

Layered social influence promotes multiculturalism in the Axelrod model

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ABSTRACT

Despite the presence of increasing pressure towards globalisation, the coexistence of different cultures is a distinctive feature of human societies. However, how multiculturalism can emerge in a population of individuals inclined to imitation, and how it remains stable under cultural drift, i.e. the spontaneous mutation of traits in the population, still needs to be understood. To solve such a problem, we propose here a microscopic model of culture dissemination which takes into account that, in real social systems, the interactions are organised in various layers corresponding to different interests or topics. We show that the addition of multiplexity in the modeling of our society generates qualitatively novel dynamical behavior, producing a new stable regime of cultural diversity. This finding suggests that the layered organisation of social influence typical of modern societies is the key ingredient to explain why and how multiculturalism emerges and thrives in our world.

Introduction

Our understanding of societies, and social dynamics more generally, is increasingly taking advantage of concepts, models and methods from statistical and computational physics.^{1,2} Within this framework, the validation of sociological theories relies on the construction of appropriate mechanistic agent-based models³ which can be tested against existing and upcoming large data sets.

The existence of multiculturalism and group boundaries is a well-established feature of social systems^{4,5} and much effort has been devoted into explaining possible mechanisms able to reproduce such empirical finding. In particular, cultural diversity appears as a striking phenomenon, supposedly in contrast with the widely acknowledged principle of social influence.⁶ According to such mechanism, changes in the cultural traits of an agent are influenced by the acquaintances and friends of that agent facilitating the local convergence towards a set of common cultural traits and promoting homogeneity across the populations. For instance, homogenous absorbing states are typical of imitative dynamics where the state of the individuals is described through a scalar binary variable, as for the voter model⁷ in finite size populations.

An interesting solution to reconcile social influence and the emergence of cultural diversity (polarization) commonly observed in human societies was suggested by Robert Axelrod, who proposed in 1997 a simple agent-based mechanistic model of dissemination of cultural traits.⁸ In the original model the state of individuals is not described through a scalar variable, but each agent is endowed with a set of cultural features F , each of them taking one of a number of different cultural traits q . Based on pairwise interactions among agents, together with social influence the model mimics another important social principle, known as homophily,⁹⁻¹¹ i.e., the tendency of individuals to connect and interact preferentially with similar ones. As a result of pairwise interactions, imitation still occurs but is limited to the update of one feature at a time. Homophily and social influence acting together constitute a self-reinforcing dynamics leading to local homogenization. However, a main result of the analysis of Axelrod was that in spite of local convergence, global polarization was possible: agents become more similar by local interactions, but cleavages among different cultural groups are created so that these groups no longer interact. The overall result is the possible emergence of a globally polarized or multicultural state with coexistence of different cultural groups. A quantitative analysis of the model¹² unveiled a non-equilibrium phase transition at a critical number of cultural traits q_c from a mono-cultural phase (global culture) to a polarized or multicultural phase, where several groups with different cultural traits survive. Such phenomenon has been observed also for realistic interaction patterns, such as small-world networks and heterogeneous distributions of the connections among the individuals.¹³

However, it was found that the multicultural phase is achievable only for a high initial number of traits q in the population. In addition, the multicultural phase is not stable under cultural drift, meaning that the spontaneous tendency of agents to modify their cultural traits independently of their environment drives the system towards a monocultural state.^{14,15} Different mechanisms have been invoked to account for robust cultural diversity, such as the plasticity of the social relations among agents, so that the social networks coevolves with the dynamics of the cultural state.¹⁶ Other alternative proposals focus on

