# Open Research Online 

The Open University's repository of research publications and other research outputs

## Coverage Characteristics of Symmetric Topologies for Pervasive Sensor Networks

## Conference or Workshop Item

## How to cite:

Iqbal, Mudasser; Gondal, Iqbal and Dooley, Laurence S. (2005). Coverage Characteristics of Symmetric Topologies for Pervasive Sensor Networks. In: Second IFIP International Conference on Wireless and Optical Communications Networks (WOCN '05), 6-8 Mar 2005, Dubai.

For guidance on citations see FAQs
(c) [not recorded]

Version: [not recorded]
Link(s) to article on publisher's website:
http://dx.doi.org/doi:10.1109/WOCN.2005.1436082
http://ieeexplore.ieee.org/search/wrapper.jsp?arnumber=1436082

Copyright and Moral Rights for the articles on this site are retained by the individual authors and/or other copyright owners. For more information on Open Research Online's data policy on reuse of materials please consult the policies page.

# Coverage Characteristics of Symmetric Topologies for Pervasive Sensor Networks 

\#Mr. M. M. Iqbal, Dr. I. Gondal, Prof. L. Dooley<br>GSCIT, Faculty of IT, MONASH University, Churchill, 3842 Australia<br>[\#Mudasser.Iqbal, Iqbal.Gondal, Laurence.Dooley]@infotech.monash.edu.au


#### Abstract

The success of pervasive computing environments comprising ubiquitous loco-dynamic sensing devices is very dependent upon the Coverage Characteristics (CCs) of the network topology. These characteristics include blanket coverage, network density, affects on surrounding environments and intra-sensor coverage overlaps. This paper presents a systematic mathematical model to quantitatively investigate the effects of CCs and provides a comparison with other well used topologies e.g. Hexagonal, Triangular and Square grid. The paper uses connectivity, density saturation, conflict regions and effectiveness of the topology as quality parameters in simulation studies for a disaster recovery network in various irregular terrains. Numerical as well as simulation results confirm the improved performance of Hexagonal Topology (HT) in terms of the above mentioned quality parameters which can be used to tune the network design to ensure the required QoS throughout the life of the network.


Keywords- Ad-hoc Wireless Sensor Networks, Network Topology, Performance Evaluation, Network Coverage, QoS

## I. INTRODUCTION

A pervasive network is usually composed of numerous locodynamic sensing devices integrating information processing into wirelessly connected everyday objects [3]. These fully distributed sensor/actuator networks are frequently deployed to achieve sophisticated and critical tasks in biological, chemical and physical sensing scenarios. The autonomous operation, limited resources for computation in terms of energy and storage pose the requirement of optimized topology design for these wireless sensor networks operating in the acoustic, seismic, infrared (IR) and electromagnetic [4] modes.
To satisfy these operational requirements, a network backbone consisting of cluster head nodes, known as Parent Nodes (PNs), is established to support the communication across the area under monitoring. A usual strategy is to deploy a planned network backbone, to which stationary and mobile devices connect latter on the fly, in a plug and play manner. The coverage characteristics of the topology used to deploy the backbone are non-trivial to the effectiveness of the whole network. These include the description of dimensions, location and density (number) of sensing and control devices (like PNs), coverage estimation, surrogate localization, interference with surrounding networks and intra-parent conflict regions. These parameters are crucial to achieve the best QoS. Quality is defined as the availability of backbone PNs to maximum sensing devices so that routing and information loss is minimum. Other QoS measures include effective coverage of PNs so that the topology does not become saturated for a particular number of backbone nodes
and the minimum number of conflict zones (where more than one PNs are accessible to a subset of sensing devices) so that computational overhead on the backbone for resolving resource ownership issues is avoided / minimized.
Unfortunately existing techniques rarely discuss the coverage issues at infrastructural level and target the application development techniques on top of an assumed topology. Narayanan [6] developed a load balancing technique with this flaw. Though Krishnendu [7] and Sameera [8] have recognized the importance of considering the coverage characteristics of the underlying topology, but they fail to present generalized measures for quantifying and analyzing the coverage related issues. These techniques result in uncovered areas for different neighboring requirements.

