An Evaluation of the Further Mathematics Support Programme : Research Report

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An Evaluation of the Further Mathematics Support Programme

Research Report

Undertaken 2014-2016 and reported in May 2016

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Glossary

A level: Advanced Level

AEA: Advanced Extension Award in Mathematics offered by Edexcel examination board

AS level: Advanced Subsidiary Level

Centre: School/College

CPD: Continuing Professional Development

EAL: English as an additional language

FM: Further Mathematics

FMSP: Further Mathematics Support Programme

FSM: Free School Meals

GCSE: General Certificate of Secondary Education

HE: Higher Education

HEI: Higher Education Institution

KS3: Key Stage 3 (Years 7 to 9)

KS4: Key Stage 4 (Years 10 to 11)

KS5: Key Stage 5 (Years 12 and 13)

Level 3 mathematics education: a qualification designed for students who have achieved a grade C or above at GCSE

LIL: Live interactive lectures, fortnightly lectures in an online classroom by an FMSP tutor

LPP: Low participation providers

LOT: Live online tuition

LOPD: Live online professional development

MAT: Mathematics Admissions Test developed by Oxford University and used for admission to Oxford’s and Imperial College London's mathematics degrees

MEI: Mathematics in Education and Industry

NCETM: National Centre for Excellence in the Teaching of Mathematics

NPD: National Pupil Database

PD: Professional development

SEN: Special educational needs

SAP: School Action Plus

TAM: Teaching Advanced Mathematics

TFM: Teaching Further Mathematics

UCAS points: A means of differentiating students based upon grades from post-GCSE qualifications

1. Executive summary

The Further Mathematics Support Programme

The Further Mathematics Support Programme (FMSP) is managed by the MEI through an extended partnership with the NCETM (Tribal), and the University College London Institute of Education. It was established in 2009, following on from the Further Mathematics Network (2005-2009), which was also managed by MEI. A central team supports a network of Area Coordinators and others to deliver the programme nationally.

It aims to:

- increase participation in AS/A level Mathematics and Further Mathematics, particularly that of girls and those from other under-represented groups
- increase capacity within schools and colleges to provide high quality mathematics teaching
- increase demand from students to study AS/A level Mathematics and Further Mathematics post-16
- support improvements in Level 3 mathematics education

To further these aims, the FMSP programme goes beyond focusing only on AS/A level Further Mathematics and A level Mathematics qualifications. It also seeks to address how to engage KS4 and other learners in developing enthusiasm for and a positive attitude to mathematics, through targeted professional development and other programmes.

The principal components of the FMSP are:

- tuition support for AS/A level Further Mathematics through both face to face and on-line programmes
- continuing Professional Development (CPD) for teachers to support teaching of Further Mathematics and A level Mathematics, as well as professional development focused on GCSE mathematics that fosters teaching approaches for younger students to prepare them for further study and increase their motivation to study more mathematics post-16
- enrichment activities that aim to increase demand from students to study AS/A level Mathematics and Further Mathematics post-16, by enriching the experience of learning mathematics and supporting positive engagement in mathematics

- regional support from Area Coordinators who coordinate regional activity and offer tailored face to face support and advice to teachers and mathematics leaders in relation to FMSP aims and provision
- on-line support including the FMSP website and access to MEI's Integral Virtual Learning Environment containing online resources
- mathematics competitions
- support for high level mathematical problem solving including support for university entrance exams in mathematics

Evaluation of previous phases of the FMSP programme found that the FMSP's work is effective and of high quality.

Evaluation methodology

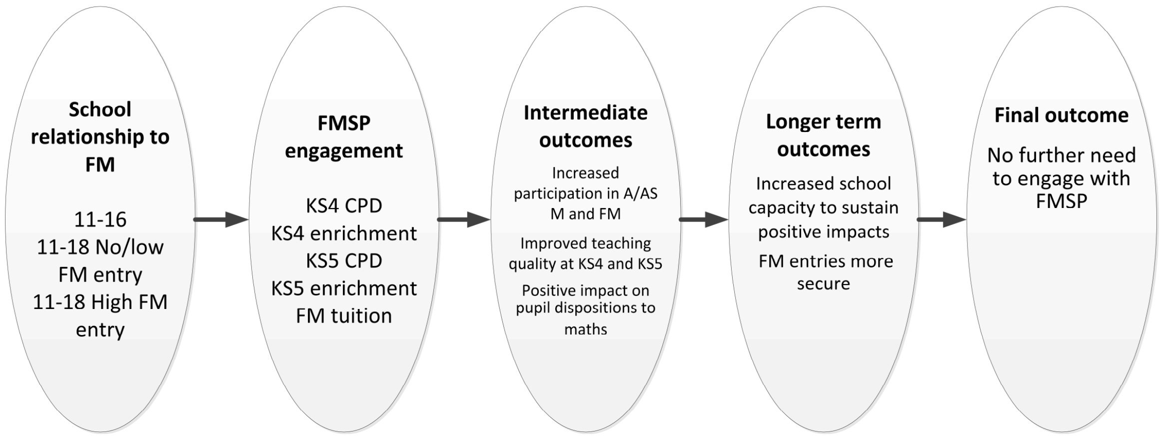
The four key focus areas for the 2014-16 evaluation were to consider:

- capacity and capability building
- reach to schools/colleges, teachers and students
- effectiveness of the programme (quality and impact)
- sustainability beyond the end of the programme

The evaluation also sought to assess the viability and value of quantitative modelling of Further Mathematics and FMSP activity, including identifying any relevant issues in drawing together disparate databases, to provide a quantitative baseline for future evaluation and to develop tools for future evaluation, including assessing the value of focus group interviews with students as a data collection method.

The FMSP is a complex programme with a large number of components. A simplified summary 'theory of change' diagram is presented below. An important part of the FMSP's purpose is for centres' (schools/colleges) Further Mathematics provision to become self-sustaining. Thus, a positive outcome for the FMSP as currently conceived is for a centre to cease to engage with the programme, or at least components of it, when they have developed their capacity for self-sustaining improvement.

Figure (i) Overarching Theory of Change



The evaluation used a mixed methods research design comprising:

- quantitative analysis of school and pupil participation and attainment in A and AS level Further Mathematics and Mathematics
- analysis of School Census data and data from the National Pupil Database, and of internal data held by the Further Mathematics Support Programme on participation in FMSP activities
- qualitative analysis of data from telephone and face to face interviews with teachers and focus groups with A level Mathematics and Further Mathematics students, supported by a survey of Maths Hub leads

Analysis of the school level census data was used to develop a ‘security of provision’ construct of A level Further Mathematics entries over a 3 year period as a measure of the extent to which the programme is effective in developing 'capacity and capability' as well as sustainability. Security of provision was cross-tabulated with school characteristics to model how A level Further Mathematics entries vary according to school characteristics. The relationship between security of provision status and records of centre involvement in FMSP CPD, enrichment and tuition was analysed.

Student participation and attainment in A level Mathematics and A level Further Mathematics were modelled using multivariate multilevel regression. Data from the National Pupil Database (NPD) was retrieved and used for the complete cohort of pupils in England who (a) completed Key Stage 4 in 2010/11 and (b) are recorded as taking a KS5 assessment between 2011/12 and 2013/14. Engagement with the FMSP was brought in as a school level explanatory variable. However, the engagement data were for one year only (2013/14). These data were obtained by synthesising a number of data sets developed for distinct purposes and so represent a limited range of support and engagement. Therefore this analysis is limited in reflecting the full relationship between school level involvement with the FMSP and pupil participation in Further Mathematics. Historical data including, if possible, data at pupil level would be needed to fully investigate this relationship.

A total of 42 interviews (37 by telephone and 5 face to face) were conducted with teachers from centres with varying of numbers of Further Mathematics entries and a variety of levels of involvement in FMSP activities. Five focus groups involving a total of 44 students were conducted in centres from two English regions. To supplement the teacher interviews a short survey, including some open questions was conducted with Maths Hub leads to gain their views on how the FMSP and the Hubs currently work together and priorities for the coming year.

Further Mathematics security

Since 2004 both the numbers taking AS and A level Mathematics and AS and A level Further Mathematics have increased (Table i).

Table (i) Participation in A & AS level Maths & Further Maths 2004-2015

	Total number of A level entries	A level Maths entries as a % of total entries	A level Further Maths entries as a % of total entries	**Total number of AS level entries**	AS level Maths as a % of total entries	AS level Further Maths as a % of total entries
2004	675,924	6.8%	0.8%	878,794	5.8%	0.3%
2015	758,768	10.8%	1.8%	1,086,702	10.2%	1.5%

Source: Adapted from the Statistical First Release - SFR 38 2015 Table 14 (see www.gov.uk/government/statistics/a-level-and-other-level-3-results-2014-to-2015-provisional)

In line with this, since 2004 there has been a 10% increase in the number of establishments offering A level Mathematics, but an 87% increase in the number offering Further Mathematics (Table ii). This is during the period of the FMSP and its predecessor the Further Mathematics Network (2005-2009), indicating a positive association between the FMSP programme and the growth of Further Mathematics entries and provision.

Table (ii) State establishments with candidates taking Further Mathematics

	State establishments with candidates taking A level Further Mathematics	State establishments with candidates taking A level Mathematics	Percentage access to Further Mathematics
2004/05	762	1926	40%
2009/10	1171	1874	63%
2014/15	1428	2115	68%

Source: DfE/MEI

A "security status" for centres was constructed that considered the stability in the number of A level Further Mathematics entries over time. Table (iii) below shows how these security statuses were constructed from the number of A level Further Mathematics entries.

Table (iii) Construction of the security status

Number of A level Further Mathematics entries over 3 years (2011/12, 2012/13, 2013/14)	**n**	**%**	**Assigned Security Status**
No entries in all 3 years	674	29.9	None
Two years with 0 entries, 1 or 2 in other year	226	10.0	Least secure
One year with 0 entry, 1 or 2 in other year	114	5.1	Least secure
3 or more in any one year, low or none in other years	216	9.6	Less secure
At least 1 in all 3 years, no more than 3 in any year	44	2.0	Less secure
At least 1 in all 3 years, 3 or more in any one year	310	13.7	Less secure
Three or more entries in all 3 years	530	23.5	Relatively secure
10 or more in all 3 years	142	6.3	Highly secure
Total	**2256**	**100.0**	

Source: adapted from 16-18 qualification and subject level results
http://www.education.gov.uk/schools/performance/download_data.html

Using data from 2013/14, 2012/13 and 2011/12 entries, security status for 2013/14 was considered in relation to school/college characteristics. The following was found:

- selective schools are most likely to have secure numbers of students taking Further Mathematics A level
- schools with higher numbers of Further Mathematics students have lower levels of students eligible for FSM
- there is little difference in Further Mathematics uptake amongst schools with varying proportions of students with EAL
- schools with higher proportions of students taking Further Mathematics have a slightly lower proportion of students with SEN or SAP

- schools with a male only intake (at KS5) have significantly higher numbers of students taking Further Mathematics compared with female only schools and mixed schools. This is also reflected when looking at the average proportion of males on roll compared with average proportion of females on roll: highly secure schools have an average of 70% of boys on roll and 30% of girls on roll, whereas for relatively secure schools and less secure schools the average percentage of boys on roll and girls on roll is very similar (close to 50/50)
- higher attaining schools at KS4 (based on % attaining 5+ A*- C including English and maths GCSEs) have higher proportions of students doing A level Further Mathematics
- similarly schools with higher average point scores at KS5 have a larger proportion of students doing Further Mathematics

Security status was calculated for a further two periods of 3 years and patterns of change identified. The table below summarises changes from 2012/13 (based on 2010/11, 2011/12, 2012/13 entries) to 2014/15 (based on 2012/13, 2013/14, 2014/15 entries):

Table (iv) Change in security 2013 to 2015

2012/13 (2010/11, 2011/12, 2012/13 data)	**2014/15** (2012/13, 2013/14, 2014/15 data)						
		None	Least secure	Less secure	Relatively secure	Highly secure	**Total**
	None	487	104	56	0	0	**647**
	Least secure	76	132	108	0	0	**316**
	Less secure	13	56	334	138	1	**542**
	Relatively secure	0	0	91	368	35	**494**
	Highly secure	0	0	0	28	107	**135**
	Total	**576**	**292**	**589**	**534**	**143**	**2134**

- in total 706 centres experienced a change in status (33% of the total)
- 442 centres moved towards a more secure status (63% of those experiencing a change in status)
- of the 647 centres who had no records for A level Further Mathematics in 2010/11, 2011/12 or 2012/13, 160 (25%) moved into the more favourable categories of "least secure" and "less secure"
- of the 316 centres in the "least secure" category in 2012/13, 108 (34%) moved into "less secure" whilst 76 (24%) moved into "none"

The general pattern of movement of centres towards a more secure status reflects the general increase in numbers of Further Mathematics students. However, the analysis shows that this increase is not restricted to centres that already had relatively high levels of Further Mathematics entries. Further, the increase in entries is not merely due to an increase in the number of centres offering Further Mathematics. The analysis also reveals that the number of entries is less stable in less secure centres; there is a continuing fragility of entries in some of these centres. In summary, there has been an increase in both capacity and sustainability of Further Mathematics provision, although there is much potential for further improvements.

Modelling pupil participation in A level Mathematics and Further Mathematics

Data from the National Pupil Database were used to analyse participation in A level Mathematics and Further Mathematics for the pupil cohort in England who completed KS4 / Y11 in 2010/11 (N=637,594) and were recorded as taking at least one KS5 assessment in 2011/12, 2012/13 or 2013/14 (this was the case for 64.7% of the 2010/11 KS4 cohort, N=412,743).

Multivariate, multilevel models of participation and attainment in A level Mathematics and Further Mathematics were developed. The models were used to examine differences and similarities between participation and attainment in A level Mathematics and Further Mathematics by considering the following explanatory variables:

- prior attainment at KS4 (overall and in mathematics)
- gender
- whether a pupil was eligible for, and parents claimed for Free School Meals (FSM) in 2010/11 (as a rough proxy for socioeconomic position)
- ethnicity
- institution type
- engagement with a subset of FMSP activity for which data was available

The models echoed what has been found by others around influences on participation in A level Mathematics and Further Mathematics, particularly around the key role of attaining A* in GCSE Mathematics in determining participation in both. The models also revealed new important detail around participation and attainment in A level Mathematics and Further Mathematics relating to gender, ethnicity and institutional type. However, the models were unsuccessful in identifying whether institutional-level engagement with FMSP (in 2013/14) was associated with a measured student-level impact in terms of participation and attainment in A level Mathematics and Further Mathematics (between 2011/12 and 2013/14). This may reflect that, for FMSP, 'impact' is more likely to be observed at an institutional or (education) system rather than individual student level.

Reviewing the FMSP's activity

During the period April 2014 to March 2016 there were over 22,800 occasions on which 11-16 students participated in FMSP enrichment events. Students and teachers judged these to be of high quality and they had a positive impact on student thinking about post 16 mathematics study. Events were varied including focusing on problem solving as well as a range of other events such as mathematical competitions.

The FMSP delivered more than 2000 days of CPD for A level Mathematics and Further Mathematics across a range of both face to face and on line courses in 2014/15 and in 2015/16. In addition, 670 days of CPD were delivered to KS4 teachers in 2014/15 and over 1400 in 2015/16. The feedback for courses ranged from good to excellent.

The number of students who access FMSP tuition has declined over time as school capacity to offer Further Mathematics in house has increased. In 2014/15 201 students accessed 519 units of tuition, comprising 202 face to face units and 317 units of Live Online Tuition.

Currently 3446 schools/colleges are registered with FMSP consisting of 672 that have students up to KS4, 2585 that have students up to KS5, 177 with KS5 students only, and 12 other.

The FMSP has engaged with HEI departments, encouraging the uptake of AS/A level Mathematics and Further Mathematics. In 2015, meetings/discussions with 99 HE departments took place resulting in 18 indicating a willingness to increase encouragement for Further Mathematics and 4 for Mathematics.

Considering this activity as a whole, it is clear that the FMSP has considerable reach relevant to its aims.

School/college FMSP engagement and student and teacher perspectives on the FMSP

Internal data at school level collected by the FMSP was matched into the school level census data (from 2011/12, 2012/13 and 2013/14) for 2256 centres. The FMSP data matched into this was from 2013/14 only. In total 50.8% of centres had at least 1 record of involvement with the FMSP from the year 2013/14. Note that this does not reflect total involvement of centres with the FMSP since this is restricted to one year and certain activities. Of the 1110 centres that had no records of involvement in 2013/14, only 122 did not appear on the FMSP administrative database of schools/colleges registered with the FMSP. All centres registered with the FMSP are sent regular updates on activities and opportunities, through electronic communications and hard mailings.

The sample for the teacher interviews was drawn from centres who had varying levels of security of provision, including those who had improved their 'security rating' based on data from the quantitative analysis. The security rating of these centres in 2013/14 is shown below (Table v):

Table (v) Sample of interviews by security status (2013/14)

	None	Least secure	Less secure	Relatively secure	Highly secure	**Total**
Number of centres	7	5	16	10	4	**42**

Out of the 42 teacher interviews conducted, 18 in total indicated that their centres were involved in some form of tuition. The teachers who were involved in tuition were in general positive about it, typical comments were 'excellent' and 'very valuable for students'. Five out of the ten teachers from those schools classified as low or least Further Mathematics secure using tuition expressed various concerns related generally to their capacity to provide additional support to their students which they considered necessary to supplement on-line tuition, in particular. This indicates the importance of support through CPD and on-line materials for teachers in centres that are involved in tuition. However, all teachers interviewed in low or least secure centres were generally of the view that tuition was important to allow students in their centres to access Further Mathematics at all. This was important for some to retain their strongest students and so additionally supported A level provision. Teachers at the 8 centres with secure or highly secure Further Mathematics stated that tuition provided an enriched experience for students, two of these also pointed to cost saving implications.

Thirty three interviewees had personally recently participated in FMSP CPD or their colleagues had. All interviews were positive about the quality of CPD, with a number describing it as excellent. The subject knowledge of the presenters and the high quality of the resources were specifically highlighted. Reported benefits were enabling the teaching of Further Mathematics; improved pedagogical and subject knowledge; and access to new high quality resources.

Nearly all teacher interviewees had accessed elements of the FMSP enrichment programme for their students with all aspects of enrichment activities represented in the sample. The enrichment activities accessed were reported to have improved student motivation; enabled students to explore mathematics in context; engage with problem solving; gain an insight into what careers were possible with mathematics qualifications; provide positive role models for girls. Team challenges had allowed the students to develop their capacity to work together.

In general, teacher interviewees were also positive about Area Coordinators and on-line support. Irrespective of their roles, engagement with the programme or Further Mathematics security, teachers were overwhelmingly positive about the FMSP and the programme continues to be held in high regard by teachers. A teacher at one centre reported that the FMSP support was important to achieving the current relatively secure status. Four of the interviewees from less secure and least secure categorised centres who engaged with CPD but not tuition suggested that without the FMSP CPD offer Further Mathematics may not be undertaken at all. Thus in total some 57% (12 of 21) of interviewees who were from centres with lower security pointed to the FMSP as essential to offering Further Mathematics.

Students' engagement with, and experience of the FMSP, was explored as part of the five focus groups conducted with 44 students. Students had varying opportunities and experiences of enrichment, depending on the extent to which teachers in their centres had engaged with local opportunities. Students overwhelmingly valued and enjoyed enrichment experiences. Some expressed pride at being selected to take part in junior and senior mathematics challenges, but the extent to which these were offered varied between centres. Some students expressed concern about equity in who was picked to take part in activities.

Overall, enrichment tended to positively influence their enjoyment of mathematics over the years, giving them opportunities to experience problem solving and team based activities that required them to apply and develop their skills in new ways. Their enjoyment of mathematics generally increased as a result, which then impacted on their choice of A levels.

Engagement with Maths Hubs

A short online survey was conducted in January 2016 with Maths Hub leads to gain their perspectives on how they work with the FMSP and how they view priorities for the coming year (25 out of 35 Hub leads responded).

Most respondents (83%) indicated that the FMSP worked with the Hubs as often as needed and in general Hub leads reported that the FMSP and the Hubs worked together effectively on most areas of work. Approximately a quarter of Hub leads considered there could be better collaboration on planning and publicising joint student and teacher events. In 61% of Hubs that responded, the FMSP Area Coordinator was a member of the strategic group for their Hub, whilst 87% indicated that the FMSP Area Coordinator was a member of one or more working groups. Hub leads were asked in the survey about what the future priorities should be for FMSP. Perhaps unsurprisingly the main priorities identified related to KS5 with activity in KS4 being less of a priority. Hub leads were asked about barriers to engagement with the FMSP and identified the following: time, geography (including mismatch between FMSP areas and Hub areas); capacity in centres to get involved; the need for a shared vision and role clarity.

Reasons for choosing and not choosing A level Mathematics and Further mathematics

Student focus group interviews, supported by analysis of teacher interviews, identified the following features as positively influencing choice of A level Mathematics and Further Mathematics:

- enjoyment and self-efficacy
- influences of parents/family, and teachers
- aspirations

Of these, enjoyment was ranked most highly.

Further Mathematics capital

The concept of Further Mathematics capital is an extension of the concept of science capital - a means to conceptualise the interplay of social, cultural and familial practices, knowledge and relationships that support engagement in science and influence patterns of participation.

In addition to mathematics ability and attainment, teachers also highlighted that students' intrinsic, personal characteristics such as self-motivation, capacity for 'independent study' and their 'work ethic' were important for their successful participation in Further Mathematics. Parental valuing and support for mathematics were also cited, particularly by higher security centres, as being important in ensuring a student’s individual success in Further Mathematics.

Teachers identified the characteristics, context and attainment profile of the school and its student intake as the primary factor that either enabled or in some cases impeded Further Mathematics engagement. Senior leadership support was important, as were personal success stories of students who had excelled in Mathematics/Further Mathematics.

Key aspects of teacher capital found were:

- enjoyment of mathematics
- commitment, energy, accessibility in relation to mathematics
- capacity to teach Further Mathematics (in post 16 settings) or to offer additional support for those taking "Live Interactive Lectures (LiL)" and similar provision
- a Further Mathematics enthusiast/champion (including in 11-16 settings someone who encourages engagement with mathematical problem solving and so on)

Departmental features contributing to organisational capital were:

- cohesive and supportive relationships
- specialist mathematics teachers
- contribution to A level teaching being the norm
- commitment to enrichment and a 'love' of mathematics
- investment in students interested/capable of doing Further Mathematics

Impediments to Further Mathematics participation

The following were identified as barriers to participation or success in Further Mathematics:

- lack of confidence was related to less positive previous learning experiences in mathematics
- a lack of 'bridging' qualifications, leading to a significant jump from GCSE to A level Mathematics/Further Mathematics
- school timetabling issues meaning students were not able to take Further Mathematics in combination with non-science subjects (more frequently cited by girls)
- lack of encouragement by universities for those wanting to study medicine or biological/health related courses
- squeeze on budgets

- the impact and the availability of Further Mathematics in-house most keenly experienced by centres with lower security ratings
- recruitment and retention of experienced/qualified teachers
- distance from or links with enrichment at local universities

Most teachers were unsure what the impact of future changes to qualifications and curricula would be, but a few saw this as a potential barrier to future take up of Further Mathematics as a two year A level course in a challenging subject is likely to be thought of as too risky - particularly for less confident students.

FMSP influence on Further Mathematics capital and ecologies

The FMSP positively impacts on Further Mathematics capital through the following aspects of the programme.

- Enrichment experiences. Positive enrichment experiences through KS3 and KS4 were identifiable as a recurring element across a number of student timelines of their mathematical experience and choices. Enrichment experiences also helped support positive mathematical and so Further Mathematics cultures
- Tuition. Making Further Mathematics more available including supporting high security centres to extend module options
- CPD. Strengthening teaching skills
- On-line resources. For other centres (including those with low/no Further Mathematics security), the MEI Integral resources were identified as providing access to materials that helped less experienced staff develop their skills at an individual level

The FMSP is an important part of school-level Further Mathematics ecologies in contexts where the internal ecologies do not support secure or in some cases any Further Mathematics entries.

Evaluation

1. Capacity and capability

The FMSP continues to enhance the capacity and capability of centres to offer Further Mathematics in the following ways:

- contribution to on-going increases in overall key measures:
  - participation in A level Further Mathematics
  - the number of centres offering Further Mathematics

    - improvements in Further Mathematics security
- teacher CPD that helps to enhance the organisational mathematics capital of centres as well as, indirectly, of individual students. The CPD offer is being taken up to a greater extent in centres that are likely to have the greatest need
- enrichment and tuition also support capacity and capability. This happens for both centres with higher and lower levels of Further Mathematics security
- the FMSP provides a safety net for the least Further Mathematics secure and less Further Mathematics secure centres, which can be at risk of losing Further Mathematics capacity due to low numbers

2. Reach

The FMSP has considerable reach, but there is scope for developing this in a targeted way:

- overall reach is strong as indicated by:
    - the number of centres registered
    - the number of centres engaging in one or more FMSP activities
    - the number of teachers participating in CPD
    - the number of students participating in enrichment
- the FMSP also successfully engages centres that are a priority – those centres offering A level Mathematics but not yet offering Further Mathematics. At the same time there are still considerable numbers of centres, particularly 11-16 schools that have not yet registered with the FMSP who could be targeted
- the evaluation highlights groups of priority students for whom the FMSP may need to make particular efforts to reach. The model presented in this report suggests that Further Mathematics is taken by a less diverse group of students than those that take A level Mathematics
- The FMSP is actively engaged with other key organisations including the new Maths Hub networks

3. Effectiveness (quality and impact)

Views of quality

Activity undertaken by the FMSP is overwhelmingly viewed positively by teacher participants. There is evidence from student focus groups and student feedback that, in general, students also view FMSP support favourably.

Impact

The key measures of impact relate to participation in A level Mathematics and Further Mathematics more generally. It is notable that the increase in Further Mathematics is faster than the increase in A level Mathematics and for centres with low Further Mathematics entries the FMSP is viewed as essential to continue to offer Further Mathematics. Improvements in 'security' status are also an indicator of success in regard to making these increases sustainable. Intermediate impact measures reported by teachers included increase confidence in teaching, improved subject knowledge, and strengthened pedagogical approaches.

4. Sustainability

The FMSP's reach, quality, capacity/capability-building and impact mean that the enhancement of Further Mathematics cultures and capital are supporting centres to develop secure and so sustainable levels of Further Mathematics entry.

However, the same analysis shows that increases are not uniform. Whilst the numbers of Further Mathematics secure and highly secure centres have increased overall, there are centres where Further Mathematics has become less secure. The qualitative analysis, particularly of enablers and barriers provides indications of why this is the case.

Notably, the most secure centres with high numbers of A level entrants are independent schools and those 11-18 state schools with more advantaged student populations. In these centres there is a considerable degree of Further Mathematics capital and an internal Further Mathematics ecology that is self-sustaining. In other centres, even those with increasing numbers of Further Mathematics entries, there is still a degree of fragility. Given changes in both A level courses and in funding arrangements for A level study, there is some risk to Further Mathematics entries. Whilst the general trend was towards increased Further Mathematics security, the security analysis showed that for every 2 centres that improved security, one centre became less secure over the period analysed. Given this, and the relatively low levels of mathematics capital in some centres, interventions and enhancements such as the FMSP or similar will be needed on an ongoing basis. The concept of 'sustainability' of Further Mathematics provision as an evaluation criteria for the FMSP may be problematic if this is taken to mean that in the short to medium term all centres will achieve fully sustainable provision.

The need for ongoing support

The FMSP and its programme components have grown organically with different aspects of the programme being developed in relation to particular issues as they emerge. The evaluation suggests that more certainty of long term support for the FMSP would greatly assist in enabling the programme to strategically plan and deliver success against its stated aims. The strength of the programme partly lies in its own longevity, and related to this, the positive regard it is held in, flexibility, and therefore ability to support schools and colleges over time, bearing fruit in the ongoing increases in Further Mathematics participation, and Further Mathematics security.

Wider engagement

The FMSP is actively engaged in Maths Hubs and has also created and stimulated teacher networks. However, the potential for those centres with secure Further Mathematics entries to support others is a possible means to extend the Further Mathematics support that can be provided, and therefore enhance the sustainability of FMSP programme impacts. In addition, whilst the Maths Hub leads are key players in support for improvements in mathematics education, teaching schools are another potential resource, since they have remit and access to resources to support other schools in improving their teaching, and school improvement more broadly.

Future evaluation

The evaluation has indicated that student focus groups are a potentially useful method for assessing the quality and impact of FMSP activity. Further, quantitative modelling has identified a number of issues that are important for the FMSP to consider. A challenge for future evaluation is to consider how to model participation in FMSP activity to identify and measure impact at the student level. Whilst the evaluation has given some indications of what influences changes in Further Mathematics security, this could be further investigated.

Recommendations

Based on the evaluation findings, recommendations are made below. These largely relate to programme structure and targeting the programme, with an evaluation recommendation and an overarching recommendation on the need to maintain the FMSP.

Programme structure

Recommendation 1: Consider offering tailored CPD focused on 'supporting Further Mathematics students engaged in tuition' as part of tuition packages, given that some teachers interviewed identified a lack of confidence to support students accessing on-line tuition.

Recommendation 2: The recently developed on-line (LIL) tuition has begun to be used by centres with a variety of Further Mathematics security profiles. The FMSP should consider ways in which this can be further enhanced or encouraged.

Recommendation 3: Consider ways that the 'crossover' benefits for the different components of the FMSP offer, such as tuition and teacher professional development can be enhanced, and how existing promotion of these benefits to centres can be improved.

Recommendation 4: The FMSP should consider enhancing materials and information to address students whose primary motivations are related to the enjoyment of mathematics itself as a reason for studying Further Mathematics, in addition to educational and career aspirations.

Targeting the programme

Recommendation 5: Maintain focus on the centres which are least Further Mathematics secure and use this classification as a means to prioritise support.

Recommendation 6: The FMSP should investigate finding ways to further engage 11-16 centres, including increasing the number registered with the FMSP, in order to influence post-16 participation.

Recommendation 7: The FMSP should consider how to further focus efforts - and future research - on understanding and providing appropriate support for students from backgrounds underrepresented in those taking Further Mathematics.

Working with schools

Recommendation 8: Consider further ways to engage with and influence school leaders directly, since the evaluation has underlined the importance of school leadership to support Further Mathematics participation.

Recommendation 9: The FMSP should consider how to engage further with current developments in the school self-improvement agenda beyond work with Maths Hubs. Potentially, those centres with high Further Mathematics security are an underused resource to support others.

Evaluation

Recommendation 10: Measuring long term impact of professional development and enrichment activities is, in general, more challenging and attention should be paid to this in future evaluations, potentially this might be linked to further investigation of change in security status.

Maintaining the programme

Recommendation 11: Given the ongoing risks to Further Mathematics sustainability, especially for centres with less secure provision, but even for more secure centres, the FMSP - and its principal funder, the Department for Education - should continue to make the case for the necessity of the programme to support Further Mathematics, and so the FMSP should consider ways to secure long term and more stable funding. The DfE should consider this recommendation.

1. Introduction

The Further Mathematics Support Programme (FMSP) was initiated and is managed by Mathematics in Education and Industry (MEI) with government funding. It aims to improve both the availability and quality of provision for AS/A Level Further Mathematics[1] and promote the study of mathematics to 14-19 year old students. AS/A Level Mathematics and Further Mathematics are usually taken by students who are 16-18 years old and whilst the qualifications have wider value and purpose, they are most notably pre-university qualifications that are important as a route to study of mathematics and mathematics related subjects at university. The study of AS level Further Mathematics increases the breadth of mathematical study and A level Further Mathematics, the depth of study and so is related to accessing the study of mathematics and related degree courses.

The FMSP offers and supports a range of activities to further these aims, including:

- professional development activities for teachers to support the teaching of Further Mathematics, A level and GCSE Mathematics
- direct support for Further Mathematics study through a range of face to face and on-line provision
- enrichment activities for A level and GCSE students
- on-line resources and materials for teachers

Previous evaluations of the FMSP (phases 1-4) have indicated that activities are of a high quality, that the programme is well regarded by teachers and other stakeholders, that the programme has had a positive impact on the number of centres offering Further Mathematics A level and is associated with an increase in the number of students taking Further Mathematics AS and A level (Searle, 2012; 2014).

Phase 5 of the evaluation occurs at a significant moment for the FMSP. Externally, changes in both GCSE and the A level curriculum and assessment have been noted as a potential challenge to the growth in engagement with A level Mathematics (ACME 2013). At the same time, this may

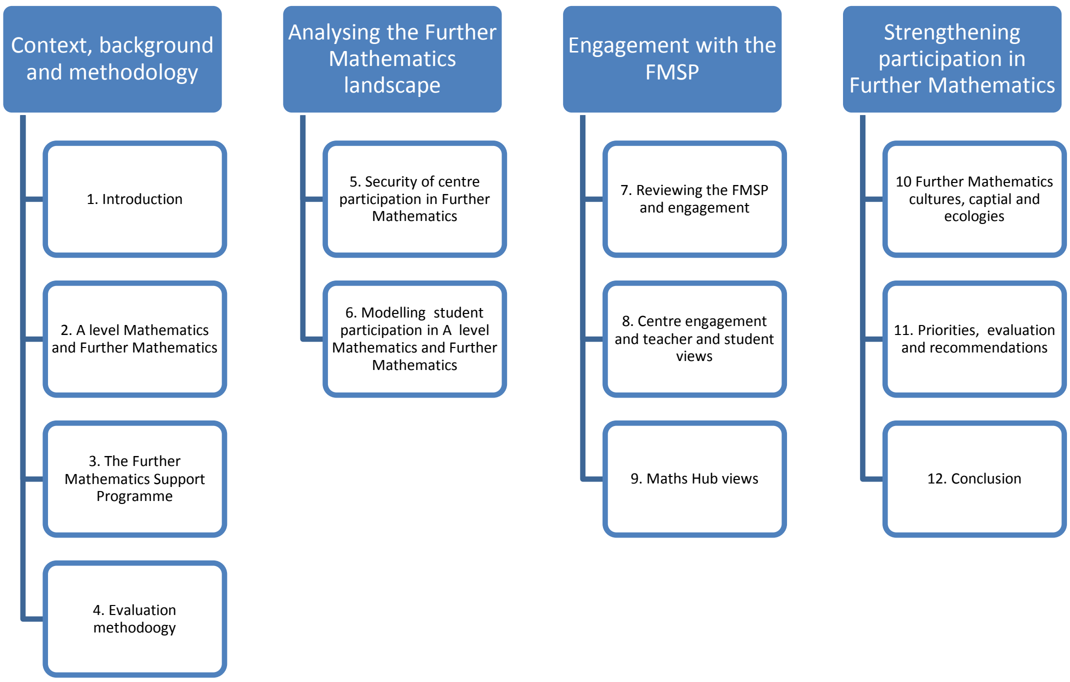
[1] A level qualifications are typically but not exclusively taken as pre-university qualifications at 18 years old. AS or Advanced Subsidiary level qualifications are of a similar level but less content than an A level qualification. During the period relevant to the data reported in this evaluation, AS level modules could be taken as the first year of a two year A level course.

potentially lead to an increased demand for professional development in relation to both KS4 and KS5, as centres and teachers review and develop their practices in response to new demands. Further, the 2014 development of the National Centre for Excellence in Teaching Mathematics (NCETM) led Maths Hub initiative presents new challenges of coordination and partnership for the FMSP as well as significant opportunities to develop its reach, capacity, effectiveness and sustainability. In addition, proposed changes to post-16 funding may impact negatively on numbers taking Further Mathematics as it is generally taken as a fourth A level and there are potentially disincentives for centres to support study of more than three A levels. In addition changes in AS may also be significant.

Phase 5 of the evaluation, reported here, builds and extends previous findings of the evaluation of the FMSP by statistical modelling of patterns of centre and student participation and the relationship of this to activities of the FMSP. Drawing on qualitative data it considers enablers and barriers to participation in Further Mathematics using concepts of Further Mathematics capital, culture and ecology to guide further development in promoting post compulsory mathematical study at Advanced levels. It considers the extent to which the FMSP is successful in meeting its aims in terms of capacity and capability building, reach, effectiveness and sustainability and identifies issues to consider both for future evaluation and more importantly for the Further Mathematics Support Programme.

The chapters of the report are organised into the sections represented in the figure below.

Figure 1 Structure of the evaluation report



2. A level Mathematics and Further Mathematics in England

2.1 AS and A level Mathematics and Further Mathematics qualifications in England

Further Mathematics A level is taken as an additional A level to Mathematics. Students studying Further Mathematics A level have the opportunity to study mathematics in greater depth by studying further and more challenging mathematical content as well as broadening study by additional study of mathematical applications than is possible to choose within single A level Mathematics. Further Mathematics can also be studied as an AS level.

The study of A level Mathematics has been promoted in educational policy in England, in part due to its perceived economic benefits nationally and evidence for an A level Mathematics premium in terms of individual income (Adkins and Noyes, 2016) and concern about the relatively lower proportion of students who study mathematics post-16 compared to other countries (Hodgen, et al., 2013). In addition, arguments for the study and so promotion specifically of Further Mathematics include:

- concern about the content, depth and demand of the main A level syllabi (Jones et al, 2016)
- the value of Further Mathematics qualifications in access to university, particularly to study mathematics and at selective institutions (Searle, 2014)
- the association between the introduction of Further Mathematics and rises in A level participation and outcomes at school level (Searle, 2014; Golding and Smith, 2016)
- evidence of wider positive impacts on departments introducing Further Mathematics on student retention and support for a range of aspirational pathways, teacher retention and development, enhancement of pedagogy and students' mathematical experience (Golding and Smith, 2016)
- studying Further Mathematics is associated with increase in A level Mathematics grade between half and one grade for individual students (Lord and Stripp, 2014)

2.2 Participation in A level Mathematics and Further Mathematics

A level Mathematics and Further Mathematics entries over time

Table 2-1 summarises A and AS level entries in Mathematics and Further Mathematics for students in England 2001 to 2015 taken from Governmental Statistical First Release publications.

Table 2-1 Participation in A & AS level Mathematics & Further Mathematics 2001-2015

Year	Total number of A level entries	A level Mathematics entries as a % of total entries	A level Further Mathematics entries as a % of total entries	Total number of AS level entries	AS level Mathematics as a % of total entries	AS level Further Mathematics as a % of total entries
2001	681,553	7.9%	0.7%	772,359	7.7%	0.3%
2002	645,033	6.8%	0.7%	839,141	6.8%	0.3%
2003	662,670	6.7%	0.7%	879,858	6.5%	0.3%
2004	675,924	6.8%	0.8%	878,794	5.8%	0.3%
2005	691,371	6.7%	0.8%	907,029	6.1%	0.4%
2006	715,203	7.0%	0.9%	919,764	6.3%	0.4%
2007	718,756	7.4%	1.0%	943,877	6.7%	0.5%
2008	741,356	7.8%	1.1%	937,337	7.1%	0.6%
2009	757,696	8.5%	1.2%	966,857	7.6%	0.9%
2010	783,347	8.9%	1.4%	973,589	8.2%	1.0%
2011	782,771	9.7%	1.5%	1,181,498	8.9%	1.1%
2012	779,479	10.0%	1.6%	1,131,886	9.5%	1.2%
2013	773,645	10.4%	1.7%	1,128,322	9.7%	1.3%
2014	742,147	10.6%	1.8%	1,131,917	9.9%	1.4%
2015	758,768	10.8%	1.8%	1,086,702	10.2%	1.5%

Source: Adapted from the Statistical First Release - SFR 38 2015 Table 14 (see www.gov.uk/government/statistics/a-level-and-other-level-3-results-2014-to-2015-provisional)

It should be noted that the SFR statistics relate to all A or AS level entrants within a single academic year. This will mean that they will include a wide range of students including those who retake an A / AS level and students returning to study following a break from education.

