Playing with the Edge: Tipping Points and the Role of Tonality

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THIS ARTICLE CENTERS ON THE PHENOMENON OF tipping points—a case of extreme pulse elasticity—in music performance, and the dynamic interplay between tonal structure and musical timing. The article presents the idea and principles of tipping points. Examples illustrate three types of global and local tipping points: melodic, boundary, and cadential. Focusing on cadential tipping points, the article considers the role of tonality in a number of examples, thus bridging the subject of tipping points and prior work on the modeling of tonality. The spiral array model for tonality is described, including how the model traces the dynamics of tonal perception. A real-time implementation of the model is applied to cadential tipping point examples to visualize the effect of tipping points on tonal perception. The analyses show how tipping points influence tonal perception—clarifying, focusing, and exploiting harmonic function, in the case of cadential tipping points, to evoke tension and shape narrative structure.

A STORY OF THE ANATOMY OF A PERFORMANCE provides an introduction to the idea of tipping points. The example highlights the interplay between tonality and timing in music performance. When confronted with the challenge of preparing for a performance of a new piece of music, there are numerous decisions to be made with respect to the shaping of tempo, timing, loudness, timbre, and articulation.

Wrinkles in Time

Consider the "Epilogue" from *Doubles* (1998-1999), a post-tonal composition by Peter Child. The piece is a slow rag, written in memory of William Albright. Embedded in the notation are several cues for performance. The characteristic swing in ragtime music—giving a sequence of equal duration (usually eighth) notes

a long-short long-short rhythm—has been explicitly written into the score in triplet notation (see Figure 1). More interestingly, the composer has carefully placed phrase markings in the score, indicating asynchronous (between the left and right hands) boundaries in the sequence of notes as highlighted in Figure 1(a).

In order to make the asynchronous boundaries apparent in performance, like in speech, a common strategy is to introduce micro pauses at the boundaries, or slowdowns towards the ends of phrases and slight accelerations into phrases. A rarer, and perhaps more sophisticated, plan would be to sometimes have a stress (elongation of a note) occur just before the phrase ending and for the stressed note to act as a pickup, effectively turning the penultimate note into a launch pad for the next phrase. Either method would result in a deliberate warping of the metrical grid of the performed music.

Furthermore, for the sake of interest, not all phrases are equal, and the musician must decide on some hierarchical grouping of these phrases to project, such as that shown in Figure 1(b). Higher-level phrase groupings would require deeper pauses or stresses for boundary indicators, further introducing wrinkles into the time fabric of the piece.

Focusing now on the dynamic markings, the crescendos followed by subito piano (**sub. *p***) and pianissimo (***pp***), respectively, demand dynamic variation like that indicated by the teardrop shapes in Figure 1c. The sudden drops from the loudest end of the crescendo (<) to a piano (***p***) or pianissimo (***pp***) require time as the musician must allow the sound from the dynamic peak to dissipate sufficiently for the next chord (at ***p*** or ***pp***) to be heard, thus introducing agogic (time-based) accents to the dynamic ones.

The loudest parts of the crescendos, compounded with the natural tendency to make prominent the melody peaks, make the dynamic peaks more extreme— thus needing more time for the subsequent drops—and lead to dynamic accent structures as shown by the inverted triangles in Figure 1(c). Further composer-marked accents are indicated by inverted triangles in Figure 1(d). The sizes of the triangles designate the hierarchy of the accent strengths. The two reversed teardrops in the first bar of Figure 1(d) signify the sound dissipation following the accented mezzo forte (***mf***) figure in the right hand to the next notes in the right- and

FIGURE 1. Crafting a performance of the first four bars of "Epilogue" from Child's *Doubles*: (a) phrase markings in the score; (b) hierarchical organization of phrases; (c) dynamic markings and accent structures; and (d) more phrases, dynamic shapings, and accents.

left-hand parts, respectively. At this point, the fast accented chords of the right-hand terminating in the acciaccatura on the dominant in the left hand in bar four, we reach the culmination of the introductory bars, in terms of the dynamics, the development of the piece, and most importantly the tonality. The nebulous tonality of the opening bars finally gives way to the V of an unambiguous perfect (V-I) cadence. The left hand strikes the B♭ with aplomb, and stands ready, poised to spring back into the swing of the piece.

Drawing upon schematic knowledge about tonality, the listener expects the dominant to resolve to the tonic (E♭) at this point. Because the outcome is known and highly expected, the performer may choose to delay the outcome—and the listener will wait for its resolution. The delay of the expected outcome heightens the suspense—when will the chord resolve?—increases the tension, and escalates the emotion response. This deliberate and acute timing delay creates a *tipping point* at the juncture between the end of the introduction and the beginning of the piece proper. In particular, because the tipping point occurs at a cadence, this is called a *cadential tipping point*. It is this kind of extreme elasticity of pulse, such tipping points, that forms the focus of this article.

The remainder of the paper will introduce the concept and principles of tipping points, and discuss in detail one class of tipping points: cadential tipping points. Following the description of a model for tonality and how it can be used to trace the dynamics of tonal perception, the model is applied to the case of cadential tipping points. Conclusions and discussions complete the paper.

Tipping Points in Music

In classical mechanics in physics, an object is stable when a line through its center of gravity falls within the base of the object. The object is unstable when it is tilted or shaped so that the line through its center of gravity falls through an edge of its base; any small perturbation will tip the balance and cause the object to fall. More formally, the tipping point is the point beyond which a line through the center of gravity lies outside the base of the object. Because they are common physical experiences, tipping points have become a favored metaphor in domains ranging from politics to economics (Lamberson & Page, 2012). In a book devoted to examples of social tipping points, Gladwell (2000) defines a tipping point as "the magic moment when an idea, trend, or social behavior crosses a threshold, tips, and spreads like wildfire."

Taking advantage of the close affinity between music and motion, the mechanics of a musical tipping point— which I first proposed in a talk at the Performance Studies Network Conference 2 in Cambridge—can be described as follows: "A musical tipping point is a timeless moment of suspended stillness, of stasis, beyond which a small perturbation will tip the balance and set in motion the inevitable return of the pulse." Next, we discuss some of the principles underlying the execution and choice of tipping points.

A metaphor. A musical tipping point simulates the feeling of a physical object balanced at its tipping point, like a train atop the hill of a roller coaster, before it falls back into motion. At this point, the pulse exhibits an extreme form of elasticity that stretches far beyond the norm. A tipping point can be considered a performance heuristic, a powerful metaphor to shape naturally the changes to expressive parameters at a high level (Leech-Wilkinson & Prior, 2013). For accompanying musicians, tipping points can be a useful metaphor for predicting when the soloist's next note will fall.

Consider Puccini's aria "O mio babbino caro" from his opera Gianni Schicchi. Figure 2(a) shows the eighth note lengths extracted from performances of the aria by Kathleen Battle (1989), Maria Callas (1997/2007), and Kiri te Kanawa (1984). Eighth note onsets were annotated using Sonic Visualiser (sonicvisualiser.com)(Cannam et al. 2010) and the duration (in seconds) of the *i*-th eighth note given by:

$$e_i = o_i - o_{i-1},$$

where o_i is the onset time of the *i*-th eighth note pulse (in seconds). The default Matlab spline function connects the dots.

While tempo graphs constitute common ways to represent the shaping of time in performance, the duration information (the inverse of tempo) gives a better visual depiction of the tipping point analogy. When there is an extreme slowdown toward a tipping point, the values register a severe rise to an acute peak, reminiscent of a roller coaster slowing to a stop at the apex of a hill before it tips over and gathers momentum back into the run. The eighth note duration data is plotted in score time, with the corresponding score excerpt shown below the graph.

In Figures 2(a), and (b), cue balls mark the tipping points in Callas' performance, with the size of each cue ball corresponding to the degree of the tipping point. A shorter excerpt from Callas' performance, boxed in Figure 2(a), is shown in Figure 2(b), this time plotted in performance time. It is important to consider both score time plots, which allow multiple performances to be compared simultaneously, and performance time plots, which allow a graph to be aligned to the audio (Chew &
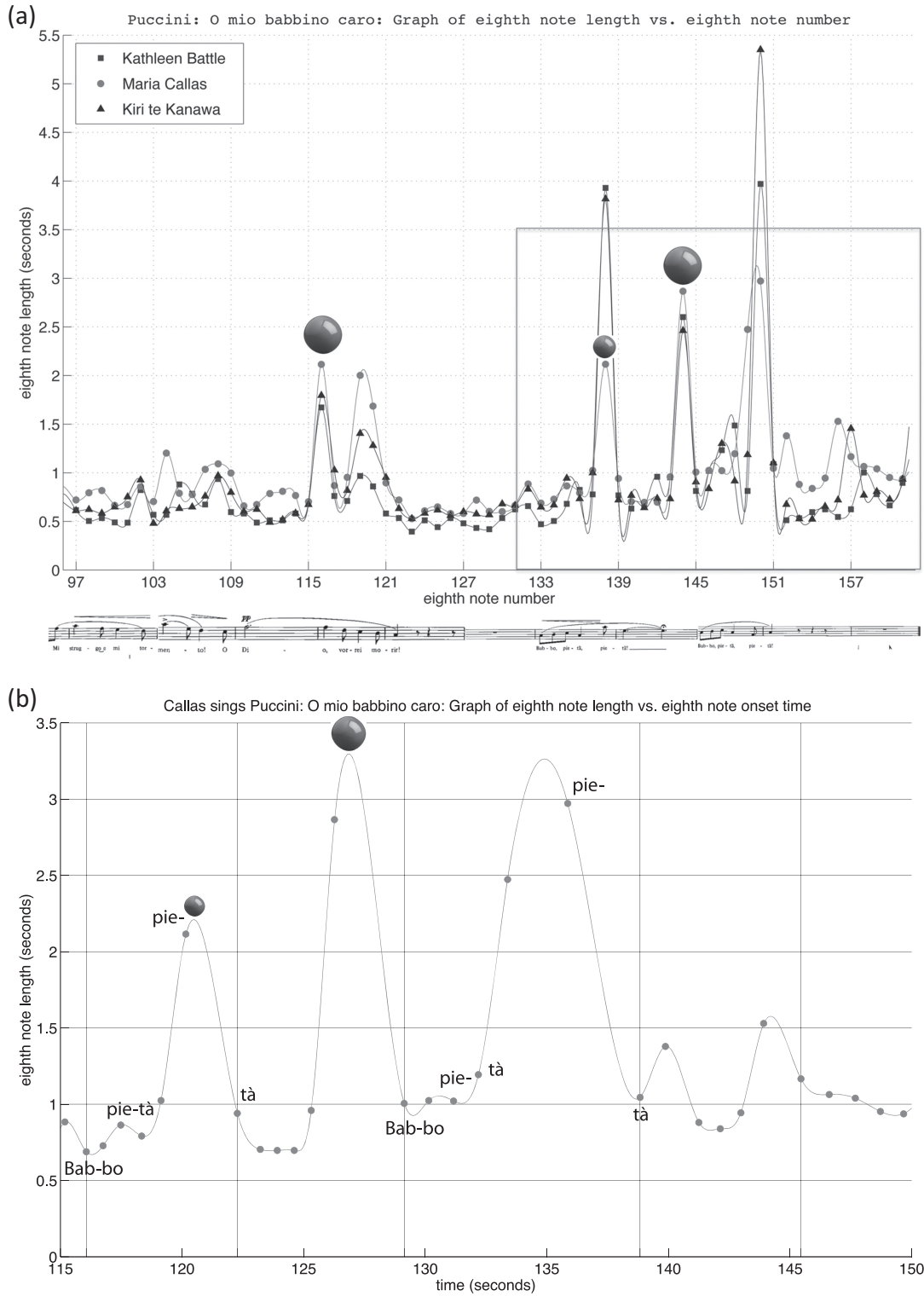
FIGURE 2. Tipping points in performances of Giacomo Puccini's "O mio babbino caro" from his opera Gianni Schicchi. (a) Eighth note lengths in score time, performances by Maria Callas, Kathleen Battle, and Kiri te Kanawa; (b) eighth note lengths in performance time, excerpt from Callas' performance (to hear excerpt, see video at vimeo.com/127507105).