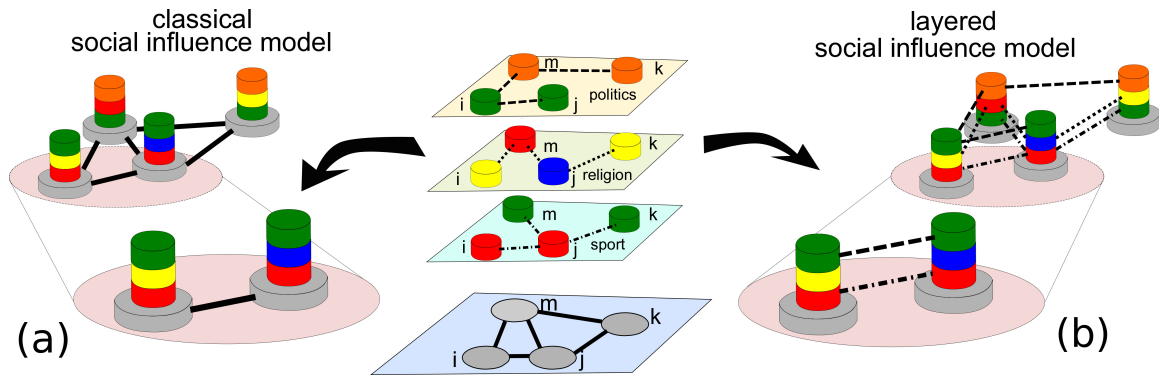


Figure 1. Social influence is inherently layered. Social systems are multiplex networks because individuals tend to differentiate their social contacts according to the different subjects of the interaction. In the classical social influence model, the specific type of each interaction is neglected and all connections are treated together (a). Conversely, we propose here a multilayer cultural influence model in which each interaction can only affect the corresponding cultural features (b). As an example, in the classical model the bond between agents i and j is active at each cultural feature: the two agents have a different religious trait but according to social influence, one of the two agents will eventually absorb the religious trait of the other one. The same bond is instead frozen in the layered model, as imitation can not affect religious beliefs, since the two agents are not linked on the corresponding layer.

modifications of the form of the local interactions, for instance based on assimilation-contrast theory,¹⁷ also known as bounded confidence.^{18–20} It has been shown how by integrating metric cultural states,^{21–23} as opposed to nominal states, the bounded confidence mechanism can lead to robust cultural diversity.²⁴ Another type of modification²⁵ addresses the nature of social influence in line with other models of social contagion:^{26–29} the dyadic interpersonal influence of the original model is replaced by a mechanism in which an agent modifies its cultural state depending on the state of all agents in her neighborhood. Finally, the mechanism of social differentiation in which agents tend to increase cultural differences has also been analyzed in.^{30,31}

In this work we propose an alternative mechanism to account for robust multiculturalism that naturally brings together social influence and the nature of the social network of interactions with no need of introducing any additional dynamical feature. In real societies the relationships among individuals are inherently layered, i.e. individuals tend to interact with different neighborhoods on different topics. Hence, interactions in social systems are better described in terms of multiplex networks,^{32–34} i.e., networks composed of many layers, each one of them describing a particular type of interaction, with a wide range of overlap between the links at the different layers as found in multiplex empirical data.^{35–39} We model cultural dissemination associating each cultural feature to a different layer of a multiplex network. Social influence becomes now a layered mechanism in which an agent is allowed to imitate only a subset of all the cultural features of its neighbors, namely those for which there exists a link on the corresponding interaction layer. The consequences of layered social influence are strongly dependent on the structural overlap among different layers, so that the overlap becomes the control parameter for the nonequilibrium transition in the system. We remark that the multiplex topology has been already successfully applied in other types of social dynamics, such as opinion formation, giving rise to novel critical behavior.^{40,41}

We find that layered social influence in synthetic and empirical multiplex networks with heterogeneous layers easily leads to a global state of cultural diversity. This state exists for any number of cultural traits provided that the interaction patterns into the population are sufficiently layered, i.e. the value of the structural overlap is below a critical point. We remark that this is a qualitative shift in the behavior of the system, only achievable in multiplex networks, and it is in agreement with empirical evidence of fragmented states even in social contexts with a limited number of available cultural choices q . Moreover, unlike the fragmented states found by Axelrod, such new multicultural state is robust against cultural drift, thus providing an explanation for the persistence of multiculturalism we experience in real-world society. In addition, we find novel phases of cultural diversity in which a global culture for a number of cultural features coexist with polarization in other features, a situation reminiscent of the so-called chimera states.^{42–44} More in general, we observe that different levels of homogeneity may be achieved on different cultural features, depending on the heterogeneity of the structure of interactions across the layers. Finally, we investigate two layered social networks from the real-world, showing how considering the nature of the different interactions promotes cultural fragmentation in the population.

