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Skewness, Tax Progression, and Demand for Redistribution: Evidence from the UK

By Kirill Pogorelskiy and Stefan Traub*

We introduce a skewness-based approach to measure tax progression and demand for redistribution. Adapting a novel, quantile-based statistical measure of skewness to right-skewed income distributions, we uncover its political economy foundation, by simultaneously relating the same *measure to the classical model of income redistribution due to Meltzer and Richard (1981), to the Prospect Of Upward Mobility (POUM) mechanism due to Bènabou and Ok (2001), and to the progressivity of a tax schedule. In an empirical analysis of UK income distributions in 1979 – 2013, we find that skewness has increased over time, with the rich moving further away from the median. While the magnitude of the increase has remained small enough so that observed redistribution (or lack thereof) could be consistent with POUM hypothesis, more recent periods show an increase in tax progression.*
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Most income distributions have the mean greater than the median, and so are commonly called "right-skewed", or "positively-skewed", implying that they have positive skewness. However, this relation does not always uphold for some of the existing statistical measures of skewness, and the scholarly consensus is still lacking as to which skewness measure should be generally used. For example, the conventional measure of skewness is given by the standardized third central moment. For very skewed distributions, this measure can be so sensitive to the extreme tails of the distribution that it might be difficult to estimate accurately[1], and for some distributions like Pareto, it is only well-defined for certain parameter values. This might be a particularly important concern for analyses that rely on the very top incomes (e.g., Piketty and Saez (2006), Atkinson, Piketty and Saez (2011), Alvaredo et al. (2013)).

In this paper, we make the first attempt to define an "economic" measure of skewness, which takes into account the observed relation between the mean and the median. We take a novel, quantile-based right-skew-sensitive measure

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[1] See, e.g., Hosking (1990) for a general approach to summarizing distribution statistics.

of skewness from statistics (Groeneveld and Meeden, 2009) and adapt it to economic applications. We show that this, purely statistical, measure has a deep political economy foundation by uncovering the relation of the *same* measure simultaneously to the classical general equilibrium model of income redistribution in democracies due to Meltzer and Richard (1981), to the Prospect Of Upward Mobility (POUM) mechanism due to Bènabou and Ok (2001), and to the progressivity of a tax schedule. More specifically, we prove that 1) the mean over the median inequality measure is a special case of the quantile measure of skewness; 2) if the quantile skewness is sufficiently small and positive, it is possible, for fixed discount factor and the number of periods the tax remans in force, to construct a social mobility function under which the majority winning tax is zero on a whim of the median voter whose income is less than the mean, i.e., the POUM hypothesis holds; and 3) under relatively mild conditions, a quantile-skewness-reducing tax is progressive in the conventional sense.

In the empirical application, we apply the quantile measure of skewness to the analysis of tax progression and demand for redistribution in the UK using nine waves of Luxembourg Income Study data covering 1979 – 2013, and document important patterns in the change of skewness over time. In particular, we show that skewness has increased over time, with the rich moving further away from the median. While the demand for redistribution has remained positive, the magnitude of increase was small enough so that the observed redistribution (or lack thereof) could be consistent with a deterministic POUM mechanism, in which individuals expect a tax to persist for at least three years, and are sufficiently patient. More recent periods, however, show an increase in tax progression.

After briefly describing the related literature in the following subsection, the remainder of the paper is organized as follows. Section I presents the theory, formally defining the skewness measure (Subsection I.B), exploring its properties (Subsection I.D), and relating it to political economy models (Subsections I.E-I.F) and measuring tax progression (Subsection I.G). Section II describes the application of the theory to the UK data, explaining the LIS data in Subsection II.A and the analysis of the empirical patterns in Subsection II.B. Section III concludes. All proofs are in Appendix A. Appendix B contains numerical simulations for several standard distribution families: Lognormal, Exponential, and Pareto. Additional data, figures, and tables are in Appendix C.

A. Related Literature

This paper is related to several major literatures. The quantile-based measure of skewness follows the approach from theoretical statistics (Groeneveld and Meeden, 2009). This literature, starting from Groeneveld and Meeden (1984) and Hosking (1990) generalized Bowley's interquartile range (Bowley, 1901) proposing a flexible approach to measure skewness, which, to the best of our knowledge, has not been used in economics[2] and has been mostly studied in statistics using simulations and examples from standard distribution families. The family of quantile measures proposed in Groeneveld and Meeden (2009) is parameterized by a one-dimensional parameter which determines how far from the median the value of skewness is computed. We take it as a starting point, and single out the value of the parameter relevant for economic applications, by relating this measure to the classical political economy models of majority voting over income redistribution that build upon Meltzer and Richard (1981).[3] The key prediction from those models is a positive relation between the median voter's preferred linear tax (the majority demand for redistribution) and the mean over the median ratio. This relation has been tested empirically and experimentally by numerous papers, so far having produced mixed results.[4] We show that the mean over the median ratio is a special case of the quantile measure of skewness.

One possible explanation for why income inequality has not resulted in a large-scale redistribution lies in social mobility considerations. In particular, the POUM hypothesis, formalized in Bènabou and Ok (2001), suggests that the poor majority may decide not to demand extreme taxes because they expect that they (or their children) will become richer in the future, and so will be affected by these redistributive policies.[5] We show that the quantile skewness of the initial gross income distribution has to be sufficiently small for the POUM mechanism to work, if one is not allowed to vary the discount factor and the tax persistence period.[6]

We also relate to the voluminous literature on measuring tax progression (see Seidl, Pogorelskiy and Traub (2013) for a book-length overview), by showing that under convexity and non-positive taxation of the lowest incomes, a quantile-skewness-reducing tax schedule is a conventionally progressive tax.

Finally, our empirical analysis uses the UK income distributions from nine waves of LIS data, and here we relate to the applied literature on patterns in the change of income inequality and tax progression over time. This literature has shown that income inequality has risen in most OECD countries since the mid-1980s, in particular, due to a faster increase of top incomes (OECD, 2008, 2011). For a recent critical review of the literature on the evolution of top incomes over time see Guvenen and Kaplan (2017).

[2]One exception is a recent paper of Krämer and Dette (2016), who propose to define skewness in terms of average net deprivation relative to the Gini coefficient. Their measure does not have a simple interpretation and may fail to satisfy the convex ordering axiom (see Subsection I.D).

[3]The special case of this framework was first studied by Romer (1975) and Roberts (1977). The static framework of Meltzer and Richard (1981) is extended to a dynamic setting in Alesina and Rodrik (1994) and Persson and Tabellini (1994), relating redistribution to economic growth. See Persson and Tabellini (2002) for an overview and related extensions.

[4] See, e.g., Meltzer and Richard (1983), Krusell and Ríos-Rull (1999), Rodríguez (1999), Milanovic (2000), Agranov and Palfrey (2015).

[5]Other relevant studies of social mobility include Piketty (1995), Gottschalk and Spolaore (2002), Alesina and La Ferrara (2005), Agranov and Palfrey (2016).

[6]If one can freely vary these parameters, then for any finite and positive initial skewness it is possible to construct a social transition function that delivers the POUM result, so the restriction on skewness loses its bite.

I. Theory

In this section, we describe a theoretical basis for our measure of quantile skewness and uncover its relation to tax progression and demand for redistribution.

A. Notation

Let y denote gross income, distributed with cdf G on $[y_{\min}, y_{\max}]$, where $y_{\min} > 0$. Let x denote net income, distributed with cdf F on $[x_{\min}, x_{\max}]$. We assume that G and F are unimodal, absolutely continuous, strictly monotone, and have differentiable densities, g and f, positive everywhere on their respective supports. It is understood that F is generated from G by applying a fixed tax schedule to every income recipient. Let $D \in \{G, F\}$ be an income distribution. For $p \in [0, 1]$ let $D^{-1}(p) := \inf\{z \in \mathbb{R} | D(z) \geq p\} = \mu_p^D$ be the quantile function of D evaluated at p. From our continuity and monotonicity assumptions, for every $p \in [0, 1]$ there is a unique income z_p such that $D(z_p) = p$. The median of distribution D is given by $\mu^D := D^{-1}(0.5)$.

B. A quantile measure of skewness

As we mentioned in the introduction, the real world income distributions are ubiquitously, and sometimes to a very large degree, right-skewed, and an "economic" measure of skewness must take this into account. Such an alternative skewness measure, based on quantile ratios, was examined in a recent paper of Groeneveld and Meeden (2009).[7] This new measure does not require existence of higher moments of a distribution and is sensitive to right-skewed distributions, which becomes particularly useful in the analysis of tax progression and demand redistribution, as discussed in more detail below.

To motivate this new measure of skewness, take an individual, Alice, with median income μ^D and consider her relative income shortfall (RIS) in comparison to a richer individual, Bob, with income $\mu_{1-p}^D > \mu^D$ for $p \in (0, 0.5)$:

$$\text{Relative Income Shortfall} = \frac{\mu_{1-p}^D - \mu^D}{\mu^D} \tag{1}$$

If Alice viewed Bob's income as a reference point, she would be probably willing to decrease this difference in incomes. If we assume that she cannot increase her own income, then the only way this can be done is by decreasing Bob's income downwards. Bob's income, however, is not the only reference point Alice might have. Consider a poorer individual, Clark, with income $\mu_p^D < \mu^D$. Alice's relative

[7] Kotz and Seier (2009) describe a similar approach to define a kurtosis measure. Quantile measures of skewness generalize the so-called Bowley coefficient (Bowley, 1901). See also Groeneveld and Meeden (1984).

income excess (RIE) in comparison to Clark is

$$\text{Relative Income Excess} = \frac{\mu^D - \mu_p^D}{\mu^D} \tag{2}$$

If Alice viewed Clark's income as a reference point (i.e., she had some sort of social preference), she would be probably also willing to decrease this difference in incomes. If we assume she is unwilling to decrease her own income to do this, then the only way this can be done is by increasing Clark's income upwards.

Thus if we put Alice in charge of income redistribution in the society, she could kill two birds with one stone: by redistributing some of Bob's income towards Clark. This would result in a less right-skewed distribution. This procedure also has a clear psychological interpretation: a typical survey question about redistribution is how the government should redistribute income from the rich to the poor.[8] This puts the respondent in the middle of the perceived income distribution, since she is unlikely to consider herself either very rich or very poor. But what are the reference incomes then? One possibility is to think of "the better-off" as a hypothetical person richer than oneself, and of "the less well-off" as a hypothetical person poorer than oneself. The relative importance of the two is precisely what our skewness measure captures.

Formally, we define a quantile-based measure of skewness $\lambda_p(D)$ for $D \in \{F, G\}$, and a fixed skewness parameter $p \in (0, 0.5)$ as follows:

Definition 1. *Let*

$$\lambda_p(D) = \frac{RIS}{RIE} - 1 = \frac{\mu_{1-p}^D - \mu^D}{\mu^D - \mu_p^D} - 1 \tag{3}$$

We say that D is skewed to the right, *if $\lambda_p(D) > 0$. This is equivalent to having*

$$D^{-1}(1-p) - D^{-1}(0.5) > D^{-1}(0.5) - D^{-1}(p)$$

C. Choosing the skewness parameter

The quantile measure of skewness depends on the value of the skewness parameter, p, which determines how far from the median income the reference incomes are located. For p close to 0.5, the measure captures only the central part of the distribution, and for p close to 0 the measure captures almost the entire distribution and so becomes very sensitive to the tails.

Thus, the choice of p involves a tradeoff. For example, Groeneveld and Meeden (2009) suggested to use $p = 0.05$, arguing that doing so excludes the absolute

[8] E.g., the British Social Attitudes Survey has been asking the following question for many years. [On a scale from 1 to 5, how much do you agree with the following statement]: "Government should redistribute income from the better-off to those who are less well-off."

extremes while capturing most of the distribution support. In economic applications, however, it makes less sense to choose a small p because reference incomes are more likely to be located closer to one's own income. This view also receives support from social psychology: According to Festinger's theory of social comparison processes, "[t]he tendency to compare oneself with some other specific person decreases as the difference between his opinion or ability and one's own increases", and "[g]iven a range of possible persons for comparison, someone close to one's own ability or opinion will be chosen for comparison" (Festinger, 1954, p. 120–121). In Subsection I.E we show that there is a unique choice of p that is determined by the demand for redistribution.

D. Properties of the quantile skewness measure

To distinguish between different skewness measures, van Zwet (1964) introduced an axiomatic approach. We reformulate these axioms here as properties of $\lambda_p(D)$ for a fixed skewness parameter $p \in (0, 0.5)$ and distribution $D \in \{F, G\}$.

To state the axioms formally, we will need just one more definition from van Zwet (1964) (see also Groeneveld and Meeden (1984)).

Definition 2. *We say that distribution G is* at least as skewed to the right *as distribution F, if $R := G^{-1} \circ F$ is convex. In this case we write $F <_c G$ and say that "F c-precedes G", where c stands for convex ordering.*

This definition says that a non-decreasing convex transformation of a random variable should be skewness-increasing (and a concave transformation skewness-reducing). If $F <_c G$, it can be shown that Y is a convex function of X, and their standardized cdfs cross exactly twice (Oja, 1981). This property is also the driving force behind the famous social mobility model of Bènabou and Ok (2001) as explained in Subsection I.F. The list of axioms for a skewness measure follows.

($A1$) *[Convex Ordering].* if $F <_c G$ then $\lambda_p(F) \leq \lambda_p(G)$

($A2$) *[Translation and Scale Invariance].* $\lambda_p(a + bD) = \lambda_p(D)$ where $b > 0, a$ are constants.