Callender 2013). The dots on the undulating line mark the onset of each eighth note pulse; the vertical gridlines mark the first eighth note pulse of each bar. The corresponding audio can be heard in a video at vimeo.com/127507105.

The elasticity of the pulse is evident in the peaks in the graph. Having a model of when and how the pulse falls back in step allows the accompanist(s) to catch each return following a time stretch. It is important to stress that tipping points constitute more than a performance heuristic. The following paragraphs will show that they form a principled means by which to construct the critical junctures that are essential to the architecting of a successful performance.

Form and magnitude. Tipping points range in size and vary in significance. A global tipping point occurs when the piece reaches a turning point, when all is spent, and the tip initiates the return. Local tipping points form smaller-scale gestures at the melodic or note-to-note level. While local tipping points may occur more frequently, global tipping points are relatively rare over the time span of a piece.

Tipping points are characterized by extreme pulse elasticity, but not all time stretches represent tipping points. The first peak in the Callas excerpt in Figure 2(b) is an example of a (local) *melodic tipping point*. The second one is a (local/phrase) *boundary tipping point*. A third kind of tipping point, a *cadential tipping point*, was featured in the opening example and will be discussed at length later in this article. In the Callas excerpt, the second peak is higher than the first—more time is taken at this phrase boundary, and time is further extended by the fermata over the note—thus producing the effect of a more severe tipping point. The third peak is not a tipping point because, while there is a high degree of pulse elasticity, structurally there is insufficient tension carrying through the peak to warrant the label of a tipping point.

Experience shows that listeners and musicians largely agree on what is and is not a tipping point, and are able to point to many examples of tipping points in the music (recordings or scores) that they know. While yet another kind of tipping point (not based on elasticity of pulse) exists in compositions, our focus here is on tipping points as employed and exploited in expressive performance.

Structure. As alluded to in the previous paragraphs, tipping points leverage and enhance inherent musical structures. At the highest level, they constitute the means through which to choreograph critical turning points in the narrative of a performance—we shall see

an example of this later in this article in Strauss' "Burleske." At the mid to high level, they are tools for navigating important transitions. At the mid to low level, they ascribe prominence to particular parts of a phrase; and at a low level, they highlight certain notes. Regardless of level, tipping points can influence the function of the notes in the pulse they bend, and shape the trajectory of the musical narrative. This article focuses on tipping points that are aligned with tonal structures, that augment and influence tonal function, and that facilitate continuations.

Expectation. The hyperbole of a tipping point—the exaggerated pulse elongation and the uncertainly of when the pulse is to return—is made possible by the certainty of highly expected events. The integrity of the music, in light of the complete derailment of the pulse, is sustained by the certainty of events to follow.

As expounded in Huron (2006), such expectations can be schematic (based on general patterns derived from exposure to many pieces) or veridical (based on knowledge of how a particular piece goes). Examples of schematic expectations include a registral return following an upward leap, and the closure of a I chord following the V chord in a perfect cadence. The melodic tipping point in the Callas example of Figure 2 plays on the expectation of a registral return, while cadential tipping point examples take advantage of the expectation of a return to tonal stability via a perfect cadence.

An example of a play on a veridical expectation can be found in Fritz Kreisler's performance of his "Schön Rosmarin" (Kreisler & Rupp, 1936/1993). The excerpt, with the quarter note lengths in Kreisler's performance, is shown in Figure 3(a). The vertical gridlines, as well as the x-axis tick marks indicate the first beat of each bar. Figure 3(b) shows the same data, plotted in performance time (see corresponding video at vimeo.com/127499857). The dots on the undulating line still mark the quarter note beats, and the vertical gridlines indicate the first beat of each bar. Cue balls indicate tipping points of varying degrees.

Remarkably, Kreisler begins this performance with a significant tipping point, at the onset of the very first note. The veridical expectation of how the melody progresses, as indeed the schematic expectation that a piece must continue beyond the first note, allows the performer to sustain the listener's attention while taking an audacious amount of time at the outset. The smaller tipping points take place at the extremities of the melodic line, and exploit the expectation that the melody will regress back to its mean from the unstable sixth scale degree.

Information. Tipping points are linked with highly expected outcomes. In information theoretic terms, there is minimal information when tipping points are invoked. In a sense, this is a highly predictable state in the scheme of the composition. Tipping points represent a means for the performer to create instability when the future is certain—by suspending the pulse and causing uncertainty regarding the moment of the tip before the pulse returns. It is when continuation is most sure, that musical interest, drama even, can then be created by drastic manipulation of time.

Tension. Apart from its structural functions, tipping points serve to generate tension. By delaying the expected, tipping points heighten anticipation and increase tension, so that the return becomes a cathartic moment of release and pleasure. For example, a significant tipping point can often be found near the end of a cadenza, a drawn out elaboration on the V of a V-I cadence. After the technical displays of the cadenza, the completion of the prototypical cadential sequence provides a long awaited path back to stability. Lengthening the V just before this completion serves to escalate the tension before the catharsis of the resolution to I.

There exists an extensive body of studies on musical tension, linking tension to musical features such as loudness, timbre, pitch height, harmony, rhythmic density, and tempo—see review in Farbood (2012), which also sets forth a model for predicting musical tension based on these parameters. While tempo has been linked to perception of tension, interestingly, a study by Fredrickson and Johnson (1996) showed that tension could not be ascribed to variations in performers' use of rubato. The present article sets forth a theory that extreme timing deviations in the form of tipping points can serve as a powerful force for evoking tension.

Lerdahl and Krumhansl (2007) propose a model of tonal tension based on intervallic dissonance, harmonic instability, and melodic attraction. While anticipation, the principle of attraction, and tension borne of anticipation intensified are common themes running through the tipping point examples, this attraction extends beyond melodic to harmonic attraction (and instability) in the form of cadential closure. The high expectedness of harmonic closure, such as the attraction between the V and I chords in a perfect cadence, thus also acts as a source of tension.

In Toiviainen and Krumhansl (2003), it was found that dominant key areas were associated with especially high tension. This finding is consistent with the premises and properties of the cadential tipping point effect, which exploits the tension generated by the dominant. Because of the strong tendency for the dominant to move on to the tonic, theorists regard the dominant as unstable, even though it is closely positioned to the tonic in tonal space—this is in contrast to overtly dissonant intervals such as the tritone, which tend to be unstable and far apart in tonal space—because of its leanings towards, and tendency to resolve to, the more stable tonic.

Related to the idea that tipping points occur where continuations are highly expected, Bigand and Parncutt (1998) equated musical tension at the end of a chord sequence with a compelling feeling that the sequence must continue. Their experiments determined that tension was more strongly influenced by local rather than global harmonic structure. The cadenza example later in this article will demonstrate how global tonal structure, together with the use of a tipping point, can evoke tension and shape narrative structure.

The goal of this article is not to define a model of tension, but rather to consider a special case of extreme timing deviation—the where, when, why, and how of tipping points—and its link to tonality. The most obvious connections reside in the cases of cadential tipping points, which form the core of the discussion in the next section.

Cadential Tipping Points

In this section, we revisit the cadential tipping point in a performance of Child's "Epilogue," a local tipping point. Then we consider cadential tipping points, both local and global, in performances of the cadenza in Strauss' "Burleske."

CADENTIAL TIPPING POINT EXAMPLE

Here, we present some analyses of a recording of Chew's performance of Child's "Epilogue" in (Chew 2009), which can be heard at soundcloud.com/elaine-chew/child-doubles-epilogue. In this performance, a tipping point is invoked in the fourth bar.

Figure 4(a) shows the instantaneous tempo (in bpm, beats per minute), and Figure 4(b) the log(tempo) (in log(bpm)) at each note event, where the instantaneous beat duration is given by the inter-onset-interval between the current and the previous note event, divided by the score duration between the two events. The instantaneous beat duration (in seconds) at the i-th event is given by:

$$b_i = \frac{o_i - o_{i-1}}{d_{i-1}},$$

where d_i is the score duration (in beats) between the (i–1)-th and i-th events, o_i is the onset time of the i-th event
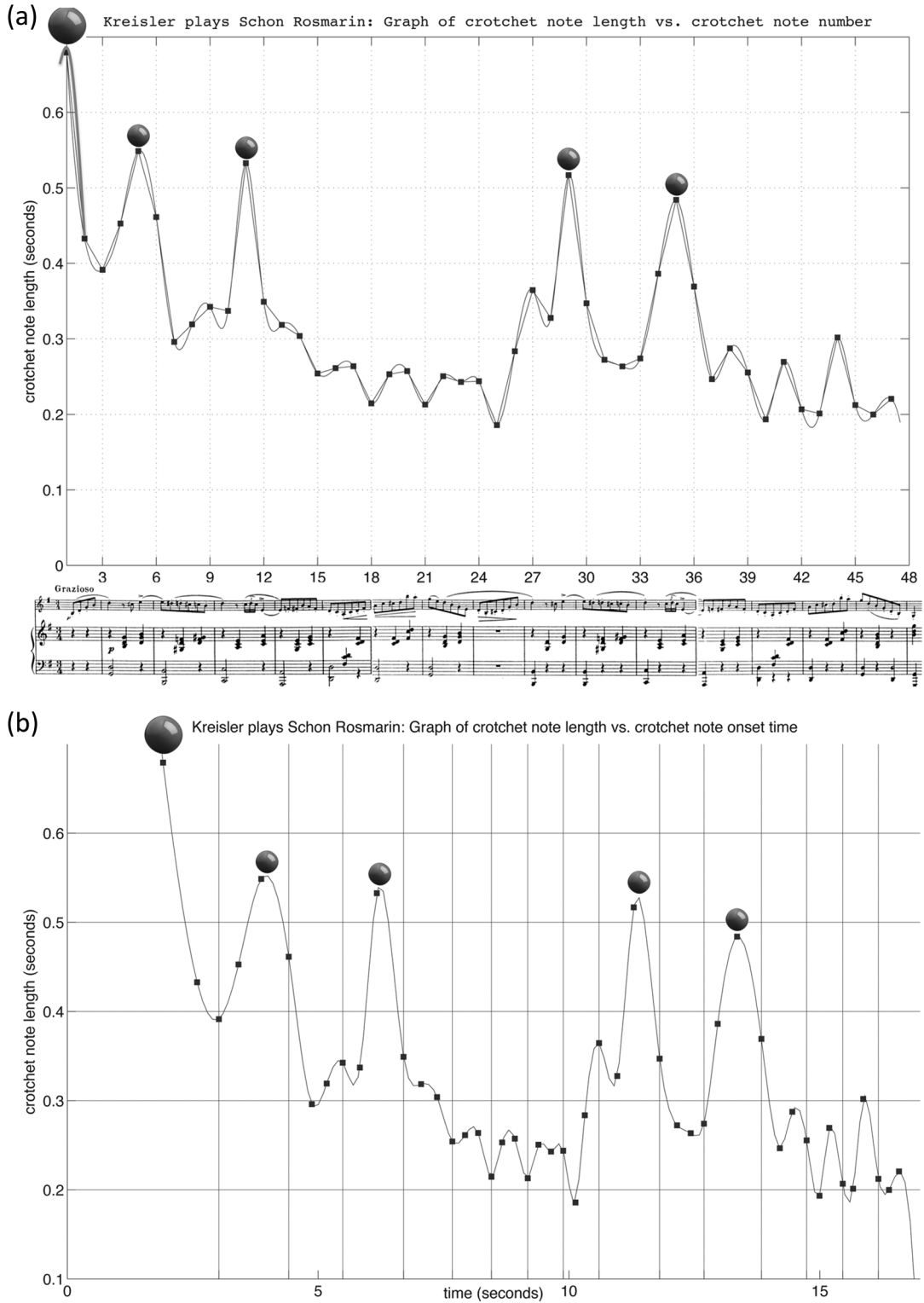
FIGURE 3. Tipping points in performances of Fritz Kreisler's "Schön Rosmarin" as performed by Kreisler. (a) Quarter note lengths in score time; (b) quarter note lengths in performance time (to hear excerpt, see video at vimeo.com/127499857).
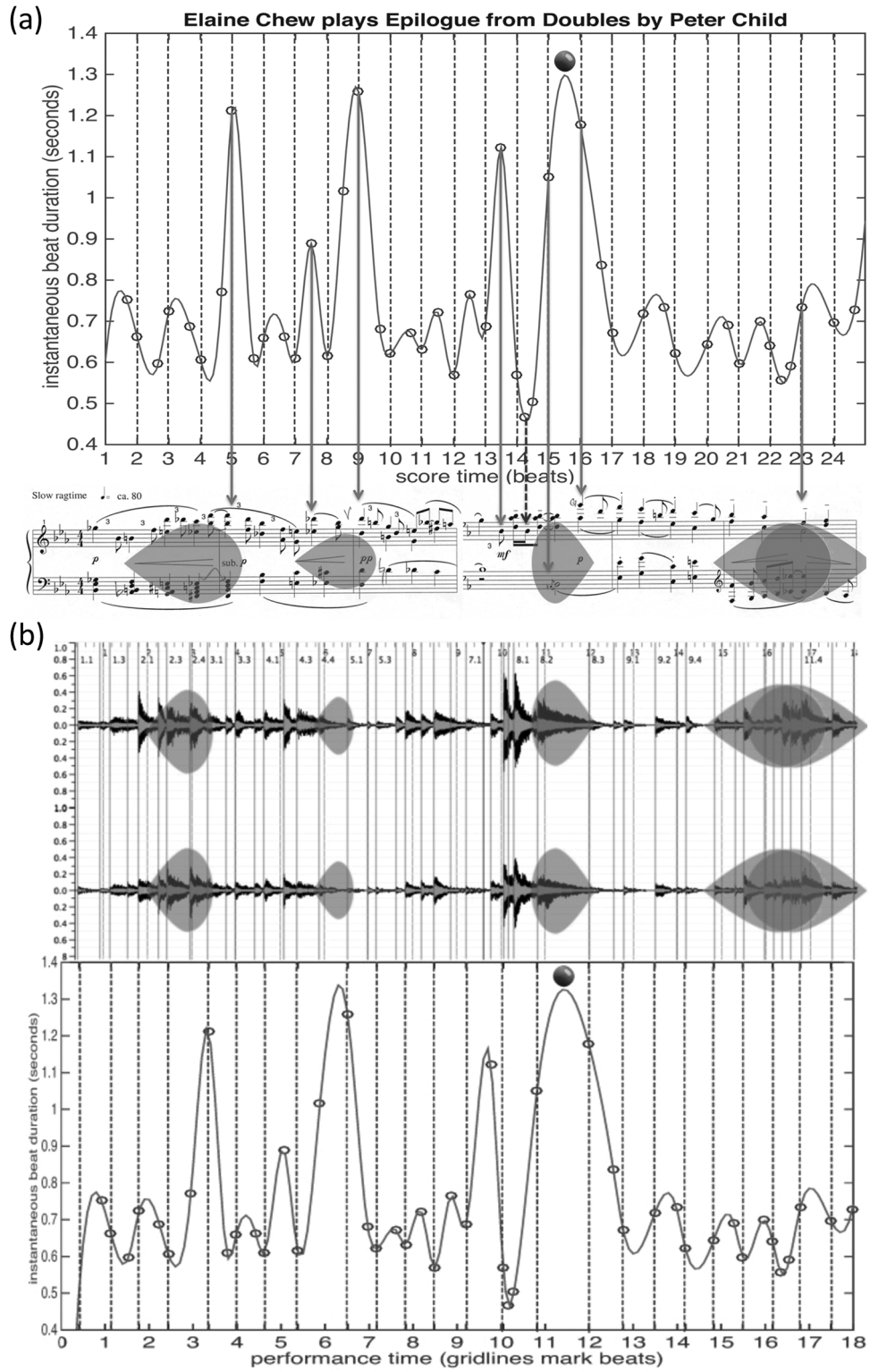
FIGURE 4. Cadential tipping point in Chew's performance of the initial bars of Child's "Epilogue" from *Doubles*: (a) instantaneous beat durations in score time; (b) instantaneous beat durations in performance time (to hear excerpt, see video at vimeo.com/127997355).

(in seconds). In the plots, the circles on the undulating lines mark note onsets and the vertical gridlines mark the beat onsets; solid arrows indicate score positions corresponding to peaks in beat duration (i.e., the largest tempo dips), while the dotted arrow marks the point of lowest beat duration (i.e., the point of greatest speed).

The alignment of the score time instantaneous beat duration to the notated score in Figure 4(a) reveals that the three highest local maxima in the beat duration plots correspond to time taken for the tipping point (marked by the cue ball) at the authentic cadence in the second half of bar four, and to time taken for achieving dynamic contrast (in the *mf*-to-*p* dynamic drop, and the crescendo peak-to-*pp* dynamic drop). Other local maxima are due to time taken to make prominent a melodic peak, and for setting up the tipping point. The rightmost arrow shows a little time is taken to achieve the hairpin dynamic effect.

Figure 4(b) shows the same information, this time plotted against performance time, and aligned to the audio signal, showing the correspondence between the beat duration maxima and the dynamic drops (the two forward teardrops) and the tipping point (at the end of the reversed teardrop, marked by the cue ball). In real time, the scale or the largest of the tipping point gesture is apparent in the width of the hill generated by the severe pulse stretching invoked to create the effect. The musical excerpt can be heard in a video at vimeo.com/127997355.

CADENZA TIPPING POINT

A prototypical place to look for tipping points is near the end of a concerto cadenza, when the listener has been kept on tenterhooks of suspense over brilliant technical displays elaborating the dominant chord, expecting and awaiting the return of the tonic in the V-I cadence.

Consider the cadenza from Richard Strauss' "Burleske" for Piano and Orchestra (1958), shown in Figure 5 (starting from the red arrow and ending the bar after the larger red cue ball.) This very example was used in the opening of Chew (2014) to demonstrate the link between tonality and expectation. We now take the illustration one step further to connect it to the idea of tipping points, and to show different degrees of tipping points.

As mentioned earlier, a cadenza is a prolonged elaboration on the dominant of a perfect (V-I) cadence. The tonic ahead is D (minor), and the V of D is A. In the cadenza, the prominence of A is established in the doubled octaves (circled, see Figure 5) near the opening of the cadenza. The octave A's (next five pairs of circles) punctuate sequences of alternating chords and sweeping arpeggios. First, the arpeggios are only ascending, then they ascend and descend. The time between the octave A's get shorter and shorter, the octave A's become more emphatic, ratcheting up the excitement.

Then, the alternating chords stop altogether, and the tide turns with respect to the time between the octave A's. The fortissimo (*ff*) dynamic falls back to mezzo forte (*mf*) to start the next climb. The ascending and descending arpeggios (shown by the arched arrows) still punctuated by the octave A's but now with increasing distance, the gestures growing in extravagance, the arpeggios reach ever higher until the first tipping point, the highest notes played so far. A lesser peak, another chord with impact, which could be interpreted to be a less significant tipping point, follows.

The sturm and drang of the earlier passages dies down, and the mood becomes pensive. The solemn ringing of the A's, no longer in octaves, persists with increasing stillness, the notes alternating between the leading tone diminished seventh chord and the A major chord.

Finally, the left-hand A comes to a standstill, a note that is sustained over four bars. The pitch range narrows and the passing A's in the ascending triplet eighth notes eventually reverse direction and slow to minims (half notes) to reveal the hidden A in the right hand as well.

At this most still and quiet moment, when both hands align, we have the main tipping point (indicated by the larger cue ball). Time is suspended; we have come to the edge of the cadenza, and are on the brink of the return. It is up to the performer to choose the moment of return, to initiate the tip to start the return of the pulse and the final descent to the tonic.

Consider this global tipping point as performed by Glenn Gould (Gould & Golschmann, 1966), Martha Argerich (Argerich & Abbado, 1992), and Sviatoslav Richter (Richter & Georgescu, 1961). Figure 6(a) represents the quantitative performed durations of each bar of Strauss' cadenza in these three performances, aligned to score time. Figure 6(b) shows Argerich's performance in real time (see corresponding video at vimeo.com/127997185).

In the graph of Figure 6(a), the drive to a first (local) tipping point (with a smaller local tipping point shortly after) is visible in Gould's and Argerich's performances, although Richter makes little of it. Perhaps more apparent, and certainly more unanimous, is the main tipping point, where all three performers prolong the bar to extraordinary lengths before initiating the tip, a little nudge, to start the return of the pulse.

The drama of the global tipping point is driven by the most basic convention of tonality—the schematic expectation of the I following the V in a V-I cadence.

FIGURE 5. Cadential tipping points (indicated by cue balls) in the cadenza of Strauss' "Burleske," with dominants (circled) highlighted (score from Strauss, 1894, p. 68).
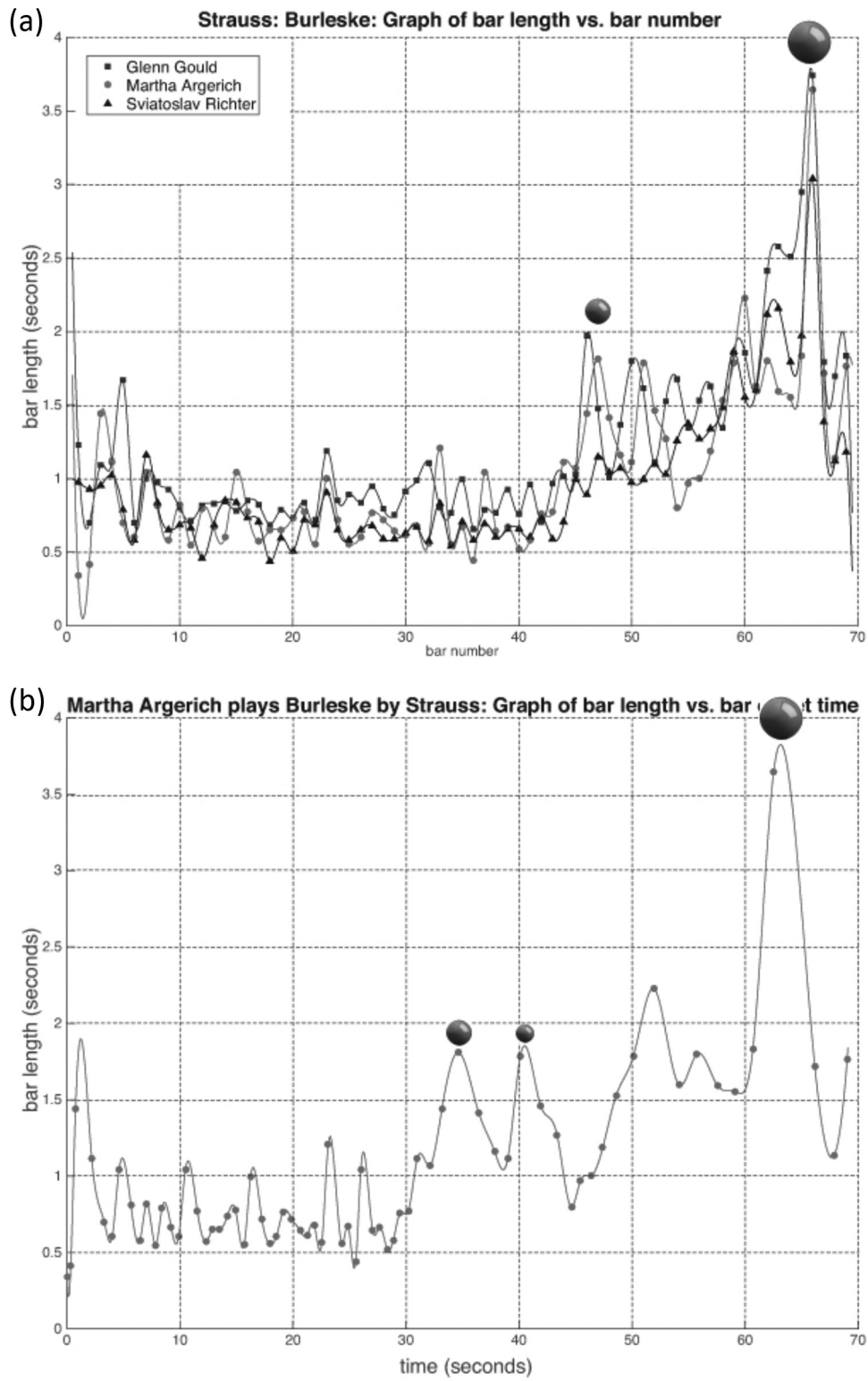
FIGURE 6. Cadential tipping points in performances of the cadenza of Strauss' "Burleske." (a) Bar lengths of performances by Martha Argerich, Glenn Gould, and Sviatoslav Richter in score time; (b) bar lengths of Argerich's performance in real time (to hear excerpt, see video at vimeo.com/127997185).