Our analyses of A and AS level participation take a 'within cohort' perspective similar to that adopted by others (Noyes and Adkins, 2016).

Specifically, for the 'within cohort' perspective we follow a complete cohort of young people who took GCSEs at the end of Y11 in summer 2011 and are recorded as taking at least 1 Key Stage 5 assessment between 2011/12 and 2013/14 (these three years are highlighted in green in Table 2-1 above). Section 5 below provides more detail on how we measured A and AS level participation and compares this with the SFR statistics. In summary, what we found was that the SFR participation statistics notably understate the popularity of A and AS level Mathematics and

Further Mathematics. For Mathematics we found that 16.7% of the 2011 KS4 cohort who stayed in education are recorded as taking the A level (compared with between 10.0% and 10.6% from the SFRs) and 22.3% are recorded as taking the AS level (compared with between 9.5% and 9.9% from the SFRs). For Further Mathematics we found that 2.7% of the 2011 KS4 cohort who stayed in education are recorded as taking the A level (compared with between 1.6% and 1.8% from the SFRs) and 2.7% are recorded as taking the AS level (compared with between 1.2% and 1.4% from the SFRs).

Therefore arguably our 'within cohort' perspective provides the more accurate perspective on participation in A and AS level Mathematics and Further Mathematics because it reflects a structural reality of the English education system - students are commonly grouped into year groups (or cohorts) as they progress through educational Key Stages. Whilst this cohort grouping will begin to dissipate following Y11, our analyses revealed that it still remained strong in the three years following Y11.

Whether a 'within A / AS level' (SFRs) or a 'within cohort' approach is taken, the trends over time consistently show the growth in popularity of Mathematics and Further Mathematics at A and AS level. Table 2-2 presents the SFR statistics as odds-ratio changes over time.

Table 2-2 Participation in A & AS level Mathematics & Further Mathematics 2001-2015 (change over time)

Year	Of those who took 1+ A level:		Of those who took 1+ AS level:	
	Odds-Ratios			
	A level Mathematics	A level Further Mathematics	AS level Mathematics	AS level Further Mathematics
2000/01 to 2001/02	0.85	1	0.87	1
2001/02 to 2002/03	0.98	1	0.95	1
2002/03 to 2003/04	1.02	1.14	0.89	1
2003/04 to 2004/05	0.98	1	1.06	1.33
2004/05 to 2005/06	1.05	1.13	1.03	1
2005/06 to 2006/07	1.06	1.11	1.07	1.25
2006/07 to 2007/08	1.06	1.1	1.06	1.2
2007/08 to 2008/09	1.1	1.09	1.08	1.5
2008/09 to 2009/10	1.05	1.17	1.09	1.11
2009/10 to 2010/11	1.1	1.07	1.09	1.1
2010/11 to 2011/12	1.03	1.07	1.07	1.09
2011/12 to 2012/13	1.04	1.06	1.02	1.08
2012/13 to 2013/14	1.02	1.06	1.02	1.08
2013/14 to 2014/15	1.02	1	1.03	1.07
2000/01 to 2014/15	**1.41**	**2.6**	**1.36**	**5.06**

Source: Adapted from the Statistical First Release - SFR 38 2015 Table 14 (see www.gov.uk/government/statistics/a-level-and-other-level-3-results-2014-to-2015-provisional)

In 2014/15 A level students are 1.4 times as likely to have taken A level Mathematics and 2.6 times as likely to take A level Further Mathematics compared to A level students in 2000/01. In 2014/15 AS level students are 1.4 times as likely to have taken AS level Mathematics and 5.1 times as likely to take AS level Further Mathematics compared to A level students in 2000/01.

Noyes and Adkins (2016) found a similar upward trend using a 'within cohort' approach.

Potential reasons for changes in participation rates

Following a revision of A level Mathematics specifications in 2000 there was a drop in participation in A level Mathematics relative to cohort size seen in 2002. Following further revision of the specification, participation in A level Mathematics has increased year on year since 2003 (Matthews and Pepper, 2007, Noyes and Adkins, 2016).

Further Mathematics has historically been taken by a relatively small minority of students taking A level Mathematics. However, Further Mathematics too, has seen a sustained increase in the number of students taking Further Mathematics A level and AS level. The rate of increase has been greater than for A level Mathematics, thus the ratio of FM students to the number of A level Mathematics students has increased

In 2009, MEI undertook a survey of teachers for reasons for increase in A level Mathematics participation. Factors identified were grouped in the following categories: motivations due to university entry and career progression; the enhancement of a conducive learning environment for mathematics; curriculum influences in particular the appropriateness of the A level syllabus; and the reputation and encouragement for take up at school level (ACME, 2009). However, a recent analysis suggests that at least a significant contributory factor since 2009 and possibly the main reason is that more students are achieving the highest GCSE grades - A* and A. Prior attainment and in particular attainment at A* and A is the stronger predictor of A level Mathematics participation (DfE, 2011; Noyes, 2009; Noyes and Adkins, 2016) and the analysis presented in section 7 of this report supports this. This may be because in many centres students need an A* or A grade to take A Level Mathematics hence little change in the profile of A level cohort. The proportion of students taking A level Mathematics who have these grades has remained relatively stable (Noyes and Adkins, 2016) with the increase in A level participation following a similar pattern to the increase in achievement at the highest grades. However, these analyses do not allow inferences to made about the extent to which the motivation to take A level Mathematics is influencing students effort to attain the highest GCSE grades.

2.3 Patterns of participation

Although there is limited research specifically on reasons for participation in Further Mathematics there is a substantial body of literature on both mathematics participation beyond 16 and, related to this, participation in science subjects. Notable features about participation and in some cases outcomes in Advanced level Mathematics in England are described below.

Prior attainment

As stated above, the strongest predictive variable is prior attainment (DfE, 2011; Noyes, 2009; Noyes and Adkins, 2016) with the majority of A level participants achieving A* at GCSE. Further, prior-attainment relative to attainment in other subjects is significant (DfE, 2011; Noyes, 2013).

Gender

There are higher rates of participation in Advanced level Mathematics by males than females even when attainment is accounted for (DfE, 2011; Noyes, 2009; Noyes and Adkins, 2016). This is not found in similar cultures, for example the USA, (College Board, 2013). Thus indicating that this can be influenced (Smith, 2016).

Tables 2-3 and 2-4 summarise A and AS level entries in Mathematics and Further Mathematics for male and female students in England 2001 to 2015 taken from Governmental Statistical First Release publications.

Table 2-3 Participation in A level Mathematics & Further Mathematics 2001-2015 by gender

Year	A level Mathematics			A level Further Mathematics		
	M	F	M:F OR	M	F	M:F OR
2001	10.6%	5.6%	2.02	1.2%	0.4%	3.24
2002	9.3%	4.7%	2.05	1.1%	0.4%	3.02
2003	9.1%	4.7%	2.02	1.1%	0.4%	3.07
2004	9.1%	4.9%	1.96	1.2%	0.4%	3.08
2005	9.0%	4.7%	2.02	1.2%	0.4%	3.02
2006	9.3%	5.0%	1.98	1.4%	0.5%	2.85
2007	9.8%	5.4%	1.88	1.5%	0.5%	2.84
2008	10.2%	5.7%	1.88	1.7%	0.6%	2.7
2009	11.0%	6.4%	1.81	1.9%	0.7%	2.61
2010	11.5%	6.7%	1.8	2.0%	0.8%	2.52
2011	12.5%	7.2%	1.85	2.2%	0.9%	2.57
2012	13.1%	7.4%	1.89	2.4%	0.9%	2.76
2013	13.8%	7.5%	1.97	2.6%	0.9%	3.01
2014	14.4%	7.6%	2.05	2.8%	0.9%	3.06
2015	14.7%	7.6%	2.09	2.9%	0.9%	3.23

Source: Adapted from the Statistical First Release - SFR 38 2015 Table 14 (see www.gov.uk/government/statistics/a-level-and-other-level-3-results-2014-to-2015-provisional)

Table 2-4 Participation in AS level Mathematics & Further Mathematics 2001-2015 by gender

Year	AS level Mathematics			AS level Further Mathematics		
	M	F	M:F OR	M	F	M:F OR
2001	10.20%	5.50%	1.93	0.40%	0.20%	2.1
2002	8.90%	5.00%	1.85	0.40%	0.20%	2.09
2003	8.40%	4.80%	1.82	0.40%	0.20%	2.26
2004	7.60%	4.30%	1.84	0.40%	0.20%	2.25
2005	8.00%	4.50%	1.84	0.50%	0.20%	2.19
2006	8.10%	4.70%	1.79	0.60%	0.30%	1.98
2007	8.60%	5.10%	1.76	0.70%	0.40%	2.02
2008	9.00%	5.50%	1.71	0.80%	0.40%	2.05
2009	9.60%	6.00%	1.67	1.20%	0.60%	1.91
2010	10.30%	6.30%	1.69	1.30%	0.70%	1.97
2011	11.20%	6.90%	1.7	1.50%	0.70%	2.26
2012	12.10%	7.30%	1.75	1.70%	0.70%	2.36
2013	12.40%	7.30%	1.81	1.90%	0.80%	2.48
2014	13.00%	7.40%	1.88	2.10%	0.80%	2.68
2015	13.40%	7.50%	1.9	2.30%	0.90%	2.72

Source: Adapted from the Statistical First Release - SFR 38 2015 Table 14 (see www.gov.uk/government/statistics/a-level-and-other-level-3-results-2014-to-2015-provisional)

The percentages shown in Table 2-3 and Table 2-4 show the increasing popularity in A and AS level Mathematics and Further Mathematics between 2001 and 2015. In most cases this increase has been similar for males and females. A level (male or female) students in 2015 are 1.4 times as likely to take A level Mathematics and 2.5 times as likely to take A level Further Mathematics compared with A level students in 2001. A similar pattern is observed for AS level mathematics - male or female students in 2015 are 1.4 times as likely to take AS level Mathematics. For AS level Further Mathematics, the change is sharper for male compared with female students. Male AS students in 2015 are nearly 6 times as likely to take AS level Further Mathematics compared with male AS students in 2001. Female AS students in 2015 are 4.6 times as likely to take AS level Further Mathematics compared with female AS students in 2001.

Tables 2-3 and 2-4 also present gender differences within each academic year using odd-ratios. Given that the rise in participation is observed to be common for males and females in A level Mathematics and Further Mathematics and in AS level Mathematics, the gender differences are observed to remain largely static. Male students are observed to be twice as likely to take A level Mathematics, three times as likely to take A level Further Mathematics and 1.9 times as likely to take AS level Mathematics compared female students. For AS level Further Mathematics, a gender gap in participation is seen to widen from males being 2.1 times as likely to take the AS level in 2001 compared with females to males being 2.7 times as likely to take the AS level in 2015 compared with females.

Ethnicity

Differences in patterns of participation by ethnicity: Chinese, Indian, Black African, Bangladeshi and Pakistani students were more likely to enter A level Mathematics than White British students; of those who do take A level Mathematics Chinese and Indian students are more likely to achieve A and B grades than White British students and other ethnic groups less likely (DfE, 2011; Noyes, 2009).

Socio-economic factors

Those who are socio-economically disadvantaged (using eligibility for Free School Meals - FSM, as a proxy) are less likely to take A level Mathematics. For example in 2009 students eligible for FSM in England were only a quarter as likely to take A level Mathematics as their non-eligible peers (DfE, 2011). However, it appears that this is largely accounted for by differences in GCSE attainment that are also linked to FSM status (Noyes, 2009).

School level effects

Independent schools and selective maintained schools have a disproportionate number of A level entries (DfE, 2011). There is considerable 'between school variation' in participation and outcomes in A level Mathematics, including when other variables are accounted for. There are also indications that this is at least in part due to departmental level effects which are independent of success in 11-16 progression (Noyes, 2013). The implication of this is that participation rates and outcomes may be impacted by school and departmental level change.

Factors influencing participation

Prior research on choices about the study of STEM subjects including mathematics are influenced by a complex range of factors, a brief discussion of some of these is provided here. Many are external to students' educational experience including social, cultural and familial practices, knowledge and relationships that support engagement in science and mathematics post 16 (Archer De Witt et al., 2012; Archer, Dawson et al., 2015; Brown, Brown and Bibby, 2008; Homer et al., 2014; Reiss et al., 2011). Recently, the notion of 'science capital' - extending Bourdieu's concepts of social and cultural capital - has been posited as a means to conceptualise both the interplay of these different factors and their influence on patterns of participation (Archer De Witt et al., 2012; Archer, Dawson et al., 2015).

Important too is the influence of the experience of school mathematics with student disaffection from the subject being extensively investigated (see for example, Nardi and Steward, 2003; Brown, Brown and Bibby, 2008). Investigation of student choice of A level Mathematics indicates the positive influence of teachers and varied pedagogies (Golding and Smith, 2016). Important too are availability of A Level Mathematics and programmes and interventions to support participation (Lord & Stripp, 2014; Searle, 2012, 2014).)

Others have focused on the way mathematics itself is constructed both in school and in society. This has been an important area of interest particularly for those investigating gender differences in participation and relationship to mathematics (see for example, Boaler, Alterndorff and Kent, 2011; Mendick, 2006), examining specifically A level participation and gender posit the notion of masculinities of mathematics, identifying different core reasons for choosing A level Mathematics of a sample of students. One of these reasons, for a minority, was being good at it embedded in a discourse of effortless achievement, though interestingly some of her research participants took A level Mathematics and Further Mathematics even though they did not describe themselves as good at mathematics. Others took mathematics either to prove something to themselves or to others.

3. The Further Mathematics Support Programme

3.1 Overview

The Further Mathematics Support Programme (FMSP), funded by the DfE and managed by MEI, continues the work of the Further Mathematics Network (2005-2008). FMSP was established in 2009 and addresses the need to increase the participation in Advanced level study of mathematics post-16, including Further Mathematics. The FMSP programme is based on the principle that all state-educated students should be able to access the mathematics education they need to fulfil their aspirations. It aims to:

- increase participation in AS/A level Mathematics and Further Mathematics, particularly that of girls and those from other under-represented groups
- increase capacity within schools to provide high quality mathematics teaching
- increase demand from students to study AS/A level Mathematics and Further Mathematics post-16
- support improvements in Level 3 mathematics education

To further these aims, the FMSP programme goes beyond focusing only on AS/A level Further Mathematics and A level Mathematics qualifications. It also seeks to address how to engage KS4 and other learners in developing an enthusiasm for and positive attitude to the subject, through targeted professional development and other programmes.

The FMSP has developed organically, as funding has grown over time, with specific activities being added to the programme to address identified needs or interim outcomes in relation to programme aims. The relative importance of activities and number participating has changed over time. There has been a refreshed programme in place since April 2014 and this is described below.

The FMSP is now delivered by an extended partnership of MEI, NCETM (and Tribal), and the University College London Institute of Education, with some programme components led by members of the partnership. It has a central team of 17, and 32 part-time Area Coordinators across the 9 regions of England, supported by 6 Assistant Area Coordinators. In addition, there are over 200 FMSP associates who support the area coordinators and contribute to activities such as tuition and CPD courses, as and when necessary.

3.2 Components of the FMSP programme

Tuition support for AS/A level Further Mathematics

The FMSP provides a variety of direct support for tuition of AS/A level Further Mathematics students. Tuition support is used by centres and students in different ways:

- to study Further Mathematics in centres where Further Mathematics is not offered due to difficulties timetabling or staffing it
- to study Advanced Level modules that are not offered by the students' centres to provide access to a broader A level Further Mathematics curriculum particularly in applied mathematics
- as a supplementary learning experience

Current forms of tuition are:

- Further Mathematics Tuition: The area coordinator, or an FMSP associate, visiting a centre to teach a small Further Mathematics group from that centre or students from different sites attending a central venue (e.g. a host centre or university)

- Live Interactive Lectures (LIL): A series of 7-12, fortnightly lectures in an online classroom by an FMSP tutor. Designed to enhance in-house provision where there may be limited contact time or limited teacher expertise. Centres support their own students between sessions with study materials supplied by the FMSP. Centres receive access to materials on the Integral Virtual Learning Environment (see below) and other support materials. It is low cost at £50 for the first student and £10 for each additional student per unit

- Live Online Tuition (LOT): A series of between 7-12 LIL sessions and 7-12 online tutorial sessions (groups of 1-6) taught by an FMSP tutor who is responsible for the marking and support of the students

Continuing Professional Development

Professional development activities for teachers to support teaching of Further Mathematics and A level Mathematics, as well as professional development focused on GCSE mathematics that fosters teaching approaches for younger students to prepare them for further study and increase motivation to study mathematics.

A level professional development and teacher support

- Live online PD (LOPD): A series of online sessions improving subject knowledge and pedagogy in a particular area of A level Mathematics or Further Mathematics
- One day A level PD course: Typically a one day course organised by an Area Coordinator to improve teacher subject knowledge in Pure Mathematics, Mechanics, Statistics or Decision Mathematics
- Access to Further Mathematics Conferences: These take place annually in March, teachers attend for Friday and Saturday to look at strategies for offering Further Mathematics and the support available
- Teaching Advanced Mathematics (TAM): Year long, funded university accredited CPD courses for teachers wishing to teach A level Mathematics or develop their skills in doing so
- Teaching Further Mathematics (TFM): Year long, CPD courses for teachers wishing to teach Further Mathematics or to improve their capacity to do so (accreditation optional)
- Supporting STEP/AEA/MAT: A one day course for teachers, sharing resources and approaches supporting students preparing for STEP/AEA/MAT[2] exams and generally developing problem solving in the sixth form
- FMSP Teacher Networks: Typically a termly twilight meeting for local teachers focusing on sharing practice or a CPD session, organised by FMSP Area Coordinator or Associate.

GCSE professional development and teacher support

- KS4 Extension and Enrichment: A two day course for teachers, sharing resources and approaches for enhancing the teaching of GCSE
- GCSE Higher Tier: A one day course for teachers, sharing resources for teaching the new content in Higher tier GCSE

[2] STEP: Sixth Term Examination Papers in Mathematics are university admissions tests for undergraduate Mathematics courses developed by the University of Cambridge and required or encouraged by a number of Universities. AEA: Advanced Extension Award in Mathematics offered by Edexcel examination board. MAT: Mathematics Admissions Test developed by Oxford University and used for admission to Oxford and Imperial College London's mathematics degrees.

- KS4 problem solving: A one day course for teachers, sharing resources and approaches for enhancing problem solving at GCSE.

Enrichment

The FMSP offers or supports a range of enrichment activities that aim to increase demand from students to study AS/A level Mathematics and Further Mathematics post-16, by enriching the experience of learning mathematics and supporting positive engagement in mathematics.

A level student support and enrichment

- One day events for year 12 and 13 students: Promoting problem solving, STEP/AEA/MAT preparation and sometimes looking at entry to HE
- Senior Team Mathematics Challenge (STMC): A team competition in November for a team of 4 sixth form students answering mathematics problems, run in partnership with United Kingdom Mathematics Trust (UKMT)
- Regular problem solving or STEP classes: Students attending weekly or fortnightly classes at a local university or other institution to get help with preparing for STEP/AEA/MAT exams or problem solving practice more generally
- Revision videos: students can use free revision videos available via the FMSP website and YouTube
- A level Revision event: Students attend a revision day at a university to go over content in A level Mathematics or Further Mathematics modules

KS4 student support and enrichment

- KS4 enrichment events: One day (or half day) events at a university or centre offering workshops, enrichment sessions, challenges and some guidance on careers using mathematics
- Year 10 Maths Feast: A mathematics team competition for year 10 students which takes place in February and March
- Year 10 and 11 enrichment talks: Area Coordinator or Associate visits to centres to talk to students about studying A level Mathematics and Further Mathematics.
- GCSE revision days for A/A*: Students attend a revision day at a university or centre for help preparing for GCSE examinations.

Regional support

Area Coordinators coordinate regional activity and offer tailored face to face support and advice to teachers and mathematics leaders in relation to FMSP aims and provision.

On-line support

- Integral virtual learning environment containing online resources: All registered centres can access teaching materials for A level Further Pure Module and Application modules – Mechanics, Statistics and Decision Mathematics
- FMSP Website: Access to information and resources about teaching and enriching A level Mathematics and Further Mathematics as well as details of events for students and teachers

3.3 Previous evidence of quality and impact of activities

Evaluation of previous phases of the FMSP programme provided evidence that:

- participating teachers view both online and face to face professional development as worthwhile and effective
- the various forms of tuition support participation in FM for students who otherwise might not be able to access it
- tuition is flexible and used by centres in a variety of ways to support students
- the FMSP has succeeded in registering most centres which have post-16 provision and more than half of 11-16 centres
- A level participation in Further Mathematics has steadily grown
- more than half of centres who have A level provision now offer Further Mathematics
- take up of Further Mathematics is associated with a general increase in A level Mathematics participation
- the Area Coordinator network provides effective support to centres
- registered users make flexible use of centrally provided materials via the FMSP Integral site
- the FMSP is well regarded by a range of stakeholders

Methods used in previous evaluations have included analysis of quantitative data, visits to programme events, surveys with students and with stakeholders.

4. Evaluation Methodology

4.1 Overview

Phase 5 of the evaluation, reported here, builds and extends previous findings of the evaluation of the FMSP by:

- reviewing the A level mathematics and Further Mathematics landscape reporting a quantitative analysis of centre and student participation in Further Mathematics through analysis of School Census data and data from the National Pupil Database supported by qualitative data on student and teacher views on participation in Further Mathematics
- analysing participation in the Further Mathematics Support Programme through analysis of School Census data, the National Pupil Database and internal data held by the FMSP, supported by data from telephone and face to face interviews with teachers, A level Mathematics and Further Mathematics students and Maths Hub leads
- considering enablers and barriers to participation in Further Mathematics posits a model of Further Mathematics culture, capital and ecology to guide further development in promoting post compulsory mathematical study at Advanced levels.
- reporting on the extent to which the FMSP is successful in meeting its aims in terms of capacity and capability building, reach, effectiveness and sustainability
- identifying issues to consider both for future evaluation and more importantly for the Further Mathematics Support Programme

The four key focus areas for the 2014-16 evaluation were to consider:

1. **Capacity and capability** building
2. **Reach** to centres, teachers and students
3. **Effectiveness** of the programme (quality and impact)
4. **Sustainability** beyond the end of the programme

The aims of the evaluation, mapping to these focus areas, were as follows:

(i) analysis of participation and engagement in FMSP (reach)

(ii) modelling the Further Mathematics landscape (capacity and capability)

(iii) qualitative evaluation of the FMSP (all four aims)

These were underpinned by the development of an underlying theory of change.

The evaluation also had three specific methodological aims:

(iv) to assess the viability and value of quantitative modelling of Further Mathematics and FMSP activity, including identifying any relevant issues in drawing together disparate databases

(v) to provide a quantitative baseline for future evaluation

(vi) to develop tools for future evaluation including assessing the value of focus group interviews with students as a data collection method.

4.2 Design

A mixed methods (Johnson, Onwuegbuzie, & Turner, 2007) evaluation design was developed with the FMSP to address these foci and aims, and further refined as evaluation activity proceeded. The study's mixed methods approach took a pragmatic position of aiming to answer the research questions with the most appropriate set of methods, rather than a 'paradigmatic' stance of coming from a particular philosophical standpoint. Underpinning the mixed methods design was an FMSP 'theory of change', developed in the early stages to inform the evaluation including sampling.

Research was conducted in accordance with institutional ethical approval and in keeping with the guidelines of the British Educational Research Association, with informed consent obtained from participants in qualitative data collection activities.

The evaluation used a mixed methods research design comprising:

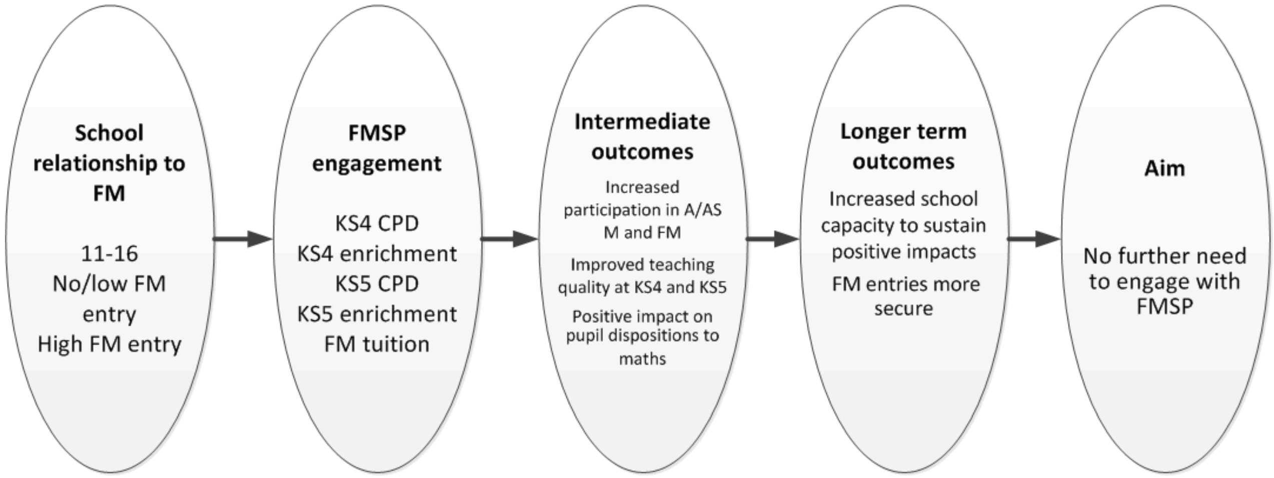
- quantitative analysis of centre and student participation and attainment in A and AS level Further Mathematics and Mathematics
- analysis of School Census data and data from the National Pupil Database, and of internal data held by the Further Mathematics Support Programme on participation in FMSP activities
- qualitative analysis of data from telephone and face to face interviews with teachers and focus groups with A level Mathematics and Further Mathematics students, supported by a survey of Maths Hub leads

4.3 The initial FMSP theory of change

As described in Section 3, the FMSP is a complex programme with a large number of components. To inform the evaluation, members of the FMSP central team worked with the evaluation team to develop an understanding of the interaction of different programme components with each other and their relationship to the programme aims. This understanding is represented visually in a simple 'theory of change' (Blamey and Mackenzie, 2007) diagram below. The complexity of the relationships between elements in the model, and the different emphases for different school types (11-16, no/low FM entries, high FM entries) were worked through, and are presented in Annexe 1.

Note that an important part of the FMSP's aim is for centres' Further Mathematics provision to become self-sustaining. Thus, a positive outcome for the FMSP, as currently conceived, is for a centre to cease to engage with the programme, or at least components of it, when they develop their capacity for self-sustaining improvement.

Figure 4-1 Overarching theory of change



4.4 Analysis of participation and engagement in Further Mathematics and FMSP

Analysis of the school level data focused firstly on developing a security construct as a measure of the extent to which the programme is effective in developing capacity and capability as well as sustainability. The number of A level Further Mathematics entries in a centre was reviewed over a 3 year period in order to consider stability of entries over time. Following discussion with the

FMSP and building on previous evaluations (Searle, 2014), boundary points of 3 and 10 entries were adopted. From this, five levels of "security" were developed as follows (see section 5):

- none - no entries of A level Further Mathematics in all 3 years
- least secure - one or more years with 0 entries and only 1 or 2 in any other year
- less secure - 3 or more entries in any one year two or fewer in other years
- relatively secure - 3 or more entries in all 3 years
- highly secure - 10 or more entries in all 3 years

Security characteristics were calculated as follows:

- security status for 2013 : based on Further Mathematics entries for 2012/13, 2011/12, 2010/11
- security status for 2014 : based on Further Mathematics entries for 2013/14, 2012/13, 2011/12
- security status for 2015: based on Further Mathematics entries for 2014/15, 2013/14, 2012/13

The security construct was cross-tabulated with school characteristics (including school type and school contextual factors) to model how A level Further Mathematics entries vary according to school characteristics.

Datasets held by the FMSP containing records of centre involvement in CPD, enrichment and tuition were brought together and merged into the longitudinal centre census data. This allowed an exploration of the relationship between engagement in FMSP and school characteristics, and of the relationship between the FMSP and the security constructs (see section 5/Annexe 1 for detail).

4.5 Modelling the Further Mathematics landscape

Student participation and attainment in A level Mathematics and A level Further Mathematics were modelled using multivariate multilevel regression. Data from the National Pupil Database (NPD) was retrieved and used for the complete cohort of students in England who (a) completed Key Stage 4 in 2010/11 and (b) are recorded as taking a KS5 assessment between 2011/12 and 2013/14. Engagement with the FMSP was brought in as a school level explanatory variable. The stages of this phase were:

- descriptive analyses of participation and attainment in A and AS level Mathematics and Further Mathematics to identify outcome variables
- identification of explanatory variables at the student[3] and school/institution levels and descriptive analyses of how they are statistically associated with participation and attainment in A and AS level Mathematics and Further Mathematics
- multivariate multilevel analyses of participation and attainment in A level Mathematics and Further Mathematics

Statistical analyses of student participation and attainment in A and AS level Mathematics and Further Mathematics was undertaken. Using data obtained from the National Pupil Database (NPD), these analyses focused on the cohort of young people in England who completed KS4 at the end of Y11 in summer 2011 (N=637,594) linked to NPD KS5 data for the 2011/12, 2012/13 or 2013/14 academic years to form a longitudinal data file.

In total, 65% (N=412,743) of the 2011 KS4 cohort were recorded as taking at least one KS5 assessment between 2011/12 and 2013/14. This created a 4-year longitudinal data file which we named the '2011 KS4 NPD cohort'. This 4-year longitudinal cohort approach reflects that taken by other research (Noyes and Adkins, 2016).

Descriptive statistical analyses were undertaken to explore potential outcome and explanatory variables and to select those for inclusion in the multivariate, multilevel analyses of A level participation and attainment. Participation in A level Mathematics and Further Mathematics was modelled using multilevel logistic regression. Attainment in A level Mathematics and Further Mathematics was modelled using multivariate multilevel modelling. The modelling approach and the model specification can be found in Annexe 2.

4.6 Qualitative evaluation of the FMSP

Teacher interviews

A total of 42 telephone interviews were conducted[4] with teachers from centres with a range of Further Mathematics entries. The sample was drawn from the FMSP database, focusing on

[3] At Key Stage 4, the term 'pupil' is appropriate and widely used but the 'student' term is more appropriate for young people who study at Key Stage 5 or beyond. The 2011 KS4 NPD data file spans Key Stages 4 and 5 - which explains the use of the 'pupil / student' description.

[4] One interviewee was interviewed twice.

centres which had records of involvement in one or more of the FMSP activities. A total of 21 interviews were conducted in Spring term 2015 and a second set of 21 interviews were conducted in the Autumn term of 2015. The second set consisted of 16 telephone interviews and 5 teacher interviews conducted in the case study centres alongside the student focus groups. The second wave of interviews were conducted after the development of the security status construct and analysis, and focused on those who had experienced a "change" in security status. As part of the analytical process the security rating was calculated of centres sampled in the first wave of interviews. Annexe 1 describes in detail the relationship of the sample to security and FMSP involvement for 2013/14. In summary, from FMSP data at the point of sampling 28 had engaged in CPD, 17 in enrichment and 5 in tuition. It is important to note that involvement is for a single year and centres may have participated in FMSP in other years, this was confirmed by analysis of the telephone interviews. Teachers were asked about their centres' participation in Mathematics and Further Mathematics A level, and progression to A level Mathematics from GCSE. They were also asked about their involvement with the FMSP including their views on the programme, implementation in their centre and its effectiveness. Teacher interviews were analysed thematically (Ryan and Russell Bernard, 2003).

To supplement the teacher interviews a short survey including some open questions was conducted with Maths Hub Leads to gain their views on how the FMSP and the Hubs currently work together and priorities for the coming year. The survey was sent to the Leads in all 35 Maths Hubs with 25 (71%) responding.

Student engagement - outcomes and choices

A total of five student focus groups were conducted in centres. Given this relatively small sample conclusions drawn are tentative, one purpose of including this activity was to evaluate focus groups as an evaluation method for FMSP, which might be used more extensively in the future.

The purpose of the student focus group strand was to explore the decision making and experiences of FMSP for young people who had chosen to study Mathematics and Further Mathematics at A level in centres where the numbers were low and Further Mathematics security was at potential risk. Based on their security status, a sample of centres were identified as being least secure or as less secure in terms of their numbers of Further Mathematics entries over recent years.

Five centres from Yorkshire, Humberside and the Midlands agreed to take part in student focus groups which were conducted during visits to the centre between December 2015 and February 2016. Focus groups included a mix of students in Y12 and Y13 and those studying both A level Mathematics and Further Mathematics.

In total, 44 students took part in the focus groups across the five centres. The priority was to include as many Further Mathematics students from each centre as possible, as well as a broad cross section of those studying A level Mathematics with other subjects.

Of the 44:

- 25 were studying A level Mathematics *and* FM
- 19 were studying A level Mathematics (with other subjects, not FM)
- 61% of the overall sample was male (27/44), 39% were female (17/44)
- 4 students (3 male and 1 female) from BME backgrounds (c.10%)

The A level Mathematics /Further Mathematics lead in each centre was also interviewed during the focus group visit (or in one case, over the telephone), and reference to information provided by the teacher is included in analysis of the student focus groups (section 8).

The interview tool for the student focus group included a 'Diamond 9' decision-making exercise for ranking their reasons for choosing A level Mathematics/Further Mathematics; a timeline for students to map the key moments in their decision making and questions on their experiences of the FMSP enrichment and tuition (see Annexe 3).

4.7 Synthesis of evaluation data

As a final stage in the evaluation, relationships between quantitative and qualitative findings were analysed by looking across data sets and findings, using the pragmatic mixed methods approach outlined above, focused on evidence in relation to capacity and capability, reach, effectiveness and sustainability. This method includes but goes beyond a more standard 'explanatory' approach (Johnson and Turner, 2003) whereby qualitative interview data are used to help illuminate and help interpret quantitative findings: additionally, we treat qualitative data and quantitative data as being particularly suited to answering different questions, not merely acting as adjuncts to one another. Therefore, our synthesis uses combinations of different forms of quantitative data; combinations of different forms of qualitative data; and combinations of qualitative and quantitative data, with the choice of data to be synthesised dependent on the specific research issue; in this case, capacity, reach, effectiveness and sustainability. By drawing on theoretical constructs of science capital and previous empirical and theoretical research, a revised model of Further Mathematics culture, capital and ecology was developed. Thus, again the overall methodology accords with that suggested by adaptive theory (Layder, 1998). Final evaluation focused on evidence in relation to capacity and capability, reach, effectiveness and sustainability.

5. Security of school participation in Further Mathematics

5.1 State establishments that offer Further Mathematics

Since the FMSP began the number of state establishments that offer Further Mathematics has increased both in absolute terms and as a proportion of those that offer A level Mathematics. Since 2004 there has been a 10% increase in the number of establishments offering A level Mathematics but an 87% increase in the number offering Further Mathematics (Table 5-1).

Table 5-1 State establishments that offer Further Mathematics

	State establishments that offer Further Mathematics	State establishments that offer A level Mathematics	Percentage access to Further Mathematics
2004/05	762	1926	40%
2005/06	882	1904	46%
2006/07	962	1896	51%
2007/08	1059	1882	56%
2008/09	1131	1893	60%
2009/10	1171	1874	63%
2010/11	1226	1960	63%
2011/12	1303	1994	65%
2012/13	1306	2048	64%
2013/14	1371	2071	66%
2014/15	1428	2115	68%

Data source: For recent years from DfE website (i.e. 16-18 qualification and subject line data: www.education.gov.uk/schools/performance/download_data.html) prior to this direct from NPD when MEI were granted access to it.

5.2 Developing the Further Mathematics secure construct

A "security status" for schools was constructed that considered the stability in the number of A level Further Mathematics entries over time. Security is taken to be an interaction of numbers of entry and entries over time. It is important to note that teachers' perceptions of the security of Further Mathematics entries in their school may not accord with the classification used here. For example, one teacher interviewed from a school that had moved from less secure to least secure, considered Further Mathematics entries to be stable providing FMSP tuition was available and the reduction in number of entries a result of being more selective.

In addition, the 'no Further Mathematics entries' category in particular does not record centres where there are students who have begun to study Further Mathematics but have not yet been entered, or centres where there is study of Further Mathematics AS level but this does not lead to A level. Nevertheless, the security construct developed here is a means to consider changes in stability of entry as well as giving more detailed insight into patterns of school characteristics and engagement with the FMSP in relation to, what are described later as, Further Mathematics cultures.

For the purpose of developing an initial definition, three years of census data were considered - 2011/12, 2012/13, and 2013/14. In discussion with the FMSP, a measure of security was constructed using three threshold measures to look at changes over time (in Further Mathematics A level entries) between 2011/12 and 2013/14. A general important feature of Further Mathematics entries and the FMSP impact is that influences are long term and are not easily seen by considering a single year. This perspective is also supported by the pilot telephone interviews.

Following analysis of the pattern of entries, as well as consideration of what level of entry might be considered to be stable, the key marker was taken to be 3 or more entries. Note that the choice of 3+ marker has been supported so far by the interview data where both schools sampled had small cohorts. Fewer than 3 will be subject to vagaries of cohorts and less commitment to FM. In addition, the pattern of entries where 10 or more students were entered was also considered.

Using data from the KS5 school level census for 2011/12, 2012/13 and 2013/14, schools were grouped into categories of Further Mathematics security under the "status" column as shown in Table 5-2 below. Note that at this point the analysis focused on the state sector only, therefore independent schools are excluded in the below table. The total number of state schools was 2256 who have complete records over the 3 years.

Table 5-2 Towards a Further Mathematics A level 'secure' scale (state sector only)

Number of A level Further Mathematics entries over 3 years (2011/12, 2012/13, 2013/14)	**n**	**%**	**Assigned Security Status**
No entries in all 3 years	674	29.9	None
Two years with 0 entries, 1 or 2 in other year	226	10.0	Least secure
One year with 0 entry, 1 or 2 in other year	114	5.1	Least secure
3 or more in any one year, low or none in other years	216	9.6	Less secure
At least 1 in all 3 years, no more than 3 in any year	44	2.0	Less secure
At least 1 in all 3 years, 3 or more in any one year	310	13.7	Less secure
Three or more entries in all 3 years	530	23.5	Relatively secure
10 or more in all 3 years	142	6.3	Highly secure
Total	**2256**	**100.0**	

Table 5-3 below also summarises these definitions, separating out the "none" category into schools whose A level Mathematics entries are less than or equal to 3 over the 3 years or none, and A level Mathematics entries which are greater than 3 over the 3 years.