($A3$) *[Symmetric Normalization].* $\lambda_p(D) = 0$ for a symmetric D.

($A4$) *[Antisymmetry].* $\lambda_p(-D) = -\lambda_p(D)$

Axiom ($A1$) is particularly important, since it introduces a partial ordering on distributions. Some skewness measures, e.g., Pearson's second skewness coefficient $3(\bar{y} - \mu)/\sigma$ may violate this axiom. Groeneveld and Meeden (2009) showed that λ_p satisfies ($A1$) – ($A3$), but not ($A4$). Hence we propose a slightly modified axiom, ($A4'$).

($A4'$) *[Right-skew Sensitivity].* $\lambda_p(-D) \geq -\lambda_p(D)$

Proposition 1. *$\lambda_p(D)$ satisfies (A4′) for all $p \in (0, 0.5)$. If D is not symmetric, $\lambda_p(D)$ satisfies (A4′) strictly for some p.*

While we do not currently have a full axiomatic characterization of $\lambda_p(D)$, leaving it for future research, we provide in Appendix B numerical simulations with examples from several distribution families: Lognormal, Exponential, and Pareto, truncated to the support $[y_{\min}, y_{\max}]$ of the gross income distribution, in order to illustrate the properties of the quantile measure of skewness. We assume a convex tax schedule with 50% of the total tax revenue redistributed as a lump-sum transfer. We show how progressive taxation reduces quantile skewness for all values of p as one moves from the gross to the net income distribution. Interestingly, in those numerical examples, $\lambda_p(D)$ is a convex and decreasing function of p for $D \in \{F, G\}$ and $p \geq 0.05$.[9]

E. Skewness and Demand for Redistribution

It turns out that the quantile measure of skewness can be related to the "demand for redistribution" in the classical theoretical framework of Meltzer and Richard (1981).

The model in brief works as follows. There is a measure one of individuals, who differ along a single characteristic (e.g., productivity). The individuals make private choices (labor supply) taking the government policy (linear tax and lump-sum transfers) as given. This generates induced preferences (indirect utility) over taxes. With a linear tax, each individual's preference for redistribution hinges on their position relative to mean income: an individual with below-mean productivity receives below-mean income, so she would benefit from a linear redistribution scheme and prefers a more redistributive tax schedule, subject to the deadweight loss due to taxation. The concern about the deadweight loss prevents the poor majority from setting the tax rate to 100%.[10]

With pairwise majority voting, which could be thought of as a result of two-candidate electoral competition, the optimal tax policy hinges on the median voter's income. In particular, assuming a reasonably-behaved utility specification, like the Stone-Geary utility from consumption c and leisure ℓ, $u(c, \ell) = \ln(c+\gamma) + \alpha \ln(\ell)$, which has been widely used in empirical analyses, the model predicts that the purely redistributive expenditure is positively correlated with $m^G := \bar{y}/\mu^G$, the ratio of the mean income to the median income, also called the Meltzer-Richard inequality measure (Meltzer and Richard, 1981, 1983).[11]

[9] Lognormal distribution behaves differently for $p < 0.05$, so in general, $\lambda_p(D)$ can behave non-monotonically as a function of p.

[10] The linear tax assumption is restrictive but standard in this literature, since it makes it possible to use the median-voter theorem to determine the majority-preferred equilibrium tax rate. Even with the linear tax, the tax-and-transfer schedule is *de facto* progressive, since the rich are getting back a smaller fraction of their income as lump-sum transfers. As explained in Subsection I.G, the application of the quantile measure of skewness to tax progression does not rely on linear tax at all.

[11] There are additional implicit assumptions required for this interpretation, e.g., that the median voter has median income. Depending on voter turnout, this may not be the case.

We will now relate the Meltzer-Richard ratio to our skewness measure. Notice first, that from (3), for any distribution D and any $p \in (0, 0.5)$ we can express the median as follows.

$$\mu^D = \frac{\mu^D_{1-p} + \mu^D_p(\lambda_p(D) + 1)}{\lambda_p(D) + 2} \tag{4}$$

Consider now a right-skewed gross income distribution. By our regularity assumptions, there exists a $\bar{p} \in [0, 0.5)$ such that $\bar{y} = G^{-1}(1 - \bar{p})$. That is, the mean gross income is given by a quantile to the right of the median income, and for very right-skewed distributions, this $1 - \bar{p}$ quantile will be closer to the top incomes than to the median income. Thus,

$$\bar{p} := 1 - G(\bar{y}) \tag{5}$$

We can now rewrite (4) as follows:

$$\mu^G = \frac{\bar{y} + \mu^G_{\bar{p}}(\lambda_{\bar{p}}(G) + 1)}{\lambda_{\bar{p}}(G) + 2} = \mu^G_{\bar{p}} + \frac{\bar{y} - \mu^G_{\bar{p}}}{\lambda_{\bar{p}}(G) + 2}$$

Dividing both sides by a positive median income, μ^G, and re-arranging terms, we obtain

$$m^G = (\lambda_{\bar{p}}(G) + 1)\underbrace{\left(1 - \frac{\mu^G_{\bar{p}}}{\mu^G}\right)}_{=\text{RIE}} + 1 \tag{6}$$

Hence *the Meltzer-Richard ratio is a special case of the quantile measure of skewness*, when evaluated at $\bar{p}$. This justifies our focus on the skewness measure evaluated at a particular value of $p = \bar{p}$ in the remainder of the paper. The scaling factor in (6), $\left(1 - \frac{\mu^G_{\bar{p}}}{\mu^G}\right)$, is precisely the median voter's relative income excess, see (2). For right-skewed distributions, $\lambda_{\bar{p}}(G) > 0$ and $\frac{\mu^G_{\bar{p}}}{\mu^G} < 1$. If a right-skewed distribution becomes more right-skewed, both $\lambda_{\bar{p}}$ and $\frac{\mu^G_{\bar{p}}}{\mu^G}$ increase, with $\frac{\mu^G_{\bar{p}}}{\mu^G}$ remaining less than 1.

Under the additional assumption that partial elasticites of mean income with respect to taxes and lump-sum transfers are constant, Meltzer and Richard (1981) show that the optimal tax for the voter with the median income is increasing in m^G. Together with (6), this implies that an increase in quantile skewness due to a decrease in RIE may not affect the equilibrium tax rate; only an increase in skewness due to RIS should lead to an increase in the equilibrium tax.

F. Quantile skewness and the prospect of upward mobility

In this section we describe an even more intimate relation that exists between the quantile measure of skewness and the POUM hypothesis. The POUM argues that the poor majority may decide not to demand extreme taxes because they expect to have incomes greater than the mean in the future, and so will be adversely affected by these redistributive policies. This argument may seem paradoxical, because, clearly, the majority of the population cannot all become richer than the mean. However, Bènabou and Ok (2001), by means of a simple social mobility model, demonstrate how the POUM hypothesis can actually hold with fully rational economic agents. The key model assumptions concern concavity of the expected transition function, which maps today's incomes to future incomes, and right-skewness of the initial income distribution.

Suppose that all individuals are risk-neutral, and starting from the initial income distribution, G, a linear redistribution scheme $r_t(y) := (1-t)y + t\bar{y}$ is applied to each individual's gross income y, subject to the budget-balance condition: $\bar{y} = \int_{y_{\min}}^{y_{\max}} r_t(y)dG(y)$. A policy of no redistribution ("laissez-faire") is given by r_0, and a full redistribution policy is given by r_1. Suppose a tax schedule is chosen at date 0 to remain in force for $T \geq 1$ periods, *starting from period 0*, and all individuals discount the future with the same discount factor $\delta \in (0,1]$. Individuals' gross income in period 0 comes from distribution G, and for any period $\tau > 0$ it is obtained by applying the same transition function $s : [y_{\min}, y_{\max}] \to [y_{\min}, y_{\max}]$, to the previous period gross income. s is assumed to be a continuous, strictly increasing, and concave but not affine transition function (denote the class of such functions $\mathcal{S}$).[12] So $y_\tau = s(y_{\tau-1}) \equiv (s(y))^\tau, \tau = 1, \ldots, T$. Thus, we momentarily step out of the general equilibrium framework of Meltzer and Richard (1981), ignoring deadweight losses from redistribution and optimal labor supply decisions, and model the change in the income distribution as a primitive.[13]

Notice that the income distribution in any period $\tau > 0$ is $F_\tau = F_{\tau-1} \circ s^{-1}$. Since s is concave, $s^{-1} \equiv (F_{\tau-1})^{-1} \circ F_\tau$ is convex. Therefore, $F_\tau <_c F_{\tau-1}$. Given axiom (A1) of a skewness measure, each period income distribution is less right-skewed than the previous one. Since the convex ordering is transitive (e.g., Oja (1981, Theorem 5.1)), $F_\tau <_c G$. So if the transition function is applied sufficiently many times, the demand for redistribution that may exist in the initial distribution will disappear as the mean income will approach the median or even go below that. Therefore, depending on δ (how patient all individuals are), they may vote against redistribution already in period 0. This is how the basic POUM mechanism works.

Denote the conditional mean gross income of the poorer population half as $\mathbb{E}\left[Y|Y \leq \mu^G\right] \equiv 2\int_{y_{\min}}^{\mu^G} ydG(y)$. Building on Bènabou and Ok (2001, Theorem

[12]In general, s can be stochastic, and depend on individual income shocks, as described in Bènabou and Ok (2001). We will only consider deterministic transition functions here.

[13]This departure is mainly for simplicity: one could add deadweight losses and endogenize transition functions, and under specific assumptions about the income distribution, still obtain the POUM result. See Bènabou (2000) and Agranov and Palfrey (2016).

2), we can prove the following

Proposition 2. *Let G be right-skewed, $\bar{p}$ defined by* (5), *and suppose that*

$$\mu_{\bar{p}}^{G} \geq \mathbb{E}\left[Y|Y \leq \mu^{G}\right]. \tag{7}$$

If, for given δ and T,

$$\lambda_{\bar{p}}(G) < \frac{\delta(3-\delta^{T})-2}{2(1-\delta)} \tag{8}$$

then there exists a transition function $s \in \mathcal{S}$ such that the median voter is indifferent between laissez-faire (zero tax) and full redistribution, everyone to the left of the median strictly prefers full redistribution, and everyone to the right of the median strictly prefers laissez-faire. For any integer $T' > T$, the majority-winning tax becomes zero.

Proposition 2 says that *for fixed δ and T*, if the initial skewness is sufficiently small but positive (and the conditional mean income is not too high), it is possible to construct a continuous, strictly increasing, and concave but not affine transition function under which the majority winning tax is zero on a whim of the median voter, who is poorer than the mean. Bènabou and Ok (2001, Theorem 2b) show that *for large enough T and δ close to 1*, one can find a transition function ensuring that the majority winning tax is zero for any initial positive skewness, i.e., the POUM argument indeed may prevent redistribution. Their conclusion carries over in full in our case – for sufficiently large T and δ close to 1, (7)-(8) are not required for the POUM mechanism.[14] However, for fixed (and possibly, small) δ and T, the initial skewness must be sufficiently small for the construction in the proof to work. For example, if tax persists for $T = 3$ years and $\delta \geq 0.95$, $\lambda_{\bar{p}}(G)$ is small enough for consistency with POUM in terms of satisfying inequality (8), in every LIS wave we used (see Section II.A). If $T = 4$ years, it is sufficient for full consistency to have $\delta \geq 0.85$.

G. *Quantile skewness and tax progression*

In this section we demonstrate the connection between the quantile measure of skewness and the well-known measures of tax progression. Suppose F c-precedes G, as defined in Section I.D. Then $R = G^{-1} \circ F$ is convex. It will be more convenient to work with the inverse of R. Define $H := F^{-1} \circ G$, then H is concave whenever R is convex. Hence we can equivalently write that $F <_c G$ if H is concave. Notice that we have a one-to-one relation between gross and net incomes: for any $x \in [x_{\min}, x_{\max}]$ we have $x = H(y)$ for some $y \in [y_{\min}, y_{\max}]$.

[14]In fact, as mentioned in Bènabou and Ok (2001), the reason that T and δ have to be large enough in their Theorem 2b is because the chosen redistribution scheme is applied already in period 0. We show that an alternative to this is to have a sufficiently small initial skewness.

This is equivalent to writing $x = y - \underbrace{(y - H(y))}_{=:t(y)} = y - t(y)$. Hence, the tax-and-transfer schedule, given by $t(y)$, is a convex function. We are going to refer to $t(y)$ as a tax schedule, with the understanding that redistributive transfers may be also included in $t(y)$. In order to warrant such an interpretation, we impose an additional "no re-ranking" condition, assumed to hold throughout the paper.

Assumption 1. The marginal tax rate, $t'(y)$, satisfies $0 \leq t'(y) \leq 1$ for all $y \in [y_{\min}, y_{\max}]$.

Under this additional assumption, we can formally define the conventional tax progressivity.

Definition 3. *(Conventional progression) A tax schedule $t(y)$ is* progressive*, if the average tax rate, $t(y)/y$, is increasing in y.*

Our next definition relates tax progression to a reduction in skewness of the net income distribution as compared to the gross income distribution.

Definition 4. *(Progression in terms of skewness). A tax schedule is progressive in terms of skewness, if $\lambda_{\bar{p}}(G) - \lambda_{\bar{p}}(F) > 0$ for $\bar{p}$ defined in (5).*

The idea behind this definition is that a progressive tax schedule should be reducing inequality and thus lowering the demand for redistribution. Loosely speaking, the median voter in the gross income distribution should have a higher ideal tax rate than the median voter in the net income distribution.[15]

It is important to understand the relation between progressivity according to the above two definitions and convexity of the tax schedule, as the following proposition shows.