The prolonging of the V, whether through compositional elaboration or via a tipping point, exacerbates the longing for closure that is only resolved with the return of the tonic.

In general, global tipping points constitute relatively rare moments. Because they often represent moments of catharsis, they can become cloy with overuse. As in any art form, the relationship between tonality and timing does not constitute a rule for employing tipping points in performance. The association represents an option, a decision that can be exercised at will—where in the piece to invoke a tipping points, and when to initiate the tip.

Before we digress from cadential tipping points, it is worth noting that music theorists have referred to cadences as signifiers of energy gain (Hepokoski & Darcy, 2006) and release (Kurth & Rothfarb, 1991); furthermore, Hatten (2014) has discussed temporal emphasis at cadence points in Chopin. None as yet have considered extreme cases of timing deviations and conceptualized them as tipping points. It is also important to point out that tipping points extend beyond cadential closures, as demonstrated in the examples in the earlier section *Tipping Points in Music*.

Tonality

This section presents the spiral array model and its associated key finding algorithm (Chew, 2014), which will be used in the next section to visualize tonal evolution in the two cadential tipping point examples. The section also seeks to explain, through a concrete example, why key perception is highly malleable and subject to deviations in performance timing.

THE SPIRAL ARRAY MODEL

The spiral array comprises of a series of nested helices, and its construction is based on the principle of generating centers of effect (c.e.) in the interior of the model.

The outermost helix, representing pitch classes, is a three-dimensional configuration of the *tonnetz* (see review in Cohn, 1998) or the Harmonic Network (Longuet-Higgins, 1962a, 1962b). The helical model removes recurrences of pitch names, while preserving pitch spellings (i.e., F♯ ≠ A♭). The Harmonic Network and the periodicity in the representation are shown in Figure 7(a). The pitch class helix is shown in Figure 7(b), with the interior space (the inner cylinder) that will be used in the generation of higher-level constructs highlighted. Each pitch representation, indexed by its position on the line of fifths can be defined as follows:

(a)

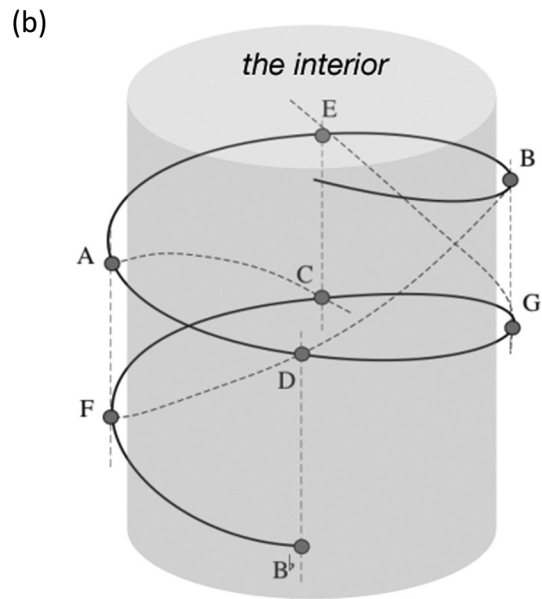| E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

(b)

FIGURE 7. (a) Periodicity in the Harmonic Network; (b) The helical configuration of the Harmonic Network and interior space for the representation of higher level entities.

$$\mathbf{P}(k) = \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} r\sin(k\pi/2) \\ r\cos(k\pi/2) \\ kh \end{bmatrix}.$$

The resulting helix is essentially a line of fifths wrapped around this cylinder, with elements positioned every quarter turn, and lining up vertically above each other every four-quarter turn.

From the pitch class exo-skeleton, representations of higher order tonal entities are created by aggregating the positions of their lower level components to generate subsequent helices. For example, triads are represented as the weighted combination of their component pitches, like the following definition for major and minor chords:

$\mathbf{C}_M(k) = w_1 \cdot \mathbf{P}(k) + w_2 \cdot \mathbf{P}(k+1) + w_3 \cdot \mathbf{P}(k+4),$

where $w_1 \geq w_2 \geq w_3 > 0$ and $w_1 + w_2 + w_3 = 1$, and

$\mathbf{C}_m(k) = \omega_1 \cdot \mathbf{P}(k) + \omega_2 \cdot \mathbf{P}(k+1) + \omega_3 \cdot \mathbf{P}(k-3),$

where $\omega_1 \geq \omega_2 \geq \omega_3 > 0$ and $\omega_1 + \omega_2 + \omega_3 = 1,$

and keys are summarized as the weighted combination of their defining chords:

$\mathbf{T}_M(k) = u_1 \cdot \mathbf{C}_M(k) + u_2 \cdot \mathbf{C}_M(k+1) + u_3 \cdot \mathbf{C}_M(k-1),$

where $u_1 \geq u_2 \geq u_3 > 0$ and $u_1 + u_2 + u_3 = 1$, and

$\mathbf{T}_m(k) = \upsilon_1 \cdot \mathbf{C}_m(k) + \upsilon_2 \cdot [\alpha \cdot \mathbf{C}_M(k+1) + (1 - \alpha) \cdot \mathbf{C}_m(k+1)] + \upsilon_3 \cdot [\beta \cdot \mathbf{C}_m(k-1) + (1 - \beta) \cdot \mathbf{C}_M(k-1)],$

where $\upsilon_1 \geq \upsilon_2 \geq \upsilon_3 > 0$ and $\upsilon_1 + \upsilon_2 + \upsilon_3 = 1$.

Figure 8 shows the layered construction of the major key representations. The pitches of each major triad forms a triangle in the interior space as shown in Figure 8(a); the series of major triads, each represented by its weighted centroid (marked by a dot) of its triangle, forms an inner major triad helix. Three adjacent major triads, whose representations outline a triangle as shown in Figure 8(b), uniquely define the pitches of a major key. Again, the major key can be represented as a weighted centroid of its defining triads. The series of major key representations then form yet another inner helix inside the spiral array structure as shown in Figure 8(c). Minor triad and minor key representations are generated in a similar fashion.

The weights are chosen so that distance in the model amongst entities in the same and across hierarchical levels reflects perceived distance between the elements. Further details on the spiral array model, including the selection of these weights, can be found in Chew (2014). The pitch class, chord, and key representations in the spiral array model share attributes with Krumhansl's (1990) pitch class cone, chord space, and key space and Lerdahl's (2001) pitch class cone, chordal space, and regional space—see details in Chew (2008). This is not surprisingly, since the models describe the same musical phenomenon.

KEY FINDING

The center of effect principle in the spiral array can also be used to determine key, among applications ranging from pitch spelling to boundary detection (Chew, 2014). In the spiral array center of effect generator (CEG) key-finding method, any collection of notes can generate a center of effect (c.e.) in the spiral array, for example by calculating the center of gravity of the pitch representations weighted by their respective durations. The key representation in
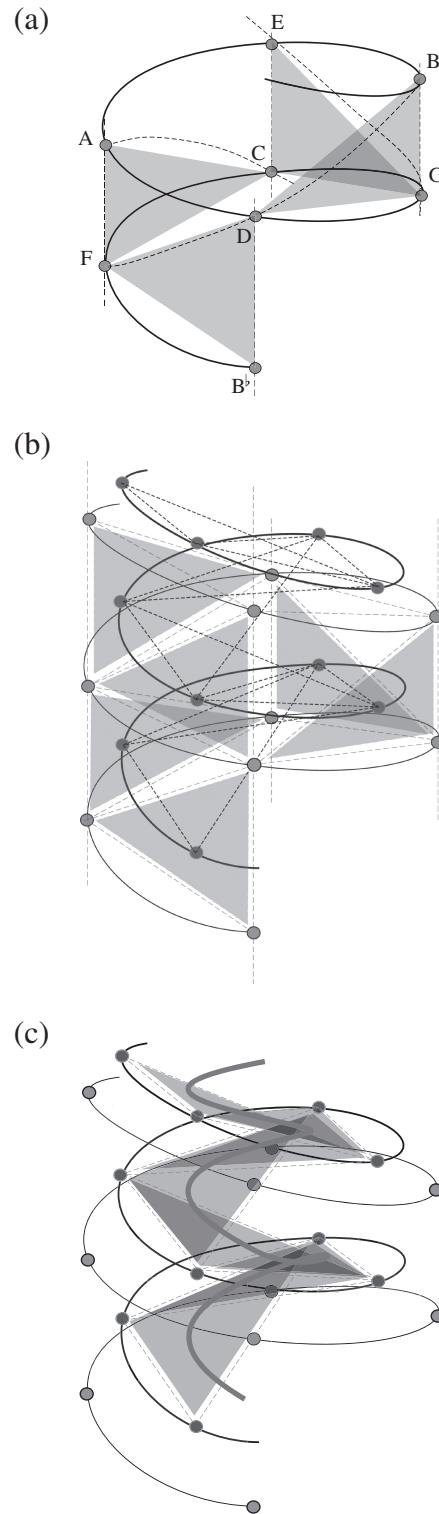

FIGURE 8. Representing tonal elements in the interior of the Harmonic Network helix. (a) From pitch classes to major triads, (b) from triads to keys, (c) the nested pitch class, major triad, major key helices in the spiral array model (figure adapted from Chew 2005).
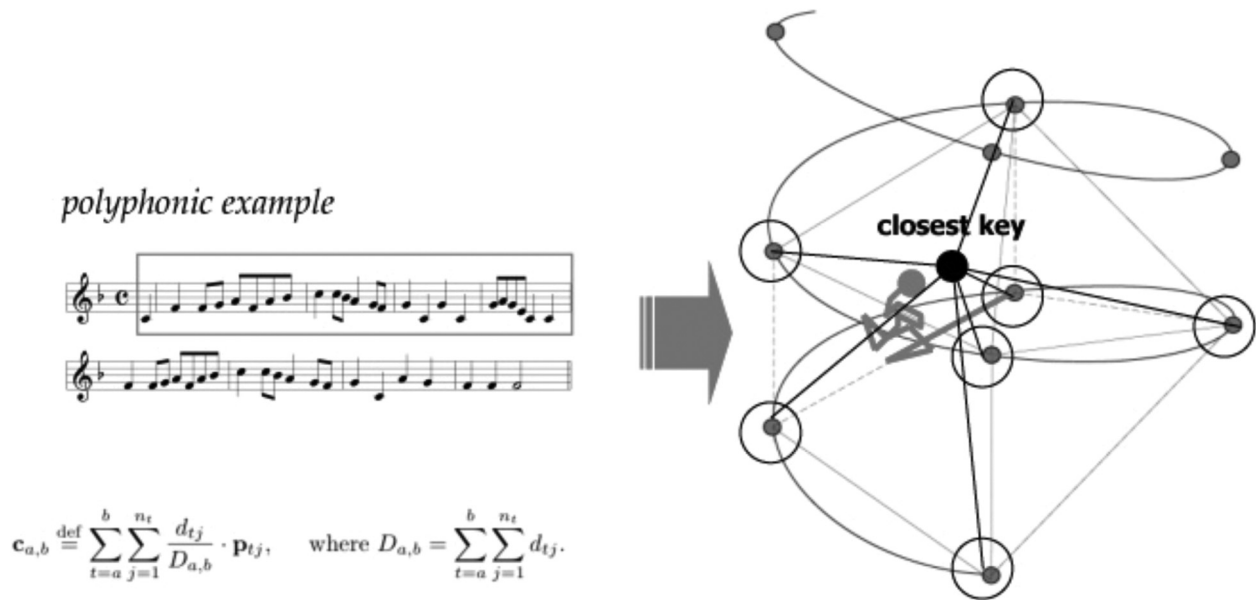
FIGURE 9. The Spiral Array CEG key-finding method.

the spiral array space nearest to this c.e. is chosen as the key of the excerpt.