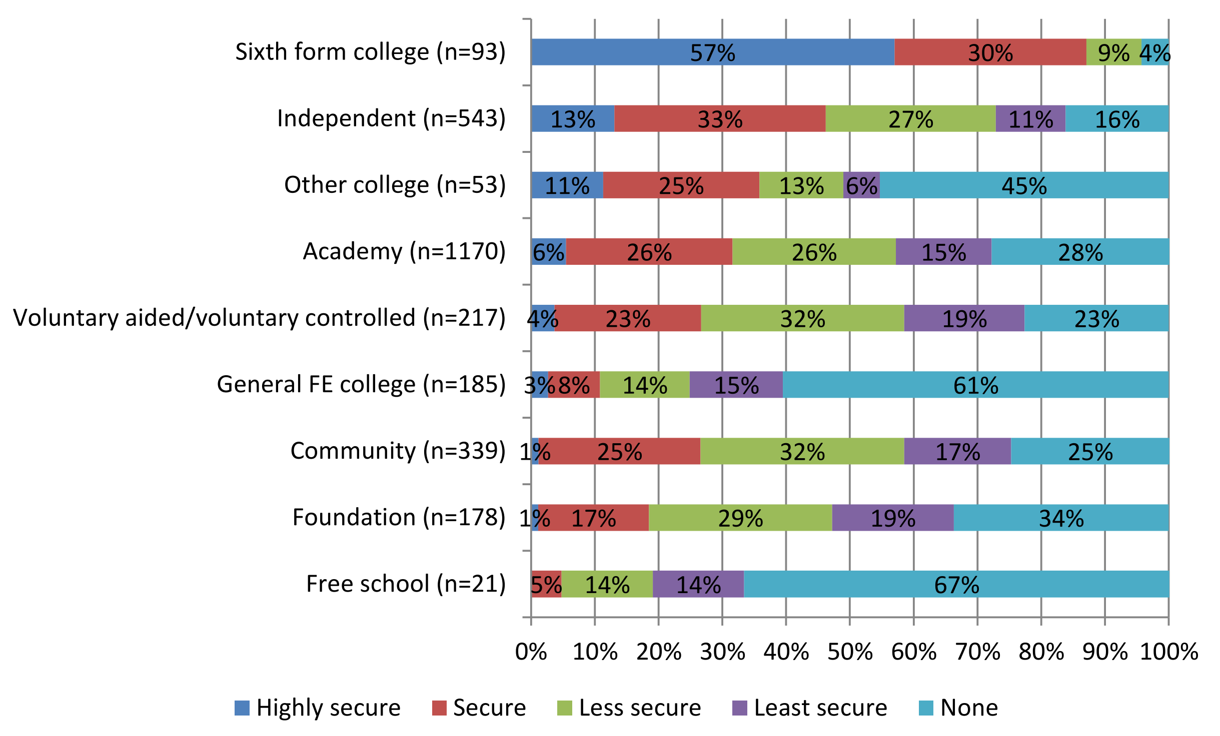
Table 5-3 Security status of schools for 11/12, 12/13 and 13/14 data (maintained schools only)

Tertiary with no AL FM A level Mathematics less than or equal to 3 over the 3 years or no entries for A level Mathematics	Tertiary with no AL FM A level Mathematics greater than 3 in total over the 3 years	Further Mathematics Less secure Low numbers of Further Mathematics and/or recently were in priority category		Further Mathematics Secure A level and Further Mathematics numbers stable		**Total**
		Least secure	Less secure	Relatively secure	Highly secure	
239	435	340	570	530	142	**2256**

5.3 School characteristics and Further Mathematics security

As can be seen from Figure 5-4 sixth form colleges appear to have the highest level of entries of Further Mathematics students; 57% of the 93 sixth form colleges have a security rating of "highly secure". Nearly half (46%) of independent schools have schools defined as "highly secure" or "secure". KS5 location and its impact on entries for Further Mathematics is discussed in more detail in the modelling in section 6.

Figure 5-4 Relationship between school type and security status (school level)



Using data from 2013/14, 2012/13 and 2011/12 entries, security status for 2013/14 was considered in relation to school/college characteristics. The following was found:

- selective schools are most likely to have secure numbers of students taking Further Mathematics A level
- schools with higher numbers of Further Mathematics students have lower levels of students eligible for FSM
- there is little difference in Further Mathematics uptake amongst schools with varying proportions of students with EAL
- schools with higher proportions of students taking Further Mathematics have a slightly lower proportion of students with SEN or SAP

- schools with a male only intake (at KS5) have significantly higher numbers of students taking Further Mathematics compared with female only schools and mixed schools. This is also reflected when looking at the average proportion of males on roll compared with average proportion of females on roll: highly secure schools have an average of 70% of boys on roll and 30% of girls on roll, whereas for relatively secure schools and less secure schools the average percentage of boys on roll and girls on roll is very similar (close to 50/50)
- higher attaining schools at KS4 (based on % attaining 5+ A*- C including English and maths GCSEs) have higher proportions of students doing A level Further Mathematics
- similarly schools with higher average point scores at KS5 have a larger proportion of students doing Further Mathematics thus it is also true that highly and relatively secure schools have higher average points score at KS5.

5.4 Change in Further Mathematics security status

Security status was calculated for a further two periods of 3 years and patterns of change identified. The table below (Table 5-5) summarises changes from 2012/13 (based on 2010/11, 2011/12, 2012/13 entries) to 2014/15 (based on 2012/13, 2013/14, 2014/15 entries):

Table 5-5 Change in security 2013 to 2015

2012/13 (2010/11, 2011/12, 2012/13 data)	**2014/15** (2012/13, 2013/14, 2014/15 data)						
		None	Least secure	Less secure	Relatively secure	Highly secure	**Total**
	None	487	104	56	0	0	**647**
	Least secure	76	132	108	0	0	**316**
	Less secure	13	56	334	138	1	**542**
	Relatively secure	0	0	91	368	35	**494**
	Highly secure	0	0	0	28	107	**135**
	Total	**576**	**292**	**589**	**534**	**143**	**2134**

- in total 706 centres experienced a change in status (33% of the total)
- 442 centres moved towards a more secure status (63% of those experiencing a change in status)
- of the 647 centres who had no entries for A level Further Mathematics in 2010/11, 2011/12 or 2012/13, 160 (25%) moved into the more favourable categories of "least secure" and "less secure"
- of the 316 centres in the "least secure" category in 2012/13, 108 (34%) moved into "less secure" whilst 76 (24%) moved into "none"

Table 5-6 below summarises the frequency of security status for 3 years 2013, 2014 and 2015 for schools for which there is data across all five years (2010/11 to 2014/15)[5]. This and the change in security status shows a general trend towards increasing security over time.

Table 5-6 Frequency of security status 2013, 2014 and 2015

	2013		2014		2015	
	n	%	n	%	n	%
None	647	30.3	592	27.7	576	27.0
Least secure	316	14.8	321	15.0	292	13.7
Less secure	542	25.4	551	25.8	589	27.6
Relatively secure	494	23.1	528	24.7	534	25.0
Highly secure	135	6.3	142	6.7	143	6.7
Total	2134	100.0	2134	100.0	2134	100.0

The general pattern of movement of centres towards a more secure status reflects the general increase in numbers of Further Mathematics students. However, the analysis shows that this increase is not restricted to centres that already had relatively high levels of Further Mathematics entries. Further, the increase in entries is not merely due to an increase in the number of centres offering Further Mathematics.

The analysis also reveals that the number of entries is less stable in less secure centres; there is a continuing fragility of entries in some of these centres. In summary, there has been an increase in both capacity and sustainability of Further Mathematics provision, although there is much potential for further improvements.

[5] Thus it does not represent the overall increase in number of centres offering Further Mathematics A level.

6. Modelling student participation and attainment in A level Mathematics, Further Mathematics and the FMSP

In this section the approach to modelling student participation and attainment is briefly described and key findings from the modelling reported. Annexe 2 provides details of both the modelling methodology and more detailed findings.

6.1 Measuring participation and attainment in A level Mathematics and Further Mathematics

Data from the National Pupil Database were used to analyse participation in A level Mathematics and Further Mathematics for the pupil cohort in England who completed KS4 / Y11 in 2010/11 (N=637,594) and were recorded as taking at least one KS5 assessment in 2011/12, 2012/13 or 2013/14 (this was the case for 64.7% of the 2010/11 KS4 cohort, N=412,743).

Multivariate, multilevel models of participation and attainment in A level Mathematics and Further Mathematics were developed. The models were used to examine differences and similarities between participation and attainment in A level Mathematics and Further Mathematics by considering the following explanatory variables:

- prior attainment at KS4 (overall and in mathematics)
- gender
- whether a pupil was eligible for, and parents claimed for Free School Meals (FSM) in 2010/11 (as a rough proxy for socio-economic position)
- ethnicity
- institution type
- engagement with a subset of FMSP activity for which data was available

6.2 Modelling participation and attainment

Prior attainment

As in previous research, prior KS4 attainment was found to be the main influence in determining participation and attainment in A level Mathematics and Further Mathematics:

- attaining A* in KS4 Mathematics is important in determining participation in A level Mathematics and Further Mathematics, but it is more important for Further Mathematics

- o students with A* in GCSE Mathematics were found to be 9 times as likely to take A level Mathematics and 19 times as likely to take A level Further Mathematics compared with students who attained lower than A*

- attaining A* in GCSE Mathematics is important in determining attainment in A level Mathematics and Further Mathematics, but it is more important for Mathematics. This reflects greater variation in GCSE Mathematics attainment amongst students who took A level Mathematics (52% attained A* at GCSE) compared with students who took A level Further Mathematics (84% attained A* at GCSE).
  - o once other factors are controlled for, attaining A* in GCSE Mathematics is associated with a +18.5 UCAS point[6] advantage for A level Mathematics and +8.6 UCAS points advantage for A level Further Mathematics.

Gender

In terms of gender, a male bias in A level Mathematics and Further Mathematics participation was found - which was stronger in A level Further Mathematics. This bias increases when the relatively higher KS4 attainment of females taking A level Mathematics or Further Mathematics is controlled for in the model - females were observed to not participate in A level Mathematics or Further Mathematics in the proportions that would be expected given gendered patterns of attainment at GCSE. A male bias in A level Mathematics and Further Mathematics attainment was also found - but this was stronger in A level Further Mathematics. This male A level attainment advantage did not emerge until the relatively higher KS4 attainment of females was controlled for in the models:

- once KS4 attainment and other factors were controlled for, females were observed to be 35% as likely to take A level Mathematics and 22% as likely to take A level Further Mathematics compared with males
- once KS4 attainment and other factors were controlled for, males were observed to have a +6.5 UCAS points advantage in A level Mathematics and +4.1 UCAS points advantage in A level Further Mathematics

[6] UCAS points are calculated by attaching a value to an A or AS level grade. At A level, the following points are applied: grade A* (140 points); A (120); B (100); C (80); D (60); E (40). At AS level, the following points are applied: grade A (60); B (50); C (40); D (30); E (20). See www.ucas.com/ucas/undergraduate/getting-started/entry-requirements/tariff/tariff-tables/946

Gender was found to significantly interact with GCSE Mathematics attainment in determining participation in A level Mathematics and Further Mathematics. Attaining A* in GCSE Mathematics is important in determining participation for males and females - but was found to be more important for females. Gender also was found to significantly interact with GCSE Mathematics attainment in determining attainment in A level Mathematics but not in A level Further Mathematics, but this was found to be more important for males.

- males who attained A* in GCSE Mathematics were 9.1 times as likely to take A level Mathematics and 19.0 times as likely to take A level Further Mathematics compared with males who did not attain A*
- females who attained A* in GCSE Mathematics are 10.5 times as likely to take A level Mathematics and 38.5 times as likely to take A level Further Mathematics compared with females who did not attain A*
- males who attained A* in GCSE Mathematics had a +18.5 UCAS point advantage in A level Mathematics attainment compared with males who did not attain A*
- females who attained A* in GCSE Mathematics had a +17.8 UCAS point advantage in A level Mathematics attainment compared with females who did not attain A*

Free School Meals

In terms of FSM, once KS4 attainment and other factors were controlled for, students who were identified as eligible and claiming FSM in 2011 were slightly (but significantly) less likely to take A level Mathematics compared with their non-FSM peers. For A level Further Mathematics, once KS4 attainment and other factors were controlled for, participation was not significantly different for FSM and non-FSM students. Prior attainment at KS4 was seen to account for most of the differences in participation between FSM and non-FSM students in A level Mathematics and nearly all of the difference in A level Further Mathematics. Because of their much lower attainment in GCSE Mathematics, FSM students are less likely to participate in A level Mathematics and Further Mathematics. Once KS4 attainment and other factors were controlled for, FSM students attained slightly (but significantly) lower in A level Mathematics and Further Mathematics compared with their non-FSM peers. Again, prior attainment at KS4 was seen to account for most of the differences in attainment between FSM and non-FSM students in A level Mathematics and Further Mathematics.

- at KS4, in 2011 non-FSM students were over 6 times as likely to attain A* and over 4 times as likely to attain A or A* in GCSE Mathematics compared with their FSM peers:

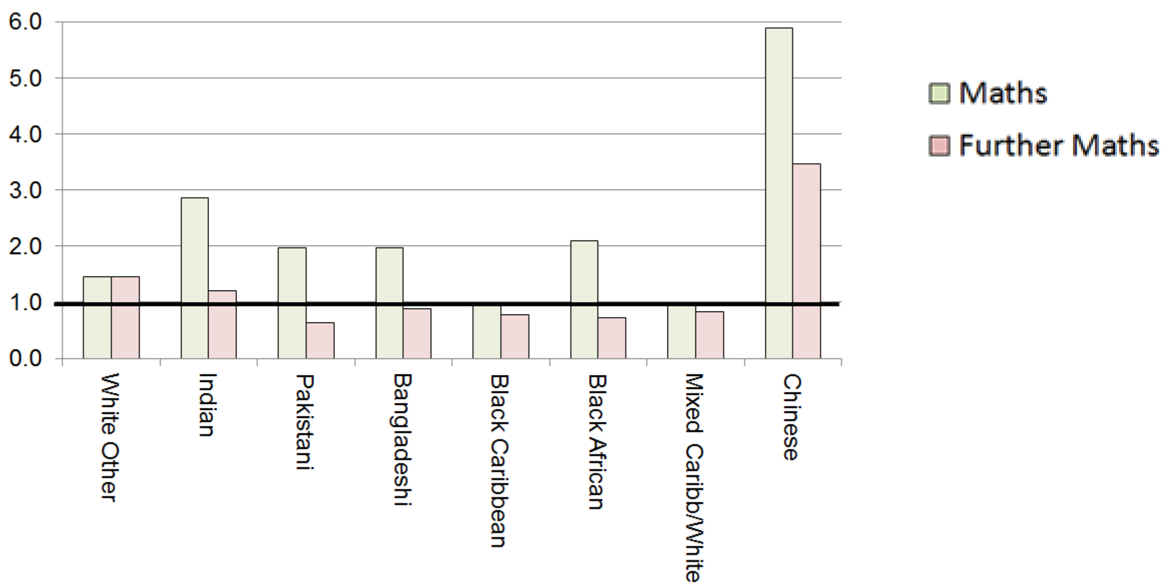
- o 8.5% of non-FSM students attained A* compared with 1.5% of FSM students; 22.3% of non-FSM attained an A or A* in Mathematics compared with 6.7% of FSM students
- once KS4 attainment and other factors were controlled for, FSM students were 90% as likely to take A level Mathematics and 97% as likely to take A level Further Mathematics compared with their non-FSM peers
- once KS4 attainment and other factors were controlled for, non-FSM students had a +1.1 UCAS point advantage in A level Mathematics and a +2.5 UCAS point advantage in A level Further Mathematics compared with their FSM peers

Ethnicity

In terms of ethnicity, once KS4 attainment and other factors were controlled for, differential patterns of participation and attainment in A level Mathematics were found to be notably different from those seen with A level Further Mathematics. Low levels of participation and attainment amongst Black Caribbean and mixed Black Caribbean and White students were found to relate to very low levels of attainment at KS4 for these ethnic groups. Two figures 6-1 below summarise the differences in participation and attainment in A level Mathematics and Further Mathematics across ethnic groups once KS4 attainment and other factors are controlled for.

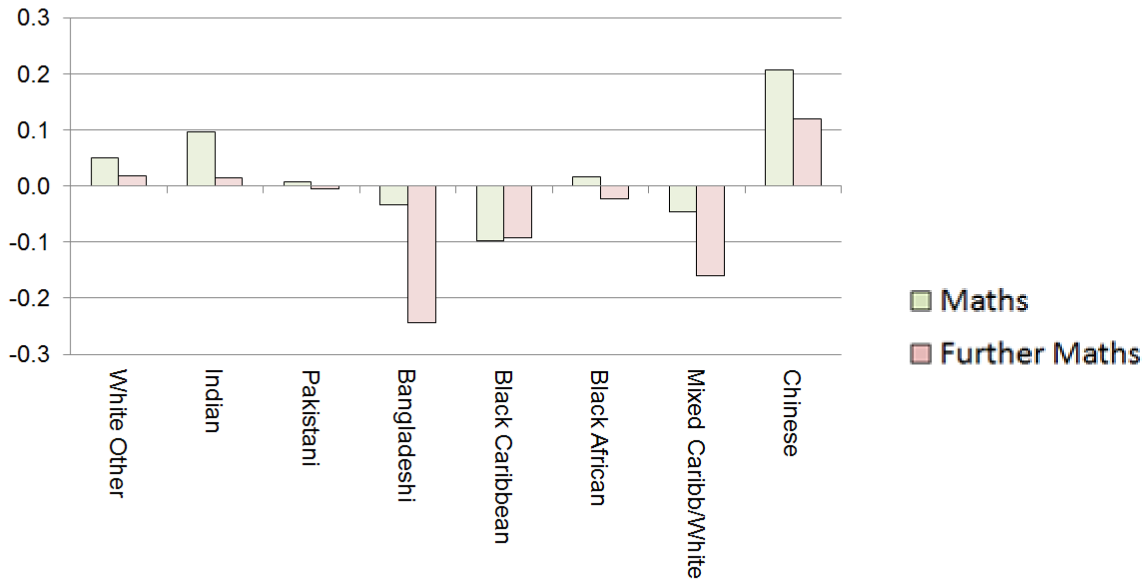
Figure 6-1 Participation in A level Mathematics & Further Mathematics by ethnic group

Odds-ratios (relative to the white British reference group).



Attainment in A level Mathematics & Further Mathematics by ethnic group

Cohens d effect size statistics (relative to the white British reference group).



School type

In terms of the type of KS5 institution a student attended, once KS4 attainment and other factors were controlled for, students in state school sixth forms or sixth form colleges were the most likely to participate in A level Mathematics and Further Mathematics. Students in FE colleges were the least likely to participate. Students in independent / fee paying KS5 institutions were found to be less likely to participate than students in state school sixth forms but more likely than students in FE colleges. This finding links to KS4 attainment and independent / fee paying KS5 institutions being less successful in getting GCSE Mathematics A* students to take A level Mathematics or Further Mathematics when compared with state funded KS5 institutions. Once KS4 attainment and other factors were controlled for, students in independent / fee paying KS5 institutions were found to attain the highest in A level Mathematics and Further Mathematics, students in FE colleges were found to attain the lowest.

- 14.7% of students in state school sixth forms had attained A* in GCSE Mathematics in 2011 - 78.3% of these A* students took A level Mathematics and 20.4% took A level Further Mathematics
- 38.9% of students in independent / fee paying institutions had attained an A* in Mathematics in 2011 - 69% of these took A level Mathematics and 17.8% took A level Further Mathematics

- once KS4 attainment and other factors were controlled for, students in independent / fee paying were found to be 57% as likely to take A level Mathematics and 86% as likely to take A level Further Mathematics compared with students in state school sixth forms
- once KS4 attainment and other factors were controlled for, students in FE colleges were found to be 16% as likely to take A level Mathematics and 31% as likely to take A level Further Mathematics compared with students in state school sixth forms
- once KS4 attainment and other factors are controlled for, students in independent / fee paying institutions attained +3.3 UCAS point higher in A level Mathematics and +3.9 UCAS points higher in A level Further Mathematics compared with students in state school sixth forms
- once KS4 attainment and other factors are controlled for, students in sixth form colleges attained -2.0 UCAS point lower in A level Mathematics and -0.5 UCAS points lower in A level Further Mathematics compared with students in state school sixth forms
- once KS4 attainment and other factors are controlled for, students in FE colleges attained -5.3 UCAS point lower in A level Mathematics and -3.4 UCAS points lower in A level Further Mathematics compared with students in state school sixth forms

Engagement with FMSP

Engagement with FMSP was included into the model at the KS5 institution level. The institutional level FMSP engagement variables all related to a single academic year (2013/14). Specifically, engagement relating to part of the FMSP support offer for which data was available and could be matched were included. These were Tuition, Live Online Professional Development (LOPD), Teaching Further Mathematics (TFM) and Teaching Advanced Mathematics (TAM) were included into the model. Findings from analyses including the FMSP engagement variables need to be treated with caution for two reasons: First, the variables are measured at the institutional level whilst the outcomes are at the student level. A student may be located within a KS5 institution that is identified as having some FMSP engagement but this does not mean that this student had any direct personal support from the FMSP. Second, the FMSP engagement variables relate to a single academic year (2013/14) whilst the analyses relate to three KS5 academic years (2011/12 to 2013/14). The combined temporal mismatch and distant position of the FMSP engagement variables from the outcomes under study need to be kept in mind when interpreting the model coefficients. In most instances, institutional engagement with FMSP was found to not be related to patterns of participation or attainment in A level Mathematics or Further Mathematics. There were two exceptions that relate to participation in A level Further

Mathematics - and can be (cautiously) read as evidence that the FMSP is targeting appropriate institutions for Tuition and TFM support - that is institutions where participation in A level Further Mathematics is relatively low.

- once KS4 attainment and other factors were controlled for, students in KS5 institutions engaged with FMSP TFM were found to be 74% as likely to take A level Further Mathematics compared with students in other KS5 institutions
- once KS4 attainment and other factors were controlled for, students in KS5 institutions engaged with FMSP Tuition were found to be 64% as likely to take A level Further Mathematics compared with students in other KS5 institutions

6.3 Modelling FMSP Engagement

It is important to note a set of considerations should be taken into account which taken together serve to undermine any causal interpretations that might be drawn from the model analyses and so caution is advised. The indirect nature of the FMSP variables needs to be borne in mind when interpreting these findings. A student may be located within an institution that had involvement with FMSP in terms of LOPD, TAM, TFM or Tuition but might not actually have any direct personal FMSP experience. A further important consideration is the time difference. The (institutional level) FMSP variables relate to just the 2013/14 academic year whilst the 2011 cohort data set spans three academic years between 2011/12 and 2013/14. These provisions should be taken into account when interpreting the findings that, within the 2011 KS4 NPD cohort, once all variables are controlled for, there is limited evidence for a strong association between institutional level engagement with FMSP and student level participation or attainment in A level Mathematics or Further Mathematics.

LOPD - Live Online Professional Development

A very small but consistently positive effect is observed with LOPD in relation to participation in A level Mathematics and Further Mathematics. Prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with LOPD are observed to be 1.6 times as likely as students in other KS5 institutions to take A level Mathematics - but this is seen to considerably narrow when all factors are controlled for the final model (1.03 times as likely). A similar pattern is observed with A level Further Mathematics from before (1.8 times as likely) and after (1.1 times as likely) controlling for KS4 attainment and other variables.

Prior to controlling for KS4 attainment and other variables, students within KS5 institutions involved in LOPD from the FMSP attained higher than students within other institutions in A level Mathematics and Further Mathematics - but this changes once KS4 attainment and other variables are controlled for. Prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with LOPD are observed to attain +4.3 UCAS points higher in A level Mathematics and +0.9 points higher in A level Further Mathematics compared with students in other KS5 institutions. This is observed to change once KS4 attainment is controlled for such that students located within a KS5 institution involved with LOPD are observed to attain +1.1 UCAS points higher in A level Mathematics and - 0.6 points lower in A level Further Mathematics compared with students in other KS5 institutions.

TAM

For TAM, prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with TAM are observed to be 93% as likely as students in other KS5 institutions to take A level Mathematics - and this remains consistent in the final model. A similar pattern is observed with A level Further Mathematics from before (93% as likely) and after (1.02 times as likely) controlling for KS4 attainment and other variables.

Prior to controlling for KS4 attainment and other variables, students within KS5 institutions involved in TAM from the FMSP attained similarly in A level Mathematics and relatively lower in Further Mathematics compared to students within other KS5 institutions. Prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with TAM are observed to attain 0.1 UCAS points lower in A level Mathematics and 3.6 points lower in A level Further Mathematics compared with students in other KS5 institutions. This is observed to change once KS4 attainment is controlled for such that students located within a KS5 institution involved with TAM are observed to attain 1.3 UCAS points higher in A level Mathematics and 0.7 points lower in A level Further Mathematics compared with students in other KS5 institutions.

TFM

For TFM, prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with TFM are observed to be 1.5 times as likely to take A level Mathematics and 1.3 times as likely to take A level Further Mathematics compared with students in other KS5 institutions. This is observed to notably change once KS4 attainment and KS5 location are controlled for such that students located within a KS5 institution involved with TFM are observed to be 94% as likely as to take A level Mathematics and 74% as likely to take A level Further Mathematics compared with students in other KS5 institutions.

Students within KS5 institutions involved in TFM from the FMSP attain higher than students within other institutions in A level Mathematics and (particularly) A level Further Mathematics. Most of this relative attainment advantage can be statistically accounted for by variations across KS4 attainment and other variables (most notably with A level Mathematics) but not all. Prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with TFM are observed to attain 3.5 UCAS points higher in A level Mathematics and 6.2 points higher in A level Further Mathematics compared with students in other KS5 institutions. This is observed to change once KS4 attainment is controlled for such that students located within a KS5 institution involved with TFM are observed to attain 0.1 UCAS points higher in A level Mathematics and 2.8 points higher in A level Further Mathematics compared with students in other KS5 institutions.

Tuition

For Tuition, prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with FMSP Tuition are observed to be 72% as likely as to take A level Mathematics and 38% as likely to take A level Further Mathematics compared with students in other KS5 institutions. This difference is observed to narrow once KS4 attainment and KS5 location are controlled for such that students located within a KS5 institution involved with FMSP Tuition are observed to be 91% as likely as to take A level Mathematics and 64% as likely to take A level Further Mathematics compared with students in other KS5 institutions.

Prior to controlling for KS4 attainment and other variables, on average, students within KS5 institutions involved in Tuition from the FMSP attained relatively lower in A level Mathematics and Further Mathematics compared to students within other KS5 institutions. Prior to controlling for KS4 attainment and other variables, students located within a KS5 institution involved with Tuition are observed to attain 6.2 UCAS points lower in A level Mathematics and 7.4 points lower in A level Further Mathematics compared with students in other KS5 institutions. This difference is again observed to narrow once KS4 attainment is controlled for such that students located within a KS5 institution involved with Tuition are observed to attain 1.2 UCAS points lower in A level Mathematics and 2.9 points lower in A level Further Mathematics compared with students in other KS5 institutions.

This makes some intuitive sense. If a KS5 institution draws on support from FMSP, this indicates that they require some support to deliver A level Further Mathematics - and this may account for the negative participation findings discussed above. Positive associations were found with respect to A level attainment - most clearly with respect to TFM and Further Mathematics A level. Positive association with attainment is also evident with LOPD but this is less clear than TFM and is more evident with A level Mathematics rather than Further Mathematics attainment. Negative associations with attainment were found with respect to TAM and Tuition. This does reduce once KS4 attainment and other variables are controlled for but not completely. Tuition is targeted at centres where there is a need to improve attainment, which explains the negative association with attainment and schools involved in tuition.

Finally, in terms of contributing to the explanatory power of the models, the FMSP engagement variables did NOT tend to reach a level of statistical significance in the final main effects models. The only exception was with TFM and Tuition and participation in A level Further Mathematics.

6.4 Summary of participation and attainment analyses

- ***Are patterns of differential participation in A and AS level Mathematics and Further Mathematics comparable?***

The descriptive analyses and multivariate multilevel analyses (Annexe 2) reveal a great deal of consistency in terms of differential patterns of participation relating to the type of KS5 institution attended, KS4 attainment (overall and in GCSE Mathematics), gender and proximity to poverty. However, the relationship between ethnicity and participation in A level Mathematics was notably different to that seen with participation in A level Further Mathematics. For A level Mathematics, participation rates were observed to be higher for most BME groups compared with White British students with two exceptions (Black Caribbean and mixed Black Caribbean and White students). For A level Further Mathematics, participation rates were higher for the Chinese, White Other and Indian compared with White British students with students in other BME groups being less likely to take the A level. The influence of institutional-level engagement with the FMSP on student-level participation was weak but consistent for both A level Mathematics and Further Mathematics.

- ***Are patterns of differential attainment in A and AS level Mathematics and Further Mathematics comparable?***

The descriptive analyses and multivariate multilevel analyses (Annexe 2) also reveal a great deal of consistency in terms of differential patterns of attainment relating to the type of KS5 institution attended, KS4 attainment (overall and in GCSE Mathematics), gender and proximity to poverty. However, again, the relationship between ethnicity and attainment in A level Mathematics was notably different to that seen with attainment in A level Further Mathematics. For A level Mathematics, attainment was observed to be higher for Chinese, Indian, White other and Black African students compared with White British students. For A level Further Mathematics, attainment was observed to be higher only for Chinese compared with White British students.

- ***To what extent can these differential participation and attainment patterns be statistically accounted for by differential patterns in KS4/GCSE attainment (in Mathematics and overall KS4 attainment)?***

Attainment at Key Stage 4 in Mathematics and overall is a key determinant to participation and attainment in A and AS level Mathematics and Further Mathematics. The inclusion of KS4 attainment into the multivariate multilevel analyses had an impact on model coefficients of other explanatory variables (see Annexe 2 for details). In some instances, this might be seen as KS4 attainment statistically 'accounting' for differential patterns of participation and/or attainment. For example, the inclusion of KS4 attainment within the A level Mathematics and A level Further Mathematics participation models results in reducing the differential participation rates for students identified as FSM. This suggests that some of the differential participation rates relating to FSM might be a result of the relatively lower levels of KS4 attainment for FSM students compared with their peers not classed as FSM.

The inclusion of KS4 attainment also resulted in some interesting changes in model coefficients that are more complex. For example, taking account of the relatively higher KS4 attainment of females taking A level Mathematics or Further Mathematics revealed a hidden gender attainment gap. When KS4 attainment was not taken into account, the attainment of females was observed to be fairly comparable to males. However, once KS4 attainment was included into the model, a gender difference (higher attainment of males) is observed.

Perhaps the most striking impact of KS4 attainment on model coefficients was seen with the A level participation models and relates to the type of KS5 institution a student took the A level in. Prior to taking KS4 attainment into account, students located in independent/fee paying KS5 institutions were observed to be over twice as likely to take A level Mathematics and Further Mathematics compared with their peers located in state school sixth forms. However, once KS4 attainment was included, this pattern changed to show that students located in independent/fee paying KS5 institutions were less likely to take the A level. This pattern was consistent in both Further Mathematics and Mathematics A level. It seems that the higher levels of KS4 attainment (in Mathematics and overall) amongst students within independent/fee paying KS5 institutions are more than sufficient to statistically account for the higher participation rates. In fact, once the relatively higher KS4 attainment is taken into account, students in independent/fee paying institutions are found to be LESS likely to take A level Mathematics (57% as likely) or Further Mathematics (86% as likely) compared with students located in state school sixth forms.

- ***What are the independent effects of student and school characteristics on participation and attainment in A level Mathematics and Further Mathematics?***

The type of KS5 institution, gender, ethnicity and (to a lesser extent) FSM were all found to independently influence differential patterns of participation and attainment in A level Mathematics and Further Mathematics (see Annexe 2 for details).

- ***At the institutional level, how is engagement with FMSP associated with participation and attainment in A level Mathematics and Further Mathematics?***

Evidence of institutional-level engagement with FMSP (with respect to LOPD, TAM, TFM and Tuition) being associated with student-level participation and attainment in A and AS level Mathematics and Further Mathematics is slim / weak and not of the same magnitude as other institution and student level influences. This may reflect issues of validity relating to these FMSP engagement variables - for example, the variables are not temporally aligned with the participation and attainment outcome variables (see Annexe 2 for details).

7. Reviewing participation in the FMSP activity

This section reviews participation in FMSP activity drawing on data collected and provided by the FMSP. This shows that **the FMSP has considerable reach**, principally to centres who offer KS5 provision but also beyond that and though its CPD programme but also other activity is **developing capacity and capability of centres to offer Further Mathematics** . Participants' views on provision indicate it is **effective and of high quality**.

7.1 FMSP enrichment activity

The number of 11-16 students who attend FMSP enrichment sessions

During the period April 2014 to March 2015, the FMSP organised enrichment events for 11-16 students that were attended by 10,385 students (against a 10,000 target). These events covered a range of formats from very large events hosted by universities to smaller centre-based events.

During the period April 2015 to March 2016, 12,436 11-16 students attended enrichment events (against a target of 12,000).

11-16 student enrichment sessions are of a high quality

Feedback was collected from students and teachers attending larger enrichment events. For small in centre events only teachers were asked to provide feedback. For 2014/15 the average feedback was 3.25, by the 6497 students who completed a form, which is between 'good' (3) and 'excellent' (4) on a scale of '1-4'. Of these students, 39% had already decided to study mathematics beyond GCSE, two thirds of these saying that the event made them more interested. 34% of the students overall described themselves as initially not sure about studying mathematics or not intending to study mathematics but had been encouraged to think about it by the event. A small proportion (5%) described themselves as not intending to study mathematics beyond GCSE and not persuaded by the event. Overall, 77.5% of the students said that they would recommend the event to others.

For 2015/16 the average feedback by 5849 students was 3.4. Of the student forms, 32% had already decided to study mathematics and 86% of these said that the event had made them more interested in studying mathematics. 45% stated that they had initially been uninterested, but made more likely to study mathematics by the event. It was good to see that centres were opening these events to more students who were not certain about studying mathematics beyond GCSE. 88% of students would recommend the event they attended to other students.

For the 2014/15 in centre sessions, feedback was sought from teachers, 80% of the 101 teachers completing the form thought that the session would encourage their students to study mathematics beyond GCSE. In 2015/16, 71% of the 76 teachers so far providing feedback thought the same.

Problem solving enrichment for A level Mathematics and Further Mathematics students based on STEP, AEA and MAT examinations

For the academic year September 2014 to August 2015 regular mathematical problem solving classes took place in 18 venues with support from the FMSP. With class sizes of 15-20 students, over 300 students took part. In addition the FMSP organised 44 events designed to support students with higher level problem solving. 905 students attended these types of events during the academic year 2014/15.

Since September 2015 regular mathematical problem solving classes have been set up at 28 venues with support from the FMSP. These are mostly universities but some satellite classes are taking place in centres. Not all of the attendance information from the providers has been received and classes continue until June 2016. Over 500 are expected to receive support through these classes. In addition the FMSP has organised 27 one-day events so far this year which have been attended by 635 students.

Engagement in FMSP competitions

The below tables show the numbers of centres and students participating in FMSP competitions; the "Maths Feast" (Year 10 competition) and the "Senior Team Mathematics Challenge" (Year 12/13 competition).

Table 7-1 Maths Feast entries (Year 10 competition)

	Maths Feast (Year 10 competition)		
Year	Events	State-funded schools involved	Number of teams (4 students per team)
2014/15	77	706	884
2015/16	86	693	842

Table 7-2 Senior Mathematics Team Challenge entries (Year 12/13 competition)

	STMC (Year 12/13 competition)	
Year	Events in England	English state-funded schools involved
2014/15	53	734
2015/16	55	782

7.2 Teacher professional development

The number of teacher days of CPD for A level Mathematics and Further Mathematics

The target of 2000 total teacher days has successfully been met each 'year' as have had sub-targets related to specific areas:

Table 7-3 Teacher days of CPD

	Period	Number of teachers from state-funded institutions	Number of teacher days	Number of courses
Live on line Professional Development (LOPD)	Sept 2014 to Aug 2015	226	414	40
	Sept 2015 to Mar 2016	222	374	32
Local one-day CPD courses for A level Mathematics and Further Mathematics	Sept 2014 to Aug 2015	1093	1040	95
	Sept 2015 to Mar 2016	600	600	53

Teaching Further Mathematics (TFM): 162 teachers enrolled on TFM in 2014/15 and 152 enrolled for 2015/16.

Teaching Additional Mathematics (TAM): 191 teachers enrolled on TAM in 2014/15 and 183 enrolled for 2015/16.

ITT in A level mathematics: Following a successful pilot phase in 2014/15, over 150 trainee teachers will have taken the FMSP's extended course on A level Mathematics (carried out by the UCL IOE on behalf of the FMSP) in 2015/16.

CPD for the extension and enrichment of KS4 Mathematics: The 2014/15 sessions of the two-day KS4 Extension and Enrichment courses were attended by 355 teachers. In addition, 70 teachers attended a one day Extension and Enrichment conference. The course was revised in 2015/16 following three successful previous cohorts to take account of changes to GCSE Mathematics. In 2015/16, 338 teachers attended the two-day course and a further 34 attended Day 1 only. In addition, 122 teachers attended a one day KS4 Extension and Enrichment conference in March 2016.

CPD to support problem solving based on STEP, AEA and MAT examinations: In 2014/15 the FMSP ran 48 events that provide CPD for higher level problem solving (STEP/AEA/MAT). These events provided 403 teacher days of CPD. Between September 2015 and March 2016, the FMSP has provided 15 one-day CPD courses in preparing students for STEP, AEA or the MAT and 11 problem solving days for students that included PD elements for their teachers. In 2015/16 the FMSP successfully delivered a new LOPD course for STEP/AEA for three groups of participants. At this point in 2016 the FMSP has provided over 200 days of teacher CPD for higher level problem solving since September 2015.

Quality of FMSP CPD

The feedback for course delivery and content for the above courses are between Good and Excellent in all cases. The feedback for venue and information in advance are Good.

7.3 FMSP Tuition

The number of students who access tuition has declined over time as centre capacity to offer Further Mathematics in-house has increased (Table 7-4).

Table 7-4 Students involved in tuition

Year	Students	Units	Face-to-face units	LOT units
2009/10	816	1977	1780	197
2010/11	607	1525	1207	318
2011/12	435	1108	891	217
2012/13	373	913	711	203
2013/14	270	702	475	227
2014/15	201	519	202	317 *

*The number of units is a combination of Live Online Tuition (LOT), 248 units, and FMSP supported Live Interactive Lectures (LIL), 69 units

7.4 Registered users

Table 7-5 shows the number of centres registered with FMSP. This represents a majority of those which have KS5 students.

Table 7-5 Number of centres registered with FMSP

Age range	Number of centres registered with the FMSP
Up to KS4	672
Up to KS5	2585
KS5 only	177
Other	12

7.5 School level engagement in different components

The table below summarises data held by the FMSP on number of centres that have students or teachers who engage in different programme components in 2013/14. Findings from the telephone interviews with teachers (see Section 9), indicate a fluid relationship to engagement in CPD and enrichment from year to year, so the data represents a 'snapshot' for a single year. Again from telephone interviews, the figures for enrichment activities may be an underestimate and not include all Area Coordinator or Associate activity in-centre, for example attendance at career or A level options evenings.

