Modeling of unsaturated granular materials in flexible pavements

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Abstract. The unsaturated granular material (UGM) is found to exhibit the moisture-sensitive and stress-dependent nonlinear cross-anisotropic behaviour in flexible pavements. This paper aims at developing a finite element (FE) model for pavement structure, which takes into account this behaviour of UGM. First, the Lytton model is employed to characterize the moisture-sensitive and stress-dependent behaviour of UGM, which incorporated a matric suction term to the existing stress-dependent constitutive model. The Lytton model is validated by the laboratory resilient modulus tests on the selected UGMs at different moisture contents. Second, the nonlinear cross-anisotropic constitutive equation of UGM is derived from the generalized Hooke's Law. The coefficients of the constitutive model are determined by the rapid triaxial test. Third, a User-Defined Material (UMAT) subroutine is developed to characterize this constitutive behaviour in the FE software ABAQUS. The UMAT subroutine adopts the secant stiffness approach with multiple damping factors. The UMAT subroutine is then implemented in the FE model of flexible pavement structures. The FE simulation results indicate the nonlinear cross-anisotropic model predicts greater pavement responses than the isotropic model. When the UGM is suction sensitive, it is found that the moisture content of UGM significantly affects the moduli distribution of base layer and the critical strains (i.e., tensile strain at the bottom of asphalt concrete, and compressive strains in base and subgrade layers) of pavement structures.

1 Introduction

Unsaturated granular materials (UGMs) are often used as base layers for flexible pavements. An unbound granular base provides the foundational support to the pavement structure, and dissipates the stresses induced by traffic loading to the underlying subgrade. Understanding the constitutive behaviour of UGM is crucial to the accurate performance prediction of the pavement structures. In the conventional pavement design, the granular base is assumed to be linear elastic. By using this assumption, the linear isotropic model predicted an unexpected tensile stress at the bottom of the base layer, which conflicts with the fact that the UGM cannot transfer the tensile stress among the aggregate particles. A number of recent studies have revealed that the UGM exhibits the nonlinear cross-anisotropic behaviour, which means the resilient modulus of the granular base is stress-dependent, and its horizontal modulus is smaller than the vertical modulus [1-2]. It was found that modeling the UGM as a cross-anisotropic material can significantly reduce or eliminate the tensile stresses in base layer [3-4]. Field studies further concluded that the nonlinear cross-anisotropic model provided better agreement with the field measurements [5-6]. Hence, modeling UGM as nonlinear cross-anisotropic material should be taken into account for the pavement design and analysis. However, a review of these existing studies showed that the moisture condition of the granular base was often

assumed at the optimum, or the same condition that the granular material was tested in the laboratory. This assumption ignores the fact that the moisture condition of the UGM is significantly affected by the weather, the groundwater table level, the drainage condition, and the surface properties in the field.

The moisture content affects the constitutive behaviour of UGM, which further influences the performance of pavement structures in the field. It is reported that the resilient modulus of UGM is moisture-sensitive, i.e., the modulus decreases with the growing saturation level [7]. Salour and Erlingsson [8] investigated the pavement response to variations of moisture content of base layers using falling weight Deflectometer tests. They concluded that increasing the water content of UGM significantly reduces the back-calculated modulus of base layer. These studies therefore suggest that the moisture-sensitive behaviour of UGM should be taken into account for modeling the pavement structures.

To address the aforementioned problems, this study aims at proposing a new constitutive model for UGM considering both nonlinear cross-anisotropic behavior and moisture-sensitive characteristics, and incorporating the proposed constitutive model into the finite element model of the base layer to quantify the influence of moisture content on the pavement performance. More specifically, the saturation factor and the matric suction of the UGM will be applied to the proposed constitutive

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model to reflect the moisture dependency. A new user-defined material subroutine (UMAT) will be developed to characterize the moisture-sensitive and stress-dependent nonlinear cross-anisotropic behavior of base material in the software ABAQUS.

The paper is organized as follows. The next section presents the proposed constitutive model to capture the moisture-sensitive characteristic of UGM. The following section develops a UMAT to define the UGM in the finite element model. The developed UMAT is also verified by the designed numerical experiment. After that, the finite element models for flexible pavement structures are setup, which consider asphalt concrete as a viscoelastic material, granular base as a moisture-sensitive and stress-dependent nonlinear cross-anisotropic material, and subgrade as an elastic material. The importance of nonlinear cross-anisotropic model for predicting the pavement response, the effect of moisture content of unbound base material on the pavement response, are investigated in this section. The final section summarizes the findings of this paper.