Figure 9 shows the melody "Simple Gifts." Given a fragment of "Simple Gifts," the notes are mapped to their corresponding representations in the pitch class spiral, and a c.e. of the notes generated by taking the average of the pitch positions weighted by their durations. As increasing numbers of notes from "Simple Gifts" are heard, the c.e. is updated and traces a trajectory inside the spiral array space like that shown on the right side of Figure 9. At any point in time, the process may be stopped, and the key determined by a nearest neighbor search for the closest key representation amongst all major and minor keys.

More formally, if each time slice has n_i notes, and the excerpt of interest is from time a to time b, then the c.e. for the excerpt is computed as:

$$c_{a,b} = \sum_{i=a}^{b} \sum_{j=1}^{n_i} \frac{d_{ij}\mathbf{p}_{ij}}{D_{a,b}}, \text{ where } D_{a,b} = \sum_{i=a}^{b} \sum_{j=1}^{n_i} d_{ij}.$$

If T_k represents the k-th key representation, then the distance minimizing k gives the key of the excerpt: arg-min$_k \parallel c_{a,b} - T_k \parallel$.

Figure 10 shows the evolving distances to the keys F major, C major, F minor, and C minor. As shown in the graph, F major quickly becomes the closest key, with F minor taking second place. As the melody nears the end of the first phrase, which strongly suggests the
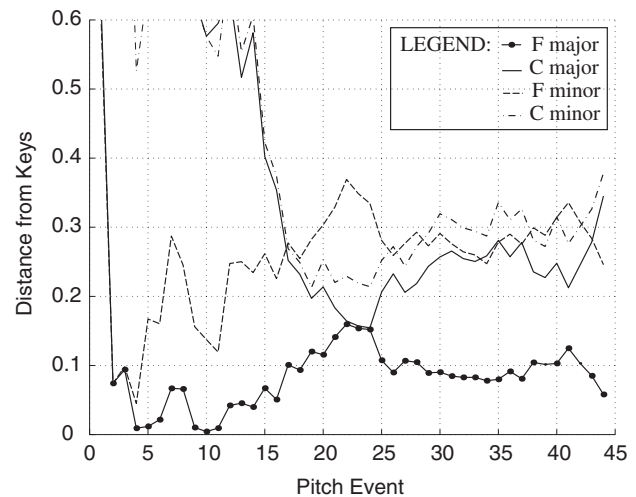

FIGURE 10. CEG key finding results for "Simple Gifts": evolution of distance to keys (figure reproduced from (Chew, 2014, p. 71).

dominant (C major) chord, the key of C major draws near, and vies for first place. When the second phrase begins, F major asserts itself as the closest key again.

The CEG key finding method has been compared favorably to that of Longuet-Higgins and Steedman (1971) and Krumhansl and Schmuckler (as described in Krumhansl, 1990); see Chew (2014). The CEG algorithm and the Krumhansl-Schmuckler method and its variants have been used in both symbolic and audio key
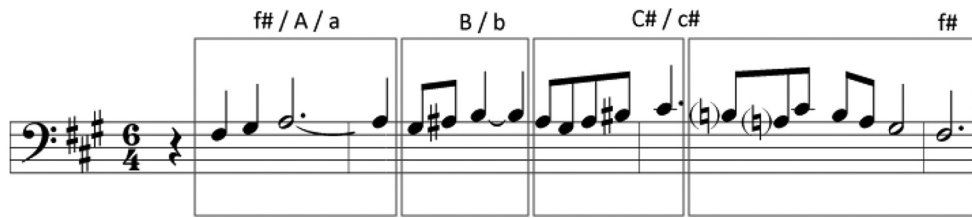
FIGURE 11. Subject of Fugue No. 14 in F♯ minor of Bach's *Well-tempered Clavier Book I.*

finding—see, for example, Aarden (2003), Bellman (2005), Chuan and Chew (2012, 2014), Madsen and Widmer (2007), Noland and Sandler (2006), and Temperley (2007).

EVALUATION OF KEY FINDING

Scientific evaluation remains elusive because of the time-sensitive and interpretive nature of ground truth in key finding. Early evaluation methods used the forty-eight fugue subjects of Bach's *Well-tempered Clavier*, with the key of the fugue subject taken to be the designated key of the fugue, see Longuet-Higgins and Steedman (1971). Mardirossian, Chuan, and Chew (2005) proposed evaluation criteria that allowed for partial credit for closely related key answers, and Chuan and Chew (2012) used the beginnings and ends of pieces with keys designated in their titles.

These methods fail to account for the difficulty that lies at the core of key finding, which is that, depending on the particular parsing (or segmentation) of the notes, one can arrive at a very different answer for the perceived key. It is this mobility of key interpretation that performers manipulate when playing with time. Strategic placement of stress can create boundaries that influence and change a listener's perception of tonality, and by extension the perception of tension as well.

Consider Bach's fourteenth fugue subject from the *Well-tempered Clavier Book I*, shown in Figure 11, for which the CEG required seven and the K-S method eighteen note events to arrive at the key of the fugue, and the shape-matching algorithm only when invoking the tonic-dominant rule after five note events (see Chew, 2014, p. 80). All three algorithms found this to be a "difficult" subject for finding the F♯ minor key, as well they should.

To the human ear, even though F♯ minor is a candidate key after the first three notes, it could just as well be A major or A minor. An A major interpretation would be akin to the starting notes of the popular Disney song from *Frozen*, "Let It Go" (Andersen-Lopez & Lopez, 2012). In fact, because the written-in
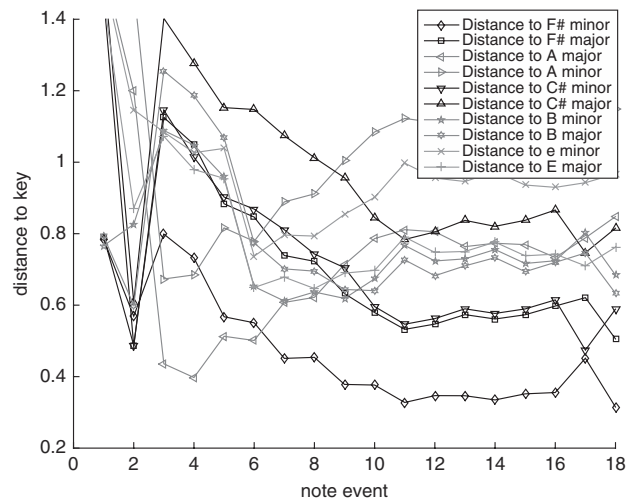

FIGURE 12. Charting distance to various keys for c.e. of unfolding subject of Fugue No. 14 in F♯ minor in Bach's *Well-tempered Clavier Book I.*

stress (by virtue of its length) is on the third note (A), without further information, the interpretation of an A tonic would supplant that of an F♯ tonic. Rather than confirming any one of the initial candidate keys, the next three notes instead posit the new key of B major or minor. The next five notes ratchet it up another notch to push on to the key of C♯ major or minor. Easing back down to naturalize the notes of B♯ and A♯, we finally have confirmation that we are actually in the key of F♯ minor.

The sense of key can evolve quickly, within a span of even a few notes, and at varying timescales, as evidenced by the different box sizes in Figure 11. Using the spiral array weights as defined in Chew (2014, p. 264), we reconsider the CEG algorithm's key finding results for this fugue subject. Figure 12 shows the changing distance to various keys as the melody unfolds, with the list of keys as shown in the legend. F♯ minor first becomes the closest key at the seventh note event. However, from the third to the sixth note event, A major is the clear winner for closest key.
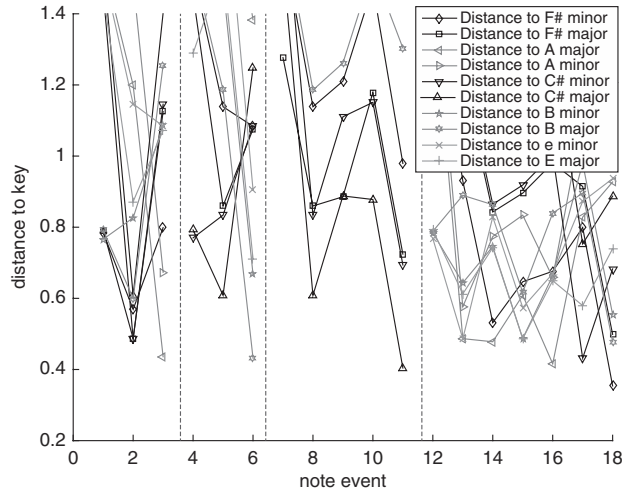
FIGURE 13. Charting distance to various keys for c.e. of segments of fugue subject No. 14.

Suppose the melody is segmented according to the possible key areas as shown in Figure 11, i.e., {(1,3),(4,6),(7,11),(12,18)}. We re-calculate and visualize the results of the CEG algorithm for each segment independently.

The results of spiral array CEG key finding on the segmented melody are shown in Figure 13. The graph shows the distance from the c.e. to a family of related keys, with each line representing the distance from the c.e. to a key as described in the legend. The boundaries are marked by dotted lines, and the model's memory is reset across the boundaries. Following the legend, the closest key at the end of the first segment is A major; that at the end of the second segment is B major; that concluding the third segment is C♯ major; and, the final segment finishes with F♯ minor. This sequence is consistent with the perceived keys described in earlier paragraphs.

These results show that segmentation and grouping are important determinants of keys. They define the time span within which to regard the key context, thus influencing the level of attending. This particular segmentation explains and generates the {A major, B major, C♯ major, F♯ minor} sequence of perceived keys. Thus, a particular conceptualization of key sequence produces a segmentation solution, and a particular segmentation leads to a certain sequence of perceived keys. Herein lies the difficulty with ground truth for key finding.

INFLUENCE OF PERFORMANCE TIMING ON TONAL PERCEPTION

Performed timing can directly influence the segmentation, which in turn impacts tonal perception. Suppose the notes of the subject of Bach's Fugue No. 14 are of equal duration. Lengthening the precise notes Bach did in performance would lead to the segmentation and perception of tonality as discussed in the previous section. In essence, Bach's writing incorporates the tonal groupings he wishes the listener to hear. Subjects like the Bach example are the reason for Palmer and Krumhansl's (1987) finding that pitch and rhythm patterns contributed to perceptions of phrase groupings, but performance timing had little effect on listeners' ratings.

A simple twist to the equal-duration Bach fugue example shows that performance timing can, in fact, be as significant as note durations in influencing perception of tonal grouping. For example, elongating (and thereby stressing) the F♯ in the equal-duration version of the fugue would tilt the perception toward F♯ minor rather than A major.