Proposition 3. *Suppose tax schedule t is convex, then the following holds: 1) t is progressive in terms of skewness (Def. 4). 2) If, additionally, $t(y_{\min}) \leq 0$, then t is progressive (Def. 3).*

This result immediately implies

Corollary 1. *Suppose tax schedule t is convex and $t(y_{\min}) \leq 0$. Then t is progressive by either definition of tax progression.*

Hence under convexity, a skewness-reducing tax is progressive.

Concluding the discussion of the theoretical basis for the quantile measure of skewness, we also state a definition of "more progressivity" in terms of skewness, which allows one to compare different tax schedules operating on different income distributions.

[15] Of course, the tax rate for the net income distribution can only be hypothetically conceived, and is not determined by the first order conditions in Meltzer and Richard (1981, Eq (13)). For the sake of comparability, $\bar{p}$ is obtained using the gross income distribution for both gross and net quantile measures of skewness in Definition 4.

Definition 5. *We say that* (F_1, G_1) *is more progressive than* (F_2, G_2)*, if* $\lambda_{\bar{p}^1}(G_1) - \lambda_{\bar{p}^1}(F_1) \geq \lambda_{\bar{p}^2}(G_2) - \lambda_{\bar{p}^2}(F_2)$ *for* $\bar{p}^i$ *defined in* (5) *for distribution* $G_i, i = 1, 2$.

In words, a more progressive tax reduces gross distribution skewness relative to net distribution skewness more. This approach to comparing tax progression across populations, which takes into account corresponding income distributions, is an alternative to the one proposed in Seidl, Pogorelskiy and Traub (2013). Using Definition 5, we'll be able to compare tax progressivity in the UK over time.

II. An application to the UK data

In this section, we apply the quantile measure of skewness to an empirical analysis of the UK data. Using the data on gross and net income distributions in the UK stretching over several decades, we compute the quantile measure of skewness and investigate the demand for linear redistribution, consistency with the POUM hypothesis, and tax progression.

A. Data

Our empirical analysis resorts to micro data drawn from the Luxembourg Income Study database (LIS, 2011). It is a cross-national data archive located in Luxembourg.[16] Currently it includes micro data from about 50 countries, most of which are OECD member states. The data sets are organized into 'waves' of about five years each, starting with Wave I around 1980 and the most recent being Wave IX (around 2013). The micro data from the different surveys is harmonized and standardized by LIS in order to facilitate comparative research. In the present paper, we apply the quantile measure of skewness to UK data only. The UK datasets for Waves I to III originate from the Family Expenditure Survey (FES), the Wave IV dataset originates from both FES and the Family Resources Survey (FRS), and Waves V to IX rely on FRS data. The nine waves of UK data correspond to the years 1979, 1986, 1991, 1995, 1999, 2004, 2007, 2010, and 2013. We use LIS data instead of original survey data from the UK because we intend to carry out comparative research in future works.

LIS reports several household income aggregates out of which we only employ disposable household income (DHI), defined as gross income (HI) minus income taxes (HXITI) and social security contributions (HXITS): DHI = HI − (HXITI + HXITS). We do not consider indirect taxes in our analyses. All observations are selected for which DHI > 0, HI > 0, HXITI ≥ 0 and HXTIS ≥ 0. The analysis is carried out at the household level as well as for equivalized data. The household-level data are the original data provided by

[16]For more information about the LIS database see Smeeding (2004) and Atkinson (2004). A thorough discussion of issues in the use of LIS micro data for measuring effective tax progression, such as re-ranking, is provided by Seidl, Pogorelskiy and Traub (2013).

LIS, weighed by household weights (HPOPWGT). These weights are intended to secure representativeness of the results for the entire country population. For individual-based analyses, equivalized data are used, which are obtained from the household data as follows. First, all monetary variables are multiplied by the Luxembourg equivalence scale $n^{-0.5}$, where n represents the number of household members (NPERS). Second, household weights are replaced by person weights which are computed as NPERS$\times$HPOPWGT.

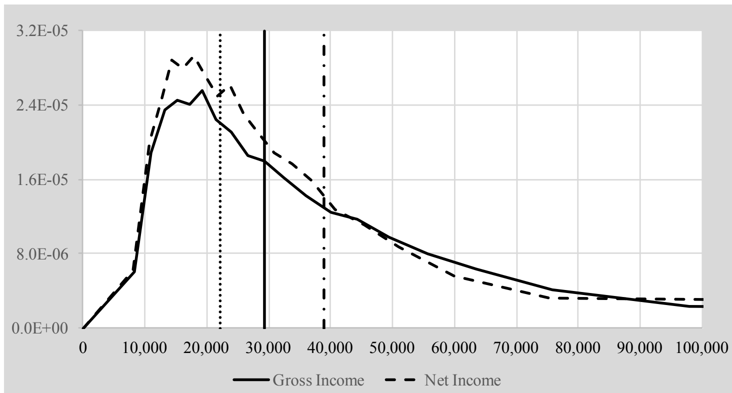
Direct access to LIS micro data is not permitted. Hence, we wrote a program in SPSS that computed μ_p^D, $D \in \{G, F\}$ for 20 equally spaced values of p in the unit interval (i.e., twenty quantiles) and printed back these results for every dataset, for both household and equivalized data. The program also computed the $1 - \bar{p}$-quantiles and the $\bar{p}$-quantiles for gross and net incomes, the quantile measure of skewness evaluated at $\bar{p}$ for gross incomes, and the mean of the net income distribution (recall that the mean of the gross income distribution is given by the $1 - \bar{p}$-quantile). The mean to the median ratio, the quantile measure of skewness for each vigintile, and the conditional mean were then computed in a spreadsheet.

B. Analysis

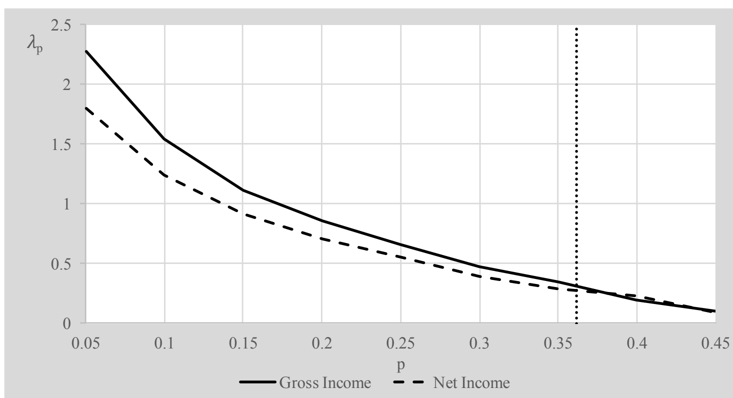
All analyses in this section are using household data only. Appendix C contains figures for all LIS Waves for the UK using both household and equivalized data. The data underlying all figures in this section can be found in Tables C1 – C6 in Appendix C.

Figure 1 is based on the most recent LIS data for the UK (2013). The top panel of Figure 1 depicts the gross (solid line) and net income (dashed line) density over gross income support. The horizontal axis is truncated after the 19th vigintile of gross income. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$ ($\bar{p}$-quantile), 0.5 (median income), and $1 - \bar{p}$ (mean), respectively. At the first glance, it is clear that both the gross and the net income distribution are skewed to the right. The median is to the left of the mean indicating that some demand for redistribution is present, according to the Meltzer-Richard model. The relative income shortfall (the distance between the right-most and the middle vertical line, normalized by the median) is greater than the relative income excess (the distance between the middle and the left-most vertical line, normalized by the median). Thus, the gross income distribution is also skewed to the right according to the quantile measure of skewness. A comparison of gross and net distributions shows that some probability mass has been shifted to lower incomes by the tax-and-transfer system.

The bottom panel of Figure 1 depicts the quantile measures of skewness, $\lambda_p(G)$ (solid line) and $\lambda_p(F)$ (dashed line) as functions of the skewness parameter p. The vertical line represents $\bar{p}$. Both quantile measures monotonically decrease in p, indicating less skewness if measured closer to the median. For lower values of p, we observe greater skewness of the gross income distribution. This also holds



(a) Gross and Net Income Distribution



(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1 - \bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure 1. : Household data, UK'13

for the skewness measure evaluated at $p = \bar{p}$. Since $\bar{p} = 0.362$ is closer to 0.5 than to zero, the reference incomes for the median, $\mu^G_{\bar{p}}$ and $\mu^G_{1-\bar{p}} \equiv \bar{y}$, are located in the central parts of the distribution rather than its tails.

The household datasets between 1999 and 2013 show similar skewness patterns (see Figures C5 – C8 in Appendix C). The earlier datasets (see Figures C1 – C4) are a bit "bumpier", showing multiple peaks. As a result, quantile skewness is less pronounced.

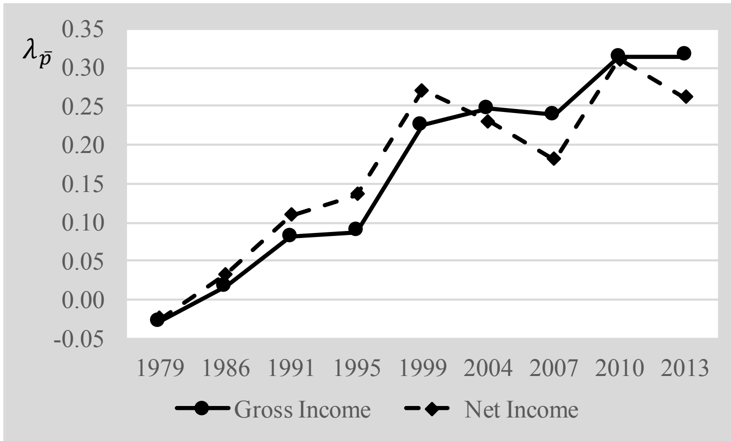
Next, we document skewness patterns that arise over time in Figure 2.[17] The top panel of Figure 2 shows changes in the quantile measure of skewness over time for gross (solid line) and net income (dashed line) distributions. Both lines indicate a clear trend towards more skewness over time. In fact, they start from values close to zero in 1979. Surprisingly, net incomes were more right-skewed than gross incomes until the early 2000s. From then on, gross income distributions became more right-skewed.

The middle panel shows the relation between quantile skewness of gross income evaluated at $\bar{p}$ (solid line, left y-axis) and the demand for redistribution in terms of the mean over the median ratio featured in Meltzer and Richard (1981) (dashed line, right y-axis). Both increase over time and their Pearson correlation coefficient is 0.889. So, looking at these pre-tax measures, we can conclude that the increase in quantile skewness is consistent with increased demand for redistribution in the Meltzer-Richard framework. Note, however, that the skewness of the net income distribution shows that the actual outcome of this redistribution falls short of fulfilling these demands. As already mentioned, only in the later LIS Waves is net income skewness lower than gross income skewness.

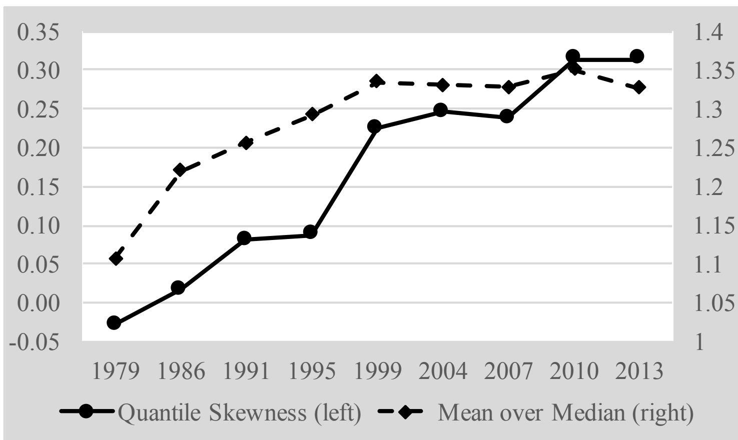
The bottom panel shows the median's relative income excess (solid line) and relative income shortfall (dashed line) over time (for gross incomes). This graph allows us to disentangle the influence of RIE and RIS on the quantile measure of skewness. We see that both started from the same value in 1979 and then RIS increased more than RIE, giving rise to the overall increase in quantile skewness. This means that those richer than the median income individual have moved further away over time while those poorer than the median have kept their distance. This pattern is also consistent with recent empirical evidence from the US, see Guvenen and Kaplan (2017).

Next, we investigate consistency of our data with the POUM hypothesis. Fixing δ and T, Proposition 2 shows that POUM holds if i) quantile skewness of gross income is sufficiently small (see (8)) and, in addition, ii) the conditional mean income amongst those poorer than the median does not exceed the $\bar{p}$-quantile (see (7)). Figure 3 illustrates ii) by comparing the $\bar{p}$-quantile (solid line) with the conditional mean income (dashed line) over time (for equivalized data see Figure C19). The figure reveals that the conditional mean has been strictly smaller than the $\bar{p}$-quantile in every LIS Wave, so (7) holds. The gross income skewness in condition i) varies between -0.029 and 0.314 (see Table C3 in Appendix C). The

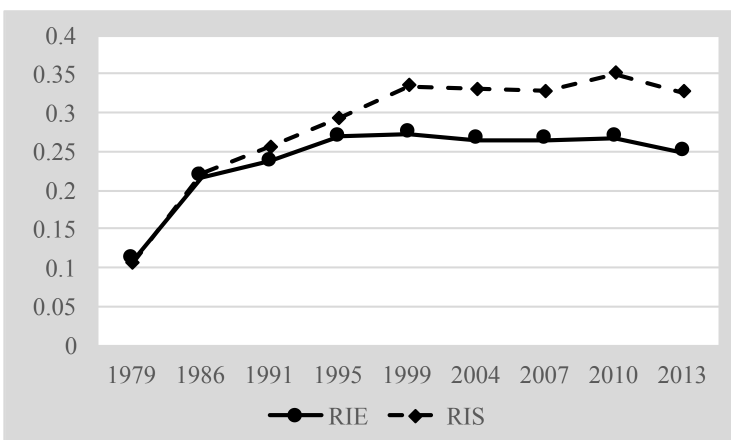
[17]Figure C18 in Appendix C presents the same measures using equivalized data.