2 Constitutive Model for Unsaturated Granular Materials

The generalized Hooke's law is used to define the cross-anisotropic behaviour of UGM for an axisymmetric problem, which is shown in Equation 1 [3].

$$
\begin{bmatrix}
\dfrac{1}{E_x} & -\dfrac{v_{xy}}{E_x} & -\dfrac{v_{xx}}{E_x} & 0 \\[2mm]
-\dfrac{v_{xy}}{E_x} & \dfrac{1}{E_y} & -\dfrac{v_{xy}}{E_x} & 0 \\[2mm]
-\dfrac{v_{xx}}{E_x} & -\dfrac{v_{xy}}{E_x} & \dfrac{1}{E_x} & 0 \\[2mm]
0 & 0 & 0 & \dfrac{1}{G_{xy}}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\ \sigma_y \\ \sigma_x \\ \tau_{xy}
\end{bmatrix}
=
\begin{bmatrix}
\varepsilon_x \\ \varepsilon_y \\ \varepsilon_x \\ \gamma_{xy}
\end{bmatrix}
\quad (1)
$$

where E_x is the horizontal modulus; E_y is the vertical modulus; G_{xy} is the shear modulus; v_{xy} is the Poisson's ratio to characterize the effect of vertical strain on horizontal strain; v_{xx} is the Poisson's ratio to characterize the effect of horizontal strain on horizontal strain.

In ABAQUS, this constitutive model needs to be rewritten as a strain-stress relationship. Converted from Equation 1, the strain-stress relationship for the cross-anisotropic material can be expressed as,

$$
\begin{bmatrix}
\sigma_x \\ \sigma_y \\ \sigma_x \\ \tau_{xy}
\end{bmatrix}
=
\dfrac{E_y}{\alpha\beta}
\begin{bmatrix}
n\left(1-nv_{yx}^2\right) & nv_{yx}\alpha & n\left(v_{xx}+nv_{yx}^2\right) & 0 \\[1mm]
nv_{yx}\alpha & 1-v_{xx}^2 & nv_{yx}\alpha & 0 \\[1mm]
n\left(v_{xx}+nv_{yx}^2\right) & nv_{yx}\alpha & n\left(1-nv_{yx}^2\right) & 0 \\[1mm]
0 & 0 & 0 & m\alpha\beta
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\ \varepsilon_y \\ \varepsilon_x \\ \frac{1}{2}\gamma_{xy}
\end{bmatrix}
\quad (2)
$$

where $n = \dfrac{E_x}{E_y}$; $m = \dfrac{G_{xy}}{E_y}$; $\alpha = 1 + v_{xx}$; $\beta = 1 - v_{xx} - 2nv_{yx}^2$.

This strain-stress relationship is used to compute the incremental stress for a given incremental strain in ABAQUS. In Equation 2, the vertical modulus E_y is dependent on both the stress state and the moisture content. In order to characterize this behavior, the Lytton model is used as shown in Equation 3, which incorporates a matric suction term into the generalized resilient modulus model [9-10].

$$
E_y = k_1 P_a \left(\frac{I_1 - 3\theta f h_m}{P_a} \right)^{k_2} \left(\frac{\tau_{oct}}{P_a} \right)^{k_3}
\quad (3)
$$

where I_1 = the first invariant of the stress tensor; P_a = the atmospheric pressure; θ = the volumetric water content; h_m = the matric suction in the aggregate matrix; f = the saturation factor, $1 \le f \le \dfrac{1}{\theta}$; τ_{oct} = the octahedral shear stress; and k_1, k_2 and k_3 are regression coefficients.