Figure 14 shows the progression of the closest keys for the first three notes of the F♯ minor fugue when the longest note is the A (the third note event) as written, when the notes are of equal duration, and when the longest note is the F♯ (the first note event). The symbols representing the various keys remain the same as in Figures 12 and 13.
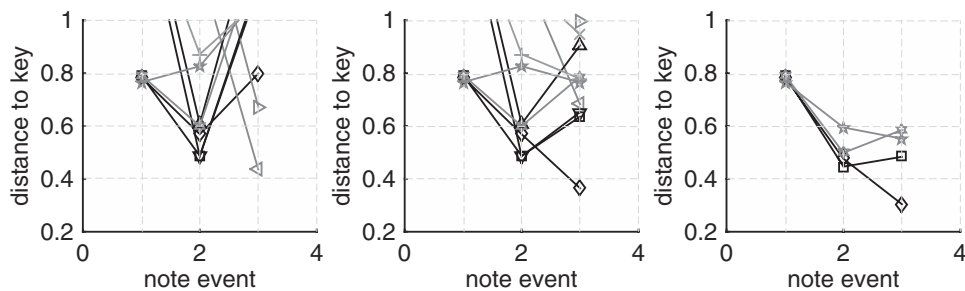

FIGURE 14. Distance to keys for first three notes of the F♯ minor fugue when (a) the longest note is A (the third note event); (b) all three notes are of equal duration; and, (c) F♯ (the first note event) is the longest note (legend as in previous Figures 12 and 13).

When A is the longest note, A major comes to the fore as the closest key by the third note. When the three notes are of equal duration, F♯ major and C♯ minor vie for first place on the second note event, before F♯ minor wins as the closest key on the third note event. When F♯ is the longest note (as long as the A in the original score), then the only true contenders for the closest keys are F♯ minor and major, and C♯ minor and major, with F♯ minor winning out by the third note, even closer in distance than in the equi-duration version.

Thus, the model reflects what one would expect to see when note durations change, which is the effect of performance timing, and its potential for impacting tonal perception.

Key Perception Dynamics

Realistic models of key finding need to take into account dynamic variations in key perception. This section reviews the MuSA.RT visualization system and its algorithm for continuous assessments of key, and applies the system to visual analysis of the cadential tipping point examples.

VISUALIZING KEY EVOLUTION

Chew and François (2005)'s MuSA.RT (Music on the Spiral Array . Real-Time) system animates the three-dimensional spiral array, computing and visualizing continuous key and chord information. The spiral array model can be used not only for key finding, but also for chord tracking. The principle of mapping the notes that are active at any given time to their corresponding positions in the pitch class helix, and updating the c.e. with this new information, and determining the latest key (or chord) via a nearest neighbor search remains the same.

The system maintains two c.e.'s for summarizing the current tonal context for key finding and chord tracking, respectively. Because key tends to evolve over a longer time span and chords change more quickly, the c.e. for key finding is more stable and less influenced by new pitches that are heard, while the c.e. for chord tracking must react more quickly to new information so as to be able to more accurately determine the current chord.

Formally, the system tracks the local key context using a long-term memory c.e.:

$$CE_\alpha(t) = \alpha \cdot c_{t-1,t} + (1 - \alpha) \cdot CE_\alpha(t - 1),$$

for some small α, and the local chord region using a short-term memory c.e.:

$$CE_\beta(t) = \beta \cdot c_{t-1,t} + (1 - \beta) \cdot CE_\beta(t - 1),$$

where $\beta \gg \alpha$. The closest keys and chords are identified through a nearest-neighbor search for the closest key and chord representations, respectively.

A screenshot of the MuSA.RT display can be seen in Figure 15. The outermost light grey helix has pitch classes (with pitch names) marked at every quarter turn. Rather than showing the major and minor triad helices, major triads appear as upward triangles, as in Figure 15(a), and minor triads appear as downward triangles, as in Figure 15(b). In the software, the major triad triangles are colored a light red, and the minor triad triangles are colored a light blue. The root of the current triad is further highlighted by a ball of the corresponding color at the end of a spoke radiating out from the appropriate pitch representation: a light colored ball attached to C for the C major triad (with "C" highlighted) in Figure 15(a), and a light colored ball attached to F♯ (with "F♯" highlighted) for the F♯ minor triad in Figure 15(b).

At the core of the model is the double helix of the major and minor key representations, colored red and blue in the software, respectively (here the minor key helix is the darker one). The closest key is shown as a ball, of the corresponding color, on the major or minor key helix. The size of this ball is inversely proportional to the distance from the long-term memory c.e. to the key representation. The key is also marked by a dodecohedron, of the corresponding color, at the end of the spoke radiating out from the tonic pitch representation—"C" in Figure 15(a), and "A" in Figure 15(b).

Two traveling balls inside the model trace the trajectories of the evolving long-term and short-term c.e.'s; the lighter (lilac in the software) ball, representing the short-term c.e., updates (and moves) faster, while the darker (purple) ball, representing the long-term c.e. updates (and moves) more slowly. The current chord is determined by finding the triangle centroid (a point on a chord helix) closest to the short-term c.e.; the current key is determined by finding the key representation (on the double helix) closest to the long-term c.e.

The visualizations in this section were created by playing the appropriate music passages on a piano with a Moog piano bar that turns each keystroke into MIDI information. The MuSA.RT software reads in the MIDI information, maps the note numbers to pitch names using a pitch spelling algorithm (Chew & Chen, 2005), updates the c.e.'s to create the long-term and short-term memory trajectories inside the model, and computes the closest chords and keys. The display is updated accordingly to show the current state of the system. In auto pilot mode, the model spins smoothly to put the current chord triangle in the background
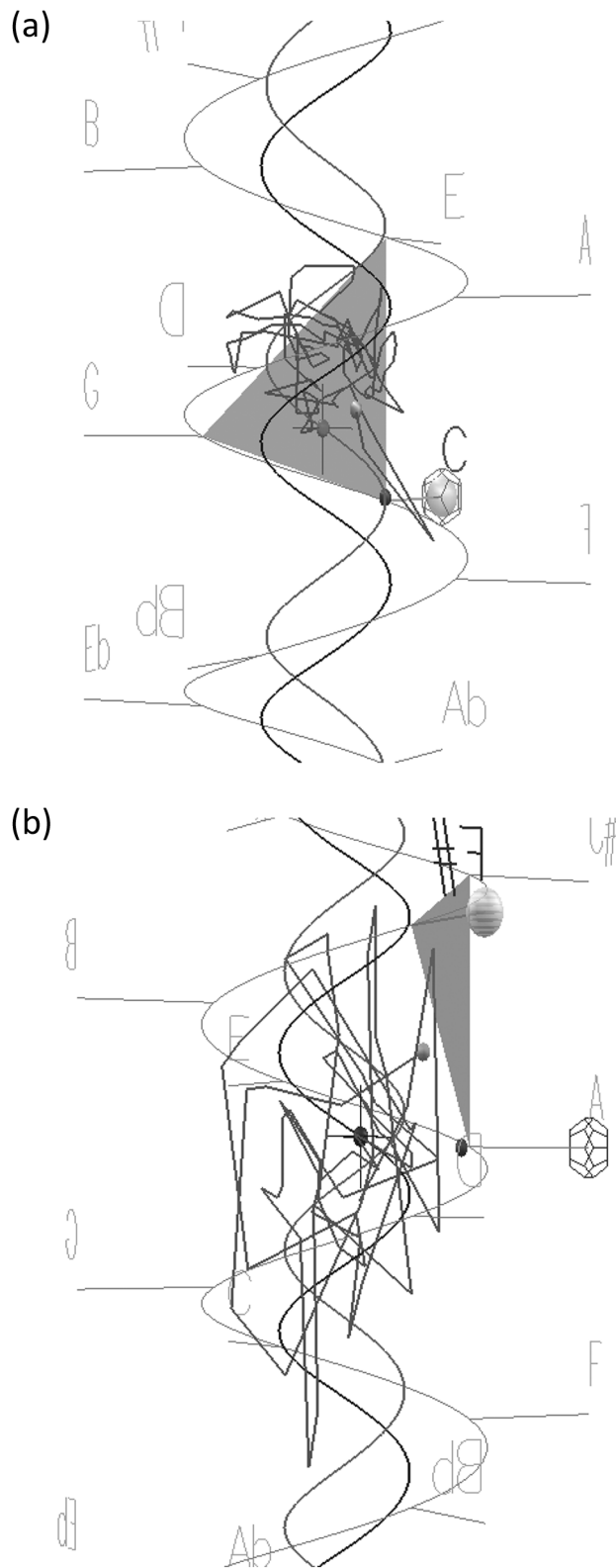
(a)

(b)

FIGURE 15. MuSA.RT screenshots of (a) expected tonal patterns
(Bach's "Prelude No. 1 in C major" from *WTC II*), and (b) unexpected
tonal patterns (Bach's "Prelude No. 20 in A minor" from *WTC II*).

behind the c.e. trajectories; in manual mode, the user
can turn and tilt the model, and zoom in and out, to
obtain the best view. A video describing how MuSA.RT
works can be seen at youtu.be/4GPwVNPuKuA.

In MuSA.RT, expected tonal patterns trace close-knit
trajectories like that shown in Figure 15(a), a visual
analysis of the initial bars of Bach's "Prelude No. 1 in
C major" from the *Well-tempered Clavier Book II*.
Unexpected tonal sequences make wild leaps to far-
ranging regions: Figure 15(b) gives an example of unex-
pected (highly chromatic) tonal patterns that hint at
numerous other keys in the initial bars of Bach's "Pre-
lude No. 20 in A minor," also from the *Well-tempered
Clavier Book II*. This kind of distinction between tonal
clarity and ambiguity is also explored in (Toiviainen,
2005) using high and low contrast color maps.

Inspired by Huron (2004)'s analysis of P. D. Q. Bach's
humor devices, many of which have to do with viola-
tions of expectations, Chew and François (2009) used
MuSA.RT to track and visualize unexpected tonal devia-
tions in selected pieces from the *Short-tempered Clavier*.
More recently, MuSA.RT was used to visualize chord
patterns in the theme and variations of Sergei Rachma-
ninoff's *Rhapsody on a Theme of Paganini for Piano
and Orchestra* (1934), and in particular to visualize the
wide ranging tonal meandering at the end of the seven-
teenth variation before arriving at the stability of the
new key of D♭ in the famous eighteenth variation (see
youtu.be/vsuFtEoqync).

Alexandre François has created a newer version of the
MuSA.RT software, called MuSA_RT, which uses
a slightly different method for updating the c.e.'s, which
can be downloaded freely from the Mac App Store (itu-
nes.apple.com/gb/app/musa-rt/id506866959?mt=12).

TONAL EVOLUTION IN CADENTIAL TIPPING POINTS

In the iconic perfect cadence, the dominant resolves to
the tonic; this is one of the classic idioms of Western tonal
music. Cadential tipping points emphasize the dominant
function, concentrating the inherent tension of the dom-
inant. In a short-term memory model, as the dominant is
elongated, and more of the past is forgotten, the present
becomes increasingly focused on the dominant. Thus,
a cadential tipping point, by prolonging the dominant,
serves to intensify the dissonance of the dominant and
the associated musical tension.

Consider MuSA.RT's analysis of the cadenza in
Strauss' "Burleske" (score pictured in Figure 5); the
significant moments of the visualization are shown in
Figure 16. Figure 16(a) shows the state of the visual
display at the point of the first (minor) tipping point.
Despite the predominance of the emphatic octave A's,

the pitches surround the key of D, making a tight tangle in the region of D minor. The intervening passage alternates between the leading-tone diminished seventh chord and the A major chord, as shown in Figure 16(b). Finally, at the main tipping point, only the A major chord sounds, the longer the A is held in the right hand, the clearer the A major tonality is heard. Eventually, as all the past fades away, only the A major chord is left, as shown in Figure 16(c). Following the tip, the D minor chord (completing the perfect cadence) and the key of D minor returns, and together with it the lyrical theme. The V-I transition from A major to D minor is highlighted in Figure 16(d).

The time taken at the tipping point allows the sound and memory of previous tones to dissipate, leaving bare the essence of the cadence, the dominant, before tipping into the tonic. The clarity of the dominant-tonic transition also allows the new section in the tonic to begin afresh. The cadential tipping point thus serves the function of clarifying, of distilling, the essential structure and function of the perfect cadence.