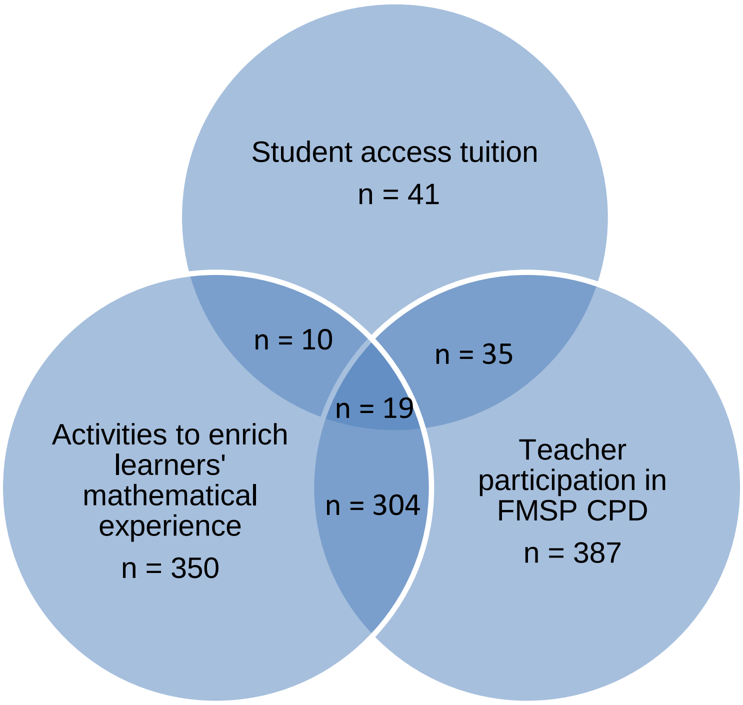
The table below (Table 7-6) and subsequent Venn diagram (Figure 7-1) shows the numbers of centres participating in each activity. Note that this is based on FMSP data that we were able to match into centre census data for the academic year 2013/14 only and therefore only includes the activities listed in the table below.

Table 7-6 Summary of numbers participating in different programme component

FMSP activity	**Number of state centres involved in 13/14 (which matched into census data by postcode)**	**Type of activity**
Senior Team Mathematics challenge (STMC)	683	Enrichment
Tuition (Standard, LOT, LIL, Supported LIL)	105	Tuition
Teaching Advanced Mathematics (TAM)	112	CPD
Live online professional development (LOPD)	161	CPD
Teaching Further Mathematics (TFM)	40	CPD
Other CPD	571	CPD

The figure below shows the interrelationship of engagement of centres in different programme components shown in the table for 2013/14. These do not include participation in 11-16 enrichment events, year 10 Maths Feasts and CPD for KS4 Extension and Enrichment.

Figure 7-1 Interrelationship of engagement in different programme component



7.6 Engagement with HEI departments

The FMSP conducted research on HEI departments' recommendations and/or entry requirements for entry to mathematics and other degree courses involving mathematical study. Following this the FMSP has engaged with HEI departments encouraging the uptake of AS/A level Mathematics and Further Mathematics.

For 2014, 116 courses were identified on a 'target list' for contact and to initiate discussion, 37 focused on A level Mathematics and 79 on A level Further Mathematics. 32 meetings/discussions with HE departments took place resulting in 10 indicating a willingness to increase encouragement for Further Mathematics and 5 for Mathematics.

For 2015, 146 courses were identified on a 'target list' for contact and to initiate discussion, 78 focused on A level Mathematics and 68 on A level Further Mathematics. Meetings/discussions with 99 HE departments took place resulting in 18 indicating a willingness to increase encouragement for Further Mathematics and 4 for Mathematics.

8. School engagement and teacher, and student perspectives on the FMSP

8.1 School engagement with the FMSP

This section examines the relationship between centre participation in the Further Mathematics Support Programme and looks at the relationship between this, school characteristics and Further Mathematics security. It is important to note that the FMSP participation data used under-records the level of participation. This is because FMSP data was collected for a variety of purposes with databases developed in response to programme developments as they occurred. Further, some participation data is held at area level. In addition, FMSP data was matched into the school level census data. The school census data used records from 11/12, 12/13 and 13/14 (2256 records in total); the FMSP data matched into this was from 13/14 only. For these reasons analysis should be taken as indicative rather than a reliable description of the situation.

The data was matched in by postcode with total matching centres as shown in Table 8-1.

Table 8-1 FMSP records matched into census data

	Total centres with FMSP records matched into census data
Teaching Advanced Mathematics (TAM)	112
Senior Team Mathematics challenge (STMC)	683
Tuition (Standard, LOT, LIL, LIL supported)	105
Live online professional development (LOPD)	161
Teaching Further Mathematics (TFM)	40
CPD	571

In total 50.8% of matched centres had at least 1 record of involvement from the year 13/14 with 32.1% having 1 record of involvement, 14.8% had 2 records, 3.2% had 3 records and 0.7% (15 centres) had 4 records. One centre was involved in 5 activities. Of the 1110 centres that had no records of involvement in 13/14, only 122 did not appear on the FMSP administrative database.

Table 8-2 Total number of FMSP activities in which centres are involved

Number of FMSP activities involved in (13/14)	n	%
0	1110	49.2
1	725	32.1
2	333	14.8
3	72	3.2
4	15	.7
5	1	.0
Total	**2256**	**100.0**

8.2 FMSP engagement and security status

Figures 8-1 and 8-2 below show the relationship between FMSP involvement and security status. Figure 8-2 shows the relationship between the number of FMSP activities that centres are involved in and security status. Centres who did not have any involvement with FMSP in 2013/14 are more likely to have lower numbers of Further Mathematics A level entries overall. Note that this does not imply a causal link between FMSP involvement and number of entries or security. So for example, centres that are less secure are more likely to be involved in tuition. For some centres tuition ensures they have some entries rather than no entries. Figure 8-2 shows the relationships between specific FMSP activities and security. Key findings are:

- centres involved in the STMC in 13/14 were much more likely to have secure numbers of Further Mathematics compared with those not involved
- centres involved in TFM in 13/14 were more likely to have secure numbers of FM, although this does not reach statistical significance
- centres who were involved in LOPD in 13/14 are more likely to have secure numbers of Further Mathematics students
- a higher proportion of centres involved in CPD in 13/14 had a "secure" Further Mathematics status compared with centres not involved in CPD
- centres involved in TAM in 13/14 were as likely to have secure numbers of Further Mathematics as those not involved in TAM
- centres involved in tuition were more likely to be centres whose numbers of Further Mathematics were "least secure" compared with those who were not involved in tuition in 13/14. However it is also notable that centres with high or relatively secure status also

make use of tuition. This is discussed further in section 9 when the evidence from teacher interviews is considered

Figure 8-1 Relationship between number of FMSP activities involved in and security status

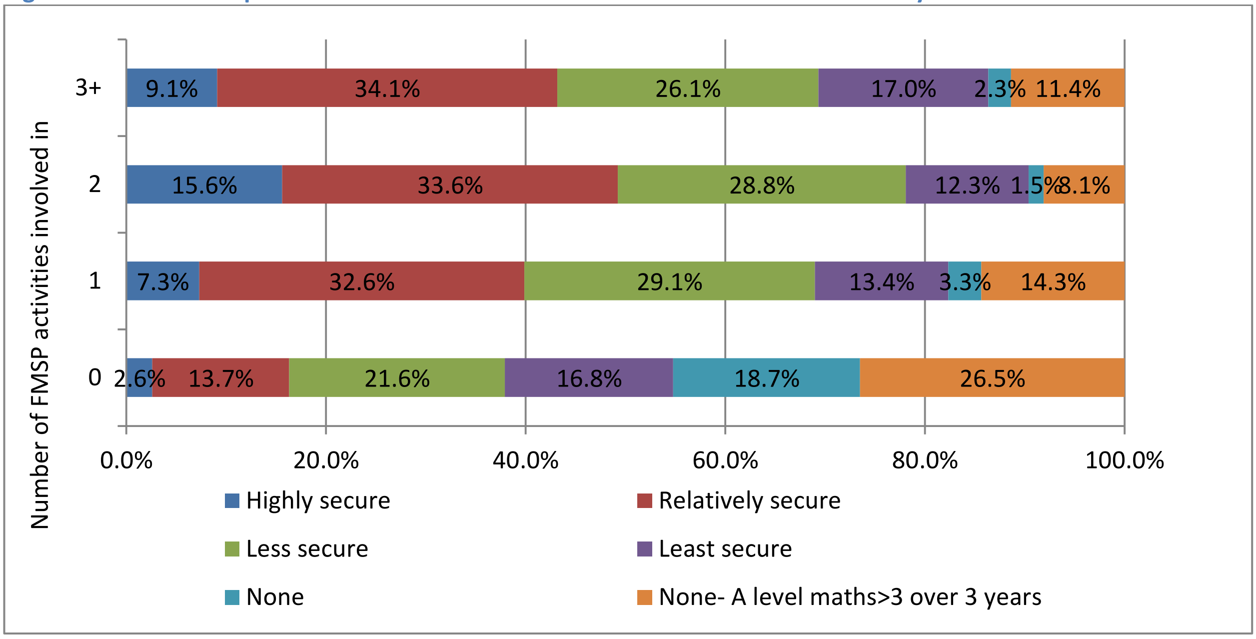
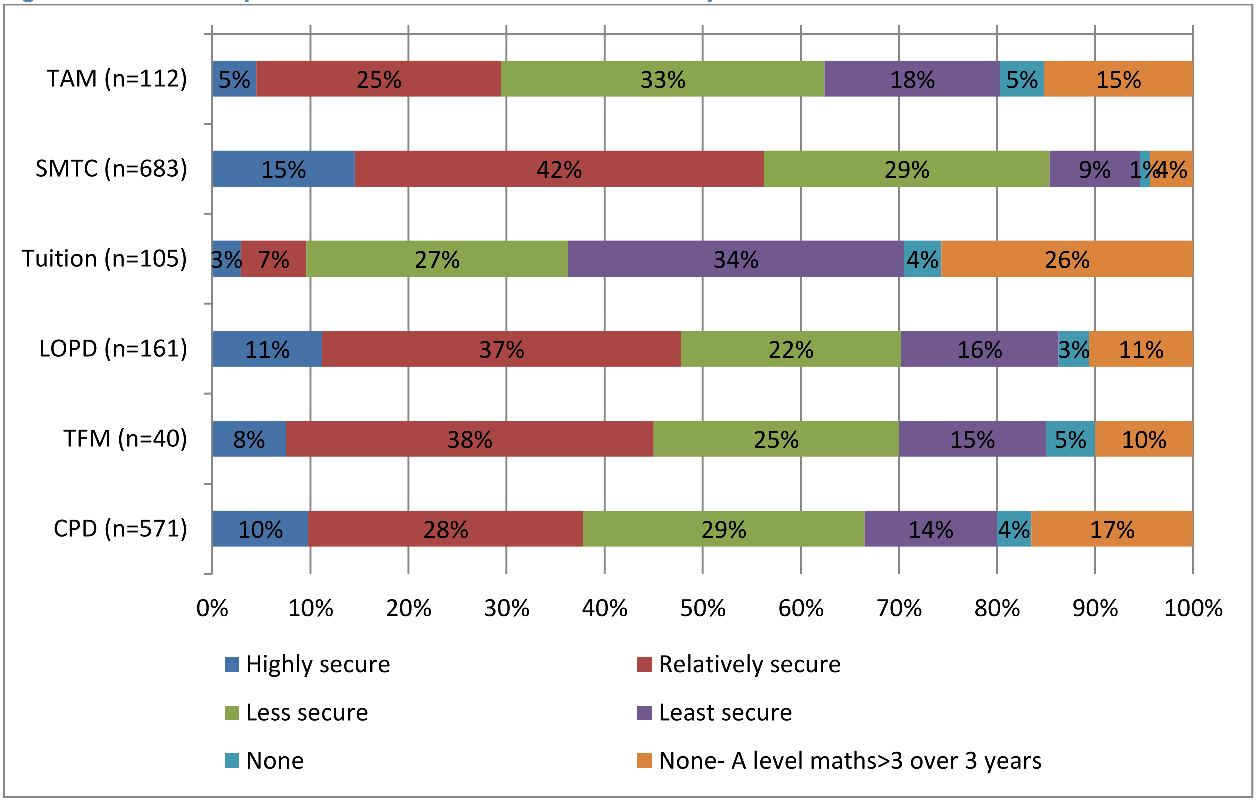


Figure 8-2 Relationship between FMSP involvement and security status



It is notable that 31 centres who have records of tuition in 13/14 have a "none" status for FM A level entries in 13/14.

Of these 31:

- 14 have entries for A level FM in 14/15

Of the 17 remaining:

- 7 have entries for AS FM in 13/14
- a further 5 have entries for AS FM in 14/15
- all apart from 1 of the rest have entries for AS Mathematics/A level Mathematics

This indicates that some centres may be using tuition to enhance the range of A level modules offered.

8.3 Teacher views of the FMSP

Tuition

18 teachers of the 42 interviewed were from centres that were currently using FMSP tuition (44% of the sample of interviewees). Five were using face to face tuition and 13 on-line tuition. Views of tuition were generally similar regardless of the form of tuition, including for those whose students accessed face to face tuition whether this was on site or offsite. An exception is some indication that on-line tuition is viewed as more demanding, with one participant commenting that this type of tuition only worked well for the most able students and another that students had to be very highly self-motivated.

For centres with low Further Mathematics entries the reasons for choosing face to face rather than on-line tuition appear to be due to availability and historical use.

The table below (Table 8-3) summarises reasons and perceived benefits of access for tuition considered by the centres 2014/15 Further Mathematics security status, text in the centre represents reasons/benefits cited by centres with all types of Further Mathematics security rating.

Table 8-3 Reasons/benefits for accessing tuition

Centres with least or low Further Mathematics security or previously no Further Mathematics (10)	**Centres with secure or highly secure Further Mathematics security (8)**	
• Students would not otherwise be able to access Further Mathematics • Keep Further Mathematics open to all • Retain strongest students/competition with other centres • Teacher watched LOT, or observed face to face tuition of students and got PD benefit • Lack of experienced/Further Mathematics capable/Further Mathematics confident staff • Benefits of students experiencing different teacher • Benefits of being part of a larger class (on-line tuition)	• Cost saving / Senior Leaders would not fund a normal teaching group including for specific modules • Difficulties timetabling specific modules • Access individual modules in on-line tuition to provide greater student choice • Staff expertise to teach specific A level and FM modules	• Enhancement - additional tuition/revision

Generally, the interviewees were positive about the quality of tuition the most common description being 'excellent' and as 'very valuable' for students, the feedback to both the students and themselves was good; and that the students could go back and re-watch the lectures in their own time if needed.

Issues raised or were:

- limited tuition time required further in-house support for the students to be successful and in one case concerns were expressed about the capacity in centre to provide this
- a teacher from one centre reported that their students did not like the online tuition because they found it too fast, they could not access it at home to re-watch the lectures (this appears contradictory to views reported above) and there was no time for them to consolidate their learning without extra support provided by the centre. After one term, they stopped using the online tuition
- another teacher suggested that online tuition only worked well for the most able students and another that students had to be very highly self-motivated
- two teachers commented that a shortcoming for them was not being able to monitor students' progress whilst they undertook online tuition, which meant students had to identify if and what they required additional support with, rather than teachers being able to assess this and intervene appropriately

Professional Development

Of the 42 participants, 33 reported some engagement with the FMSP CPD programme recently[7].

Participants or their colleagues had accessed a range of the FMSP CPD. The universal response was that all CPD accessed was of excellent quality, with the subject knowledge of the presenters and the high quality of the resources being specifically highlighted.

Participants or their colleagues from centres across the whole range of Further Mathematics security levels had used CPD to support their development.

Reported benefits were that it:

- enabled them or colleagues to teach Further Mathematics (note that this was reported by teachers from both lower and higher security status)
- increased their confidence
- enabled them to rethink their approaches to teaching mathematics
- increased subject knowledge
- provided time and space to meet and interact with other Mathematics teachers
- diversified the skills base of the departmental teams and so built capacity
- and offered access to new high quality resources, that were immediately applicable in the classroom and provided ideas to use and also to share with others

With regard to resources and activities accessed via CPD courses, participants reported that use of these had:

- improved teaching
- made teaching more interactive
- had engaged students more
- had been used to challenge brighter students
- and allowed a more flexible approach to teaching

The access to these resources was thought to be invaluable to new or less experienced staff. Two participants also mentioned the Integral resources as being excellent.

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[7] Note that data provided prior to sampling indicated 27 had engaged with CPD in 2012/13.

Enrichment activities

The majority of participants (37 centres) had accessed elements of the FMSP enrichment for their students. Note that is considerably higher than indicated in data from FMSP for 2013/14 only which indicated 17 centres at the point of sampling. There was also evidence of some confusion about who provided particular activities for example between MEI and FMSP.

The table below (Table 8-4) summarised enrichment activities by Further Mathematics security status:

Table 8-4 Summary of security status and enrichment activities

Further Mathematics Security status	Type of enrichment activities accessed
No FM	• Attended Year 11, 12 and 13 Mathematics conferences; • Year 12 and 13 students visit to a local university for problem solving and revision sessions; • KS4 enrichment day; • Area coordinator talk in centre about Further Mathematics.
Least secure	• One-off sessions at local universities for revision; • Talks on complex numbers; • Problem solving and statistics enrichment event; • Master classes; • Mathematics challenges for years 9 and 10 (Maths Feast).
Less secure	• Senior Team Mathematics Challenge; • Individual Mathematics challenges; • Year 10 and 11 inspiration days; • Sessions at local universities about Mathematics in the wider world; • Year 10 and 11 courses on, for example, complex numbers; • The FMSP coming into the centre to give a talk about Further Mathematics; • Year 9 females attending a university Master class; • The year 10 Maths Feast.
Relatively secure and highly secure	• Senior Team Mathematics Challenge; • Revision days at local Universities; • 1 day specialist courses at local Universities; • Speakers from the FMSP in school; • STEP revision/preparation.

The enrichment activities accessed were reported to have:

- improved student motivation
- enabled students to explore mathematics in context
- engaged students with problem solving and complex numbers
- helped students to see ‘real’ mathematics at work and in context

- given students an insight into what careers were possible with mathematics qualifications
- provided positive role models for females
- increased student understanding and achievement
- allowed the students to develop their capacity to work together in team challenges

One participant reported that the sessions at a local university were enjoyable and that they helped student learning in class. They felt that their students seeing and interacting with other confident students could help them to become more confident about their own mathematical skills. Another participant reported that attending problem solving workshops allowed students to see that Mathematics was not just theoretical and that it increased their resilience. Participants felt that KS4 enrichment activities were particularly important for high achieving GCSE students who often became bored with the mathematics content of GCSE.

Although generally enrichment activities were viewed positively, one teacher reported being disappointed following the FMSP visit to the centre for a talk that had not covered what they wanted; they wanted more interaction for the students and a focus on problem solving. However, the FMSP had responded to this criticism and so the centre said they would use this service again, but would ensure that a clearer outline of what was needed was agreed before the visit.

Team challenges were mostly viewed positively. Although one interviewee commented that the competitive environment might impact negatively on some females, this was not an issue raised by others.

At one centre, the students had to apply for places on the revision courses as places were limited so the students saw these courses as being very valuable. However, at one of the centres with high numbers of Further Mathematics students, the fact that the revision sessions were held on a Saturday had proved problematic so students preferred in house revision sessions.

Some participants raised issues that might impact on any out of centre activity; namely cost of travel and time out of class.

Area coordinators

All participants who had had contact with Area Coordinators spoke positively about the support they received. It was notable that often Area Coordinators would be referred to by name. As well as providing direct support (for example, attending careers and similar events) they also act as important 'signposters' to various components of the programme.

On-line support

Free teacher access by the FMSP to the Integral online resources was very positively received. Such access allowed teachers to engage students through a wide variety of resources. One participant commented that these resources were better than a text book, because they explained the processes rather than just giving the answers. Another commented that the students loved the problems on the website. The fact that these resources were novel and that the answers were not readily available was also seen as a bonus.

8.4 Student views on FMSP activity

Students' engagement with, and experience of the FMSP, were explored as part of the five focus groups conducted with 44 students. The focus group centres were purposively sampled to include those that were least secure in terms of patterns of Further Mathematics entries. A summary of the main themes emerging from the analysis of comments about enrichment and tuition are given below:

Enrichment activities

Students had variable opportunities and experiences of enrichment, depending on the extent to which teachers in their centres had engaged with local opportunities.

Students were often unclear about the organisers or funders of different enrichment activities (e.g. MEI, UKMT and FMSP events at local centres or universities), so often mentioned a broad range of mathematics-related activities outside of centre, including revision sessions, maths degree taster days, and team and individual challenges.

Small numbers of the most able mathematicians tended to be picked to take part in team challenges which gave them a wider range of experience that built their confidence over the years (many of whom were now also studying FM). Whilst valuable for them, one student who was interested and keen to take part expressed the unfairness of never being picked or given the opportunity to attend, by their centre, because she wasn't 'one of the best'.

Students overwhelmingly valued and enjoyed enrichment experiences; some expressed pride at being selected to take part in junior and senior maths challenges. However, the extent to which these were offered varied between centres - from those where individual challenges are offered lower down the centre, to those engaging only with senior challenges in the 6th form. Smaller numbers of students had experienced many enrichment events and challenges throughout their time at the school where they had studied GCSEs.

Team activities were generally seen as a positive, fun and challenging experience, but these depended on the commitment of the teacher in organising these and lunch time clubs for them to prepare beforehand.

Competing against grammar and independent centres was described as overwhelming or off-putting for a small number of students, echoing comments also made by some of the teachers interviewed. However, most students saw them as an enjoyable challenge - enabling them to reassess their skills and strengths from a wider perspective.

A small number of students commented that compared to their other A level subjects, Mathematics offered more opportunities for enrichment, which they valued and enjoyed. Overall, enrichment tended to positively influence their enjoyment of mathematics over the years, giving them opportunities to experience problem solving and team based activities that required them to apply and develop their skills in new ways. Their enjoyment of Mathematics generally increased as a result, which then impacted their choice of A levels, including Further Mathematics (see Section 10.1 on students' choice making). So although enrichment experiences did not directly inform their A level decision making, it contributed to their enjoyment and feelings of wanting to further extend their subject knowledge.

Tuition

None of the five focus group centres had formally engaged in online tuition. Some teachers had used FMSP materials in some lessons, or pointed students to online Further Mathematics resources or exercises for revision, but in general there was little awareness, engagement and use of FMSP materials.

In three centres, teachers and/or students mentioned that they had, or were about to access online STEP tuition (or classes) for possible Oxbridge applications.

In one centre, students travelled to the local university to access off-site Further Mathematics tuition. The travel time and hassle involved was seen as very disruptive and difficult for the students. They described lessons as being very fast-paced which necessitated a lot of additional work outside of lessons which the teacher in centre tried to support at the end of their regular A level lessons - but they felt that this was not always enough. The overall experience was off-putting for some, and their awareness of other Further Mathematics students' experience in the year above had discouraged some A level students from also studying FM. Others had dropped Further Mathematics after AS level because of the travel inconvenience.

One centre had an external tutor through FMSP, based at the local university, coming into centre to offer face to face teaching for specific modules. Students described him as being very experienced, bringing a different skillset and expertise to the Further Mathematics teaching offered in centre. Overall they were very positive about their experience and the tuition support offered. Their teacher explained in his interview that he had the skills to teach more Further Mathematics modules but the centre w/couldn't timetable it for a small group. It was cheaper to bring in the external tutor and for the teacher to provide the additional ad hoc support in his non-timetabled time.

Common to both on and off-site tuition was the need for additional support by their class teachers - not all of whom had the skills or capacity to do this.

8.5 View of the FMSP as a whole

Teacher views

Overwhelmingly positive

The Phase 5 evaluation confirms that the new FMSP programme continues to be held in high regard by teachers. Irrespective of their roles, engagement with the programme or Further Mathematics security, teachers were overwhelmingly positive about the FMSP. Indeed all 38 of the interviewees that provided a comment on their overall view of FMSP were positive in general about the quality, usefulness and appropriateness of the programme.

Those centres with large numbers of Further Mathematics entries and existing staff expertise tended to make limited use of programme components, yet they still recognised the importance of the programme to others, with one, for example, stating they would recommend FMSP activities to colleagues in centres in different circumstances. Those centres who were classified as secure commented that the FMSP tuition support allowed flexible delivery if needed. One centre reported that the FMSP support was important to achieving their current secure status. Those with low number or no Further Mathematics entries recognised the excellent quality of the tuition and the CPD offered.

In addition to the specific issues below, interviewees highlighted the benefits of a significant programme focused on higher level mathematics, as opposed to being spread thinly across many areas; the reputation for quality, engaging resources and support. In the representative words of one teacher:

> *"They're the people to go to help your school with FM." (Teacher in centre with secure Further Mathematics status)*

Enabling Further Mathematics participation

Perhaps most importantly, some 57% (12 of 21) of interviewees who were from the less secure and least secure categorised centres suggested that without the FMSP Further Mathematics may not be undertaken at all.

By providing resources, tuition, support and professional learning, FMSP enabled some centres to be able to provide FM, which might not have been possible otherwise. Importantly, this group of interviewees included respondents from three centres identified as having the "least secure" Further Mathematics status:

> "Really useful place to find resources and ask for support - would have struggled without them for A level teaching."

> "FMSP has been invaluable. … Without FMSP there would be no uptake of Further Mathematics at all at this school. The quality of support and training is very high. FMSP is a label for guarantee of quality."

> "It is reassuring that FMSP is there. The coordinator sends CPD info (even though it is not taken up), and helped organise the online tuition - without it the Y12 would have gone elsewhere for FM."

Others in centres that were more secure felt similarly, either from the previous experience in other centres, their earlier experience in their current centre or in relation to the system more broadly:

> "For other schools, including my previous school, FMSP means that they can offer Further Mathematics to students even if they have a small number and it is not financially viable to have taught classes." (Teacher in centre with highly secure Further Mathematics status)

> "Resources have helped the school to believe that they could offer Further Mathematics - until two years ago the school didn't feel confident to offer further maths, but directed students to other schools/colleges." (Teacher in centre with no Further Mathematics status since new to the programme)

> "FMSP is vital for other schools who have fewer specialist teachers. Students would be disadvantaged if not able to take up Further Mathematics courses and the FMSP provides a support mechanism to ensure that schools/colleges can offer FM." (Teacher in centre with highly secure Further Mathematics status)

A related point, expressed by 6 respondents was the importance of the role of the FMSP as a safety net enabling centres to continue to offer FMSP if there are temporary issues with staffing, or concerns about delivery, for example:

"It has allowed the school to offer a more flexible pathway to Further Mathematics and has widened the number of students who can take it by using both in house delivery and online lectures." (Teacher in centre with secure Further Mathematics status)

"FMSP are good - able to help and responsive to needs, provide personal support to students if needed. Will be calling on their support next year if needed as only one or two students do FM." (Teacher in centre with no Further Mathematics status since new to the programme)

FMSP staff viewed positively

A further significant theme emerging in relation to respondents' view of the FMSP was the positive view held of the staff, especially, as noted above of Area Coordinators. Ten respondents explicitly mentioned the importance of these roles in directing them to resources, brokering support and generally providing help when needed:

"Short of coming into school to teach maths, can't think of anything else FMSP could do to help. They're the people to help your school with FM. The area coordinator is always good, efficient at sorting things out." (Teacher in centre with secure Further Mathematics status)

"It is really important to have the area coordinator - for connection and support. I miss the revision days that used to be offered, this was an opportunity to meet the coordinator, there is less contact/relationship building now as a result of only online tuition support." (Teacher in centre with relatively secure Further Mathematics status)

Quality provision

Finally, it is worth noting the areas of provision that were highlighted by interviewees. 11 interviews approvingly mentioned the quality of resources, for example:

"Very useful resources, used a lot of the website materials used in lessons - unseen materials where answers can't be googled." (Teacher in centre with relatively secure Further Mathematics status)

A further 5 discussed quality of tuition, for example:

> "Quality of tuition and overall co-ordination of the programme is excellent and has allowed students to be successful." (Teacher in centre with less secure status)

And two also discussed CPD:

> "FMSP is a comprehensive programme run by knowledgeable staff. I am very grateful for support and want to encourage more of my staff to attend CPD and build up links with other schools and Unis." (Teacher in centre with the least secure Further Mathematics status)

9. Engagement with Maths Hubs

9.1 FMSP engagement with Hubs

A short online survey was conducted in January 2016 with Maths Hub leads to gain their perspectives on how they work with the FMSP and how they view priorities for the coming year (see section 9.2 for priorities). In total 25 out of 35 Hub leads responded to one or more questions on the survey who were for the most part the Maths lead within the Hub. Respondents were asked how often the FMSP currently works with their Hub in terms of the FMSP providing information and advice, and working on joint projects. Of the 24 respondents who completed the question on FMSP work with Hubs, most indicated that this happened as often as they felt needed (Table 9-1).

Table 9-1 How often and in what ways FMSP works with the Hubs

	As often as needed	Less often than needed	Never/not at all	Unsure	**Total n**
FMSP provides information to our Hub	83%	8%	4%	4%	**24**
FMSP is working with our Hub on joint projects	83%	8%	4%	4%	**24**
FMSP provides advice to our Hub	83%	4%	4%	8%	**24**

Respondents were then asked in turn how often and in what ways they work with the FMSP. There was some indication from Hub leads that they would like to see the planning and commissioning of FMSP student/teacher events to happen more often. Less than half of respondents (41%) cited that their Hub provided advice to FMSP as often as needed, with 32% citing that this did not happen at all. The majority of respondents (around 80%) felt that the following aspects happened as often as needed:

- our Hub signposts centres to FMSP support
- our Hub provides information to FMSP
- our Hub helps host/organise FMSP student/teacher events

Table 9-2 How often and in what ways the Hubs work with FMSP

	As often as we believe is needed	Less often than we believe is needed	Never/not at all	Unsure	**Total n**
Our Hub and FMSP plan joint student/teacher events	57%	26%	13%	4%	**23**
Our Hub commissions FMSP student/teacher events	54%	25%	17%	4%	**24**
Our Hub publicises FMSP student/teacher events	82%	21%	0%	0%	**22**
Our Hub provides advice to FMSP	41%	14%	32%	14%	**22**
Our Hub signposts centres to FMSP support	78%	13%	9%	0%	**23**
Our Hub provides information to FMSP	77%	9%	14%	0%	**22**
Our Hub helps host/organise FMSP student/teacher events	79%	8%	13%	0%	**24**

As can be seen from Table 9-3, 61% of respondents indicated that the FMSP Area Coordinator was a member of the strategic group for their Hub, whilst 87% indicated that the FMSP Area Coordinator was a member of one or more work groups.

Table 9-3 FMSP co-ordinator

	Yes	No	**Total n**
The FMSP Area Coordinator is a member of the strategic group for our Hub	61%	39%	**23**
The FMSP Area Coordinator is a member of one or more work groups	87%	13%	**23**

9.2 Hub and teacher views on priorities

Hub leads were asked in the survey about what the future priorities should be for FMSP. Outcomes are shown in Table 9-4 in ranked order.

Table 9-4 Priorities

	Essential	High priority	Medium priority	Low priority	Not a priority	**Total n**
Support for teachers of A level Mathematics and Further Mathematics	63%	25%	13%	0%	0%	**24**
Supporting teaching of GCSE mathematics	42%	33%	21%	4%	0%	**24**
Promoting study of A level Mathematics to KS4 students	38%	46%	17%	0%	0%	**24**
Initiatives to improve local provision for A level Further Mathematics	33%	50%	13%	4%	0%	**24**
Initiatives to improve participation for A level Mathematics and Further Mathematics	33%	46%	21%	0%	0%	**24**
Targeting support on centres with low provision/low participation in A level Mathematics and Further Mathematics	33%	42%	21%	4%	0%	**24**
Initiatives to improve the gender gap in participation at A level	21%	54%	21%	4%	0%	**24**

Open comments focused on a small number of areas. Comments that related to A level Mathematics and Further Mathematics (mentioned by 5 respondents) included helping teachers access information about the new A Level, and providing support including schemes of work to ensure 'deep learning' (one respondent in each case). Specific curriculum areas mentioned (by one respondent in each case) were statistics and mechanics.

In addition, three teacher interviewees in the main body of the research mentioned extending and developing resources available for A Level Mathematics and Further Mathematics modules.

Comments related to the new GCSE in Mathematics (mentioned by 7 respondents) focused on supporting teachers' confidence and effectiveness, especially for non-specialists (1 comment), with 3 respondents suggesting a focus needed on the GCSE to A Level transition.

Six respondents discussed the need to increase or maintain post-16 participation in Mathematics, especially in centres with small sixth forms (one respondent), with two mentioning broadening the gender and social balance at post-16 and potentially having specific FMSP enrichment activities to promote this. One teacher interviewee also mentioned recruitment and retention of students.

Other areas mentioned were working together to provide enrichment activities (two respondents) such as "STEM Days, Maths days for LPP centres, Extension provision, providing links with projects in different regions, providing visiting inspiring teachers for enrichment talk across an area", working together on broad mathematics teacher skills development (one respondent) and requests for more proactive engagement of FMSP in sharing their offer with the Hub (two respondents).

Other areas mentioned by teachers involved in qualitative interviews included more face to face contact with coordinators, perhaps meeting the Headteacher, attending options evenings or reintroducing revision days, and extending competitions to other year groups beyond Y10 and Y12.

9.3 Enablers and Barriers to engagement with the FMSP by Maths Hubs

Barriers to engagement with FMSP

Maths Hub survey respondents were asked about barriers to engagement with the FMSP.

The most commonly mentioned barrier was time, identified by 7 respondents: "for everybody, the Hub staff, the work group leaders and the teachers" in the words of one respondent. The issue of teacher time was emphasised by others: "This is simply not our time and the time of the FMSP but more of an issue is the time of maths teachers."

A further issue was geography, with issues of lack of intersection of Maths Hub and FMSP areas, noted by these two respondents:

> *"Different geographical interests. So our hub crosses over two FMSP areas - which can make it slightly harder to contact the right people"*

> *"Connecting with all the schools in the Hub's region, as [our region] has a large region of schools to cover"*

Centre capacity was mentioned by three respondents, with one noting the problem was "finding sufficient people in the schools and colleges with the capacity to get involved with this [leading or hosting events or being part of working groups]."

A set of five comments identified the need for a shared vision and role clarity, being able to both develop on an agreed position and a shared understanding of differences in role:

> "Doubling up of work: unclear guidance on who is responsible"

> "Different aims Sharing of data 'Stepping on toes'"

> "Shared vision on what effective PD looks like"

> "Clarity about the roles of each organisation. Hubs are focused on teacher development whereas FMSP offer teacher and student development. There are still ways the two can work together still with different roles. There is no need for it to be both if there are distinct roles for each, for example, student enrichment by FMSP is highly valued by schools."

10. Further Mathematics capital and ecologies

This section considers students' reasons for choosing or not choosing Further Mathematics using the concept of Further Mathematics capital, and the influence of FMSP activity on this. The concept of capital is extended to the consider organisational capital and the variety of Further Mathematics cultures - both those which are more enabling and those in which there are greater barriers. The concept of Further Mathematics ecology is posited as a means to model the interplay between culture, capital, Further Mathematics entries and the opportunity to study Further Mathematics. In this section, the findings of the evaluation are synthesised using the concepts of capital, ecology and cultures in a revised model of how the FMSP impacts Further Mathematics (and A level Mathematics) participation and outcomes. This provides the basis for reconceptualising the role of the FMSP to one of developing positive FM ecologies.

The analysis draws on the findings from the 42 interviews with teachers and focus groups with 44 students doing A level Mathematics and/or Further Mathematics, and outcomes of the modelling analyses reported earlier as well as other relevant research. In Annexe 3 a further analysis of these themes is provided organised in terms of barriers and enablers to participation.

10.1 Reasons for choosing and not choosing Further mathematics

In a Diamond 9 activity students were given 9 cards with different factors or reasons for choosing A level Mathematics/Further Mathematics. They were asked to sort these in order of importance and stick them on a five-tier diamond template (in a 1-2-3-2-1 format with the most and least important reasons at the top and bottom). Students were also asked to annotate a timeline to identify the significant points at which they became aware of and chose A level Mathematics/Further Mathematics. These activities formed the basis for further focus group discussion about students' relationships to Further Mathematics.

Enjoyment and self-efficacy

The following reasons for choosing A level Mathematics/Further Mathematics were ranked as most important in the Diamond 9 activity:

- I enjoy mathematics (47% - 21 students)
- Maths is useful for my future career or future study (36 % - 16 students)
- I am good at maths (11% - 5 students)

- My past experience of studying maths was positive (5% - 2 students)

Although there were differences between centres in the relative prominence of these reasons (see Annexe 3), enjoyment of mathematics was the most important factor overall in influencing students' choice at A level followed by its usefulness for future career or study. Enjoyment, being good at mathematics and having positive past experiences were often closely ranked and linked to each other in students' fuller explanations, as were competence, often through exam success; internalised positive learning experiences and confidence. Some (particularly those studying Further Mathematics) traced their enjoyment of Mathematics or sense of their relative competence back to primary centre. KS2 SATs results and early enrichment (for example through Gifted and Talented programmes) were key influences in their notion of themselves being 'good at Maths'.

In the timeline exercise, examination results (especially at GCSE and AS) were key decision-making points for nearly all students, confirming their strengths and their 'best options' for choosing and continuing with A levels Mathematics. Additional qualifications (e.g. Additional Mathematics, Further Mathematics GCSE) 'bridged the gap' between GCSE and A level and encouraged decisions to take Mathematics further.

Influences of parents/family, and teachers

The influence of parents/family and teachers were secondary to these student-centric reasons, often forming their second and third tier reasons. Parents were often described as being supportive of whatever choices they made. Family influences were sometimes identified as fuelling early motivation (e.g. parents' mathematics interest/skills/encouragement, competing with older siblings) but this varied across the centres depending on socio-economic profile and parental backgrounds. Older siblings choosing Further Mathematics and open days in other centres also raised awareness in some cases.

Most students first became aware of A level Further Mathematics through their teachers in lessons or during the options process. Teachers' encouragement and feedback about their abilities and subjects choices appeared more influential when they were discussing their decision making processes in relation to their timelines, than was suggested by their ranked reasons in the Diamond 9 exercise. One student said, 'all teachers want you to do their subject so you shouldn't be influenced by them'. In further discussion teachers were often part of the reason they had a positive past experience of Mathematics.

This may highlight the subtlety of multiple, sometimes indirect external influences on their internalised sense of enjoyment and feeling that they are 'good at Maths'.

Aspirations

In addition, their career/goal focus, self-motivation and determination also emerged as important themes influencing their personal engagement. Clear degree and/or career goals guided the decisions they made about A level combinations, including whether to do Further Mathematics or not, which in some cases seemed to be gender related. For example, females studying a wider range of other sciences were more likely to be considering medicine or health-related routes that did not require FM; whereas males tended to be orientated towards engineering, economics, accountancy and banking careers where Further Mathematics was seen as a more important pre-requisite. For some students, careers advice about degrees courses and further study also had a bearing on their final A level subject choices and whether or not Further Mathematics was appropriate for them or not.

