y, k, m, p, p^e are logarithms of output, real capital, nominal money stock, price level and expected price level; x is the real rate of interest; Q is an exogenous vector and u_i are white noise terms).

Equation (1) is the "surprise" supply function of Lucas (1973). Equation (2) gives the IS relation, equation (3) corresponds to the LM schedule and equation (4) gives the path of capital stock over time. Note that $(p^e_{+1} - p^e)$ is an index of the expected rate of inflation, π.

Sargent and Wallace have the nominal interest rate 'r' as an argument. We have simply used the relation $r = x + (p^e_{+1} - p^e)$, implicit in their paper, to rewrite the functions in terms of x. It is easy now to show that a deterministic money supply rule is neutral provided RE holds.

Begg claims that real balance effects are absent from the aggregate demand curve (2), and it is this assumption which ultimately gives neutrality. In other words rewriting (2) as

$$y = b_1 k_{-1} + b_2 x + b_3 Q + b_4(m-p) + u_2 \qquad (2')$$

($b_4 > 0$), then real balances are incorporated in the expenditure function and money is no longer neutral. However, we must be careful to distinguish between two separate (though related) propositions:

(i) Weak neutrality - output in the current period is independent of changes in money supply;

(ii) Strong neutrality - output in all future periods is independent of current changes in money supply.

Note that proposition (i) still holds even if real balance effects are present and the system to be analysed is (1), (2'), (3), (4). Given RE, $(p - p^e)$ is a function of random terms and exogenous processes alone. Thus current output is uninfluenced by changes in the money supply, from equation (1). This satisfies weak neutrality. However, from (2') real rate of interest x is dependent on money stock 'm'. From (4), current capital stock is affected by x and therefore by 'm'. Thus output in the next period is influenced by current capital formation, hence by current money supply.

$$y_{+1} = a_1 k + a_2(p_{+1} - p^e_{+1}) + u_1 \qquad (1')$$

There is a lag in the effectiveness of monetary policy. The strong neutrality proposition ceases to hold as soon as (2) is modified to incorporate the role of real money balance in aggregate demand.

Begg compares the standard RE model with a simplified monetary growth model where

$$\dot{K} = Y(K) - C(Y(K),K,Z) - \phi K \qquad (5)$$

$$Z = M/P = L(Y(K),x(K) + \Pi) \qquad (6)$$

$$\dot{Z}/Z = \dot{M}/M - \dot{P}/P = \theta - \Pi \qquad (7)$$

K is capital stock, Y is output, C consumption, Z the stock of real money balance, x the real rate of interest, M is stock of nominal money and P the price level. Given perfect foresight actual price inflation $\dot{P}/P$, is equal to expected price inflation Π. The rate of growth of money supply is θ, and is perfectly anticipated.

Here, consumption C depends on real balance Z given $C_3 \neq 0$. Comparing between steady states, it is shown that a change in θ will have real effects in the sense that steady state values of K will be affected provided $C_3 \neq 0$ and $L_2 \neq 0$. If real balance effects do not exist ($C_3 = 0$) then money is superneutral.

It is difficult to accept the contention that the basic difference between these two strands in the theoretical literature depends on what after all is an empirical matter. Monetarists have occasionally claimed that the dividing line between their models and Keynesian ones is an empirical one dependent on values of relevant elasticities. In this case, for example, if $C_3 = 0$, then monetary policy is relatively impotent. However, as Hahn (1980) states on the effectiveness of interventionist policy, "the difference of opinion is really ..... on a grand scale concerned with what is the most appropriate model of the whole economy". One suspects more fundamental analytical reasons for rational expectations and monetary growth models to give different policy conclusions, than simply the existence of real balance effects.

We suggest that the first major reason for neutrality of money is that the market clearing equilibrium equation for output ($y^s = y^d$) is made independent of the rate of inflation, by suitable choice of assumptions. In every single model that claims to have neutrality, appropriate assumptions are made such that the reduced form for output ($y^s = y^d = y$) is not influenced by expected inflation rate Π. Since growth rate of money affects Π in the first instance, and output is independent of Π, anticipated changes in money supply never feeds into the determination of output. Hence we have neutrality.

Consider first the basic RE model given by (1),(2),(3),(4). The assumption of rationality implies that $p = p^e + v$ where 'v' is a white noise term, independent of anticipated monetary policy. Therefore the aggregate supply of output (after substituting for $(p-p^e)$ in equation (1)) is not influenced by the rate of inflation. Similarly in equation (2), the demand for output is made dependent on real rate of interest x and is not affected by Π or $(p^e_{+1} - p_e)$.

However, when the real balance term is introduced in the aggregate demand equation as in (2') then 'y' is a function of M/P or (m-p). It can be shown that (m-p) is itself functionally related to $(p^e_{+1} - p^e)$, the expected rate of inflation. Solving (1) (2') (3) in the three unknowns y, p,x, we get the following reduced form expression for p:

$$p = J_0 p^e + J_1 p^e_{+1} + J_2 m + R \qquad (8)$$

where: $J_0 = (b_2 c_2 + a_2 c_2 + c_1 b_2 a_2)/D$

$J_1 = -b_2 c_2/D$

$J_2 = (b_4 c_2 + b_2)/D$

and $D = b_2 + b_4 c_2 + a_2 c_2 + c_1 b_2 a_2$

Note that $J_0 + J_1 + J_2 = 1$.

Taking expectations of (8), assuming rationality and perfectly anticipated money supply

$$E(p) = p^e = J_0 p^e + J_1 p^e_{+1} + J_2 m + E(R) \qquad (9)$$

from which

$$m = (1-J_0)p^e/J_2 - J_1 p^e_{+1}/J_2 - E(R) \qquad (10)$$

From (8)

$$m-p = (1-J_2)m - J_0p^e - J_1p^e_{+1} - R \qquad (11)$$

Substitute (10) in (11) and simplify

$$m-p = (p^e_{+1} - p^e)(-J_1/J_2) + \hat{R} \qquad (12)$$

(where $\hat{R}$ is a function of R, E(R) and can be ignored for our purpose).

Hence from (12), an index of real balance is uniquely related to the rate of expected price inflation.

Thus, real balance effects in (2') is one way of introducing an inflation rate term in the function determining output. Its importance lies not directly in the role that real money plays in aggregate demand. Rather, it is a proxy for Π which is the main vehicle through which money growth affects output. This can be seen clearly in the traditional argument about the slope of the expectations augmented Phillips curve. Friedman (1968) suggested that the relationship between unemployment (u), expected and actual price inflation should be

$$\dot{P}/P = f(u) + \Pi \qquad (13)$$

Given perfect foresight $\dot{P}/P = \Pi$, thus $f(u) = 0$ is independent of expected price inflation. In this model of the long run vertical Phillips curve, money is neutral. However, taking Solow's (1969) specification

$$\dot{P}/P = f(u) + \lambda\Pi \qquad \lambda \neq 1 \qquad (14)$$

Even with perfect foresight,

$$f(u) = (1-\lambda)\Pi$$

$$u = f^{-1}\,[(1-\lambda)\Pi] \qquad (15)$$

Changes in perfectly anticipated money supply will influence Π and thereby affect unemployment u. Thus money will not in general be neutral.

Finally, let us look at Sidrauski (1967), the most important example of superneutrality in growth models with money. As mentioned earlier, real balance effects are present in this model. Assume for simplicity that labour force is constant and Sidrauski's equations containing per capita variables can be rewritten in absolute terms. Households maximise an additive utility function over an infinite life span,

$$V = \int_0^{\infty} u(C_t Z_t)e^{-\rho t}\ dt$$

The production function is $Y_t = F(K_t)$. There are three first order conditions for an optimal path of which only the households budget constraint need concern us here. We have

$$\dot{K} + \dot{Z} + C = F(K) + T - Z\Pi \qquad (16)$$

The left hand side of (16) gives the expenditure side of household's budget where net wealth is $W = K + Z$ and $\dot{W} = \dot{K} + \dot{Z}$ is the net change in wealth. C is once again total consumption. The right hand side gives the income which consists of output Y plus two other variables. T is the rate of real money transfers and $-Z\Pi$ is the capital loss due to inflation.

The crucial assumption that ensures superneutrality is that

$$T = \theta Z \qquad (17)$$

i.e. transfers are made according to real cash balance held by households. Note that the rate of return by which transfers are calculated is not an arbitrary number such that $T = qZ$. Rather growth rate of money θ, determines how much transfer households will receive.

Substituting (17) in (16)

$$\dot{K} + \dot{Z} + C = F(K) + (\theta - \Pi)Z \tag{18}$$

Since $\dot{Z}/Z = \theta - \Pi$, we get finally

$$\dot{K} + C = F(K) \tag{19}$$

Thus the final form of the budget constraint is independent of Π and money can be shown to be superneutral. This is in spite of real balance affecting optimal consumption.

It should be clear that models of neutrality invariably use assumptions to vet out the effects of inflation on output (or other real variables) and thus stop the channel through which monetary policy can work. Patinkin's model (1965) does not need this mechanism since it has no inflation rate and Π does not appear in any functional form. Thus both Sidrauski (1967) and Patinkin (1965) can assume the existence of real balance effects and simultaneously get neutral money. These two conditions coexist since real variables are independent of rate of inflation.

The opposite is true of almost all growth models with money as Tobin's (1965) classic case will show. Tobin does not explicitly assume real balance effect in expenditure functions. In his model, real wealth and real net disposable income are defined by

$$W = K + Z \tag{20}$$

$$Y = F(K,L) - \phi K + \dot{Z} \tag{21}$$

Given $F(K,L) = C + \phi K + \dot{K}$

we have

$$Y = C + \dot{K} + \dot{Z}$$

$$Y = C + \dot{K} + (\theta - \Pi)Z \quad (22)$$

Thus equilibrium output is made to depend on rate of inflation Π. Using money market equilibrium conditions, it is shown that a change in the growth rate of money θ will affect output. Money is not superneutral. Once again let us stress that this non-neutrality does not require any explicit assumption of real balance affect.

It is safe to presume that real balance effects are neither necessary nor sufficient by themselves to prove results on neutrality or non-neutrality. The following table shows this clearly.

	Real balance effect present	Real balance effect absent
Sargent & Wallace		Neutral
Tobin		Non-neutral
Sidrauski	Superneutral	
Begg	Not superneutral	

In every single case, the existence or otherwise of rate of inflation in the output equation determines the nature of policy conclusions.

The second important respect in which RE and MG models differ is the way they tackle portfolio choice. Consider the standard case of non interest bearing money distribured free of charge. If x is the real interest rate and Π the rate of expected rate of inflation, the yield on holding capital is $x + \Pi = r$. Given total wealth, $W = K + Z$, the demand for capital is a function of its yield, i.e. $K = K(x+\Pi)$, $K' > 0$. Thus the desired ratio of holding the two assets in a portfolio and the demand for real balance are both dependent on $x + \Pi$. We have

$$Z = W - K(x + \Pi) = L(x + \Pi), \; L' < 0.$$

With transaction demand for money included one gets a standard liquidity preference function $Z = L(Y, x + \Pi)$ where $L_2 < 0$.

The active role of monetary policy on real variables (in MG models) can be seen clearly through the wealth constraint. An increase in the growth rate of money increases the rate of inflation Π. This reduces the demand for real cash balance and alters the portfolio in favour of capital stock. Alternatively, an increase in Π increases the yield and attractiveness of capital and its demand (and supply in equilibrium) increases. Thus expansionary monetary policy is associated with higher steady state value of capital stock.

The essential point to remember is that in MG models, the expected (and actual) rate of inflation affects capital accumulation. If we look at RE models, however, this mechanism is absent. Equation (4) shows that capital stock is not a function of total yield on capital ($r = x + \Pi$), but only on the real interest rate x. Thus Π is conspicuous by its absence in the functional form for capital stock. Note that the wealth constraint $W = K + Z$ which is an essential feature of growth models is violated too since demand for real balance is a function of Π (equation (3)) while demand for capital is not. Thus RE models deny an important channel through which monetary policy can be effective, while MG models stress its importance.

The portfolio choice-wealth constraint conduit of monetary policy will be shown to be even more crucial in a result proved in the next section. We show that even if real balance effects are absent and money is superneutral, yet perfectly anticipated changes in money growth has real effects out of steady state equilibrium. This result crucially depends on $L_2 \neq 0$, i.e. the proper functioning of the wealth constraint and portfolio behaviour.

To sum up, changes in money growth, initially affect the rate of expected inflation. If output and capital accumulation are made independent of the inflation rate by suitable assumptions, then money is neutral or superneutral. On the other hand, if disposable income is suitably defined or portfolio choice is allowed its proper role, the path of capital and output depend crucially on rate of change of expected prices. Then we get standard non-neutrality conclusions. The role of real money balance is relatively peripheral in this respect and is just one of many alternative assumptions that gives money an important role in the economy.

Section 3: Monetary Growth Model

The simple monetary growth model that we propose eschews the use of real balance effects in the aggregate consumption (demand) function. We also impose conditions such that the model obeys superneutrality in long run equilibrium. Thus it is made to conform closely to the new classical structure; the stock of real money does not affect real expenditure directly; nor does the growth rate of money influence steady or stationary state capital stock. However, in the light of our previous discussion, the rate of inflation is introduced (indirectly) in the aggregate demand function and this, as we shall see, causes real effects of monetary change. Thus money is non-neutral outside of steady state. Since the labour force plays no crucial role in the analysis we assume a constant supply of labour (which is never used up fully) and ignore the labour market. All attention is focussed on the behaviour of capital stock, investment and, of course, the rate of inflstion. We also assume perfect foresight so that the actual and expected inflation rates are equal.

The supply of output is a function of the capital stock, $F(K)$. There is no independent investment function; so net investment is the time derivative of capital stock, $\dot{K}$. Consumption is a function of disposable income as well as capital stock (giving the effect of wealth). Disposable income, following Tobin, is defined as aggregate output plus the change in the stock of real money balance ($\dot{Z}$). In principle, depreciation should be netted out from disposable income but this makes no difference to the final results (provided the rate of depreciation, ϕ, is small, specifically $F'(K)>\phi$) and is therefore left out from the consumption function. The equilibrium in the commodity market is given by, following (5),

$$F(K) = \dot{K} + C + \phi K$$

or

$$\dot{K} = F(K) - C(F(K) + \dot{Z}, K) - \phi K \qquad (23)$$

We stress the fact that C does not depend on real balance directly; there is no real balance effect in the sense of Patinkin (1965).

Money market equilibrium is characteristed by the equality of the demand and supply of real balance. Thus,

$$Z = M/P = L(F(K), x(K) + \pi) \qquad (24)$$

where $x(K) = F'(K)$ is the real rate of interest or marginal productivity of capital ($x'<0$); Π is both the expected and actual rate of inflation; thus $x + \Pi$ is the nominal interest rate. It should be noted that demand for money (L) should also, in principle, be made a function of disposable income; equation (24) shows that L depends only on F(K). This is only a matter of convenience and tractability; basic results remain unchanged but the cost of algebra becomes large. Since the central points of the chapter are unconnected with the (income) specification of the money demand function, we take the simpler function given by (24) rather than the following alternative:

$$Z = M/P = L\,(F(K) + \dot{Z},\ x(K) + \Pi) \qquad (24)'$$

Finally, from definition,

$$\dot{Z} = (\theta - \Pi)Z \qquad (25)$$

where $\theta = \dot{M}/M$ is the rate of growth of money supply, given exogenously.

The signs of the partial derivatives of the functions in (23) and (24) are conventional.

$$F' > 0,\ C_1 > 0,\ C_2 > 0,\ L_1 > 0,\ L_2 < 0,\ x' < 0, \qquad (26)$$

Long run equilibrium or stationary state is given by $\dot{K} = \dot{Z} = 0$. It is clear that the model has superneutrality since equilibrium K^* is independent of θ and is defined by

$$F(K^*) - C(F(K^*),K^*) - \Phi K^* = 0 \qquad (27)$$

However, our main purpose is to show that the behaviour of K out of long run equilibrium is crucially influenced by the money growth parameter θ; a fortiori, the rate of investment ($I = \dot{K}$) is also effected by changes in θ on the transition path to stationery state. Thus if the economy is not in full equilibrium monetary policy does have an affect on real variables such as investment, and hence growth.

To determine the behaviour of the variables consider first equation (24). Taking total differentials,

$$dZ = BdK + L_2 d\Pi \qquad (28)$$

where

$$B = L_1 F' + L_2 x' > 0 \tag{29}$$

Then, $L_2 d\Pi = dZ - BdK$

and

$$\partial\Pi/\partial Z = 1/L_2 < 0, \ \partial\Pi/\partial K = -B/L_2 > 0 \tag{30}$$

We now need to analyse the two stationaries $\dot{Z} = 0 = \dot{K}$, at steady state equilibrium $Z = Z^*$, $K = K^*$. From (25), using (29)

$$\partial\dot{Z}/\partial K = - Z\partial\Pi/\partial Z = - Z^*/L_2 > 0 \tag{31}$$

$$\partial\dot{Z}/\partial K = - Z\partial\Pi/\partial K = Z^* B/L_2 < 0 \tag{32}$$

Similarly, from (23), and (31),

$$\partial\dot{K}/\partial K = A - (C_1 B Z^*/L_2) \tag{33}$$

$$\partial\dot{K}/\partial Z = C_1 Z^*/L_2 < 0 \tag{34}$$

where

$$A = F'(1-C_1) - C_2 - \Phi \tag{35}$$

In effect the linearised system, around the equilibrium, given $k = K - K^*$, $z = Z - Z^*$, is the following:

$$\begin{bmatrix} \dot{k} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A - (C_1BZ^*/L_2) & (C_1Z^*/L_2) \\ BZ^*/L_2) & -(Z^*/L_2) \end{bmatrix} \begin{bmatrix} k \\ z \end{bmatrix} \qquad (36)$$

Note that all partial derivatives are evaluated at $\Theta = \Pi$ where Θ is an exogenous parameter giving money growth.

To get an idea of the signs of the characteristic roots we need to sign the determinant of the Jacobian, $|J|$. This is

$$|J| = -(AZ^*/L_2) \qquad (37)$$

The trace of the Jacobian matrix, J, is

$$\text{trace } J = A - (C_1BZ^*/L_2) - (Z^*/L_2) \qquad (38)$$

Suppose both roots are real. Consider first the case where $A > 0$. Then, from (37), $|J| > 0$. Therefore, either both characteristic roots are positive or both are negative. But we note from (38) that trace J is also positive. Thus, at least one root is positive. Hence both roots must be positive - a totally unstable case. We neglect the possibility of $A > 0$. Consider now the alternative where A may be negative. If in addition $[A - (C_1BZ^*/L_2)] > 0$, then the trace is again positive. At least one characteristic root must be positive. But note from (37) that the determinant of J is negative for this case. Thus one characteristic root is positive while the other is negative. We have a saddle point equilibrium. On the other hand, even if $[A - C_1BZ^*/L_2] < 0$ and the trace is negative, implying at least one root to be negative, we still have a saddle point equilibrium given $|J| < 0$. The existence of a unique pair of paths leading to long run equilibrium, a property common to perfect foresight models, is now guaranteed. Remember, for consistency and stability, $A < 0$; hence from (33), the sign of $\partial\dot{K}/\partial K$ is indeterminate.

We are now in a position to draw the phase diagrams. Using equations (31) to (34),

$$(dZ/dK)\Big|_{\dot{K}=0} = B - (AL_2/C_1Z^*) \qquad (39)$$

$$(dZ/dK)\Big|_{\dot{Z}=0} = B > 0 \qquad (40)$$

The slope of the $\dot{Z} = 0$ line in the (K, Z) plane is always positive. The slope of $\dot{K} = 0$ may be positive or negative. But given that A and L_2 are both negative, slope of $\dot{K} = 0$ is always less than the slope of $\dot{Z} = 0$ (see (39) and (40)).

Using (6.33) we note that when $[A - (C_1BZ^*/L_2)]$ is positive then $\dot{K} = 0$ slopes upwards. Then the phase diagram is given by Figure 7.1. Alternatively, $\dot{K} = 0$ may slope downwards and we have the phase lines shown in Figure 7.2. The saddle paths are denoted by SS.

As discussed earlier, the model has been constrained to obey superneutrality in long run equilibrium. The impact of a change in money growth, Θ, falls totally on equilibrium real balance Z^* and capital stock in steady (stationary) state remains unchanged. This is obvious from the definition of steady state:

$$F(K^*) = C(F(K^*),K^*) - \Phi K^* \tag{41}$$

$$Z^* = L(F(K^*),\ x(K^*)) + \Theta \tag{42}$$

(where $dK^*/d\Theta = 0$ and $(dZ^*/d\Theta) < 0$)

Since the effect of the rate of inflation Π has been removed from the market clearing condition of output, at steady state (see(4)), monetary growth has no effect on capital stock and a fortiori on output in long run equilibrium. Changes in Θ work via Π and this transmission mechanism is effectively suppressed.

Consider now the effect of an unanticipated change in Θ, first comparing only the two long run equilibria. If the economy is initially in steady state at E_1, (see figures 7.3, 7.4), and Θ falls (contractionary monetary policy) then the new long run equilibrium is at E_2. Capital stock is a "backward looking" state variable whose current value is given by past decisions and which cannot respond to current information or parameter changes. The instantaneous inflation rate $(\Delta P/P)$ is, on the other hand, a "foreward looking" or "jump" variable which can respond rapidly to the movement of Θ. It falls rapidly at a rate faster than the new growth rate of money. Thus the real value of money holding (Z) rises until the new equilibrium at E_2 is reached. Note that the existence of the saddle path, or unique trajectory to long run equilibrium, requires that the economy moves rapidly to E_2. If the full jump, from E_1 to E_2, is not made then the new stationary state can never be reached. All other points on the vertical through K^* are unstable.

The economic rationale behind the postulated movements should be clarified. There are two assets in this model --- real money balance and capital stock. The rational investor wishes to keep both assets in his "portfolio"; capital stock is required to produce output which gives consumption; money balance facilitates transactions and has its own utility. The proportion and amounts of the two forms of wealth in his portfolio will depend, at the margin, on the relative rate of return. Capital has its own marginal productivity as well as capital gains from inflation. A reduction in money growth is expected to reduce the rate of inflation; thus the return, at the margin, of capital declines relative to real money; the holding of the latter asset (real balance) increases in the portfolio.

Since there are positive returns from the two assets, it is expected that positive quantities of both assets will be held by rational investors in their portfolio. Given the nature of the saddle point equilibrium, all paths except the stable trajectory diverge away from long run equilibrium; the final outcome is either zero holding of capital or real money (unstable paths always hit the axes in the limit). This is undesirable. Thus the agents in the economy wish to attain long run equilibrium characterised by positive holding of K^* and Z^*. For an unanticipated change in O, leading to a shift in equilibrium from E_1 to E_2, the only way to reach the new stationary state (equilibrium) is for the economy to jump i.e. rapidly transfer financial demand towards the holding of money. thus reducing the inflation rate for the aggregate commodity.

It should be noted that the jump itself is a response to a discontinuity. The behavioural functions, in the market clearing equilibrium relations for commodity and money, represent agents' optimisation. At the point of parametric change these equilibrium conditions do not apply; hence behavioural demands are not satisfied and optimality is violated. Though the jump per se is not desirable it has to be made to attain the new equilibrium and concomitant optimum situation. Since the jump is not desired for its own sake, the fewer the number of such movements and smaller the "length" or amount by which variables have to adjust rapidly, the more preferable it is. Thus, in this case given by figures 7.3 and 7.4, the economy will jump only once to attain the new long run equilibrium and bridge the discontinuity.

The foregoing analysis considered an unanticipated and permanent change in O. We can also analyse the various cases of (a) anticipated and permanent, (b) unanticipated and temporary, as well as (c) anticipated and temporary parametric shifts. Figures 7.5, 7.6, 7.7 show the dynamic movements of K and Z for these possibilities. In each case, O falls and the equilibrium shifts from E_1 to E_2. The basic points remain invariant; multiple jumps are undesirable; a smaller jump is preferred to a larger one.

If the policy shift is anticipated and permanent, the monetary authorities may announce that O will change T_1 periods hence, Figure 7.5 shows that a smaller jump will occur instantaneously from E_1 to F. At F the laws of motion of K and Z pertain to the initial equilibrium

E_1. Thus capital stock will fall and real balance rise. It will take the economy T_1 periods to reach the new saddle path S'S' at the point G. The shift in policy parameter now takes place (T_1 time periods after the initial announcement) and the relavant saddle path for the new and permanent long run equilibrium (at E_2) is clearly S'S'. Capital and real balance traverse along this path to move the economy towards E_2. It is important to note that capital stock falls and then rises during the movement from E_1 to E_2. Thus we have a business cycle with negative investment as the initial response to monetary contraction.

Similarly analysis can be done for the two cases when the change in Θ is expected and believed to be temporary. The main motivation of the dynamics lies in the efficient traverse of the economy when the equilibrium shifts to E_2 only temporarily; it is known that the final stationary state will be at E_1. The time period when Θ is lower temporarily is given by T_2 in the diagrams. Once again we observe the possibility of real business cycle in response to monetary shocks. Capital stock falls and rises; net investment is negative and positive; all of these occour for a change in the rate of growth of nominal money and the corresponding dynamics of the inflation rate.

It is clear that even under superneutrality, there are quite a few possibilities where monetary shocks have real effects particularly when the economy is out of long run equilibrium. Markets always clear; expectations are held rationally; announced policy changes are perfectly anticipated; yet cyclical behaviour of capital stock conot be ruled out consequent to activist monerary policy. Real balance effects play no direct role in the sense of influencing aggregate (consumption) demand.

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Let us now return to the first and possibly the most important case; here the change in Θ is unanticipated and known to be permanent. The analysis until now has been confined to transition from one steady state to another (see figures 7.3 and 7.4) We would now like to investigate formally what happens if initially the economy is not at steady state. Would the parameter shift in Θ have any effect on the transition of the economy to its long run equilibrium capital stock ? Therefore, the economy is, suppose, on the saddle path SS but not at E_1. Rate of growth of monetary stock, Θ, falls and the stock of real balance held jumps to the corresponding level given by S'S'; capital stock of course behaves as a state variable given by its initial level. The formal analysis, described below, tells us how investment and capital stock behaviour changes as the economy attains K^* from the initial K_0 (see figures 7.8)

Consider the linearised system given by (36). Let the characteristic root be λ. Remember that Z^* is a function of the parameter Θ and that all partial derivatives are evaluated at $\Theta=\Pi$, then the characteristic equation can be written as a function of λ and Θ. We have (see 36)

$$\Psi(\lambda,\Theta) = \lambda^2 + \lambda\left[(Z^*/L_2) - A + (C_1 B Z^*/L_2)\right] - \left[AZ^*/L_2\right] = 0 \qquad (43)$$

We know there are two real and distinct values of λ; concentrate henceforth on the convergent case given by $\lambda < 0$. Since λ characterises the path of $K(t)$, where t is time, we need to find how a change in Θ affects λ and hence $K(t)$. Thus it is necessary to sign $d\lambda/d\Theta$.

First we need to prove the following:

Lemma: $(A - \lambda) < 0$

The characteristic equation for the system is (see also (36))

$$\begin{bmatrix} A - (C_1 BZ^*/L_2) - \lambda & (C_1 Z^*/L_2) \\ (BZ^*/L_2) & (-Z^*/L_2) - \lambda \end{bmatrix} = 0 \qquad (44)$$

$$\left[A - (C_1 BZ^*/L_2) - \lambda\right]\left[(-Z^*/L_2) - \lambda\right] = BC_1(Z^*/L_2)^2 \qquad (45)$$

Thus

$$\left[A - (C_1 BZ^*/L_2) - \lambda\right] = -\left[BC_1(Z^*/L_2)^2\right]/\left[(Z^*/L_2) + \lambda\right] > 0 \qquad (46)$$

Then, after simplification, we have

$$(A - \lambda) = \left[C_1 BZ^*\lambda/(\lambda L_2 + Z^*)\right] < 0 \qquad (47)$$

given λ_1, L_2 negative.

The derivation of the sign of $d\lambda/d\Theta$ proceeds in a number of stages. First we note that

$d\lambda/d\Theta \neq 0$ provided $L_2 \neq 0$ or $dK^*/d\Theta \neq 0$

This can be formally established. We have from the characteristic equation, (43), $\Psi(\lambda, \Theta) = 0$,

$$d\lambda/d\Theta = -(\partial\psi/\partial\Theta)/(\partial\psi/\partial\lambda) \qquad (49)$$

Observing (43), for a given Θ, $\Psi(-\infty, \Theta) > 0$

Further, there is only one negative root since the other is positive by saddle point properties. Therefore, $\partial\Psi/\partial\lambda < 0$, at the appropriate (negative) root. The diagram (Figure 7.9) shows this clearly. Given $\partial\psi/\partial\lambda < 0$, from (49) it is clear that

$$\text{sign } d\lambda/d\Theta = \text{sign } \partial\Psi/\partial\Theta \qquad (50)$$

We need now an expression for $\partial\Psi/\partial\Theta$.