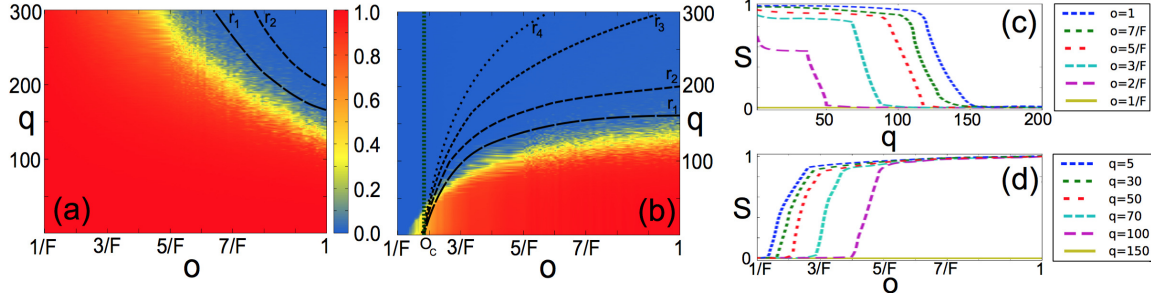


Figure 2. Modeling classical versus layered social influence. The size of the largest cultural component S in the classical (a) and in the layered (b) social influence model is shown as function of the number of cultural traits q , and for multiplex networks with $F = 10$ layers and a tuneable structural overlap o . In the case of classical social influence, a decrease in o increase the density of the aggregated networks, hence increasing the value of $q_c(o)$. Conversely, for layered social influence, the critical number of cultural traits $q_c(o)$ separating globalisation (red region) from multiculturalism (blue region) decreases with the overlap, and goes to 0 for o smaller than a critical value. This means that, for sufficiently small values of the overlap, multiculturalism is always achieved independently from the values of q . Under the presence of drift r , the critical overlap sustaining globalisation depends both on q and r . Noticeably, the polarised phase obtained for $o < o_c(q, r = 0) \approx 2/F$ is stable under the presence of noise. This is shown in (b), where we plot with different dashes the critical lines relative to four increasing values of noise $r = r_1, \dots, r_4$. Conversely, the multicultural states on the classical model are not stable in presence of drift. The two projections of the phase diagram for layered social influence are reported in (c) and (d). Transitions from globalisation to multiculturalism are steep both as a function of the number of cultural traits and the overlap.

Model

Let us consider a social systems where agents are involved on interactions on different topics, such as politics, religion and sport, as shown in Fig. 1. As an example the couple of agents i and j discusses sports and politics, the couple j and k discusses sports and religion, while m and k can only discuss politics. In classical models of social influence such as the Axelrod model, the cultural profile of each individual is a vector of different features (sport, religion, politics), shown as a stack of cylinders, and each feature can take one of a series of different traits, represented as the colors of the cylinders. However, the peculiar nature of each interaction is neglected and all connections are treated together. Consequently, interactions at any level can potentially impact (and thus modify) any cultural trait, as in Fig. 1(a). For instance, the existence of a link between nodes i and j , and the different traits of the pair on the feature “religion”, suggest that this bond is still active. According to social influence, one of the two agents will eventually absorb the religious trait of the other one, making the pair equal on all the three features. Conversely, we propose a layered cultural influence model where the structure of the relationships pertaining to different social spheres is preserved and explicitly taken into account, assuming that each layered interaction can affect the corresponding cultural features, as shown in Fig. 1(b). Agents i and j are not linked at the religious layer. As a consequence, since the two individuals already have the same sport and politics traits, the interaction between i and j is frozen. Indeed, the two individuals already reached consensus at all levels where they are linked, and imitation can not concern religious beliefs, since religion is not discussed by them.

We propose in general to take into account the multilayer structure of real social interactions by modelling layered social influence between pairs of individuals. Formally, we can describe the specific pattern of social influence at each cultural level f by mean of a *multilayer network*, i.e. a set of F adjacency matrices $A^{[f]} = \{a_{ij}^{[f]}\}$, $f = 1, \dots, F$, where $a_{ij}^{[f]} = 1$ if agents i and j influence one another on topic f .^{32,35}

Given two individuals i and j , the normalized total number of connections between them is measured by the so-called edge overlap, $o_{ij} = \frac{1}{F} \sum_{f=1}^F a_{ij}^{[f]}$ which is a number ranging in the interval $[0, 1]$.^{35,45} We can then define the *structural overlap* o of a multilayer network as the average number of links among connected individuals:

$$o = \frac{\sum_i \sum_{j \neq i} o_{ij} \Theta(o_{ij})}{\sum_i \sum_{j \neq i} \Theta(o_{ij})}, \quad (1)$$

where $\Theta(x)$ is the Heaviside function, i.e. $\Theta(x) = 1$ if $x > 0$, and $1/F \leq o \leq 1$. When $o \approx 1$ the layered nature of social influence will be negligible. Indeed, if the peculiar nature of the links is neglected, the corresponding aggregated network of social interactions (formally defined at the end of this Section) will be similar to the interaction networks relative to each feature. Conversely, when $o \approx 1/F$ interactions are extremely diversified across the different topics. As a consequence, the

aggregated network will be very different from the interactions occurring at each layer, hinting at possible different outcomes between the layered model and the original model on the aggregated network, to which we refer as the classical case.