(a) Quantile skewness of gross and net income distributions



(b) Left y axis: quantile skewness of gross income. Right y axis: gross mean to median ratio

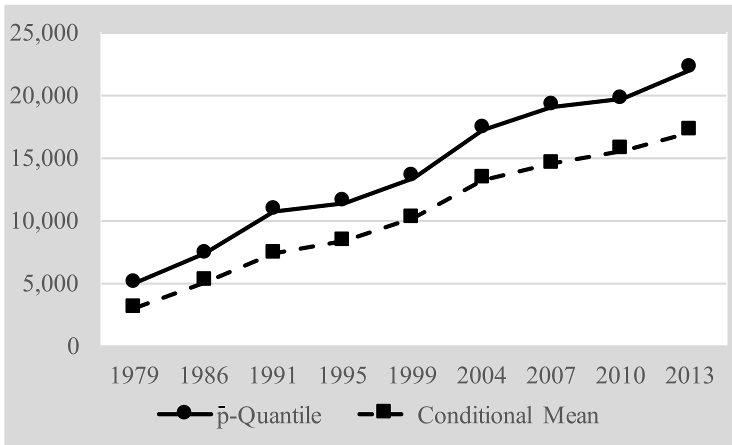


(c) Income excess/shortfall relative to gross median

Notes. All panels use household data. Top panel: quantile measure of skewness over time for gross and net incomes. Middle panel: quantile skewness of gross income evaluated at $\bar{p}$ (left y-axis) and the mean over the median ratio (right y-axis). Bottom panel: Relative income excess and relative income shortfall over time.

Figure 2. : Quantile Skewness and Demand for Redistribution

right hand side of (8) is non-negative for $T > 2$ and $\delta > 0.815$, and increases fast in δ for fixed T. E.g., for $T = 3$ years and $\delta \geq 0.94$, it is greater than the largest $\lambda_{\bar{p}}(G)$ in our data. Hence, the skewness of the gross income distribution has been sufficiently small in order to prevent poor voters from renouncing income redistribution in every LIS wave.



Notes. Based on household data. $\bar{p}$-quantile (solid line) versus conditional mean (dashed line) over time.

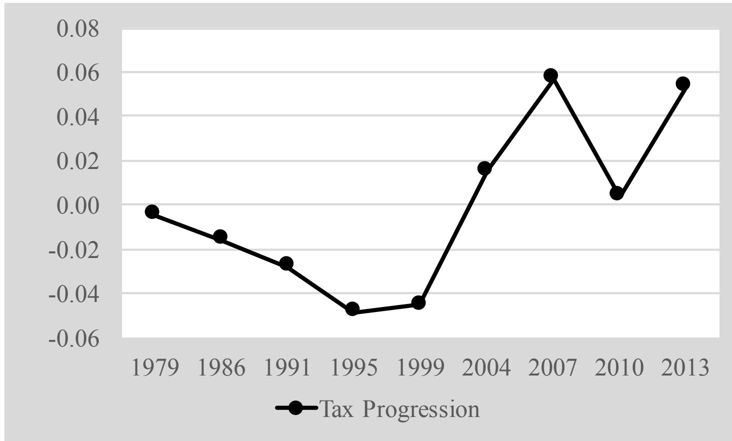
Figure 3. : Condition (7) for POUM hypothesis holds in household data

Finally, we turn to changes in tax progression. Figure 4 shows the difference between the quantile skewness of gross and net incomes evaluated at $\bar{p}$ over time.[18] According to Definition 4, a positive (negative) difference indicates a progressive (regressive) tax-and-transfer system. Moreover, positive (negative) pairwise differences between two time periods indicate an increase (decrease) in the degree of progression according to Definition 5. Between 1979 and 1999, we observe a steady decline of progressivity, ending up in a regressive tax-and-transfer system (from the perspective of the median income). After 1990, tax progression increases. The "dip" in progression in 2010 could be due to abolishment of the 10% starting rate and reduction in the basic rate from 22% to 20% in 2008-09.

III. Concluding remarks

Since the mid-1980s, developed countries have encountered a sharp rise in income inequality (OECD, 2008, 2011). The tax-benefit systems of many OECD countries have become more redistributive, but their effectiveness in reducing income inequality declined. An important byproduct of these developments – the

[18]For equivalized data, see Figure C20 in Appendix C. Equivalized data show a few notable discrepancies (although not in the overall trend), which could probably be due to re-ranking.



Notes. Based on household data. Difference between the quantile skewness of gross and net incomes evaluated at $\bar{p}$.

Figure 4. : Tax progression over time

pronounced increase in the skewness of OECD countries' income distributions – has been largely ignored in both scholarly and public debate.

In this paper, we put the spotlight back on the role of skewness in the distribution and redistribution of income and propose an "economic" measure of skewness, which explicitly takes into account the right-skewness property of income distributions. We take a novel, quantile-based right-skew-sensitive measure of skewness from theoretical statistics (Groeneveld and Meeden, 2009) and adapt it to public economics applications. We prove that it fulfils a number of desirable axioms, the most important being convex ordering, which allows one to meaningfully compare skewness between different distributions.[19] We discover that the quantile measure of skewness, evaluated at a certain quantile, has a deep political economy foundation by uncovering the relation of the *same* measure simultaneously to the Meltzer-Richard model of voting over redistribution, to the Bénabou-Ok POUM mechanism, and to the progressivity of a tax schedule.

In the empirical application, we apply the quantile measure of skewness to the analysis of tax progression and demand for redistribution in the UK using nine waves of Luxembourg Income Study data covering 1979 – 2013. We show that skewness has increased over time, with the rich moving further away from the median. While the demand for redistribution has remained positive, the magnitude of increase was small enough so that the observed redistribution (or lack thereof) could be consistent with a deterministic POUM mechanism, in which individuals expect a tax to persist for at least three years, and are sufficiently

[19]Developing a full axiomatic characterization of the quantile measure of skewness remains an important direction for future research.

patient. More recent periods, however, show an increase in tax progression.

The quantile measure of skewness can be easily and robustly applied to other datasets, and can become a natural alternative to popular inequality measures, like the Gini coefficient and the ratio of the top to the bottom decile.

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Appendix A. Proofs

Proof of Proposition 1. Let

$$\gamma_p(D) := \frac{\mu^D_{1-p} + \mu^D_p - 2\mu^D}{\mu^D_{1-p} - \mu^D_p}$$

This is a measure of skewness proposed in Groeneveld and Meeden (1984), where they also show that $\gamma_p(-D) = -\gamma_p(D)$ and $\gamma_p(D) \in [-1, 1]$. One can easily check (Groeneveld and Meeden, 2009, p.4) that

$$\lambda_p(D) = \frac{2\gamma_p(D)}{1 - \gamma_p(D)}$$

Then using the properties of γ_p we can write

$$\lambda_p(-D) = \frac{2\gamma_p(-D)}{1 - \gamma_p(-D)} = \frac{-2\gamma_p(D)}{1 + \gamma_p(D)} \geq \frac{-2\gamma_p(D)}{1 - \gamma_p(D)} \equiv -\lambda_p(D)$$

The last inequality holds with a strict sign as long as $\gamma_p(D) \neq 0$, but if D is not symmetric, we cannot have $\gamma_p(D) = 0$ for all $p \in (0, 0.5)$. □

Proof of Proposition 2. The proof builds upon the proof of Bènabou and Ok (2001, Theorem 2), with a few changes. For any $\eta \in (y_{\min}, \bar{y})$ and $\alpha \in [0, 1]$, define a family of piecewise linear income transition functions as follows:

$$s_{\eta,\alpha}(y) := \min\{y, \eta + \alpha(y - \eta)\} \tag{A1}$$

Each $s_{\eta,\alpha}$ is continuous, strictly increasing, and concave. Since G is right-skewed, $\bar{y} > \mu^G$, and so $s_{\mu^G,\alpha}$, transition function relative to the median, is also in the family defined by (A1). The τ-th iteration of $s_{\eta,\alpha}$ represents gross income after τ periods of redistribution, and is given by

$$(s_{\eta,\alpha})^\tau (y) = \min\{y, \eta + \alpha^\tau(y - \eta)\} \equiv s_{\eta,\alpha^\tau}(y) \tag{A2}$$

Clearly, for $\alpha = 0$ and $\alpha = 1$, the transition function stays the same after any number of iterations. This will simplify expected utility comparisons when $s_{\eta,1}$ and $s_{\eta,0}$ are used.

Suppose a transition function s is used, then the expected utility of an individual with initial income y after T periods under laissez-faire policy r_0 is

$$V^T(s(y)) := \sum_{\tau=0}^{T} \delta^\tau s^\tau(y),$$

and under the full redistribution policy r_1, when every individual receives the same income, it is

$$W^T(s) := \sum_{\tau=0}^{T} \delta^\tau \bar{y}_\tau = \int_{y_{\min}}^{y_{\max}} V^T(s(y))dG(y),$$

where $\bar{y}_0 = \bar{y}$. The second equality holds since the redistribution in each period has to satisfy the budget-balance constraint.

Suppose transition function $s_{\eta,1}$ is used. The expected utility of an individual with initial income η after T periods under r_0 is

$$V^T\left(s_{\eta,1}(\eta)\right) = \sum_{\tau=0}^{T} \delta^\tau \eta < \sum_{\tau=0}^{T} \delta^\tau \bar{y} = W^T\left(s_{\eta,1}\right), \tag{A3}$$

since $\eta < \bar{y}$. Thus, if the transition function is $s_{\eta,1}$, any individual with initial income below the average strictly prefers r_1 to r_0.

Now, suppose transition function $s_{\eta,0}$ is used. The expected utility of an individual with initial income η after T periods under r_0 is strictly greater than under r_1 if and only if

$$\begin{aligned}
&V^T\left(s_{\eta,0}(\eta)\right) > W^T\left(s_{\eta,0}\right) && \text{(A4)}\\
&\Updownarrow\\
&\eta + \sum_{\tau=1}^{T} \delta^\tau \eta > \bar{y} + \sum_{\tau=1}^{T} \delta^\tau \int_{y_{\min}}^{y_{\max}} \min\{y,\eta\} dG(y)\\
&\Updownarrow\\
&\frac{\bar{y}-\eta}{\eta - \int_{y_{\min}}^{y_{\max}} \min\{y,\eta\}dG(y)} < \sum_{\tau=1}^{T} \delta^\tau = \frac{\delta(1-\delta^T)}{1-\delta}\\
&\Updownarrow\\
&\frac{\bar{y}-\eta}{\eta - \int_{y_{\min}}^{\eta} ydG(y) - \eta(1-G(\eta))} < \frac{\delta(1-\delta^T)}{1-\delta}\\
&\Updownarrow\\
&\underbrace{\frac{\bar{y}-\eta}{\eta G(\eta) - \int_{y_{\min}}^{\eta} ydG(y)}}_{:=Q(\eta)} < \frac{\delta(1-\delta^T)}{1-\delta} && \text{(A5)}
\end{aligned}$$

Since $\frac{dQ(\eta)}{d\eta} = -\frac{\bar{y}G(\eta) - \int_{y_{\min}}^{\eta} ydG(y)}{\left(\eta G(\eta) - \int_{y_{\min}}^{\eta} ydG(y)\right)^2} < 0$, Q is continuous and strictly decreasing in η for $\eta < \bar{y}$. Therefore, if (A5) holds for $\eta = \mu^G$, it also holds for any

$\eta \in (\mu^G, \bar{y})$. At $\eta = \mu^G$, (A5) is equivalent to

$$\frac{\bar{y} - \mu^G}{\mu^G - 2\int_{y_{\min}}^{\mu^G} y dG(y)} < \frac{\delta(1-\delta^T)}{2(1-\delta)}$$
$$\Updownarrow$$
$$\frac{\bar{y} - \mu^G}{\mu^G - \tilde{y}} - 1 < \frac{\delta(1-\delta^T)}{2(1-\delta)} - 1 \tag{A6}$$

where $\tilde{y} := 2\int_{y_{\min}}^{\mu^G} y dG(y) < \mu^G$. This condition for the median voter income is implicitly invoked in the proof of Bènabou and Ok (2001, Theorem 2), and it is guaranteed to hold by choosing a sufficiently large T and δ close to 1. Comparing the left hand side of (A6) with $\lambda_{\bar{p}}(G) \equiv \frac{\bar{y}-\mu^G}{\mu^G - \mu^G_{\bar{p}}} - 1$, we see that the two coincide if and only if $\tilde{y} = \mu^G_{\bar{p}}$; for $\tilde{y} > \mu^G_{\bar{p}}$, $\lambda_{\bar{p}}(G) < \frac{\bar{y}-\mu^G}{\mu^G-\tilde{y}} - 1$; and for $\tilde{y} < \mu^G_{\bar{p}}$, $\lambda_{\bar{p}}(G) > \frac{\bar{y}-\mu^G}{\mu^G-\tilde{y}} - 1$. Therefore, for $\tilde{y} \leq \mu^G_{\bar{p}}$, as guaranteed by (7), (8) implies (A6) (and for $\tilde{y} \geq \mu^G_{\bar{p}}$, (A6) implies (8), so if we are allowed to choose T and δ, we don't need to impose an extra condition on the degree of skewness).