It is important to realise that due to linearisation we are studying the properties of the dynamic system in a close neighbourhood of equilibrium. Accordingly, it is reasonable to assume that the partial derivatives of (23) to (25) are invariant to changes in Θ. This is by virtue of linearisation. However

$$Z^* = L(F(K^*),\ x(K^*) + \Theta)$$

is obviously dependent on Θ and we get

$$dZ^*/d\Theta = B\ dK^*/d\Theta + L_2 \qquad (51)$$

(where B is given by (29)).

We are now in a position to derive the value of $\partial\Psi/\partial\Theta$. From (43)

$$\partial\Psi/\partial\Theta = \left[(\lambda/L_2) + (\lambda C_1 B/L_2) - (A/L_2)\right]\ (dZ^*/d\Theta) \qquad (52)$$

Further, (43) can be written as

$$\Psi = \lambda(\lambda - A) + \left[(\lambda/L_2) + (\lambda C_1 B/L_2) - (A/L_2)\right]Z = 0 \qquad (53)$$

The first term of the RHS in (53) is negative since $\lambda < 0$ (stable root) and $(A - \lambda) < 0$ (by lemma). Hence the second term must be positive. Given $Z > 0$,

$$\left[\lambda/L_2 + (\lambda C_1 B/L_2) - (A/L_2)\right] > 0 \qquad (54)$$

Comparing (54) with (52) we observe

$$\text{sign}\ \partial\Psi/\partial\Theta = \text{sign}\ dZ^*/d\Theta \qquad (55)$$

Using (50), (51) and (55),

$$\text{Sign } (d\lambda/d\theta) = \text{sign}\left[B(dK^*/d\theta) + L_2\right] \tag{56}$$

This expression in (56) proves the assertion made earlier in (48). Given $B > 0$, we need to have both $(dK^*/d\theta) = 0$ and $L_2 = 0$ to get $(d\lambda/d\theta) = 0$. Alternatively, if either $dK^*/d\theta \neq 0$ or $L_2 \neq 0$, then $d\lambda/d\theta$ is not zero.

We therefore find that the rate of growth of money stock will affect the stable characteristic root, and hence the dynamic path of capital stock as well as the economy, provided liquidity preference holds ($L_2 \neq 0$) or money is not superneutral (steady state capital is affected by monetary growth and $(dK^*/d\theta) \neq 0$). Since our model presupposes superneutrality, we do indeed have $dK^*/d\theta = 0$. However, the negativity of L_2 allows money growth so affect capital stock outside of steady state. We now turn to formalising this relation.

It is clear from (56), given $dK^*/d\theta = 0$ and $L_2 < 0$, that

$$d\lambda/d\theta < 0 \tag{57}$$

Given $\lambda < 0$, an increase in θ will raise the absolute value of the negative root λ. Solving for the paths of K we have

$$K(t) - K^* = (K(0) - K^*)e^{\lambda t} \tag{58}$$

Further,

$$\dot{K} = \lambda(K(0) - K^*)e^{\lambda t}$$

Given, from (58),

$$(K(0) - K^*) = (K(t) - K^*)e^{-\lambda t} \tag{59}$$

We get

$$\dot{K} = (-\lambda)(K^* - K(t)) \tag{60}$$

(where $K(0)$ is an arbitrary initial value of capital stock)

Consider the case where $K(t) < K^*$ for any given time period. Thus, on the stable path, investment $I = \dot{K}$ is positive. An increase in 0, using (57), will increase $(-\lambda) > 0$ and thus raise investment (from equation (60). Thus, if investment is positive, the rate of investment is faster, higher is the growth rate of money. We have therefore demonstrated that even though money growth cannot affect steady state capital stock, it can increase investment on the transitional path. Hence the economy traverses faster to long run equilibrium.

We can also formally derive the behaviour of the time path of real balance Z from (36),

$$\dot{K} = [A - (C_1 BZ^*/L_2)][K(t) - K^*] + (C_1 Z^*/L_2)[Z(t) - Z^*] \qquad (61)$$

or using (60),

$$[Z(t) - Z^*] = \{ [\lambda - A + (C_1 BZ^*/L_2)][K(t) - K^*]\}/\{[C_1 Z^* L_2]\} \qquad (62)$$

The characteristic equation for the system, and (46), tells us that $[Z(t) - Z^*]$ has the same sign as $K(t) - K^*$. Thus $Z(t)$ and $K(t)$ move in the same direction and the saddle path is positively sloping. This can be verified by looking at the phase diagrams.

Let me summarise. The model assumes superneutrality; thus a change in the growth rate of money cannot affect long run capital stock, though it will change the stock of real money balance. However, monetary growth does have an effect out of long run equilibrium even if it is unanticipated. In particular, an unexpected but permanent shift in the growth rate of money will change the rate at which investment takes place and the speed with which current capital reaches the stationary (steady) state value. If investment is positive then contractionary monetary policy will reduce the amount of investment and thereby have real effects. The latter are strengthened with anticipated and or temporary changes in money growth. These movements end up in business cycles with capital stock rising and falling. All this has been proved without assuming the presence of the standard real balance effect whereby the stock of real money affects aggregate demand.

Section 4: A Monetary Disequilibrium Model:

It is of interest to note how the growth model analysed in the previous section would change if we introduced disequilibrium behaviour -- for example, by assuming that markets do not clear instantaneously and that demand and supply in some aggregate market(s) may not be always equal. Once again, let us assume superneutrality and concentrate attention on the transition path out of steady (stationary) state.

In principle, both the money and the aggregate commodity market may have disequilibrium. Since this is a monetary growth model, we assume that it is possible for the money market to have disequilibrium. Excess supply of money raises the inflation rate in monetarist fashion while excess demand for money lowers inflation. We also assume that inflation is equal to a "core" rate given by the growth of money supply (see Buiter and Miller (1981)). Thus

$$\Pi = \Theta + \gamma\left[Z - L(F(K),\ x(K) + \Pi)\right] \qquad (63)$$

The formulation has a number of advantages: in long run equilibrium, demand and supply of money are equal; so also is Π and Θ; thus we can contrast the behaviour of the economy on the transition path given that the equilibrium and disequilibrium models have the same stationary state; the comparisons then can focus on the short run, our main subject of interest. We also keep close to the central concerns of the monetarists i.e. superneutrality and the ability of the money market to cause inflation. Changes in structure are thus kept to a minimum. Yet, as we shall see later, quite fundamental differences emerge and capital stock behaviour, including the possibility of business cycles, may be dissimilar to standard monetarist predictions.

Proceeding as before, we need to find the effect of the two endogenous variables (K, Z) on the rate of inflation Π. Totally differentiating (63),

$$d\Pi = dO + \gamma\left[dZ - BdK - L_2 d\Pi\right] \quad (64)$$

Simplifying,

$$d\Pi = \left[dO/1+\gamma L_2\right] + \left[\gamma(dZ/1+\gamma L_2)\right] - \left[(\gamma B/1+\gamma L_2)(dK)\right] \quad (65)$$

Thus

$$\partial\Pi/\partial Z = \gamma/1+\gamma L_2 = 1/D \quad (66)$$

where $D = \left[(1/\gamma) + L_2\right]$

and

$$\partial\Pi/\partial K = - B/D \quad (67)$$

The monetary disequilibrium model is given now by

$$\dot{K} = F(K) - C(F(K) + \dot{Z}, K) - \Phi K \quad (68)$$

$$\dot{Z} = (O - \Pi)Z \quad (69)$$

$$\Pi = O + \gamma\left[Z - L(F(K), x(K) + \Pi))\right] \quad (70)$$

with the partials of the Π derived from (66) and (67).

We need to draw the phase diagrams from this pair of differential equations. Following usual practice, the equations are linearised in the neigbourhood of equilibrium (Z^*, K^*). Remember the long run equilibrium values have same properties as those in the previous sections.

Using (70), (69) can be written as

$$\dot{Z} = \gamma\left[L(F(K), x(K)+\Pi) - Z\right] \quad (71)$$

so that

$$\partial \dot{Z}/\partial Z = \gamma Z[\partial L/\partial Z - 1] \quad (72)$$

$$\partial \dot{Z}/\partial K = \gamma Z[\partial L/\partial K] \quad (73)$$

Note, all partials are evaluated at $\dot{Z} = o = \dot{K}$)

We know that

$$\partial L/\partial Z = L_2(\partial \Pi/\partial Z) = L_2/D \quad (74)$$

$$\partial L/\partial K = B + L_2 \partial \Pi/\partial K = B(1-(L_2/D)) \quad (75)$$

Therefore, from (72) to (75) we get

$$\partial \dot{Z}/\partial Z = - Z/D \quad (76)$$

$$\partial \dot{Z}/\partial K = BZ/D \quad (77)$$

$$dZ/dK\Big|_{\dot{Z}=0} = B \quad (78)$$

Turning to the equation for capital stock, (68), we get

$$\partial \dot{K}/\partial K = A - C_1(\partial \dot{Z}/\partial K) \quad (79)$$

$$\partial \dot{K}/\partial Z = - C_1(\partial \dot{Z}/\partial Z) \quad (80)$$

where A is defined in the previous section by equation (35). Note A is negative as assumed earlier. Using (76) and (77), we get from (79) and (80) the following:

$$\partial \dot{K}/\partial K = A - (C_1 ZB/D) \quad (81)$$

$$\partial \dot{K}/\partial Z = C_1 Z/D \quad (82)$$

$$dZ/dK\Big|_{\dot{K}=0} = B - AD/C_1 Z \quad (83)$$

The composite parameter D is crucial. Recall the expression

$$D = \left[(1/\gamma) + L_2\right] \qquad (84)$$

where $\gamma > 0$ is the speed of adjustment, of inflation to its core equilibrium rate, consequent to disequilibrium in the money market; L_2 is the standard parameter for liquidity preference. It is clear that D can be positive or negative depending on the strength of the adjustment parameter γ.

Consider first the case where γ is "small", thus $D > 0$. From (78) and (82) we note that both $\dot{K}=0$ and $\dot{Z}=0$ lines slope upwards (remember $A<0$, $D>0$) and the former has a higher slope. Using this, plus the information embodied in (76), (77), (79) and (80) we can draw the phase diagrams as shown in Figure 7.10.

It can be easily demonstrated that the trace of the Jacobian is negative and its determinant positive. We can also prove that both roots are real. We have two negative and real roots; the model is stable (in the Rothian sense) with convergent movements towards long run equilibrium. Thus cycles, emanating from disequilibrium initial starting polints, can be ruled out.

If now the rate of growth of money supply, θ, is reduced then the $\dot{K}=0$ and $\dot{Z}=0$ lines shift to the position given by the dotted lines in Figure 7.10. The new long run equilibrium is at E_2. If the economy was at initial stationary state E_1, then capital stock first falls and then rises as shown by the arrowed lines. We do observe a business cycle since the capital labour declines initially but increases subsequently. This only occurs because the initial situation was one of equilibrium and stable behaviour precludes "jumps" in variable and causes a steady, but slow, movement towards the new long run steady state.

If we contrast this model with the previous one the fundamental difference is in the stability properties. It is seen that disequilibrium, money demand not equal to supply, is stabilising even under rational expectations. In a sense rapidly changing expectational variables (as would happen under rational, as contrasted with adaptive, expectations) tends to impart a degree of instability into the system. Thus we have saddle point properties with actual (and expected) inflation rate jumping to catch the convergent paths. When markets do not clear and the speed of adjustment is low (remember γ is small) then the model becomes stable. Thus we have now two negative characteristic root rather then one as in the last section's model.

One the other hand, if γ, the speed of adjustment in the face of market-clearing disequilibrium, is large then it is possible for D (given by (84), may be negative. The limiting case is obviously that of $\gamma \to \infty$ so that Z=L and we are back to the equilibrium model. Even with disequilibrium however a large enough γ will make D<0. Then the determinant of the Jacobian, which is (-AZ/D), is negative; thus we once again get saddle point equilibra. Two cases, similar to those previously discusseed, are shown in figures, 7.11 and 7.12. Disequilibrium responses are not slow enough to compensate for rapidly changing expectations. The resultant instability can only be compensated by having jump variables such as the rate of inflation.

Section 5: Conclusion

This chapter had two purposes. First, we wished to analyse the conditions under which superneutrality -- the invariance of capital stock (or output) to changes in the rate of growth of money supply -- will hold. We conclude that if the dynamic equations for the real variables of the model can be made invariant to the rate of inflation, then these real variables will not be affected by monetary growth. This is because discretionary monetary policy affects initially the rate of inflation. If inflation has real effects, in the short and long run, money (growth) will also have real effects. The role of inflation, and thus money, can be brought about in various ways. One method, proposed by Tobin, (1965) and used here, is to use real disposable income as a determinant of the consumption function. This includes current output and change in the real value of money stock; the latter is clearly affected by inflation; thus money growth and concomitant price changes have real effects.

The second purpose of the chapter was to study various non steady state and disequilibrium behaviour of the economy under monetary growth, given that money was superneutral. We have found that monetary policy, both anticipated and unanticipated, can cause real effects. In particular capital stock and investment are affected by changes in money growth when the economy is in disequilibrium or out of steady state. Specifically, monetary policy can cause business cycles in the sense that capital stock may fall (rise) initially and then rise (fall) later consequent to monetary contraction (expansion). Long run neutrality may not stop the real economy from being affected in a major way by changes in money growth rates, particularly in the short run and under disequilibrium.

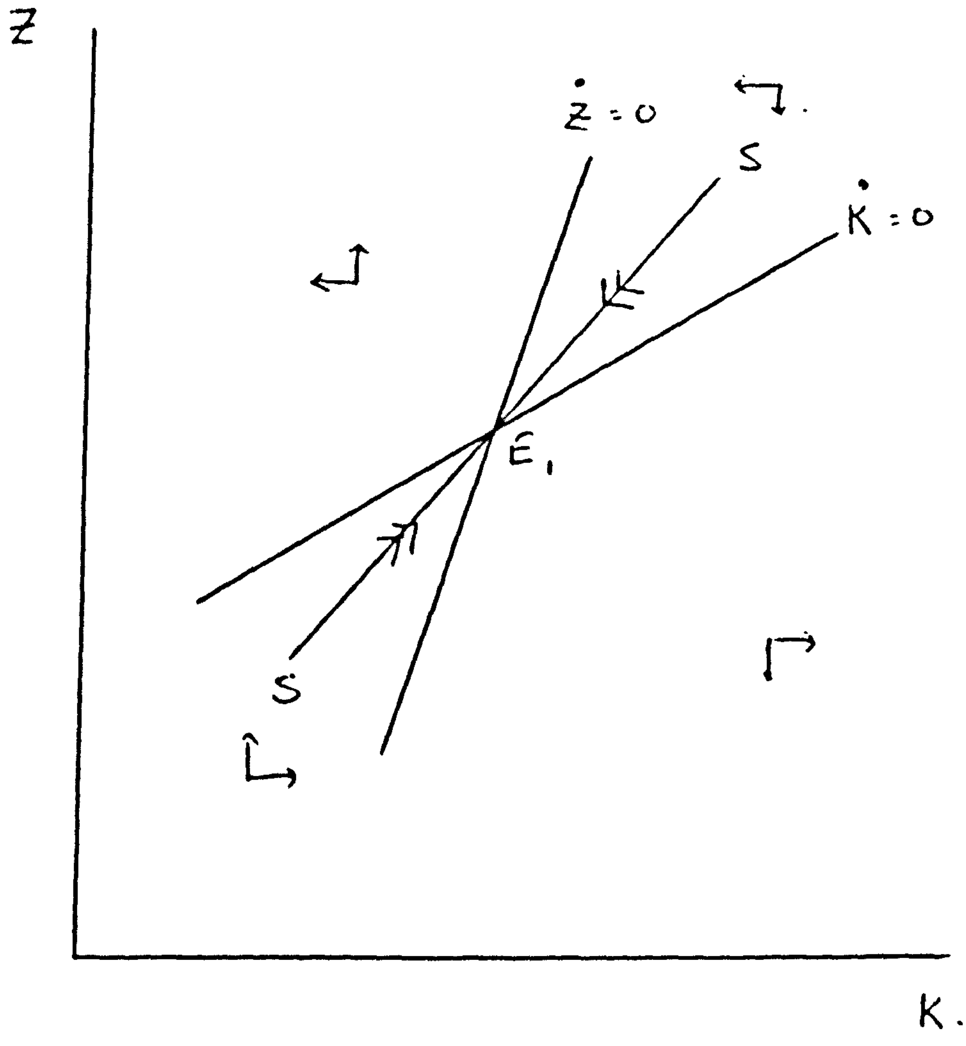


Figure 7.1

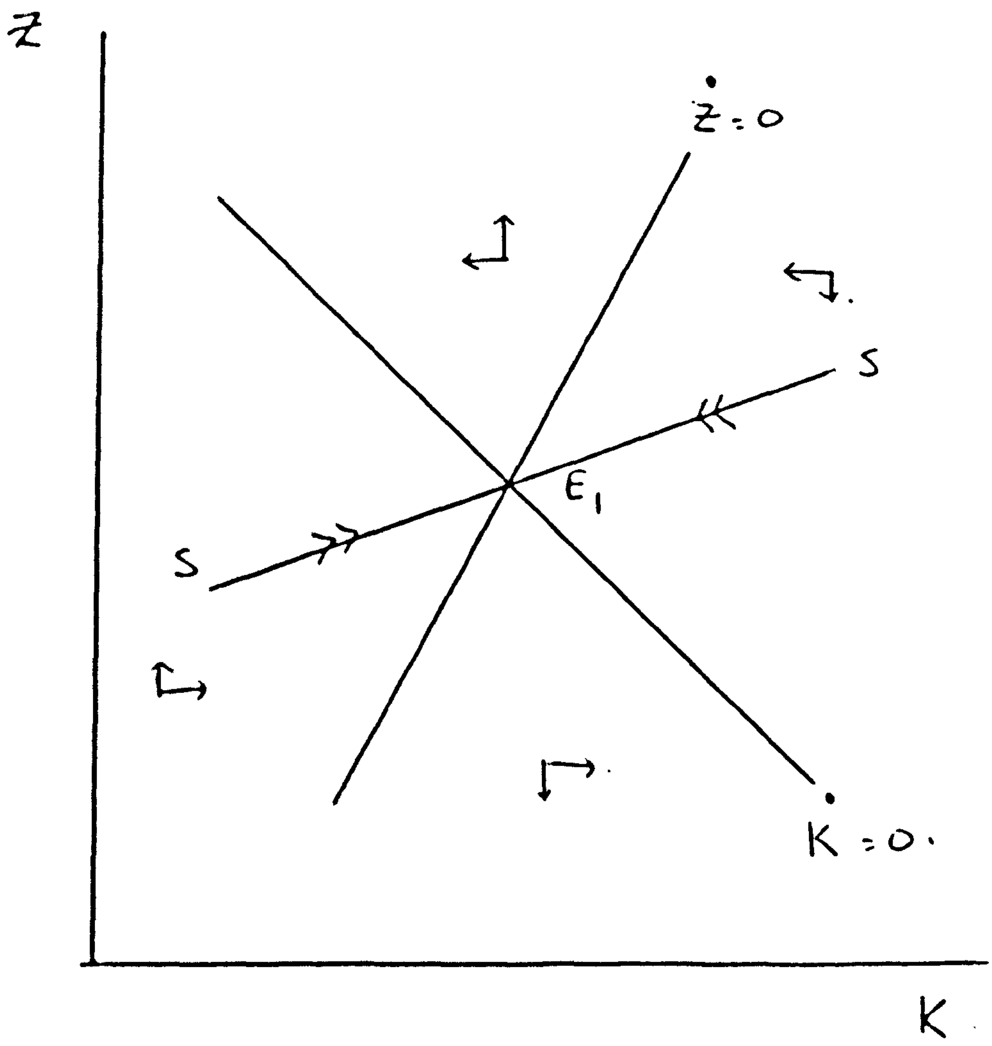


Figure 7.2

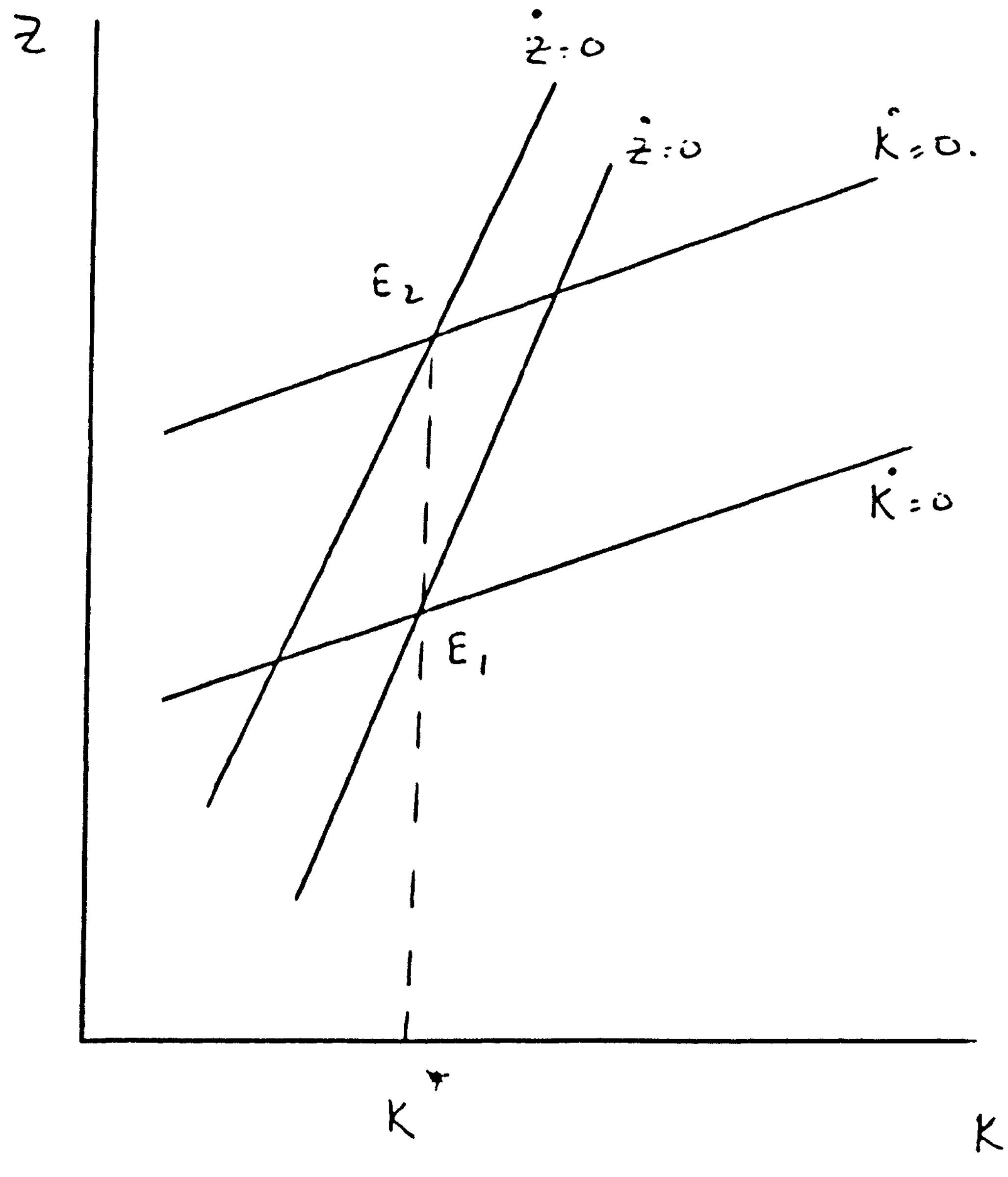
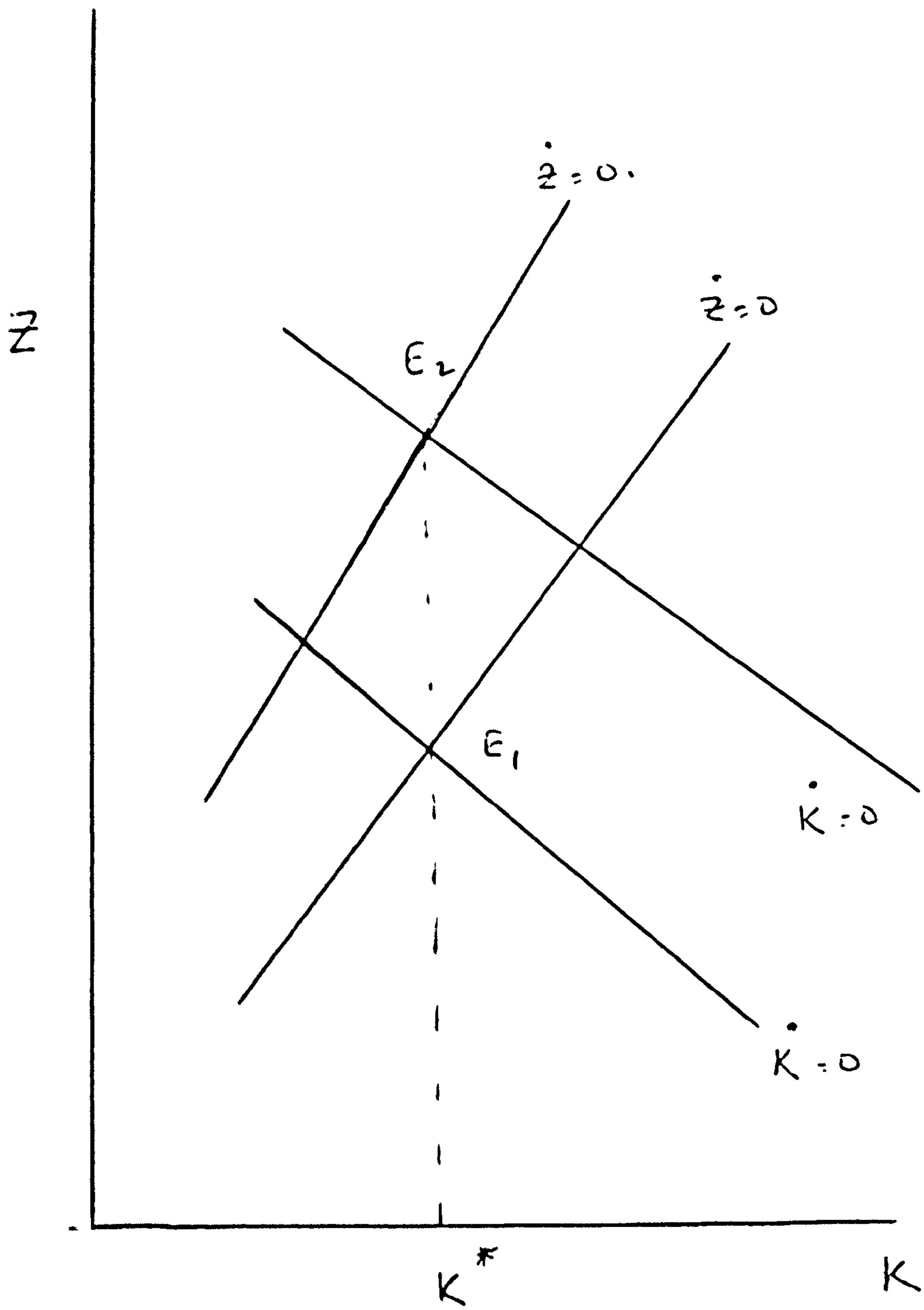


Figure 7.3.