Significant research is done on location tracking without a comprehensive investigation of the affects of coverage characteristics of topology. Trianglular topology is considered by Y. Tseng [2] for agent-based location tracking. The techniques described by H.T. Kung [9] and Congzhou [10] for parent selection without defining the actual coverage parameters appears to be hypothetical. Nirupama [5, 6] assumed a square-grid topology for extension of existing sensor network, but did not define the original topology.

This paper considers the importance of above mentioned coverage characteristics of network topology. A systematic mathematical model is presented in order to comparatively quantify and analyze the area coverage parameters of three different topologies; hexagonal, triangular and square grid. Apart from the general belief that hexagonal topology usually performs better in terms of coverage, our model analyzes the actual impacts of different topologies on the QoS parameters. This provides a clear guideline to the designer to select the best topology parameters. Also the community focusing on developing applications on top of a topology would be able to see the impact of network coverage characteristics on the applications and, therefore, would be able to tune the services according to the support provided by the network backbone.
Rest of the paper is organized as follows: Section II describes the proposed HT model, while Section III mathematically compares the three topologies; Hexagonal, Square and Triangular. Section IV discusses the simulation results, to highlight the comparative QoS provided by each model, and conclusions are presented in Section VI.

## II. Proposed Model

The proposed HT model for analyzing the network coverage characteristics is based upon the geometrical characteristics of hexagon [1][11] that will reduce the overall computational, power and memory requirements. It is assumed that all nodes
in the network backbone have isotropic radial coverage and use radio frequency to communicate with neighboring nodes.

## A. Network Design

Network design is based on the optimal selection of number and locations of Parent Nodes (PNs) in a virtual Hexagonal Topology (HT). The design is optimized to achieve the best QoS by; ensuring the availability of PN to a maximum number of Pervasive Sensors (PS), minimizing Grey Region (GR) areas (to reduce many-hop routing) and minimizing confusion / conflict zones.
In the design, it is assumed that PS-PS and PS-PN communications are RF based and the average range of PS nodes is the sensor $R F$ range $\bar{\gamma}$ which is always greater than the sensing range $(\gamma)$ and is given by:-

$$
\begin{equation*}
\bar{\gamma}=\{k \gamma \mid 2 \leq k \leq M\} \tag{1}
\end{equation*}
$$

Where $M$ is the maximal communication range factor and depends upon the available power and node-to-node distances. Similarly the average communication range of the PN used for PN-PS communication is the parent $R F$ range $\gamma_{p}$. This is tuned so that a PN placed in a Hexagonal Region covers the surrounding hexagonal orbits in an incremental way as shown in Fig. 1A which also illustrates that the RF range of PN must be an odd multiple of the side length $R$ of an HR. $\gamma_{p}$ is given by:

$$
\begin{equation*}
\gamma_{p}=\{\gamma \bar{k} \mid(2 \leq \bar{k} \leq M) \wedge(\bar{k} \bmod 2=1)\} \tag{2}
\end{equation*}
$$

where $M$ is as in (1).
If all the nodes in the network communicate directly with the Central Commanding Infrastructure, the communication load-especially over long distances will quickly exhaust the network's resources. Therefore the network is arranged as a series of decentralized federations, where a federation is a basic cluster that drives the entire parent localization model and defines the norms of the network. Its dimensions, as shown in Fig. 1A, are the concentric hexagonal orbits having a PN at its centre. These orbits are bound to a federation and define its density. It also ensures the availability of PN to the PS nodes in these orbits. The density of a federation is:

$$
\begin{equation*}
F_{d}=\left[1+\sum_{i=2}^{\gamma_{p}-1} 3 i \mid(i \in \text { EvenIntegers }]\right. \tag{3}
\end{equation*}
$$

Federations define some critical network characteristics in terms of PN availability areas, where PS nodes have direct connection to a PN, routing areas where an intermediate routing PS node is required due to lack of PN availability and confusion zones where the presence of more than one PNs confuse a PS node for selection between the PNs [12]. As the federation establishes its control over the area in a localized manner, this setup is scaled up for illustration purposes to a major disaster site in Australia, the Granville train collapse as shown in Fig. 2.
Fig. 1B shows a federation with $\gamma_{p}=3 R, \bar{\gamma}=2 R, F_{d}=7$ and two randomly placed PS nodes A and B. It is assumed that if a bi-directional link exists between PN and PS nodes, they are in-range of each other. Fig. 1B also shows that


Fig. 1. A: Coverage of a PN in the form of Hexagonal Orbits B: Federation Structure
although both PS nodes are within the federation, A is inrange while B is not. The area of PN availability is referred to as the "PN Availability Region (PNAR)" while the area where no PN is available and routing is required is referred to as the GR. After careful analysis of parent availability points and routing points in a federation, it is found that these areas are defined by $\bar{\gamma}$. Numerical estimations of PNAR and GR are discussed in detail in our earlier work [12].