Thus, we have established that (A5) holds for any $\eta \in [\mu^G, \bar{y})$, which implies

$$W^T(s_{\eta,1}) > V^T(s_{\eta,1}(\eta)) = \sum_{\tau=0}^{T} \delta^\tau \eta = V^T(s_{\eta,0}(\eta)) > W^T(s_{\eta,0}). \tag{A7}$$

Since by construction, $W^T(s_{\eta,\alpha})$ is continuous and strictly increasing in α, there exists a unique $\alpha(\eta) \in (0,1)$ such that $W^T\left(s_{\eta,\alpha(\eta)}\right) = \sum_{\tau=0}^{T} \delta^\tau \eta$, so that the agent with initial income η is indifferent between r_0 and r_1. Under $s_{\eta,\alpha(\eta)}$ and r_0, any agent with $y < \eta$ receives y in every period τ, while an agent with $y > \eta$ receives $\eta + \alpha^\tau(y-\eta) > \eta$. Therefore, η separates those who support r_0 (richer voters) from those who support r_1 (poorer voters), under transition function $s_{\eta,\alpha(\eta)}$. This holds for any $\eta \in [\mu^G, \bar{y})$, and, in particular, for the median voter. Increasing T will push the cutoff income of those in support of r_0 below η, as proved in Bènabou and Ok (2001, Theorem 2(a)), resolving the median voter's indifference in favor of r_0. □

Proof of Proposition 3. To prove the first part, note that since t is convex, F *c*-precedes G. Since quantile measure of skewness satisfies (*A*1), $\lambda_p(F) - \lambda_p(G) \leq 0$ for any $p \in (0, 0.5)$, in particular, for $p = \bar{p}$. Hence t is progressive in terms of skewness by Definition 4.

To prove the second part, note that if t is convex, then $t'(y)$ is increasing in y.

t is progressive by Definition 3 if and only if

$$\frac{d(t(y)/y)}{dy} = \frac{t'(y)y - t(y)}{y^2} = \frac{t'(y)}{y} - \frac{t(y)}{y^2} > 0 \Leftrightarrow t'(y) > \frac{t(y)}{y}$$

By the mean value theorem, for any $y \in (y_{\min}, y_{\max})$

$$\exists\, \tilde{y} \in (y_{\min}, y) : t'(\tilde{y}) = \frac{t(y) - t(y_{\min})}{y - y_{\min}}$$

By convexity of t, $t'(y) > t'(\tilde{y})$ for $y > \tilde{y}$. Combining, and using the assumption that $t(y_{\min}) \leq 0$ and $y_{\min} > 0$,

$$t'(y) > t'(\tilde{y}) = \frac{t(y) - t(y_{\min})}{y - y_{\min}} > \frac{t(y) - t(y_{\min})}{y} \geq \frac{t(y)}{y}$$

Hence t is progressive by Definition 3. □

Appendix B. Numerical computations

In this Appendix, we demonstrate some properties of the quantile measure of skewness when applied to three families of standard distributions: exponential, lognormal and Pareto. Assume that the tax functional is given by

$$H(y, \alpha) = y - \frac{y^2}{3y_{\max}} + \alpha\theta,$$

where θ is the average tax and α is "efficiency" (how much of the tax return is redistributed as a lump-sum transfer to the households).

We start with an exponential distribution, truncated from both sides at $y_{\min}$ and $y_{\max}$. Its pdf is

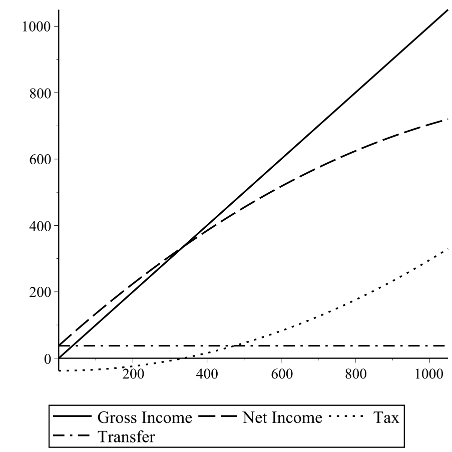
$$g(y) = \ell \exp(-\ell y) \frac{I_{(y_{\min}, y_{\max})}(y)}{\tilde{G}(y_{\max}) - \tilde{G}(y_{\min})},$$

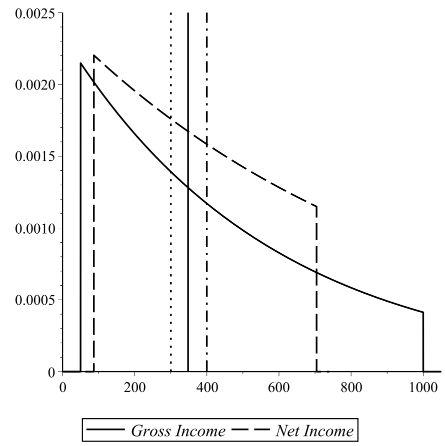
where $I_{(a,b)}(y) = 1$ if $a < y < b$ and 0 otherwise, $\tilde{G}(\cdot)$ is the cdf of the non-truncated exponential distribution with support $(0, +\infty)$, and ℓ is the parameter of the truncated exponential.

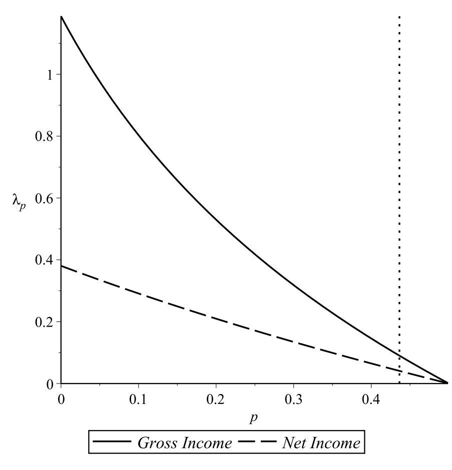
Table B1 presents the computation results, and Figure B1 illustrates them graphically.

Next, we look at the lognormal distribution. Its pdf is

$$g(y) = \frac{\exp -\frac{(\ln(y)-m)^2}{2\sigma^2}}{\sqrt{2\pi}\sigma y} \frac{I_{(y_{\min}, y_{\max})}(y)}{\tilde{G}(y_{\max}) - \tilde{G}(y_{\min})},$$







The upper left figure shows gross income (the diagonal) and net income, tax (including transfers), and lump sum transfer as functions of gross income. The upper right figure depicts gross and net income density over gross income support. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$ *(*$\bar{p}$*-quantile),* 0.5 *(median income), and* $1-\bar{p}$ *(mean), respectively. Note that the horizontal axis refers to gross income in the first case and to net income in the second. The lower figure depicts quantile measures of skewness,* $\lambda_p(G)$ *(solid line) and* $\lambda_p(F)$ *(dashed line) as functions of the skewness parameter* p*. The vertical line represents* $\bar{p}$*. For parameters see Table B1.*

Figure B1. : Exponential Distribution

Table B1—: Numerical Results: Exponential Distribution

	Gross Income	Net Income
ℓ	0.0017	0.0011
Max	1000	704
Min	50	87
Mean	400	362
Median	348	346
$\bar{p}$-Quantile	300	309
Conditional Mean	304	—
Variance	65988	31114
Transfer	—	38
Skewness	0.090	0.041

Notes. Efficiency: $\alpha = 0.50$; $\bar{p} = 0.44$.

where $\tilde{G}(\cdot)$ is the cdf of the non-truncated lognormal distribution with support $(0, +\infty)$, m and σ are the parameters of the truncated lognormal. The estimation results are given in Table B2 and illustrated by Figure B2.

Table B2—: Numerical Results: Lognormal Distribution

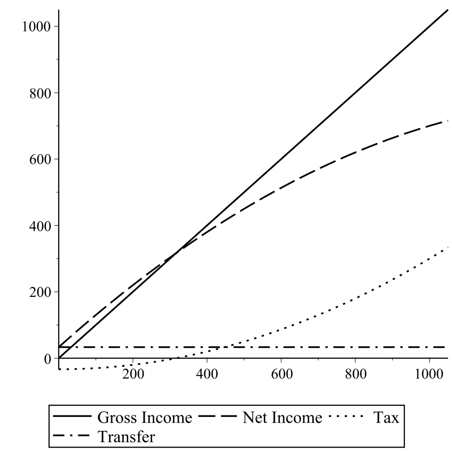
	Gross Income	Net Income
m	5.9168	5.9054
σ	0.5776	0.4645
Max	1000	700
Min	50	82
Mean	400	367
Median	360	350
$\bar{p}$-Quantile	323	322
Conditional Mean	263	
Variance	40000	18775
Transfer	—	33
Skewness	0.101	0.066

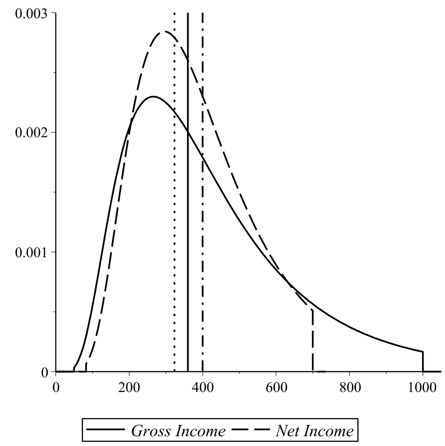
Notes. Efficiency: $\alpha = 0.50$: $\bar{p} = 0.42$.

Finally, we turn to the Pareto distribution. Its pdf is

$$f(y) = \rho \frac{y_{\min}^{\rho}}{y^{\rho+1}} \frac{I_{(y_{\min}, y_{\max})}(y)}{\tilde{G}(y_{\max})},$$

where $\tilde{G}(\cdot)$ is the cdf of the non-truncated Pareto distribution with support $[y_{\min}, +\infty)$, and ρ and $y_{\min}$ are the parameters of the truncated Pareto. Estimation results are given in Table B3 and illustrated by Figure B3.





The upper left figure shows gross income (the diagonal) and net income, tax (including transfers), and lump sum transfer as functions of gross income. The upper right figure depicts gross and net income density over gross income support. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$ ($\bar{p}$-quantile), 0.5 *(median income), and* $1 - \bar{p}$ *(mean), respectively. Note that the horizontal axis refers to gross income in the first case and to net income in the second. The lower figure depicts quantile measures of skewness,* $\lambda_p(G)$ *(solid line) and* $\lambda_p(F)$ *(dashed line) as functions of the skewness parameter* p*. The vertical line represents* $\bar{p}$*. For parameters see Table B2.*

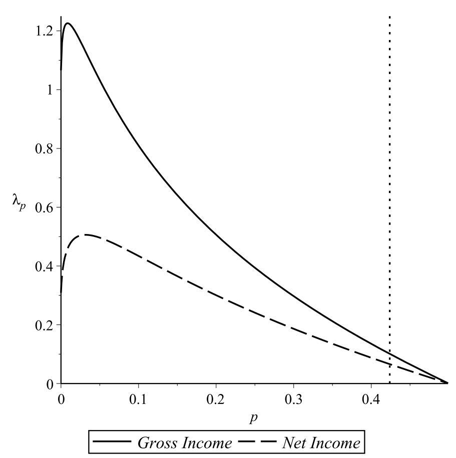
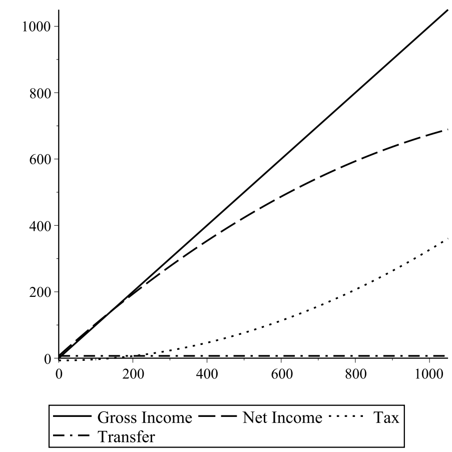
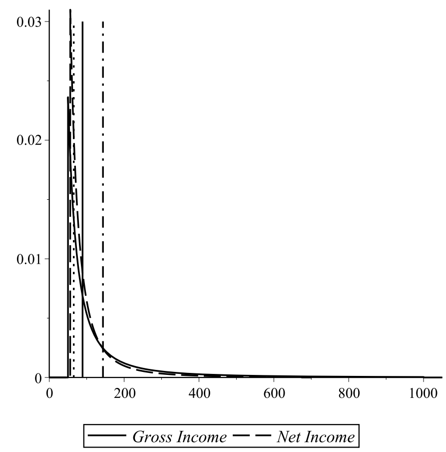
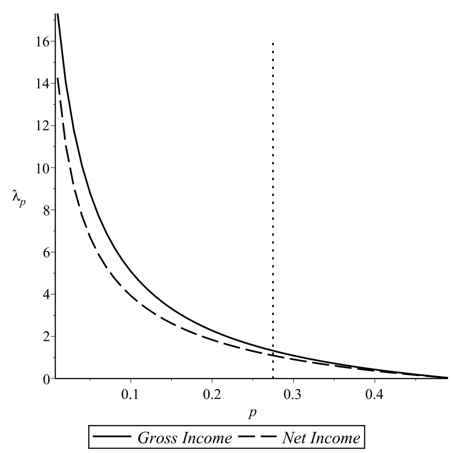


Figure B2. : Lognormal Distribution







The upper left figure shows gross income (the diagonal) and net income, tax (including transfers), and lump sum transfer as functions of gross income. The upper right figure depicts gross and net income density over gross income support. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$ ($\bar{p}$-quantile), 0.5 (median income), and $1-\bar{p}$ (mean), respectively. Note that the horizontal axis refers to gross income in the first case and to net income in the second. The lower figure depicts quantile measures of skewness, $\lambda_p(G)$ (solid line) and $\lambda_p(F)$ (dashed line) as functions of the skewness parameter p. The vertical line represents $\bar{p}$. For parameters see Table B3.