In order to validate the Lytton model, the repeated load triaxial tests are conducted on the 3 selected materials at 3 different moisture contents (i.e., optimum moisture condition, and 1.5 percent below and above the optimum moisture condition). Figure 1 shows the comparison between the predicted moduli using Equation 3 and the measured moduli from the triaxial tests. The model prediction provides a good agreement with the test measurements. This indicates that the constitutive model proposed in Equation 3 is able to reflect the moisture-sensitive and stress-dependent behavior of UGM.
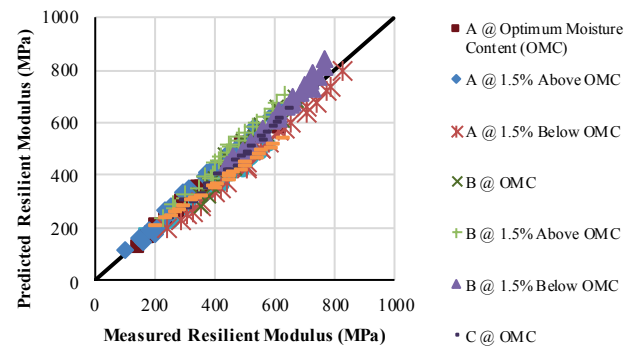

Figure 1. Comparison of predicted and measured resilient moduli for selected materials

3 Development of a User-Defined Material Subroutine for Unsaturated Granular Material

Several studies have been carried out to program different UMAT subroutines to define the stress-dependent behavior of UGM. One UMAT subroutine was developed for the UGM based on the tangent stiffness method [11]. The nonlinear stress-dependent resilient modulus model was formulated as a function of the strain state. Kim et al. [12] adopted a direct secant stiffness approach to

determine the nonlinear resilient modulus solution in each iteration. This nonlinear solution technique is less complicated than the tangent stiffness approach and Newton-Raphson approach, but it is accurate enough to provide convergence of the iterations. Based on the secant stiffness approach, Wang and Al-Qadi [6] programmed a UMAT subroutine for the UGM by incorporating an anisotropic constitutive model. Using this nonlinear cross-anisotropic UMAT subroutine, they successfully analyzed the response of a 3-Dimensional pavement model under the moving vehicular loading. In this study, a similar UMAT subroutine is also programmed to define the moisture-sensitive and stress-dependent behavior by using the secant stiffness technique with the damping factor λ. The trial vertical modulus is computed by Equation 4 in each iteration.

$$E_y^i = (1-\lambda) E_y^{i-1} + \lambda E_{ycomputed}^i \qquad (4)$$

where E_y^i is the vertical modulus output from the ith iteration; E_y^{i-1} is the vertical modulus output from the $(i-1)^{th}$ iteration; λ is the damping factor (e.g., initial λ is 0.95); $E_{ycomputed}^i$ is the vertical modulus computed from Equation 3 at the ith iteration [12]. The convergence criteria used in this study are shown in Equations 5 and 6.

$$Error_i = \frac{\left| E_y^i - E_y^{i-1} \right|}{\left| E_y^i \right|} \le 2\% \qquad (5)$$

$$Error_c = \frac{\sum_{i=1}^{n} \left(E_y^i - E_y^{i-1} \right)^2}{\sum_{i=1}^{n} \left(E_y^i \right)^2} \le 0.5\% \qquad (6)$$

where $Error_i$ is the individual error for each node; $Error_c$ is the cumulative error for the entire model; n is the number of nodes in the model. The moisture-sensitive and stress-dependent cross-anisotropic constitutive models, as shown in Equations 2 and 3, are coded into the UMAT. The Mohr-Coulomb failure theory is also applied to adjust the initially computed horizontal stresses so that the yield stress of the material will not be exceeded. This method originally proposed by ILLI-PAVE and KENLAYER is incorporated in the development of the UMAT for nonlinear cross-anisotropic unbound aggregates in this study. Figure 2 is the flowchart of the developed UMAT subroutine.

4 Finite Element Modeling of Flexible Pavement Structures

As shown in Figure 3, the axisymmetric pavement structures analysed in this section consist of a 15-cm hot mix asphalt (HMA) layer, a 25-cm granular base and 1.4-meter subgrade. The pavement structures are subjected to a half-sine impact load with a loading amplitude of 40.03 kN and a pulse duration of 0.1 second. The load is assumed as a uniform pressure over a 0.15m radius of circular area at the left edge of the axisymmetric pavement structures. The meshed finite element model is constructed according to the pavement structures in Figure 3. Fine mesh is used in the loading area. The 8-node biquadratic axisymmetric elements with reduced integration are used in the whole finite domain. The interfaces between the HMA layer, granular base and subgrade are assumed to be fully bonded.
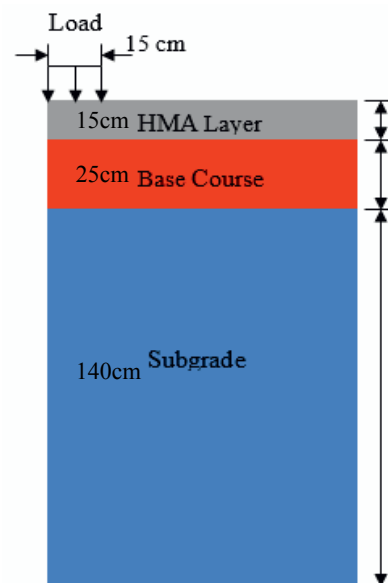