## B. Number of Parent Nodes

The fundamental design principle of the HT model is to achieve maximum coverage across the disaster site with minimum number of PNs. This is achieved by deploying PNs at the regions where maximum neighborhood density can be achieved. The total number of federations required to cover the area under surveillance is equal to the total number of parent nodes required and depends on $\gamma_{p}$. Since the parents are assumed to have nearly identical $\gamma_{p}$, an estimate of the minimum number of parent nodes can be calculated by:

$$
\begin{equation*}
F_{n}=H_{d} / F_{d} \tag{4}
\end{equation*}
$$

Were $H_{d}$ is the density of hexagonal regions in the HT and $F_{d}$ is the density of each federation. $F_{n}$ is the cardinality of federations and defines the minimum number of parents required to provide area-wide communication support to the sensors. Since $H_{d}$ is a key parameter that determines $F_{n}$, its value depends upon the number of regions covering the area to be monitored. As shown in Fig. 2, after the design of hexagonal grid, some regions are turned off which are either outside the subject area or where network component devices cannot be installed at all. The remaining hexagonal regions contribute to the estimation of $F_{n}$, which is further tuned depending upon the network load and number of sensors [12].
After the HT is formed and a communication backbone of parent nodes is established, sensors nodes are deployed in the area, usually in a random fashion, to bring the disaster site swiftly under monitoring and control. The decision on the type of the sensors to deploy depends upon the nature of rescue services solicited. These could include human detection, chemical leakage detection, guiding rescue squad throughout the disaster site etc.

## III. Coverage Characteristics Of The Network

The coverage characteristics are vital for such a network topology in order to describe the performance of the network in terms of non-sporadic monitoring, interference with


Fig. 2 The disaster recovery network simulated for Granville train collapse
surrounding pervasive environments and computational overheads incurred due to issues like ownership resolution and coexistence. This section investigates these parameters for the HT model.

## A. Blanket Coverage ( $B C$ )

The HT model formation mechanism described in the previous subsections: A and B makes a hexagonal grid on the whole surface making sure that no area is left uncovered. Secondly the grid is formed in such a way that hexagons fit together like a jigsaw. Next the PNs are placed by maximizing the neighborhood coverage making sure that whole HT blanket covers the entire area to be monitored. This blanket coverage provided by HT model ensures nonsporadic monitoring of loco-dynamic objects / events at the area coverage level.

## B. Affect on Surrounding

The objective is to minimize the effect of the deployed network on the surrounding network environments. Although disaster site networks are usually adhoc, mobile and stand independent in the area, this issue is also investigated to mitigate any prospective interference. This can be achieved if it can be demonstrated that maximum PNs are placed only within the area to be monitored.
According to the model, the hexagons guide the placement of PNs and specify the regions covered by each PN. Since HT does not take any area into consideration that is beyond the area marked boundary in Fig. 2, It is highly likely that most of the PNs will be placed inside the boundary of the area to be monitored. But there could be some PNs placed on or near the boundary of the area due to the irregularity of the terrain as shown in Fig. 3.
Let $S_{n}^{\prime}$ be the number of boundary PNs with each parent having radius of coverage given by (2), then:

$$
\begin{equation*}
A^{\prime} \geq\left(S_{n}^{\prime}\right)\left(\pi \gamma_{p}^{2}\right) / 2 \tag{5}
\end{equation*}
$$

is the approximate surrounding area that will be affected.
This area can be significantly minimized by:

1. Keeping $\gamma_{p}$ to the minimum possible.
2. Keeping $S_{n}^{\prime}$ at the minimum.

Fact (2) can be illustrated using a "Boundary Distribution Function (BDF)". It is based on the fact that for a given
symmetric topology, the number of boundary PNs is directly proportional to the total number of PNs. BDF investigates a particular number $S_{n}^{\prime}$ of boundary sensors for an area under surveillance of $\mathrm{X}, \mathrm{Y}$ dimensions and total number of sensors $S_{n}$. It is defined as:

$$
B D F\left(S_{n}^{\prime}\right)=1-\left(S_{n}^{\prime} / S_{n}\right)
$$

which implies that:

$$
\begin{equation*}
A^{\prime} \propto B D F\left(S_{n}^{\prime}\right) \tag{6}
\end{equation*}
$$

The simulation results quantify the above relationship by deploying sensors of different ranges in various test scenarios and evaluate the comparative performance of hexagonal, triangular and square grid topologies.