Gender

As evidenced in the literature and this qualitative data, a significant barrier to wider participation in Further Mathematics is gender. In all the centres and focus groups, there were fewer females involved in Further Mathematics than males. This was something most teachers were asked about explicitly in their interviews, and an issue that some students referred to in their focus groups, or a pattern that emerged indirectly from the thematic analysis.

10.2 Further Mathematics capital

Extending the concept of science capital to Further Mathematics

The concept of science capital has been posited as means to conceptualise the interplay of social, cultural and familial practices, knowledge and relationships that support engagement in science and influence of patterns of participation (Archer et al. 2012; Archer et al. 2015). Components of science capital (Archer et al 2015) are:

- aspirations to a future science job
- valuing science and scientists
- parental attitudes and practices (including attitudes to science)
- "informal" science activities
- everyday science/media engagement

- valuing museums/museums experiences
- science teachers and lessons
- self-efficacy in science

This can be extended to consider mathematics capital (see Williams and Choudry, 2006; Noyes, 2016) and in relation to the FMSP, Further Mathematics capital. Clearly, mathematics and science capital can be expected to overlap given the close relationship between further study of mathematics and participation in A level science and/or aspirations to science degrees.

Some of the components of science capital can be translated directly to mathematics - for example, self-efficacy in mathematics or parental attitudes towards mathematics; others have parallels or may point to underlying variables for example, a theme enjoying mathematics and presumably enjoyment of museums and museum experiences underlying the valuing of them. It is outside the scope of this evaluation to fully explore the nature of mathematics capital. Nevertheless, this concept can help to explain the value of the FMSP enrichment experiences and potentially to guide future developments in them.

Some aspects of Further Mathematics capital as currently conceived are problematic. For example, the notion of 'capable of doing Further Mathematics' is somewhat doubled edged, with other students who do not take Further Mathematics identifying in some places a group of elite students. This can lead to some resentment of those excluded from this status. Such constructions are also highly gendered. Thus some of the factors that support some to take Further Mathematics may hinder the participation of others.

Science capital is found to be unevenly distributed and patterns are related to wider cultural capital, and so social and economic patterns of inequity. The nature of the evaluation study does not support drawing conclusions at an individual level. However, the pattern of school level participation, indicate that access to centres with high levels of Further Mathematics capital is unequal.

Personal Further Mathematics capital

In addition to mathematics ability and attainment, teachers also highlighted that students' intrinsic, personal characteristics such as self-motivation, capacity for 'independent study' and their 'work ethic' were important for their successful participation in Further Mathematics. Some of this relates to the acknowledgement that timetabled lessons and any additional FMSP tuition alone is insufficient - a number of teachers highlighted the need for students to be hard working and commit to self-study given the lack of time or support often available in centre to meet the demands of the Further Mathematics course. Parental valuing and support for Mathematics was also cited - particularly by highly secure centres - as being important in ensuring a student's individual success in Further Mathematics.

Organisational capital: schools, departments and teachers

A second way that the concept of science capital can be extended is to look beyond the individual student and their preferences and to consider capital at a departmental level. The analysis of enablers and barriers to participation in Section 9 above indicates constituent aspects of Further Mathematics capital at a departmental and school level.

Teachers identified the characteristics, context and attainment profile of the school and its student intake as the primary factor that either enabled Further Mathematics engagement (or, as discussed below, could also be barrier to [more] engagement). These characteristics often related to the schools' security rating: those with higher status were often associated with higher-achieving cohorts as schools attracted 'the brightest students' and maintained entries and results over time by tightening selection criteria. Components of Further Mathematics capital in those with high security were:

- A level entry base
- high GCSE attainment
- a critical mass of students interested/capable of doing Further Mathematics

Personal success stories of students who had excelled in Mathematics/Further Mathematics e.g. those who went on to study Mathematics/STEM subjects at prestigious universities were cited by centres as role models to help promote Mathematics and Further Mathematics. This was particularly the case where females or students from less privileged backgrounds had done well. Two centres commented that they thought these students had positively impacted the take up of A level Mathematics and Further Mathematics.

In addition the support of senior leaders for Further Mathematics was identified as important including, in some cases, not being driven by cost analysis of cohort size.

Teachers in centres across the range of security classifications identified that their own skills, confidence and interest in Further Mathematics – or those of their departmental colleagues -

were cited as the key factor in enabling the department to be able to offer a full range of Further Mathematics modules. Key aspects of teacher capital found were:

- enjoyment of mathematics
- commitment, energy, accessibility in relation to mathematics
- capacity to teach Further Mathematics (in post 16 settings) or to offer additional support for those taking LiL and similar provision
- a Further Mathematics enthusiast/champion (including in 11-16 settings someone who encourages engagement with mathematical problem solving and so on)

This enabled them to offer timetabled Further Mathematics in-house by qualified and experienced staff retained by the centre as they have opportunities to use and extend their skills.

Teachers had varying experience, different strengths and motivations towards upskilling to cover skills gaps. In centres with higher security, interviewees were more likely to report that skills and confidence in Further Mathematics was strong across all/most module areas, with larger Further Mathematics entry numbers providing opportunities to gain additional experience and access CPD. Departmental features contributing to organisational capital were:

- cohesive and supportive relationships
- specialist mathematics teachers
- contribution to A level teaching the norm
- commitment to enrichment and a 'love' of mathematics
- investment in students interested/capable of doing Further Mathematics

The enthusiasm and commitment of Further Mathematics teachers was cited by staff (and students) in being key to inspiring a love of mathematics through prioritising and organising enrichment activities. This often took time and energy above and beyond their demanding classroom responsibilities, but in some centres (more often those with lower security status) where timetabling pressures and KS4 provision was prioritised, enrichment and additional support for Further Mathematics was often more challenging.

Impediments to FM capital or its translation into positive engagement

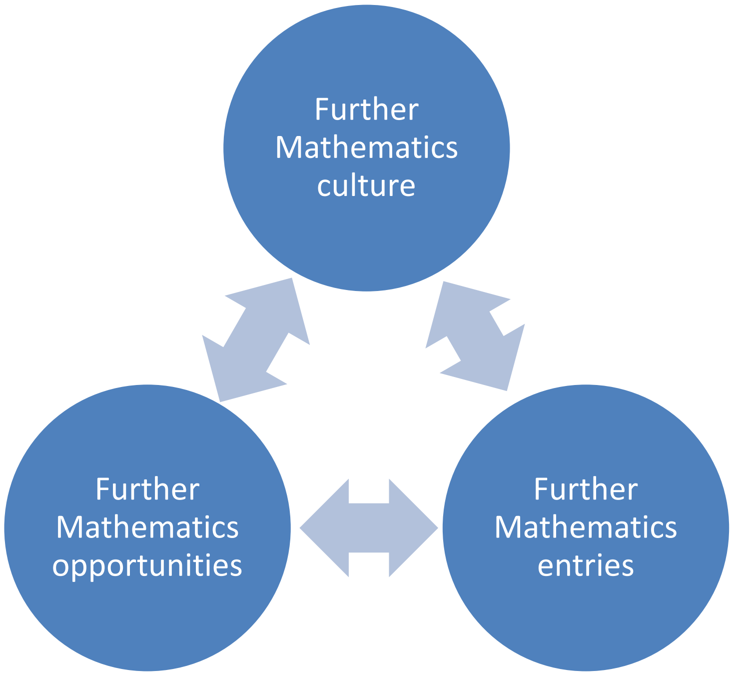
For some students, a personalised sense of lack of confidence was related to less positive previous learning experiences in mathematics (this was both internalised as them not being as able as other students, or externalised to identifying lack of teacher support as the underlying issue). For some of those who did not have as many positive enrichment experiences or did not have the opportunity to take Level 2 Further Mathematics or other additional 'bridging' qualifications, the jump from GCSE to A level Mathematics/Further Mathematics was more challenging than they had anticipated. If FMSP tuition or teacher skills/support was not adequate to meet their learning needs, these confounding barriers were likely to lead to poorer exam performance at AS, which partly explains the high drop-out rates reported by many centres between Y12/AS and Y13/A2. Female students often stated that they had chosen a wider range of A levels, including mathematics and sciences - and often in combination with non-science subjects, which had for many, ruled out Further Mathematics from their options. In one centre, a female student and her teacher both commented that timetabling difficulties was the frustrating barrier to her studying Further Mathematics in combination with her other choices. At a university admissions level, teachers and students cited that females studying sciences tended to pursue medicine or biological/health related courses which did not specifically require Further Mathematics in the same way that engineering or physics degrees - more often cited by male students - did.

Further Mathematics culture

The Further Mathematics culture refers to the student and teacher dispositions towards Further Mathematics. As such it is embedded within a broader A level Mathematics culture and that in turn a general mathematics culture. The concept of Further Mathematics culture offers holistic way to consider the way in which enablers and barriers to participation interact. It also shifts attention from participation as being a result of individual choice alone but rather identifies the way in which that is impacted by context. The analysis of the current evaluation is supported also by recent case studies of departments that are successful in encouraging students to participate in A level Mathematics including in relation to gender (Golding and Smith, 2016).

A positive Further Mathematics culture is supported by, and in turn supports opportunities to study Further Mathematics and the number of Further Mathematics entries in a centre, as shown in Figure 10-1.

Figure 10-1 Further mathematics culture



Three aspects of a strong Further Mathematics culture amongst at least a minority of students are that Mathematics is perceived as:

- intrinsically enjoyable
- challenging:
  - either as a personal challenge - proving something to oneself (Mendick, 2006)
  - or, and more common in the focus group sample, a competitive challenge - proving something to others (Mendick, 2006)
- career enhancing, echoing the FMSP tagline- 'Let Maths take you further"

Such a culture can exist in both 11-16 and 11-18 schools and colleges.

Considering the data and analysis in Section 8, the FMSP positively impacts on Further Mathematics capital through the following aspects of the programme.

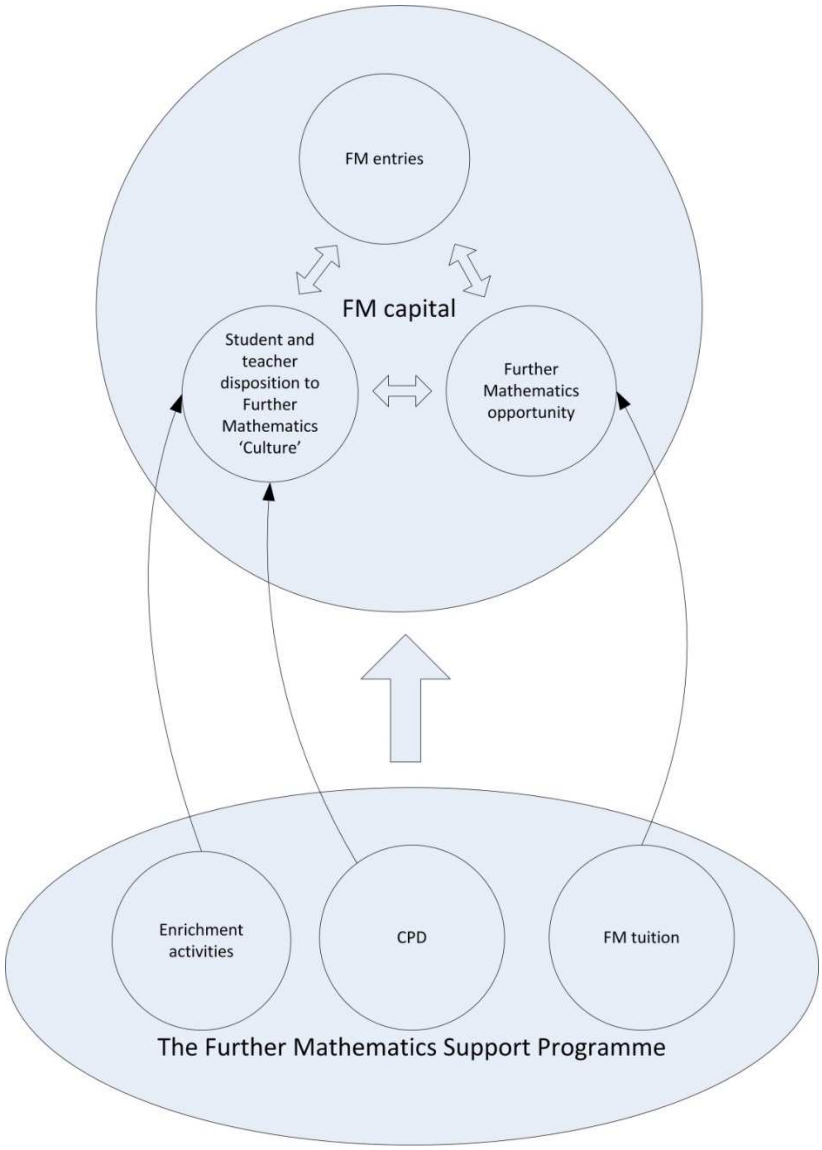
- enrichment experiences-positive enrichment experiences through KS3 and KS4 were identifiable as a recurring element across a number of student timelines of their Mathematical experience and choices. Enrichment experiences also helped support positive mathematical and so Further Mathematics cultures

- tuition-making Further Mathematics more available including supporting high security centres to extend module options
- CPD- identified as strengthening teaching skills
- on-line resources- for other centres (including those with low/no Further Mathematics security), the MEI Integral resources were identified as providing access to materials that helped less experienced staff develop their skills at an individual level.

10.3 Further Mathematics ecologies

One way to conceptualise the role of the FMSP is through the concept of a Further Mathematics ecology. A simplified diagrammatic representation is shown below in Figure 10-2.

Figure 10-2 the FMSP ecology



The concept of a Further Mathematics ecology is flexible and can be applied at both a school level but also a system wide concept. This elucidates the role of the FMSP in being an important part of a school level Further Mathematics ecology in contexts where the internal ecology does not support secure or in some cases any Further Mathematics entries.

11. Evaluation, priorities and recommendations for the Further Mathematics Support Programme

In this section the FMSP programme is evaluated in terms of capacity and capability, reach, effectiveness (quality and impact) and sustainability. Priorities are identified and recommendations in relation to each of these are proposed. In addition, implications for future evaluation are considered.

Evaluation

1. Capacity and capability

The FMSP continues to enhance the capacity and capability of centres to offer Further Mathematics in the following ways:

- contribution to on-going increases in overall key measures:
  - participation in A level Further Mathematics
  - the number of centres offering Further Mathematics
  - improvements in Further Mathematics security
- teacher CPD that helps to enhance the organisational mathematics capital of centres as well as, indirectly, of individual students. The CPD offer is being taken up to a greater extent in centres that are likely to have the greatest need
- enrichment and tuition also support capacity and capability. This happens for both centres with higher and lower levels of Further Mathematics security
- the FMSP provides a safety net for the least Further Mathematics secure and less Further Mathematics secure centres, which can be at risk of losing Further Mathematics capacity due to low numbers

2. Reach

The FMSP has considerable reach, but there is scope for developing this in a targeted way:

- overall reach is strong as indicated by:
  - the number of centres registered
  - the number of centres engaging in one or more FMSP activities
  - the number of teachers participating in CPD
  - the number of students participating in enrichment

- the FMSP also successfully engages centres that are a priority – those centres offering A level Mathematics but not yet offering Further Mathematics. At the same time there are still considerable numbers of centres, particularly 11-16 schools that have not yet registered with the FMSP who could be targeted
- the evaluation highlights groups of priority students for whom the FMSP may need to make particular efforts to reach. The model presented in this report suggests that Further Mathematics is taken by a less diverse group of students than those that take A level Mathematics
- the FMSP is actively engaged with other key organisations including the new Maths Hub networks

3. Effectiveness (quality and impact)

Views of quality

Activity undertaken by the FMSP is overwhelmingly viewed positively by teacher participants. There is evidence from student focus groups and student feedback that, in general, students also view FMSP support favourably.

Impact

The key measures of impact relate to participation in A level Mathematics and Further Mathematics more generally. It is notable that the increase in Further Mathematics is faster than the increase in A level Mathematics and for centres with low Further Mathematics entries the FMSP is viewed as essential to continue to offer Further Mathematics. Improvements in 'security' status are also an indicator of success in regard to making these increases sustainable. Intermediate impact measures reported by teachers included increase confidence in teaching, improved subject knowledge, and strengthened pedagogical approaches.

4. Sustainability

The FMSP's reach, quality, capacity/capability-building and impact mean that the enhancement of Further Mathematics cultures and capital are supporting centres to develop secure and so sustainable levels of Further Mathematics entry.

However, the same analysis shows that increases are not uniform. Whilst the numbers of Further Mathematics secure and highly secure centres have increased overall, there are centres where

Further Mathematics has become less secure. The qualitative analysis, particularly of enablers and barriers provides indications of why this is the case.

Notably, the most secure centres with high numbers of A level entrants are independent schools and those 11-18 state schools with more advantaged student populations. In these centres there is a considerable degree of Further Mathematics capital and an internal Further Mathematics ecology that is self-sustaining. In other centres, even those with increasing numbers of Further Mathematics entries, there is still a degree of fragility. Given changes in both A level courses and in funding arrangements for A level study, there is some risk to Further Mathematics entries. Whilst the general trend was towards increased Further Mathematics security, the security analysis showed that for every 2 centres that improved security, one centre became less secure over the period analysed. Given this, and the relatively low levels of mathematics capital in some centres, interventions and enhancements such as the FMSP or similar will be needed on an ongoing basis. The concept of 'sustainability' of Further Mathematics provision as an evaluation criteria for the FMSP may be problematic if this is taken to mean that in the short to medium term all centres will achieve fully sustainable provision.

The need for ongoing support

The FMSP and its programme components have grown organically with different aspects of the programme being developed in relation to particular issues as they emerge. The evaluation suggests that more certainty of long term support for the FMSP would greatly assist in enabling the programme to strategically plan and deliver success against its stated aims. The strength of the programme partly lies in its own longevity, and related to this, the positive regard it is held in, flexibility, and therefore ability to support centres over time, bearing fruit in the ongoing increases in Further Mathematics participation, and Further Mathematics security.

Wider engagement

The FMSP is actively engaged in Maths Hubs and has also created and stimulated teacher networks. However, the potential for those centres with secure Further Mathematics entries to support others is a possible means to extend the Further Mathematics support that can be provided, and therefore enhance the sustainability of FMSP programme impacts. In addition, whilst the Maths Hub leads are key players in support for improvements in mathematics education, teaching schools are another potential resource, since they have remit and access to resources to support other schools in improving their teaching, and school improvement more broadly.

Future evaluation

The evaluation has indicated that student focus groups are a potentially useful method for assessing the quality and impact of FMSP activity. Further, quantitative modelling has identified a number of issues that are important for the FMSP to consider. A challenge for future evaluation is to consider how to model participation in FMSP activity to identify and measure impact at the student level. Whilst the evaluation has given some indications of what influences changes in Further Mathematics security, this could be further investigated.

Recommendations

Based on the evaluation findings, recommendations are made below. These largely relate to programme structure and targeting the programme, with an evaluation recommendation and an overarching recommendation on the need to maintain the FMSP.

Programme structure

Recommendation 1: Consider offering tailored CPD focused on 'supporting Further Mathematics students engaged in tuition' as part of tuition packages, given that some teachers interviewed identified a lack of confidence to support students accessing on-line tuition.

Recommendation 2: The recently developed on-line (LIL) tuition has begun to be used by centres with a variety of Further Mathematics security profiles. The FMSP should consider ways in which this can be further enhanced or encouraged.

Recommendation 3: Consider ways that the 'crossover' benefits for the different components of the FMSP offer, such as tuition and teacher professional development can be enhanced, and how existing promotion of these benefits to centres can be improved.

Recommendation 4: The FMSP should consider enhancing materials and information to address students whose primary motivations are related to the enjoyment of mathematics itself as a reason for studying Further Mathematics, in addition to educational and career aspirations.

Targeting the programme

Recommendation 5: Maintain focus on the centres which are least Further Mathematics secure and use this classification as a means to prioritise support.

Recommendation 6: The FMSP should investigate finding ways to further engage 11-16 centres, including increasing the number registered with the FMSP, in order to influence post-16 participation.

Recommendation 7: The FMSP should consider how to further focus efforts - and future research - on understanding and providing appropriate support for students from backgrounds underrepresented in those taking Further Mathematics.

Working with schools

Recommendation 8: Consider further ways to engage with and influence school leaders directly, since the evaluation has underlined the importance of school leadership to support Further Mathematics participation.

Recommendation 9: The FMSP should consider how to engage further with current developments in the school self-improvement agenda beyond work with Maths Hubs. Potentially, those centres with high Further Mathematics security are an underused resource to support others.

Evaluation

Recommendation 10: Measuring long term impact of professional development and enrichment activities is, in general, more challenging and attention should be paid to this in future evaluations, potentially this might be linked to further investigation of change in security status.

Maintaining the programme

Recommendation 11: Given the ongoing risks to Further Mathematics sustainability, especially for centres with less secure provision, but even for more secure centres, the FMSP - and its principal funder, the Department for Education - should continue to make the case for the necessity of the programme to support Further Mathematics, and so the FMSP should consider ways to secure long term and more stable funding. The DfE should consider this recommendation.

12. Conclusion

The evaluation affirms previous findings about the quality and value of FMSP activity and the positive regard of stakeholders for the programme. It supports a continuation of a varied programme offer. The analysis highlights the important role that the FMSP has and can play in supporting Further Mathematics culture particularly in contexts and centres in challenging circumstances.

The evaluation confirms the impact that the FMSP has had on widening participation in Further Mathematics, but also indicates that access to Further Mathematics - and so to both the intrinsic benefits of this as well as access to further opportunities - continues to be more available to students who are socially and economically advantaged.

13. References

Advisory Committee on Maths Education, (2009). 'Rises in A-Level Mathematics - some preliminary thoughts by ACME' (viewed on 27th June 2016)

Advisory Committee on Maths Education, 2013 URL: http://www.acme-uk.org/home

Adkins, M, & Noyes, A (2016) 'Reassessing the economic value of advanced level mathematics'. *British Educational Research Journal*, *42*(1), 93-116

Boaler, J, Altendorff, L, & Kent, G (2011) 'Mathematics and science inequalities in the United Kingdom: when elitism, sexism and culture collide' *Oxford Review of Education*, *37*(4), 457-484

Blamey, A and Mackenzie, M, (2007) 'Theories of change and realistic evaluation peas in a pod or apples and oranges?' *Evaluation*, *13*(4), pp.439-455

Brown, M, Brown, P, & Bibby, T (2008) '"I would rather die": Reasons given by 16-year-olds for not continuing their study of mathematics' *Research in Mathematics Education*, *10*(1), 3-18

Hodgen, J, Marks, R, & Pepper, D (2013) 'Towards Universal Participation in Post-16 Mathematics: Lessons from High-performing Countries'. Nuffield Foundation

Homer, M, Ryder, J, & Banner, I (2014) 'Measuring determinants of post-compulsory participation in science: a comparative study using national data' *British Educational Research Journal*, *40*(4), 610-636

Golding, J & Smith, C (2016) 'Wider school effects of engaging with the Further Mathematics Support Programme to introduce Further Mathematics: Final Report' London: UCL IoE

Johnson, R. B, Onwuegbuzie, A. J, & Turner, L. A (2007) 'Toward a definition of mixed methods research' *Journal of mixed methods research*, *1*(2), 112-133

Johnson, B and Turner, L.A (2003) 'Data collection strategies in mixed methods research' *Handbook of mixed methods in social and behavioral research*, pp.297-319

Jones, I, Wheadon C, Humphries S and Inglis, M (2016) 'Fifty years of A-level mathematics: have standards changed?' British Educational Research Journal, forthcoming.

Layder D (1998) 'Sociological theory: linking theory and social research' London: Sage

Lord, K & Stripp, C (2014) 'Improving access for state-school students' The Institute of Mathematics and Its Applications (*IMA) Mathematics TODAY, Vol. 51 Issue 2*, p80-82 (viewed on 27th June 2016)

Matthews, A and Pepper, D (2007) 'Evaluation of participation in A level mathematics: Final report' London: Qualifications and Curriculum Agency

Mendick, H (2006) 'Masculinities in mathematics' McGraw-Hill Education (UK)

Nardi, E, & Steward, S (2003) 'Is mathematics TIRED? A profile of quiet disaffection in the secondary mathematics classroom' *British Educational Research Journal*, *29*(3), 345-366

Noyes, A (2009) 'Exploring social patterns of participation in university-entrance level mathematics in England' *Research in Mathematics Education*, *11*(2), 167-183

Noyes, A (2013) 'The effective mathematics department: adding value and increasing participation?' *School Effectiveness and School Improvement*, *24*(1), 1-17

Noyes, A (2016) 'Bringing Bourdieu to mathematics education: a response to Williams and Choudry' Research in Mathematics Education, 18(1), 22-26

Noyes, A and Adkins, M (2016) 'Reconsidering the rise in A-Level Mathematics participation' *Teaching Mathematics and its Applications* 35.1 (2016): 1-13

Reiss, M, Hoyles, C, Mujtaba, T, Riazi-Farzad, B, Rodd, M, Simon, S, & Stylianidou, F (2011) 'Understanding participation rates in post-16 mathematics and physics: Conceptualising and operationalising the UPMAP Project' *International Journal of Science and Mathematics Education*, *9*(2), 273-302.

Ryan, G and Bernard, H.R (2003) 'Techniques to identify themes' *Field Methods 15(1)*, 85-109

Searle, J (2012) 'Evaluation of the Further Mathematics Support Programme 2009-2012: Summary Report' Durham: Centre for Evaluation and Monitoring

Searle, J (2014) 'Evaluation of the Further Mathematics Support Programme Phase 4' Durham: Centre for Evaluation and Monitoring

Williams, J, & Choudry, S (2016) 'Mathematics capital in the educational field: Bourdieu and beyond' *Research in Mathematics Education*, *18*(1), 3-21

Annexe 1: Detail of the evaluation methodology

Overview

A mixed methods (Johnson, Onwuegbuzie, & Turner, 2007) evaluation design was developed with the FMSP to address the evaluation aims and further refined as evaluation activity proceeded. The study's mixed methods approach took a pragmatic position of aiming to answer the research questions with the most appropriate set of methods, rather than a 'paradigmatic' stance of coming from a particular philosophical standpoint (for example, an exclusively realist or interpretivist standpoint) on the nature of the social world. From this perspective, we agree with Johnson and Turner (2003 p299) that 'methods should be mixed in a way that has complementary strengths and non-overlapping weaknesses…..[in order] …(a) to obtain convergence or corroboration of findings, (b) to eliminate or minimize key plausible alternative explanations for conclusions drawn from the research data, and (c) to elucidate the divergent aspects of the phenomenon'.

Research was conducted in accordance with institutional ethical approval and in keeping with the guidelines of the British Educational Research Association, with informed consent obtained from participants in qualitative data collection activities.

Evaluation strands

The evaluation had the following principal components:

1. Developing an initial understanding of the tacit FMSP theory of change to inform the evaluation including sampling.

2. School participation in FMSP:

- developing a measure of security of Further Mathematics entries based on longitudinal school census data
- analysing Further Mathematics participation in relation to engagement with the FMSP drawing on FMSP datasets
- analysing Further Mathematics participation in relation to school characteristics (school type, attainment level, contextual factors)

3. Modelling student participation and attainment in Mathematics and Further Mathematics A level using data from the National Pupil Database (NPD) for the complete cohort of students in England who completed Key Stage 4 in 2010/11 and are recorded as taking a KS5 assessment between 2011/12 and 2013/14. Engagement in FMSP was brought in as a school level explanatory variable. The stages of this phase were:

- descriptive analyses of participation and attainment in A and AS level Mathematics and Further Mathematics to identify outcome variables
- identification of explanatory variables at the student[8] and school/institution levels and descriptive analyses of how they are statistically associated with participation and attainment in A and AS level Mathematics and Further Mathematics
- multivariate multilevel analyses of participation and attainment in A level Mathematics and Further Mathematics

4. A qualitative analysis to understand engagement in A level Mathematics, Further Mathematics and in FMSP activity and views of the FMSP, by teachers and students through collection and analysis of:

- 42 individual interviews with teachers and
- 5 case studies of schools including student focus group interviews
- a survey of Maths Hub leads

5. Synthesising outcomes of the analysis to address evaluation aims including developing a conceptual model of Further Mathematics participation drawing on empirical and theoretical findings from related research.

Modelling FMSP 'theories of change'

As described in Section 3, the FMSP is a complex programme with a large number of components. To inform the evaluation, members of the FMSP central team worked with the evaluation team to develop an understanding of the interaction of different programme components with each other and their relationship to the programme aims.

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[8] At Key Stage 4, the term 'pupil' is appropriate and widely used but the 'student' term is more appropriate for young people who study at Key Stage 5 or beyond. The 2011 KS4 NPD data file spans Key Stages 4 and 5 - which explains the use of the 'pupil / student' description.

This understanding is represented visually in three 'theory of change' (Blamey and Mackenzie, 2007) diagrams. These represent change processes for three types of schools that the FMSP support: 11-16 schools who do not offer A level Mathematics; centres that have post 16 A level provision but no or low Further Mathematics entries; and centres with higher number of Further Mathematics entries. The theory of change diagrams show the initial understanding prior to evaluation activities.

Note that an important part of the FMSP's aim is for centres to become self-sustaining. Thus, a positive outcome for the FMSP, as currently conceived, is for a centre to cease to engage with the programme, or at least components of it, when they develop their capacity for self-sustaining improvement. For simplicity, the change diagrams do not represent visually the way in which the three core types of programme components are supported by the Area Coordinator network and online resources support provision.

Figure A1-1 Theory of change for 11-16 schools

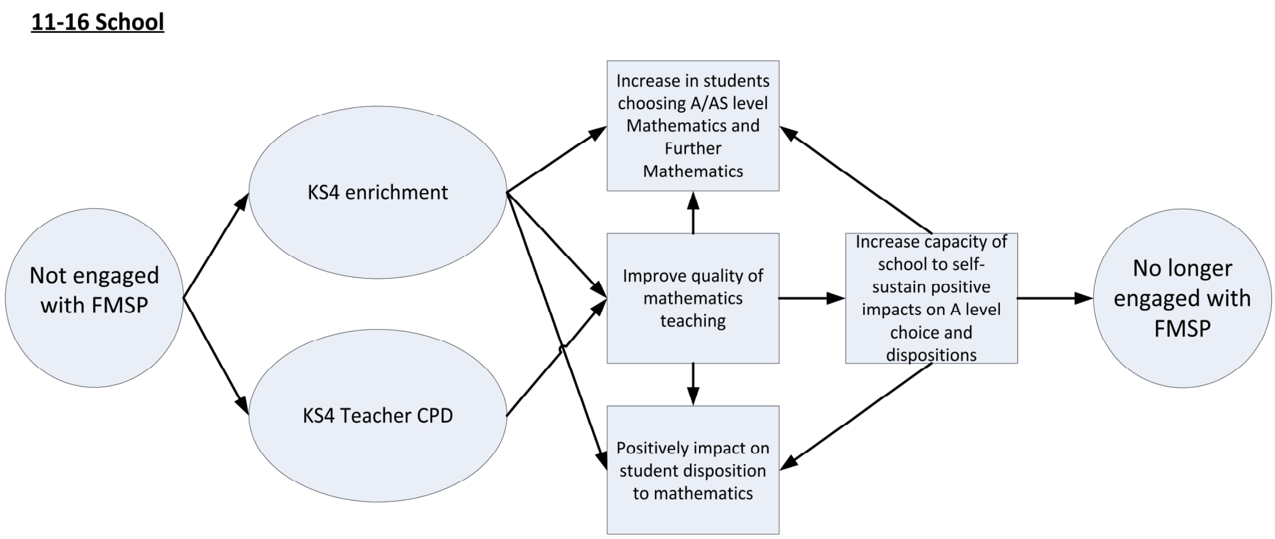
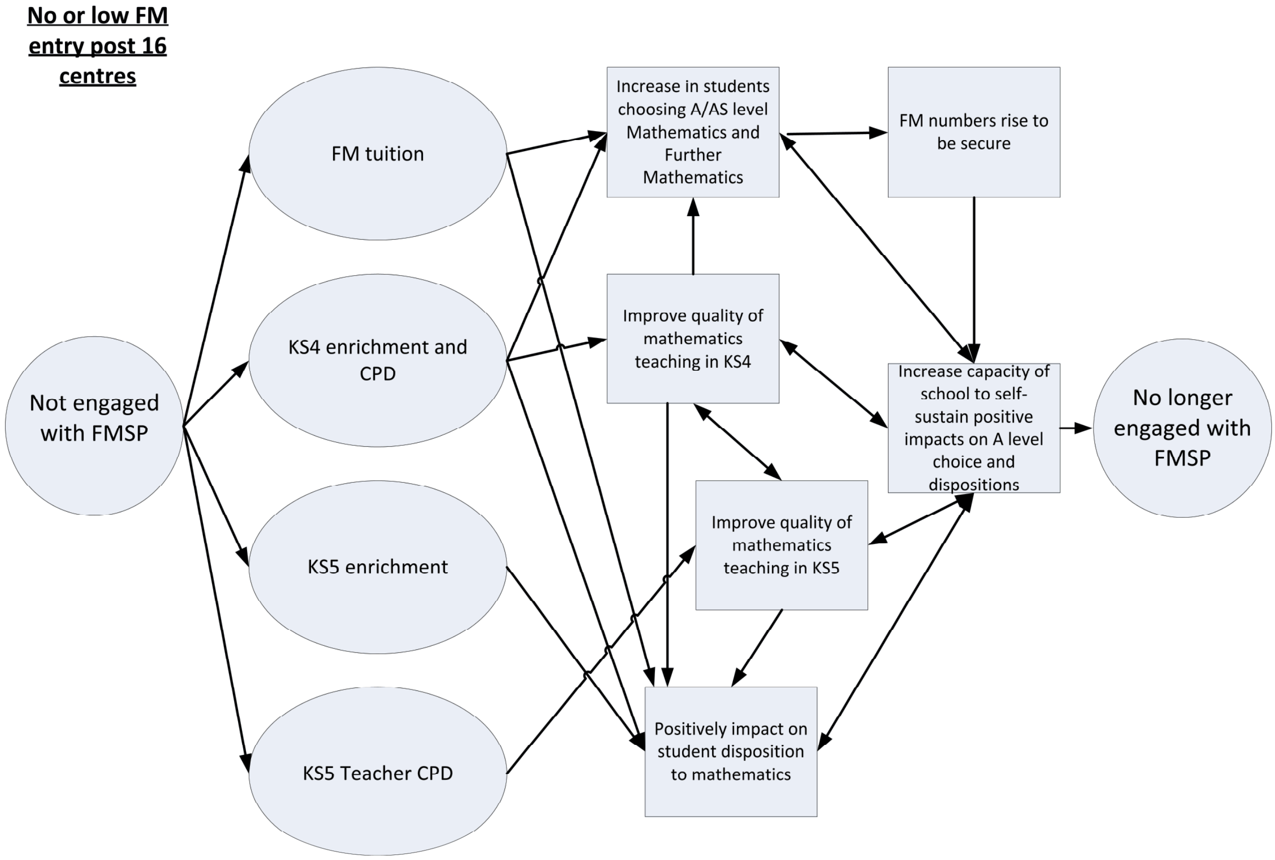
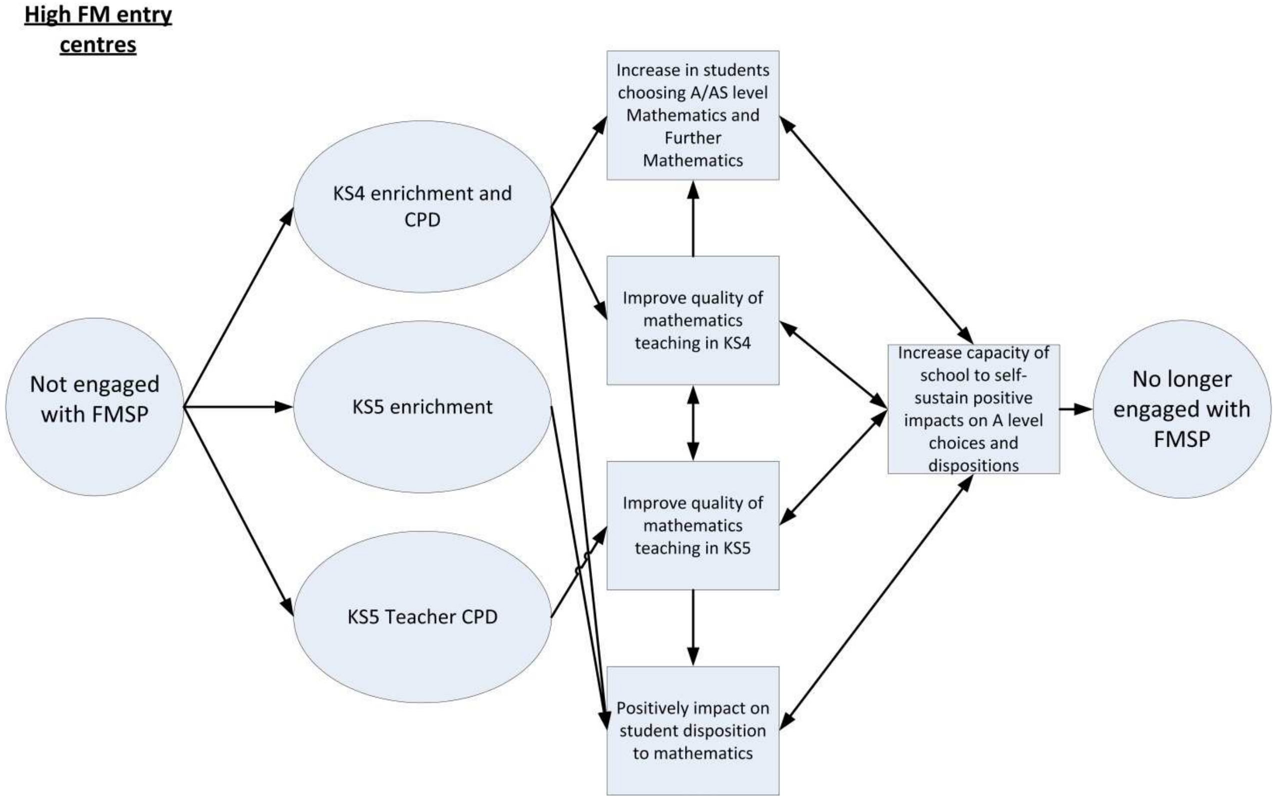


Figure A1-2 Theory of change for no or low FM entry school[9]



9 In for simplicity, KS4 enrichment and CPD are conflated into a single aspect of provision

Figure A1-1 Theory of change for higher further mathematics entry centres[10]



10 *Note that this model, in particular needs revising in the light of the evaluation in that it does not represent the way in which some high Further Mathematics entry centres have begun to use the on-line tuition offer to supplement or broaden the student learning experience.