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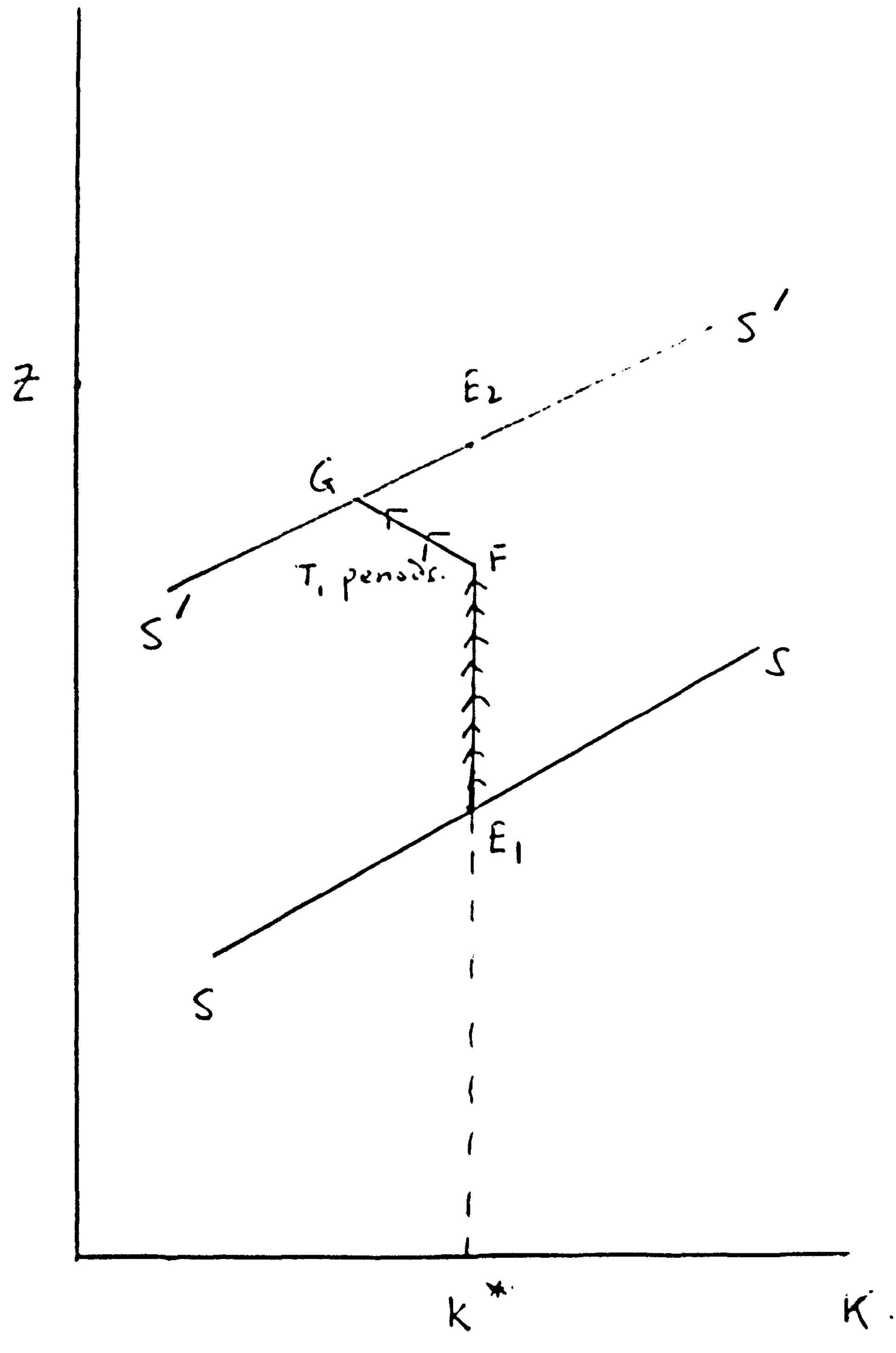

Figure 7.5

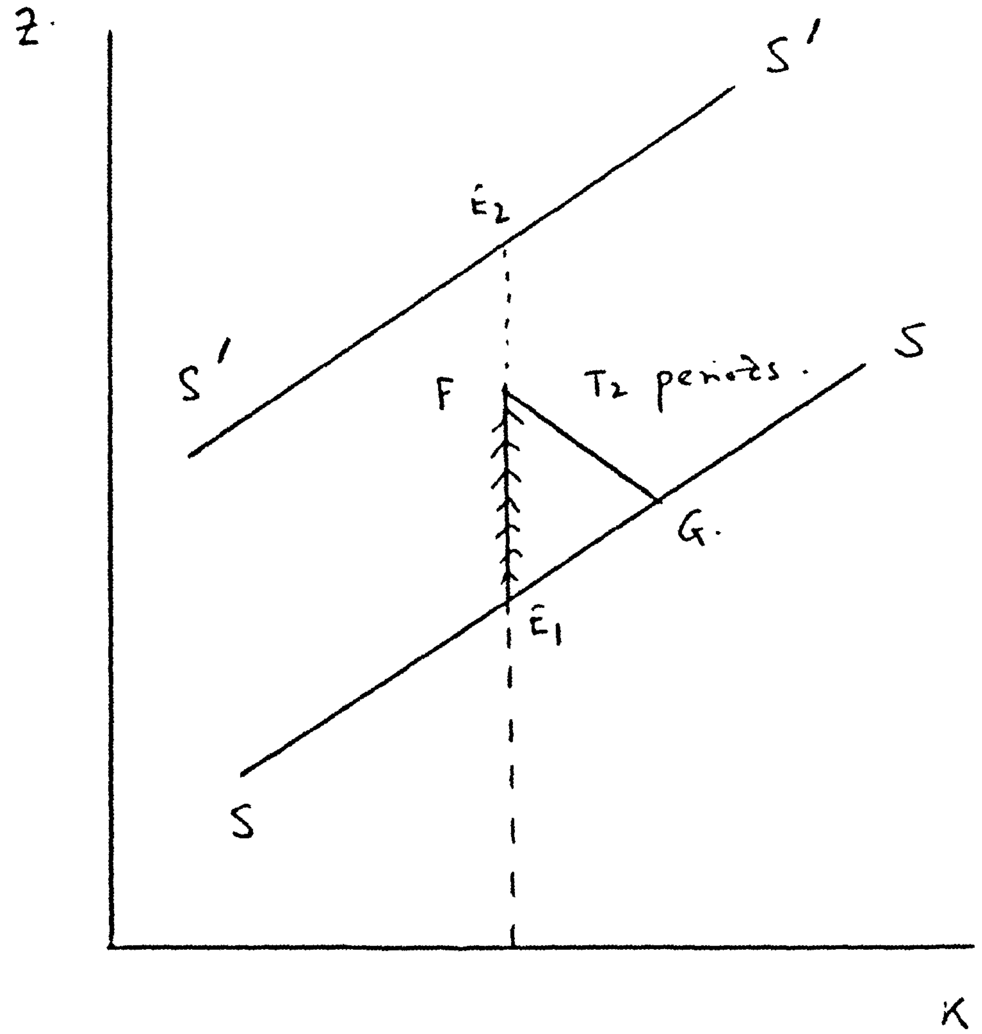

Figure 7.6.

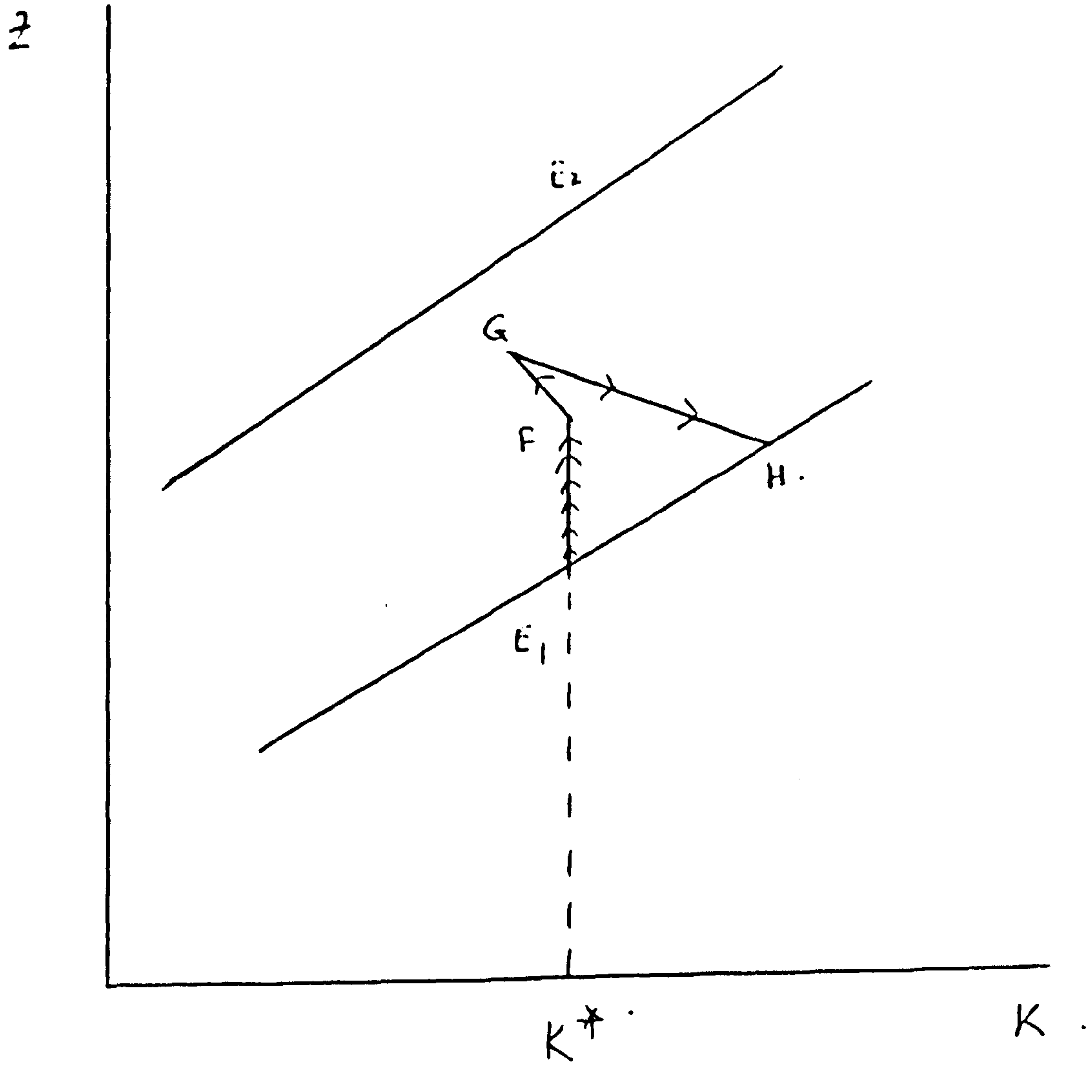

Figure. 7. 7.

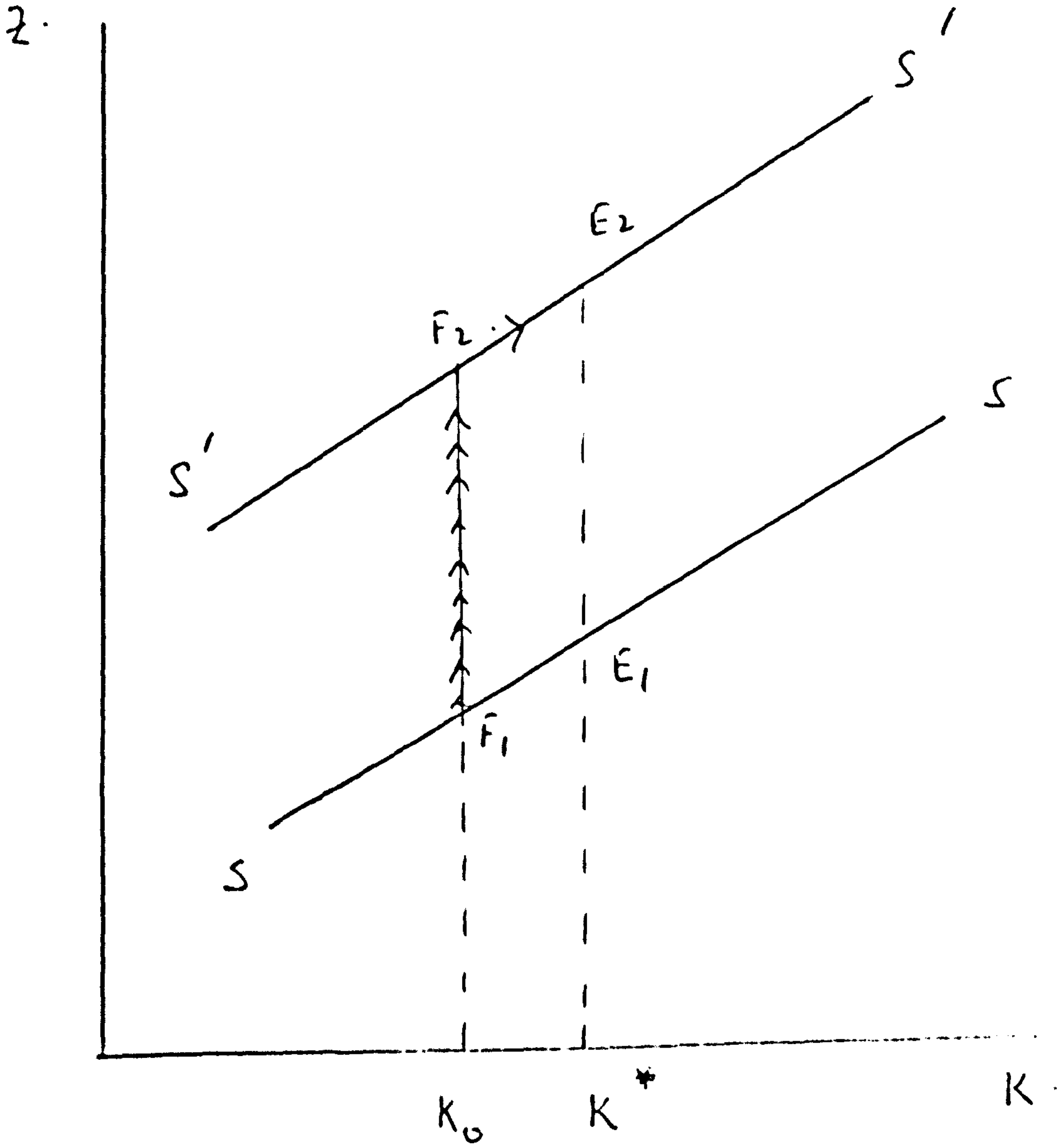

Figure 7.8

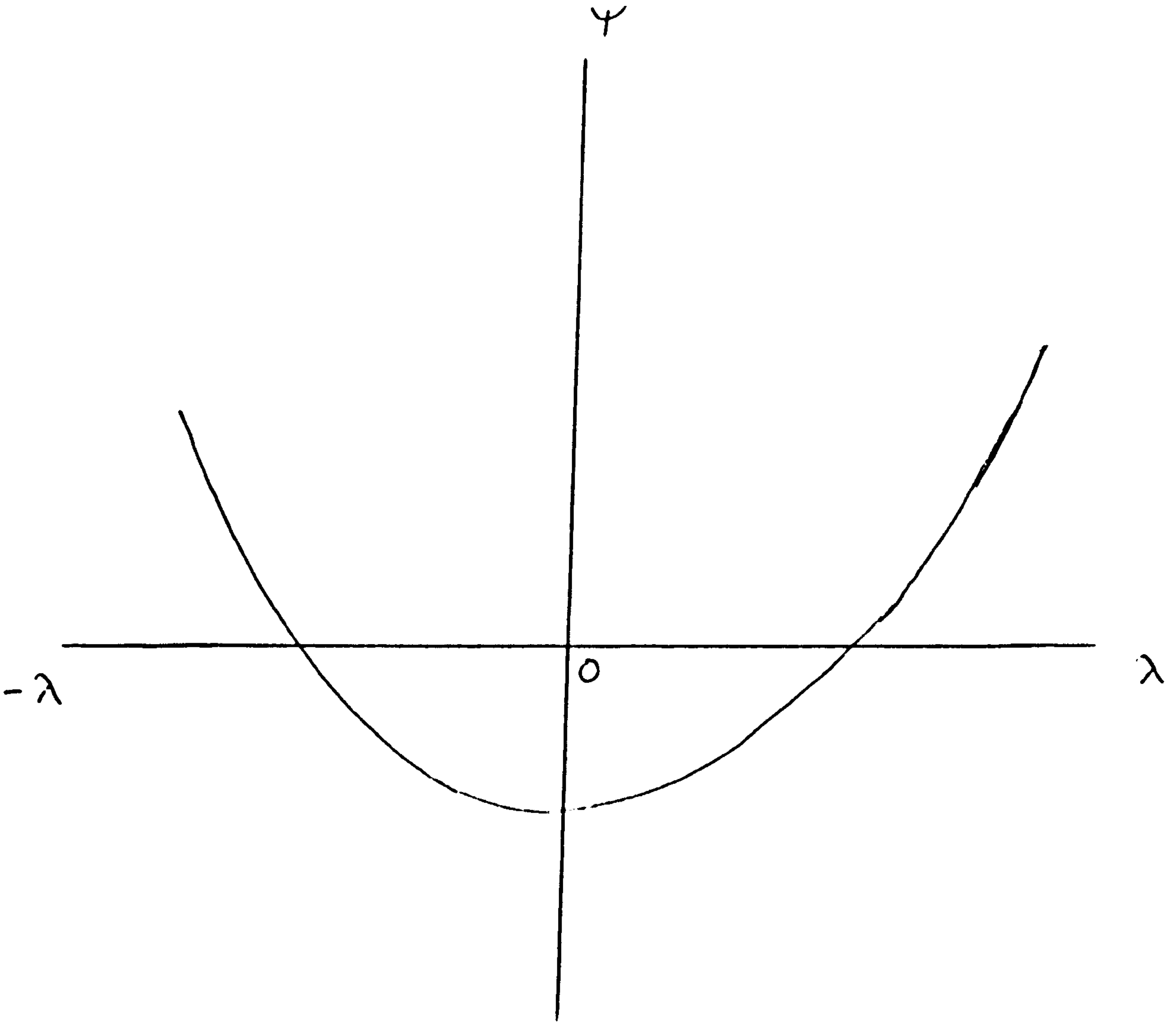

Figure 7.9

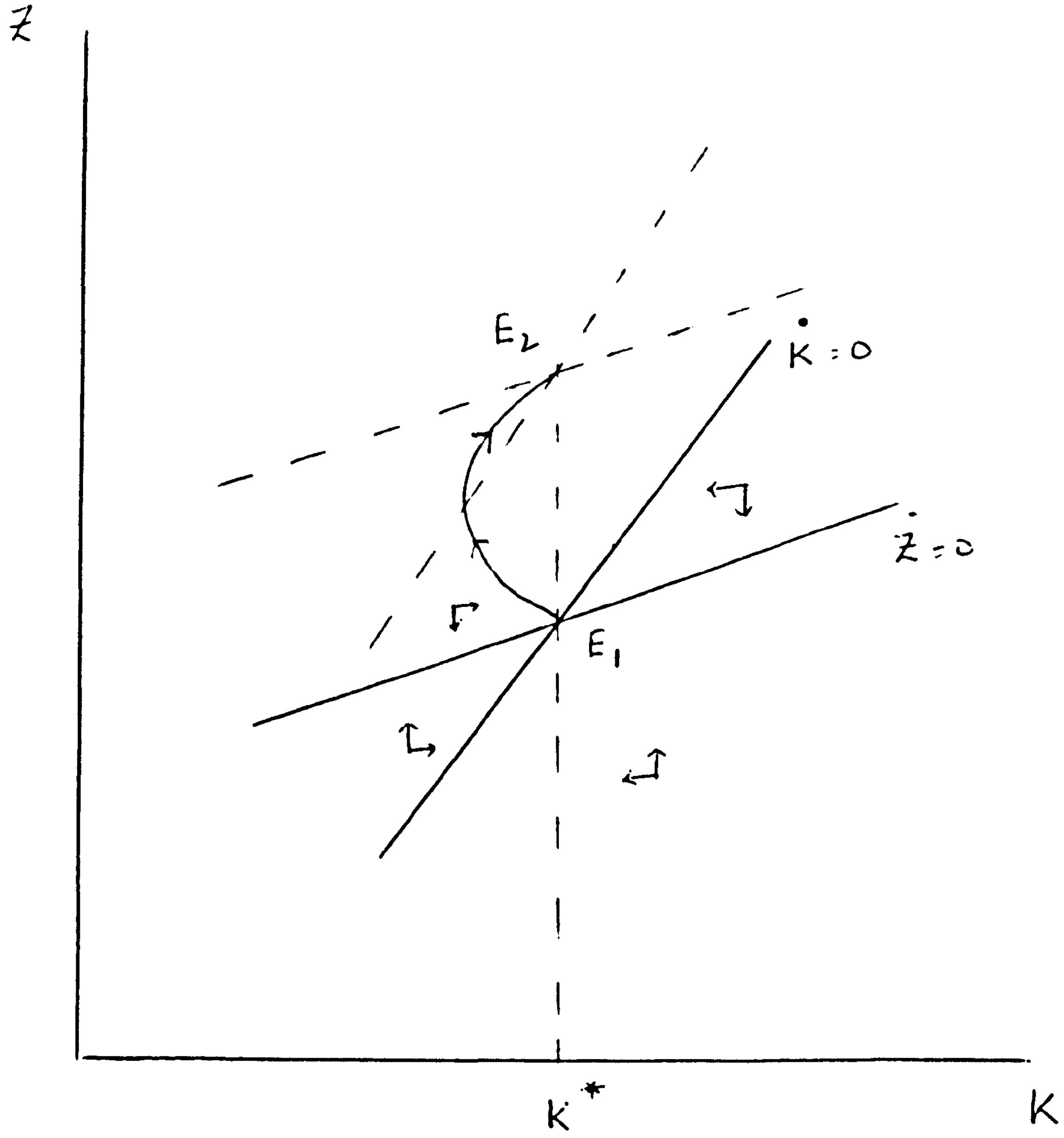

Figure 7.10

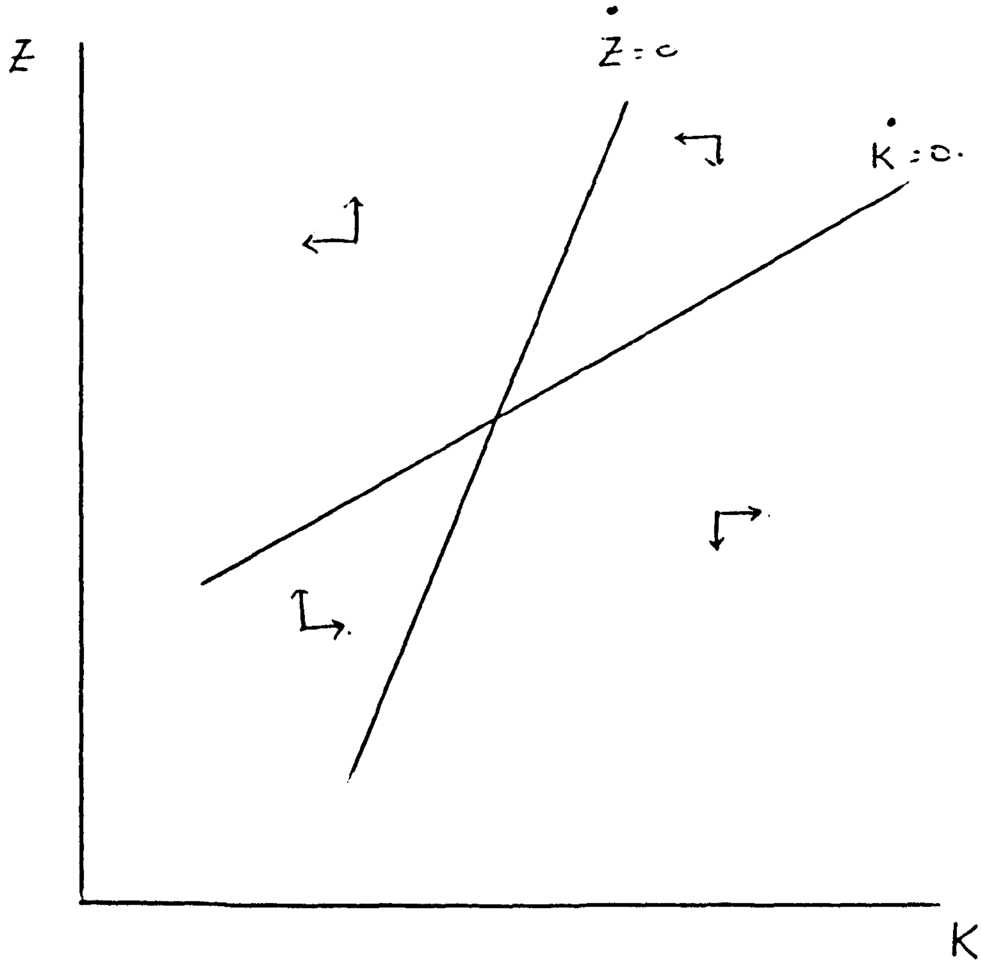

Figure 7.11

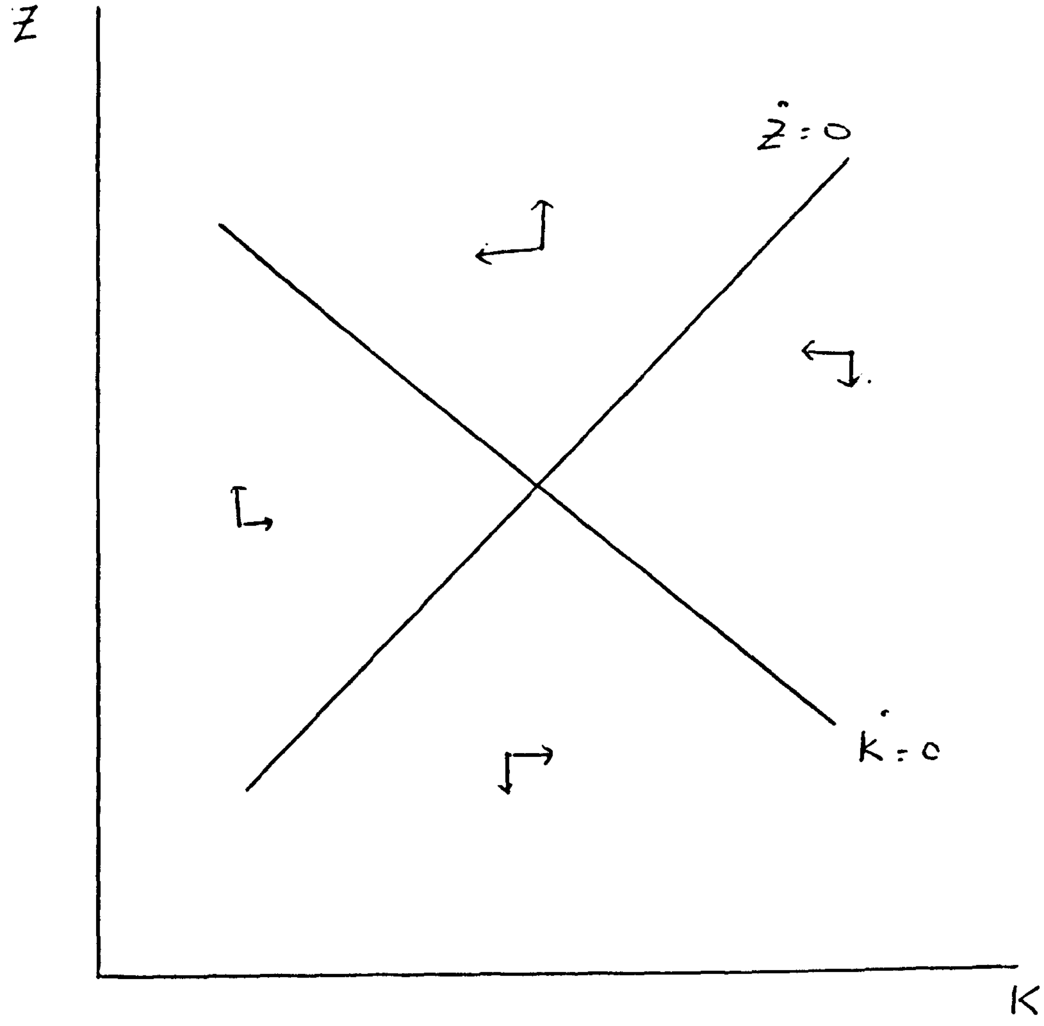

Figure 7.12

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Chapter VIII

Rational Expectations and Neutrality under Classical Adjustment Rules

Section 1: Introduction

The important Rational Expectations (hereafter called RE) literature of the past decade has generated three fundamental contributions to the analysis and evaluation of macroeconomic theory and policy. Firstly, the important contribution originating from Lucas (1976), that structural (or reduced form) parameters of the model might not remain invariant when policy regimes change. Thus government policy rules, based on econometric evaluation of past performance, might alter the coefficients of the underlying economic model itself and make it difficult to anticipate possible effects. Added to this, there may be a further crucial issue not apparent to the protagonists of equilibrium RE models, but extremely important to theorists studying non-Walrasian or "disequilibrium" models (Malinvaud, 1977). Even under very simplified assumptions, there may be different types of non-Walrasian equilibria (for example, Keynesian, classical or repressed inflation). Thus there is no one single model on which expectations are based and this might cause problems of forming any macro rational expectations for all agents taken together (see Sen, 1978). Even if one imposes aggregate RE, policy effectiveness may crucially depend on the structure of the regime concerned and no general conclusions are possible.

However, these issues pose deep theoretical questions regarding the relevance of macroeconomic policy and need much more intensive study of the subject. Our analysis in this paper is limited to the other two aspects of RE theory - neutrality of perfectly anticipated monetary policy; uniqueness (or multiplicity) and convergence (or stability) of RE paths. From the early papers of Sargent and Wallace (1975), neutrality or policy ineffectiveness of systematic monetary policy has been an important topic for discussions. Similarly, Taylor (1978), Gourieroux, Laffont, Montfort (1980) and others, have shown that RE paths may exhibit a very large number of solutions.