Figure B3. : Pareto Distribution

Table B3—: Numerical Results: Pareto Distribution

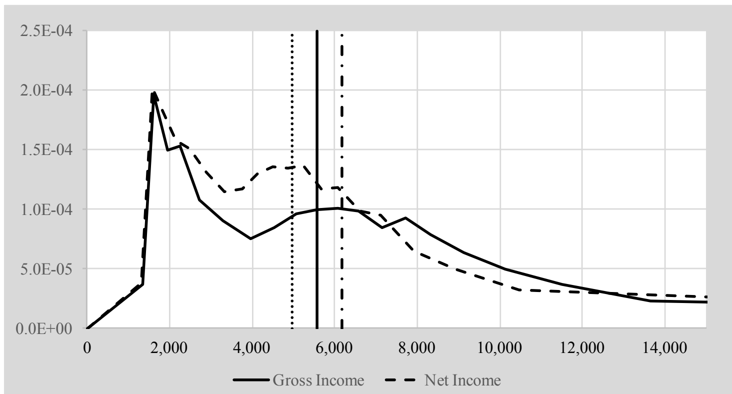
	Gross Income	Net Income
ρ	1.1483	1.7124
Max	1000	674
Min	50	56
Mean	143	113
Median	89	83
$\bar{p}$-Quantile	65	67
Conditional Mean	109	—
Transfer	—	7
Skewness	1.319	1.092

Notes. Efficiency: $\alpha = 0.50$; $\bar{p} = 0.27$.

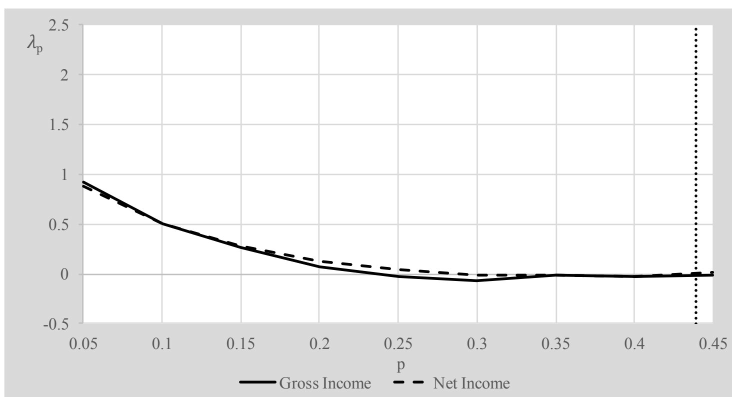
Appendix C. Income distributions and quantile measures of skewness: additional data (for online publication)

C1. Household data

Figures C1 – C8 that follow, cover UK data from 1979 to 2010.



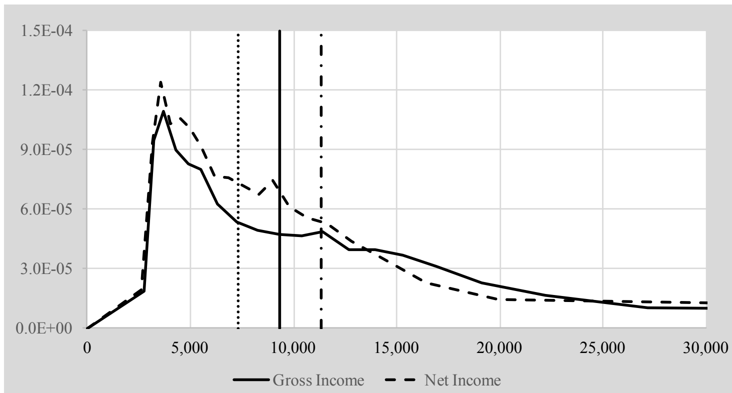
(a) Gross and Net Income Distribution



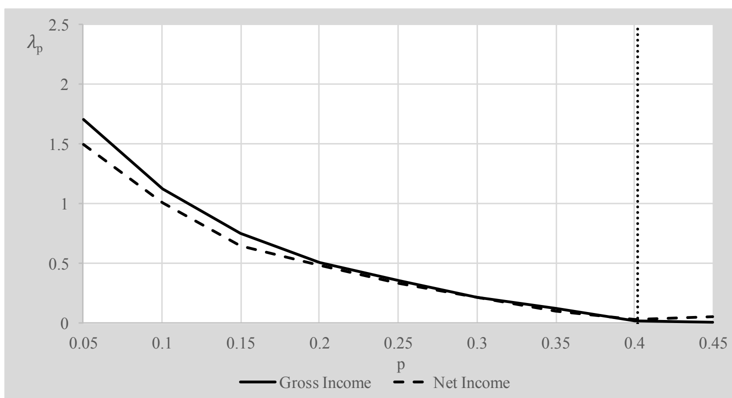
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C1. : Household data, UK'79



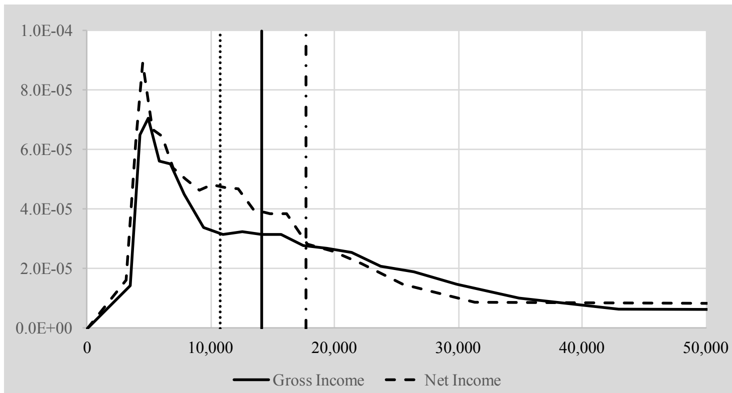
(a) Gross and Net Income Distribution



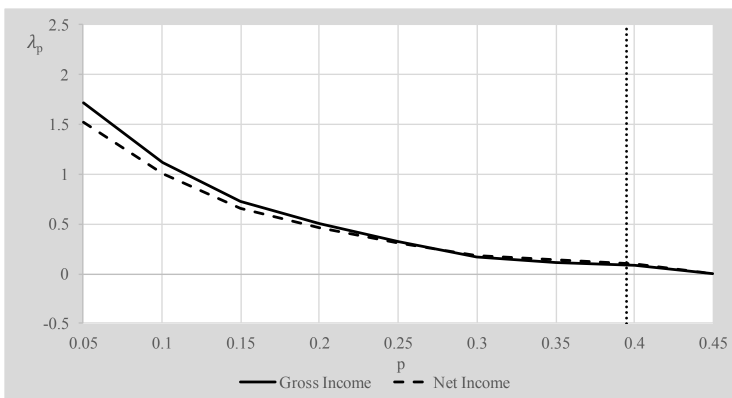
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C2. : Household data, UK'86



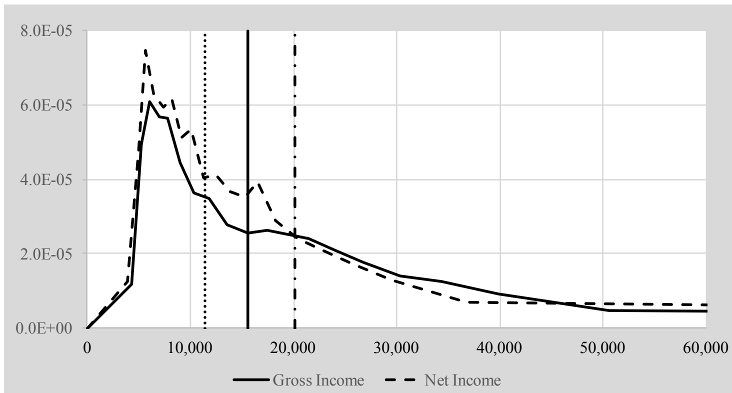
(a) Gross and Net Income Distribution



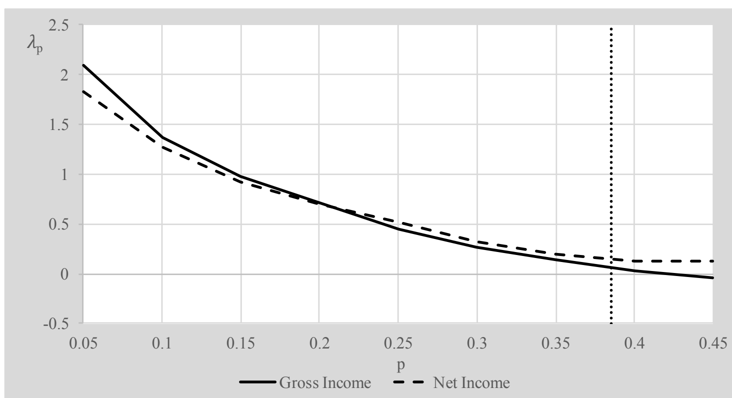
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C3. : Household data, UK'91



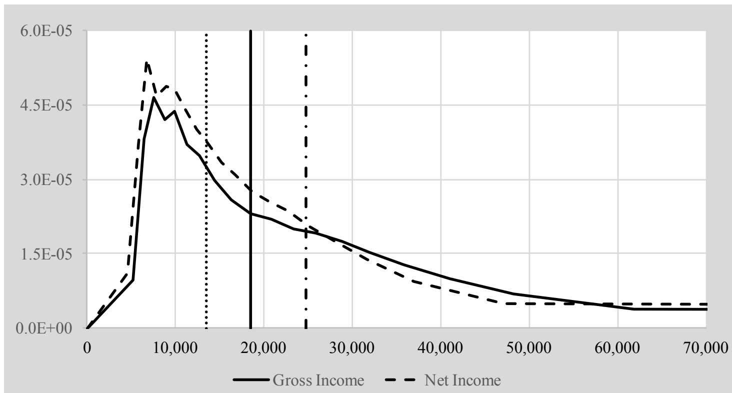
(a) Gross and Net Income Distribution



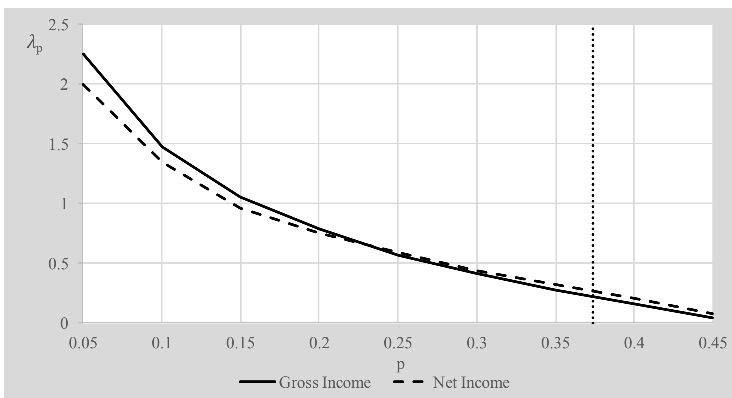
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C4. : Household data, UK'95



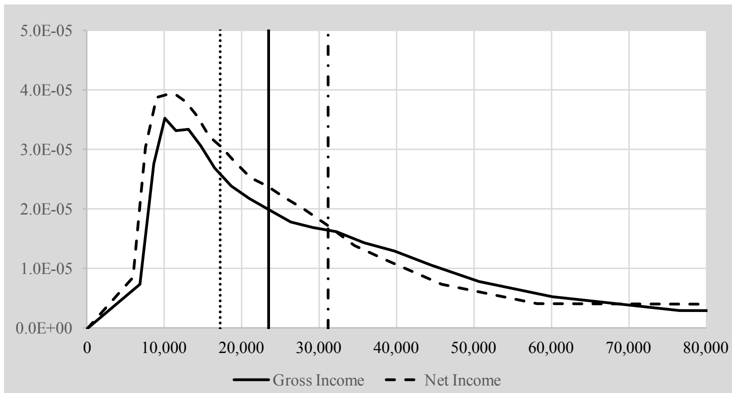
(a) Gross and Net Income Distribution



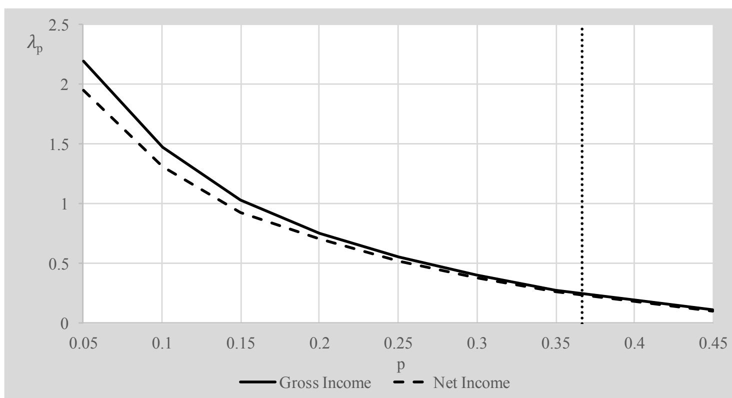
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C5. : Household data, UK'99