School participation in Further Mathematics and FMSP

Analysis of the school level data focused firstly on developing a security construct as a measure of the extent to which the programme is effective in developing capacity and capability as well as sustainability. The number of A level Further Mathematics entries in a centre was considered over a 3 year period in order to consider stability of entries over time. Following discussion with the FMSP and building on previous evaluations (Searle, 2014), boundary points of 3 and 10 entries were adopted. From this, five levels of "security" were developed as follows:

- none - no entries of A level Further Mathematics in all 3 years
- least secure - one or more years with 0 entries and only 1 or 2 in any other year
- less secure - 3 or more entries in any one year two or less in other years
- relatively secure - 3 or more entries in all 3 years
- highly secure - 10 or more entries in all 3 years

Security characteristics were calculated as follows:

- security status for 2013: based on Further Mathematics entries for 2012/13, 2011/12, 2010/11
- security status for 2014: based on Further Mathematics entries for 2013/14, 2012/13, 2011/12
- security status for 2015: based on Further Mathematics entries for 2014/15, 2013/14, 2012/13`

The security construct was cross-tabulated with school characteristics (including school type and school contextual factors) to gain a picture of how Further Mathematics entries vary according to school characteristics.

Datasets held by FMSP containing records of school involvement in CPD, enrichment and tuition were brought together and merged into the longitudinal school census data. This allowed an exploration of the relationship between engagement in FMSP and school characteristics, and of the relationship between FMSP and the security constructs.

Modelling student participation in Further Mathematics and FMSP

Statistical analyses of student participation and attainment in A and AS level Mathematics and Further Mathematics was undertaken. Using data obtained from the National Pupil Database

(NPD), these analyses focused on the cohort of young people in England who completed KS4 at the end of Y11 in summer 2011 (N=637,594) linked to NPD KS5 data for the 2011/12, 2012/13 or 2013/14 academic years to form a longitudinal data file.

In total, 65% (N=412,743) of the 2011 KS4 cohort were recorded as taking at least one KS5 assessment between 2011/12 and 2013/14. This created a 4-year longitudinal data file which we named the '2011 KS4 NPD cohort'. This 4-year longitudinal cohort approach reflects that taken by other research (Noyes and Adkins, 2016).

Descriptive statistical analyses were undertaken to explore potential outcome and explanatory variables and to select those for inclusion in the multivariate multilevel analyses of A level participation and attainment. Participation in A level Mathematics and Further Mathematics was modelled using multilevel logistic regression. Attainment in A level Mathematics and Further Mathematics was modelled using multilevel linear regression. The modelling approach is summarised below.

The objective of these statistical analyses was to examine evidence of differential student-level participation and attainment in A and AS level Mathematics and Further Mathematics with respect to both centre / KS5 institution and student level factors. At the institution level, the types of KS4 centres, KS4 to KS5 routes, KS5 institutions and the engagement of KS5 institutions with the FMSP were included in these analyses. At the student level, KS4 / GCSE attainment, gender, whether a student was eligible and claiming Free School Meals (FSM), Income Deprivation Affecting Children Index (IDACI) and Ethnicity were included in these analyses.

The analyses set out to answer the following research questions using the 2011 KS4 NPD cohort data file:

1. Are patterns of differential participation in A and AS level Mathematics and Further Mathematics comparable?
2. Are patterns of differential attainment in A and AS level Mathematics and Further Mathematics comparable?
3. To what extent can these differential participation and attainment patterns be statistically accounted for by differential patterns in KS4/GCSE attainment (in Mathematics and overall KS4 attainment)?

4. What are the independent effects of student and school characteristics on participation and attainment in A level Mathematics and Further Mathematics?
5. At the institutional level, how is engagement with FMSP associated with participation and attainment in A level Mathematics and Further Mathematics?

Participation in A and AS level Mathematics and Further Mathematics

Participation in A and AS level Mathematics and Further Mathematics is measured by identifying whether a student was recorded on the NPD as taking the A / AS level in 2011/12, 2012/13 or 2013/14. Students who began the A / AS level but did not complete, would not be included in these analyses. The descriptive analyses provide a broad perspective on participation in A and AS level Mathematics and Further Mathematics in 2011/12, 2012/13 and 2013/14. The multivariate, multilevel analyses focus on participation in A level Mathematics and Further Mathematics during the three KS5 years combined (2011/12 to 2013/14).

Attainment in A and AS level Mathematics and Further Mathematics

Attainment in A / AS level Mathematics and Further Mathematics was measured using both grade-thresholds (attaining a grade A or A*; attaining a grade B or higher) and by converting the grades into an UCAS points score[11] scale. The descriptive analyses examined evidence of differential A and AS level attainment across the explanatory variables with respect to both the grade-threshold and UCAS points score whilst the multivariate, multilevel analyses focus in on differential A level attainment relating to UCAS points score.

KS5 institutional level engagement with FMSP

The longitudinal 4-year cohort approach is comprehensive in that it reflects a structural reality within the English education system - students are commonly clustered into year groups (or cohorts) as they progress through Key Stages. This approach also echoes that taken independently by others (Noyes and Adkins, 2016).

Engagement with the FMSP was drawn from the FMSP administrative database and relates to a single academic year (2013/14). There are two noteworthy features of the FMSP engagement variables. First, because they are measured at an institutional level they have an indirect nature. For example, a student located may take an A or AS level in Mathematics or Further Mathematics

[11] See www.ucas.com/ucas/undergraduate/getting-started/entry-requirements/tariff/tariff-tables/946

in a KS5 institution identified as having some FMSP engagement but this does not necessarily mean that this student had personal / direct experience of this FMSP engagement. Second, the FMSP engagement variables are not aligned with respect to time with the NPD data file because they relate to a single academic year (2013/14) rather than the three KS5 years within the longitudinal NPD cohort (2011/12 to 2013/14). For these reasons, caution is advised when interpreting and drawing conclusions from the student-level analyses relating to FMSP engagement compared with other institution and individual level explanatory variables. Table A1-1 provides an overview of the statistical analyses into participation and attainment in Mathematics and Further Mathematics undertaken.

Table A1-1 Analyses of Participation and Attainment in A and AS level Mathematics and Further Mathematics.

2011 KS4 NPD Cohort

	Descriptive (Bivariate) Statistical Analyses	***Multivariate, Multilevel Modelling Analyses***
Participation and Attainment Outcome Variables		
AS Level Mathematics	**X**	-
AS Level Further Mathematics	**X**	-
A Level Mathematics	**X**	**X**
A Level Further Mathematics	**X**	**X**
Level 2 (Institution) Explanatory Variables		
Further Maths Security Scale	**X**	-
KS4 School Type (2010/11)	**X**	-
KS5 Institution Type (2011/12 to 2013/14)	**X**	**X**
KS4 through KS5 Route (2010/11 to 2013/14)	**X**	-
FMSP Engagement	**X**	**X**
Level 1 (Student) Explanatory Variables		
Overall KS4 Attainment (2010/11)	**X**	**X**
KS4 Maths Attainment (2010/11)	**X**	**X**
FSM (2010/11)	**X**	**X**
IDACI (2010/11)	**X**	-
Gender (2010/11)	**X**	**X**
Ethnicity (2010/11)	**X**	**X**

Qualitative analysis of engagement in and with Further Mathematics and the FMSP

Teacher interviews

A total of 42 telephone interviews were conducted[12] with teachers from centres with a range of Further Mathematics entries. The sample was drawn from the FMSP database, focusing on centres which had records of involvement in one or more of the FMSP activities. A total of 21 interviews were conducted in Spring term 2015 and a second set of 21 interviews were conducted in the Autumn term of 2015. The second set consisted of 16 telephone interviews and 5 teacher interviews conducted in the case study centres alongside the student focus groups. The second wave of interviews were conducted after the development of the security status construct and analysis, and focused on those who had experienced a "change" in security status. As part of the analytical process the security rating was calculated of centres sampled in the first wave of interviews. Table A1-2 describes in detail the relationship of the sample to security and FMSP involvement for 2013/14. In summary, from FMSP data at the point of sampling 28 had engaged in CPD, 17 in enrichment and 5 in tuition. It is important to note that involvement is for a single year and centres may have participated in FMSP in other years, this was confirmed by analysis of the telephone interviews.

Table A1-2 Sample of interviews by security status and FMSP involvement (13/14)

	None	Least secure	Less secure	Relatively secure	Highly secure	**Total**
CPD only	3	1	4	6	0	**14**
Enrichment only	0	0	2	2	0	**4**
Tuition only	1[13]	1	0	0	0	**2**
CPD and Enrichment	1	1	4	1	4	**11**
CPD and Tuition	0	0	1	0	0	**1**
CPD, Enrichment and Tuition	0	0	1	1	0	**2**
None	2	2	4	0	0	**8**
Total	**7**	**5**	**16**	**10**	**4**	**42**

Teachers were asked about their centres' participation in Mathematics and Further Mathematics A level, and progression to A level Mathematics from GCSE. They were also asked about their involvement with FMSP including their views on the programme, implementation in their centre

[12] One interviewee was interviewed twice.

[13] This centre had 1 FM student in 14/15 but none prior to that going back to 11/12

and its effectiveness. Teacher interviews were analysed thematically (Ryan and Russell Bernard, 2003).

To supplement the teacher interviews a short survey including some open questions was conducted with Maths Hub Leads to gain their views on how the FMSP and the Hubs currently work together and priorities for the coming year. The survey was sent to the Leads in all 35 Maths Hubs with 25 (71%) responding.

Student engagement - outcomes and choices

A total of five student focus groups were conducted in centres. Given this relatively small sample conclusions drawn are tentative, one purpose of including this activity was to evaluate focus groups as an evaluation method for the FMSP, which might be used more extensively in the future.

The purpose of the student focus group strand was to explore the decision making and experiences of the FMSP for young people who had chosen to study mathematics and Further Mathematics at A level in centres where the numbers were low and Further Mathematics security was at potential risk. Based on their security status, a sample of centres were identified as being least secure or at less secure in terms of their numbers of Further Mathematics entries over recent years.

Five centres from Yorkshire, Humberside and the Midlands agreed to take part in student focus groups which were conducted during visits to the centre between December 2015 and February 2016. Focus groups included a mix of students in Y12 and Y13 and those studying both A level Mathematics and Further Mathematics.

In total, 44 students took part in the focus groups across the five centres. The priority was to include as many Further Mathematics students from each centre as possible, as well as a broad cross section of those studying A level Mathematics with other subjects.

Of the 44:

- 25 were studying A level Mathematics *and* FM
- 19 were studying A level Mathematics (with other subjects, not FM)
- 61% of the overall sample was male (27/44), 39% were female (17/44)
- 4 students (3 male and 1 female) from BME backgrounds (c.10%)

Although not generalisable, there appeared to be patterns in the range of subjects chosen. Students studying A level Mathematics and Further Mathematics tended to also specialise in sciences (often studying Physics and Chemistry) - or Economics. This was particularly the case for males. Those who chose A level Mathematics without Further Mathematics tended to study a wider range of other subjects - a pattern more often seen amongst the females.

None of the female students studying Mathematics (without FM) were studying all other sciences - they tended to choose either all non-Sciences or a mix of sciences and other subjects. The wider range of non-science subject combinations for those not studying Further Mathematics included Economics, Business, Languages, Geography, Psychology.

The A level Mathematics /Further Mathematics lead in each centre was also interviewed during the focus group visit (or in one case, over the telephone), and reference to information provided by the teacher is included in analysis of the pupil focus groups

The interview tool for the student focus group included a 'Diamond 9' decision-making exercise for ranking their reasons for choosing A level Mathematics/Further Mathematics A level; a timeline for students to map the key moments in their decision making and questions on their experiences of the FMSP enrichment and tuition. Focus groups varied in length from 20 minutes to over an hour, depending on the numbers participating and the extent of their experience of FMSP activities.

Focus groups were recorded to capture the key issues during discussions. The Diamond 9 exercise rankings were recorded and transferred onto excel spreadsheets, together with their timetable data, to allow for some quantification as well as thematic comparisons across centres and groups of students (e.g. by gender, year, Mathematics/Further Mathematics A level). Thematic analysis (Ryan and Russell Bernard, 2003) of students' experiences and views of enrichment and tuition was conducted from the summaries. Initial analysis was inductive considering emergent themes, then further analysis was informed by considering previous research on student participation in mathematics and science. Thus the approach accorded with an adaptive approach (Layder, 1998). Selective quotes were then transcribed from the recordings.

Synthesis of evaluation data

As a final stage in the evaluation, relationships between quantitative and qualitative findings were analysed by looking across data sets and findings, using the pragmatic mixed methods approach outlined above, focused on evidence in relation to capacity and capability, reach, effectiveness and sustainability. This method includes but goes beyond a more standard 'explanatory' approach (Johnson and Turner, 2003) whereby qualitative interview data are used to help illuminate and help interpret quantitative findings: additionally, we treat qualitative data and quantitative data as being particularly suited to answering different questions, not merely acting as adjuncts to one another. Therefore, our synthesis uses combinations of different forms of quantitative data; combinations of different forms of qualitative data; and combinations of qualitative and quantitative data, with the choice of data to be synthesised dependent on the specific research issue; in this case, capacity, reach, effectiveness and sustainability. By drawing on theoretical constructs of science capital and previous empirical and theoretical research, a revised model of Further Mathematics culture, capital and ecology was developed. Thus, again the overall methodology accords with that suggested by adaptive theory (Layder, 1998). Final evaluation focused on evidence in relation to capacity and capability, reach, effectiveness and sustainability.

Annexe 2: Measuring Participation and Attainment in A level Mathematics and Further Mathematics

PART I:Explanatory Variables

This Appendix provides details on the explanatory variables used within the statistical analyses into participation and attainment in A and AS level mathematics and further mathematics.

The explanatory variables were at two levels: individual student (level 1) and KS5 institution (level 2). The appendix is organised into two sections. The first section provides a brief summary of the analyses. This is followed by the tables referred to within the summary discussion.

Level 1 (Student) Explanatory Variables.

Two groups of variables are examined:

- KS4 / GCSE attainment in 2011
- Student background (FSM, gender & ethnicity)

Level 2 (Institution) Explanatory Variables.

Three groups of variables are examined:

- The Further Mathematics A level Security Scale
- School / Institution type (KS4 and KS5, and KS4 through KS5 routes)
- KS5 institution engagement with the FMSP

KS4 / GCSE Attainment (Y11 in 2011).

Table 1a introduces the KS4 attainment measures and shows that the likelihood of progressing to KS5 following Y11 is correlated with KS4 / GCSE attainment. This is seen across all measures of attainment:

- 89% who attained 5+ A*-C including Mathematics and English in 2011 are recorded as taking a KS5 assessment between 2012 & 2014.

- 96% who attained an A* in KS4 / GCSE Mathematics 2011 are recorded as taking a KS5 assessment between 2012 & 2014.

Tables 1b illustrates that, amongst those who did progress to KS5, access to A and AS maths or further maths A level is correlated with KS4 attainment.

- Of the 11,045 young people recorded as taking **A Level Further Mathematics:** 96% had attained 5+ A*-C GCSEs or equivalent at key stage 4 in 2011; 84% had attained a grade A* in KS4/GCSE Mathematics and 98% had attained a grade A or A*.
- Of the 69,048 young people recorded as taking **A Level Mathematics:** 98% had attained 5+ A*-C GCSEs or equivalent at key stage 4 in 2011; 52% had attained a grade A* in KS4/GCSE Mathematics and 92% had attained a grade A or A*.
- Of the 11,236 young people recorded as taking **AS Level Further Mathematics:** 96% had attained 5+ A*-C GCSEs or equivalent at key stage 4 in 2011; 71% had attained a grade A* in KS4/GCSE Mathematics and 97% had attained a grade A or A*.
- Of the 92,229 young people recorded as taking **AS Level Mathematics:** 97% had attained 5+ A*-C GCSEs or equivalent at key stage 4 in 2011; 36% had attained a grade A* in KS4/GCSE Mathematics and 81% had attained a grade A or A*.

It seems that attaining an A or A* grade in KS4 maths is a key determinant in who takes Further Mathematics A (or AS) level - although only 8% of those who did attain this at KS4 in 2011 went on to take the Further Mathematics A level.

In other words, *It is highly likely that a student taking A level Further Mathematics will have a grade A or A* in KS4 maths (98% do). However, it is highly unlikely that a student with a grade A or A* in KS4 maths will take A level Further Mathematics (92% did not).*

Attaining an A or A* grade in KS4 maths is also strongly associated with taking Mathematics A level - 48% of those who did attain this at KS4 in 2011 took the A level, 57% took a maths AS level.

Table 1c presents the association between KS4 / GCSE attainment and A / AS level attainment in Mathematics and Further Mathematics.

A positive correlation between KS4 attainment (overall and in Mathematics) and A/AS level attainment was observed with all four measures.

Correlations between A & AS level attainment and overall KS4 attainment are seen to be of a more consistent magnitude (r ranging between +0.5 and +0.6) compared with correlations with KS4 Mathematics attainment ($+0.2 < r < +0.6$). This relates to the relatively low variation in KS4 GCSE Mathematics attainment for the group of students who took Further Mathematics A level (84% attained A*, 98% attained A/A*).

It seems apparent that both overall and Mathematics KS4 attainment have a relatively strong statistical relationship with participation and attainment in A level Mathematics and

Further Mathematics. For this reason, both attainment measures were carried forwards to the multilevel, multivariate stage of the analyses.

This, however, brings a methodological problem known as multicolinearity. KS4 maths and overall attainment are highly correlated (r = 0.89) and so it will be difficult (and perhaps impossible) to disentangle the unique influences of these KS4 attainment measures on the A level participation and attainment outcomes. To avoid this problem, a threshold measure for KS4/GCSE Mathematics attainment was selected (identifying whether a young person attained a grade A* or not at KS4 in 2011) and a scale measure for overall KS4 / GCSE attainment (Points per KS4 assessment). These two measures are still correlated but not to same degree as was observed with the two scale KS4 attainment measures (r=0.44) and so this circumnavigates the multicolinearity issue.

Student Background

The explanatory variables looking at student background include students identified as receiving free school meals (FSM) when in Y11 in 2011, gender and ethnicity.

Free School Meals (FSM)

The FSM measure identifies young people with parents who are in receipt of certain benefits and registered to receive free school meals in the 2010/11 academic year. This means that there will be some FSM eligible (but unregistered) young people who will be hidden by this measure (see Iniesta & Evans, 2012).

Table 2a shows that young people classed as FSM in 2011 are less likely to be on the 2012-14 KS5 data file. In terms of odds-ratios, those not classed as FSM in 2011 are nearly 3 times as likely to take a KS5 assessment 2012 to 2014 compared with their peers classed as FSM.

Table 2b summarises the statistical association between FSM and the A and AS level participation outcome measures.

Socioeconomic bias is observed to be stronger with participation in Further Mathematics compared with access to Mathematics - at A and AS level

- **3%** of those taking **A Level Further Mathematics,** were recorded as FSM in 2011. Young people classified as not-FSM are 3.0 times as likely to take A level Further Mathematics compared to their peers classed as FSM.
- **5%** of those taking **A Level Mathematics,** were recorded as FSM in 2011. Young people classified as not-FSM are 2.2 times as likely to take A level Mathematics compared to their peers classed as FSM.

- **4%** of those taking **AS Level Further Mathematics,** were recorded as FSM in 2011. Young people classified as not-FSM are 2.2 times as likely to take AS level Further Mathematics compared to their peers classed as FSM.
- **6%** of those taking **AS Level Mathematics** were recorded as FSM in 2011. Young people classified as not-FSM are 1.7 times as likely to take AS level Mathematics compared to their peers classed as FSM.

Table 2c presents associations between FSM and the A and AS level attainment outcome variables.

Amongst those who do take A / AS level Mathematics and Further Mathematics, average attainment for young people classed as FSM in 2011 were observed to be lower than those not classed as FSM.

Gender

Table 3a introduces the gender explanatory variable. In terms of odds-ratios, females are over 1.5 times as likely to be in KS5 compared with males.

Table 3b presents the associations between gender and participation in A and AS level Mathematics and Further Mathematics. It seems that the male bias is stronger for access to Further Mathematics compared with access to Mathematics - and strongest at A Level compared with AS level

- **27%** of those taking **A Level Further Mathematics** were female. Males were 3.2 times as likely to take A level Further Mathematics compared to their female peers
- **39%** of those taking **A Level Mathematics** were female. Males were 1.9 times as likely to take A level Mathematics compared to their female peers.
- **31%** of those taking **AS Level Further Mathematics** were female. Males were 2.5 times as likely to take AS level Further Mathematics compared to their female peers.
- **42%** of those taking **AS level Mathematics** were female. Males were 1.8 times as likely to take AS level Mathematics compared to their female peers.

Table 3c presents the associations between gender and attainment in A and AS level Mathematics and Further Mathematics.

Ethnicity

Table 4a introduces the ethnicity variable.

Ethnic groups listed in rank order based on the ***relative likelihood of progressing to KS5 following Y11:***

- Chinese (91%, 6.3 times as likely as White British);
- Indian (86%, 3.8 times as likely as White British);
- Black African (80%, 2.4 times as likely as White British);

- Bangladeshi (75%, 1.8 times as likely as White British);
- Pakistani (71%, 1.5 times as likely as White British);
- White Other (68%, 1.3 times as likely as White British);
- Black Caribbean (67%, 1.2 times as likely as White British);
- White British (63%);
- Mixed Black Caribbean & White (58%, 0.8 times as likely as White British)

Tables 4b and 4c present the associations between ethnicity and the A and AS level participation and attainment outcome variables.

55% of those taking **A level Further Mathematics** identified as being white British. Students who identified as Chinese were observed to be 7.6 times as likely to take the A compared with their white British peers. The Indian (2.1 times as likely) and white other groups (1.3 times as likely) are the only other BME groups to be more likely to take A level Further Mathematics compared with their white British peers. Bangladeshi (0.8 times as likely to take the A level), Black African (0.5 times as likely), Pakistani (0.5 times as likely), mixed Black Caribbean & white (0.4 times as likely) and Black Caribbean (0.3 times as likely) all are seen to be less likely to participate in A level further maths compared with the white British group.

Average attainment in **A level Further Mathematics** varies across ethnic groups from the relatively higher attaining Chinese group (mean UCAS points = 110.7) to the relatively lower attaining Bangladeshi group (93.4).

55% of those taking **A level Mathematics** identified as being white British. Those who identified as Chinese were observed to be 7.0 times as likely to take the A level compared with their white British peers. Most BME groups are seen to be more likely to take A level maths compared with their white British peers; the Indian group (3.2 times as likely to take the A level), Bangladeshi (1.3 times as likely), white other (1.3 times as likely), Black African (1.2 times as likely) and Pakistani (1.2 times as likely). There are two exceptions; the Black Caribbean (0.5 times as likely) and the mixed Black Caribbean and white group (0.6 times as likely to take the A level) are both less likely to take A level Mathematics compared with their white British peers.

Average attainment in **A level Mathematics** varies across ethnic groups from the relatively higher attaining Chinese group (mean UCAS points = 106.3) to the relatively lower attaining Black Caribbean group (85.5).

63% of those taking **AS level Further Mathematics** identified as white British. Those who identified as Chinese were observed to be 5.9 times as likely to take the AS level compared with their white British peers. The Indian (2.0 times as likely) and white other groups (1.2 times as likely) are the only other BME groups to be more likely to take AS level Further Mathematics compared with their white British peers. Bangladeshi (0.9 times as likely to take the AS level), Black African (0.7 times as likely), Pakistani (0.6 times as likely), mixed Black Caribbean & white (0.5 times as likely) and Black Caribbean (0.3 times as likely) all are seen to be less likely to participate in AS level Further Mathematics compared with the white British group.

Average attainment in **AS level Further Mathematics** varies across ethnic groups from the relatively higher attaining Indian group (mean UCAS points = 47.3) to the relatively lower attaining Black African group (38.1).

60% of those taking **AS level Mathematics** identified as white British. Those who identified as Chinese were observed to be 6.3 times as likely to take the AS level compared with their white British peers. Most BME groups are seen to be more likely to take AS level Mathematics compared with their white British peers; the Indian group (3.2 times as likely to take the AS level), Bangladeshi (1.5 times as likely), Black African (1.4 times as likely), Pakistani (1.4 times as likely) and white other (1.3 times as likely),. There are, once again, two exceptions; the Black Caribbean and the mixed Black Caribbean and white group (both around 0.6 times as likely to take the AS level compared with their white British peers) are both observed to be less likely to take AS level Mathematics compared with the white British group.

Average attainment in **AS level Mathematics** varies across ethnic groups from the relatively higher attaining Chinese group (mean UCAS points = 43.6) to the relatively lower attaining Black Caribbean group (29.1).

The pupil background variables selected to be carried forward into the multilevel multivariate stage are; overall KS4 / GCSE attainment (KS4 points per assessment), KS4 / GCSE Mathematics attainment (attaining a grade A* or not), FSM, gender and ethnicity.

Level 2 (Institution) Explanatory Variables.

Further Mathematics A level Security Scale

The Further Mathematics A level security scale was created using institutional-level data on A level Further Mathematics entrants in 2012, 2013 and 2014. Attaching this to pupil-level data provides a different perspective. Whilst we already know the proportion of institutions delivering KS5 have sufficient FM A level entries to be classed as having a highly secure Further Mathematics A level provision, the pupil level detail sheds some light on the proportion of young people located within these 'highly secure' KS5 institutions.

Table 5a presents the Further Mathematics security distribution at both the institutional and student levels. Table 1a shows that around 8% of KS5 institutions are classed as 'highly secure' and around 20% of young people in KS5 2012-14 are located in these KS5 institutions. In other words, KS5 assessments are more concentrated in institutions with secure Further Mathematics A level provision.

Table 5b presents the association between the Further Mathematics security scale and participation in A and AS level Mathematics and Further Mathematics. Students located in KS5 institutions with a highly secure Further Mathematics A level provision are more likely to take A and AS level Mathematics and Further Mathematics compared with students located within KS5 institutions with lower security in Further Mathematics A level provision.

A level Mathematics: Over 70% of entries were from young people located in KS5 institutions with a relative or high secure Further Mathematics rating. Young people located in a KS5 institution with a relative or high secure Further Mathematics rating 4.8 times as likely to take maths AS level compared with young people in a KS5 institution with no Further Mathematics A level or less secure provision.

A level Further Mathematics: Over 81% of entries were from young people located in KS5 institutions with a relative or high secure Further Mathematics rating. Young people located in a KS5 institution with a relative or high secure Further Mathematics rating are over 20 times as likely to take Further Mathematics A level compared with young people in a KS5 institution with no Further Mathematics A level or less secure provision.

A level Mathematics: Over 66% of entries were from young people located in KS5 institutions with a relative or high secure Further Mathematics rating. Young people located in a KS5 institution with a relative or high secure FM rating 3.7 times as likely to take Mathematics AS level compared with young people in a KS5 institution with no Further Mathematics A level or less secure provision.

AS level Further Mathematics: Over 72% of entries were from young people located in KS5 institutions with a relative or high secure Further Mathematics rating. Young people located in a KS5 institution with a relative or high secure Further Mathematics rating over 7 times as likely to take Further Mathematics AS level compared with young people in a KS5 institution with no Further Mathematics A level or less secure provision.

Table 5c shows that for the 2011 cohort, the FM secure scale is also positively correlated with all 4 outcomes (A and AS level Mathematics and Further Mathematics attainment). In other words, the more secure that A level FM provision at the institutional level, the higher the student level average attainment in AS and A level Mathematics and Further Mathematics.

KS4 School Type

Table 6a shows that the vast majority of the 2011 cohort took KS4 in a state school. The Table also shows the differential staying on rates for pupils across different KS4 institutions. The 'progress to KS5' rate for the 2011 cohort was 65% but this is seen to vary by type of institution. Young people who took KS4 in an independent / fee paying school were more likely to take a KS5 assessment between 2012 and 2014 (87%) compared with their peers who took KS4 in a state secondary school (66% overall but varies between 54% with *sponsored* academies and 77% with *converter* academies)

Table 6b shows the statistical associations between KS4 location and participation in A and AS level Mathematics and Further Mathematics.

- Of those 11,045 young people in the 2011 NPD cohort are recorded as taking **A level Further Mathematics** between 2012 & 2014, 73% took KS4 in a state school in 2011.
- Of those 69,048 young people in the 2011 NPD cohort are recorded as taking **A level Mathematics** between 2012 & 2014, 76% took KS4 in a state school in 2011.
- Of those 11,236 young people in the 2011 NPD cohort are recorded as taking **AS level Further Mathematics** between 2012 & 2014, 83% took KS4 in a state school in 2011.
- Of those 92,229 young people in the 2011 NPD cohort are recorded as taking **AS level Mathematics** between 2012 & 2014, 83% took KS4 in a state school in 2011.

KS5 Location

Table 7a shows that the vast majority of the 2011 cohort took their A or AS level Mathematics or Further Mathematics in a state KS5 institution.

Participation rates for AS level Mathematics and Further Mathematics are highest amongst those located in a state school 6th form. Whilst for A level Mathematics and Further Mathematics, participation rates are highest amongst those located in an independent / fee paying KS5 institutions.

In terms of odds-ratios, young people in state school 6th forms are 1.4 times as likely to take AS level Mathematics and 1.2 times as likely to take AS level Further Mathematics compared with their peers in independent / fee paying KS5 institutions.

Young people in independent / fee paying KS5 institutions are 3.1 times as likely to take A level Mathematics and 3.3 times as likely to take A level Further Mathematics compared with their peers in state funded KS5 institutions (school 6th forms; sixth form colleges & FE colleges).

- Of those 11,045 young people in the 2011 NPD cohort are recorded as taking **A level Further Mathematics** between 2012 & 2014, 78% were located in a state KS5 institution.
- Of those 69,048 young people in the 2011 NPD cohort are recorded as taking **A level Mathematics** between 2012 & 2014, 82% were located in a state KS5 institution.
- Of those 11,236 young people in the 2011 NPD cohort are recorded as taking **AS level Further Mathematics** between 2012 & 2014, 90% were located in a state KS5 institution.
- Of those 92,229 young people in the 2011 NPD cohort are recorded as taking **AS level Mathematics** between 2012 & 2014, 91% were located in a state KS5 institution.

Table 7c shows that, on average, A and AS level attainment levels are highest for students located in independent fee/paying KS5 institutions compared with students located in state funded KS5 institutions.

Within state KS5 institutions, attainment is highest amongst those located within school 6th forms and lowest amongst those located within FE colleges.

KS4 to 5 Route

Using a simplified version of the KS4 location variable, along with the four KS5 location variables, a series of 'KS4 into KS5 route' variables were derived - these are summarised in **Table 8a**.

- Over 80% of young people take a KS4 through KS5 route through state institutions.
- Around 7% take a route purely through independent / fee paying institutions.
- Around 3% take a route that involves both state and independent / fee-paying institutions
- Around 0.5% switched from State institutions(KS4) to independent / fee-paying (KS5)
- Around 2.5% switched from independent / fee-paying institutions(KS4) to State (KS5)

Table 8b presents patterns of A and AS level participation for students taking the different KS4 through KS5 routes.

Participation in **A Level Mathematics** is most likely for students who switched from a state school at KS4 to an independent / fee paying institution for KS5 (47%) and least likely for students who went from a state school for KS4 into an FE college for KS5 (4%).

Of the 69,048 young people recorded as taking **A level Mathematics,** …

- 79% were located in a state funded KS4 AND KS5 institution
- 16% were located in an independent / fee paying KS4 AND KS5 institution
- 3% were located in an independent / fee paying institution in 2011 (KS4) but switched to a state funded KS5 institution.
- 1% were located in a state funded KS4 institution in 2011 but switched to an independent / fee paying KS5 institution.

Participation in **A Level Further Mathematics** is most likely for students who took KS5 in an independent / fee paying institution (7.7%) and least likely for students who went from a state school for KS4 into an FE college (0.4%).

Of the 11,045 young people recorded as taking **A level Further Mathematics,**

- 76% were located in a state funded KS4 AND KS5 institution
- 21% were located in an independent / fee paying KS4 AND KS5 institution
- 2% were located in an independent / fee paying institution in 2011 (KS4) but switched to a state funded KS5 institution.
- 1% were located in a state funded KS4 institution in 2011 but switched to an independent / fee paying KS5 institution.

Participation in **AS Level Mathematics** is most likely for students who switched from and independent / fee paying school for KS4 to a state school 6th form (36%) and least likely for students who went from a state school for KS4 into an FE college (6%).

Of the 92,229 young people recorded as taking **AS level maths**…

- 88% were located in a state funded KS4 and KS5 institution
- 8% were located in an independent / fee paying KS4 AND KS5 institution
- 3% were located in an independent / fee paying institution in 2011 (KS4) but switched to a state funded KS5 institution.
- 1% were located in a state funded KS4 institution in 2011 but switched to an independent / fee paying KS5 institution.

Participation in **AS Level Further Mathematics** is most likely for students who switched from and independent / fee paying school for KS4 to a state 6th form college (4.4%) and least likely for students who went from a state school for KS4 into an FE college (0.6%).

Of the 11,236 young people recorded as taking **AS level Further Mathematics:**

- o 87% were located in a state funded KS4 AND KS5 institution
- o 9% were located in an independent / fee paying KS4 AND KS5 institution
- o 3% were located in an independent / fee paying institution in 2011 (KS4) but switched to a state funded KS5 institution.
- o 1% were located in a state funded KS4 institution in 2011 but switched to an independent / fee paying KS5 institution.

Table 8c presents patterns of A and AS level attainment for different KS4 to KS5 routes.

On average, attainment in **A Level Mathematics** is highest for students who were located within an independent / fee paying institution at both KS4 and KS5 (Mean UCAS score =113.5) and lowest for students who went from a state school for KS4 into an FE college (89.3).

On average, attainment in **A Level Further Mathematics** is highest for students who were located within an independent / fee paying institution at both KS4 and KS5 (Mean UCAS score =120.5) and lowest for students who went from a state school for KS4 into an FE college (101.6).

On average, attainment in **AS Level Mathematics** is highest for students who were located within an independent / fee paying institution at both KS4 and KS5 (Mean UCAS score =47.2) and lowest for students who went from a state school for KS4 into an FE college (29.7).

On average, attainment in **AS Level Further Mathematics** is highest for students who were located within an independent / fee paying institution at both KS4 and KS5 (Mean UCAS score =51.8) and lowest for students who went from a state school for KS4 into an FE college (39.5).

Whilst these analyses reveal that whilst there are some differences in A and AS level participation and attainment that are associated with the KS4 to KS5 route, it seems that the key variable of influence is KS5 location. Where a student takes the A or AS level matters more than where they took their KS4 assessments.

The KS4 to 5 routes variable combines the KS4 and KS5 location variables to show some movement over time. The analyses reveal some detail on 'switchers' from state to independent (or vice versa) but also highlights how rare this actually is.

Essentially, the cohort were either taught in state schools for KS4 and in then in state KS5 institutions (83% of the 2011 KS5 cohort) or were either taught in independent schools for KS4 and in then in independent KS5 institutions (7%). Distinguishing between the type of KS5 institution (school 6th form, sixth form college, FE college, independent) therefore seems more important than keeping details on where KS4 was taken. For this and parsimonious reasons, just the KS5 location variable is carried forward into the multivariate multilevel stage of the analyses

FMSP engagement variables.

Whilst these measures are not perfectly aligned with respects to time[1], the analyses do provide a perspective on 'reach' with respects to the various activities that FMSP are involved in. Specifically, detail on KS5 institution engagement with FMSP (in 2013/14) relating to Live Online Personal Development (LOPD), Teaching Advanced Mathematics (TAM), Teaching Further Mathematics (TFM) and Tuition were examined.

At the institution level, this was already known from previous analyses but Table 5a also provides a pupil-level perspective.

Live Online Professional Development (**LOPD**) was the most common FMSP Activity - involving around 8% of KS5 institutions. 9% of the 2011 KS4 NPD cohort were located within a KS5 institution identified as being involved with FMSP LOPD

Teaching Advanced Mathematics (**TAM**) was the next most common - involving around 4% of KS5 institutions. 3% of the 2011 KS4 NPD cohort were located in a KS5 institution identified as involved with TAM in 2013/14.

FMSP Tuition was the next most common - involving around 4% of KS5 institutions. 3% of the 2011 cohort were located within a KS5 institution identified as involved FMSP tuition in 2013/14.

Teaching Further Mathematics (**TFM**) was the least common - involving around 2% of KS5 institutions. 2% of the 2011 KS4 NPD cohort were located within a KS5 institution identified as involved with TFM in 2013/14.

[1] The measures relate to the 2013/14 academic year whilst the 2011 cohort spans the 2012, 2013 and 2014 academic years - with a majority taking an A or AS level in 2013.

LOPD - Live Online Professional Development:
Young people located in institutions involved with LOPD are more likely to take A or AS level Mathematics & Further Mathematics compared with the cohort average. Students located within KS5 institutions involved with LOPD have higher average attainment in A and AS level Mathematics and Further Mathematics compared with the cohort average.

TAM - Teaching Advanced Mathematics:
Young people located in institutions involved with TAM are more likely to take A or AS level Mathematics or Further Mathematics compared with the cohort average. Students located within KS5 institutions involved with TAM have lower average attainment in A and AS level Mathematics and Further Mathematics compared with the cohort average.

Tuition:
Young people located in institutions involved with FMSP Tuition are less likely to take A or AS level Mathematics or Further Mathematics. This reflects how FMSP tuition is concentrated within KS5 institutions identified as having A level FM provision at 'least secure'. Students located within KS5 institutions involved with FMSP Tuition have lower average attainment in A and AS level Mathematics and Further Mathematics compared with the cohort average.

TFM - Teaching Further Mathematics:
Young people located in institutions involved with TFM are more likely to take A and AS level maths & further maths. Students located within KS5 institutions involved with TFM have higher average attainment in A and AS level Mathematics and Further Mathematics compared with the cohort average.

In summary, in terms of A and AS level participation, positive correlations between institutional-level engagement with LOPD & TFM and student-level participation in A and AS level Mathematics and Further Mathematics have been observed. A negative correlation between institutional-level engagement with FMSP tuition and student-level participation in A and AS level Mathematics and Further Mathematics has also been observed. The association between institution engagement with TAM and participation seems to relate more to AS than A level Mathematics and Further Mathematics.

In terms of A and AS level attainment, positive correlations between institutional-level engagement with LOPD & TFM and average student attainment in A and AS level Mathematics and Further Mathematics have been observed. Negative correlations between institutional-level engagement with FMSP tuition & TAM and average student attainment in A and AS level Mathematics and Further Mathematics have also been observed.

In all, a complex collection of relationships are observed between the FMSP engagement variables and the A and AS level participation and attainment outcome variables. This complexity needs to be considered alongside the additional caution advised for the FMSP variables. This additional caution relates to time and level. In terms of time, the FMSP engagement measures stem from a single academic year (2013/14) whilst the A and AS level participation and attainment outcome variables relate to three KS5 academic years (2011/12 to 2013/14) - this means that the FMSP engagement measures are not temporally aligned with the outcomes. In terms of level, the FMSP engagement variables are located at the institutional level whilst the A and AS level participation and attainment outcome variables are all at the individual student level. A student may be located within a KS5 institution that is identified as having some engagement with FMSP (in 2013/14) but this does not necessarily mean that this student will have personal / direct experience of this FMSP engagement - this means that the FMSP engagement measures are not aligned in terms of 'level' with respect to the outcomes.