However, the economic models purporting to discuss these concepts have generally been put in an equilibrium framework. Market clearing Walrasian equilibrium has become an article of faith among members of the new classical school and has been elevated to as important a position as the assumption of RE itself. But there seems to be no logical interconnection between the two postulates. It is conceivable that agents may form expectations in a rational manner even if disequilibrium prevails and prices are sticky (see Burmeister (1980)). It is possible that given rationing, transactors may form rational quantity expectations in addition to expectations on price variables. However, this may be easily integrated into the theory. Thus, it is a natural extension to analyse solution paths and possible neutrality in RE models, with disequilibrium, and excess demand (supply) in markets as well as prices not reacting instantaneously to clear all markets.

Disequilibrium analysis can be of two types. One can assume a tatonnement process by which no transactions may take place unless the "auctioneer" has called the "correct" price. A better assumption is to use the minimum condition such that actual transactions is equal to the minimum of effective demand and supply in aggregate. Coupled with this we need a price adjustment rule (assuming sticky but not rigid prices) which states that the change in price is a function of excess demand in relevant markets. The speed of adjustment will obviously be assumed to be finite (since infinite speed implies that equilibrium always prevails).

Recently, Lucas (1980) has attacked the use of so-called "free parameters". Speed of adjustment of any variable in response to market disequilibrium is a "free parameter" in the sense that it is exogenously specified (at a positive finite level). There is no microeconomic foundation which could determine its value as a function of other endogenous variables. Disequilibrium theory postulates a speed of adjustment but does not say why it should exist or what its value should be. On the other hand, in a defence of free parameters in adjustment equations, Burmeister (1980) has pointed out extremely important reasons why they can be utilised to achieve internal consistency of the dynamic models. The convergence, stability and uniqueness of RE models can be strengthened by judicious use of adjustment variables and thus are essential if we are to get logically satisfactory models. Further, there seems to be equal validity in assuming that speeds of adjustments are all equal to infinity (equilibrium) as in assuming that they are constants but less than infinity (disequilibrium). Finally, one can ideally construct a model with these speeds as a function of prices (having given a preliminary story), and then study their properties. There seems to be good enough justification for the use of dynamic adjustment equations in a disequilibrium framework.

Some recent work on disequilibrium RE models seems to claim that the policy ineffectiveness proposition fails when markets do not clear (see particularly the paper by Honkapohja (1979) and his comment: "Some differences of opinion remain, but the results suggest that the neutrality proposition in general fails to hold when markets do not continuously clear"). This is certainly true of work by Fischer

(1977) and Phelps and Taylor (1977). However, McCallum (1978, 1979, 1980) shows that money can still be proved to be neutral under suitable assumptions regarding disequilibrium. The position seems to be far from settled and needs more research. Secondly, there is no consensus among the contestants of the debate about the proper specification of the dynamic adjustment rules. Given the ad hoc nature of macroeconomic policy analysis, this is to be expected. The question therefore remains as to how sensitive these results (of non-neutrality) are in Honkapohja (1979) (and others mentioned in his paper) to specific adjustment rules.

We propose to analyse a set of disequilibrium RE models with adjustment equations which may be termed "classical" (alternatively pre-Keynesian). Evidence for the use of such adjustments may be found in Metzler (1950) and Patinkin (1965) (as well as the extensive literature summary given there). The two price variables singled out for analysis are the aggregate price level and the real rate of interest. We assume that the price level adjusts to disequilibrium in the money market such that excess supply of money raises price. This accords with the monetarists' interpretation of the effect of an increase in the supply of money in increasing the price level. If inflation is a monetary phenomenon then sooner or later an increase in the supply of money over demand will lead to an increase in prices. The effect may not be direct. For example an increase in money stock might simply increase nominal income and purchasing power in the first instance. But given supply, after a while prices must rise. We take the transmission mechanism to be between money supply and price though there may be intermediate steps in between.

On the other hand, interest rate reacts to conditions prevailing in the bond market. It can be shown that under classical transmission mechanism, the interest rate adjusts positively to the excess demand for output. If aggregate demand for output exceeds supply, then investment is greater than saving. In a classical world, investment can be interpreted as the demand for loanable funds while saving may be interpreted as the supply of loanable funds. Thus investment minus saving is equal to excess demand for loanable funds. Given an excess demand for loanable funds, we have an equivalent excess supply of bonds. (Demand for bonds = supply of saved funds. Supply of bonds = demand for investible funds). Thus the price of bonds falls and the interest rate (inversely related to bond price) rises.

The quantity theory of money and the loanable funds theory of saving investment can be used to yield the type of adjustment equations we have proposed. These assumptions were commonly used to analyse the behaviour of money, commodity and bond markets, in the writings of the old classical school. It is instructive to analyse their behavioural equations in the light of the workings of the new classical macroeconomics (Buiter (1980)).

Essentially, we wish to retain the spirit of classicism even within the disequilibrium framework. Thus our aggregate price and interest rate adjustments follow theories proposed by classical (monetarist) authors (see Patinkin (1965)). Quite often they arrived at conclusions regarding the long run neutrality of money, even though on the transition path, money could have real effects. Our purpose is to evaluate the conclusions reached by new classical macroeconomics (Lucas et al) in the light of some of the assumptions made by old classical macroeconomics. The quantity theory of money and loanable fund theory were important contributions made by the latter. How would they affect RE conclusions?

Note that an important underlying proposition of classical theory was that real variables clear real markets while nominal (monetary) variables clear nominal markets. In our model real rate of interest equilibrates the commodity/real bond markets while the money price level equilibrates the money market.

The purpose of the chapter may therefore be briefly summarised. A disequilibrium model of RE must postulate suitable adjustment rules for price level and interest rate, in the presence of excess demand (or supply). Since there is no rigorous foundation, the choice of rule is a matter of judgement. We wish to remain as close as possible to the central tenets of classical macro models and see whether standard RE conclusions hold or not. We invoke two basic theories - the quantity theory of money and the loanable funds theory of bond financing - to specify the structural characteristics of the disequilibrium model. Given this classical framework we intend to analyse solution paths of models with RE and study the neutrality question.

In the next two sections, we construct models which have disequilibrium in both commodity and money markets. Adjustment rules follow classical tradition. Three types of models are used. Firstly, anticipatory price setting is postulated and prices depend on expected excess demand. Secondly, we assume that prices change according to actual excess demands. In both these models real balance effects are ignored. In the third type we assume that the stock of real balance affects aggregate demand. In Section 2 these three types of models are discussed given a simplifying assumption that expected price inflation is equal to zero. In the next section, we relax this simplification and the expected inflation rate is taken to be non-zero. This is merely to focus attention on essentials in the first stage and then move on to more complicated structures.

Our primary purpose in these models is to study the multiplicity (or uniqueness) of solution paths and then see whether anticipated money is neutral or not for some or all these paths. It is shown that disequilibrium (of the classical type) per se does not necessarily give non-neutrality. Rather it is the structure of the model which produces relevant policy effectiveness conclusions. Models without real balance effects always show money is neutral. Models with real balance effects invariably give non-neutral results. Thus, neither RE nor disequilibrium determine whether policy is effective or not. It is usually the assumptions of the underlying model that produce relevant policy conclusions.

Section 2: The Basic Models

As mentioned earlier, we first want to analyse basic models with expected rate of inflation assumed equal to zero. We would like to focus attention on the central causes in the policy ineffectiveness debate, within a disequilibrium structure which is shorn of all unnecessary assumptions. As will be shown in the next section, this does not affect the final results and all conclusions follow through in models with non-zero expected price inflation.

The first model in this section has anticipatory price setting rules. We have the following system of equations:

$$y_t = a_1(p_t - {}_{t-1}p_t^*) + u_{1t} \qquad (1)$$

$$e_t = b_1 r_t + u_{2t} \qquad (2)$$

$$m_t^d = p_t + c_1 r_t + c_2 e_t + u_{3t} \qquad (3)$$

$$p_t - p_{t-1} = \lambda(m_t^s - {}_{t-1}m_t^{d*}) + v_t \qquad (4)$$

$$r_t - r_{t-1} = \Omega({}_{t-1}e_t^* - {}_{t-1}y_t^*) + w_t \qquad (5)$$

$$m_t^s = m_t + u_{4t} \qquad (6)$$

$$a_1 > 0,\ b_1 < 0,\ c_1 < 0,\ c_2 > 0,\ \lambda > 0,\ \Omega > 0$$

(where y_t, e_t, m_t^d, m_t^s and p_t are logs of output supplied, output demand, money demand, money supply, and price level; r_t is the rate of interest and no distinction is made between real and nominal rates since expected price inflation is zero. A star (*) indicates an expected value).

Expectations for all variables are formed rationally. Thus for any variable x_t, its expected value formed at period t-1 is given as

$$_{t-1}x_t^* = E(x_t/\text{information available at time t-1})$$

(where E is the mathematical expectations operator). In our model rational expectations are assumed for money demand, effective demand (e_t), aggregate supply (y_t) and price level (p_t). To simplify the notation, we shall write from now on, x_t^* instead of $_{t-1}x_t^*$, for variables whose expectations are formed in t-1. Care will be taken to explicitly state the date of expectations formation, if it is not t-1.

Equation (1) is the Lucas supply function expressed in deviation form from the natural rate. Thus in all subsequent analysis we drop an extra term invoking factors which determine the natural rate of unemployment. Similarly, aggregate demand (in deviation form) is given by equation (2) as a function of the rate of interest (real or nominal). Equation (3) gives the demand function for money. (A model similar to (1) (2) (3), but only for an equilibrium situation, has been used by Woglom (1979). It can be shown that if $y_t = e_t$ and $m_t^d = m_t^s$, money is neutral). The supply of money in equation (6) is kept arbitrary to make the monetary rule as general as possible. It is easy to use specific rules and conclusions remain invariant. Money supply is perfectly anticipated at level m_t except for white noise terms.

Equation (4) uses the classical monetarist rule that prices respond positively to an increase in the supply of money. If the supply of money (perfectly forecast except for error u_{4t}) is greater than the expected demand for money then the price level will rise from its previous value. Thus inflation depends on the expected excess supply of money. Note that price setting is anticipatory in the sense that p_t is fixed at the beginning of period t with information from t-1 used to forecast rationally the expected value of m_t^d. In exactly analogous fashion, the interest rate is determined at the start of period t and e_t^* and y_t^* are predicted rationally. It is assumed that expected excess demand for output (implying investment greater than saving) will cause an expected excess supply of bonds. This will lower the price of bonds and raise the rate of interest.

The error terms u_{it}, v_t, w_t are white noise terms, mean zero, constant variance and serially uncorrelated. We may assume without loss of generality that $u_{it} \equiv 0$ and concentrate on the errors v_t and w_t in the price/interest rate formation equation.

The semi reduced form of the equation with endogenous variables p_t, y_t, e_t, m_t^d and r_t in terms of exogenous and expected variables can be written as:

$$p_t = p_{t-1} + \lambda(m_t - m_t^{d*}) + v_t \tag{7}$$

$$r_t = r_{t-1} + \Omega(e_t^* - y_t^*) + w_t \tag{8}$$

$$y_t = a_1 p_{t-1} + a_1\lambda(m_t - m_t^{d*}) - a_1 p_t^* + U_{1t} \tag{9}$$

$$m_t^d = p_{t-1} + \lambda(m_t - m_t^{d*}) + (c_1 + c_2 b_1)\,[r_{t-1} + \Omega(e_t^* - y_t^*)] + U_{2t} \tag{10}$$

$$e_t = b_1 r_{t-1} + b_1\Omega(e_t^* - y_t^*) + U_{3t} \tag{11}$$

We now invoke the minimum condition from disequilibrium theory. Assume actual output (y_t^a) is the minimum of supply and demand. Thus

$$y_t^a = \min (y_t, e_t) \tag{12}$$

Now taking expectations, from (7) we get

$$p_t^* = p_{t-1} + \lambda[m_t - m_t^{d*}] = p_t \tag{13}$$

Thus, using (1) it is clear that y_t is unaffected by money supply m_t since $y_t = u_{1t}$

Similarly from (1) or (9),

$$y_t^* = 0 \tag{14}$$

From (11), taking expectations of e_t we get

$$e_t^* = b_1 r_{t-1} + b_1 \Omega e_t^* - b_1 \Omega y_t^*$$

or

$$e_t^* = (b_1/1-b_1\Omega) r_{t-1}$$

Substituting in (8),

$$r_t = r_{t-1} + \Omega[b_1/1-b_1\Omega] r_{t-1} + w_t$$

$$r_t = [1/1-b_1\Omega] r_{t-1} + w_t \tag{15}$$

From (2) therefore,

$$e_t = (b_1 /1-b_1 \Omega) r_{t-1} + u_{2t} \tag{16}$$

It is clear that r_t is a function of its past values (from (15)) and this is not influenced by money supply m_t. Therefore, e_t dependent on

r_{t-1} is not affected by <u>any</u> m_{t-1} (i=0, 1, 2,...). We therefore see from (1), (12), (15), (16) that <u>both</u> y_t and e_t cannot be influenced by anticipated monetary policy. Actual output y_t^a being independent of m_t we have neutrality, and anticipated money rules are ineffective in affecting output.

We now turn to the other part of our enquiry - the analysis of solution paths. From (4) take expectations, substitute for e_t^* and y_t^* and solve for m_t^{d*} to get

$$m_t^{d*} = \frac{p_{t-1}}{1+\lambda} + \frac{\lambda}{1+\lambda} m_t + \left(\frac{c_1+c_2b_1}{1+\lambda}\right)\left(\frac{1}{1-b_1\Omega}\right) r_{t-1} \tag{17}$$

Given

$$p_t^* = p_{t-1} + \lambda(m_t - m_t^{d*})$$

$$= p_{t-1} + \lambda m_t - \frac{\lambda}{1+\lambda} p_{t-1} - \frac{\lambda^2 m_t}{1+\lambda} + \left(\frac{c_1+c_2b_1}{1+\lambda}\right)\left(\frac{1}{1-b_1\Omega}\right) r_{t-1} \tag{18}$$

Using (15) and (18) we can express r_t and p_t^* as functions of r_{t-1} and p_{t-1} <u>plus</u> exogenous and random variables.

$$\begin{bmatrix} r_t \\ p_t^* \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{1-b_1\Omega}\right) & 0 \\ \lambda\left(\frac{c_1+c_2b_1}{1+\lambda}\right)\left(\frac{1}{1-b_1\Omega}\right) & \frac{1}{1+\lambda} \end{bmatrix} \begin{bmatrix} r_{t-1} \\ p_{t-1} \end{bmatrix} + \text{exogenous and random terms} \tag{19}$$

Call the Jacobian matrix

$$\begin{bmatrix} \left(\frac{1}{1-b_1\Omega}\right) & 0 \\ \lambda\left(\frac{c_1+c_2b_1}{1+\lambda}\right)\left(\frac{1}{1-b_1\Omega}\right) & \frac{1}{1+\lambda} \end{bmatrix} = A$$

We may now use a theorem due to Blanchard and Kahn (1980) regarding solution paths of the difference equation system (19). Note that in our model, the rate of interest in any period t, i.e. r_t, is fully determined by information available at period t-1. Since r_t depends on e_t^* and t_t^* which are expectations formed in t-1, the interest rate variable r_t is a predetermined variable. On the other hand p_t depends on current shocks at period t (through m_t^s) and is therefore a "jump" or non-predetermined variable. Therefore in our model, we have one predetermined variable and one jump variable. To analyse the possible solutions to (19) we have to find the number of unstable (or stable) characteristic roots of (19).

Clearly the trace of the matrix is $\left(\frac{1}{1-b_1\Omega}\right) + \left(\frac{1}{1+\lambda}\right)$ and the determinant of the matrix is $\left(\frac{1}{1-b_1\Omega}\right)\left(\frac{1}{1+\lambda}\right)$.

Therefore it is obvious that the roots are $(1/1-b_1\Omega)$ and $(1/1+\lambda)$. Since $\lambda > 0$, $b_1 < 0$, $\Omega > 0$, both roots are less than unity (1). Therefore, the number of unstable roots (zero) is less than the number of jump or non predetermined variables (one). Under these conditions, there are an infinite number of solution paths to the system.

We may summarise our conclusions in the following proposition.

Proposition 1

There are an infinite number of possible solutions to the model set out in equations (1) to (6). However, all solution paths exhibit neutrality, i.e. perfectly anticipated monetary policy does not affect either aggregate demand or supply.

We now turn to our next model (within the basic framework) and assume that price setting is no longer anticipatory. Now price changes depend on actual rather than expected excess demands in respective markets. Once again we wish to analyse the neutrality proposition as well as study the existence (and possible multiplicity) of RE solution paths.

We have

$$y_t = a_1(p_t - p_t^*) \tag{20}$$

$$e_t = b_1 r_t \tag{21}$$

$$m_t^d = p_t + c_1 r_t + c_2 e_t \tag{22}$$

$$m_t^s = m_t \tag{23}$$

$$p_t - p_{t-1} = \lambda(m_t - m_t^d) + v_t \tag{24}$$

$$r_t - r_{t-1} = \Omega(e_t - y_t) + w_t \tag{25}$$

Structurally, it is similar to the first model except that in price and interest adjustment equations (24) and (25), actual variables have replaced anticipated ones. Once again the dynamic adjustments follow classical modes in that the price level responds to excess demand in the money market and interest rate changes are a function of excess demand in the output (bond) market.

From (21) and (22)

$$m_t^d = p_t + (c_1 + c_2b_1)r_t \tag{26}$$

From (24) and (25)

$$p_t = \frac{p_{t-1}}{1+\lambda} + \frac{\lambda}{1+\lambda} m_t - \frac{\lambda}{1+\lambda}(c_1+c_2b_1)r_t \tag{27}$$

From (25), (20) (21) and (27) we get

$$B_1r_t = r_{t-1} - \frac{\Omega a_1}{1+\lambda} p_{t-1} - \frac{\Omega a_1\lambda}{1+\lambda} m_t + \Omega a_1p_t^* \tag{28}$$

where $$B = 1 - \Omega b_1 - \frac{\Omega a_1\lambda}{1+\lambda}(c_1+c_2b_1) > 0$$

and $B > 1$

Thus (28) may be rewritten as

$$r_t = J_1r_{t-1} + J_2p_{t-1} + J_3m_t + J_4p_t^* \tag{29}$$

where $J_1 = 1/B$, $J_2 = -[\Omega a_1/B(1+\lambda)]$, $J_3 = -[\Omega a_1\lambda/B(1+\lambda)]$, $J_4 = \Omega a_1/B$ and $J_1 > 0$, $J_2 < 0$, $J_3 < 0$, $J_4 > 0$.

Substituting (29) into (27) we get

$$p_t = K_1p_{t-1} + K_2m_t + K_3r_{t-1} + K_4p_t^* \tag{30}$$

where $$K_1 = \frac{1}{1+\lambda} - \frac{\lambda}{1+\lambda}(c_1+c_2b_1)J_2$$

$$K_2 = \frac{\lambda}{1+\lambda} - \frac{\lambda}{1+\lambda}(c_1+c_2b_1)J_3$$

$$K_3 = -\frac{\lambda}{1+\lambda}(c_1+c_2b_1)J_1$$

$$K_4 = -\frac{\lambda}{1+\lambda}(c_1+c_2b_1)J_4$$

Taking expectations of (30) we get

$$p_t^* = \frac{K_1}{1-K_4} p_{t-1} + \frac{K_2}{1-K_4} m_t + \frac{K_3}{1-K_4} r_{t-1} \qquad (31)$$

Substituting in (29) we get

$$r_t = (J_1 + \frac{J_4K_3}{1-K_4})r_{t-1} + (J_2 + \frac{J_4K_1}{1-K_4}) p_{t-1}$$

$$+ (J_3 + \frac{J_4K_2}{1-K_4})m_t + \text{random terms} \qquad (32)$$

Once again equations (32) and (31) may be investigated to find the nature of RE solution paths. The relevant Jacobian is now

$$A_2 = \begin{bmatrix} J_1 + \frac{J_4K_3}{1-K_4} & J_2 + \frac{J_4K_1}{1-K_4} \\ \frac{K_3}{1-K_4} & \frac{K_1}{1-K_4} \end{bmatrix} \qquad (33)$$

Observe that the determinant of A_2 (giving the product of the roots) is

$$\det A_2 = J_1/(1+\lambda)(1-K_4)$$

Since $J_1 = 1/B$ and $B > 1$, therefore $J_1 < 1$

$(1+\lambda) > 1$ since $\lambda > 0$

$(1-K_4) = 1 + \lambda(1-(c_1+c_2h_1)J_4/1+\lambda > 1$

since $(c_1+c_2h_1) < 0$

Therefore $\det A_2 < 1$.

Thus either both roots are less than unity (both roots stable) or one root is less than unity (one root stable). So the number of unstable roots is either zero or one.

However, note that unlike our previous case both variables are non predetermined in this model given by (20) to (25). This is because both p_t and r_t can respond to current shocks and news and are determined on the basis of events in period t. We have therefore the number of non predetermined variables (2) greater than the number of unstable roots (1). Thus there must be an infinite number of solutions.

To analyse neutrality, observe that e_t is a function of r_t alone. Therefore effect of money supply increase (m_t) must operate through r_t. We have to evaluate in (32) the coefficients $[J_3+J_4K_2/1-K_4]$ to get the current effect of m_t and the coefficient $[J_2+J_4K_1/1-K_4]$ to get the past effects of m_{t-1} via its influence on price level.

It can be easily calculated that

$$J_3 + \frac{J_4K_2}{1-K_4} = \frac{J_3+\lambda J_3+J_4\lambda}{1-K_4} = 0 \qquad (34)$$

and

$$J_2 + \frac{J_4K_1}{1-K_4} = \frac{J_2+J_2\lambda+J_4}{1-k_4} = 0 \qquad (35)$$

Thus from (32), (34), (35),

$$r_t = (J_1 + \frac{J_4K_3}{1-K_4})\, r_{t-1} + \text{random errors}$$

or

$$r_t = (J_1/1-K_4)r_{t-1} + \text{random errors} \qquad (36)$$

Once again it is clear that r_t depends on its past history and is not influenced by monetary shocks. Since

$$e_t = (h_1 J_1 / 1 - K_4) r_{t-1} + \text{random errors} \tag{37}$$

therefore aggregate demand is uninfluenced by monetary policy. It is obvious that y_t is also uninfluenced by anticipated money, since $p_t - p_t^*$ is equal to random errors. Thus actual output

$$y_t^a = \min(y_t, e_t)$$

is not affected by systematic monetary rules.

We sum up the results in the proposition:

<u>Proposition 2</u>

Even if price setting is not anticipatory and all prices adjust to current events, there are an infinite number of RE solution paths in our classical model. However, all these paths exhibit neutrality.

The two disequilibrium models studied up till now highlight an important conclusion. Neutrality (or non neutrality) does not necessarily depend on disequilibrium behaviour. We set classical price adjustment rules and found that given RE (of all price and quantity variables), perfectly anticipated money stock does not affect the structure of output.

We now proceed to a third model which is essentially similar to the first one (given by equations (1) to (6) except that a real balance effect is introduced. Since our adjustment rules are classical in spirit, it is quite appropriate to assume that the stock of real balance affects aggregate demand (see Patinkin (1965)). The Pigou effect has been used by traditional neutralists to counter

Keynesian objections to the existence of full employment equilibrium output. It seems logical and valid therefore to analyse the neutrality propositions within RE assumptions in a model where real balance effects are present.

We therefore have

$$v_t = a_1(p_t - p_t^*) \quad (38)$$

$$e_t = b_1 r_t + b_2 m_t - b_2 p_t \quad (39)$$

$$m_t^d = p_t + c_1 r_t + c_2 e_t \quad (40)$$

$$m_t^s = m_t \quad (41)$$

$$p_t - p_{t-1} = \lambda[m_t - m_t^{d*}] + v_t \quad (42)$$

$$r_t - r_{t-1} = \Omega[e_t^* - y_t^*] + w_t \quad (43)$$

$(b_2 > 0)$

$$\text{Actual output} = \text{minimum}\ (y_t, e_t) \quad (44)$$

Once again price and interest rate setting is anticipatory. The new innovation is that positive real balance effects are assumed given $b_2 > 0$.

From (37), (38)

$$m_t^d = p_t + c_1 r_t + c_2 b_1 r_t + c_2 b_2 m_t - c_2 b_2 p_t \quad (45)$$

$$\text{Therefore } p_t = \frac{m_t^d}{1-c_2 b_2} - \frac{c_1 + c_2 b_1}{1-c_2 b_2} r_t - \frac{c_2 b_2}{1-c_2 b_2} m_t \quad (46)$$

From (42),

$$p_t = p_{t-1} + \lambda m_t - \lambda m_t^{d*} + v_t \qquad (47)$$

Equating (46) and (47) and using (43) we get

$$m_t^d = (c_1+c_2b_1)(r_{t-1} + e_t^*\Omega-\Omega y_t^*) + m_t(c_2b_2+\lambda-\lambda c_2b_2)$$

$$+ p_{t-1}\ (1-c_2b_2)-\lambda(1-c_2b_2)m_t^{d*}$$

(For simplicity we drop from now on all terms involving errors v_t and w_t. This does not affect the final conclusions).

From (39) and (43),

$$e_t = b_1(r_{t-1} + \Omega e_t^* - \Omega y_t^*) + b_2m_t - b_2p_{t-1} - b_2\lambda m_t - b_2\lambda m_t^{d*}$$

$$(49)$$

Taking expectations of (48) and (49) we get two equations for m_t^{d*} and e_t^* which may be written as

$$m_t^{d*}(1+\lambda(1-c_2b_2)) - (c_1+c_2b_1)\Omega e_t^* = \hat{L}_1 \qquad (50)$$

$$m_t^{d*}(\lambda b_2) - (1-b_1\Omega)e_t^* = \hat{L}_2 \qquad (51)$$

where

$$\hat{L}_1 = m_t(c_2b_2+\lambda-\lambda c_2b_2) + (c_1+c_2b_1)r_{t-1} + (1-c_2b_2)p_{t-1} \qquad (52)$$

and $\hat{L}_2 = (b_2\lambda-b_2)m_t - b_1r_{t-1} + b_2p_{t-1}$ (53)

Equations (50) and (51) can be used to solve for m_t^{d*} and e_t^*. When these values are substituted into equation (42) we get the expression for p_t. Taking expectations of p_t we can derive p_t^*.

Similarly substituting the value of e_t^* in equation (43) we get an equation in r_t (note $v_t^* = 0$). These are extremely tedious to solve but one can derive in exactly analogous fashion as in the first model, a Jacobian matrix A_3 such that

$$\begin{bmatrix} p_t^* \\ r_t \end{bmatrix} = A_3 \begin{bmatrix} p_{t-1} \\ r_{t-1} \end{bmatrix} + \text{exogenous and random terms}$$

Unlike the first model, it is not possible to predict the nature of the characteristic roots derived from matrix A_3. However, the roots depend crucially on the values of λ and Ω, the speed of adjustment terms. Thus existence of solution, multiplicity or uniqueness, and convergence depends explicitly on values of λ and Ω (similar results have been obtained by Burmeister (1980)).

It is possible however to study the neutrality proposition in some detail. From the values of m_t^{d*} and e_t^* we can get the values of p_t and r_t. Substituting these into the expression for e_t in (39) we can get the value of e_t as a function of r_{t-1}, p_{t-1} and m_t. The coefficient of m_t is given by the following:

$$[b_1\Omega+b_1\Omega\lambda(1-c_2b_2)+b_2\lambda(c_1+c_2b_1)\Omega-b_2\lambda]b_2(\lambda-1)$$

$$+ [b_1\Omega\lambda b_2\Omega(c_1+c_2b_1)-b_2\lambda(1-b_1\Omega)][c_2b_2+\lambda-c_2\lambda b_2]$$

$$= \frac{\partial e_t}{\partial m_t} > 0$$

after suitable substitution.

Thus if the stock of money increases, the aggregate demand increases. If actual output is equal to demand, then it will increase too. Therefore money is non neutral and perfectly anticipated money has real effects. Note that this coefficient is positive given $h_2 > 0$. If $h_2 = 0$, $\partial e_t/\partial m_t = 0$ (h_2 gives the real balance effect).

We have proved:

Proposition 3

If real balance effects are present then the existence and uniqueness of RE solution paths depends explicitly on the speed of adjustment; nothing specific can be predicted unless the actual values of λ and Ω are known. However, for any solution path, money is non neutral and anticipated monetary policy is effective.

Section 3: The Complete Models

We can now turn our attention to the complete model where expected rate of inflation appears as an explanatory variable. It can be demonstrated that this does not change any of the essential conclusions. Once again if real balance effects are present and the stock of real money affects the aggregate demand function, then money is non neutral. On the other hand, without real balance effects, money is neutral and perfectly anticipated money does not affect the structure of either aggregate demand or aggregate supply.