Turning to MuSA.RT's analysis of the "Epilogue" from Child's *Doubles* (score pictured in Figure 1); the moments just preceding and immediately after the tip are shown in Figure 17. The opening bars trace a butterfly shape in the spiral array space as shown in Figure 17(a). The tonality is amorphous, as demonstrated by the widely straying paths. The first sign of a clear tonal harmony occurs at the tipping point—on the V (B♭) chord of the perfect cadence as flagged in Figure 17(a)). The tip on the dominant leads to closure of the cadence on the I (E♭) chord, thus confirming the E♭ key of the piece. A trace of the V chord leading to the I is shown in Figure 17(b).

It is worth noting that if the dominant is sufficiently prolonged, it would seem like a new tonic, and thus reduce the tension. As can be seen in Figure 16(c), I had held the "A" just a little too long in the "Burleske" example so that the tonic had tipped from "D" over to "A" (hint: look for the dodecohedron). In Figure 17, the tonic stays at E♭, thus sustaining the attraction and tension of the V-I progression. In effect, the performer must strike a balance of holding the dominant just long enough for suspense to peak, yet not long enough for the tension to dissipate. The goal is to play the edge, to maximize the suspense, without losing the tension.

Conclusions and Discussion

In this article, I have introduced the concept of tipping points in music, and considered how they relate to performance timing. Tipping points constitute an acute form of timing deviation characterized by extreme pulse elasticity. While local tipping points can occur fairly frequently, global tipping points are quite rare in the scheme of a performance. Tipping points leverage and enhance inherent music structures; exploiting the certainty of continuation, tipping points derail the pulse and generate instability, thereby creating tension. Several examples illustrated different kinds of global and local tipping points: melodic tipping points, phrase boundary tipping points, and cadential tipping points.

Focusing on cadential tipping points, which play on the dominant-tonic attraction in a perfect cadence, we applied the MuSA.RT tonal analysis and visualization system to examples of cadential tipping points. We have described and shown how these extreme timing deviations serve to clarify and focus the dominant function in the dominant-tonic structure of the perfect cadence, drawing out the dissonance and escalating the tension of the dominant in preparation for the release of the tonic arrival. The analyses revealed the fragility of the balance between instability and stability in tipping points; if the dominant is overly prolonged, it then becomes the tonic, and the tension is lost.

In the course of the discussions, we have also touched upon perceived grouping as a source of divergence in tonal perception. Not only can the same score give rise to different interpretations of structure (and hence to different performances), the same performance can generate divergent perceptions of structural grouping. When considering disparate structural analyses of the same improvised performances, Smith, Schankler, and Chew (2014), found that these divergent opinions can be attributed to the listener's attention (musical features to which they are attending), state of information (the degree to which the listener knew the performance), expectation (for example, whether sections should be of similar sizes), and ontological commitment (what the listener considered a unit, say, 'A' vs. 'B') at the outset of the piece. Because there can be multiple plausible segmentations of any piece of music or performance, there can also be multiple interpretations of key.

We have focused on the dynamics of tonal perception in the article, addressed predominantly by short-term memory models. An aspect of tonal perception not addressed in this paper is that of sudden tonal shifts. In Chew and Chen (2005), it was found that simply having a short-term memory window was not sufficient to capture sudden tonal shifts for pitch spelling. Thus, a two-step bootstrapping method, allowing for the influence of a local contextual reset (spelling without influence of previous keys), was proposed to allow for abrupt key changes. The presence of sudden shifts in
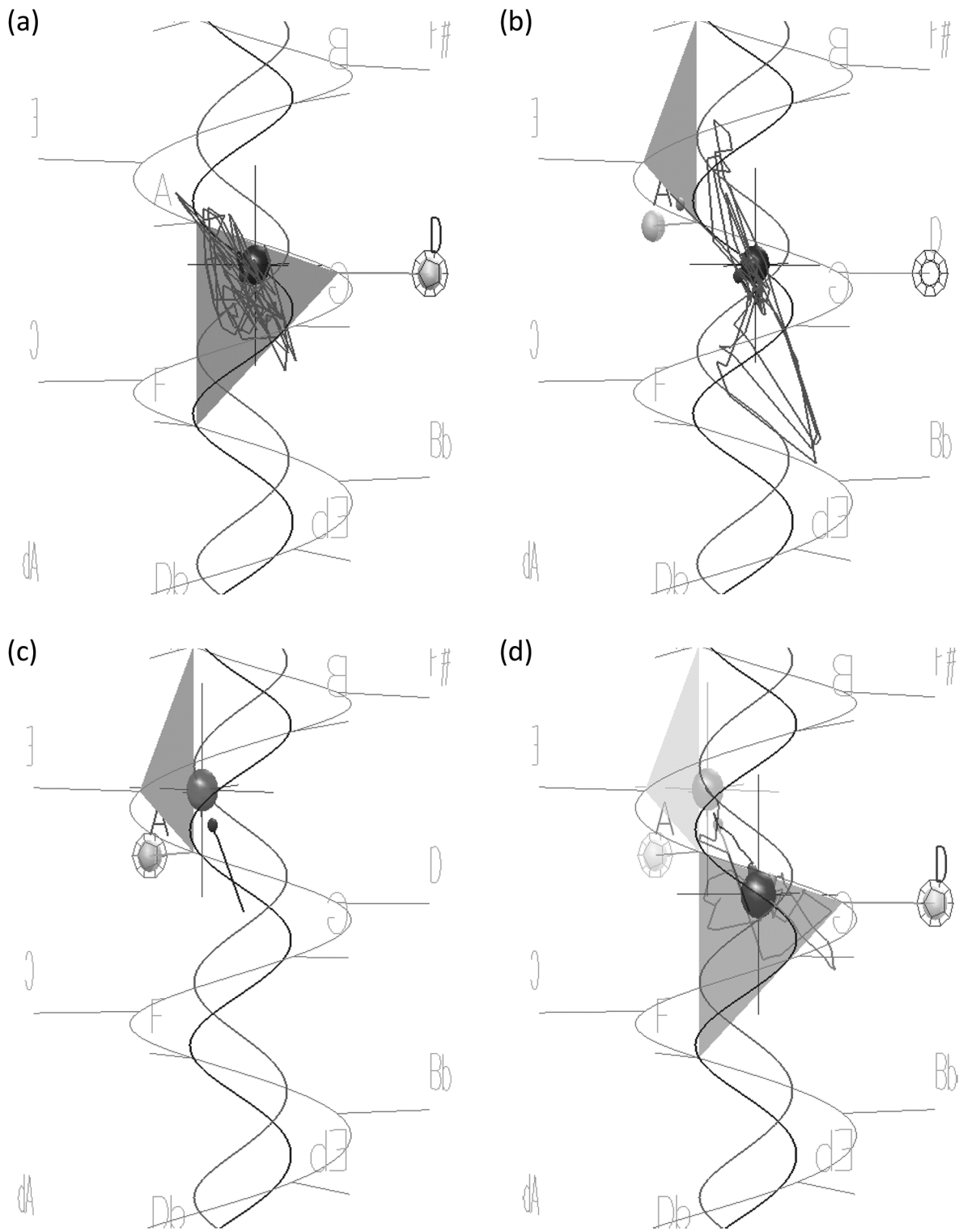
FIGURE 16. MuSA.RT analysis of cadenza from Strauss' "Burleske": (a) at the first (minor) tipping point; (b) see-saw interlude between tipping points; (c) at the main tipping point; (d) back into lyrical theme in D minor.
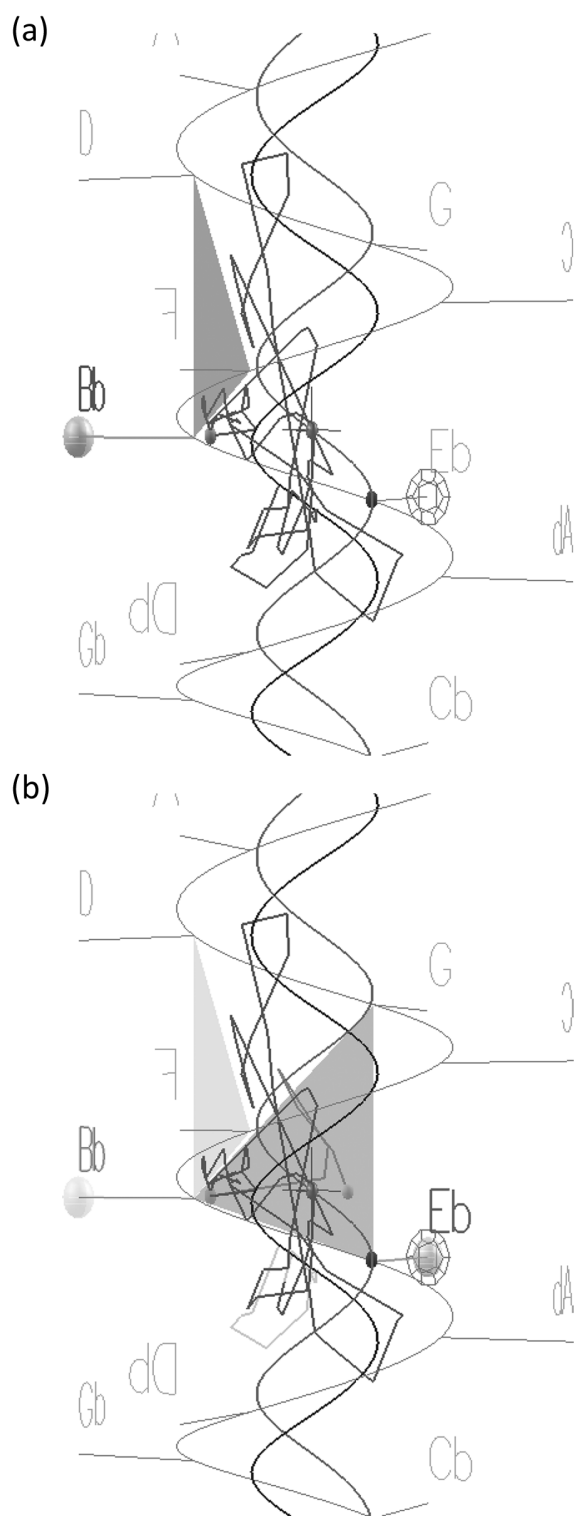
FIGURE 17. MuSA.RT analysis of the "Epilogue" from Child's *Doubles*: (a) at the tipping point on the dominant; (b) after the tip, the E♭ major key is confirmed.

perceived tonality points to the need for a reset or memory wipe function in systems for tracking the dynamics of tonal perception, like the one implemented in the Mimi system, see Schankler, Chew, and François (2014). Performance timing also serves to clarify these transitions from one key to the next, as evidenced in Chew (2012) where Schnabel's tempo arcs are shown to consistently outline and reinforce the tonal groupings of the opening fifteen bars of Beethoven's Moonlight Sonata (Op. 27, No. 2).

As pointed out in Palmer and Hutchins (2006), performance timing serves the function of clarifying tonal structure. This points to the opportunity for defining a new paradigm for key finding (and in fact for music analysis in general) using the structures projected in performance. Performance timing can provide structural constraints, the segmenting of music into tonally meaningful chunks, for key finding algorithms. In Smith and Chew (2013), listeners' annotations of musical structure served as the basis of extracting information on features to which they were attending in order to explain their particular analysis. In the context of tonal perception, segmentations obtained from performances can provide clues to the performer's perceived groupings. As performers can have divergent perceptions of groupings and structure, not only can this approach provide information on the timespan of the performer's attention, it can also reveal alternate "right" answers for key finding.

The focal point of these discussions has been the creativity inherent in perception, in performance, and in listening. Tipping points serve as an extreme example of moments of creative interpretation in performance timing motivated by the need to derive, architect, and project meaningful processed information, such as tonality, in performance. Tipping points, and other moments of musical sense making, can be linked to the creativity of the performer's perceptions, and to that of listeners' perceptions. Differences that arise from the dynamics of information processing calls for new paradigms for scientific models that focus on meaningful differences in individual perception, performance, and listening, to complement existing models that tend toward grand aggregations that focus on large-scale patterns and trends.