## C. Intra-Parent Coverage Overlaps

Intra-parent coverage overlaps are the common areas which can be monitored by adjacent PNs. These areas are formed by the overlapping radial coverage of adjacent PNs due to sufficiently close distance between them. The overlaps are necessary for continuous monitoring of loco-dynamic objects and events. The amount of overlap affects the overall surveillance performance in the following ways:
Condition 1: Larger the overlap, more PNs will be involved in target monitoring thereby increasing the energy consumption and computational overhead. This reduces the overall network lifetime.
Condition 2: Smaller the overlap more will be the chances of losing certain events / objects which are dynamic in their location.
The above conditions imply that the PNs must be deployed in such a way that the overlap between adjacent sensor coverage areas must be smaller enough so that only a unique subset of PNs is involved in monitoring the target area most of the time. Also the overlap must be larger enough for smooth Handover of monitoring responsibility from one PN to the other, keeping the target under continuous surveillance.
Lemma 1 defines an optimal / minimal intra-PN coverage that avoids both conditions.
Lemma 1: If PNs are placed on a grid of regular, adjacent hexagons, the relation between the coverage radius of a PN $\gamma_{p}=\alpha R$ and the side length of a hexagon must be $\gamma_{p}=R$ in order to achieve minimal intra-parent coverage overlaps.


Boundary PN also covering some area outside the boundary
Fig. 3 Boundary Parent Nodes causing interference in the surrounding environments

Proof : The blanket coverage of HT model implies that the overlap is larger enough that no space is left uncovered. This means that there is no chance that any loco-dynamic object / event is missed from the surveillance thereby avoiding condition 2. But this overlap must be shorter enough that it does not impose a computational overhead on the network.
According to HT model PN placement technique, if a PN of coverage radius $\gamma_{p}=\alpha R$, where $\alpha=1$, is placed in a regular hexagon of circumradius $R$, the relationship between PN coverage radius and hexagon side length that makes the PN cover the whole hexagon and minimizes the extra area covered is given by $\gamma_{p}=R$. Since the PNs deployed by this method cover the least possible extra area on all sides of the hexagonal regions, the overlaps emerging between adjacent hexagons will be minimum too. In this case, an event occurring outside the overlapping area will be monitored by exactly one PN while those occurring within the overlap will be monitored by the two PN in the adjacent hexagons as shown in Fig. 4. In this way the HT model avoids condition 1 as well and uses the overlap region for Handover. The amount of total extra coverage with six adjacent hexagons is given by: $\quad \beta_{h}=\pi-3 \sqrt{3} / 2$
Therefore overlap area between two adjacent hexagons is given by:

$$
\beta_{h}^{\prime}=2 \beta_{h} / 6=\beta_{h} / 3
$$

## iv. Comparison With Other Models

The performance of coverage characteristics of HT model presented in the previous section has been compared with triangular and square grid models. This section presents the performance comparison of the triangular and square models with the hexagonal model on numerical grounds.

## A. Sensors Density

Krishnendu et al [7] placed nodes on alternate square grid points assuming that each node can cover four adjacent grid points including the central one (Fig. 5c). But their basic density formula did not explicitly include node coverage. A modified density formula that includes the range factor which is:

$$
S_{n}^{p} \leq(2 n+1) K^{p}(n, 2)
$$

where $K^{p}(n, 2)$ is the minimum number of code-words which are candidates for node placement. This technique places nodes in symmetric way but does not explain how this placement will be able to monitor the whole area. Also this technique puts a restriction of highly symmetrical deployment of nodes which is not a viable solution in emergency situations.
The proposed HT model was modified for placing nodes on a square, instead of a hexagonal grid. Mathematically, let $\gamma$ is the circumradius of a radial coverage that inscribes a hexagon of side length $R$ and a square of side length $E$. Let $A$, $H$ and $S$ be the areas of the circle, hexagon and square respectively then: $\quad A=\pi \gamma^{2}, H=\frac{3 \sqrt{3}}{2} R^{2}, S=E^{2}$