Figure 2. Flowchart of developed UMAT subroutine

Figure 3. Schematic plot of pavement structures

HMA is considered as a viscoelastic material in the numerical analysis. In ABAQUS, the Prony-Series models are used to characterize the time-dependent behaviour of HMA, which are shown in Equations 7 and 8.

$$G(t) = G_0\left(1 - \sum_{i=1}^{n} G_i\left(1 - e^{-t/\tau_i}\right)\right) \quad (7)$$

$$K(t) = K_0\left(1 - \sum_{i=1}^{n} K_i\left(1 - e^{-t/\tau_i}\right)\right) \quad (8)$$

where $G(t)$ and $K(t)$ are the relaxation shear modulus and bulk modulus, respectively; G_0 and K_0 are the corresponding instantaneous shear modulus and bulk modulus ; G_i, K_i and τ_i are the input coefficients. The coefficients of the Prony-Series model are determined by fitting the dynamic modulus test results. Table 1a lists the coefficients of the Prony-Series model for the HMA. A constant Poisson's ratio is assumed during the analysis. The nonlinear cross-anisotropic properties of UGM are presented in Table 1b. Figure 4 is the soil-water characteristic curve to characterize the moisture-sensitivity of UGM. As shown in Table 1c, subgrade is simplified as a linear-elastic material with constant Poisson's ratio.

Table 1. Determined Model Inputs for Paving Materials

a. Determined Prony-Series Model Coefficients for HMA

Series Number	Prony-Series Coefficients		
i	Gi	Ki	τi
1	0.362	0.362	4.09E-06
2	0.363	0.363	2.56E-04
3	0.177	0.177	7.71E-03
4	0.074	0.074	2.10E-01
5	0.017	0.017	3.88E+00
6	0.006	0.006	6.53E+01
Note: Instantaneous modulus=18,130MPa, Poisson's ratio=0.35			

b. Nonlinear Cross-Anisotropic Properties for UGM

Input Variables	k_1	k_2	k_3	n	m	v_{xy}	v_{xx}
Values	1281	0.81	-0.08	0.45	0.35	0.17	0.43

c. Linear-Elastic Material Properties for Subgrade

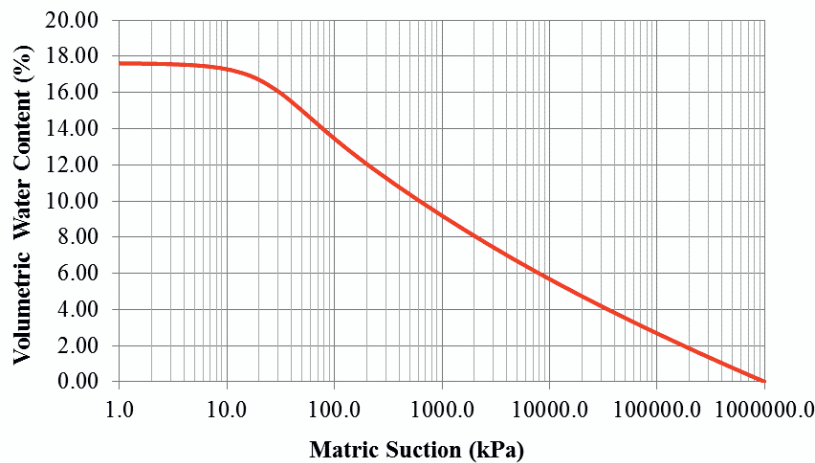| Input Variables	E (MPa)	v
Values	56	0.4