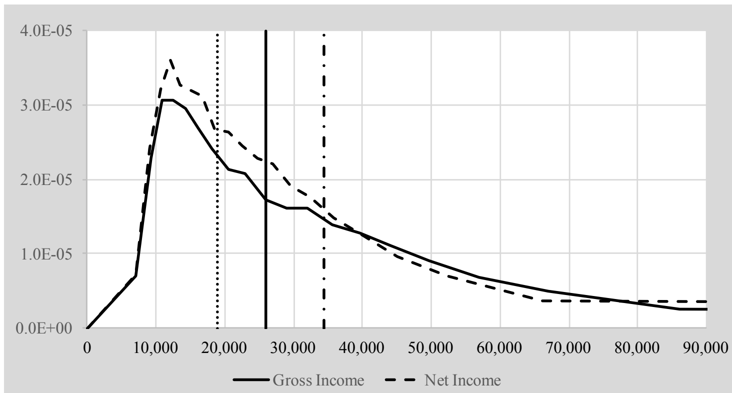
(a) Gross and Net Income Distribution



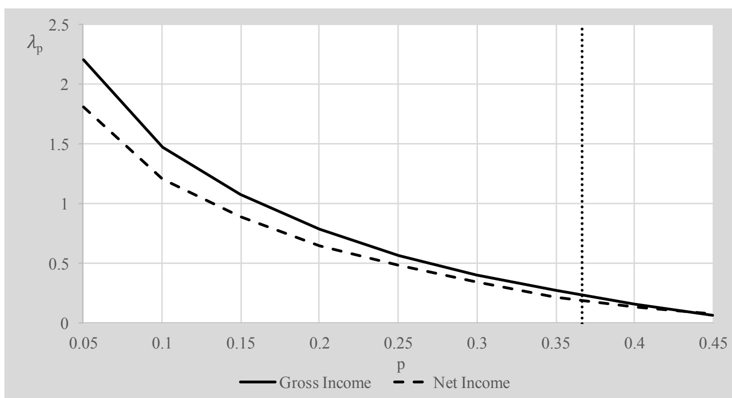
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C6. : Household data, UK'04



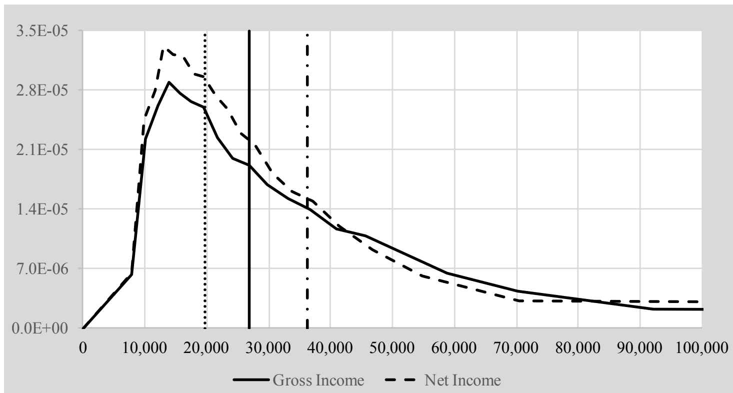
(a) Gross and Net Income Distribution



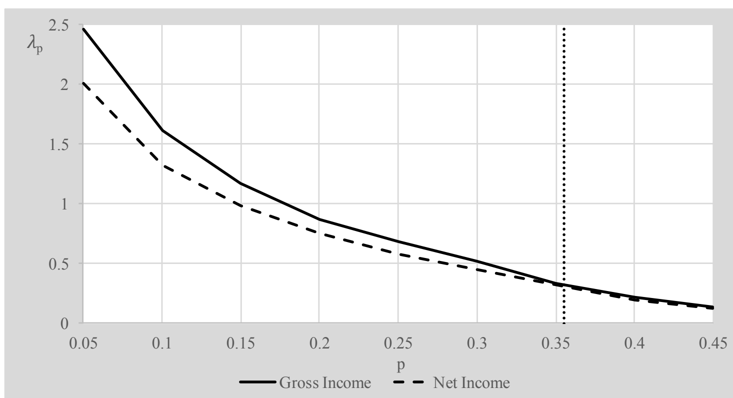
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C7. : Household data, UK'07



(a) Gross and Net Income Distribution



(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C8. : Household data, UK'10

Table C1—: Gross income distributions over time, household data.

	Gross Income (G)				
Year	Skewness Parameter	Mean	Median	$\bar{p}$-Quantile	Conditional Mean
	$\bar{p}$	$\mu^G_{1-\bar{p}}$	μ^G	$\mu^G_{\bar{p}}$	
1979	0.439	6,158	5,563	4,950	2,949
1986	0.402	11,360	9,307	7,289	5,078
1991	0.395	17,676	14,075	10,742	7,296
1995	0.386	20,095	15,546	11,361	8,282
1999	0.373	24,677	18,483	13,426	10,177
2004	0.367	31,146	23,400	17,181	13,214
2007	0.367	34,354	25,869	19,015	14,486
2010	0.355	36,147	26,754	19,602	15,552
2013	0.362	38,913	29,328	22,035	17,077

Table C2—: Net income distribution over time, household data.

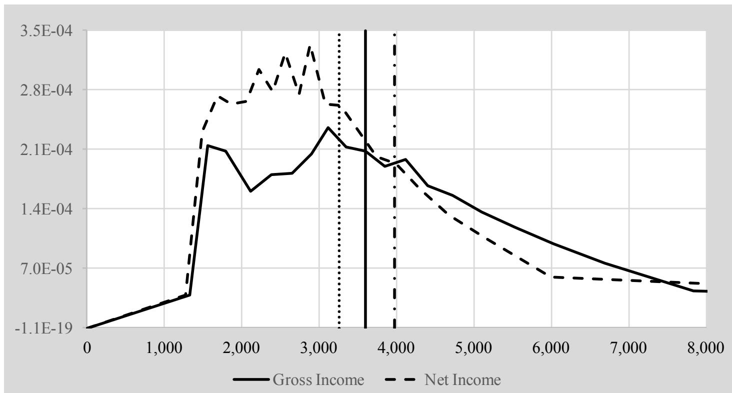
	Net Income (F)				
Year	Skewness Parameter	Mean	$1-\bar{p}$-Quantile	Median	$\bar{p}$-Quantile
	$\bar{p}$	$\bar{x}$	$\mu^F_{1-\bar{p}}$	μ^F	$\mu^F_{\bar{p}}$
1979	0.439	4,942	4,943	4,496	4,038
1986	0.402	8,909	8,929	5,073	6,217
1991	0.395	13,571	13,586	7,717	8,897
1995	0.386	15,740	15,659	8,784	9,722
1999	0.373	19,620	19,630	10,875	11,615
2004	0.367	24,827	24,666	13,674	14,653
2007	0.367	28,485	28,703	16,241	17,277
2010	0.355	29,894	30,247	16,955	18,036
2013	0.362	32,564	32,934	18,260	20,206

Table C3—: Gross and net income skewness over time, household data.

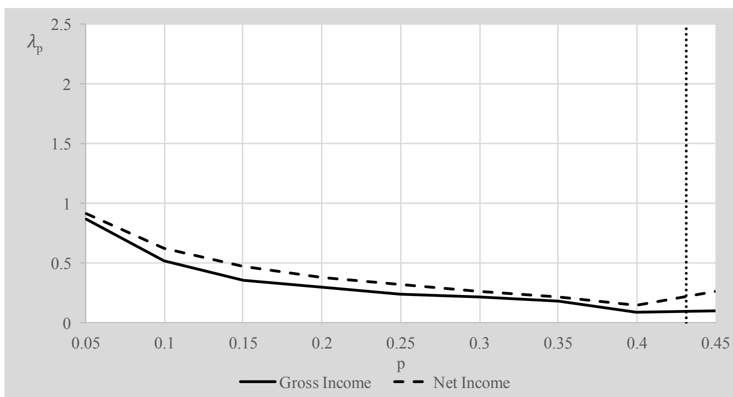
Year	Skewness Parameter	Quantile Skewness: Gross Income (G)	Quantile Skewness: Net Income (F)	Mean over Median	Relative Income Excess	Relative Income Shortfall
	$\bar{p}$	$\lambda_{\bar{p}}(G)$	$\lambda_{\bar{p}}(F)$	m^G	$1-\mu^G_{\bar{p}}/\mu^G$	$\mu^G_{1-\bar{p}}/\mu^G-1$
1979	0.439	-0.029	-0.024	1.107	0.110	0.107
1986	0.402	0.017	0.033	1.175	0.217	0.221
1991	0.395	0.080	0.108	1.207	0.237	0.256
1995	0.386	0.087	0.136	1.213	0.269	0.293
1999	0.373	0.225	0.270	1.253	0.274	0.335
2004	0.367	0.246	0.230	1.266	0.266	0.331
2007	0.367	0.238	0.181	1.251	0.265	0.328
2010	0.355	0.313	0.310	1.270	0.267	0.351
2013	0.362	0.314	0.261	1.275	0.249	0.327

C2. Equivalized data

Figures C9 – C17 that follow, cover UK data from 1979 to 2013. Equivalence scale is $n^{-0.5}$.



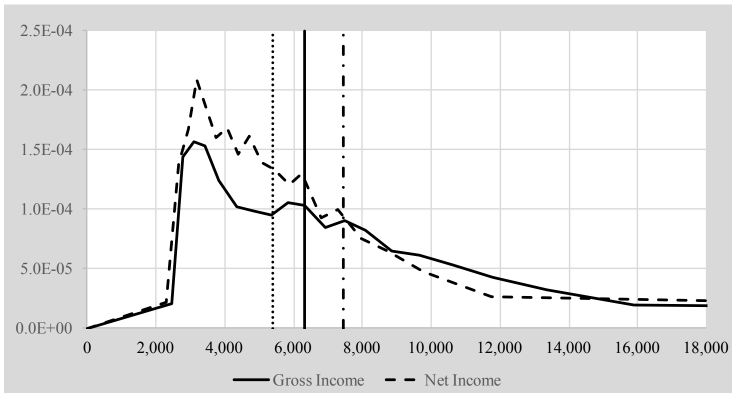
(a) Gross and Net Income Distribution



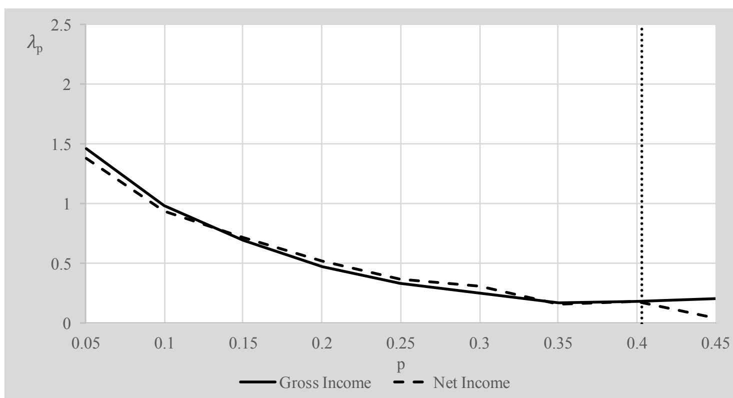
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C9. : Equivalized data, UK'79



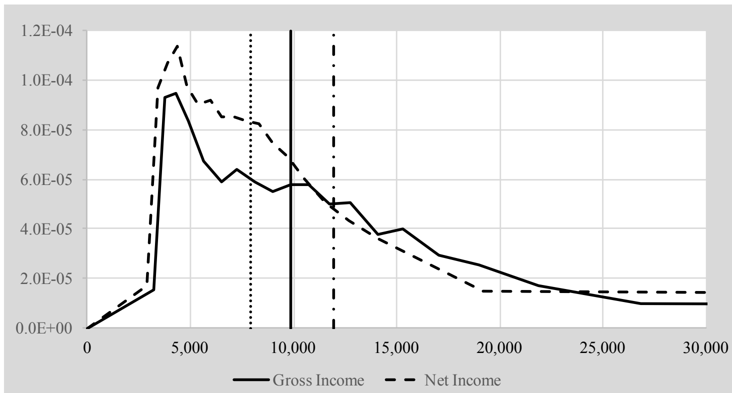
(a) Gross and Net Income Distribution



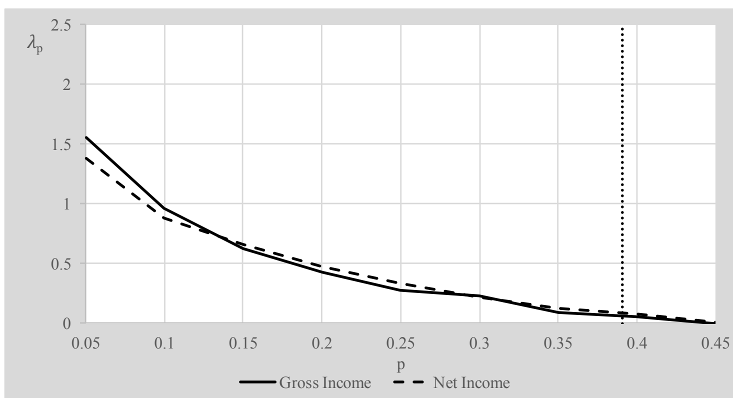
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C10. : Equivalized data, UK'86