However, characterisation of solution paths becomes much more complex. It is no longer possible to use the Blanchard Kahn (1980) method to derive the existence or uniqueness of RE paths. In the absence of such methods we rely on the technique proposed by Muth (1960) originally and analysed by Taylor (1978). We check for non uniqueness of solution paths and analyse whether money is neutral or not on all such paths.

It is possible to repeat the exercise for all the three models analysed in Section 2 after adding terms containing the expected rate of inflation. However, the algebra is tedious and serves no analytical purpose. We shall therefore investigate the detailed solution for the first model and state the results for the other two. Since the methods of solution are similar, it is easy to see how they may be applied to any other case.

The model is given by

$$y_t = a_1(p_t - {}_{t-1}p_t^*) \qquad (54)$$

$$e_t = b_1 i_t \qquad (55)$$

$$m_t^d = p_t + c_1(i_t + {}_{t-1}p_{t+1}^* - {}_{t-1}p_t^*) + c_2 e_t \quad (56)$$

$$m_t^s = \overline{m}_t \quad (57)$$

$$p_t - p_{t-1} = \lambda[\overline{m}_t - {}_{t-1}m_t^{d*}] + v_t \quad (58)$$

$$i_t - i_{t-1} = \Omega[{}_{t-1}e_t^* - {}_{t-1}y_t^*] + w_t \quad (59)$$

Further actual output = minimum of output demand (e_t) and supply (y_t).

The model is essentially similar to Sargent and Wallace (1975) except for the adjustment rules for price level and interest rate. This anticipatory price setting behaviour has already been justified in our first model. Note that i_t is the <u>real</u> rate of interest. Thus the real variable (i_t) responds to real market disequilibrium in equation (59) and nominal variable (p_t) responds to money market disequilibrium. This is palpably classical.

Since $y_t^* = 0$ from (54) we can write equation (59) as

$$i_t = i_{t-1} + \Omega({}_{t-1}e_t^*) + w_t \quad (60)$$

and

$$e_t = b_1 i_{t-1} + b_1\Omega({}_{t-1}e_t^*) + b_1 w_t \quad (61)$$

From (56), (60), (61) we get

$$m_t^d = p_t + i_{t-1}(c_1 + c_2 b_1) + e_t^*(\Omega + c_2 b_1 \Omega) \quad (62)$$

$$+ c_1\, {}_{t-1}p_{t+1}^* - c_1\, {}_{t-1}p_t^* + W_t$$

(where W_t is a combination of error terms).

Substituting for p_t from (58) we finally have

$$m^d_t = p_{t-1} + \lambda\overline{m}_t - \lambda m^{d*}_t + i_{t-1}(c_1+c_2b_1)$$

$$+ e^*_t(\Omega+c_2b_1\Omega) + c_1\ {}_{t-1}p^*_{t+1} - c_1\ {}_{t-1}p^*_t \qquad (63)$$

Taking expectations of e_t in (61) and simplifying

$$_{t-1}e^*_t = [b_1/1-b_1\Omega]i_{t-1} \qquad (64)$$

So: $$i_t = [1/1-b_1\Omega]i_{t-1} + w_t \qquad (65)$$

and $$e_t = b_1 i_t$$

$$e_t = [b_1/1-b_1\Omega]i_{t-1} + b_1w_t \qquad (66)$$

Similarly, taking expectations of m^d_t in (63) we can get an expression for m^{d*}_t. This is substituted in (58) to give us

$$p_t = A_2p_{t-1} + A_3i_{t-1} + A_4m_t + A_5\ {}_{t-1}p^*_{t+1} + A_6\ {}_{t-1}p^*_t + v_t \qquad (67)$$

where

$$A_2 = 1/1+\lambda, \text{ where } 1 > A_2 > 0$$

$$A_3 = -\lambda\ \frac{c_1+c_2b_1-c_1b_1\Omega+b_1\Omega}{(1-b_1\Omega)(1+\lambda)} > 0$$

$$A_4 = \lambda/1+\lambda, \text{ where } 1 > A_4 > 0$$

$$A_5 = -\lambda c_1/1+\lambda > 0$$

$A_6 = \lambda c_1 / 1+\lambda < 0$

The reduced form of the two equations for i_t and p_t are given by

$$i_t = A_1 i_{t-1} + w_t \tag{68}$$

(where $A_1 = 1/1-b_1\Omega > 0$) and equation (67).

To solve equations (67) and (68) we make use of the fact that the system is decomposable. Taking the value of i_{t-1} from (67) we get

$$i_{t-1} = \frac{p_t}{A_3} - \frac{A_2}{A_3} p_{t-1} - \frac{A_4}{A_3} m_t - \frac{A_5}{A_3} {}_{t-1}p_t^* - \frac{A_6}{A_3} {}_{t-1}p_{t+1}^* + \text{errors} \tag{69}$$

Leading this by one period,

$$i_t = \frac{p_{t+1}}{A_3} - \frac{A_2}{A_3} p_t - \frac{A_4}{A_3} m_{t+1} - \frac{A_5}{A_3} {}_t p_{t+1}^* - \frac{A_6}{A_3} {}_t p_{t+2}^* + \text{errors} \tag{70}$$

Using (69), (70) and substituting in (68) we have

$$\frac{p_{t+1}}{A_3} - \frac{A_2}{A_3} p_t - \frac{A_4}{A_3} m_{t+1} - \frac{A_5}{A_3} {}_t p_{t+1}^* - \frac{A_6}{A_3} {}_t p_{t+1}^*$$

$$= \frac{A_1}{A_3} p_t - \frac{A_1 A_2}{A_3} p_{t-1} - \frac{A_1 A_4}{A_3} m_t - \frac{A_1 A_5}{A_3} {}_{t-1}p_t^* - \frac{A_1 A_6}{A_3} {}_{t-1}p_{t+1}^* + \text{errors} \tag{71}$$

Lagging (71) by one period and simplifying,

$$p_t = -A_1 A_4 m_{t-1} + A_4 m_t + (A_1+A_2) p_{t-1} - A_1 A_2 p_{t-2} + A_5 \; {}_{t-1}p_t^*$$

$$+ A_{6\ t-1}p^{*}_{t+1} - A_1A_{5\ t-2}p^{*}_{t-1} - A_1A_{6\ t-2}p^{*}_{t} + u_t \qquad (72)$$

(where u_t is a random term; and $_{t-j}p^{*}_{t+i} = E(p_{t+1}$ information at t-j)

We can rewrite (72) as

$$p_t = J_0m_{t-1} + J_1m_t + J_2p_{t-1} + J_3p_{t-2} + J_{4\ t-1}p^{*}_{t} + J_{5\ t-1}p^{*}_{t+1}$$

$$+ J_{6\ t-2}p^{*}_{t-1} + J_{7\ t-1}p^{*}_{t} + u_t \qquad (73)$$

The solution path to the model can be derived by solving equation (72). Following Muth and Taylor, assume that p_t can be expressed as a linear combination of current and lagged values error term u_t. The contribution of exogenous variables m_t and m_{t-1} is captured by a term p. Thus

$$p_t = p + \sum_{i=0}^{\infty} \Pi_i u_{t-i} \qquad (74)$$

Similarly,

$$p_{t-1} = p + \sum_{i=0}^{\infty} \Pi_i u_{t-i-1} \qquad (75)$$

$$p_{t-2} = p + \sum_{i=0}^{\infty} \Pi_i u_{t-i-2} \qquad (76)$$

$$p_{t+1} = p + \sum_{i=0}^{\infty} \Pi_i u_{t-i+1} \qquad (77)$$

$$p_{t-3} = p + \sum_{i=0}^{\infty} \Pi_i u_{t-i-3} \qquad (78)$$

Taking appropriate expectations,

$$_{t-1}p^*_t = p + \sum_{i=0}^{\infty} \Pi_i u_{t-i} \tag{79}$$

$$_{t-1}p^*_{t+1} = p + \sum_{i=2}^{\infty} \Pi_i u_{t-i+1} \tag{80}$$

$$_{t-2}p^*_{t-1} = p + \sum_{i=1}^{\infty} \Pi_i u_{t-i-1} \tag{81}$$

$$_{t-2}p^*_t = p + \sum_{i=2}^{\infty} \Pi_i u_{t-1} \tag{82}$$

These expressions must satisfy equation (72) for p_t. Thus multiplying (74) to (80) by their respective J_i and equating to the expression in (73) we have

$$p = [J_0 m_{t-1} + J_1 m_t] / (1 - \sum_{i=2}^{7} J_i) \tag{83}$$

Further equating terms containing u_{t-i} (for all i) we get

$$\Pi_0 = 1 \tag{84}$$

$$\Pi_1 = J_2 \Pi_0 + J_4 \Pi_1 + J_5 \Pi_2 \tag{85}$$

and $$\Pi_{i+1} = \left(\frac{1-J_4-J_7}{J_5}\right)\Pi_i - \left(\frac{J_2+J_6}{J_5}\right)\Pi_{i-1} - \frac{J_3}{J_5}\Pi_{i-2} \tag{86}$$

(for i = 2, 3, 4, ...)

Thus it is clear that Π_1 may be classified as an unknown free parameter which can take on any possible values. Once Π_1 is specified, Π_2 is derived from (84) and subsequently all other Π_i from (85). However, for each value of Π_1, pt can take on a special value, thus p_t in general will have infinite possible values.

Using the information in (83), (84), (85) one can express p_t as an ARMA (3,2) process of the following type:

$$J_5 p_t = Kp + (1-J_4-J_7)p_{t-1} - (J_2+J_6)p_{t-2} - J_3 p_{t-2}$$
$$+ J_5 u_t + (J_5\Pi_1 - 1 + J_4 + J_7)u_{t-1}$$
$$+ [J_5\Pi_2 - (1-J_4-J_7)\Pi_1 + (J_2+J_6)]u_{t-2} \qquad (87)$$

Note that the solution path is non unique since it depends on Π_1 which can take on any value. Thus in general there will be an infinite number of solution paths for p_t and thus for the model.

However, the crucial point carries over from the basic model of the previous section. Observe from (66) and (65) that aggregate demand depends on the real rate of interest and is independent of money stock m_t. Thus money is neutral. Obviously y_t is independent of anticipated money, giving neutrality for supply.

We claim the following:

Proposition 4

Distinguishing between real and nominal interest rates, having an expected rate of inflation in the model and assuming classical adjustment equations for anticipatory price setting, we see that there are an infinite number of solution paths to the model. However, on all these paths, money is neutral.

This is exactly the same result as in our basic model. Thus extending and complicating the model does not change the basic conclusions. In analogous fashion the results of Propositions 2 and 3 from the last section can be shown to be true in a model with expected price inflation. The method of analysis can be extended to characterise solution paths and exhibit neutrality or otherwise.

Section 4 : Conclusion

Analysis of classical macroeconomic transmission mechanisms has had a long and distinguished history. Policy shocks were transmitted through the economy and affected major macro variables through two fundamental methods, embodied in the quantity theory of money and the theory of loanable funds. One of the major purposes of this chapter was to study the implications of rational expectations in a framework put forward by pre-Keynesian economists. We analysed the conclusions of new classical macroeconomic in the light of the principles of old classical macroeconomics.

The other strand of the paper was to study alternative adjustment rules under non-clearing market disequilibrium. Since in general, adjustment rules are ad hoc ("free parameters"), it is important to see how sensitive major conclusions are to different specifications. As mentioned earlier, our adjustment rules followed classical postulates, thus integrating disequilibrium methods with traditional concepts.

The general supposition that disequilibrium per se leads to non-neutral effects with perfectly anticipated money, is found to be incorrect. The structure of the model determines the nature of solution paths and gives suitable results regarding policy effectiveness (or otherwise). Models with real balance effects show that even under RE, perfectly anticipated stock of money can influence effective demand, and thus under Keynesian unemployment, can have real effects in the short run. On the other hand, without real balance effects affecting aggregate demand, money is neutral for all possible solution paths. Assumptions regarding the existence of non-zero expected rate of inflation do not influence the final results. These disequilibrium conclusions agree broadly with those found for equilibrium models.

These results strengthen the policy issues discussed in Sen (1980) (though in a completely different context). Neither RE nor disequilibrium by itself is sufficient to give neutral (or non-neutral) monetary policy. The structure of the model and underlying behavioural assumptions are crucial in the debate as to whether anticipated money can influence output and employment.

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Chapter IX

Medium Term Dynamics and Disequilibrium.

Section 1. Introduction

Dynamic models of the open macroeconomy have generally emphasised disequilibrium in the sense that certain aggregative markets fail to clear, in the short run, due to price rigidity or stickiness. It is generally assumed that financial markets equilibrate quickly given the flexibility of relevant variables such as exchange and interest rates. On the other hand the labour and/or commodity markets exhibit disequilibrium since the wage and price level do not adjust rapidly. Various forms of contracts, implicit or explicit, constrain the movement of wage rate and aggregate price; thus disequilibrium persists until long run equilibrium (stationary state) is reached.

The literature has handled these issues in two conceptually separate ways, though there are formal similarities in the method of solution. The first was initiated by Dornbusch (1976) who extended the Mundell-Fleming model by assuming a price adjustment mechanism whereby the aggregate price level rises (falls) in response to excess demand (supply) in the commodity market. Exchange rate dynamics obey covered interest parity. The speed of adjustment of price is finite; it is a "state" variable (in the language of control theory) and can not jump when new information or news is available in the current period; this defines a "backward-looking" variable. The exchange rate on the other hand is a "forward-looking" or jump variable and responds rapidly to current information. The exchange rate is also a major expectational variable and its expected appreciation (in the presence of interest rate differentials between the home country and the exogenous rest of the world) plays an important role in the analysis. Under perfect foresight (rational expectations without uncertainty) the jump in the exchange rate allows the economy to achieve a stable trajectory when parametric changes disturb an initial long run equilibrium. The transition path from one equilibrium to another is characterised by disequilibrium in the product market. One can assume nominal wage rigidity, in Keynesian fashion, and unemployment in the labour market (perfecly elastic labour supply at the fixed money wage rate). But this is not strictly necessary as Dornbusch (1980) shows. Major extensions have been made of this basic model, see for example Buiter and Miller (1981); the fundamental structure however remains similar.

The supply of output, in the foregoing models, is given by an exogenously determined full employment level and is assumed fixed at the "natural rate". The alternative way of analysing these issues is through an aggregate supply <u>function</u> dependent on the real wage rate via the labour demand curve. (Sachs 1980). The advantage of this method is that it can handle real wage resistance or rigidity more explicitly. But more important, it emphasises the supply side and relates wage-price dynamics to the behaviour of aggregate supply (and the firm); in principle, it gives as much importance to supply as to aggregate demand emanating from the traditional IS/LM structure. Unfortunately, this class of models <u>assume</u> aggregate supply equals demand so that there is no necessity for a price adjustment equation. The price level changes rapidly to equate demand and supply in the commodity market. Thus, an essential tenet of disequilibrium analysis, the divergence of desired supply and demand, is removed. Essentially, the model therefore becomes an equilibrium one (except for the labour market which remains implicit in any case). In what follows therefore we confine attention to the Dornbusch type model, assuming explicitly the possibility that aggregate demand and supply are not equal except in long run equilibrium.

Two problems remain however. The assumption of fixed desired supply of output, at "full employment" level, is unreasonable since as prices changes the profit maximising firm would wish to change its output level. Fixed input-output coefficients, mark-up pricing and the aggregate firm supplying the amount demanded at the going price, do not seem to be consistent with the postulated price dynamics. We need to explicitly model the supply side; this is simillar to Sachs (1980) but with the fundamental difference that disequilibrium will be stressed. Of even greater importance however is that when desired demand and supply are not equal then the actual output should be the minimum of the two. This requires some consideration of the dual decision hypothesis and recalculation of the parameters and responses of the system.

Thus, the innovations of our model are the following: (a) analysis of an explicit supply function; (b) use of that supply function to determine price movements; (c) modelling disequilibrium formally by invoking the dual decision hypothesis. This is done within a Dornbusch framework by keeping nominal wage constant and assuming that the supply of labour is perfectly elastic at the current wage. The case of fixed real wage is easily handled and the method is pointed out where necessary though no explicit analysis is done. Labour supply, and modelling household behaviour constrained by unemployment, is beyond the scope of this chapter.

The fundamental assumption made here is that actual output is constrained by the minimum of planned demand and supply. An alternative way of handling disequilibrium would be to assume inventory holding (so that demand can always be met) and analyse the firms supply behaviour in terms of inventory stocks held. This however needs a separate treatment due to its importance and we discuss it more fully in the next chapter.

Section 2. The Basic Model

The usual way of modelling the supply side, in the short run, is to assume fixed capital stock and determine profit maximising employment level of the firm; this in turn gives the optimum output. We use a non-standard, though realistic, procedure by taking a production process which requires an intermediate imported input (call it oil), together with capital and labour, to produce the aggregate domestic output. Following Findlay and Rodriguez (1977) we distinguish between gross output (Q) and net output (q). The former is a function of (fixed) capital, labour and oil inputs. Net output however is gross output minus the real value of the intermediate input which is used up fully in the production process. The production function is of the mixed neoclassical Leontief type and is given by :

$$Q = \min\,[F(K,L), \alpha \quad R] \qquad (1)$$

where F(.) is neo-classical 'α' is a fixed input-output ratio (between gross output Q and the quantity of intermediate input oil, R); choice of units makes $\alpha = 1$. Since this is a small open economy, it can buy any quantity of oil R in the world market at the fixed price P^* (denominated in the world currency). The domestic factors of production are the limiting ones, thus $F(K,L) \leq R$. Efficiency demands that $F(.) = R$ (remember $\alpha = 1$).

Let P be the price of the domestic good; w is the nominal wage rate; E is the nominal exchange rate; given P^* is the dollar pice of oil, EP^* is the domestic currency price of the intermediate input imported from abroad. Firms profit is defined as

$$T = PQ - wL - P^*ER$$

$$= PQ - wL - P^*EQ$$

$$= (P - EP^*)F(K,L) - wL \qquad (2)$$

Profit maximisation implies

$$(P - EP^*)\ F_L - w = 0 \qquad (3)$$

where $F_L = \partial F/\partial L$. Note also that $F_L > 0$; thus $(P - EP^*) > 0$

Taking total differentials of (3),

$$F_L dP - F_L P^* dE - F_L E dP^* + (P - EP^*)F_{LL} dL - dw = 0 \qquad (4)$$

Noting that L is positively related to Q we have, from (4),

$$Q = Q(E,\ P,\ P^*)$$

and

$$Q_E < 0, \quad Q_P > 0, \quad Q_{P^*} < 0 \qquad (5)$$

(the subscript gives the relevant partial derivative).

Net output, measured in terms of the domestic product, is defined as

$$q = Q - (EP^*/P)R \tag{6}$$

Since Q = R by choice of units and technology we get

$$q = (1 - EP^*/P)Q \tag{7}$$

Using (5), it is clear that

$$q = q(E,P,P^*) \tag{8}$$

and

$$q_E<0, \quad q_P>0, \quad q_{P^*}<0 \tag{9}$$

The implicit assumption that has been made in the derivation of (3) to (9) is that nominal wage w is constant. Thus in (4), dw = 0 and we get the postulated relation between Q(or q) and (P,E,P^*). It is easy to see what happens when real wage, in terms of the domestic price level, is constant. We have, instead of equation (3), the following:

$$(1 - EP^*/P)F_L - w/P = 0 \tag{3'}$$

Total differentials will again give functional forms for Q and q. The only difference is that gross and net output will now be dependent on the real exchange rate (E/P), as well as the real wage, and there will be no independent price effect emanating from P. Thus we have, for example,

$$q = q(E/P, P^*) \tag{8'}$$

With perfect wage indexation, the fundamental conclusions of the model remain unchanged. Henceforth, therefore, we shall work in

Keynesian fashion assuming money wage to be constant.

Before turning to the demand side it is useful to find out the value of oil imports, in terms of domestic product, since this is a constituent part of the trade balance. Define this variable as MR. Then

$$MR = Q - q \tag{10}$$

Using (6),

$$MR = (EP^{*}/P)R = (EP^{*}/P)Q \tag{11}$$

Substituting from (7),

$$MR = (EP^{*}/P-E)q \tag{12}$$

Equation (12) gives us the value of the intermediate input in physical units of the home good (Q or q), provided the net output produced in the economy is q. In other words, if it is necesary to produce an amount q then the amount of oil required is given by (12). If q is the unconstrained supply of output, as given by profit maximization and equation (8), then q will itself depend on P and E in the manner postulated in (9). However, in a disequilibrium model there is no reason to suppose that the firm will be able to sell its desired supply. Then q will be different from the above noted relation. To understand this problem more clearly we now turn to the specification of the demand side.

Aggregate demand is determined within the IS/LM framework. Let realised or actual income be denoted by y, while planned demand, for income level y, is given by d. Trade balance, to be defined later, is denoted by B. Domestic absorption, of the private sector, is A.

Government spending G is all on the domestic output. Variables d, y, B, A, and G are all measured in terms of the home product. Commodity market equilibrium, or the IS relation, is given by

$$d = A + G + B \tag{13}$$

Specifying conventional functional forms,

$$d = A(y,r) + G + B(y, E, P, P^*) \tag{14}$$

(where $0 < A_1 < 1$, $A_2 < 0$, r is the rate of interest). We can introduce the Laursen-Metzler effect in the absorption function (see Ford and Sen (1985)) but this does not add to or change significant conclusions.

Trade balance B is defined as

$$B = X(E/P) - MC(y, E/P) - MR(E, P, P^*) \tag{15}$$

X is exports, MC is import of a consumption good whose price is set at unity, and MR is the value of oil imports; all these are measured in term of the domestic product. Exports and consumption imports are function of the real exchange rate while MR is already defined by (12). Marshall- Lerner tells us that

$$B_2 = B_E > 0, \quad B_3 = B_P < 0. \tag{16}$$

Further,

$$B_1 = B_y < 0 \tag{17}$$

The exchange rate is perfectly flexible.

The money market equilibrium condition, LM relation, is kept simple:

$$H = k(r)Py \qquad (18)$$

where H is the given stock of money and $k_r < 0$. It should be noted that we use the same rate of interest, r, in both the IS and LM schedules; thus no distinction is made between real and nominal rates of interest. This is only a matter of convenience; suitable extensions can be made to incorporate the distinction between two interest rates; tractability induces us to stick to the simpler formulation; no essential conclusions are altered.

Before introducing dynamics, we would like to solve for the equilibrium model. In this Mundell-Fleming world, domestic and world interest rates are equalised so that $r=r^*$. Long run equilibrium also means that aggregate demands, net supply of output and realised income are equal; hence $q = d = y$.

Thus we have

$$q = q(E, P, P^*) \qquad (19)$$

$$q = A(q, r^*) + G + B(q, E, P, P^*) \qquad (20)$$

$$H = k(r^*)Pq \qquad (21)$$

Substituting (19) into (20) and (21) we can solve for E and P for given values of exogenously specified G, H, and P^*. The long run equilibrium levels of E and P then give us, from (19), the value of net output q.

Four types of comparative statics excercises are possible: monetary policy, fiscal policy, change in the world price of oil, shift in the nominal wage. We deal with the first two; simple extensions can accomodate the other changes. First let us discuss fiscal policy, more specifically an increase in G.

Writing (20), (21), in excess demand form, and taking total differentials, we get

$$a_{11}dE + a_{12}dP = K_1\ dG \tag{22}$$

$$a_{21}dE + a_{22}dP = 0 \tag{23}$$

where

$$a_{11} = (A_1 - 1)q_E + B_1q_E + B_2 > 0$$

$$a_{12} = (A_1 - 1)q_P + B_1q_P + B_3 < 0$$

$$a_{21} = kPq_E < 0 \tag{24}$$

$$a_{22} = k(q + Pq_P) > 0$$

$$K_1 = -1$$

Solving (22), (23) we get

$$\partial P/\partial G = (kP/|J|)q_E \tag{25}$$

$$\partial E/\partial G = -k(q + Pq_P)/|J| \tag{26}$$

The determinant of the Jacobian J cannot be signed a priori since

$$|J| = a_{11}a_{22} - a_{12}a_{21} \tag{27}$$

Differentiating (19) with respect to G and substituting (25), (26), we get

$$\partial q/\partial G = -(kqq_E/|J|) \tag{28}$$

We know that $q_E < 0$ (see (9)). Thus government expenditure on the domestic product can increase aggregate output, i.e. $\partial q/\partial G > 0$ provided $|J| > 0$. Henceforth we assume that $|J|$ is indeed positive.

Turning now to monetary policy, we find that

$$\partial P/\partial H = a_{11}/|J| > 0 \tag{29}$$

$$\partial E/\partial H = -a_{12}/|J| > 0 \tag{30}$$

Thus under the postulated conditions, expansionary monetary policy raises the price level and causes the exchange rate to depreciate. The upward movement, for long run equilibrium, of both P and E is as expected from monetary models of flexible exchange rate determination.

Since our main interest here lies in disequilibrium dynamics we shall consider in some detail the behaviour of the economy out of stationary state (given by (19) to (21)). In what follows we assume that the economy is perturbed by a monetary shock, specifically that

H rises. The transition path of the economy towards a new long run equilibrium will be our major focus henceforth.

Section 3. Dynamics and Disequilibrium

The two endogenous variables of primary interest are the price level (P) and nominal exchange rate (E); all other variables are functionally dependent on these two. We therefore need to specify their dynamic behaviour in disequilibrium.

Exchange rate dynamics depends on covered interest parity and rational expectations. Thus we have

$$\dot{E}/E = r - r^{*} \qquad (31)$$

(where $\dot{E} = dE/dt$ and t is time). The determination of the domestic interest rate is not clear-cut however and we need to specify it carefully; but this requires a prior consideration of the aggregate commodity market and the behaviour of prices over time.

The standard form in which price dynamics is usually expressed is

$$\dot{P} = \quad [d - q] \qquad (32)$$

(speed of adjustment is set equal to unity by choice of units) where d is aggregate demand from the IS/LM model and q is supply of net output from the postulated behaviour of the firm. But, with disequilibrium, the definitions of d and q become complicated and need to be handled with care. Specifically, define realised income y as

$$y = \min [d,q] \tag{33}$$

We can now have two (or even three) sub-cases.

First, <u>Case 1:</u>

$$y = d < q \tag{34}$$

$$\dot{P} < 0$$

Secondly, <u>Case 2:</u>

$$y = q < d$$

$$\dot{P} > 0 \tag{35}$$

Figure 9.1 shows the configurations, with equilibrium ($\dot{P} = 0$) price level at $\hat{P}$. The curves are drawn on the basis of a given exchange rate E.

The line AB shows the unconstrained supply function with q rising with P. The line CED is the downward sloping macroeconomic demand schedule derived from the IS/LM model. It is essential to stress that this curve is unconstrained in the sense that for each P it shows the level of aggregate demand that is always (elastically) supplied; demand creates its own supply.

For $P > \hat{P}$, actual income or output is given by points on CE. Realised income $y = d$; q is planned but not realised. The level of national income (product) is derived from the IS/LM equations (14) and (18); so also is rate of interest; MR is found from (12) given that actual output is y. Price dynamics is regulated by

$$\dot{P} = \quad [y - q] \qquad (36)$$

$$y = d(E,P) = y(E,P)$$

Unconstrained supply (q) still affects price dynamics as in (32). The inability of the firm to sell its desired output has an effect on the labour market where a "dual decision" needs to be taken. However, since we do not consider explicitly the labour market, this behaviour is not modelled here. Prices therefore change as a response to the difference between unconstrained demand and supply as in the Dornbusch model. A major difference however is in the calculation of the oil input; there is an implicit dual decision in the intermediate input market where the firm re-calculates the value of oil imports on the basis of its constraint i.e. net output produced must equal demand.

For the second case, $P<\hat{P}$, the position is diametrically reversed. Output is now given by q; this must therefore be the realised income, y = q. The unconstrained IS/LM analysis loses its significance. Aggregate demand must be re-calculated as a constrained demand, d, given actual income is q. The constrained demand curve is drawn as EF and price movements depends on the level of d on this curve. We have therefore,

$$\dot{P} = \quad [d - y] \qquad (37)$$

$$(y = q(E,P,P^*))$$

Price dynamics therefore depends on the difference between constrained demand and actual supply of output.

In this phase, income or output is given by the supply function. We assume that the money market clears rapidly so that from (18), putting $y = q$, we can derive the actual interest rate which equilibrates the money market. This value of r, as well as the fact that $y = q$, helps us to derive the level of constrained demand d in equation (14), the IS relation. Given the values of d and y (or q), price dynamics is generated according to (37). We now derive these relations formally.