Figure 4. Soil-Water Characteristic Curve for UGM

5 Results and Discussion

5.1 Effect of Anisotropic Model on Pavement Responses

The comparison of pavement responses predicted by the anisotropic model and the isotropic model are shown in Figures 5a, 5b and 5c. As mentioned previously, the primary difference between the anisotropic model and the isotropic model is that the horizontal moduli of UGM are smaller than the vertical moduli in the anisotropic model, while the horizontal and vertical moduli of UGM are assumed to be equal in the isotropic model. It is seen from Figure 5a that the anisotropic model predicts approximately 10% higher tensile strain at the bottom of HMA layer than the isotropic model. This indicates that the pavement fatigue life estimated by the anisotropic model is shorter than the fatigue life predicted by the isotropic model. Figures 5b and 5c show that the anisotropic model also predicts 8% higher average compressive strain in base and 12% higher compressive strain at the top of subgrade when compared to the isotropic model. This demonstrates that using the anisotropic model for pavement design will predict a higher rutting depth. Hence, it is concluded that using anisotropic model to predict the pavement performance provides more conservative results for pavement ME design.
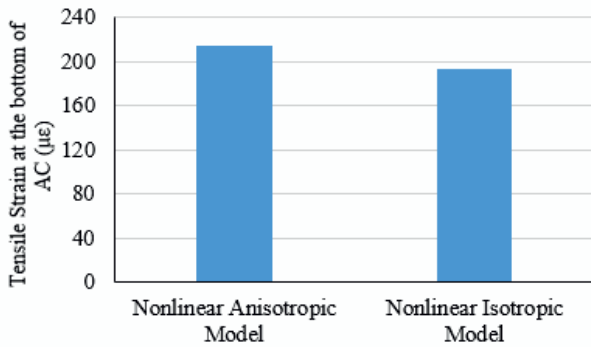
a. Tensile Strain at the Bottom of HMA Layer
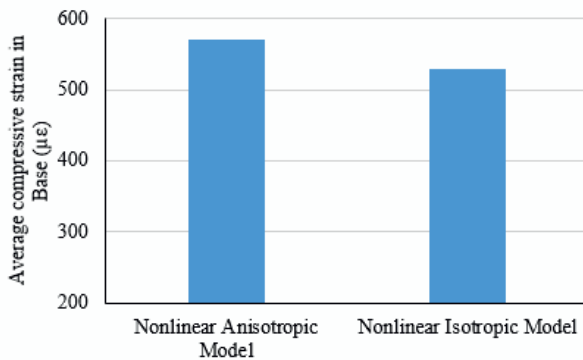
b. Average Compressive Strain in Base Layer
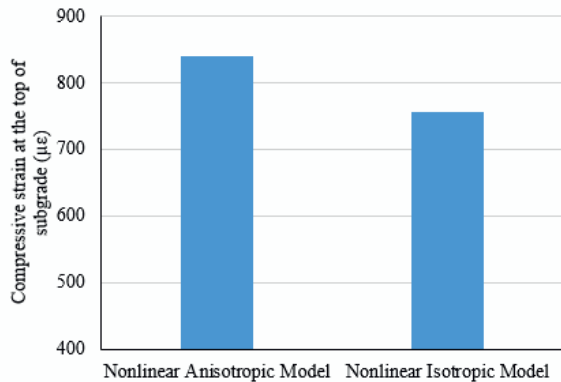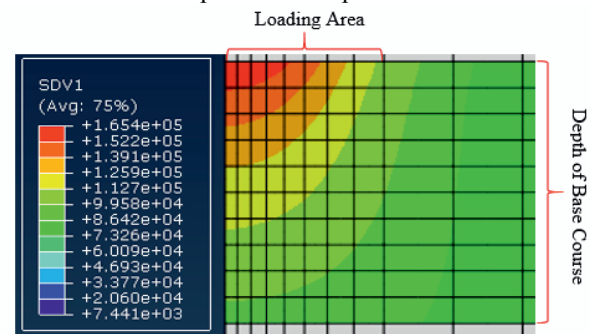

c. Compressive Strain at the Top of Subgrade
Figure 5. Effect of Anisotropic Model on Pavement Responses

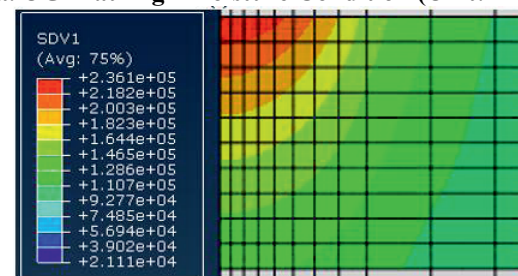### 5.2 Effect of Moisture Content of Unbound Base Layer on Pavement Responses