Table 1a: KS4 / GCSE Attainment Measures

OVERALL KS4 / GCSE Attainment Grade-Threshold Measures	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
5+ A*-Cs incl M&E (with equivs)	370,255	58.1%	327,561	79.4%	88.5%
5+ A*-Cs (not incl M&E)	129,645	20.3%	62,019	15.0%	47.8%
5+ A*-G's	90,170	14.1%	20,370	4.9%	22.6%
Lower / other	47,524	7.5%	2,793	0.7%	5.9%
Total	637,594	100.0%	412,743	100.0%	64.7%

KS4 / GCSE Attainment in Mathematics Grade-Threshold Measures	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
Grade A*	48,462	7.6%	47,737	11.6%	98.5%
A	86,743	13.6%	83,501	20.2%	96.3%
B	107,946	16.9%	96,473	23.4%	89.4%
C	169,890	26.6%	121,054	29.3%	71.3%
Grade D or lower	219,453	34.4%	63,274	15.3%	28.8%
Missing details	5,100	0.8%	704	0.2%	13.8%
Total	637,594	100.0%	412,743	100.0%	64.7%

KS4 / GCSE Attainment Overall and in Mathematics Scale Measures	Complete Cohort (n=637,594)		On KS5 File 2012-14 (n=412,743)	
	Mean	s.d.	Mean	s.d.
Mean number of KS4 Assessments	6.0	2.50	6.6	2.13
Total KS4/GCSE score	29.7	16.34	36.3	13.90
Mean KS4 Score per assessment taken	4.6	1.81	5.5	1.26
Mean KS4 Mathematics Score	4.8	2.06	5.7	1.48

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

Table 1b: KS4 Attainment - A and AS level Participation

	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond[1])
Whole Cohort (n=412,743)	22.3%	16.7%	2.7%	2.7%	16.0%
KS4 / GCSE Overall (Grade-Threshold)	**%**	**%**	**%**	**%**	**%**
5+ A*-C GCSEs or equivalents incl Eng & Maths	27%	21%	3.3%	3.2%	16%
Did not attain this threshold	3%	2%	0%	1%	-
Strength of Association (V)	0.239	0.204	0.076	0.071	0.043
KS4 / GCSE Mathematics (Grade-Threshold)	**%**	**%**	**%**	**%**	**%**
Grade A*	69%	75%	16.8%	19.3%	25.8%
Grade A or A*	57%	48%	8.3%	8.2%	17.0%
Grade B or higher	40%	30%	4.9%	4.8%	15.9%
Grade C or higher	26%	20%	3.2%	3.1%	15.9%
Below Grade C	3%	2%	0%	1%	-
Strength of Association (V)	0.600	0.655	0.323	0.376	0.376

	Maths		Further Maths		
	AS Level	A Level	AS Level	A Level (ALL)	A Level (Cond[1])
KS4 / GCSE Points Score per assessment	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
Students who took AS / A Level	6.4 (1.09)	6.7 (1.06)	6.9 (1.04)	7.2 (0.97)	7.2 (0.97)
Students who did not	5.2 (1.18)	5.3 (1.15)	5.5 (1.24)	5.5 (1.23)	6.6 (1.06)
Strength of Association (eta)	0.384	0.438	0.185	0.219	0.180
KS4 / GCSE Mathematics Points	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
Students who took AS / A Level	7.2 (0.81)	7.4 (0.80)	7.7 (0.65)	7.8 (0.84)	7.8 (0.84)
Students who did not	5.3 (1.37)	5.4 (1.35)	5.7 (1.46)	5.7 (1.46)	7.3 (0.77)
Strength of Association (eta)	0.516	0.511	0.220	0.229	0.195

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

1 - 'Cond' refers to a conditional population of students. In this context, it refers to the proportion of students taking A level mathematics that also took A level Further Mathematics. Whilst 2.7% of all students took A level Further Mathematics, 16% of students who took A level Mathematics also took A level Further Mathematics

Table 1c KS4 Maths Attainment - A & AS Level Attainment (UCAS points)

All who took A / AS level	Maths		Further Maths	
	AS Level	A level	AS Level	A level
KS4 Mathematics (Grade-Threshold)	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
GCSE KS4 grade A*	53.4 (10.56)	116.3 (22.24)	51.2 (13.2)	115.1 (24.08)
A	33.8 (18.09)	86.1 (28.38)	35.1 (19.09)	87.3 (29.11)
B	15.7 (17.44)	66.6 (29.47)	22.4 (20.97)	71.1 (34.58)
C	9.5 (15.61)	65.3 (35.22)	~	~
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (eta)	0.66	0.56	0.47	0.38
Pearson's Correlation Coefficient	**r**	**r**	**r**	**r**
Overall KS4 Attainment	0.58	0.55	0.54	0.51
KS4 Mathematics Attainment	0.60	0.43	0.38	0.15

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.
~ - number of cases < 10 so statistics supressed.

Table 2a Pupil Background - FSM

FSM - original	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
Not FSM	488,147	76.6%	332,856	80.6%	68.2%
FSM	79,400	12.5%	35,289	8.5%	44.4%
Missing Details	70,047	11.0%	44,598	10.8%	63.7%
Total	637,594	100.0%	412,743	100.0%	64.7%

FSM - 'Gorard Correction'[1]	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
Not FSM or in independent fee/paying schools without FSM details	535,684	84.0%	374,727	90.8%	70.0%
FSM	79,400	12.5%	35,289	8.5%	44.4%
State Educated but missing FSM details	22,510	3.5%	2,727	0.7%	12.1%
Total	637,594	100.0%	412,743	100.0%	64.7%

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

1 - The 'Gorard Correction' (Gorard, 2012) is a way of dealing with the notable missing values in the original FSM variable (11% of cases). This assumes that those located within independent / fee paying schools that do not have any FSM details recorded to not be FSM.

Table 2b: FSM - A and AS Level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond)
FSM	%	%	%	%	%
Not FSM	23.1%	17.5%	2.9%	2.8%	16.2%
FSM	15.2%	8.8%	1.3%	1.0%	10.9%
State educated, FSM unknown	11.7%	7.9%	1.5%	1.4%	18.1%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (V)	0.057	0.068	0.028	0.033	0.030

Table 2c: FSM - A & AS Level Attainment

	Maths		Further Maths	
	AS Level	A level	AS Level	A level
FSM	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
Not FSM	37.9 (20.68)	101.0 (30.59)	46.6 (16.97)	111.3 (26.74)
FSM	29.5 (21.39)	88.0 (33.20)	39.7 (19.49)	97.9 (30.54)
State ed, FSM unknown	35.9 (21.10)	94.4 (35.10)	42.6 (20.73)	105.1 (35.16)
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (eta)	0.094	0.088	0.081	0.086

Table 3a: Pupil Background - Gender

	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
Male	327,233	51.3%	195,633	47.4%	59.8%
Female	310,361	48.7%	217,110	52.6%	70.0%
Total	637,594	100.0%	412,743	100.0%	64.7%

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

Table 3b: Gender- A and AS Level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond)
	%	%	%	%	%
Male	27.6%	21.4%	3.9%	4.1%	19.3%
Female	17.6%	12.5%	1.6%	1.4%	10.8%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (V)	0.119	0.120	0.072	0.086	0.113

Table 3c: Gender- A & AS Level Attainment

	Maths		Further Maths	
	AS Level	A level	AS Level	A level
Gender	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
Male	36.9 (21.19)	100.6 (31.59)	45.8 (17.46)	110.7 (27.40)
Female	38.2 (20.26)	100.1 (29.65)	47.4 (16.38)	111.3 (25.84)
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (eta)	0.030	0.008	0.043	0.010 (NS)

There is little evidence of large gender differences in attainment across the four attainment outcome measures.

Table 4a: Pupil Background - Ethnicity

	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
White British	444,309	69.7%	278,231	67.4%	62.6%
White Other	17,888	2.8%	12,235	3.0%	68.4%
Indian	12,991	2.0%	11,217	2.7%	86.3%
Pakistani	16,468	2.6%	11,673	2.8%	70.9%
Bangladeshi	6,982	1.1%	5,215	1.3%	74.7%
Black Caribbean	8,068	1.3%	5,393	1.3%	66.8%
Black African	14,529	2.3%	11,599	2.8%	79.8%
Mixed Black Caribbean & White	6,466	1.0%	3,757	0.9%	58.1%
Chinese	2,314	0.4%	2,113	0.5%	91.3%
Other or missing	107,579	16.9%	71,310	17.3%	66.3%
Total	637,594	100.0%	412,743	100.0%	64.7%

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

Table 4b: Ethnicity - A and AS Level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond)
	%	%	%	%	%
White British	19.8%	13.5%	2.5%	2.2%	16.2%
White Other	24.2%	16.8%	3.1%	2.9%	17.2%
Indian	43.9%	33,4%	4.9%	4.5%	13.5%
Pakistani	26.2%	15.8%	1.5%	1.1%	7.2%
Bangladeshi	27.3%	17.1%	2.4%	1.7%	9.8%
Black Caribbean	13.3%	7.9%	0.9%	0.7%	9.0%
Black African	25.5%	16.0%	1.7%	1.1%	6.6%
Mixed Black Caribbean & White	13.1%	8.5%	1.3%	1.0%	11.2%
Chinese	61.0%	52.4%	13.3%	14.6%	27.6%
Other or missing	27.0%	26.9%	3.3%	4.7%	17.6%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (V)	0.134	0.171	0.059	0.086	0.086

Table 4c: Ethnicity - A & AS Level Attainment

N=412,743	Maths		Further Maths	
	AS Level	A level	AS Level	A level
	Mean (sd)	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
White British	36.8 (20.99)	98.7 (30.83)	46.4 (16.98)	108.8 (26.83)
White Other	35.6 (21.25)	97.6 (31.84)	45.6 (16.02)	106.7 (28.98)
Indian	38.9 (19.79)	100.9 (30.04)	47.3 (16.04)	110.4 (28.06)
Pakistani	32.3 (21.03)	91.8 (31.06)	43.9 (17.89)	103.6 (27.51)
Bangladeshi	31.8 (21.27)	91.1 (32.77)	40.3 (18.17)	93.4 (33.18)
Black Caribbean	29.1 (20.63)	85.5 (32.33)	38.9 (18.41)	97.8 (28.60)
Black African	31.3 (20.07)	87.6 (30.93)	38.1 (20.99)	99.5 (25.92)
Mixed Black Carib & White	34.5 (20.89)	92.6 (29.94)	45.3 (19.48)	101.1 (21.88)
Chinese	43.6 (18.81)	106.3 (30.09)	47.1 (17.25)	110.7 (25.94)
Other or missing	41.1 (19.87)	106.6 (29.50)	47.2 (17.27)	116.5 (25.79)
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (eta)	0.127	0.153	0.086	0.157

Table 5a: Further Mathematics A level Security Scale at institution & student level

	School Level (N=2,800 KS5 institutions)		Student Level (N=412,743 KS5 students)	
	n	%	%[1]	%[2]
None	762	27%	17%	18%
Least secure	399	14%	9%	10%
Less Secure	716	26%	19%	21%
Relatively Secure	710	25%	29%	31%
Highly Secure	213	8%	19%	20%
Missing Details	-	-	8%	-
Total	*2,800*	*100%*	*100%*	*100.0%*

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

1 - these percentages include the missing values

2 - these percentages do not include the missing values

3 - this includes independent schools

Table 5b: Institution Variables - FM Security - A & AS level Participation

N=412,743 for all except A level (cond[1])	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond[1])
	%	%	%	%	%
None	8.0%	4.6%	0.2%	0.0%	0.6%
Least Secure	16.3%	9.9%	1.3%	0.6%	6.5%
Less Secure	22.9%	16.5%	3.0%	2.3%	14.8%
Relatively Secure	29.5%	23.2%	4.1%	4.0%	17.5%
Highly Secure	33.1%	27.7%	4.5%	5.7%	20.8%
Missing Details	5.5%	2.6%	0.5%	0.3%	2.6%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (V)	0.234	0.239	0.102	0.127	0.155

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

1 - 'Cond' refers to a conditional population of students. In this context, it refers to the proportion of students taking A level mathematics that also took A level Further Mathematics. Whilst 2.7% of all students took A level Further Mathematics, 16% of students who took A level Mathematics also took A level Further Mathematics

Table 5c: Institution Variables - FM Security - A & AS level Attainment

UCAS Points Score	Maths		Further Maths	
	AS Level	A level	AS Level	A level
FM Security Scale	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**	**Mean (sd)**
None	26.9 (21.05)	81.1 (32.15)	36.1 (20.70)	101.2 (47.15)
Least Secure	29.6 (21.47)	89.6 (31.44)	38.1 (21.24)	103.0 (28.67)
Less secure	35.2 (20.88)	96.5 (31.22)	42.7 (18.67)	103.9 (29.61)
Relatively Secure	39.1 (20.25)	102.0 (30.03)	47.2 (16.62)	110.0 (26.71)
Highly Secure	41.4 (19.70)	106.0 (29.29)	49.6 (14.48)	115.3 (24.95)
Missing Details	30.0 (21.22)	90.9 (31.28)	39.2 (20.62)	99.2 (29.88)
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (eta)	0.206	0.200	0.197	0.159

School / Institutional Type (KS4 and KS5)

Table 6a: KS4 Location (in 2011)

KS4 Location	Complete Cohort		On KS5 File 2012-14		% on KS5 file
	n	%	n	%	%
Community	248,146	38.9%	157,226	38.1%	63.4%
Voluntary Aided	81,851	12.8%	60,942	14.8%	74.5%
Voluntary Controlled	17,259	2.7%	12,089	2.9%	70.0%
Foundation	168,605	26.4%	111,487	27.0%	66.1%
Sponsored Academy	41,376	6.5%	22,385	5.4%	54.1%
Converter Academy	5,030	0.8%	3,882	0.9%	77.2%
Pupil Referral Unit	7,747	1.2%	541	0.1%	7.0%
Independent / Fee Paying	48,079	7.5%	41,970	10.2%	87.3%
Other	19,501	3.1%	2,222	0.5%	11.4%
Total	637,594	100.0%	412,743	100.0%	64.7%

Source: 2011 KS4 Pupil Cohort; KS5 2012 to 2014 Data File.

Table 6b: KS4 location - A and AS level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)[1]	A level (cond)[2]
	%	%	%	%	%
Community	21%	14%	2.5%	2.0%	14.7%
Voluntary Aided	25%	18%	3.1%	2.8%	15.4%
Voluntary Controlled	24%	18%	2.9%	2.9%	16.4%
Foundation	23%	16%	2.8%	2.5%	15.5%
Sponsored Academy	16%	10%	1.6%	1.2%	12.7%
Converter Academy	31%	26%	5.6%	5.0%	19.7%
Pupil Referral Unit	6%	4%	0.4%	0.6%	15.8%
Independent / Fee Paying	26%	31%	3.2%	6.0%	19.3%
Other	11%	8%	1.4%	1.2%	15.1%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (Cramer's V)	0.065	0.145	0.030	0.076	0.049

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.
1 - percentages based on complete sample (N=412,743)
2 - percentages based on those who took A level maths (16% of those taking A level maths also took A level FM) - N=69.048

Table 7a: KS5 Location

KS5 Location	%	*n*[1]
	%	n
State School 6th Form	43-44%	176,744 - 182,125
Sixth Form College	15%	63,164 -63,674
FE College	27%	109,351 - 110,688
Independent / Fee Paying	8%	31,324 - 31,875
Other / Unclear	6-8%	23,662 - 32,160
Total	100.0%	412,743

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data Fil
1 - KS5 location is centered on where each A/AS level was taken and so these numbers vary a little - although there is very little proportionate differences in the percentages

Table 7b: KS5 location - A and AS level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (ALL)	A level (Cond)
	%	%	%	%	%
State School 6th Form	32.3%	23.0%	4.2%	3.6%	16.3%
Sixth Form College	27.4%	18.3%	3.2%	2.6%	14.9%
FE College	6.5%	3.9%	0.7%	0.5%	14.0%
Independent / Fee Paying	26.0%	37.6%	3.5%	7.7%	21.2%
Other / Unclear	0.6%	0.6%	0.1%	0.0%	0.4%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (Cramer's V)	0.287	0.285	0.100	0.127	0.113

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

Table 7c KS5 location - A & AS level Attainment

	Maths		Further Maths	
	AS Level	A level	AS Level	A level
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
State School 6th Form	37.5 (20.82)	99.5 (30.68)	46.4 (17.16)	109.2 (27.00)
Sixth Form College	35.9 (20.64)	95.2 (31.55)	45.4 (16.92)	106.4 (27.34)
FE College	29.9 (21.39)	89.5 (32.58)	39.8 (19.92)	101.7 (30.90)
Independent / Fee Paying	46.9 (16.82)	112.8 (25.97)	51.7 (13.27)	120.2 (23.32)
Other	36.4 (19.11)	89.5 (33.62)	38.8 (21.18)	101.4 (25.38)
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (eta)	0173	0.204	0.141	0.195

Table 8a: KS4 to KS5 Route

KS4 to 5 Route	%	n
State KS4 - State School 6th Form KS5	42-43%	172,804 - 177,963
Independent KS4 - State School 6th Form KS5	1%	3,940 - 4,162
State KS4 - State 6th Form College KS5	15%	59,784 - 61,108
Independent KS4 - State 6th Form College KS5	1%	3,380 - 3,475
State KS4 - State FE College KS5	26%	106,528 - 107,770
Independent KS4 - State FE College KS5	1%	2,823 - 2,918
State KS4 - Independent KS5	<1%	1,800 - 2,107
Independent KS4 - Independent KS5	7%	29,497 - 29,810
other / unclear / high flux	6-8%	23,662 - 32,160
Total	100.0%	412,743

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.

Table 8b: KS4 to KS5 Route - A and AS level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond)
	%	%	%	%	%
State KS4 - State School 6th Form KS5	32.3%	22.9%	4.2%	3.7%	16.4%
Independent KS4 - State School 6th Form KS5	36.4%	24.9%	4.1%	2.9%	12.4%
State KS4 - State 6th Form College KS5	27.1%	18.0%	3.1%	2.6%	14.9%
Independent KS4 - State 6th Form College KS5	33.4%	23.9%	4.4%	3.3%	14.8%
State KS4 - State FE College KS5	6.3%	3.7%	0.6%	0.4%	14.3%
Independent KS4 - State FE College KS5	14.2%	10.0%	1.3%	0.8%	9.9%
State KS4 - Independent KS5	28.4%	46.8%	3.8%	7.7%	20.1%
Independent KS4 - Independent KS5	25.9%	37.0%	3.4%	7.7%	21.2%
other / unclear	0.6%	0.6%	0.1%	0.0%	0.4%
Total	22.3%	16.7%	2.7%	2.7%	16.0%
Strength of Association (Cramer's V)	0.288	0.286	0.101	0.127	0.114

Table 8c KS5 location - A & AS Level Attainment

	Maths		Further Maths	
	AS Level	A level	AS Level	A level
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
State KS4 - State School 6th Form KS5	37.5 (20.81)	99.5 (30.66)	46.5 (17.11)	109.1 (27.04)
Independent KS4 - State School 6th Form KS5	36.5 (21.03)	98.0 (31.08)	43.0 (18.75)	110.9 (24.55)
State KS4 - State 6th Form College KS5	35.6 (20.66)	94.9 (31.56)	45.4 (16.78)	106.3 (27.26)
Independent KS4 - State 6th Form College KS5	39.4 (20.01)	99.2 (31.13)	45.9 (18.62)	108.3 (28.38)
State KS4 - State FE College KS5	29.7 (21.40)	89.3 (32.48)	39.5 (20.07)	101.6 (30.88)
Independent KS4 - State FE College KS5	33.1 (21.04)	92.2 (33.93)	44.1 (16.58)	104.3 (31.88)
State KS4 - Independent KS5	42.9 (18.22)	105.4 (28.74)	51.4 (12.40)	115.1 (25.49)
Independent KS4 - Independent KS5	47.2 (16.69)	113.5 (25.61)	51.8 (13.33)	120.5 (23.15)
other / unclear	36.4 (19.11)	89.5 (33.62)	38.8 (21.18)	101.4 (25.38
Total	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
Strength of Association (V)	0.175	0.207	0.144	0.1

Table 9a: FMSP Engagement Variables

	School Level (institutions) N_{School} = 2,800		Pupil Level (Individuals) N_{pupil} = 412,743	
LOPD - Live online PD	n	%	%[1]	%[2]
0	2,587	92%	83-85%	91%
1	**213**	**8%**	**8%**	**9%**
Missing Details	-	-	8-9%	-
TAM - Teaching Advanced Mathematics	n	%	%[1]	%[2]
0	2,688	96%	88-90%	97%
1	**112**	**4%**	**3%**	**3%**
Missing Details	-	-	8-9%	-
TFM - Teaching Further Mathematics	n	%	%[1]	%[2]
0	2,753	98%	90-91%	98%
1	**47**	**2%**	**2%**	**2%**
Missing Details	-	-	8-9%	-
Tuition (Standard, LOT, LIL)	n	%	%[1]	%[2]
0	2,693	96%	88-90%	97%
1	**107**	**4%**	**3%**	**3%**
Missing Details	-	-	8-9%	-
Total	2,800	100.0%	100.0%	100.0%

Source: 2011 KS4 Pupil Cohort who are also on the KS5 2012 to 2014 Data File.
1 - these percentages include the missing values 2 - these percentages do not include the missing values

Table 9b shows the A and AS level participation outcome variables and their statistical association with the various measures of FMSP engagement.

Table 9b: FMSP Engagement - A and AS level Participation

N=412,743 for all except A level (cond)	Maths		Further Maths		
	AS Level	A level	AS Level	A level (all)	A level (cond)
	%	%	%	%	%
Total 2011 KS4 NPD Cohort	22.3%	16.7%	2.7%	2.7%	16.0%
LOPD	28.5%	22.7%	3.8%	4.1%	18.5%
TAM	28.0%	18.0%	4.0%	2.8%	16.0%
TFM	28.7%	23.7%	4.6%	3.8%	16.3%
Tuition	21.7%	13.9%	2.1%	1.4%	10.5%

Table 9c: FMSP Variables - A and AS level Attainment

UCAS POINTS SCORE	Maths		Further Maths	
	AS Level	A level	AS Level	A level (all)
	Mean(s.d.)	Mean(s.d.)	Mean(s.d.)	Mean(s.d.)
ALL Respondents	37.4 (20.82)	100.4 (30.84)	46.3 (17.15)	110.8 (26.99)
LOPD	39.5 (20.25)	102.5 (30.12)	48.3 (16.10)	112.1 (26.44)
TAM	35.4 (21.51)	98.4 (30.59)	45.2 (18.00)	106.1 (28.79)
TFM	41.0 (19.76)	104.1 (29.66)	46.4 (17.79)	117.4 (23.10)
Tuition	32.6 (21.08)	92.0 (31.18)	39.7 (19.81)	101.8 (32.82)

PART II: Specifying the Multilevel Models

Participation in A level Mathematics and Further Mathematics was modelled using a series of multilevel, multivariate binary logistic regression models.

Attainment in A level Mathematics and Further Mathematics was modelled using a series of multilevel, multivariate linear regression models.

The theoretical specifications of these models are shown below but prior to doing so, the conceptual stages are discussed.

Model Stages

1. The empty or 'null' model
2. Including KS4 attainment (overall & KS4 Mathematics) alone
3. Including KS5 location alone
4. Including pupil background (FSM, gender & ethnicity) alone
5. Including FMSP engagement variables alone (TAM, TFM, LOPD & Tuition)
6. Complete main effects model (Pupil background, KS5 location, FMSP engagement & KS4 attainment all together)

Following this, an interaction between gender and KS4 attainment was included.

The summary tables for the models will focus on the complete main effect stage and including the gender * KS4 attainment interaction with further support tables provided for detail on stages 1 to 5.

The complete main effects models will show how the explanatory variables are associated with the A level participation and attainment outcome variables in multivariate space, once all other explanatory variables are statistically controlled for. For example, the models will show the likelihood of females participating in A level maths compared with males after KS4 attainment, KS5 location, FSM status, ethnicity and FMSP engagement have all been statistically controlled for.

The inclusion of an interaction between gender and KS4 attainment will enable a view on whether the relationship between KS4 attainment (in KS4 Mathematics and overall KS4 attainment) and the A level participation and attainment outcome variables is consistent for males and females. Specifically, the inclusion of the interaction will enable the following questions to be answered:

- Is the relationship between KS4 attainment and A level participation the same for males and females?

- Is the relationship between KS4 attainment and A level attainment the same for males and females?

Whilst the models will be summarised using the complete main effects and interaction stages, it is not good practice to simply take an end-point perspective. The end-point is where the explanatory variables are all brought together into multidimensional space. Within this space there will be multidimensional peaks and cleavages where explanatory variables entwine to form relatively high or low rates of participation and attainment in A level Mathematics and Further Mathematics.

Multilevel Models - theoretical specification

A level Participation Models: Stage 1: null model:

The participation models are multilevel binary logistic regression models and the empty / null model is shown in equation 1.1 below.

Equation 1.1: $\text{Log}\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) = \text{Logit}\left(\pi_{ij}\right) = \beta_0 + e_{0j}$

Where π represents the probability of taking A level Mathematics (16.7%, π = 0.167), and A level Further Mathematics (? = 0.027 & ? = 0.160). This is transformed into log-odds $\left(\frac{\pi}{1-\pi}\right)$ which linearises the probability distribution and removes the upper and lower (0 and 1) probability boundaries. This transformation is known as the logit-link function $\left(\text{Logit}\left(\pi_{ij}\right)\right)$.

The levels are individual students (level 1, denoted by the 'i' in equation 1.1) and KS5 institutions (level 2, denoted by 'j' in equation 1.1).

The intercept β_0 is constant across KS5 institutions but the random effect e_{0j} is specific to KS5 institutions. The random effect is assumed to follow a Gaussian distribution with variance σ^2_{u0}

Attainment Models: Stage 1: null model:

The attainment models are multilevel linear regression models and the empty / null model is shown in equation 2.1 below.

Equation 2.1: $Y_{ij} = \beta_0 + U_{0j} + e_{ij}$

Where Y_{ij} represents the A level UCAS points score attainment of student j in KS5 institution i.

β_0 is constant across KS5 institutions and represents the overall mean, U_{0j} is the effect of KS5 institution j on attainment and e_{ij} is a student level residual.

The KS5 Institution effects U_{0j} are random effects and assumed to follow a Gaussian distribution with a mean of zero and a variance σ_{u0}^2

For both the participation and attainment models, the model parameters are estimated using iterative methods[2]. The empty or null models are useful in assessing the structure of the A level participation and attainment outcome variables. Specifically, they are useful in assessing the degree of clustering in A level participation and attainment at the KS5 institutional level. The variance partition coefficient (vpc) otherwise known as the intra-cluster correlation coefficient (icc) statistically describe the clustering at the KS5 institution and student levels for the A level participation and attainment outcome variables. Table 1 summarises the variance partition coefficients for the null models for the A level participation and attainment outcome variables.

Table 1: summary of variance partition coefficients for the A level participation and attainment outcome variables.

A level Participation	A level Maths	A level FM[1]	A level FM[2]
variance partition coef:			
Level 2 / KS5 institution (%)	34%	35%	8%
Level 1 / Student (%)	66%	65%	92%

[1] - participation in A level further maths amongst the 2011 cohort (2.7%).

[2] - participation in A level further maths amongst the 2011 cohort who took A level maths (16.0%).

A level Attainment	A level Maths	A level FM
variance partition coef:		
Level 2 / KS5 institution (%)	17%	20%
Level 1 / Student (%)	83%	80%

From the statistics in Table 1, it seems that a multilevel approach is justified - a sizable proportion of the variation in A level participation and attainment in A level Mathematics and Further Mathematics is found to reside at the KS5 institutional level.

[2] For the A level participation logistic multilevel models, maximum likelihood estimation using adaptive quadrature is the estimation method and for the attainment linear multilevel models, estimation begins with expectation maximisation before switching to a Newton-Raphson gradient based method (Steele, F. 2009).

The participation models show a stronger degree of clustering at the institutional level compared with the attainment models. Over a third (34 - 35%) of the variation in participation rates for A level Mathematics and Further Mathematics are observed to reside at level 2 (KS5 institution).

For the attainment models, the degree of level 2 (KS5 institution) clustering is slightly stronger in A level Further Mathematics (20%) compared with A level Mathematics (17%). The level 2 clustering within the 'conditional' A level Further Mathematics participation null model is smaller (9%) than observed with the unconditional models (34-35%). *Given* that a young person took A level Mathematics, the likelihood of them also taking Further Mathematics is still clustered at level 2 but not to the same extent as seen with the unconditional participation in A level Mathematics and Further Mathematics models. In essence, the institutional difference (or variation) in offering or not offering[3] A level Mathematics (or Further Mathematics) is stronger than the institutional difference (or variation) in offering or not offering A level Further Mathematics *given* that A level Mathematics is already offered.

Following the null model and assessment whether a multilevel analytical approach is required, the models go through a series of six stages before reaching the complete main effects stage.

There are many similarities between the theoretical model specification for the A level participation and attainment models, the key difference is the need for a logit-link function for the participation models $\left(\text{Log}\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) in\ equation\ 1\right)$ in order to linearise the probability distribution. For the attainment variable, no such transformation is required and the A level UCAS point score is modelled directly in its original scale. One further difference is that the A level participation models do not have a student level residual term $(e_{ij}\ in\ equation\ 2)$. This is because, at the student level the variable is binary in nature (1 and 0) and it does not become a probability until considered across other explanatory variables (proportion of population / males etc. who took A level Mathematics).

These two (important) distinctions aside, stages 2 to 5 are conceptually similar for the A level participation and attainment models.

[3] The 2011 cohort data file identifies whether a young person took A level maths or further maths I 2012, 2013 or 2014 and the institution where they took the A level. At the institution level it makes more linguistic sense to talk of 'offering' and 'not offering' but really this is a short hand for institutions that have a young person who is recorded as taking the A level. It may be the case that some institutions offered A level maths or further maths but no students were actually entered.

Stage 2: KS4 Attainment models:

As shown in equations 1.2 and 2.2, KS4 attainment is included into the model using a one dummy variable and one scale covariate. These are all level 1 (student) variables.

Equation 1.2:

$$\text{Logit}\left(\pi_{ij}\right) \quad = \quad \beta_0 + \{\beta_1 MathsAstar\} + \{\beta_2 KS4OverallCENT\} + e_{0j}$$

Equation 2.2:

$$Y_{ij} \quad = \quad \beta_0 + \{\beta_1 MathsAstar\} + \{\beta_2 KS4OverallCENT\} + U_{0j} + e_{ij}$$

Where ..

$\beta_1 MathsAstar$ is the coefficient that identifies whether participation / attainment is higher or lower for students who passed KS4 maths with a grade A-star compared with students who attained a lower grade in KS4 maths

$\beta_2 KS4OverallCENT$ is the coefficient that identifies whether participation / attainment is correlated with overall KS4 attainment. This variable has been centred around the mean KS4 points score per assessment taken.

β_0 is the constant term that collects all of the reference groups. Here, it represents the mean participation / attainment for students who did not attain a grade A-star in KS4 maths and who had a mean overall KS4 attainment.

Stage 3: KS5 location models:

As shown in equations 1.3 and 2.3, KS5 location is included into the model using 4 dummy variables. These are all level 2 (KS5 institution) variables.

Equation 1.3:

$$\text{Logit}\left(\pi_{ij}\right) \quad = \beta_0 + \{\beta_3 6thColl + \beta_4 FEColl + \beta_5 IndFee + \beta_6 Other\} + e_{0j}$$

Equation 2.3:

$$Y_{ij} = \beta_0 + \{\beta_3 6thColl + \beta_4 FEColl + \beta_5 IndFee + \beta_6 Other\} + U_{0j} + e_{ij}$$

Where ..

$\beta_3 6thColl\ to\ \ \beta_6 Other$ are coefficient that identify whether participation / attainment is higher or lower for students within four types of KS5 institution compared with the reference group (students who took the A level within a school 6th form).

β_0 is the constant term that collects all of the reference groups. Here, it represents the mean participation / attainment for students who took the A level within a school 6th form.

<u>Stage 4: Pupil Background models:</u>

As shown in equations 1.4 and 2.4, proximity to poverty, gender and ethnicity are included into the model using a series of 11 dummy variables. These are all level 1 (student) variables.

Equation 1.4:

$$
\begin{aligned}
\text{Logit}\left(\pi_{ij}\right) \quad = \ & \beta_0 + \{\beta_7 FSM\} + \{\beta_8 Female\} + \cdots \\
& + \{\beta_9 WhiteOth + \beta_{10} Indian + \beta_{11} Pakistani + \beta_{12} Bangladeshi \\
& + \beta_{13} Black\_Caribb + \beta_{14} Black\_Afric + \beta_{15} Mixed + \beta_{16} Chinese \\
& + \beta_{17} Other\} \\
& \quad + e_{0j}
\end{aligned}
$$

Equation 2.4:

$$
\begin{aligned}
Y_{ij} \quad = \ & \beta_0 + \{\beta_7 FSM\} + \{\beta_8 Female\} + \cdots \\
& + \{\beta_9 WhiteOth + \beta_{10} Indian + \beta_{11} Pakistani + \beta_{12} Bangladeshi \\
& + \beta_{13} Black\_Caribb + \beta_{14} Black\_Afric + \beta_{15} Mixed + \beta_{16} Chinese \\
& + \beta_{17} Other\} \\
& \qquad + U_{0j} + e_{ij}
\end{aligned}
$$

Where ..

$\beta_7 FSM$ is the coefficient that identifies whether participation / attainment is higher or lower for young people classed as FSM in 2011 compared with the reference group (not classed as FSM).

$\beta_8 Female$ is the coefficient that identifies whether participation / attainment is higher or lower for females compared with the reference group (males).

$\beta_9 WhiteOth \ldots . to\ \beta_{17} Other$ are coefficients that identify whether participation / attainment is higher or lower for nine ethnic groups[4] compared with the reference group (white British): white other; Indian; Pakistani; Bangladeshi; Black Caribbean; Black African; Black Caribbean & white mixed; Chinese and other.

β_0 is the constant term that collects all of the reference groups. Here, it represents the mean participation / attainment for non-FSM males who are identified as white-British.

<u>Stage 5: FMSP engagement models:</u>

As shown in equations 1.5 and 2.5, FMSP engagement is included into the model using a series of 4 dummy variables. These are all level 2 (KS5 institution) variables.

Equation 1.5:

$$\text{Logit}\left(\pi_{ij}\right) = \beta_0 + \{\beta_{18}\ LOPD\} + \{\beta_{19}\ TAM\} + \{\beta_{20}\ TFM\} + \{\beta_{21}\ Tuition\} + e_{0j}$$

Equation 2.5:

$$Y_{ij} = \beta_0 + \{\beta_{18}\ LOPD\} + \{\beta_{19}\ TAM\} + \{\beta_{20}\ TFM\} + \{\beta_{21}\ Tuition\} + U_{0j} + e_{ij}$$

Where ..

$\beta_{18} LOPD$ is the coefficient that identifies whether participation / attainment is higher or lower for students who took the A level within an institution identified as being involved with Live Online Professional Development from the FMSP.

$\beta_{19} TAM$ is the coefficient that identifies whether participation / attainment is higher or lower for students who took the A level within an institution identified as being involved with 'Teaching Additional Maths' from the FMSP.

[4] The nine ethnic groups - White other, Indian, Pakistani, Bangladeshi, Black-Caribbean, Black-African, Mixed (Black Caribbean & White), Chinese & Other.

$\beta_{20}TFM$ is the coefficient that identifies whether participation / attainment is higher or lower for students who took the A level within an institution identified as being involved with 'Teaching Further Maths' from the FMSP.

$\beta_{21}Tuition$ is the coefficient that identifies whether participation / attainment is higher or lower for students who took the A level within an institution identified as being involved with tuition from the FMSP.

β_0 is the constant term that collects all of the reference groups. Here, it represents the mean participation / attainment for students who took the A level within an institution that had no FMSP involvement in terms of LOPD, TAM, TFM and tuition.

Stage 6: Complete Main Effects models:

Given the amount of space it would require, the complete models will not be theoretically specified here. Essentially, this model stage is when ALL of the variables discussed in stages 2 to 5 are entered into the model at the same time. The final model will include a series of 20 dummy variables and one scale covariate.

At level 1 (student), pupil background and KS4 attainment are included using a series of 12 dummy variables and one scale covariate.

At level 2 (institution), type of KS5 institution and FMSP engagement are included using a series of 8 dummy variables.

The constant term β_0 for the stage 6 main effects models collects all of the reference groups. Here it represents the mean participation / attainment for non-FSM males who are identified as white-British, attained less than A-star in KS4 maths and a mean KS4 points score per assessment and were located within a school 6th form that had no FMSP involvement.

Including the gender*KS4 interaction terms

As shown in equations 1.6 and 2.6, the gender*KS4 attainment interaction terms are included into the model using a one additional dummy variable and one additional scale covariate. These are all level 1 (student) variables. This final stage includes a total of 21 dummy variables (13 at level 1, 8 at level 2) and two scale covariates (both at level 1).

Equation 1.6:

$$\text{Logit}\left(\pi_{ij}\right) = \{\beta_1 MathsAstar\} + \{\beta_{22} MathsAstarFemale\}$$
$$+ \{\beta_2 KS4OverallCENT\} + \{\beta_{23} KS4OverallCENTFemale\} + e_{0j}$$

Equation 2.6:

$$Y_{ij} = \{\beta_1 MathsAstar\} + \{\beta_{22} MathsAstarFemale\} + \{\beta_2 KS4OverallCENT\} + \{\beta_{23} KS4OverallCENTFemale\} + U_{0j} + e_{ij}$$

Where ..

$\beta_1 MathsAstar$ is the coefficient that identifies whether participation / attainment is higher or lower for students who passed KS4 maths with a grade A-star compared with students who attained a lower grade in KS4 maths

$\beta_{22} MathsAstarFemale$ is the coefficient that identifies whether the relationship between KS4 maths attainment (attaining a grade A-star) and A level participation / attainment is the same for females as it is for males.

$\beta_2 KS4OverallCENT$ is the coefficient that identifies whether participation / attainment is correlated with overall KS4 attainment. This variable has been centred around the mean KS4 points score per assessment taken.

$\beta_{23} KS4OverallCENTFemale$ is the coefficient that identifies whether the correlation between overall KS4 attainment and A level participation / attainment is the same for females and males.

Assessing & Interpreting the models

Participation in A level Mathematics & Further Mathematics.

As specified in equations 1.1 to 1.6 above, the A level participation models are multilevel logistic regression models that involve a logit link function that converts the probabilities (of participation) into log-odds. This means that the model coefficients within the participation models are not directly readable. To aid interpretation, the exponential of the coefficients is taken to convert the coefficients into odds-ratios. These are the relative odds of one group (as captured by a dummy variable) compared with a reference group. For example, for A level maths within the interaction model, the female coefficient is estimated as '-1.05' which equates to an odds-ratio of 0.35[5] - telling us that females are 35% as likely to take A level Mathematics compared with males. This gender difference is observed once all other explanatory variables have been controlled for. It should be noted that these are odds and not probabilities but it is possible to convert from one to another by multiplying the exponential coefficient (odds-ratio) with the constant to generate an odds (rather than odds-ratio) score and then convert from odds to probabilities using equation 3 below

Equation 3: $Prob = \left(\frac{Odds}{1+Odds}\right)$; $Odds = \left(\frac{Prob}{1-Prob}\right)$

In the A level Mathematics gender example, 0.35 x 0.17 = 0.060 providing the odds for females taking A level maths within the 2011 cohort which converts to a probability of 0.056 (or 5.6%) which can be compared with the odds for males (0.17) that converts to a probability of 0.145 (or 14.5%).