Case 1 gives, from (34), (14), and (18),

$$A(y,r) + G + B(y,E,P,P^*) - y = 0 \qquad (14)'$$

$$k(r)Py = H \qquad (18)'$$

(noting aggregate demand d is equal to realised income y).
Taking total differentials of (14)′ and (18)′ we can solve for $y = y(E, P)$ and $r = r(E, P)$ in the region $\dot{P} < 0$.
This gives

$$y_E = -B_2k_1Py/|J| \quad > 0 \qquad (38)$$

$$y_P = kyA_2 - B_3k_1Py/|J| \quad < 0 \qquad (39)$$

$$r_E = kPB_2/|J| \quad > 0 \qquad (40)$$

$$r_P = [B_2kP - ky(A_1 + B_1 - 1)/|J|] \quad > 0 \qquad (41)$$

$$|J| = (A_1 + B_1 - 1)k'Py - A_2kP \quad > 0 \qquad (42)$$

The signs of (38) to (40) are obvious. The sign of r_P is ambiguous, but we follow the standard usage in the literature and assume positivity. It is quite simple to derive the analytics of the case where $r_P < 0$ (see for example Ford and Sen (1985)).

The dynamic equations for E and P are now

$$(\dot{E}/E) = r(E, P) - r^* \quad (43)$$

$$\dot{P} = [y(E,P) - q(E,P,P^*)] \quad (44)$$

It is clear, using (9), (38), (39) that the P- stationary is upward sloping. Similiarly, from (40), (41) we note that $\dot{E}/E = 0$ line slopes downwards. Further,

$$\partial \dot{P}/ \partial P = [y_P - q_P] < 0 \quad (45)$$

$$\partial (\dot{E}/E)/\partial E = r_E > 0 \quad (46)$$

All this information help us to draw the phase diagram (Figure 9.2) <u>in the range</u> $\dot{P} < 0$. If long run equilibrium is given by Z, then SZ denotes a saddle path -- the only path in this region that is convergent.

Consider now Case 2 where the supply constraint is binding. Thus

$$y = q < d \quad \text{and} \quad \dot{P} > 0 \quad (47)$$

The standard IS/LM relations, based on perfectly elastic supply at any given aggregate demand, now lose their significance. We assume that the interest rate is determined by the instantaneously equilibrating money market, so that from (18),

$$r^a = k^{-1} (H/Py)$$

$$= k^{-1}(H/pq(E,P,P^*)) \quad (48)$$

It is clear that since $k_r < 0$, $q_E < 0$, $q_P > 0$,

$$r^a_E < 0, \qquad r^a_P > 0 \tag{49}$$

We distinguish this case by writing the interest rate as r^a (actual rate of interest required to make demand and supply of money equal).

We now need to analyse the constrained demand function (see figure 9.1, curve EF). From (14), noting $y = q(E, P, P^*)$ we get

$$d = A(q(E,P,P^*),r^a) + G + B(q(E,P,P^*), E,P,P^*) \tag{50}$$

Taking total differentials,

$$dd = \{(A_1 + B_1)\, q_E + A_2\, r^a_E + B_2\}\, dE$$
$$+ \{ (A_1 + B_1)\, q_P + A_2\, r^a_P + B_3 \}\, dP \tag{51}$$

It is not possible, a priori, to sign the partials $\partial d/\partial E$ and $\partial d/\partial P$. A_1 is the propensity to spend (absorb) by the private sector on both domestic and imported goods; B_1 is the propensity to import: thus $(A_1 + B_1)$ is the propensity to spend, by domestic residents, on the home product, q. If we assume that this propensity, $(A_1 + B_1)$, is small, then we get

$$d_E > 0, \qquad d_P < 0 \tag{52}$$

The signs of the derivatives in (52) seem reasonable and we maintain them for subsequent analysis.

The dynamics in this region of excess demand is given by

$$\dot{P} = [d(E,P) - q(E, P,P^*)] \qquad (53)$$

$$(\dot{E}/E) = r^a - r^* \qquad (54)$$

where d is constrained aggregate demand whose form is given by (52); r^a is actual interest rate dependent on E and P with partial derivatives signed by (49).

It is clear that

$$\partial \dot{P}/ \partial P = [d_P - q_P] < 0 \qquad (55)$$

Further, excess demand implies that $\dot{P} > 0$.

Using (49), the E-stationary, or $\dot{E} = 0$ line, is upward sloping. Also,

$$\partial (\dot{E}/E)/\partial E = r^a{}_E < 0 \qquad (56)$$

Figure 9.3 uses the relevant information to draw the phase lines for the case $\dot{P} > 0$. It should be noted that $\dot{E} = 0$ has a positive slope greater than that of $\dot{P} = 0$. This can be formally proved. The equilibrium Z is a saddle point and S'Z is the only convergent path that leads to long run equilibrium.

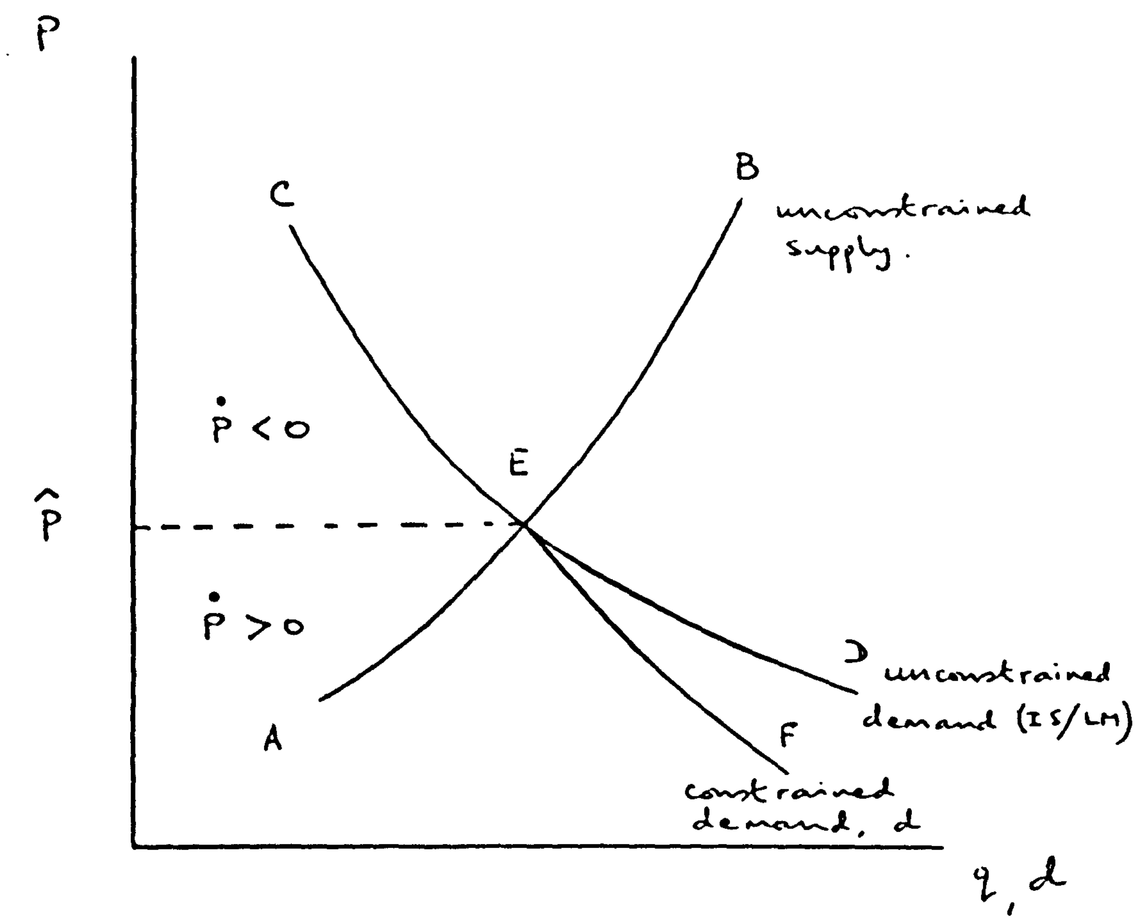
Figure 9.4 shows the complete picture with both cases superimposed on the same diagram. The $\dot{E} = 0$ line is kinked at the point of equilibrium Z; the usual models would have $\dot{E} = 0$ sloping downwards in both regions of excess demand and supply. The saddle path S'Z will have a higher absolute slope than SZ. Thus we have a kinked saddle path S'ZS -- a specific innovation of the disequilibrium model.

We make the usual assumption that P is a backward looking variable associated with the stable root while E is a jump variable related to the unstable root.

It is interesing to note the effect of expansionary monetary policy. In figure 9.5, the new equilibrium with a higher money stock, is given by Z_2. Compared to traditional models, monetary expansion causes an even larger overshooting of the exchange rate since the convergent path S˝Z has a higher absolute slope. This will increase aggregate demand even further, over and above that caused by increased money stock. However output is constrained by supply which does not respond to demand per se, except through price changes. In this specific case, the presence of the intermediate input will actually make output to fall on impact. Thus excess demand will be even greater than initially conceived. We should thus expect inflation to be much higher than forecast by traditional "overshooting" models. However, with a rise in P, ultimately net output q responds to expansion and its increase will be rapid since P is rising and E falling. Monetary expansion therefore causes high initial inflation with little output effect on impact. But, if wage claims are moderated, then the final movements of output and income can be rapid, motivated by rising prices and appreciating currency (leading to cheaper oil). Demand expansion has no direct effect since realised income or output is constrained by supply alone. All effects are indirect via price and exchange rate movements. This again is an interesting, and peculiar, conclusion of the disequilibrium analysis.

Section 4. Concluding Remarks.

Unlike the standard short run macro models of the open economy, we explicitly incorporate disequilibrium, assuming that the market clears on the short side. This gives rise to a dual decision on the part of agents which in turn has implications for the dynamics of the system. In particular, consequent to monetary expansion, the overshooting of the exchange rate is higher. This causes further excess demand and consequently more inflation. A feature of significant interest is the asymmetric behaviour of the model and a kinked saddle path.



P
C
B
unconstrained
supply.
Ṗ < 0
E
P̂
Ṗ > 0
D
unconstrained
demand (IS/LM)
A
F
constrained
demand, d
q, d

Figure 9.1

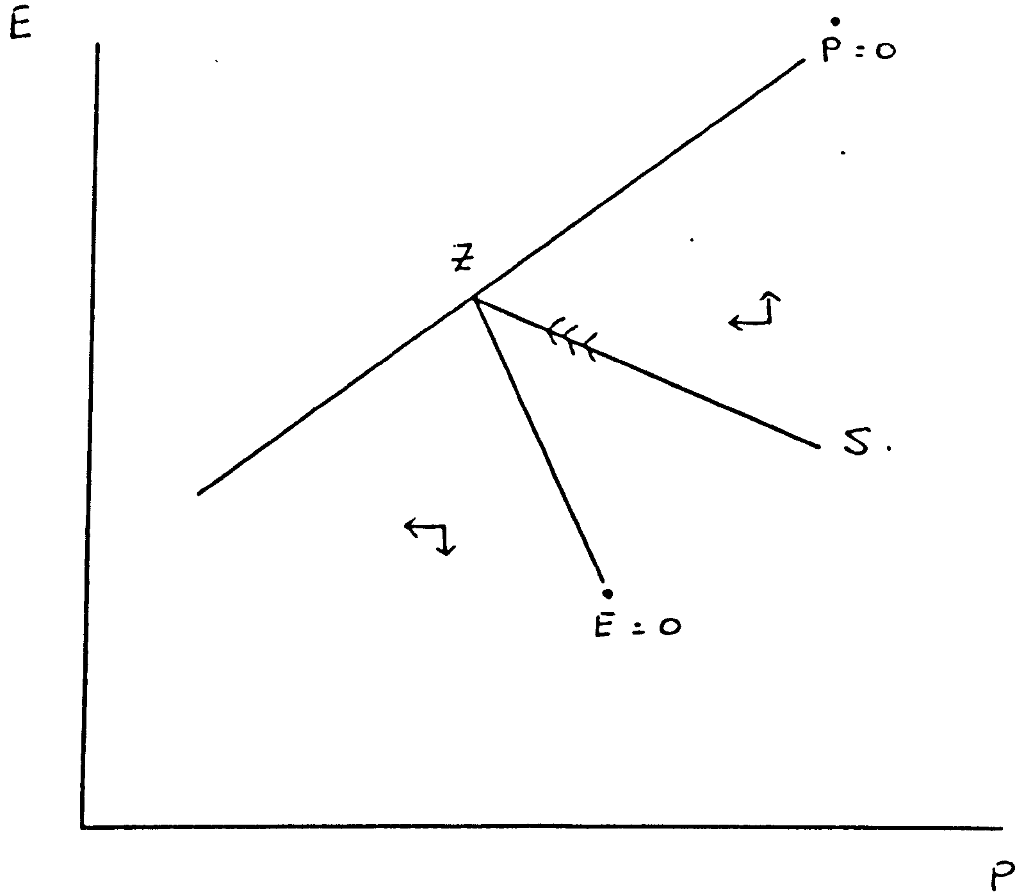


Figure 9.2

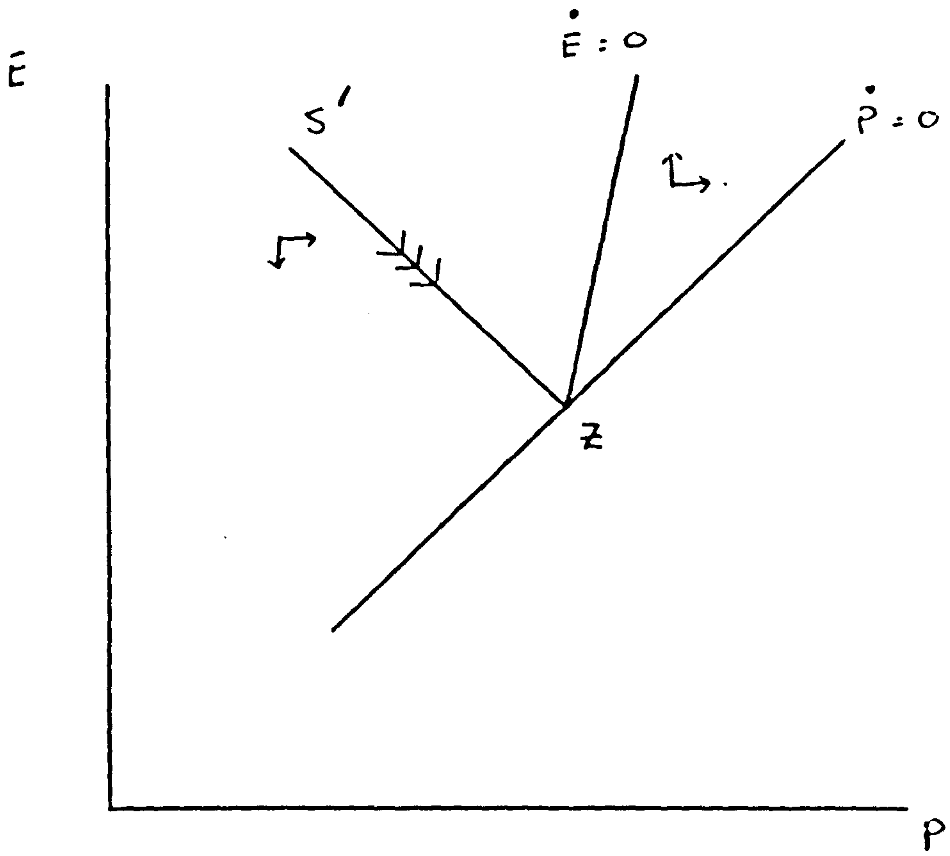


Figure 9.3

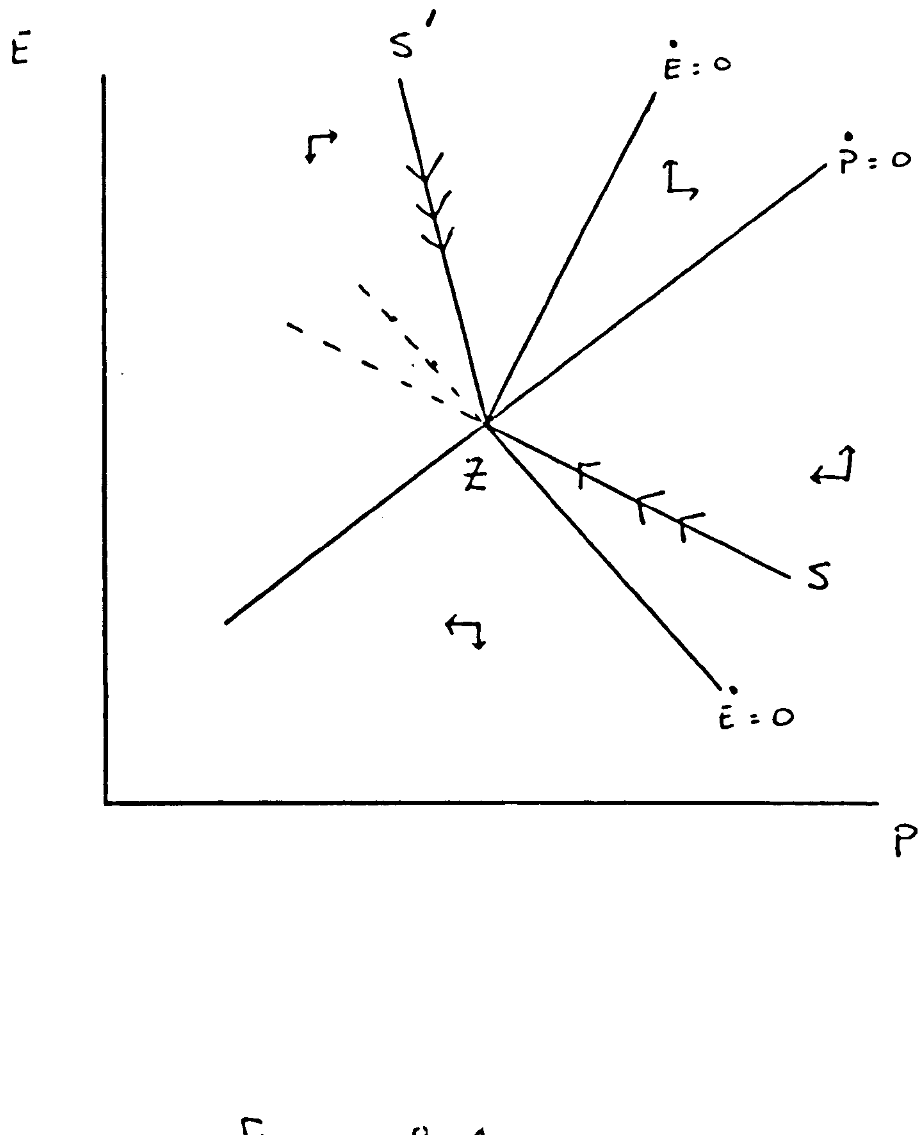


Figure 9.4

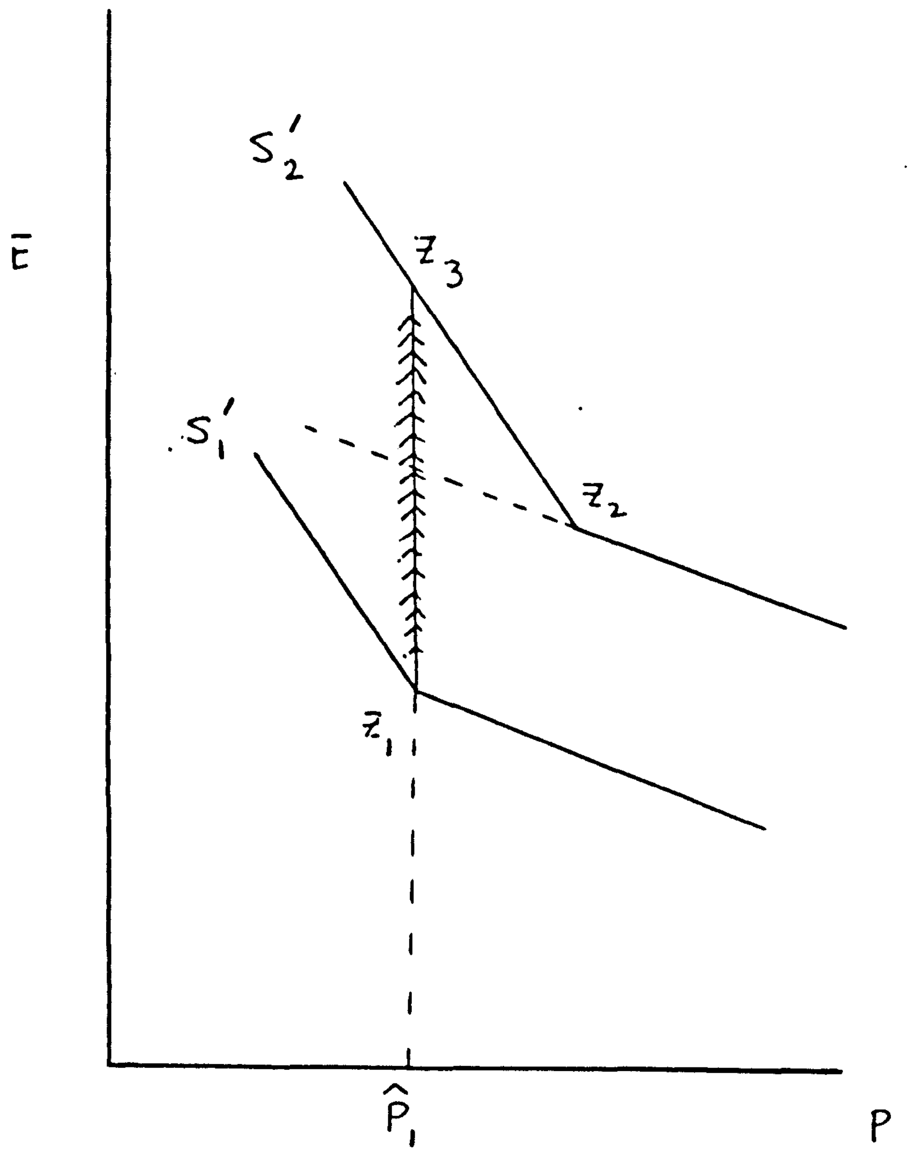


Figure 9.5

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Chapter X

Inventory Dynamics

Section 1: Introduction

It is now well known that a contractionary policy, for example in the form of an unanticipated reduction in the rate of monetary growth, may cause a discontinuous "jump" in the real exchange rate and consequent overshooting of its long run equilibrium value (Dornbusch (1976)), Buiter and Miller (1981)). A sudden appreciation of the exchange rate adds to the output depressing effects of monetary contraction and is a major cause of business cycles in the open economy. However, this analysis of policy-induced economic downturn ignores completely the role of inventory fluctuations in adjusting to disequilibrium and in the propagation of short run changes in aggregate output. The purpose of our chapter is to show within a theoretical model, how the behaviour of inventory dynamics may be one of the most important causes of output fluctuations in a small open economy.

Current research on inventories in macro models (see Blinder (1980), Blinder and Fischer (1981), Ackley (1983)) has succeeded in restoring this relatively neglected topic to its rightful position as a major explanatory variable in the analysis of output changes consequent to government policy shocks. Unfortunately, macro inventory models to date have predominantly concerned themselves with a closed economy. The open character of the macro economy exacerbates the inventory effects of changes in policy parameters. Thus it is extremely interesting to study the simultaneous movement of output, inventories and exchange rates. This chapter attempts to analyse these crucial relationships and integrate aggregate inventory analysis with macro theories of the open economy.

We wish to study the macrodynamics of inventories and output in an open economy with flexible exchange rates. In particular, the focus of our analysis will be the possible discontinuous behaviour of inventory and real exchange rate consequent to monetary shocks, when the domestic price level is sticky. It will be demonstrated that if the growth rate of money is decreased, the real exchange rate appreciates rapidly ("instantaneously"), while the stock of inventories held, rapidly falls. There may thus be "overshooting" in both exchange rate and inventory as they fall below their equilibrium levels in the short run, after which they both slowly move up to attain the new equilibrium. The transition path from one equilibrium to another is characterised by lower than desired inventory and an appreciated domestic currency following a monetary contraction. Both effects tend to depress aggregate output causing a recession and downward movement in the business cycle.

The stylised facts of the recent recessionary experience of the U.K. described by Buiter and Miller (1981) are consistent with our analytical model. Thus the chapter though theoretical, also has an additional interest in explaining some facets of the current movements of U.K. exchange rates and manufacturing output in the light of inventory dynamics.

A key relation in the models that follow is the functional relationship between initial inventory holding (the independent variable) and level of production or output (the dependent variable). More specifically, would an increase in initial stock of inventory held by firms lead to an increase or decrease in the level of output. Analytical and empirical models at the firm level give no definite answers. Holt, Modigliani, Muth and Simon (1960) consider the improved production scheduling model where additional inventories make a multiproduct firm more efficient and thus contribute positively to output. However, Arrow, Karlin and Scarf (1958) and Mills (1962) claim the opposite and show that higher inventories lead to lower output. Blinder (1980) generally tends to agree with the latter view. We have shown elsewhere (see Deger and Sen (1983)) that using an optimal control framework and stressing intertemporal optimisation, analytical models can give both types of conclusions depending on parameter values. Empirical evidence is also not clear cut. Mills (1962) provides econometric results for the firm while Deger and Sen (1983) reports regressions using U.K. aggregate data - but no definite conclusions can be reached. Given this ambiguity, we choose to work with both the alternative hypotheses and demonstrate the rather startlingly different transition paths that follow in a macro model of the open economy.

The plan of the chapter is as follows. The next section summarises the model, Section 3 analyses transition paths consequent to policy shocks and the last section draws some policy conclusions.

Section 2: The Model

The basic model reflects our interest in analysing the role of inventory fluctuations as a cause of business cycles within the framework of an open macro economy. We concentrate on monetary shocks induced by a change in government policy parameters. The interested reader can work out the implications for fiscal policy and/or other real shocks.

The model is described by the following equations:

$$Y = -\gamma(r-\dot{p}) + \delta(e-p) + G \quad (1)$$

$$m = kY - \lambda r + p \quad (2)$$

$$\dot{p} = \pi + \phi Y + \theta(\hat{N}-N) \qquad (3)$$

$$\pi = \mu = \dot{m} \qquad (4)$$

$$\dot{N} = Y - \underset{\sim}{Y} \qquad (5)$$

$$Y = -a + bN \qquad (6)$$

$$e = r - \hat{r} \qquad (7)$$

(A '•' over a variable represents a time derivative).

$\underset{\sim}{Y}$ is aggregate demand (sales) while Y is aggregate output (production). r is the nominal rate of interest. p is the log of price level P, thus $\dot{p} = \dot{P}/P$ is the rate of inflation. e is the log of the nominal exchange rate, so that $\dot{e}$ is the rate of currency appreciation. The exchange rate is defined as the domestic currency per unit of foreign currency, so that its increase signifies a depreciation of the domestic currency. m is log of nominal money stock (M), and $\dot{m} = \dot{M}/M$ is the growth rate of money, here assumed to be a policy parameter μ. π is defined as the "core" or underlying rate of inflation. N is stock of inventory and $\hat{N}$ is the optimal or desired level of inventory demand. $\hat{r}$ is the foreign (international) rate of interest exogenously given. G shows the effect of autonomous demand, for example government expenditures. γ, δ, λ, ϕ, θ, k are behavioural parameters, constant and all positive.

Equation (1) is the IS relation where aggregate demand is a function of the real interest rate $(r-\dot{p})$ and the log of the real exchange rate $(e-p)$. Equation (2) gives the LM relation demand (equal to supply) for money is a function of aggregate demand for output (transactions motive), nominal rate of interest (speculative motive) and price level. This equation is in semi-log linear form. The index of real balance is thus m-p. Equation (3) is a Phillips curve "augmented" by an inventory factor. Rate of inflation $\dot{p}$ is the sum of "core" rate of inflation π, a variable reflecting aggregate output (production) Y, and the difference between actual and desired inventory. When the desired demand for inventory is greater than actual holding, then prices are increased to reduce demand and stimulate stock creation. Equation (4) assumes that the core rate of inflation is equal to the growth rate of money ($\dot{m}=\mu$), where μ is fixed by monetary authorities. Choice of units makes full employment output equals zero.

Equation (5) is definitional where inventory accumulation is the difference between production Y and demand X. We assume that no stock-outs occur so that if demand is greater than current production, the former is met by reducing stocks. Equation (6) is crucial given the previously mentioned relation between output and inventory. As discussed in Section 1, the sign of parameter b is controversial. It has been claimed that increase in initial inventory held, may add to the efficiency of a firm leading to a higher output, thus $b > 0$. Alternatively, increasing marginal costs to inventory holding may make $b < 0$. We shall thus leave the sign of b unspecified, dealing with both positive and negative cases. Note if $b > 0$, $a > 0$, alternatively if $b < 0$, $a < 0$.

Equation (7) describes the open economy's capital accounts and represents covered interest parity. We assume rational expectations which in the absence of uncertainty is tantamount to perfect foresight. Thus the expected and actual exchange rate appreciations are equal to each other and depends on the difference between domestic and foreign rates of interest ($r-\hat{r}$).