Author Note

I first introduced the concept of tipping points in music performance in a talk at the Performance Studies Network 2 conference in Cambridge in 2013. The tipping point analogy in music has been further explored in

a number of lectures—including at the Milestones in Music Cognition (BKN) symposium in Montreal (see www.music.mcgill.ca/bkn25/videos.php#Chew), and at the International Congress on Music and Mathematics (ICMM) in Puerto Vallarta (see vimeo.com/ 112980119). A short proceedings paper (Chew, in press) defined tipping points and provided three illustrative examples, which are re-visited in this article. I am grateful to colleagues who have commented on this

work at various presentations; in particular, Simon Dixon, for noting the link between tipping points and information.

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References

AARDEN, B. (2003). *Dynamic melodic expectancy* (Unpublished doctoral dissertation). Ohio State University, Columbus, OH.

ANDERSEN-LOPEZ, K., & LOPEZ, R. (2012). *Let it go* (Idina Menzel, Singer). Burbank, CA: Hollywood Records.

ARGERICH, M., & ABBADO, C. (1992). *Strauss Burleske for piano and orchestra, New Year's Eve concert.* Germany: Berliner Philharmonika, Arthaus.

BAROLSKY, D. (2007). The performer as analyst. *Music Theory Online, 13*(1). Retrieved from www.mtosmt.org/issues/ mto.07.13.1/mto.07.13.1.barolsky.html

BATTLE, K., BOSTON POPS ORCHESTRA, & WILLIAMS, J. (1989) O mio babbino caro from Gianni Schicchi by Puccini. In *Evening at Pops*, a joint production of the Boston Symphony Orchestra and WGBH-Channel 2. Boston, MA Public Broadcasting Service.

BELLMAN, H. (2005). About the determination of key of a musical excerpt. *Proceedings of the International Conference on Computer Music Modeling and Retrieval* (CMMR) (pp. 187-203). Pisa, Italy: CMMR.

BIGAND, E., & PARNCUTT, R. (1998). Perceiving musical tension in long chord sequences. *Psychological Research, 62*, 237-254.

CALLAS, M., PHILHARMONIA ORCHESTRA, & SERAFIN, T. (1997/ 2007). Gianni Schicchi: O mio babbino caro (ASIN B001ISB5L6). EMI Classics Records.

CANNAM, C., LANDONE, C., & SANDLER, M. (2010). Sonic Visualiser: An Open Source Application for Viewing, Analysing, and Annotating Music Audio Files. In A. del Bimbo & S.-F. Chang (Eds.), *Proceedings of the ACM Multimedia 2010 International Conference* (pp. 1467-1468). Firenze, Italy: ACMMIC.

CHEW, E. (2000). Towards a mathematical model of tonality (Unpublished doctoral dissertation). Massachusetts Institute of Technology, Cambridge, MA.

CHEW, E. (2005). Regards on two regards by Messiaen: Post-tonal music segmentation using pitch context distances in the spiral array. *Journal for New Music Research, 34*(4), 341-354.

CHEW, E. (2008). Out of the grid and inside the spiral— Geometric interpretations of and comparisons with the spiral array model. *Computing in Musicology, 15*, 51-72.

CHEW, E. (2009). Epilogue. In *Doubles: Music of Peter Child* (ASIN: B00000DH69). Albany, NY: Albany Records.

CHEW, E. (2012). About time: Strategies of performance revealed in graphs. *Visions of Research in Music Education 20* [Online journal]. Retrieved January 5, 2016 from www-usr.rider.edu/ ~vrme/v20n1/visions/Chew%20Bamberger%20.pdf

CHEW, E. (2014). *Mathematical and computational modeling of tonality.* New York: Springer.

CHEW, E. (in press). The mechanics of tipping points: A case of extreme elasticity in expressive timing. *Proceedings of the International Congress on Music and Mathematics.*

CHEW, E., & CALLENDER, C. (2013). Conceptual and experiential representations of tempo: effects on expressive performance comparisons. In J. Yust, J. Wild, & A. Burgoyne (Eds.), *Mathematics and computation in music: Fourth international conference* (pp. 76-87). Berlin/Heidelberg: Springer Verlag GmbH.

CHEW, E., & CHEN, Y.-C. (2005). Real-time pitch spelling using the spiral array. *Computer Music Journal, 29*(3), 61-76.

CHEW, E., & FRANÇOIS, A. R. J. (2005). Interactive multi-scale visualizations of tonal evolution in MuSA.RT Opus 2. In N. Lee (Ed.), *Special Issue on Music Visualization and Education, ACM Computers in Entertainment, 3*(4) [Online journal].

CHEW, E., & FRANÇOIS, A. R. J. (2009). Visible humour —Seeing P. D. Q. Bach's musical humour devices in the short-tempered clavier on the spiral array space. In T. Noll & T. Klouche (Eds.), *Mathematics and computation in music—First international conference* (pp. 11-18). Berlin, Heidelberg: Springer.

CHUAN, C.-H., & CHEW, E. (2012). Creating ground truth for audio key finding: when the title key may not be the key. In P. Herrera, L. G. Martins, & M. Mueller (Eds.), *Proceedings of the International Conference of the Society for Music Information Retrieval* (pp. 247-252). Porto, Portugal: ISMIR.

CHUAN, C.-H., & CHEW, E. (2014). The KUSC classical music dataset for audio key finding. *International Journal of Multimedia and Its Applications, 6*(4), 1-18.

COHN, R. (1998). Introduction to Neo-Riemannian theory: An introduction and a historical perspective. *Journal of Music Theory, 42*(2), 167-180.

FARBOOD, M. M. (2012). A parametric, temporal model of musical tension. *Music Perception, 29*, 387-428.

FRIEDRICKSON, W. E., & JOHNSON, C. M. (1996). The effect of performer use of rubato on listener perception of tension in Mozart. *Psychomusicology, 15,* 78-86.

GLADWELL, M. (2000). *The tipping point: How little things can make a big difference.* New York: Time Warner Trade Publishing.

GOULD G., & GOLSCHMANN, V. (1966/1990). Strauss Burleske for piano and orchestra. In *Mostly Strauss and Rhapsodic Interludes, the Glenn Gould collection* (Vol. 5). New York: SONY.

HATTEN, R. S. (2014). Performing expressive closures in structurally open texts: Chopin's Prelude in A minor and the last two dances of Schumann's *Davidsbündlertänze. MTO Online, 20*(4). Retrieved January 5, 2016 from www.mtosmt.org/issues/mto.14.20.4/mto.14.20.4.hatten.html

HEPOKOSKI, J., & DARCY, W. (2006). Elements of sonata theory: Norms types and deformations in the late eighteenth century sonata. New York: Oxford University Press.

HURON, D. (2004). Music-engendered laughter: An analysis of humor devices in P. D. Q. Bach. In S. D. Lipscomb, R. Ashley, R. O. Gjerdingen, & P. Webster (Eds.), *Proceedings of the 8th International Conference on Music Perception and Cognition* (pp. 700-704). Evanston, IL: ICMPC.

HURON, D. (2006). Sweet anticipation: Music and the psychology of expectation. Cambridge, MA: MIT Press.

KIRI TE KANAWA, LONDON PHILHARMONIC ORCHESTRA, & PRITCHARD, J. (1984). Gianni Schicchi: O mio babbino caro. In *Kiri te Kanawa Sings Verdi and Puccini Arias* (ASIN: B00138F6D4). New York: SONY Classical.

KREISLER, F., & RUPP, F. (1936/1993). *Schön Rosmarin* (1993 Digital Remaster) in Kreisler plays Kreisler. Holland: EMI Classics.

KRUMHANSL, C. L. (1990). *Cognitive foundations of musical pitch.* Oxford: Oxford University Press.

KURTH, E., & ROTHFARB, L. A. (1991). *Ernst Kurth: Selected writings.* Cambridge, UK: Cambridge University Press.

LAMBERSON, P. J., & PAGE, S. E. (2012). Tipping points. *Quarterly Journal of Political Science, 7,* 175-208.

LEECH-WILKINSON, E., & PRIOR, H. (2013). Heuristics for expressive performance. In D. Fabian, R. Timmers, & E. Schubert (Eds.), *Expressiveness in music performance: empirical approaches across styles and cultures* (pp. 34-57). Oxford, UK: Oxford University Press.

LERDAHL, F. (2001). *Tonal pitch space.* Oxford, UK: Oxford University Press.

LERDAHL, F., & KRUMHANSL, C. L. (2007). Modeling tonal tension. *Music Perception, 24,* 329-366.

LONGUET-HIGGINS, H. C. (1962a). Letter to a musical friend. *Music Review, 23,* 244-248.

LONGUET-HIGGINS, H. C. (1962b). Second letter to a musical friend. *Music Review, 23,* 271-280.

LONGUET-HIGGINS, H. C., & STEEDMAN, M. (1971). On interpreting Bach. *Machine Intelligence, 6,* 221-241.

MADSEN, S., & WIDMER, G. (2007). Key-finding with interval profiles. *Proceedings of the International Computer Music Conference* (pp. 212-215). Copenhagen, Denmark: ICMC.

MARDIROSSIAN, A., CHUAN, C.-H., & CHEW, E. (2005). *MIREX audio and symbolic key finding* [Online report]. Retrieved January 5, 2016 from www.music-ir.org/mirex/wiki/2005: Audio_and_Symbolic_Key

NOLAN, K., & SANDLER, M. (2006). Key estimation using a hidden Markov model. In K. Lemström, A. Tindale, R. Dannenberg (Eds.): *Proceedings of the International Conference on Music Information Retrieval.* Victoria, Canada: ISMIR, pp. 121-126.

PALMER, C., & HUTCHINS, S. (2006). What is musical prosody? *Psychology of Learning and Motivation, 46,* 245-278.

PALMER, C., & KRUMHANSL, C. L. (1987). Pitch and temporal contributions to musical phrase perception: Effects of harmony, performance timing, and familiarity. *Perception and Psychophysics, 41,* 505-518.

RACHMANINOFF, S. (1934). *Rhapsody on a theme of paganini for piano and orchestra* (Op. 43). London, UK: Boosey & Hawkes.

RICHTER, S., & GEORGESCU, G. (1961/2013). *Best of live performances at Enescu festival 1961* [CD]. Romania: Casa Radio.

SCHANKLER, I., CHEW, E., & FRANÇOIS, A. R. J. (2014). Improvising with digital auto-scaffolding: How Mimi changes and enhances the creative process. In N. Lee (Ed.), *Digital da Vinci—Computers in music* (pp. 99-125). New York: Springer.

SMITH, J. B. L., & CHEW, E. (2013). Using quadratic programming to estimate musical attention from self-similarity matrices. In A. Jaimes, N. Sebe, & N. Boujemaa (Eds.), *Proceedings of ACM Multimedia* (pp. 113-122). Barcelona, Catalunya, Spain: ACM Multimedia.

SMITH, J. B. L., SCHANKLER, I., & CHEW, E. (2014). Listening as a creative act: meaningful differences in structural annotations of improvised performances. *Music Theory Online, 20*(3). Retrieved January 5, 2016 from www.mtosmt.org/issues/mto.14.20.3/mto.14.20.3.smith_schankler_chew.html

STRAUSS, R. (1894). *Burleske für pianoforte und orchester.* Leipzig: Steingräber.

TEMPERLEY, D. (2007). *Music and probability.* Cambridge, MA: MIT Press.

TOIVIAINEN, P. (2005). Visualization of tonal content with self-organizing maps and self-similarity matrices. *ACM Computers in Entertainment, 3*(4). Retrieved from http://cie.acm.org/articles/visualization-of-tonal-content-with-self-organizing-maps-and-self-similarity-matrices/

TOIVIAINEN, P., & KRUMHANSL, C. L. (2003). Measuring and modeling real-time responses to music: Tonality induction. *Perception, 32,* 741-766.