The cultural profile of each agent is described by a feature vector of F integer variables $\mathbf{s} = (s^{[1]}, \dots, s^{[F]})$. Each feature f , with $f = 1, \dots, F$, takes one of q possible traits, $s^{[f]} = 1, 2, \dots, q$. Because of the layered structure of interactions, when an agent interacts with a neighbor it only considers the subset of features where the two are connected. According to the principle of *homophily*, the probability of a social interaction between two connected agents i and j is assumed to be proportional to their layered *cultural overlap*, which we define as:

$$\omega_{ij} = \frac{1}{F} \sum_{f=1}^F a_{ij}^{[f]} \delta(s_i^{[f]}, s_j^{[f]}). \quad (2)$$

where $\delta(s_i^{[f]}, s_j^{[f]})$ is the Kronecker's delta function. Notice that the cultural overlap of two agents is proportional to the number of shared traits on the layers in which the two agents are directly connected. When two agents interact, their cultural profiles are updated according to *layered social influence*. In practice, when agent i interacts with one of its neighbors j , imitation occurs, and j aligns one of its cultural features to that of i , choosing only among cultural features in which i and j interact, i.e., only those features for which a link between i and j exist in the corresponding layer.

The dynamics of the model proceeds in epochs. During each epoch we consider each of the N agents in a random order, and perform the following steps:

1. Consider agent i , chosen uniformly at random.
2. Consider a neighbor j of i , chosen uniformly at random among all the neighbours of i (nodes with a link to i on at least one layer).
3. Let i and j interact with probability ω_{ij} in Eq. 2.
4. If the interaction takes place, choose at random a feature f such that $a_{ij}^{[f]} = 1$ and set $s_j^{[f]} = s_i^{[f]}$.

The dynamics stops when all pairs of connected agents i and j have either $\omega_{ij} = 0$ or $\omega_{ij} = o_{ij}$. This implies the existence and stability of interacting pairs of agents who only share a limited number of cultural features, a realistic property observed in real systems but not achievable in the classical model.

In fact, if we neglect the layered structure of interactions, we are implicitly assuming that any link between two individuals potentially allows for an exchange of traits over any feature. In that case, the structure of interactions in the systems can be described by the corresponding aggregated network, i.e. the single-layer graph $A = \{a_{ij}\}$ associated to the original multiplex such that $a_{ij} = 1$ if i and j have a link in at least one of the F layers. We note that the aggregate network does not lead to any discrepancy with the structure of the original multilayer networks for the dynamics of cultural diffusion when $o = 1$.

Results

In this Section we show how the presence of layered interactions, which mimic the multilayer structure of real-world societies,^{36,38} can explain empirical observations on the presence of multiculturalism in real systems. In order to study the impact of layered interactions on culture diffusion, we model the structure of a social network by tuning its level of structural overlap o . We considered multilayer networks with F layers, each of them being an Erdős-Renyi graph with the same number of links such that $\langle k^{[f]} \rangle = \frac{1}{2N} \sum_{i,j \neq i} a_{ij}^{[f]} = 4 \forall f = 1, \dots, F$.

In the case of F identical layers, the structural overlap is maximum and we have $o = 1$. Starting with this configuration, we then assign to each edge of each layer a probability p to be rewired at random. Since each edge is rewired independently, we can express the structural overlap o as a function of p (see Methods). In general, the larger the value of p , the lower the value of o , with $o = 1$ for $p = 0$ and $o = 1/F$ for $p = 1$, and the denser the corresponding aggregated network.

In Fig. 2 we plot the size of the normalised largest cultural component S as a function of the number of cultural traits q and for different values of the structural overlap o , both for the classical Axelrod model (a), which runs on the aggregate graph associated to the multiplex network, and for the layered social influence model (b). Two individuals are considered to be part of the same cultural component if there is a path from one to the other (paths whose edges lay on different layers are allowed) and if they have equal cultural traits for all F features. In the classical model, a lower value of overlap by making the aggregated network denser is favouring globalisation, i.e. $S \approx 1$, increasing the critical number of cultural traits $q_c(o)$ at which multiculturalism appears, i.e. $S \approx 0$.

For $o = 1$ (i.e., when all the layers are identical) the classical and layered model are undistinguishable, and multiculturalism can only be achieved for large values of q . For instance, in the population considered in Fig. 2, multiculturalism can be achieved