The above model incorporates the general features of open economy macromodels as well as macroinventory analysis (see Buiter and Miller (1981) for the former and Blinder (1980) for the latter). However, as far as we know, they have never been brought together, and thus the primary purpose here is to study the interrelationships between e, N, Y, Υ and μ.

Since our major interest is in transition paths and disequilibrium phenomenon, we assume long run neutrality. Thus the long run equilibrium value of inventory held (the optimal or desired level) is assumed to be a constant $\hat{N}$, unaffected by monetary growth rate μ. It is a simple extension to the results that follow, to assume that $\hat{N}$ is a function of nominal variables, thus allowing for non-neutrality. The disequilibrium behaviour of the economy remains similar and nothing of substance is altered. We shall also assume from now on for notational simplification that G=0. The case for G>0 will be obvious.

To solve the model for its long run equilibrium, define two new variables, an index (log) of real balance

$$\ell = m - p \tag{8}$$

and an index (log) of real exchange rate

$$c = e - p \tag{9}$$

In long run equilibrium

$$\dot{\ell} = \dot{c} = 0 \tag{10}$$

thus real balance and real exchange rates are constants. Further, the core rate of inflation is equal to the actual rate of inflation. We have

$$\dot{m} = \mu = \pi = \dot{p} \tag{11}$$

Equilibrium is also characterised by inventory attaining its desired level

$$N = \hat{N} \tag{12}$$

and

$$\dot{N} = 0 \tag{13}$$

Denoting equilibrium by starred (*) values and using (1) to (13) we have

$$Y^* = X^* = 0 \tag{14}$$

$$N^* = \hat{N} = a/b \tag{15}$$

$$\dot{p} = \mu \tag{16}$$

$$r^* = \hat{r} + \mu \tag{17}$$

$$\ell^* = -\lambda(\hat{r} + \mu) \tag{18}$$

$$c^* = (\lambda/\delta)\hat{r} \tag{19}$$

Let us now consider dynamic paths. Inventory behaviour, exchange rate movements and monetary policy are central issues here so dynamics will be analysed in terms of three variables - inventory (N), real balance (ℓ) and real exchange rate (c).

Using equations (1) to (4) and (6) we first get

$$X = (1/\Delta)[\gamma\ell + \lambda(\gamma\phi b - \gamma f)N + \delta c + \lambda q] \tag{20}$$

(where $\Delta = (k\gamma + \lambda) > 0$; $q = (\gamma\mu - \gamma\phi a + \gamma f\hat{N})$) and

$$r = (1/\Delta)[k\gamma(\phi b - f)N + k\delta c - \ell + kq] \tag{21}$$

Before proceeding further, we need to investigate more closely the relationship between aggregate demand or sales (X) and stock of inventory (N). From (20) we have

$$\partial X/\partial N = (\lambda\gamma/\Delta)(\phi b - f) \qquad (22)$$

As mentioned earlier, the sign of b is ambiguous. 'b' gives the behavioural relation between output and inventory ($b = \partial Y/\partial N$ from (6)). If b is negative as suggested by some of the micro literature on inventories (including Blinder (1980)), then it is definitely true that $\partial X/\partial N < 0$. However, this contradicts the recent macro literature (Blinder and Fischer (1981)) which claim that an increase in initial inventory will increase total sales, thus under equilibrium, $\partial X/\partial N > 0$. A positive relation between X and N is consistent with $b > 0$ but not otherwise. As we shall see later, the sign of $(\phi b - f)$ (which in turn signs $\partial X/\partial N$ from (22)) will be extremely important in studying transitional paths. Given that past literature is controversial, to circumvent this problem we shall therefore deal with two alternatives, assuming first that $(\phi b - f) > 0$ and noting its implications and then doing the opposite. To simplify the notation from hereon we define

$$\Omega = (f - \phi b) \qquad (23)$$

Noting from (8) that

$$\dot{\ell} = \dot{m} - \dot{p} \qquad (24)$$

we get after some simplification

$$\dot{\ell} = (\phi a - f\hat{N}) + \Omega N \qquad (25)$$

Similarly from (9)

$$\dot{c} = \dot{e} - \dot{p} \qquad (26)$$

and thus

$$\dot{c} = (\lambda/\Delta)\Omega N - (1/\Delta)\ell + (k\delta/\Delta)c + [(k\alpha/\Delta) - \hat{r} - \mu + \phi a - f\hat{N})] \qquad (27)$$

Finally, using the definition of $\dot{N}$ from (5) and substituting from (1) to (6),

$$\dot{N} = [b + (\lambda\gamma/\Delta)\Omega]N - (\gamma/\Delta)\ell - (\lambda\delta/\Delta)c - [a + (\lambda q/\Delta)] \tag{28}$$

The complete model consists then of the three differential equations

$$\dot{\ell} = \Omega N + Z_1 \tag{29}$$

$$\dot{c} = (-1/\Delta)\ell + (k\delta/\Delta)c + (\lambda\Omega/\Delta)N + Z_2 \tag{30}$$

$$\dot{N} = (-\lambda/\Delta)\ell + (-\lambda\delta/\Delta)c + [b + (\lambda\gamma\Omega)/\Delta]N + Z_3 \tag{31}$$

where Z_1, Z_2, Z_3 are functions of exogenous variables defined as

$$Z_1 = (\phi a - t\hat{N})$$

$$Z_2 = [(kq/\Delta) - \hat{r} - \mu + \phi a - t\hat{N})]$$

$$Z_3 = -[a + (\lambda q/\Delta)].$$

The solution of these differential equations (29), (30), (31), gives us exact time paths of $c(t)$, $N(t)$, $\ell(t)$ and thus all other endogenous variables in particular $Y(t)$, $X(t)$, $r(t)$. The qualitative characteristics of solution paths and the effect of policy shocks are dealt with in the next section.

Section 3: Transition Paths and Monetary Shocks

The dynamic behaviour of the economy is well analysed by looking at the matrix of the first derivatives (Jacobian) of the system and studying the properties of characteristic roots. We have from (29) to (31),

$$A = \begin{bmatrix} 0 & 0 & \Omega \\ (-1/\Delta) & (k\delta/\Delta) & (\lambda\Omega/\Delta) \\ (-\gamma/\Delta) & (-\lambda\delta/\Delta) & [b+(\lambda\gamma\Omega)/\Delta] \end{bmatrix} \qquad (32)$$

The determinant of this matrix

$$|A| = \Omega[(\lambda\delta+\gamma k\delta)/\Delta^2]$$
$$= \Omega\delta/\Delta \qquad (33)$$

gives the product of the characteristic roots. As can be seen from (33), the sign of $|A|$ depends crucially on that of Ω. Since this is going to play a major role in the analysis and, as discussed previously, theory a priori does not allow us to sign unambiguously, we have to deal with a number of possibilities. In essence there are three alternative cases:

(a) $b > 0$, $\Omega < 0$

(b) $b < 0$, $\Omega > 0$

(c) $c > 0$, $\Omega > 0$

We shall deal in some detail with the first two. As will be clear to the reader, the solution method for case (c) is an amalgamation of (a) and (b) and therefore need not be analysed extensively.

Case (a): $b > 0$, $\Omega < 0$

Here an increase in inventory stock at the beginning of the period makes the multiproduct firm more efficient in planning production and thus increases output. Inventory has a direct productivity effect (see equation (6)). Further b is not too small, so we also have $\Omega = f - \phi b < 0$.

Since the determinant $|A|$ is negative from (33), there are two possibilities for the signs of the characteristic roots, if they are distinct and real (we rule out complex roots). Either, all three roots are negative, or one of them is negative and two positive. In the former case stability is guaranteed and the analysis is straightforward. However, the Routh Hurwicz conditions cannot be verified since the trace of the matrix A (equation (32)) need not be negative. Be that as it may, this possibility gives rise to standard results and need not detain us long.

The more interesting case is when one characteristic root is negative while the other two are positive. In such cases the stable root (negative) needs to be associated with a predetermined variable while the unstable roots (positive) need to be related to non-predetermined variables. In terms of economic intuition, the predetermined variables are "backward looking" in the sense that they cannot move discontinuously (or "jump" à la Sargent) and their current value is given by past history. The non-predetermined variables on the other hand are forward looking and may change rapidly (discontinuously) when the exogenous variables (such as money growth rate in our model) jump. If we can identify the number of stable roots with predetermined variables and the number of unstable roots with non-predetermined variables then a saddle path exists. Initial conditions for the predetermined variables will determine the initial values of the non-predetermined variables given the requirement that the latter lie on the stable manifold (plane) (see Dixit (1980)).

Identifying variables as backward or forward looking is a matter of economic intuition. Following Buiter and Miller (1981) we assume that the price level is sluggish, thus the stock of real balance (ℓ) is predetermined. This accounts for the unique negative root. On the other hand, the real exchange rate following Dornbusch (1976) is obviously a candidate for a forward looking variable which can change rapidly ("jump") in the presence of new information or policy shocks. We claim as the innovation of this paper that the level of inventory demanded by the representative firm in the open economy is also a forward looking non-predetermined variable. Producers with rational expectations foresee the future on the basis of current information after the policy change, and adjust their inventory stock very rapidly, possibly buying (selling) from abroad, thus making inventory non-predetermined. Unlike the traditional literature which deals with inventory as a residual, we analyse it in the context of a fuller decision-making and control variable. The economy behaves like a small firm in the competitive world; thus stocks can change rapidly.

To derive the equation for the stable trajectory and the values of non-predetermined c and N in terms of predetermined ℓ,, we utilise a method due to Dixit (1980). Assume that the three characteristic roots are θ_1, θ_2, θ_3, where

$$\theta_1 < 0, \theta_2 > 0, \theta_3 > 0 \qquad (34)$$

Thus θ_1 is associated with ℓ, θ_2 with c and θ_3 with N.

Let $$x' = (\ell-\ell^*, c-c^*, N-N^*) \qquad (35)$$

be the row vector of deviations of variables from their (new) equilibrium values. Then system (29), (30), (31) can be written as

$$\dot{x} = Ax \tag{36}$$

If M is the (3x3) matrix (with elements m_{ij}) whose rows are the corresponding left eigen vectors of A then

$$MA = BM \tag{37}$$

where

$$B = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & \theta_3 \end{bmatrix} \tag{38}$$

Let

$$y = Mx \tag{39}$$

Clearly

$$\dot{y} = By \tag{40}$$

so that for non-predetermined variables corresponding to the unstable roots, the corresponding y(t)'s must be zero. In other words, $y_2(t) = 0 = y_3(t)$.

From (39) we have

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} \bar{\ell}(t) \\ \bar{c}(t) \\ \bar{N}(t) \end{bmatrix} \tag{41}$$

where barred values denote deviations from equilibrium. Given the requirements for saddle path stability,

$$y_2(t) = m_{21}\bar{\ell}(t) + m_{22}\bar{c}(t) + m_{33}\bar{N}(t) = 0 \tag{42}$$

$$y_3(t) = m_{31}\bar{\ell}(t) + m_{32}\bar{c}(t) + m_{33}\bar{N}(t) = 0 \qquad (43)$$

Equations (42), (43) can be solved to get

$$\bar{c}(t) = [(-m_{21}m_{33}+m_{23}m_{31})/(m_{22}m_{33}-m_{23}m_{32})]\bar{\ell}(t) \qquad (44)$$

$$\bar{N}(t) = [(-m_{22}m_{31}+m_{21}m_{32})/(m_{22}m_{33}-m_{23}m_{32})]\bar{\ell}(t) \qquad (45)$$

which is written in simplified form as

$$c(t) - c^* = v_1(\ell(t) - \ell^*) \qquad (46)$$

$$N(t) - N^* = v_2(\ell(t)-\ell^*) \qquad (47)$$

where v_1 and v_2 are functions of m_{ij}.

For initial values of $\ell(0)$, these two equations then give us the values of c(0) and N(0). Thus (46), (47) are the equations for the saddle path that leads the economy to its ultimate long run equilibrium.

We now need to find the characteristic vectors or elements of M. Take the second row of M initially and note that it corresponds to the root θ_2. Thus

$$(m_{21},m_{22},m_{23})((\theta_2 I-A)) = (0, 0, 0) \qquad (48)$$

(where I is the identity matrix and ((.)) denotes a matrix). Normalising $m_{22} = 1$ and after some algebra we get

$$m_{21} = -(1+\gamma m_{23})/\theta_2\Delta \qquad (49)$$

$$m_{23} = (k\delta-\theta_2\Delta)/\lambda\delta \qquad (50)$$

(note the elements of A from (32)).

Using the same method for the third row of M and the other unstable root θ_3 we get

$$m_{31} = -(1+\gamma m_{33})\theta_3\Lambda \qquad (51)$$

$$m_{33} = (k\delta - \theta_3\Delta)/\lambda\delta \qquad (52)$$

Substituting these into the corresponding expressions (46) and (47) will give us the values of v_1 and v_2.

To concentrate attention on the relationship between inventory (N) and the predetermined variable ℓ, consider (47) and write it as

$$N = v_2\ell + (N^* - v_2\ell^*) \qquad (53)$$

where v_2 after solving comes out to be

$$v_2 = [1/(m_{33}-m_{23})][\{\Delta(\theta_2-\theta_3)+\gamma\Delta(\theta_2 m_{33}-\theta_3 m_{23})\}/\theta_2\theta_3\Delta^2] \quad (54)$$

Note that from (52) and (50)

$$(m_{33}-m_{23}) = (\theta_2-\theta_3)\Delta/\lambda\delta \qquad (55)$$

Thus $(m_{33}-m_{23})$ and $(\theta_2-\theta_3)$ have the same sign and

$$m_{33} \gtrless m_{23} \Leftrightarrow \theta_2 \gtrless \theta_3 \qquad (56)$$

This implies that both the numerator and denominator of v_2 (see equation (54)) have the same sign. It is important to note that <u>both</u> θ_2, θ_3 are <u>positive</u>. Thus it is clear that on the stable arm of the saddle path

$$v_2 > 0 \qquad (57)$$

and N and ℓ move in the <u>same</u> direction.

It can be shown in precisely the same way that $v_1 > 0$ so that the real exchange rate and real balance increase(decrease) together on the unique path that leads to the long run equilibrium. The latter result has been observed elsewhere (Buiter and Miller (1981)) and the analysis need not be repeated here. What is extremely interesting is that a similar phenomenon occurs in an inventory augmented open economy macro model and alerts us to the possibility of "overshooting" of the level of inventories, in addition to that of the real exchange rate.

The behaviour of inventory (non-predetermined) and real balance (predetermined) can best be understood by eliminating c from the equation system and restricting attention to two differential equations in N and ℓ. This also helps in drawing a phase diagram for the model. Considering only the homogenous part of (29) to (31), using the fact that $\bar{c} = v_1\bar{\ell}$ (from (46)) and that $v_1 > 0$, we can substitute to get

$$\dot{N} = -(\lambda\delta v_1+\gamma)\ell/\Delta + [b+(\lambda\gamma\Omega)/\Delta]N \tag{58}$$

$$\dot{\ell} = \Omega N \tag{59}$$

It is clear that

$\partial\dot{N}/\partial\ell < 0$, $\partial\dot{N}/\partial N$ sign ambiguous,

$\partial\dot{\ell}/\partial\ell = 0$, $\partial\dot{\ell}/\partial N < 0$

(remember $\Omega = f-\phi h < 0$ and $h > 0$). The phase diagrams for the alternatives are shown in Figures 10.1 and 10.2.

Note now the interesting effects of a contractionary monetary policy. The initial equilibrium is given at $N^* = \hat{N}$ and $\ell^* = \ell_1^*$. The stable path is denoted by S_1S_1. A decrease in the growth rate of money produces a new long run equilibrium $\ell^* = \ell_2^*$ and $N^* = \hat{N}$. Since we assumed neutrality, the equilibrium inventory is at its constant (desired) level $\hat{N}$ and thus equilibrium output is the same as before the contractionary monetary change. However, from (18), the stock of real balance in equilibrium increases to $\ell^* = \ell_2^*$. The new saddle path is S_2S_2.

The transition path is crucial. As the growth rate of money changes unanticipatedly, the economy at its original equilibrium E1 is on an unstable trajectory. The price level is sticky and the stock of real balance is predetermined at $\hat{\ell}_1$. Producers have rational expectations regarding the future movements of the economy, which in the absence of uncertainty implies perfect foresight. Foreseeing the new stable path S_2S_2, they reduce inventory stocks discontinuously and this is possible since inventory is a non-predetermined variable. A rapid fall in inventory ensues, which given that $b > 0$ (see equation (6)) implies a fall in output too. After reaching F_1 on S_2S_2, inventory and real balance both move upwards continuously (note from (57) that $v_2 > 0$) and finally reaches the new equilibrium E_2.

The difference with standard Keynesian inventory analysis (including the very recent one by Blinder (1980)) is fundamentally important. The standard literature assumes that a demand shock (here it is in the form of a contraction) will cause output to exceed sales. Thus unanticipated inventory accumulates so that the actual amount of stocks is higher than desired. This prompts producers to reduce output while additional inventory is run down until the new equilibrium. However, this traditional analysis eschews the effects of (rational) expectations. Producers are passive, non forward looking and only when sales actually fall and they have unwanted stocks do they respond. Thus inventories first rise since they are residuals, after which some form of stock adjustments take place.

Our analysis is radically different. Producers have forward looking expectations with respect to shadow price and stock of inventory. They anticipate that demand will fall after a decrease in monetary growth rate. Thus given the current rate of production they will have in the near future surplus or greater than desired stocks. To forestall that possibility they reduce current inventories rapidly so that when the lower sales materialise, producers have new inventories to rebuild stocks to desired levels. "Overshooting" of inventories takes place because the price level is sluggish and cannot be adjusted in anticipation of demand changes. The shadow price of inventory, on the other hand, is a "jump" variable whose change makes it worthwhile for producers to reduce stocks at the time of contraction.

This is once again an example of the "Hahn problem". Producers make two decisions at the margin; one is to produce for inventories, the other is to satisfy demand from inventories. Thus inventory takes on the form of a capital good or durable asset. In a multiple asset world (as here), saddle path properties and discontinuous changes in time paths of variables become important.

Lower inventory leads to inefficiency and contributes to a fall in output, even though the decrease in the latter may not be by as much (b will usually be less than 1). Simultaneously, the "overshooting" of the real exchange rate à la Dornbusch and the increase in the real interest rate all contribute to a fall in demand which itself will affect output after a time. The combination of the two may cause an important recession as the current conditions in the UK demonstrate.

Case (b): $b < 0$, $\Omega > 0$

In this case, an increase in the amount of inventory held leads to a decline in production (output Y). The analysis now is more in line with traditional ideas of inventory cycles and as we demonstrate below inventory holding changes continuously after policy changes.

The transaction matrix A (see equation (32)) remains similar. However, the determinant of A has a different sign from the previous case and we now have $|A| > 0$. Since the product of the characteristic roots is positive it is possible to have all roots positive. We rule out this unstable model. Assume that at least one characteristic root is negative. Then since $|A| > 0$, we must have two roots negative and one positive (all roots are assumed to be real and distinct). Denoting roots by θ_1, θ_2, θ_3, we have say

$$\theta_1 < 0,\ \theta_2 > 0,\ \theta_3 < 0 \qquad (60)$$

In the light of the earlier discussion we need two variables to be predetermined (corresponding to θ_1, θ_3) and one to be non-predetermined (corresponding to θ_2). Once again it is plausible to assume real exchange rate is a "jump" or forward looking variable. Thus the model dictates that inventory and real balance are predetermined, and are given at any point of time by "history". The economy still exhibits saddle point equilibrium but only the real exchange rate "jumps" to reach the stable manifold given the initial values of ℓ and N. The dynamics of the state variable (ℓ,N) dictate the movements of the system while the equation of the saddle path gives the value of c which must be attained at every point of time on transition paths for the economy to reach equilibrium.

Let the left characteristic vector associated with the unstable (positive) root θ_2 be deonted by $(w_1, -1, w_2)$ where the second term has been normalised to unity. Then using Dixit's method (1980) we have

$$(w_1,-1,w_2)\begin{bmatrix} \theta_2 & 0 & -\Omega \\ 1/\Delta & \theta_2-k\delta/\Delta & -\lambda\Omega/\Delta \\ \gamma/\Delta & \lambda\delta/\Delta & \theta_2-z \end{bmatrix} = (0, 0, 0) \qquad (61)$$

(where $z = b+\lambda\gamma\Omega/\Delta$). (Note this is similar to equation (48)).

Solving (61) gives us

$$w_1 = [(\theta_2-z)/\Delta+\lambda\Omega\gamma/\Delta^2]/[\theta(\theta-z)+(\Omega\gamma)/\Delta] \qquad (62)$$

$$w_2 = [-\theta\lambda\Omega/\Delta+\Omega/\Delta]/[\theta(\theta-z)+\Omega\gamma/\Delta] \qquad (63)$$

It is clear that given $b < 0$, $\Omega > 0$, the sign of z is unknown and we cannot determine the sign of w_1 and w_2 a priori. Assume then that $z < 0$, in other words b is large enough to dominate this term. Then we have

$w_1 > 0$

but w_2 can still be either positive or negative.

The relation between predetermined and non-predetermined variables can now be written as (in deviation form from equilibrium)

$$\bar{c} = w_1\bar{\ell} + w_2\bar{N} \qquad (64)$$

Since $w_1 > 0$ on the saddle path real balance and real exchange rate move together. Thus our result conforms to the Dornbusch prediction (provided $z < 0$). With a monetary contraction, the real exchange rate will fall (appreciate) or "jump" downwards, then move upwards to its equilibrium value. Simultaneously the level of real balance will increase, since a decline in money growth rate increases the equilibrium value of real balance ℓ^* (see equation (18)).

Once again let us focus attention on the relationship between inventory and real balance noting that both are predetermined here. Substituting (65) in the differential equation for N, (31), and using (29) we have

$$\dot{N} = -[(\gamma+\lambda\delta w_1)/\Delta]\ell+[(-\lambda\delta w_2/\Delta)+b+(\lambda\gamma\Omega/\Delta)]N \qquad (65)$$

$$\dot{\ell} = \Omega N \qquad (66)$$

It is clear that $\partial\dot{N}/\partial\ell < 0$, $\partial\dot{N}/\partial N$ sign indeterminate, $\partial\dot{\ell}/\partial N = \Omega > 0$, $\partial\dot{\ell}/\partial\ell = 0$.

The two phase diagrams (depending on the signs of $\partial\dot{N}/\partial N$) are shown in Figures 10.3 and 10.4. In both cases there are inventory cycles around equilibrium E_1 (determined on the basis of an exogenously fixed money growth rate μ_1). A decrease in money growth shifts the new equilibrium to E_2. The dotted line shows a possible transition path.

Note now the fundamental difference with case (a). A contractionary demand shock (as shown in the figure) will inevitably lead to inventory stocks rising initially. This is because inventory holders are passive and not behaving in a forward looking way, simply accepting N as a residual. As demand falls, given output, inventory rises. An increase in N above its desired level causes output to fall and the economy moves into a recession. However, the trend is reversed after a time and a cycle takes place around the new equilibrium. Inventories start declining from F_1 and the destocking leads to a signal for more production. The behaviour of the model is Keynesian and diametrically opposite to the forward looking attributes of producers in the previous story.

Section 4: Conclusion

The recent theoretical literature on open economy macro models have stressed exchange rate overshooting when there is an unanticipated change in the growth rate of money. Specifically, a contraction in monetary growth leads to a sudden appreciation of the real exchange rate given an inflexible (sticky) price level. Here we have tried to integrate an inventory model with exchange rate fluctuations to show how an analysis of inventory behaviour can give us a much clearer insight of the behaviour of output on transition paths out of long run equilibrium.

Depending on the values of parameters, a number of alternative possibilities manifest themselves. In particular, when there is a positive (technological) relationship between output and inventory ($b > 0$) and the numerical values of parameters are such that $\Omega < 0$, then the amount of inventory the aggregate firm wishes to hold becomes a forward looking non-predetermined variable under rational expectations. Faced with a contractionary demand shock and knowing that future sales will be low, the aggregate firm runs down current inventory very rapidly. Price inflexibility does not allow the firm to adjust to the new long run equilibrium quickly. Thus given predetermined real balance, the level of inventory falls below its desired (equilibrium) level even before the fall in sales has taken place. The role of anticipations then is crucial here. Inventory holding is not a residual in this model as in a standard Keynesian one, rather inventory adjustment takes place faster than other variables.

Clearly, the rapid fall in inventory as analysed in the model with not take place "instantaneously". A distributed lag mechanism will operate. But our first model suggests that inventory reductions will be much faster than the changes in price level or real balance (predetermined variables). If we are asked to (implicitly) rank these three variables in terms of speeds of adjustments, then case (a) of the previous section implies that the ranking would be real exchange rate, inventory and real balance changes. It is in this sense that we claim that both real exchange rate and inventory are non-predetermined variables liable to adjust in the face of new information quite rapidly. As our analysis shows a monetary (demand) contraction then would imply a rapid appreciation of the exchange rate and a fast fall in inventory stocks both of which will "overshoot" their respective equilibria and contribute to a recession. A contractionary demand shock as here will lead to an initial fall in inventory and then rising to its desired level.

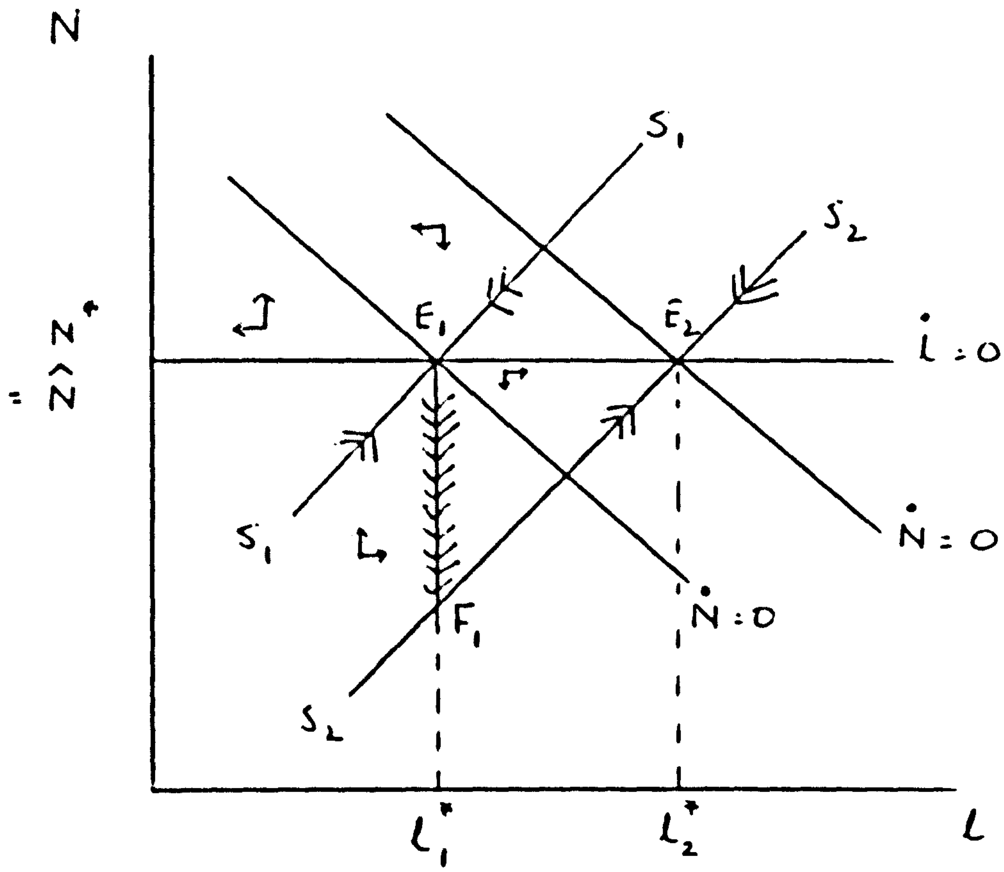
The other possibility (case (b) of the previous section) gives us the Keynesian view where inventory holding is initially a residual. Thus following a reduction in money growth and a contractionary demand shock, producers have unanticipated inventory and only after that do they try to adjust it to the desired level. Initially there is a rise in stocks and then falling towards its desired level. Inventory becomes a predetermined variable at any point of time given by past behaviour and holders of inventory exhibit backward looking behaviour. Around equilibrium we have inventory cycles.

The importance of inventory fluctuations as a stylised fact of business cycles is undeniable. Our chapter gives a theoretical formalisation of the crucial interrelationships that may exist between inventory, exchange rate and real balance (thus the money stock and price level) in the short run for a macro model of the open economy.