One way of estimating the overall statistical explanatory power of the models is McFadden's pseudo r-square which has some conceptual similarities to the coefficient of determination R-square statistic commonly used within linear regression. In addition to McFadden's pseudo r-square, the variance partition coefficient is calculated to show the structure of the variance in terms of the percentage at levels 2 (KS5 institution) and 1 (student).

For Mathematics A level participation, the empty model has a pseudo r-square of 0% (because it is the null model) and the vpc shows that 34% of the variation lies at level 2 and 66% at level 1. In the final (interaction) model, the model has a pseudo r-square value of 26.2% (just over a quarter of the variation in participation in A level maths has been accounted for by variation across the explanatory variables) and the final vpc shows that 13% of the remaining variation lies at level 2 and the remaining 87% at level 1.

Attainment in A level maths & further maths.

[5] $e^{(-1.05)} = 0.34994$

The attainment models are more standard multilevel linear regression models where the outcome (A level attainment) is measured directly (UCAS points score) without the need for a logit link function.

The coefficients are presented in their original scale (UCAS points score) which is converted into a Cohens d effect size statistic by dividing by the standard deviation of the A level attainment distribution. For example, in the summary table for the attainment models, the female coefficient is seen as -6.48 ~ females attain around 6.5 UCAS points lower than males in A level maths once all other variables are controlled for. Alongside this coefficient is the Cohens d effect size statistic of -0.21 ~   females attain around 0.2 standard deviations lower than males in A level maths UCAS points once all other variables are controlled for.

Assessing fit for the attainment models is a more straightforward process than with the A level participation models. The variation can be partitioned and the explanatory power at each level can be estimated using the standard coefficient of determination R^2.

For the A level maths attainment models, the null model found that 17% of variance in attainment lay at level 2 and 83% at level 1. The final interaction model shows that the variation remaining had 5% at level 2 and 95% at level 1. The overall explanatory power of the model ($R^2_{TOT} = 38\%$) was 38.4% with a greater proportion of variance being accounted for at level 2 ($R^2_{L2} = 81\%$) than level 1 ($R^2_{L1} = 30\%$).

PART III:

Model details for multilevel analyses into participation and attainment in A level Mathematics and Further Mathematics

Participation in A level maths:

Table III.3: Stages of the main effects A level participation models
Table III.2: Including the gender*KS4 attainment interaction

ACCESS to A level further maths:

The first set of tables look at participation in A level further maths for the whole of the 2011 cohort - 2.7%

Table III.3: Stages of the main effects A level participation models for A level further maths
Table III.4: Including the gender*KS4 attainment interaction for A level further maths

The second set of tables look at participation in A level further maths amongst those who took A level Maths - 16.0%

Table III.5: Stages of the main effects A level participation models for A level further maths
Table III.6: Including the gender*KS4 attainment interaction for A level further maths

ATTAINMENT in A level maths:

Table III.7: Stages of the main effects ATTAINMENT models
Table III.8: Including the gender*KS4 attainment interaction

ATTAINMENT in A level further maths:

Table III.9: Stages of the main effects ATTAINMENT models
Table III.10: Including the gender*KS4 attainment interaction

Table III.1: Stages of the main effects A level participation models (Maths A level (16.7%))

Participation in A level Maths Model Stages:	1. Null Model	2. KS4 Attain Logit (OR)	3. KS5 Location Logit (OR)	4. Pupil Backgrd Logit (OR)	5. FMSP Engage Logit (OR)	6. Full Main Efects Logit (OR)
KS4 Attainment						
KS4 Maths Grade A*	-	**2.38 (10.79)**	-	-	-	**2.26 (9.61)**
Overall (KS4 points per exam)	-	**0.58 (1.79)**	-	-	-	**0.71 (2.03)**
Type of KS5 Inst						
(6th Form College)	-	-	-0.16 (0.86)	-	-	-0.10 (0.90)
(FE College)	-	-	**-2.78 (0.06)**	-	-	**-1.81 (0.16)**
(Independent / Fee Paying)	-	-	**0.78 (2.19)**	-	-	**-0.56 (0.57)**
(Other)	-	-	**0.70 (2.01)**	-	-	**0.86 (2.37)**
Pupil Background						
Gender (Female)	-	-	-	**-0.77 (0.46)**	-	**-1.14 (0.32)**
Poverty (FSM)	-	-	-	**-0.37 (0.69)**	-	**-0.11 (0.90)**
Ethnicity (white other)	-	-	-	**0.27 (1.30)**	-	**0.38 (1.46)**
(Indian)	-	-	-	**0.91 (2.49)**	-	**1.05 (2.85)**
(Pakistani)	-	-	-	**0.39 (1.47)**	-	**0.67 (1.96)**
(Bangladeshi)	-	-	-	**0.54 (1.72)**	-	**0.68 (1.97)**
(black Caribbean)	-	-	-	**-0.45 (0.64)**	-	-0.02 (0.98)
(black African)	-	-	-	**0.37 (1.44)**	-	**0.74 (2.09)**
(mixed black Carib & white)	-	-	-	**-0.32 (0.73)**	-	-0.08 (0.93)
(Chinese)	-	-	-	**1.80 (6.04)**	-	**1.77 (5.86)**
(Other)	-	-	-	**0.39 (1.48)**	-	**0.44 (1.55)**
FMSP Engagement						
LOPD	-	-	-		**0.45 (1.57)**	0.03 (1.03)
TAM	-	-	-	-	-0.07 (0.93)	-0.07 (0.90)
TFM	-	-	-	-	0.37 (1.45)	-0.06 (0.94)
Tuition	-	-	-	-	**-0.33 (0.72)**	-0.09 (0.91)
Constant	-1.67 (0.19)	-2.29 (0.10)	-1.54 (0.21)	-1.44 (0.24)	-1.69 (0.18)	-1.73 (0.18)
variance partition coef:						
Level 2 / KS5 institution (%)	34%	20%	20%	32%	33%	12%
Level 1 / Student (%)	66%	80%	80%	68%	67%	88%
McFaddens R-square	-	22.2%	0.5%	2.9%	0.0%	26.1%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking at least one KS5 assessment between 2012 and 14.

Table III.2: Including the gender*KS4 attainment interaction (Maths A level - 16.7%)

Participation in A level Maths — Model Stages:	Full Main Efects — Logit (odds-ratio)	Including gender*KS4 interaction terms — Logit (odds-ratio)
KS4 Attainment		
KS4 Maths Grade A*	**2.26 (9.61)**	**2.21 (9.12)**
(Females*Grade A*) Interaction term	-	**+0.14 (1.15)**
Overall KS4 points per exam	**0.71 (2.03)**	**0.79 (2.20)**
*(Females*Overall KS4 points per exam) Interaction*	-	**-0.18 (0.84)**
Type of KS5 institution		
(6th Form College)	-0.10 (0.90)	-0.10 (0.90)
(FE College)	**-1.81 (0.16)**	**-1.81 (0.16)**
(Independent / Fee Paying)	**-0.56 (0.57)**	**-0.56 (0.57)**
(Other)	**0.86 (2.37)**	**0.88 (2.42)**
Pupil Background		
Gender (Female)	**-1.14 (0.32)**	**-1.05 (0.35)**
Poverty (FSM)	**-0.11 (0.90)**	**-0.11 (0.90)**
Ethnicity (white other)	**0.38 (1.46)**	**0.38 (1.46)**
(Indian)	**1.05 (2.85)**	**1.05 (2.86)**
(Pakistani)	**0.67 (1.96)**	**0.68 (1.97)**
(Bangladeshi)	**0.68 (1.97)**	**0.68 (1.97)**
(black Caribbean)	-0.02 (0.98)	-0.01 (0.99)
(black African)	**0.74 (2.09)**	**0.74 (2.09)**
(mixed black Carib & white)	-0.08 (0.93)	-0.07 (0.93)
(Chinese)	**1.77 (5.86)**	**1.77 (5.88)**
(Other)	**0.44 (1.55)**	**0.44 (1.56)**
FMSP Engagement Variables		
LOPD	0.03 (1.03)	0.03 (1.03)
TAM	-0.07 (0.90)	-0.07 (0.93)
TFM	-0.06 (0.94)	-0.07 (0.94)
Tuition	-0.09 (0.91)	-0.09 (0.91)
Constant	-1.73 (0.18)	-1.75 (0.17)
VPC:		
Level 2 / KS5 institution (%)	12%	13%
Level 1 / Student (%)	88%	87%
McFaddens R-square	26.1%	26.2%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking at least one KS5 assessment between 2012 and 14.

Table III.3: Stages of the main effects A level participation models for A level further maths (Further Maths A level unconditional - 2.7%)

Participation in A level Further Maths Model Stages:	1. Null Model	2. KS4 Attain Logit (OR)	3. KS5 Location Logit (OR)	4. Pupil Backgrd Logit (OR)	5. FMSP Logit (OR)	Full Main Efects Logit (OR)
KS4 Attainment						
KS4 Maths Grade A*	-	**3.39 (29.80)**	-	-	-	**3.11 (22.42)**
Overall (KS4 points per exam)	-	0.22 (1.25)	-	-	-	**0.36 (1.43)**
Type of KS5 Inst						
(6th Form College)	-	-	-0.08 (0.93)	-	-	0.10 (1.11)
(FE College)	-	-	**-2.54 (0.08)**	-	-	**-1.16 (0.31)**
(Independent / Fee Paying)	-	-	**0.89 (2.44)**	-	-	**-0.15 (0.86)**
(Other)	-	-	-0.23 (0.79)	-	-	-0.14 (0.87)
Pupil Background						
Gender (Female)	-	-	-	**-1.22 (0.30)**	-	**-1.36 (0.28)**
Poverty (FSM)	-	-	-	**-0.48 (0.62)**	-	-0.03 (0.97)
Ethnicity (white other)	-	-	-	**0.30 (1.35)**	-	**0.37 (1.45)**
(Indian)	-	-	-	**0.45 (1.57)**	-	**0.19 (1.21)**
(Pakistani)	-	-	-	**-0.47 (0.62)**	-	**-0.47 (0.62)**
(Bangladeshi)	-	-	-	0.03 (1.03)	-	-0.12 (0.88)
(black Caribbean)	-	-	-	**-0.86 (0.42)**	-	-0.26 (0.77)
(black African)	-	-	-	**-0.53 (0.59)**	-	**-0.31 (0.73)**
(mixed black Carib & white)	-	-	-	**-0.56 (0.57)**	-	-0.19 (0.83)
(Chinese)	-	-	-	**1.73 (5.63)**	-	**1.26 (3.52)**
(Other)	-	-	-	0.25 (1.29)	-	0.00 (1.00)
FMSP Engagement						
LOPD	-	-	-		**0.60 (1.82)**	0.13 (1.14)
TAM	-	-	-	-	-0.07 (0.93)	0.02 (1.02)
TFM	-	-	-	-	0.25 (1.28)	**-0.30 (0.74)**
Tuition	-	-	-	-	**-0.98 (0.38)**	**-0.45 (0.64)**
Constant	-4.03 (0.02)	-5.55 (0.004)	-3.83 (0.02)	-3.56 (0.03)	-4.05 (0.02)	-4.98 (0.01)
VPC:						
Level 2 / KS5 institution (%)	35%	15%	22%	33%	34%	13%
Level 1 / Student (%)	65%	85%	78%	67%	66%	87%
McFaddens r-square	-	25.8%	1.0%	3.9%	0.1%	29.5%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking at least one KS5 assessment between 2012 and 14.

Table III.4: Including the gender*KS4 attainment interaction (Further Maths A level unconditional - 2.7%).

Participation in A level Further Maths Model Stages:		Full Main Efects Logit (odds-ratio)	Including gender*KS4 interaction terms Logit (odds-ratio)
KS4 Attainment			
	KS4 Maths Grade A*	**3.39 (29.80)**	**2.95 (19.04)**
	(Females*Grade A*) Interaction term	-	**+0.70 (2.01)**
	Overall KS4 points per exam	**0.22 (1.25)**	**0.39 (1.48)**
	*(Females*Overall KS4 points per exam) Interaction*	-	**-0.17 (0.85)**
Type of KS5 institution			
	(6th Form College)	0.10 (1.11)	0.10 (1.10)
	(FE College)	**-1.16 (0.31)**	**-1.15 (0.32)**
	(Independent / Fee Paying)	**-0.15 (0.86)**	**-0.15 (0.86)**
	(Other)	-0.14 (0.87)	-0.14 (0.87)
Pupil Background			
Gender	(Female)	**-1.36 (0.28)**	**-1.50 (0.22)**
Poverty	(FSM)	-0.03 (0.97)	-0.03 (0.97)
Ethnicity	(white other)	**0.37 (1.45)**	**0.37 (1.45)**
	(Indian)	**0.19 (1.21)**	**0.19 (1.21)**
	(Pakistani)	**-0.47 (0.62)**	**-0.47 (0.62)**
	(Bangladeshi)	-0.12 (0.88)	-0.12 (0.88)
	(black Caribbean)	-0.26 (0.77)	-0.26 (0.77)
	(black African)	**-0.31 (0.73)**	**-0.31 (0.73)**
	(mixed black Carib & white)	-0.19 (0.83)	-0.19 (0.83)
	(Chinese)	**1.26 (3.52)**	**1.24 (3.47)**
	(Other)	0.00 (1.00)	0.00 (1.00)
FMSP Engagement Variables			
	LOPD	0.13 (1.14)	0.13 (1.14)
	TAM	0.02 (1.02)	0.02 (1.02)
	TFM	**-0.30 (0.74)**	**-0.30 (0.74)**
	Tuition	**-0.45 (0.64)**	**-0.44 (0.64)**
Constant		-4.98 (0.01)	-4.95 (0.01)
VPC:			
	Level 2 / KS5 institution (%)	13%	13%
	Level 1 / Student (%)	87%	87%
	McFaddens R-square	29.5%	29.6%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking at least one KS5 assessment between 2012 and 14.

Table III.5: Stages of the main effects A level participation models for A level (A level Further Maths *given* that a student is taking A level maths)

Participation in A level Further Maths Model Stages:	1. Null Model	2. KS4 Attain Logit (OR)	3. KS5 Location Logit (OR)	4. Pupil Backgrd Logit (OR)	5. FMSP Engage Logit (OR)	Full Main Efects Logit (OR)
KS4 Attainment						
KS4 Maths Grade A*	-	**1.80 (6.08)**	-	-	-	**1.76 (5.82)**
Overall (KS4 points per exam)	-	0.01 (1.01)	-	-	-	**0.13 (1.14)**
Type of KS5 Inst						
(6th Form College)	-	-	0.01 (1.01)	-	-	0.13 (1.14)
(FE College)	-	-	-0.14 (0.87)	-	-	0.09 (1.09)
(Independent / Fee Paying)	-	-	**0.33 (1.40)**	-	-	0.10 (1.11)
(Other)	-	-	0.06 (1.07)	-	-	0.09 (1.07)
Pupil Background						
Gender (Female)	-	-	-	**-0.71 (0.49)**	-	**-0.97 (0.38)**
Poverty (FSM)	-	-	-	**-0.18 (0.83)**	-	0.06 (1.07)
Ethnicity (white other)	-	-	-	0.11 (1.12)	-	**0.25 (1.29)**
(Indian)	-	-	-	**-0.16 (0.85)**	-	-0.12 (0.89)
(Pakistani)	-	-	-	**-0.78 (0.46)**	-	**-0.68 (0.51)**
(Bangladeshi)	-	-	-	**-0.42 (0.66)**	-	**-0.39 (0.68)**
(black Caribbean)	-	-	-	**-0.52 (0.59)**	-	-0.21 (0.81)
(black African)	-	-	-	**-0.84 (0.43)**	-	**-0.58 (0.56)**
(mixed black Carib & white)	-	-	-	-0.25 (0.78)	-	-0.11 (0.90)
(Chinese)	-	-	-	**0.75 (2.12)**	-	**0.76 (2.14)**
(Other)	-	-	-	0.03 (1.03)	-	-0.13 (0.88)
FMSP Engagement						
LOPD	-	-	-		**0.17 (1.19)**	0.08 (1.08)
TAM	-	-	-	-	-0.05 (0.95)	0.02 (1.02)
TFM	-	-	-	-	-0.06 (0.94)	**-0.30 (0.74)**
Tuition	-	-	-	-	**-0.59 (0.56)**	**-0.43 (0.65)**
Constant	-1.75 (0.17)	-2.93 (0.05)	-1.81 (0.16)	-1.47 (0.23)	-1.75 (0.17)	-2.79 (0.06)
variance partition coef:						
Level 2 / KS5 institution (%)	8%	8%	7%	7%	8%	8%
Level 1 / Student (%)	92%	92%	93%	93%	92%	92%
McFaddens R-square	-	8.7%	0.1%	1.9%	0.1%	11.4%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking an A level in maths between 2012 and 14.

Table III.6: Including the gender*KS4 attainment interaction
(A level Further Maths *given* that a student is taking A level maths)

Participation in A level Further Maths Model Stages:		Full Main Efects Logit (odds-ratio)	Including gender*KS4 interaction terms Logit (odds-ratio)
KS4 Attainment			
	KS4 Maths Grade A	**1.76 (5.82)**	**1.74 (5.69)**
	(Females*Grade A*) Interaction term	-	**+0.09 (1.09)**
	Overall KS4 points per exam	**0.13 (1.14)**	**0.16 (1.17)**
	*(Females*Overall KS4 points per exam) Interaction*	-	**-0.11 (0.90)**
Type of KS5 institution			
	(6th Form College)	0.13 (1.14)	0.13 (1.14)
	(FE College)	0.09 (1.09)	0.09 (1.09)
	(Independent / Fee Paying)	0.10 (1.11)	0.10 (1.11)
	(Other)	0.09 (1.07)	0.09 (1.07)
Pupil Background			
Gender	(Female)	**-0.97 (0.38)**	**-0.77 (0.38)**
Poverty	(FSM)	0.06 (1.07)	0.06 (1.07)
Ethnicity	(white other)	**0.25 (1.29)**	**0.25 (1.29)**
	(Indian)	-0.12 (0.89)	-0.12 (0.89)
	(Pakistani)	**-0.68 (0.51)**	**-0.68 (0.51)**
	(Bangladeshi)	**-0.39 (0.68)**	**-0.39 (0.68)**
	(black Caribbean)	-0.21 (0.81)	-0.21 (0.81)
	(black African)	**-0.58 (0.56)**	**-0.58 (0.56)**
	(mixed black Carib & white)	-0.11 (0.90)	-0.10 (0.90)
	(Chinese)	**0.76 (2.14)**	**0.76 (2.14)**
	(Other)	**-0.13 (0.88)**	**-0.13 (0.88)**
FMSP Engagement Variables			
	LOPD	0.08 (1.08)	0.08 (1.08)
	TAM	0.02 (1.02)	0.02 (1.02)
	TFM	**-0.30 (0.74)**	**-0.30 (0.74)**
	Tuition	**-0.43 (0.65)**	**-0.43 (0.65)**
Constant		-2.79 (0.06)	-2.84 (0.06)
vpc:			
	Level 2 / KS5 institution (%)	8%	8%
	Level 1 / Student (%)	92%	92%
	McFaddens R-square	11.4%	11.4%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking an A level in maths between 2012 and 14.

Table III.7: Stages of the main effects A level Attainment models. (Maths A level)

Attainment in A level Maths	1. Null Model	2. KS4 Attain	3. KS5 Location	4. Pupil Backgrd	5. FMSP Engagement	6. Full Main Effects
Model Stages:		β (Cohen's d)	β (Cohen's d)	β (Cohen's d)	β (Cohen's d)	β (Cohen's d)
KS4 Attainment						
KS4 Maths Grade A*	-	**18.73 (+0.61)**	-	-	-	**18.19 (+0.59)**
Overall (KS4 pts per exam)	-	**9.8 (+0.32)**	-	-	-	**10.43 (+0.34)**
Type of KS5 Inst						
(6th Form College)	-	-	**-3.55 (-0.11)**	-	-	**-1.97 (-0.06)**
(FE College)	-	-	**-12.64 (-0.41)**	-	-	**-5.30 (-0.17)**
(Independent)	-	-	**13.34 (+0.43)**	-	-	**3.30 (+0.11)**
(Other)	-	-	**-10.85 (-0.35)**	-	-	**-5.45 (-0.18)**
Pupil Background						
Gender (Female)	-	-	-	**-0.64 (-0.02)**	-	**-6.46 (-0.21)**
Poverty (FSM)	-	-	-	**-4.96 (-0.16)**	-	**-1.05 (-0.03)**
Ethnicity (white other)	-	-	-	-0.61 (-0.02)	-	**1.57 (+0.05)**
(Indian)	-	-	-	0.70 (+0.02)	-	**2.95 (+0.10)**
(Pakistani)	-	-	-	**-4.13 (-0.13)**	-	0.23 (+0.01)
(Bangladeshi)	-	-	-	**-2.73 (-0.09)**	-	-1.01 (-0.03)
(black Caribbean)	-	-	-	**-9.24 (-0.30)**	-	**-3.03 (-0.10)**
(black African)	-	-	-	**-5.66 (-0.18)**	-	0.52 (+0.02)
(mixed blk Carib & white)	-	-	-	-4.01 (-0.13)	-	-1.40 (-0.05)
(Chinese)	-	-	-	5.23 (+0.17)	-	**6.36 (+0.21)**
(Other)	-	-	-	**0.91 (+0.03)**	-	0.32 (+0.01)
FMSP Engagement						
LOPD	-	-	-		**4.29 (+0.14)**	1.07 (+0.03)
TAM	-	-	-	-	-0.06 (0.00)	1.34 (+0.04)
TFM	-	-	-	-	3.54 (+0.11)	0.08 (+0.00)
Tuition	-	-	-	-	**-6.20 (-0.20)**	-1.22 (-0.04)
Constant	96.13	77.84	94.77	96.78	95.93	79.45
variance partition coef:						
Level 2 (%)	17%	6%	13%	16%	17%	5%
Level 1 (%)	83%	94%	87%	84%	83%	95%
Explanatory Power:						
% level 2 (institution)	-	76.6%	28.8%	8.0%	2.0%	81.1%
% level 1	-	28.6%	-0.1%	0.2%	0.0%	29.7%
Overall	-	36.8%	4.8%	1.5%	0.3%	38.5%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking A level maths between 2012 and 14.

Table III.8: Including the gender*KS4 attainment interaction (Maths A level)

Attainment in A level Maths Model Stages:		Full Main Efects β (Cohen's d)	Including gender*KS4 interaction terms β (Cohen's d)
KS4 Attainment			
	KS4 Maths Grade A*	**18.19 (+0.59)**	**18.46 (+0.60)**
	(Females*Grade A*) Interaction	-	**-0.67 (-0.02)**
	Overall KS4 points per exam	**10.43 (+0.34)**	**10.31 (+0.33)**
	*(Females*Overall KS4 points per exam) Interaction*	-	+0.29 (+0.01)
Type of KS5 institution			
	(6th Form College)	**-1.97 (-0.06)**	**-1.97 (-0.06)**
	(FE College)	**-5.30 (-0.17)**	**-5.30 (-0.17)**
	(Independent / Fee Paying)	**3.30 (+0.11)**	**3.32 (+0.11)**
	(Other)	**-5.45 (-0.18)**	**-5.44 (-0.18)**
Pupil Background			
Gender	(Female)	**-6.46 (-0.21)**	**-6.48 (-0.21)**
Poverty	(FSM)	**-1.05 (-0.03)**	**-1.06 (-0.03)**
Ethnicity	(white other)	**1.57 (+0.05)**	**1.57 (+0.05)**
	(Indian)	**2.95 (+0.10)**	**2.95 (+0.10)**
	(Pakistani)	0.23 (+0.01)	0.23 (+0.01)
	(Bangladeshi)	-1.01 (-0.03)	-1.02 (-0.03)
	(black Caribbean)	**-3.03 (-0.10)**	**-3.03 (-0.10)**
	(black African)	0.52 (+0.02)	0.52 (+0.02)
	(mixed black Carib & white)	-1.40 (-0.05)	-1.40 (-0.05)
	(Chinese)	**6.36 (+0.21)**	**6.36 (+0.21)**
	(Other)	0.32 (+0.01)	**0.32 (+0.01)**
FMSP Engagement Variables			
	LOPD	1.07 (+0.03)	1.07 (+0.03)
	TAM	1.34 (+0.04)	1.34 (+0.04)
	TFM	0.08 (+0.00)	0.08 (+0.00)
	Tuition	-1.22 (-0.04)	-1.23 (-0.04)
Constant		79.45	79.44
variance partition coef:	Level 2 (%)	5%	5%
	Level 1 (%)	95%	95%
Explanatory Power:			
	% level 2 (institution)	81.1%	81.0%
	% level 1	29.7%	29.7%
	Overall	38.5%	38.4%

Source: Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking A level maths between 2012 and 14.

Table III.9: Stages of the main effects ATTAINMENT models (Further Maths A level)

Attainment in A level Further Maths Model Stages:	1. Null Model	2. KS4 Attain β (Cohen's d)	2. KS5 Location β (Cohen's d)	3. Pupil Backgrd β (Cohen's d)	4. FMSP Engagement β (Cohen's d)	5. Full Main Efects β (Cohen's d)
KS4 Attainment						
KS4 Maths Grade A*	-	8.82 (+0.33)	-	-	-	**8.69 (+0.32)**
Overall (KS4 points per exam)	-	**11.9 (+0.44)**	-	-	-	**11.90 (+0.44)**
Type of KS5 Inst						
(6th Form College)	-	-	**-3.52 (-0.13)**	-	-	-0.54 (-0.02)
(FE College)	-	-	**-10.43 (-0.39)**	-	-	**-3.36 (-0.12)**
(Independent)	-	-	**10.11 (+0.37)**	-	-	**3.93 (+0.15)**
(Other)	-	-	**-4.77 (-0.18)**	-	-	-0.11 (0.00)
Pupil Background						
Gender (Female)	-	-	-	1.04 (+0.04)	-	**-4.07 (-0.15)**
Poverty (FSM)	-	-	-	**-7.56 (-0.28)**	-	**-2.52 (-0.09)**
Ethnicity (white other)	-	-	-	-1.72 (-0.06)	-	0.50 (+0.02)
(Indian)	-	-	-	-0.71 (-0.03)	-	0.40 (+0.01)
(Pakistani)	-	-	-	**-4.71 (-0.17)**	-	-0.12 (0.00)
(Bangladeshi)	-	-	-	**-10.64 (-0.39)**	-	**-6.57 (-0.24)**
(black Caribbean)	-	-	-	**-9.08 (-0.34)**	-	-2.47 (-0.09)
(black African)	-	-	-	**-5.73 (-0.21)**	-	-0.61 (-0.02)
(mixed blk Carib & white)	-	-	-	-5.61 (-0.21)	-	-4.33 (-0.16)
(Chinese)	-	-	-	0.24 (+0.01)	-	**3.23 (+0.12)**
(Other)	-	-	-	**3.21 (+0.12)**	-	0.66 (+0.02)
FMSP Engagement						
LOPD	-	-	-		0.86 (+0.03)	-0.62 (-0.02)
TAM	-	-	-	-	-3.61 (-0.13)	-0.66 (-0.02)
TFM	-	-	-	-	6.15 (+0.23)	2.80 (+0.10)
Tuition	-	-	-	-	**-7.38 (-0.27)**	-2.89 (-0.11)
Constant	108.34	72.99	107.00	107.79	108.44	73.56
variance partition coef:						
Level 2 (%)	20%	9%	16%	18%	20%	8%
Level 1 (%)	80%	91%	84%	82%	80%	92%
Explanatory Power:						
% level 2 (institution)	-	68.0%	21.7%	13.2%	2.5%	72.5%
% level 1	-	18.2%	-0.4%	-0.2%	-0.1%	18.6%
Overall	-	28.2%	4.0%	2.5%	0.4%	29.4%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking A level further maths between 2012 and 2014.

Table III.10: Including the gender*KS4 attainment interaction (Further Maths A level).

Attainment in A level Further Maths		Full Main Efects	Including gender*KS4 interaction terms
	Model Stages:	β (Cohen's d)	β (Cohen's d)
KS4 Attainment			
	KS4 Maths Grade A*	**8.69 (+0.32)**	**8.61 (+0.32)**
	(Females*Grade A*) **Interaction**	-	+0.37 (+0.01)
	Overall KS4 points per exam	**11.90 (+0.44)**	**11.92 (+0.44)**
	*(Females*Overall KS4 points per exam)* ***Interaction***	-	-0.12 (0.00)
Type of KS5 institution			
	(6th Form College)	-0.54 (-0.02)	-0.55 (-0.02)
	(FE College)	**-3.36 (-0.12)**	**-3.37 (-0.12)**
	(Independent / Fee Paying)	**3.93 (+0.15)**	**3.93 (+0.15)**
	(Other)	-0.11 (0.00)	-0.12 (0.00)
Pupil Background			
Gender	(Female)	**-4.07 (-0.15)**	**-4.08 (-0.15)**
Poverty	(FSM)	**-2.52 (-0.09)**	-2.52 (-0.09)
Ethnicity	(white other)	0.50 (+0.02)	0.50 (+0.02)
	(Indian)	0.40 (+0.01)	0.40 (+0.01)
	(Pakistani)	-0.12 (0.00)	-0.12 (0.00)
	(Bangladeshi)	**-6.57 (-0.24)**	**-6.57 (-0.24)**
	(black Caribbean)	-2.47 (-0.09)	-2.49 (-0.09)
	(black African)	-0.61 (-0.02)	-0.61 (-0.02)
	(mixed black Carib & white)	-4.33 (-0.16)	-4.32 (-0.16)
	(Chinese)	**3.23 (+0.12)**	**3.22 (+0.12)**
	(Other)	0.66 (+0.02)	0.66 (+0.02)
FMSP Engagement Variables			
	LOPD	-0.62 (-0.02)	-0.62 (-0.02)
	TAM	-0.66 (-0.02)	-0.66 (-0.02)
	TFM	2.80 (+0.10)	2.80 (+0.10)
	Tuition	-2.89 (-0.11)	-2.88 (-0.11)
Constant		73.56	73.56
Variance Partition Coef:	Level 2 ICC(%)	8%	8%
	Level 1 (%)	92%	92%
Explanatory Power:			
	% level 2 (institution)	72.5%	72.5%
	% level 1	18.6%	18.6%
	Overall	29.4%	29.4%

Source:Data from the 2011 pupil-level cohort of all young people in England who took KS4 assessments at the end of Y11 in summer 2011 and are recorded as taking A level further maths between 2012 and 2014.

References:

Hutcheson, G. & Sofroniou, N. (1999) The Multivariate Social Scientist. Sage.

Menard, S. (1995) Applied Logistic Regression Analysis. Quantitative Applications in the Social Sciences series - number 106. Sage.

Steele, F. (2009) Module 7: Multilevel Models for Binary Responses Technical Appendix: A Primer on Estimation of Multilevel Binary Response Models

Annexe 3: Barriers and Enablers to participation in Mathematics and Further Mathematics

Barriers and enablers compared

From the thematic analysis of teacher interviews and student focus groups (with those doing both A level Mathematics and/or Further Mathematics), a range of barriers to Further Mathematics participation were also identified at the micro, meso and macro levels. In a number of cases these barriers can be seen as the reverse or absence of the enabling factors - and from the teacher interviews are often associated with schools with lower security status. This is best summarised in the tables below (Table A3-1).

Table A3-1 Thematic analysis of teacher interviews – enablers and barriers

Key

Factors associated with most centres, regardless of security	Factors more associated with centres with lower Further Mathematics security	Factors more associated with centres with higher Further Mathematics security

	Enablers	Barriers
Micro (individual) level factors	Strong teacher skills / confidence / interest in FM	Low or variable teacher skills / confidence / interest in FM
	Teachers Further Mathematics skills increased through using FMSP materials /Integral/FMSP CPD	
	Keen/enthusiastic Further Mathematics teacher - prioritises and organises enrichment	
	Able, self-motivated students with 'strong work ethic'	Less able/lower attaining students or cohorts Students want to focus on fewer subjects at A2
	Parental support/high regard for M/FM	
	Further Mathematics 'star' or role model(s) - used to promote FM, especially females or students from non-traditional backgrounds	
Meso (school	Drive to maintain numbers/	Budget/funding issues

level) factors	promote Mathematics/Further Mathematics at KS3&4	
		Low or falling numbers of students in 6th form, competition from local centres/centres
	Stable or growing numbers of Further Mathematics entries	Low or falling numbers studying A level Mathematics or FM
	Tougher selection criteria for Further Mathematics (e.g A*/A GCSE Mathematics + 'work ethic') to reduce drop-out lower grades	Lower/ variable prior attainment cohorts at GCSE and those choosing M/Further Mathematics
	SLT supportive of FM, even when numbers fall	SLT see all A levels as 'equally difficult'. Falling numbers leads to SLT deprioritising staff/lesson time for FM
	Further Mathematics offered on the timetable by experienced/qualified staff	Centres perceive that students struggle with Further Mathematics content so it is not offered/students sent to other partner centres for FM
		Teacher timetable issues/staffing priority at KS4 Ad hoc support for Further Mathematics /emphasis on 'independent learning' (to compensate for limited time/support)
		Timetable clashes for students / restricted options
		Larger class sizes (due to staffing shortages)
		Reduced range of modules offered (due to teacher skills gaps) Fewer modules offered in-house = fewer opportunities for teacher skill development
		Difficult jump from GCSE to A level/Further Mathematics – high drop out at AS
	Centre offers additional / accelerated/extended Mathematics qualifications (e.g. GCSE FM) – A level Further Mathematics seen as a 'natural progression'	Recruitment and retention of qualified teachers at KS5/Further Mathematics (also macro)
	Centre attracts and retains skilled Further Mathematics teachers	Tuition is off-site/requires additional travel time
Macro	Face to face tuition is	

(national/system) level factors	accessible/offered on-site	Travel/distance to enrichment activities is difficult /prohibitive
	Local university/enrichment is accessible	
	Further Mathematics seen as elite extra subject only offered to the highest achievers	Change of qualifications/curricula – lack of information, two year A level likely to be risky/deter some students

The associations and links between barriers and enablers for students at the micro, meso and macro levels are summarised in Table A3-2 below:

Table A3-2 Thematic analysis of student focus groups* - enablers and barriers

	Enablers	Barriers
Micro (individual) level factors	Enjoyment Being good at maths Positive learning experience (internalised)	Poor or variable learning experience (internalised)
	Confidence Determination/focus/self-motivation	Low confidence
	Clear career/study goals Exam success, including additional qualifications	Low grades (higher numbers of resits/drop out after AS) Jump from GCSE to A level
Meso (family/school) level factors	Family capital - parental interest/skills/encouragement Sibling competitiveness	Lack of parental interest
	Teacher encouragement, influence, inspiration	Poor/variable teacher experience in maths (externalised) Limited teacher expertise in FM/skill gaps
	Access to/teacher support for enrichment activities Positive enrichment experiences Good links with local university	Limited teacher capacity/commitment/support for enrichment activities Weak links/engagement with local university/ enrichment No/negative/off putting enrichment experiences

	Small class sizes - more 1:1 Further Mathematics support Sufficient numbers of Further Mathematics students to offer lessons on-site Additional support for Further Mathematics (outside lesson time) Access to local/convenient/quality off-site tuition or to on-line tuition	Low numbers - means timetable/options restrictions on A level choices & FM Limited/disruptive off-site tuition Limited teacher support for Further Mathematics (outside lessons/tuition)
	Positive Further Mathematics role models (in years above)	No/negative Further Mathematics role models
	Helpful careers advice	
Macro (national/system) level factors	Maths viewed as high status subject Positive jobs market for maths skills	Distance/travel difficulties to access off-site tuition

*NB: Centres with low levels of Further Mathematics security were purposively selected for the conduct of student focus groups, so unlike the teacher interviews it is not possible to highlight differences in student responses by security status of their centre. Students studying both A level Mathematics and Further Mathematics were involved in focus groups so enablers and barriers have been thematically analysed across all students combined.

An example of a Further Mathematics culture

Below a vignette from one focus group is included to exemplify the complex interaction of different factors.

Focus group vignette - Gender, enrichment opportunities, confidence and decision making

One of the focus groups was conducted in a mixed 6th form located on the site of an 11-16 all-girls centre. The Further Mathematics focus group included one female in Y12 who had been a student at the centre since Y7, and four Y13 males who had joined the 6th form after completing GCSEs at the local boys' centre.

The males in the group repeatedly referred to a very encouraging, inspirational mathematics teacher who they had for several years at their boys' centre. He pushed them, made mathematics interesting, frequently identified interesting aspects of topics that they would cover in Further Mathematics A level which promoted it to them. All four males were close friends and competitors, completing their GCSE mathematics in Y10. In Y11 they completed an additional mathematics enhancement qualification through a local university. They described this as an important bridge to A level.

Their teacher also organised a lot of enrichment - individual and team mathematics challenges from Y7 - challenging, problem solving and providing practical activities that they hadn't experienced during their GCSE course. They had attended several taster days and events at the local university and had Mathematics undergraduates visiting their secondary school to lead talks and help with revision sessions. They gained much from these experiences: the mathematics challenges enabled them to assess their abilities and skills in a wider context of able students from other centres, rather than their limited perspective of being top of their class. As a friendship group, they were enthusiastic, confident and competitive and wanted to beat the private and grammar centre students they competed against:

> "The competitions were fun. When you're competing against other people it encourages you to try harder"

They commented on the pride they felt in being 'hand-picked' to represent their centre and then defying their teachers' expectations and improving the centres' standing in the competition. They increased the range and depth of their skills beyond what was required by their curriculum. The taster days on undergraduate mathematics confirmed for one student that continuing to degree level was not for him, but he had identified another maths-related career and HE routes that made his Further Mathematics A level highly valuable. Overall, this group of males felt that this dedicated 'investment' and enrichment over the years fuelled their enjoyment in the subject - and was something they had not experienced in their other subjects. This encouraged and shaped their decision making - and directly influenced them to continue Mathematics to A level and FM:

> "Having invested that much in one subject, it sort of says 'I'm going to carry on with this as I've put so much of my time into it, invested more in it'. (Without enrichment) 'I don't think I would have chosen FM, I didn't really know much about Further Mathematics and what it was about, but doing more maths and different maths meant I could do Further Mathematics - otherwise I probably would have done another science subject".

These males joined the 6^{th} form located on the site of an 11-16 girls only school. The males' confidence and dominance in the focus group with the one other female who was also doing Further Mathematics was noticeable. Although she had gained a good grade in Further Mathematics GCSE and was more quietly confident in the way she described her abilities, she had fewer opportunities to experience the level of enrichment during KS4. She had participated in the individual maths challenges lower down the centre but had only experienced the senior team challenge with the males while in the 6^{th} form. She commented:

> "Being at an all-girls schools, we're not that competitive so no one really cared what we got (at the individual challenges)"

Now in the 6^{th} form and taking part in team competitions:

> "…makes it more enjoyable and wanting to do well instead of just waiting for your exams"

She felt that if she had the opportunity to take part in external competitions lower down the centre it would have made maths more enjoyable. However, she still identified that her decision making was influenced by her enjoyment, having always done well in Mathematics and having the encouragement of her teachers.