To investigate the effect of moisture content of UGM on pavement responses, three moisture conditions are simulated in the numerical model, which include the low moisture condition (i.e., the degree of saturation is 0.7), the optimum moisture condition (i.e., the degree of saturation is 0.85), and the saturated condition (i.e., the degree of saturation is 1.0). Figure 6 compares the vertical moduli (i.e., SDV1) distribution in base course at different moisture conditions. It is seen that the moduli of UGM in the vicinity of load area are significantly larger than those far away the load area. This is because the UGM is modelled as a stress-hardening material. It is also shown that the modulus of UGM decreases from the top to the bottom of base layer, which indicates the stress state varies in the base layer. One of objectives in this

study is to model the moisture-sensitive behaviour of UGM. The comparison among Figures 6a, 6b and 6c demonstrates that the model-predicted moduli of UGM is sensitive to the moisture condition. It is seen that the increasing of moisture content of unbound base significantly reduces the moduli of UGM. A further observation shows that the moduli of UGM at the low moisture condition are nearly twice as large as those of base material at the saturated condition.

This variation thereby results in the change of pavement responses, such as the surface deflection, the tensile strain at the bottom of HMA layer, and the compressive strains in base course and subgrade. It is seen from Figure 7 that the model-predicted surface deflections, tensile strain at the bottom of HMA layer and compressive strain in base course are significantly sensitive to the moisture condition in base course, while the model-predicted compressive strain at the top of subgrade is merely slightly affected by the moisture variation in base course. It is obvious that increasing the moisture content of base course results in larger surface deflections, higher tensile strain at the bottom of HMA layer, and higher compressive strains in base and subgrade. This indicates that the developed finite element model can properly reflect the influence of moisture content of UGM on pavement responses.
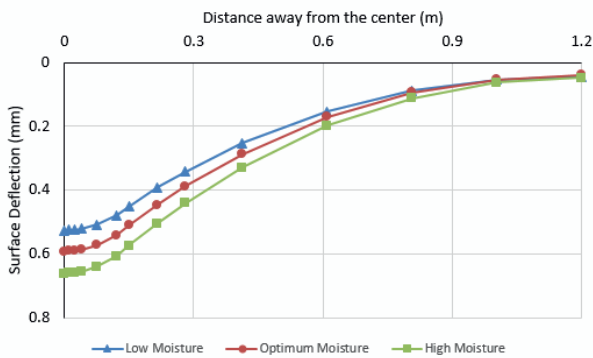

a. UGM at High Moisture Condition (Unit: kPa)
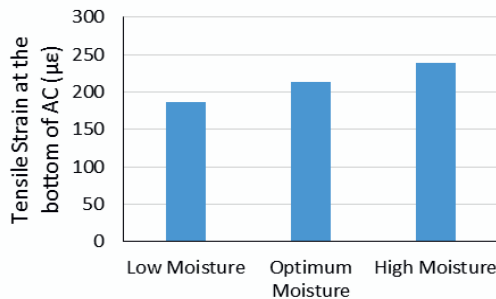

b. UGM at Optimum Moisture Condition (Unit: kPa)
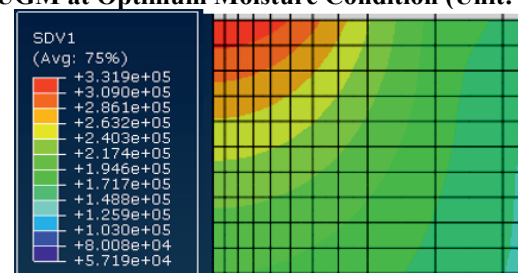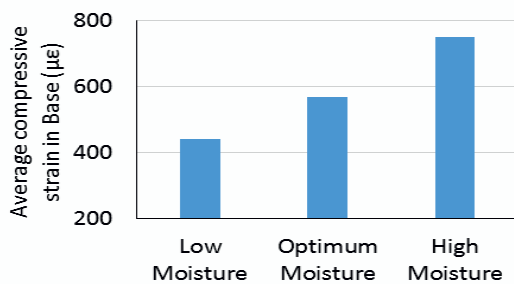