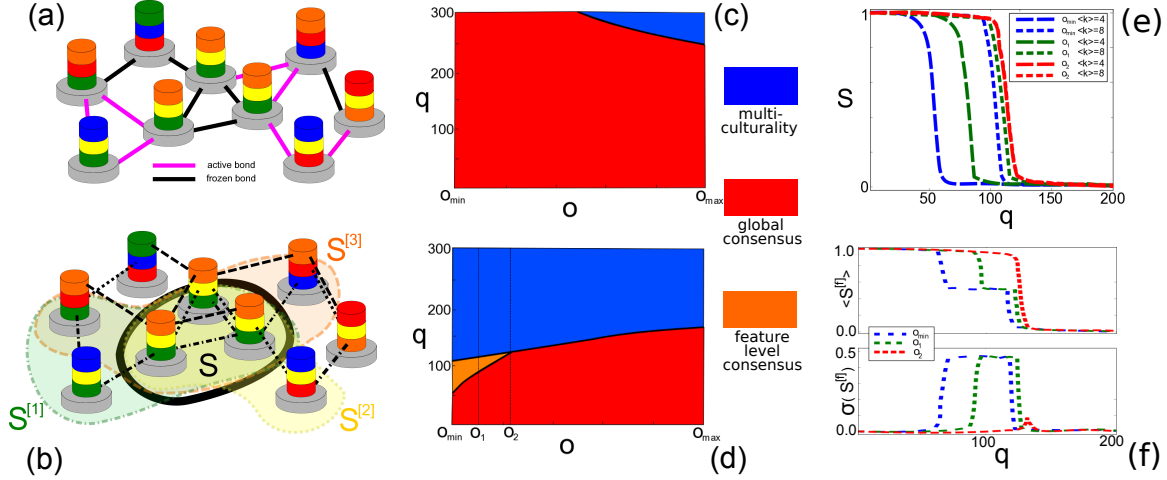


Figure 3. Global and feature-level consensus. A toy social system with three cultural features (a, b). With classical social influence (a), such system is still far from the equilibrium. Conversely, with layered social influence (b), the system is already in the absorbing state, and the largest cultural components at each layer differ from the global one. In (c, d) we show the phase diagram of the global largest cultural component of the system for classical (c) and layered (d) social influence as a function of q and for different values of the overlap o in networks with $F = 10$ and five low-density layers ($\langle k \rangle = 4$) and five high-density layers ($\langle k \rangle = 8$). In the classical case, the system reaches a state where either globalization (red region) or fragmentation (blue region) is achieved for all features. In the layered case, instead, when o is smaller than a critical value o_2 , it is possible to have globalisation in the five high-density layers together with multiculturalism in the five low-density ones (orange region). In (e), for the layered case, we plot the values of S as a function of q for one low-density and one high-density layer, and for three different values of the overlap, namely o_{\min} , o_1 and o_2 . In (f), for the same values of overlap, we show the average and the standard deviation of the largest component $S^{[f]}$ over all layers.

only for $q > 140$. In the layered social influence model, where an agent can absorb the cultural trait of his neighbor only if they are connected instead, $q_c(o)$ becomes smaller as o decreases, until it vanishes at critical value $o_c(q) > 0 \forall q \geq 2$, whose exact value depends on q , as shown in Fig. 2(c). This implies that for low structural overlap globalisation is not achievable, and the system always has a qualitative different behavior, always converging towards a multicultural state independently of the number of cultural traits $q \geq 2$. We note that in the classical model with $o = 1$ multicultural states can only be obtained for an unrealistically high number of cultural traits $q \approx 125$.

A symmetric situation is observed if we study the system as a function of o , as shown in Fig. 2(d), where it is evident that when q is small fragmentation can be obtained only for very low values of structural overlap. As q increases, the globalised phase shrinks and the critical value o_c increases. For $q > 140$, finally, the system is always in a fragmented state, even in the case of maximal structural overlap.

An important feature of social systems is the presence of cultural drift, i.e., the occasional spontaneous mutation of a cultural trait of an agent. Such phenomenon can be modelled as a noise of constant rate r , acting on the system on longer time-scales compared to the one that regulate the imitation and interactions among individuals. In the classical Axelrod model on finite populations the multicultural absorbing states (blue region in Fig. 2(a)) are known to be metastable and fragmentation is destroyed if spontaneous mutation is allowed,^{14,15} still leaving unanswered how it is possible to explain the persistence of multiculturalism in realistic societies.

In Fig. 2(b) we show with different dashed lines the position of the critical line separating globalisation from fragmentation under four increasing values of noise, namely $r_1 = 3 \times 10^{-5}$, $r_2 = 5 \times 10^{-5}$, $r_3 = 6 \times 10^{-5}$ and $r_4 = 7 \times 10^{-5}$. We note that, given a number of cultural traits q , the critical value of overlap needed to sustain globalisation depends on the amount of noise in the system, i.e. $o_c = o_c(q, r)$. In analogy to the classical case, for $o > o_c(q, r)$ the multicultural region appearing at high values of q is unstable under cultural drift. Conversely, the polarised phase obtained for $o < o_c(q, r)$ is not affected by the presence of noise and allows to explain the persistence of multicultural states that we observe in real-world societies. Noticeably, in the limit $q \rightarrow 2$ the value of $o_c(q, r)$ appears to be independent of r and approaching the value of $2/F$. Conversely, in the classical model shown in Fig. 2(a) already all multicultural states for $q < 200$ are destroyed already for a drift as small as $r_2 = 5 \times 10^{-5}$, and all multicultural states for $q < 300$ are lost for $r_3 = 6 \times 10^{-5}$, no matter the value of the structural overlap. We remark that

the instability of multicultural states is meant under the introduction of a moderate level of drift. Conversely, if drift is too high, the system enters a noisy dynamical state, describing an unrealistic society where spontaneous mutation accounts for most of the cultural chances and the effect of social influence is widely neglected.