Figure 4. Intra-Parent coverage overlaps
The relationship between $\gamma, R$ and $E$ is given by:

$$
\gamma=R, \quad \gamma=\frac{E \sqrt{2}}{2}
$$

Therefore:

Since

$$
A=H+\omega_{h}, A=E+\omega_{s}
$$

$\Rightarrow$

$$
\omega_{h}=\frac{2 \pi-3 \sqrt{3}}{2}, \omega_{s}=\frac{\pi-2}{2}
$$

$\Rightarrow \quad \omega_{h}<\omega_{s}$
$\Rightarrow \quad H>E$
It means that large number of squares will be required to blanket cover the disaster site resulting in greater number of sensors for square grid than that required for the HT model.
Triangular modal described by Y. C. Tseng et al [2], divides the whole area into equilateral triangular regions. PNs of coverage radius equal to the side length of a triangle are placed at each vertex of the triangle (Fig. 5A). Since a hexagon is formed by six equilateral triangles, PNs can also be placed according to HT model as shown in Fig. 5B. The significant difference in the number of PNs is due to the high level of redundancy caused by Triangular model. Although it makes system more fault-tolerant, simulation studies show that high redundancy leads to saturation of PNs producing more conflicts due to high intra-sensor overlaps and reduces the overall lifetime of the backbone and sensing devices due to computational overheads.
To numerically calculate the number of PNs required in triangular topology, let $X . Y$ are the dimensions of the area which need to be monitored. Let $F$ is the side length of the equilateral triangle. The number of triangular regions $T_{n x}$ along $X$-axis in one row is given by:

$$
T_{n x}=X / F
$$

Therefore the number of PNs ( $S_{n x}$ ) required along one row of triangles is given by:

$$
S_{n x}=T_{n x}+1
$$

If $T_{n y}$ is the number of rows in the whole area then the total number of PNs required is given by:

$$
\begin{equation*}
S_{n}=\left(S_{n x}\right)\left(T_{n y}+1\right)-1 \tag{9}
\end{equation*}
$$

From Fig. 6 and comparing Equation (4) with (9), it becomes evident that this PN number is greater for triangular model as compared to HT model.


Fig. 5 Node Placement Comparison for three Topologies


Fig. 6 A: Hexagonal, B: Triangular, C: Square Grid Topologies' PN Density Comparison for Granville Disaster Site

## B. Sensor Coverage Overlaps

Coverage overlap is characterized by the following:

- Blanket Cover (BC)
- Minimum Affect on Surrounding (SA)
- Minimum Intra-Sensor Overlaps (SCO)

Blanket Cover: A topology must fulfill at least the following criteria to blanket cover a given region:
a) The constituent shapes of the topology must be capable of fitting together like a jigsaw,
b) PNs could be placed on the grid in such a way that there are no uncovered areas on the grid.
These two requirements have been proved to be true for a hexagonal grid in subsection IV-A.
Due to their geometrical characteristics, the three topology models fulfill the first requirement. The radial coverage of PNs placed in the constituent regions of each of these topologies can circumscribe the region. If a node is placed in each component region of the grid formed over the disaster site, the topology would provide blanket coverage throughout the area. Therefore these topologies conform to the BC requirement.
Affect on Surrounding: It was proposed in subsection IV-B that for minimizing the interference of network with the surrounding environments, maximum nodes must be deployed only within the area to be monitored. This characteristic was also shown for the HT model.
In case of square and triangular grids, Equation (5) for estimation of effects on surrounding is valid for square grid whereas it is not valid for a triangular grid. Considering (5) for a square grid to estimate surrounding affected gives:

$$
\begin{equation*}
A_{S}^{\prime} \geq\left(S_{n S}^{\prime}\right)\left(\pi \gamma_{p}^{2}\right) / 2 \tag{10}
\end{equation*}
$$

where $S_{n S}^{\prime}$ is the number of nodes that could be placed at the boundary. Same density function (BDF) is used for estimating the surrounding interference area for a square grid. In order to minimize $A_{S}^{\prime}, S_{n S}^{\prime}$ and $\gamma_{p}$ must be optimized. As we know that:

\[

\]

where subscript ' $S$ ' stands for square grid while subscript ' $H$ ' indicates hexagon. Equation (11) illustrates that the affected surrounding area is larger for square grid than for HT due to high cardinality of parent nodes in square topology.
For a triangular grid, the scenario would be different because the nodes will be deployed at the vertices of triangular regions. In this case, some nodes will have to be placed outside the boundary of area to be monitored if the
vertices of some triangles are beyond the boundary. Also the nodes at vertices inside but close to the boundary will affect a significant area outside the boundary. These two situations direct the modification of Equation (5) for triangular topology as:

$$
\begin{equation*}
A_{T}^{\prime} \geq\left(S_{n T}^{\prime}\right)\left(\pi \gamma_{p}^{2}\right) \tag{12}
\end{equation*}
$$

Where $\gamma_{p}$ has the same affect on the area as it has for square and hexagonal grids. Equation (9) implies that:

$$
\begin{equation*}
S_{n T}^{\prime} \gg S_{n H}^{\prime} \tag{13}
\end{equation*}
$$

Since the effected area is directly proportional to the boundary nodes, the above equation illustrates that this area is larger for triangular topology than for HT.
Intra-Sensor Overlaps: From the geometry of the three topologies:

$$
\begin{gather*}
\beta_{h}=R_{h}^{2}(\pi-3 \sqrt{3} / 2) \\
\beta_{s}=R_{s}^{2}\left(\frac{\pi}{2}-1\right) \text { and } \beta_{t}=R_{t}^{2}(4 \pi-3 \sqrt{3}) / 12 \\
\Rightarrow \quad \beta_{h} \ll \beta_{s} \ll \beta_{t} \tag{14}
\end{gather*}
$$

where $R_{h}, R_{s}, R_{t}$ are side lengths of hexagon, square and triangle respectively and $\beta_{h}, \beta_{s}, \beta_{t}$ are the differences in areas covered by a PN of known radial coverage and the areas of inscribed hexagon, square and triangle respectively. If we place nodes in the adjacent hexagons, squares and triangles, the overlap in the radial coverage for a hexagonal grid and triangular grid will be nearly equal, while that for a square grid will be much larger as shown in Fig. 7.
If a hexagonal region is surrounded by other hexagonal regions with all having a PN, then from Lemma 1, the total overlapped area will be $\beta_{h}$. Also this area is less than 25 percent of $A_{h}$, which implies that for more than 75 percent of network operation time, an event / object will be under the surveillance of one parent. This reduces the number of PNs engaged over the same activity which reduces the redundant operations thereby increasing the network lifetime.
A different observation is for triangular topology. In addition to the overlaps, Fig. 8 illustrates that there are other regions formed labeled as "Free Zone (FZ)" areas, which are outside the geometry of any triangular region. This renders the disaster site in these regions under "virtually-no" symmetrical control. This means that the network will not support high accuracy of localization of objects and other objectives that base on topological measurements. The cause of free zones is due to the restriction of "minimum" nodes for providing blanket coverage. If the condition of minimum is relaxed then the number of PNs will be almost twice than that of in the HT model, leaving the triangular topology incomparable with the hexagonal.


Figure 7. Intra-Sensor Overlaps


Figure 8. Overlaps in Triangular and Square Topologies
Combining the overlapping areas and FZ areas, Fig. 8 shows that a node-less-conflict triangular region is formed between the adjacent triangular regions. The triangular grid design as shown in Fig. 5 illustrates that this node-lessconflict region area $\left(T A_{t}^{\prime}\right)$ is almost equal to the area $\left(T A_{t}\right)$ where PNs are available, i.e. :

$$
\begin{equation*}
T A_{t} \approx T A_{t}^{\prime} \tag{15}
\end{equation*}
$$

This implies that 50 percent of the network operation time, the resources will be consumed in just negotiating the ownership of events/objects thereby halving the overall network lifetime. One thing should be noted that we are not considering the PN placement on vertices of each triangle for triangular grids as assumed by [2]. The reason for doing so is implied by (15), i.e. if network performance has halved due to increased overlapping and free zones for one node placed per triangular region, it will be incomparably poor for nodes placed at every vertex where total density and overlapping area will become almost three times of that in (15).
For square grid, Equation (14) and Fig. 8 illustrate that the amount of coverage overlap is greater for square topology than for hexagonal and triangular one. If a square region is surrounded by six square regions, all having a PN , then (10) illustrates that the total overlapped area, $\beta_{s}$, is more than 57 percent of $A_{S}$. This implies that 57 percent of the network operation, the network will have to deal with the conflicts, which would reduce the effective network operation to around 40 percent. This increases the number of active PNs at a time which increases the energy consumption of the network. The efficiency of sensor placement is further affected by highly overlapped areas as shown in Fig. 8, whereby three nodes are engaged in monitoring of same activity, producing redundant information and consuming extra energy.