c. UGM at Low Moisture Condition (Unit: kPa)
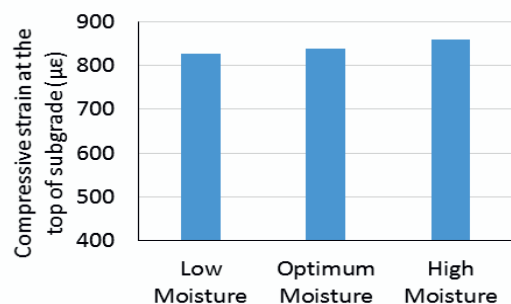**Figure 6. Vertical Moduli Distribution in Base Layer**

a. Surface Deflections of Flexible Pavement

b. Tensile Strain at the Bottom of HMA layer

c. Average Compressive Strain in Base Layer

d. Compressive Strain at the Top of Subgrade
Figure 7. Effect of Moisture Content of UGM on Pavement Responses

6 Conclusions

The Lytton model is used to characterize the stress-dependence and moisture-dependence of resilient modulus of UGM. The degree of saturation and the matric suction are incorporated to discriminate the effect of the moisture variations. The moisture dependence of the Lytton model is validated by comparing the model-predicted resilient moduli of UGM at different moisture contents to those measured from the laboratory tests. It is demonstrated that the matric suction of the UGM is a key

element to reflect the moisture dependence of the resilient modulus. The FE approach is then employed to model the stress-dependent and moisture-sensitive cross-anisotropic behaviour of UGM. The secant stiffness method with the multiple damping factors are efficient to program the UMAT subroutine. The UMAT subroutine is then implemented in the FE model of flexible pavement structures. The FE simulation results indicate the nonlinear cross-anisotropic model predicts greater pavement responses than the isotropic model. When the UGM is suction sensitive, it is found that the moisture content of UGM significantly affects the moduli distribution of base layer and the critical strains (i.e., tensile strain at the bottom of asphalt concrete, and compressive strains in base and subgrade layers) of pavement structures.

References

[1] A. Adu-Osei, D. Little, and R. Lytton. *Cross-anisotropic characterization of unbound granular materials*. Transp. Res. Rec., **1789**, 82-91 (2001)

[2] E. Tutumluer, and M. Thompson. *Anisotropic modeling of granular bases in flexible pavements*. Transp. Res. Rec., **1577**, 18-26 (1997)

[3] J. Oh, R. Lytton, and E. Fernando. *Modeling of pavement response using nonlinear cross-anisotropy approach*. J. Transp. Eng., **132**, 6, 458-486 (2006)

[4] I. Al-Qadi, H. Wang, and E. Tutumluer. *Dynamic analysis of thin asphalt pavements by using cross-anisotropic stress-dependent properties for granular layer*. Transp. Res. Rec., **2154**, 156-163 (2010)

[5] E. Tutumluer, D. Little, and S. Kim. *Validated model for predicting field performance of aggregate base courses*. Transp. Res. Rec., **1837**, 41-49 (2003)

[6] H. Wang, and I. Al-Qadi. *Importance of nonlinear anisotropic modeling of granular base for predicting maximum viscoelastic pavement responses under moving vehicular loading*. J. Eng. Mech., **139**, 1, 29-38 (2013)

[7] F. Lekarp., U. Isacsson, and A. Dawson. *State of art: Resilient response of unbound aggregates*. J. Transp. Eng., **126**, 1, 66-75 (2000)

[8] F. Salour, and S. Erlingsson. *Investigation of a pavement structural behaviour during spring thaw using falling weight deflectometer*. Road Mater. Pavement, **14**, 1, 141-158 (2013)

[9] R. Lytton. *Foundations and pavements on unsaturated soils*. Proceedings, First International Conference on Unsaturated Soils, ISSMFE, Paris, 1201-1220 (1995)

[10] F. Gu, H. Sahin, X. Luo, R. Luo, and R. Lytton *Estimation of resilient modulus of unbound aggregates using performance-related base course properties*. J. Mater. Civ. Eng., **27**, 6, 04014188 (2015)

[11] K. Hjelmstad, and E. Taciroglu. *Analysis and implementation of resilient modulus models for granular solids*. J. Eng. Mech., **126**, 8, 821-830 (2000)

[12] M. Kim, E. Tutumluer, and J. Kwon. *Nonlinear pavement foundation modeling for three-dimensional finite element analysis of flexible pavements*. IJOG, **9**, 5, 195-208 (2009)