Individuals might reach consensus on some topics and disagree on some other ones. To this scope, we introduce now the largest topical cultural component $S^{[f]}$. Two individuals i and j are considered to be part of the same topical component f if there is a path connecting them at layer f , and share the same cultural trait on feature f , i.e. $s_{f(i)} = s_{f(j)}$. We note that when $S = 1$, the condition that $S^{[f]} = 1 \forall f$ is automatically satisfied in both the classical and the layered model (assuming that each layer is connected). A remarkable property of layered social influence is that it allows for different levels of consensus on single topics to coexist, possibly resulting in the emergence of globalisation on some topics, i.e. $S^{[f]} \approx 1$ for some features f , and the persistence of fragmentation in some other ones, $S^{[\tilde{f}]} \approx 0$ for some other \tilde{f} . We call this regime *feature-level consensus*, or *topical consensus*. We remark that this realistic phenomenon can not be achieved with the classical version of the model, where at the absorbing state the largest component on each layer is always as large as the one computed taking into account all topics simultaneously.

In Fig. 3 we show how feature-level consensus may be possible only as an effect of the difference in the number of social interactions related to the different features, i.e. different layers' densities. Let us consider two sets of layers. The first set is made of five equal layers with average degree $\langle k^{[f]} \rangle = 8$, whereas the second is made of five equal layers with average degree $\langle k^{[f]} \rangle = 4$. When all edges of the layers in the second set also belong to those in the first set, the structural overlap in the system is maximised, $o = o_{\max}$. Conversely, when no edges in the low-density layers are also present in the high-density ones the overall will be minimum, $o = o_{\min}$, and the system behaves as two isolated five-layer classical models with different average degree. Hence, because of the different density, they will have a different critical values q_c for transitioning from globalisation to multiculturality. Intermediate configurations when $o_{\min} < o < o_{\max}$ are also considered.

For $o = o_{\min}$, the critical values separating globalisations from multiculturality for the low-density and high-density layers are different. While both sets of layers are globalised for $q < 60$ and fragmented for $q > 120$, in the range $60 < q < 120$ globalisation is achieved only in the high-density layers, for which $S^{[f]} \approx 1$, and the low-density ones are instead polarised, $S^{[f]} \approx 0$. Strikingly, such property of the system is not peculiar of the case $o = o_{\min}$, but it is preserved for a finite range of values of structural overlap of to a critical value o_2 . Such region is highlighted in orange in the diagram shown in Fig.3(d), and separates the globalised region (red) from the fragmented one (blue) when the value of overlap is sufficiently low.

The value of $S^{[f]}$ for low-density and a high-density layers, as well as the average value of the largest cultural components $\langle S^{[f]} \rangle$ and its standard deviation, are also shown in Fig. 3(e, f),. The standard deviation $\sigma(S^{[f]})$, approximately 0 for both the globalised and fragmented regions, takes values close to 0.5 in the mixed orange region, where consensus only exists on some layers.

We remark that higher heterogeneity in the structure of the layers produces even richer and more diverse patterns of consensus across the different cultural features. A typical case is when the activity of the nodes on the layers is heterogeneous,⁴⁶ i.e. when not all agents are involved in discussion with other individuals on all topics, preventing the spread of a cultural traits across the whole population for some given features. This is the case of many real-world systems, where the structure of interactions is often inherently layered and diverse levels of activity occur at the different layers. As a test-case, we study the dynamics of our model with layered social influence on two multilayer social systems, namely the network of Indonesian terrorists³⁵ and the collaboration network of the Pierre-Auger observatory³⁷ (see Methods).