## V. Simulation Results

Simulations were carried out to analyze the performance of the network for the coverage characteristics developed by the topologies for such irregular disaster terrains for which numerical evidences could not be established, like Granville train disaster site. Performance metrics of ineffectiveness in network coverage, network connectivity and conflicts caused by the topology were investigated for various disaster sites with other environmental parameters shown in Table I.

## A. Ineffective Network Coverage(INC)

A direct result out of the network design at Granville and other nine experimental sites was the ineffective coverage provided by the topologies under discussion. This metric analyzes the throughput of the topology while utilizing

TABLE I
Simulation Environment Parameters

| Attribute | Value |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Area under Surveillance | Ten open irregular disaster sites of approximately $100-25000 \mathrm{~m}^{2}$ area |  |  |  |
| Deployment Topology | Random for sensors <br> Hexagonal, Triangular \& Square Grid for parent nodes |  |  |  |
| Sensor Comm. Range | $3 \mathrm{~m}-11 \mathrm{~m}$ |  |  |  |
| PN Comm. Range | $3 \mathrm{~m}-13 \mathrm{~m}$ |  |  |  |
| Number of PS nodes | 125-143 Randomly Deployed |  |  |  |
| Number of PN nodes | 25-100 |  |  |  |
| Performance Metrics | Ineffectiveness of Network Coverage, Network Connectivity, Conflicts |  |  |  |
| Network Activity Time | 15 min |  |  |  |
| Power Consumption | Tx | Rx | Idle | Sleep |
|  | 14.88 | 12.50 | 12.36 | 0.016 |

minimum number of PNs and providing area-wide communication link. It is given by:

$$
\begin{equation*}
I N C=1-(\text { Density } / E A C) \tag{16}
\end{equation*}
$$

Where EAC is the Excess Area Covered by the topology within and outside the boundary of area under monitoring. This relationship estimates the maximum performance of the topologies, when the network uses minimal number of PNs while providing coverage to the whole area with minimum conflicting and interference regions and maximum availability of PNs to maximum sensing devices scattered throughout the network.
Fig. 9 illustrates the performance of four different topologies averaged over all sites. As observed numerically in Section V, HT and CTriangle (nodes at center of triangles) topologies appear well below in INC, thereby proving their coverage effectiveness over square and VTriangle (nodes at vertices of triangles) topologies. Moreover, CTriangle appears to perform a little better for middle sized networks, but latter converges to HT model for larger sites.

## B. Network Connectivity

In subsection V-A, it was shown that square and triangular topologies deploy more PNs. This design aspect can be taken as a strategy to strengthen the backbone, but the simulations have shown a different picture at the Granville. The vertical bars in Fig. 10 show the minimum PN densities that must be achieved by the three topologies to provide connectivity across the disaster site. This number is minimum for HT model. Moreover it is observed that anymore PNs deployed only served to strengthen the network backbone and finally a stage reached where the area wide connectivity approaches to


Fig. 9 Analysis of three topologies for Ineffective Network Coverage. LEFT: PN average range: 5 m , RIGHT: PN average range: 10 m


Fig. 10 Analysis of saturation state caused by increasing number of PNs
saturation. Results in Fig. 10 show that when triangular and square topologies were still well below $100 \%$ connectivity (see regions from density 23 onwards), HT model had already approached that milestone and started strengthening the PNs backbone while consuming lesser resources than other topologies.