In view of the alternative possibilities that exist and the rich variety of results that can be generated theoretically, more empirical research should be conducted to give us an idea of the signs of values of parameters involved. In particular, as we have seen, the sign of the coefficient 'b' plays a crucial role in the conclusions that follow from policy analysis. This comes from equation (6), a quasi production function, which relates inventories (as the independent variable) to aggregate output. The difficulty of estimating this function, in its specified form, lies in the close co-relationship betwen the two variables. It is not possible therefore to establish causality since from inventory theory it can be claimed that causal relations of influence can work either way. The problem is particularly acute with macroeconomic data given the national income identities.

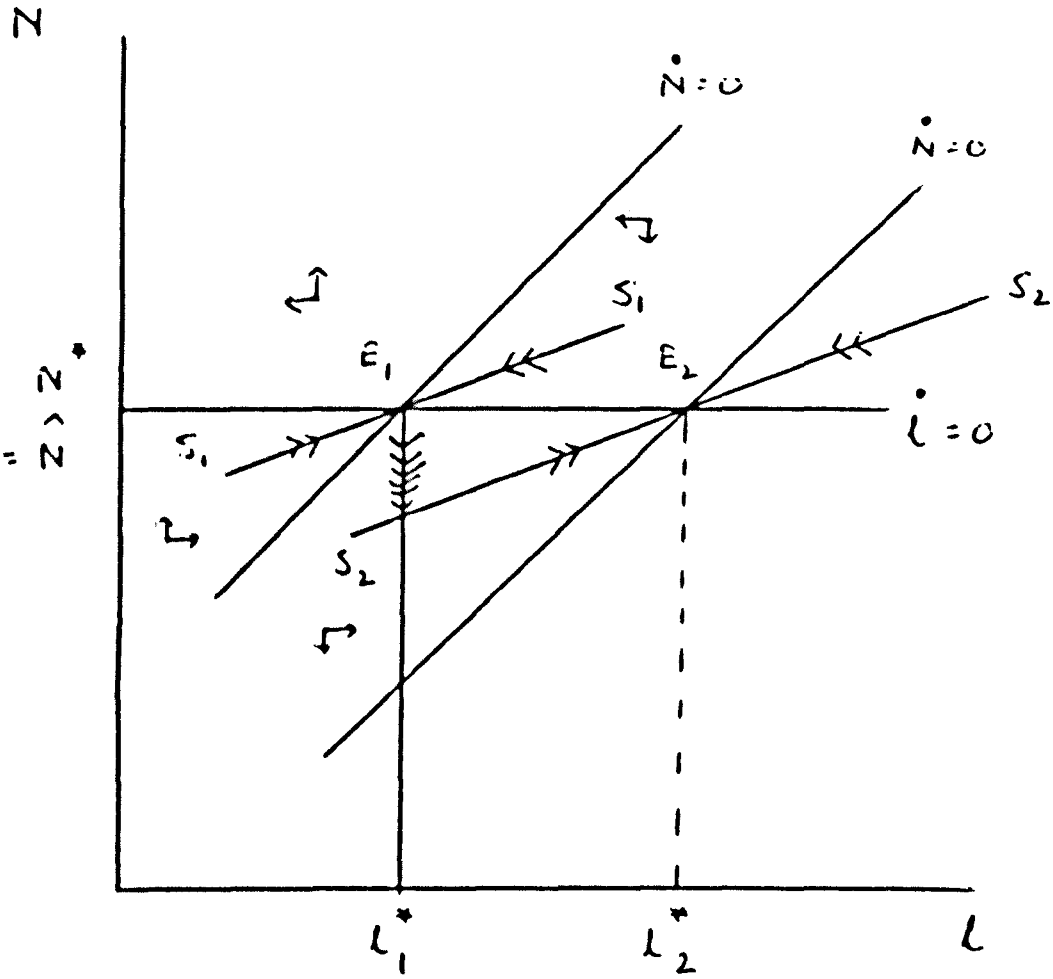
Deger and Sen (1983) do some Granger-Sims causality tests with UK quarterly macroeconomic data and find that the production relation discussed in this chapter could be justified empirically; inventory may Granger-cause output. However, one should be very careful about such tests given the recent scepticism regarding atheoretical macroeconometrics. There can be many explanations for pure data based 'causation'; without adequate theoretical foundations, it is not easy to claim that it is inventory that influences output and not the other way around.

The production scheduling model for the aggregate firm gives us a good reason why higher inventory-stock could, in principle, contribute to more efficiency and thereby raise output. An alternative (see Deger and Sen (1983)) is to use an optimal control model to solve the firms inter-temporal optimising problem and in the process get a relation whereby inventory affects output. Using these theoretical (microeconomic) results as a starting point, Deger and Sen (1983) estimate a single equation output-inventory relationship from UK quaterly data. The appropriate lag structure is chosen through diagnostic checks and the need to be parsimonious. The long run (steady state) parameter turns out to be positive, in accordance with the analysis as presented in case (a) above. Our model, in this chapter, would predict for this case that inventory would need to move relatively fast for the economy to attain the saddle path and thus achieve equilibrium. Monetary contraction may lead to inventory decumulation — a result which is the opposite of standard analysis. Thus some empirical evidence can be given for the policy implications of the analysis, though more econometric research requires to be done.



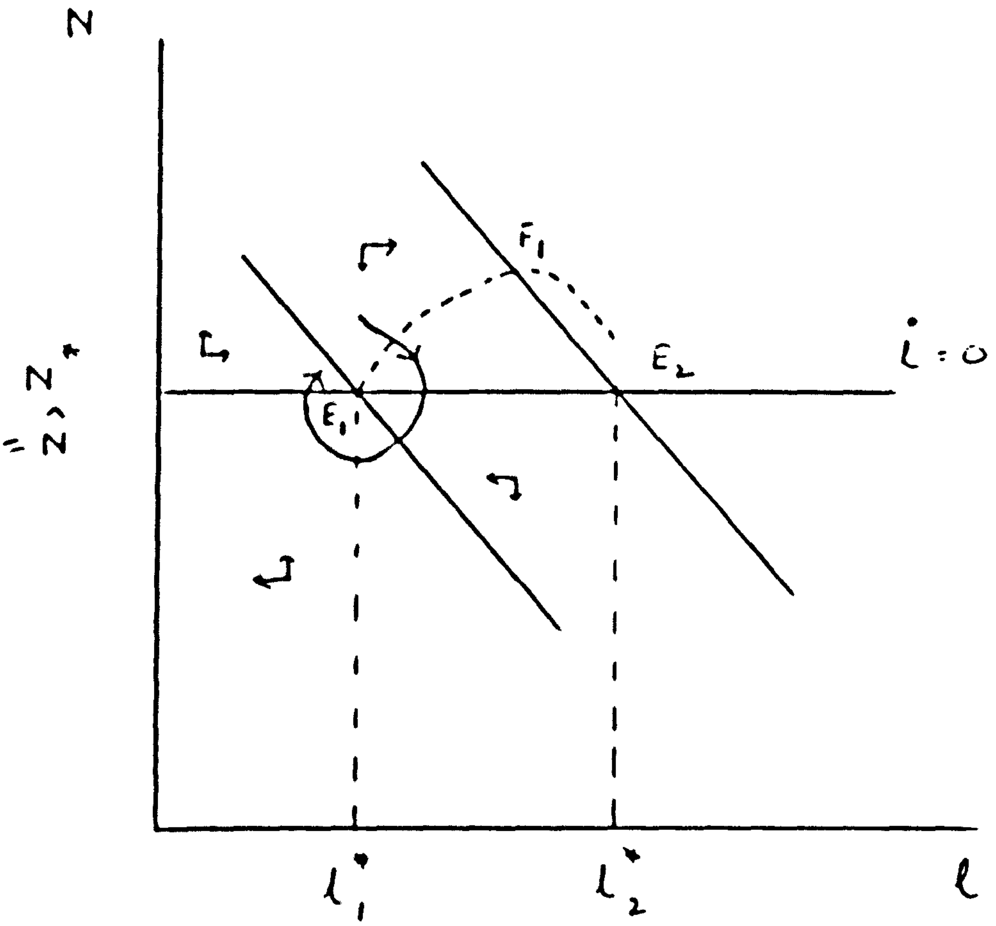
$\left(\frac{\partial \dot{N}}{\partial N} < 0 \right)$

Figure 10.1



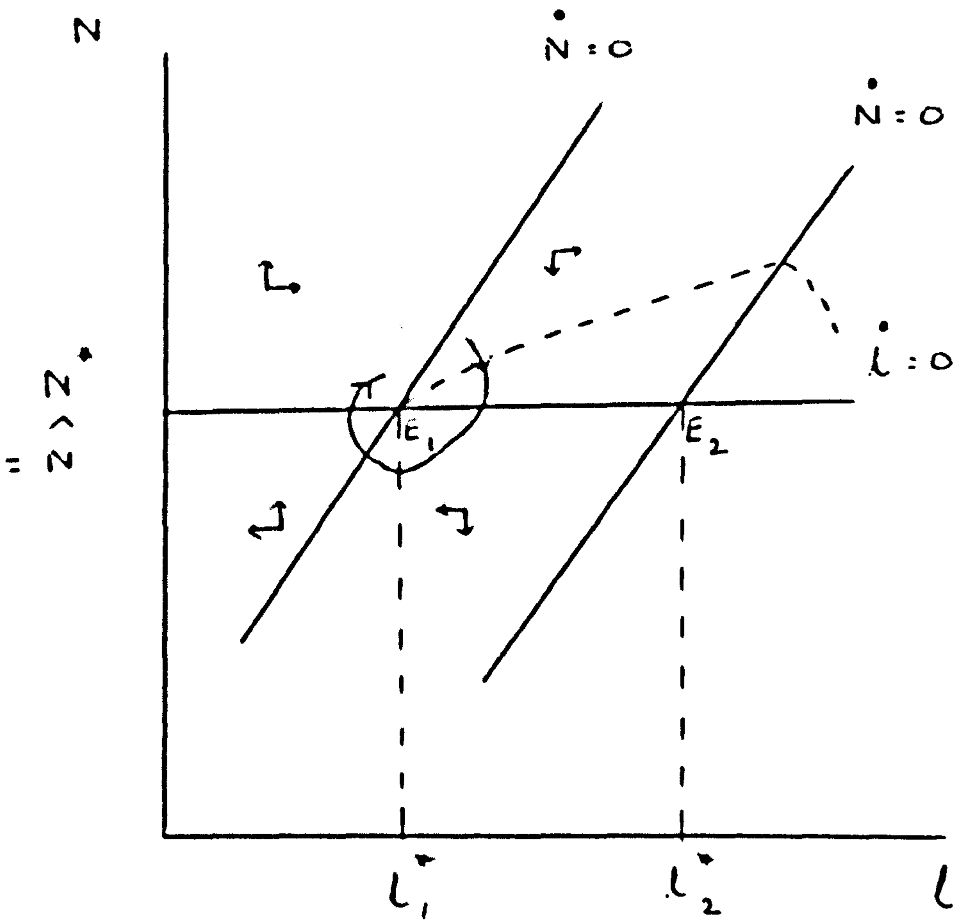
$\left(\frac{\partial \dot{N}}{\partial N} > 0 \right)$

Figure 10.2



$\left(\frac{\partial \dot{N}}{\partial N} < 0 \right)$

Figure 10.3



$\left(\frac{\partial \dot{N}}{\partial N} > 0 \right)$

Figure 10.4

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Chapter XI.

General Conclusions

The thesis covers a wide area but emphasises specific aspects of disequilibrium market behaviour in an aggregative framework. The focus is on macroeconomic models of growth and medium-term dynamics. Since the concepts are essentially diverse we cannot gain substantial leverage by trying to formulate a general theory which will encompass all the various models and relevant issues. Rather, the cost of generalisation may mean that stylised facts and policy problems are not sharply focussed. The chapters are therefore relatively independent and have their own conclusions which summarise their findings. It is however necessary to have an overall concluding perspective; this will be given here briefly.

It will be useful, in this context, to discuss some of the recent research that have appeared over the last few years and see how the work reported here fits in with the broad framework of disequilibrium growth and macroeconomics. The overall conclusions of the thesis therefore will be presented in the context of the wider literature. In Chapter I, the Introduction, we have already discussed the underlying concepts of disequilibrium dynamics and the interrelationships of the models presented here with the earlier literature. I will therefore focus attention now on recent analysis and results in the field. In the process, it will also be instructive to review some of the directions in which future research in disequilibrium macroeconomic and growth theories can proceed. In addition to being an agenda for further research, we will also get an additional general perspective on the contents of my thesis.

I will proceed in two stages. First, comes the literature review with the description of future research possibilities. This is, of course, over and above the various analyses cited earlier for the individual chapters. Secondly, I shall present the overall conclusions of the thesis, summing up the unifying concepts that link the different chapters and sections. In a sense, this will be a meta view which may give us a wider perspective.

Part A of the thesis concentrates on growth and capital accumulation, with excess capacity and unemployment. The major assumptions are related to disequilbrium adjustment and proper dynamic specifications. Traditionally, dynamic models have either assumed pure price adjustments (of the Walrasian type) or pure quantity adjustments (of the Marshallian type). By its very nature, disequilibrium growth models must have both types of adjustments over time. Growth, per se, requires capital formation; hence quantity variables change over time. On the other hand, disequilibrium and non market clearing induces price changes over time to bring about stable equilibrium. We therefore have "mixed adjustments". Mas-Colell (1986) uses a general equilibrium tatonnement model to analyse ´simultaneous price and quantity dynamics´ which combines the two methodologies mentioned earlier. He demonstrates the enormous difficulties of proving global stability even when there is only one good, one factor input and one price. The demands made on the system by nontatonnement dynamics, as analysed by Fisher (1983) for example, are even higher. The basic model in Chapter II of this thesis is a good example of this simultaneous operation of both quantity (capital stock) and price (rate of interest) dynamics in a nontatonnement framework. As we demonstrated in that chapter even this simplest disequilibrium growth model requires quite a few restrictions and specifications before one can be assured of existence and stability of temporary and steady state equilibria.

The second point relates to the nature of the saving function, an important ingredient of long run growth. We have assumed (for example in Chapters IV and V) a fixed saving (consumption) propensity given exogenously. Ginsburg, Henin and Michel (1985) construct an intertemporal optimising model using the dual decision hypothesis and allow for capital accumulation with unemployment. The aggregate household, maximising a functional of utility (integral), chooses an optimum path for the saving rate. It assumes employment as given. The aggregate firm, on the other hand, assumes the saving propensity as an exogenous parameter and tries to maximize an integral of profits, net of savings. Its control variable is labour demand for each point of time. Four disequilibrium regimes can be constructed: unemployment, over (full) employment, underconsumption, and under saving (excess consumption demand). Though the model is elegant, it suffers from a number of shortcomings. The formal derivation of the phase diagram is not always rigorous; the use of an intertemporal Nash equilibrium, with agents not taking into account the effect of their decisions on the behaviour of others, is difficult to justify under rational expectations; the problem of time consistency for this class of models remain; the firm's optimising function can be questioned as ad hoc. Some of these issues, as well as a formal model, are analysed in Sen (1987). The general consensus is that it is exceedingly difficult to construct a fully specified optimum savings model taking into account all the various facets of disequilibrium discussed earlier in the thesis.

The work of Ginsberg et al (1985) as well as that Ito (1980) and Ito (1982, referred to by Ginsburg et al) lies firmly in the tradition of neoclassical (Solow type) growth theory. They correspond to what is often termed in the thesis as the 'Classical regime'. A more Keynesian perspective is to be found in Malinvaud (1981, 1984) and Picard (1983). These are related to the 'Keynesian regime' models discussed in earlier chapters. The essential characteristics are the following: specification of an investment function; an assumption that actual capital accumulation is given by investment, rather than saving; allowing for unemployment of labour and/or excess capacity of capital stock.

This type of models explain a considerable amount in the fields of capital formation under unemployment, policy effectiveness, the existence and propagation of business cycles, as well as the trade-offs implicit between unemployment and inflation. However, lack of an optimising behaviour by firms (as well as more general optimal growth theory) constitutes a shortcoming. The integration of disequilibrium growth and intertemporal optimisation must remain a subject of considerable, and exciting, future research.

Most of the growth models, discussed in the thesis, concentrate either on a classical or Keynesian regime. But regime switching remains a distinct possibility. Stability analysis then becomes relatively intractable since the standard restrictions on the Jacobians, as well as the signs of the partial derivatives of the differential equations, may not hold at the border of adjacent regimes. In effect, the differential equations are discontinuous at the boundary of adjacent regimes. Global stability for each set of differential equations (for a _given_ regime) does not necessarily imply global stability of the "patched-up" system (Ito (1980)) of all regimes taken together. Ito (1980) and Honkapohja and Ito (1983) use the Filippov solution to demonstrate for some specific disequilibrium systems how global stability can be achieved. For more complex models, particularly with asymmetric adjustment mechanisms as postulated here, it is yet not possible to get general stability results for "mixed" regimes or regime switching.

Another interesting development in the theory of _equilibrium_ growth, but with disequilibrium features, is the application of dynamic (differential) games to study the possible conflict between distribution and growth. Using the basic two class model (workers and capitalists), a number of papers analyse the various noncooperative solutions (Nash, Stackelberg) under open loop or feedback information structures. Pohjola (1986) gives an insightful survey on the various possibilities that exist here. Of particular interest is the Keynesian flavour to many of these models. For example, workers can affect distributional parameters by choosing the propensity to save (consume); capitalists, on the other hand, decide on accumulation by choosing the share of investment in the national product. Thus the fundamental distinction between decisions to save and invest open up the possibility of a Keynesian regime. Though some of these models assume a labour surplus economy (the reserve army of labour a lá Marx), they do not generally tackle wage adjustment in response to either unemployment or the discrepancy between desired saving and investment.

The incorporation of money, and monetary issues, in disequilibrium models create additional complexities. The fundamental reason lies with the long standing problem of integrating general equilibrium theory with monetary theory (see the various essays on this issue in Hahn (1984)). There has been a recent upsurge in interest in such integrative models from the point of view of the microfoundations of the theory. The attempt is now on to construct "monetary general equilibrium asset-pricing models" (Svensson (1986)). The essential idea is that money should be treated in symmetric fashion with other assets. Two methods are proposed to do this: either, real balance enters directly into the utility function together with other goods and assets; or, money is introduced in the model through a cash-in-advance constraint, its utility depending on its liquidity rather than dividends (as for other assets). The value of money is the reciprocal of the aggregate price level and is given by a standard asset pricing equation (Lucas and Stokey (1985)). Most of the models in this genre assume perfect flexibility of all asset prices including the price of money. Implicitly this gives market clearing equilibrium even when there are shocks to the system following from new information or parameter changes. A major exception is the paper by Svensson (1986) which incorporates monopolistic competition and sticky prices thus ensuring that markets may not clear and resources remain underutilised. The emergence of unemployment therefore gives a disequilibrium flavour to this important area of current research --- integrating monetary, financial and real sectors of the economy.

The models discussed above are microeconomic in nature and are based on well specified general equilibrium theory. However the basic insight, of treating money as symmetric with other assets, has been present in monetary growth models for a considerable period of time. These are discussed, in some detail, in the first section of Chapter VII. I have surveyed the various strands of monetary growth theory, in the context of superneutrality, and emphasised the symmetry of real money and capital stock within the asset portfolio. Money in the utility function gives rise to "real balance effects", the consequences of which are extensively discussed in that chapter. The alternative, where cash-in-advance money is part of the wealth constraint, is essentially Tobinian. As mentioned earlier, the central point of monetary growth models should be that wealth must consist of both (real) money and capital stock and their demands must depend on the same set of (price) variables. The symmetric treatment of physical capital and real balance in output/disposable income, as well as in total wealth holding, can now be justified by the recent microtheoretic literature.

Explicit disequilibrium analysis in monetary growth models, with sticky prices, inflexible wages, and unemployment, is rare. The model in Section 4 of chapter VII gives a flavour of the type of issues that can be raised here. Another example is a paper by Azam (1983) which uses an IS/LM framework to deal with disequilibrium concepts in a Keynes-Tobin monetary growth model. In terms of our earlier discussion, it may be fruitful for future research to consider an optimum growth model with money and capital accumulation, as well as incorporating disequilibrium features such as unemployment.

Discussions on monetary growth theory is never complete without a reference to the question of policy effectiveness and monetary (super) neutrality. A related question is whether real business cycles can be caused by anticipated discretionary monetary policy. If the role of money (growth) is neutral or super neutral, then can it cause temporary fluctuations leading to business cycles? Within the new classical framework the answer must be no. The only reason that Lucasians can provide for the existence of cycles are random or unanticipated shocks produced through government policies under uncertainty. We have shown, in the chapter on monetary growth models, Chapter VII, that even under perfect foresight (absence of uncertainty) the transition path for capital accumulaton can be affected by monetary policy.

In a different context, Grandmont (1986) tackles these issues squarely by constructing a model of endogenous business cycles, which occur even without random shocks. The competitive monetary economy with a stationary environment can produce cyclical fluctuations. Further, government monetary policy, even when fully anticipated, does have real effects in stabilising the economy. Of particular interest are the mathematical tools employed: "the techniques employed to study the occurrence and the stability of such business cycles are partly borrowed from recent mathematical theories that have been constructed by using the notion of a 'bifurcation' of a dynamical system in order to explain the emergence of cycles and the transition of turbulent ('chaotic') behavior in physical, biological or ecological systems" (Grandmont (1986)). The basic Grandmont model does assume Walrasian equilibrium since it is in part an answer to the Lucasian objection to disequilibrium modelling. But the use of bifurcation theory, for disequilibrium growth models, must also open up exciting research possibilities.

When we turn to medium term dynamics, the subject matter of Part B of the thesis, we find that basic concepts of disequilibrium are integrated into the analysis right from the beginning. In particular, open economy macroeconomics assumes sticky prices (wages) and unemployment within its basic framework. Most of the open economy literature now try to incorporate features of the monetary, Keynesian and portfolio models (Allen and Kenen (1983), Gyalfason and Helliwell (1983)). This gives rise to models with interesting and empirically relevant characteristics: full equilibrium in the long run; the short run characterised by disequilibrium; sticky prices and contractually fixed wages; perfectly flexible exchange rates; assets markets clear rapidly while unemployment persists in the labour market. The Dornbusch model, which we have extensively used, is a classic example of these various issues. (See Obstfeld and Rogoff (1984) for recent extensions). Its particular strength lies in medium term dynamics which can be exploited, as in chapters IX and X , to achieve interesting results.

Unfortunately, one of the shortcomings of the Dornbusch type model is the failure to assimilate the dual decision hypothesis; rationing in one market has spill-overs elsewhere and this needs to be incorporated formally. Cuddington, Johansson and Lofgren (1984) do precisely that and their innovative approach demonstrates how macroeconomic models can usefully integrate the fundamental microeconomic tenets of disequilibrium. However, their analysis (except in an Appendix) is essentially short run and an extension of the fix price approach to include the foreign sector. It is really not possible to incorporate dynamic concepts such as "overshooting" in their model. In general they tend to ignore dynamics.

I have shown, in chapter IX, that the dynamics of the Dornbusch type models can be integrated with elements of the dual decision hypothesis and regime switching. Assuming perfect foresight for variables and regimes, we have formulated the concept of the kinked saddle path. This is clearly a combination of the central tenets of the two types of models mentioned earlier: saddle point equilibrium under perfect foresight and kinked boundaries between regimes, due to rationing. The existence of kinked saddle paths has also major implications for policies since the effects of government action is bound to be asymmetric.

Another advantage of having (kinked) saddle paths, in models which combine rationing with rational expectations, is that the earlier mentioned problems of global stability disappear. Since there is only one and unique path, in each regime, which leads to equilibrium we need not concern ourselves with the discontinuities inherent in a "patched-up system. If the basic tenet of this analysis, the existence of "jump" variables, is acceptable then there should be no analytical problems with demonstrating that the equilibrium is stable.

The recent upsurge of interest in macroeconomic (monetary) policy coordination (see the papers in Buiter and Marston (ed) (1985), particularly Miller and Salmon (1985a), as well as Miller and Salmon (1985b)) has produced a large literature on how small open economies can conduct co-operative games to increase their joint welfare. It would be of considerable research interest to analyse policy coordination using the tools of disequilibrium analysis, specifically the recognition of asymmetric responses and kinks in the dynamic paths. More formally, what would policy co-ordination look like for two economies of the type discussed in Chapter IX, particularly if one is characterised by excess supply and the other by excess demand?

There can be little doubt that dynamic analysis per se, under disequilibrium, is exceedingly complex. The demands of growth theory i.e. capital accumulation, coupled with the existence of unemployment and the presence of excess capacity, create many additional problems. The presence of price and quantity dynamics within the same framework, the asymmetric adjustments that can take place, as well as nontatonnement and the dual decision hypothesis all contribute to the complexity. As Mas-Colell (1986) points out " ... except in (some) cases there is nothing simple about the global dynamics of even the simplest demand and supply model".

Let us now turn specifically to the general conclusions that can be derived from the many models studied in the thesis. As we have seen, through various examples, the nature of disequilibrium, the causes of market failure, the adjustment rules, the dynamic behaviour of the economy on the transition path, stable or unstable movements, policy prescriptions if any, are all heavily dependent on the sort of regime the economy is in. For example, Keynesian unemployment may need the wage rate to go up while Classical unemployment would claim the opposite. Similarly, whether capital formation is governed by investment or saving will have crucial implications for the subsequent bahaviour of the economy. It has generally been assumed that steady (stationary) state is exogenously determined, independent of the dynamic behaviour of the economy in disequilibrium. However, even this result may not be sacrosant, particularly with increasing returns to scale.

Models of disequilibrium behaviour generally acknowledge regime specific behaviour; but the emphasis does not go far enough. In particular, adjustment equations tend to be similar, independent of the sort of disequilibrium state the economy is in. A classic example of this is the assumption that wage adjusts to clear the labour market; thus unemployment induces a fall in real wages. Yet, if unemployment is caused by a lack of effective demand, then a fall in wages is potentially destabilising since it can cause a further fall in aggregate demand. As we have seen in Chapter V, a Keynesian regime may require wage to adjust in response to disequilibrium in the goods market rather than in the labour market. One of the important conclusions of the thesis is that dynamic behaviour and adjustment must be related to the specific nature of disequilibrium.

Regime specification and concomitant adjustment rules have been major issues in the research reported here. This is particularly true for the growth chapters. Though the properties of these regimes, specifically Keynesian or Classical, may seem ad hoc they are nevertheless similar to the quantity rationing models, which are based on optimisation. In similar fashion, the adjustment equations of macrodynamics under disequilibrium are often attainable from inter-temporal optimisation models, where the Hamiltonian conditions could deliver the relevant dynamic equations. The distinction between control and state variables (when using the Maximum Principle) are essentially similar to that between backward-looking and foreward-looking variables, with similar interpretations possible for discontinuous behaviour. The problem of free parameters may remain.

However, institutions and structure can tell us about stylised facts which can be used to determine the values of such parameters. There is also the possibility of empirical testing and econometric verification which can give a data-based specification, supporting or contradicting the assumptions made in the theoretical model. Macroeconomics must remain eclectic in its choice of method and support.

The central methodological conclusion seems to be that dynamic behaviour under disequilibrium is asymmetric across regimes. This means that generalised predictions will be less meaningful compared to more specific forecasts of economic behaviour which rely on the characteristics of the regime.

Consider for example the usual "overshooting" models where a shift in a parameter causes the jump variable to move beyond its new long run equilibrium value. Inevitably, the literature shows a symmetric movement. Thus a rise or fall in the relevant variable is treated in exactly the same way. For a given positive or negative shock (say expansionary or contractionary policy) the rise or fall in the price variable is quantitatively the same. There are no discontinuities in the saddle path. Yet our analysis shows that this cannot be true. The saddle path may be kinked. Hence the quantitative jump may be quite different depending on which direction the jump is being made. The implications for the transition path, as well as the time taken to reach the new equilibrium, can be quite different.

The implications for policy are equally important. Policy rules, even if useful temporarily, may become counterproductive if there is regime switching. Even discretionary policy has to be more carefully scrutinised dependent on the regime the economy is in. In a sense we may have here a variant of the Lucas critique. As the regime shifts, so does stable dynamic behaviour; previous parameters and policies may be irrelevant or even destabilising. The current literature on neo-Keynesian policy effectiveness has not really grappled with this issue. For example, in a policy co-ordination model, if one country has excess demand (high inflation) and another country has excess supply (high unemployment) then policy rules need to be more carefully constructed since their effect will be qualitatively different. In a sense the disequilibrium models analysed here pose important questions for policy formulation and require a more specific study of the macroeconomy.

Disequilibrium economics and dynamics still lack an unifying framework since it is difficult to bring all the competing (and sometimes conflicting) hypotheses together. On the other hand, the heterogeneity and variety of concepts adds to its attractions and allows us to get a richer menu of results which have much wider relevance. Future research must aim to capture the diverse elements that constitute disequilibrium regimes; but the search for a common conceptual framework also has to continue.

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