In Fig.4 we plot for both systems, as a function of q , the size of the largest cultural components found in the model with layered social influence (dashed lines) compared with those obtained with the classical model simulated on the corresponding aggregated networks (solid lines). For the terrorists network (panel (a)), a giant component of the order of the number of nodes appear for small q even in the layered case, while for $q \approx 20$ fragmentation appears on all features. The classical model predicts instead $q_c \approx 50$. For the Pierre-Auger collaboration network (panel (b)), in the classical model we have $S \approx 1/N$ only for $q > 1000$. Conversely, the layered model predicts multiculturality already for $q = 2$, producing a qualitative different result. Such striking difference between the layered and classical dynamics is due to the relatively low level of structural overlap and to the heterogeneity in the activity and density of the layers of the multiplex. Such qualitative different behaviors highlights the importance of the underlying structure of social interactions to understand cultural patterns in real-world societies.

Discussion

Reconciling the phenomenon of social influence and imitation with the empirical evidence of multiculturality at a global scale is a long-debated problem. The model for the dissemination of culture introduced by Axelrod in 1997⁸ has proven to be able to produce socially diversified states in spite of the existence of a locally polarising rule. However, such multicultural states occur only for a very high number of cultural traits q and are not robust against cultural drift. In spite of several proposals,^{16,24,25,30,31} explaining the emergence and persistence of multicultural states that we experience in our life still appears as an open problem.

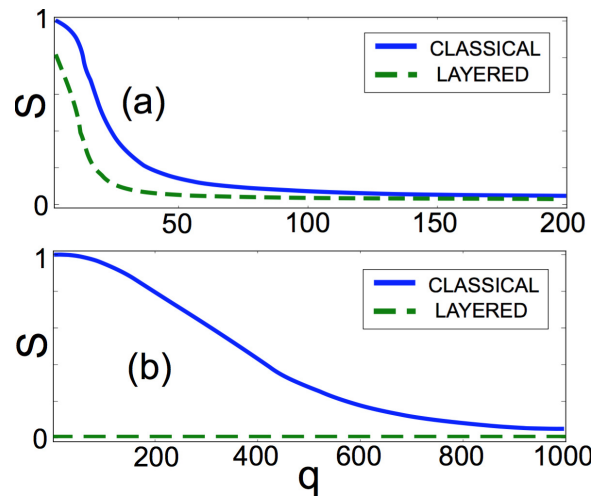


Figure 4. Multiculturalism in empirical multilayer social networks. The largest cultural component S for classical (solid lines) and layered social influence (dashed lines) is reported as a function of q for two real-world multiplex social systems, respectively the network of Indonesian terrorists in panel (a) and the collaboration network of the Pierre Augier observatory in panel (b). In the first case the outcome of the classical and layered model are similar, with layered social influence only producing a decrease in the value of q_c . This is mainly due to the relatively large amount of structural overlap in the terrorists network ($\sigma = 0.48$) and low heterogeneity in the activity of nodes. Conversely, the low value of redundancy in the Pierre Augier collaboration network ($\sigma = 0.07$), and the heterogeneity in the density and activity of the layers, produce qualitatively different results: the layered model is always fragmented, even at $q = 2$, whereas the classical model produces a largest component of order $1/N$ only for $q > 1000$.

In our work we considered agents having different interaction patterns according to the different cultural features. This is a property of many social systems, where individuals usually choose the recipients of their discussion according to the content of their messages and vice versa. By constraining the imitation of the cultural traits to the cultural features where two individuals are actually linked, we naturally introduce the notion of layered social influence, inherently connected to the average structural overlap σ of a social system. Such definition implies the existence of stable states where connected agents might not have completely equal or different cultural profiles, but instead share just a limited number of features.

A main finding is that a robust multicultural regime emerges as a natural consequence of taking into account the actual structure of social interactions, which are inherently layered. Interestingly, when the structural overlap among the layers is small, i.e. when connections are not too much redundant across layers, only multicultural states will be allowed, independently of the actual number of possible cultural traits, suggesting a qualitative shift in the behavior of the system. This finding is also in agreement with the empirical evidence of socially fragmented societies even under a limited number of cultural choices. Moreover, differently from the multicultural regime predicted when the layering of social influence is not taken into account, and achievable only for a large number of traits, such novel multicultural regime is robust to the presence of cultural drift, providing a potential explanation to the persistence of global diversity over time.

Another interesting aspect of layered social influence is the ability to explain the existence of a novel regime where globalisation appears only on a limited number of cultural features, while the remaining ones are characterised by multiculturalism. The coexistence of different levels of multiculturalism in different aspects of a cultural profile is in fact another typical property observed in human societies.