## C. Conflict Regions

Fig. 10 results give a motivation to increase the number of PNs for better connectivity, but there is another factor to be considered as well. Fig. 11 shows the other side of the picture when the number of PNs in the backbone was increased at Granville and other sites, it lead to the formation of conflict regions. As also observed numerically in subsection IV-B, HT model suffered from $25 \%$, while square and triangular topologies suffered from $57 \%$ and $50 \%$ conflict regions respectively in the minimal PN density profile. Graphs in Fig 11 show that square and triangular topologies suffered drastically from conflict regions as the number of PNs approached $100 \%$ connectivity and beyond. On the other side, HT model kept conflicts well under control (i.e. under $30 \%$ ) while it provided $100 \%$ connectivity and then graceful smooth increase in conflict regions is clear in Fig. 11 for higher densities for achieving strength in the backbone. As the saturation point reached, the HT model too lost control over conflicts, but still kept them well below other topologies. The graph also provides an avenue to the network architect for selecting the best values of conflict tolerance that can be traded-off against the network strength.

## VI. Conclusions \& Future Work

The paper has analyzed and evaluated the significance of area coverage characteristics of symmetric network topologies while studying for various irregular terrains including the major test site, the Granville train collapse area. A localized HT model has been presented and compared with square and triangular topologies quantitatively considering the effects of density, coverage, conflicts and interference at the network design level.
The mathematical and simulation results have shown that HT and CTriangle models required least number of nodes to establish area wide backbone of PNs. However HT outperforms other topologies in case of conflict regions and saturation zones caused due to increasing demands of node densities by topologies other than HT. These effects leaded networks to consume higher computational resources for square and triangular topologies. The numerical comparisons


Fig. 11 Increasing number of PNs gives strength to the network backbone, but on the other side causes conflicts in the network, reducing the life of network
and simulation results provide guideline for the network architect to decide the topology by looking at the available resources, urgency of deployment, area to be covered, surrounding network environments and the life of network solicited. Focusing on these design requirements the model guides to select best number, location and communication ranges of nodes that would maximize network availability across the area, minimize conflict regions and provide effective topology that is supportive to developing applications that base on symmetry of network design.
Since the HT model has proved to be comparatively optimal at infrastructure level, we would extend this model to support a topology having both mobile sensor and parent nodes. This will be considered for application level techniques: like location tracking, network self-configuration, and data aggregation.

## References

[1] K. R. Doheny, "On the Lower Bound of Packing Density for Convex Bodies in the Plane", Contributions to Algebra and Geometry, Vol. 36, 1995. [2] Y. C. Tseng, S.-P. Kuo, H.-W. Lee, and C.-F. Huang. "Location tracking in a wireless sensor network by mobile agents and its data fusion strategies." (IPSN), 2003.
[3] D. Estrin, D. Culler, K. Pister, and G. Sukhatme, "Connecting the physical world with pervasive networks," IEEE PervasiveComputing, vol. 1, no. 1, pp. 59-69, January-March 2002.
[4] C. Y. Chong, S.P. Kumar, "Sensor networks: Evolution, opportunities, and challenges", Proceedings of the IEEE, August 2003
[5] N. Bulusu, J. Heidemann, and D. Estrin. "Adaptive Beacon Placement". 21st International Conference on Distributed Computing Systems, pp. 489498. Phoenix, AZ, April, 2001.
[6] N. Sadagopan and B. Krishnamachari,"Decentralized Utility-based Design of Sensor Networks," WiOpt'04, University of Cambridge, UK, March, 2004.
[7] K. Chakrabarty, S. S. Iyengar, H. Qi and E. Cho, "Coding theory framework for target location in distributed sensor networks", Proc. Intl. Symposium on Information Technology: Coding and Computing , pp. 130134, 2001.
[8] S. Poduri and G. S. Sukhatme, "Constrained Coverage for Mobile Sensor Networks," IEEE International Conference on Robotics \& Automation, New Orleans, USA, April 26-May 12004
[9] H. T. Kung and Dario Vlah, "Efficient Location Tracking Using Sensor Networks", IEEE WCNC 2003, New Orleans, March 2003
[10] C. Zhou, B. Krishnamachari, "Localized topology generation mechanisms for wireless sensor networks", GLOBECOM 2003-IEEE Global Telecommunications Conference, no. 1, Dec 2003 pp. 1269-1273
[11] R. Kershner, "The Number of Circles Covering a Set", American J. Math. 61, 665-671, 1939.
[12] M. Iqbal, I. Gondal, L. Dooley. "Investigating the Dynamics of Pervasive Sensor Networks through Parent Localization", ATNAC 04