In conclusion, the model of layered social influence proposed in this paper, provides a mechanism leading to robust multiculturalism, revealing the important role played by a multilayer organisation of social interactions in avoiding globalisation, and pointing out the effect of heterogeneity of layer densities in the emergence of partially globalised regimes. While previous proposals to account for robust multiculturalism invoke different forms of interactions among agents in an aggregated social network, we provide here an alternative mechanism not based in the form of the interaction, but on the structural properties of the social network. From this new perspective, the relevant control parameter of polarization-globalization transitions is no longer the number of cultural traits per feature, but a structural property, namely the overlap parameter accounting for the heterogeneity of social links among the different layers of the social network. The relevance of the structural properties of multilayered social interactions, illustrated here for cultural dissemination, is far reaching, and should be taken into account to reconsider a number of results based on models of social interactions among agents in aggregated networks.

Methods

Constructing synthetic multilayer social networks. In this section we focus on the model to produce synthetic multilayer networks with different values of structural overlap o . Let us consider a multiplex network with F layers, each with the same number of links $K^{[f]}$, so that we have $K^{[f]}F$ links in total. We then consider the aggregated network obtained by collapsing all the layers. If all the layers are identical, the aggregated graph will have $K = K^{[f]}$ edges. Hence $o = \frac{1}{F} \frac{KF}{K} = 1$ for $p = 0$. When $p \neq 0$ and links are rewired independently at each layer, the number of edges in the aggregated networks increases. If we neglect the probability that two links of different layers are rewired in the same position, the number of edges in the aggregated network becomes equal to

$$K = \sum_{m=1}^F \binom{F}{m} (1-p)^m p^{F-m} K^{[f]} + \sum_{m=0}^F \binom{F}{m} (1-p)^m p^{F-m} K^{[f]} (F-m) \quad (3)$$

However the total number of connections in the multiplex remains $K^{[f]}F$. Hence

$$o \approx \frac{1}{\sum_{m=0}^F \binom{F}{m} (1-p)^m p^{F-m} (1 - \delta_{0,m} + F - m)}, \quad (4)$$

higher values of rewiring correspond to lower values of overlap and vice versa, with the limiting cases of $o = 1$ for $p = 0$ and $o = 1/F$ for $p = 1$. Given the average degree $\langle k^{[f]} \rangle$, the average number of neighbours $\langle k \rangle$ in the aggregated network can be obtained as $\langle k \rangle = \langle k^{[f]} \rangle / o$. Hence, the higher the p , the lower the o and the higher the density of the aggregated network. The results shown in Fig. 2 have been obtained for a system with $N = 25^2$ agents. Each interaction layer f has average degree $\langle k^{[f]} \rangle = \frac{1}{2N} \sum_{i,j \neq i} a_{ij}^{[f]} = 4$. In Fig. 3 we consider layers with different average degrees $\langle k^{[f]} \rangle = \frac{1}{2N} \sum_{i,j \neq i} a_{ij}^{[f]}$. Five of them are equal networks with $\langle k^{[f]} \rangle = 4$, the other five are equal networks with $\langle k^{[f]} \rangle = 8$. We consider different combinations of the two sets of layers such in a way to span the range of values of structural overlap $o \in [o_{\min}, o_{\max}]$. When $o = o_{\max}$, all edges in the emptier layers are also present in the denser ones. Conversely, when $o = o_{\min}$ no edge of the emptier layer is shared with the denser ones, and the system behaves as two five-layer decoupled classical models.

Datasets. In Fig. 4 we apply the classical and layered social influence models to two real-world multilayer networks with layered interaction patterns and different activities of the node and layers. A node i is defined as active on layer f , i.e. $b_i^{[f]} = 1$, if $k_i^{[f]} > 0$. Otherwise it is considered inactive, i.e. $b_i^{[f]} = 0$.⁴⁶ The first dataset is the network of Indonesian terrorists presented in,³⁵ which has $F = 3$ layers, corresponding to trust, common operations and exchanged communication across 78 individuals, most of which are active on all layers, and has a structural overlap $o \approx 0.4817$, with moderate difference in the layers' activity. The second data set is the multi-layer collaboration network of the Pierre-Auger observatory,³⁷ which consists of $F = 16$ layers. To allow, at least potentially, complete globalisation in the system, we selected the subset of individuals who belong to the giant component of the aggregated network. Such system includes 475 individuals. The system has a structural overlap of $o \approx 0.069$ and high heterogeneity in the activity of its members.

Acknowledgments

The authors acknowledge the support of the EU Project LASAGNE, Contract no. 318132 (STREP). V.L. also acknowledges support from EPSRC projects EP/N013492/1. M.S.M. also acknowledges financial support from FEDER (EU) and MINECO (Spain) under Grant ESOTECOS FIS2015-63628-C2-R.

Author contributions

F.B., V.N. V.L. and M.S.M. designed research, F.B. and V.N. performed research, F.B., V.N. V.L. and M.S.M. analysed data, F.B., V.N. V.L. and M.S.M. wrote the paper.

Competing financial interests

The authors declare no competing financial interests.

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