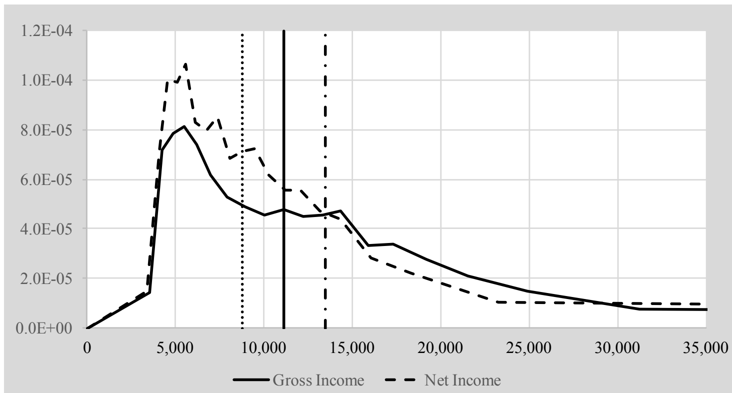
(a) Gross and Net Income Distribution



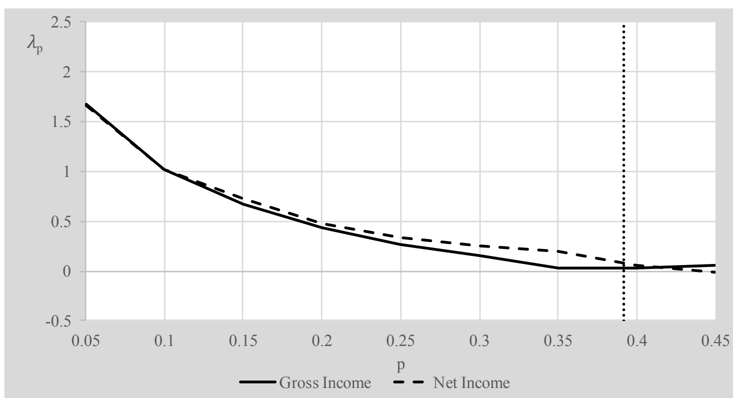
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1 - \bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C11. : Equivalized data, UK'91



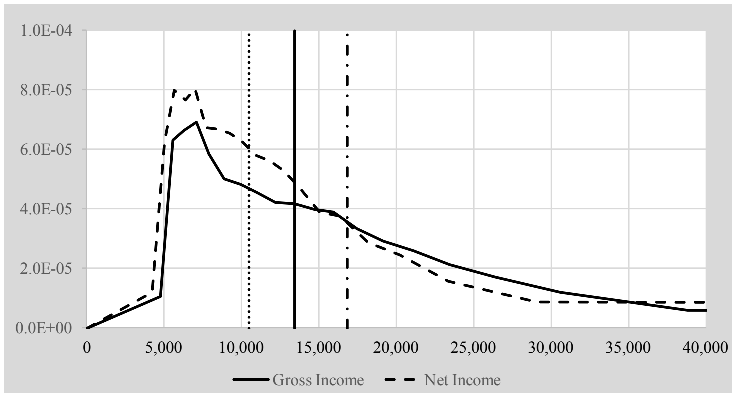
(a) Gross and Net Income Distribution



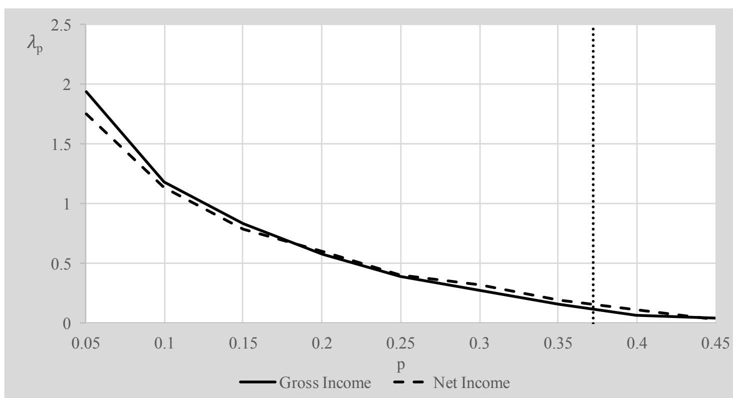
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C12. : Equivalized data, UK'95



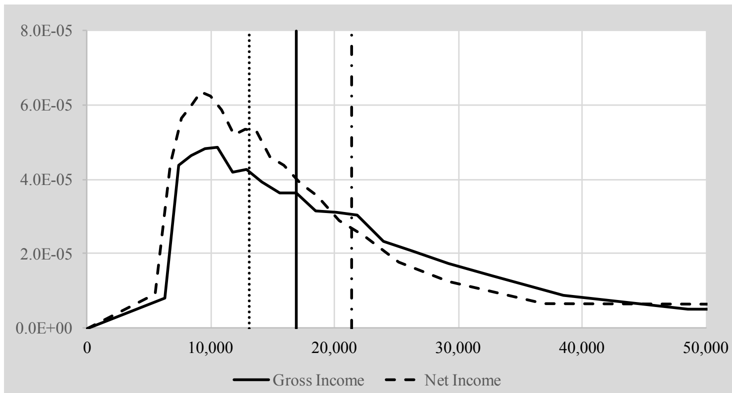
(a) Gross and Net Income Distribution



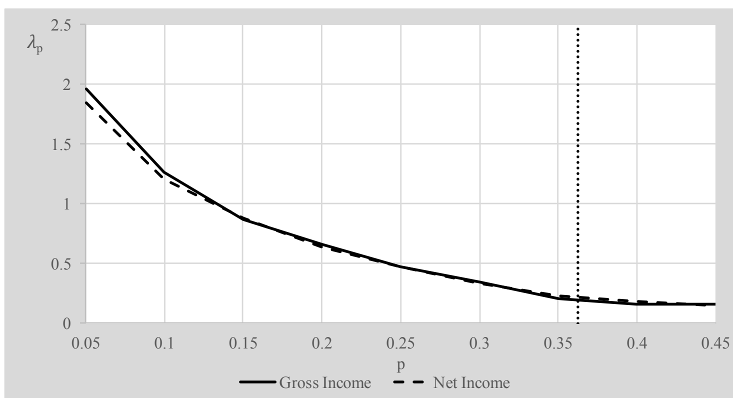
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1 - \bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C13. : Equivalized data, UK'99



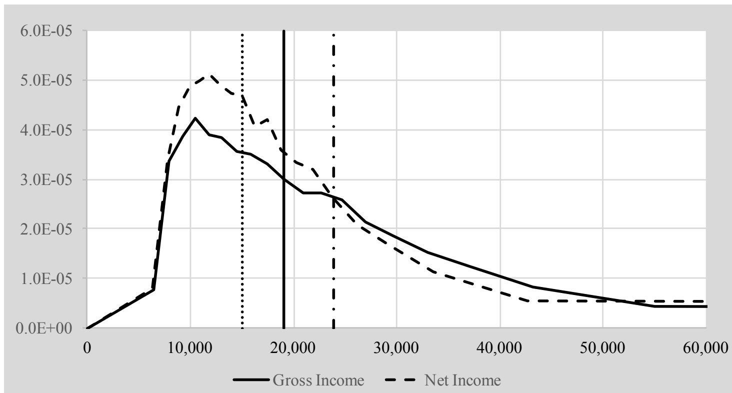
(a) Gross and Net Income Distribution



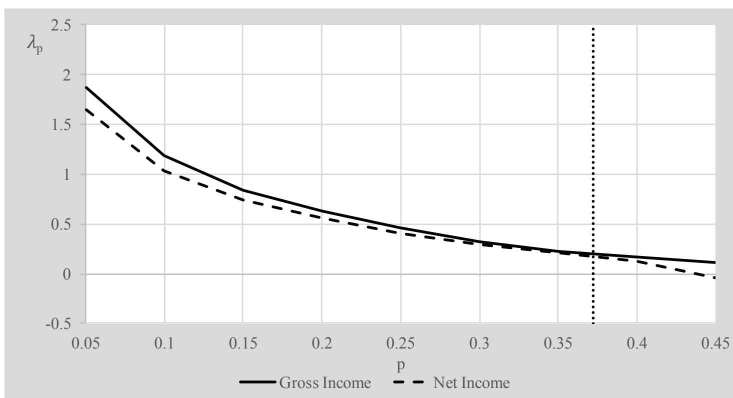
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C14. : Equivalized data, UK'04



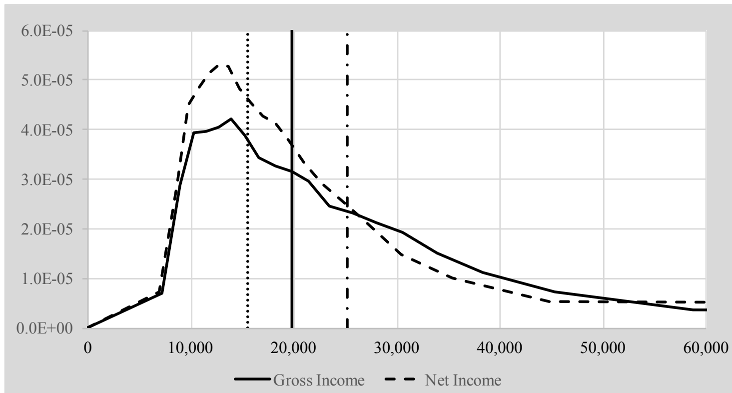
(a) Gross and Net Income Distribution



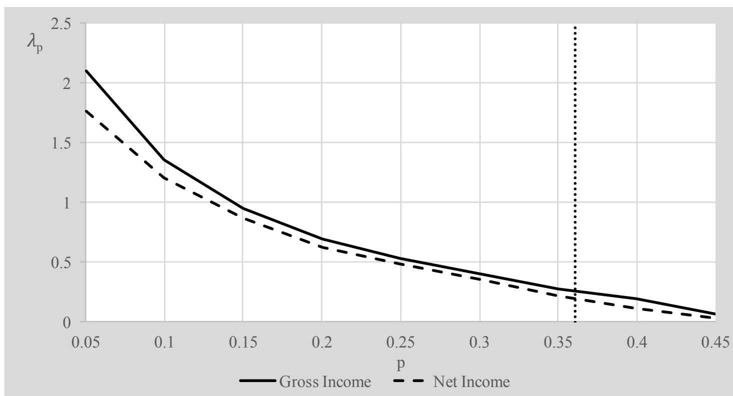
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1-\bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C15. : Equivalized data, UK'07



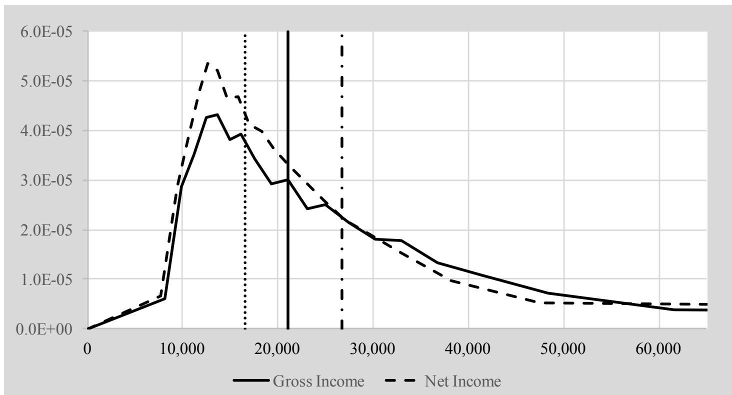
(a) Gross and Net Income Distribution



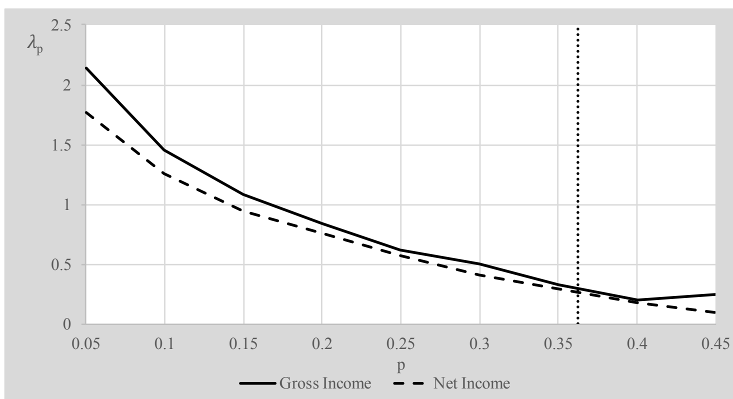
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1 - \bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

Figure C16. : Equivalized data, UK'10



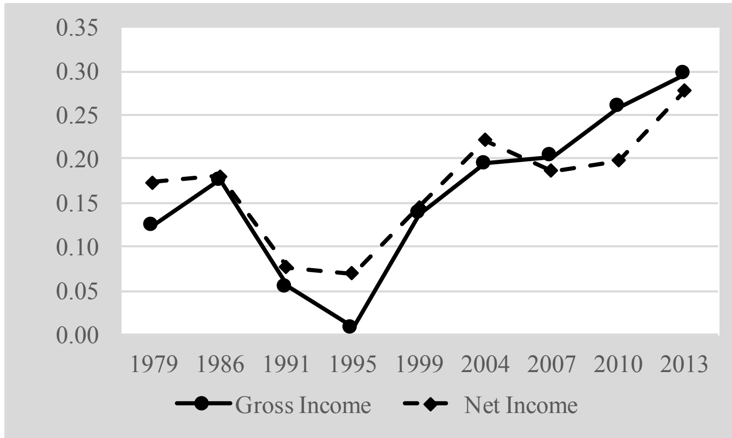
(a) Gross and Net Income Distribution



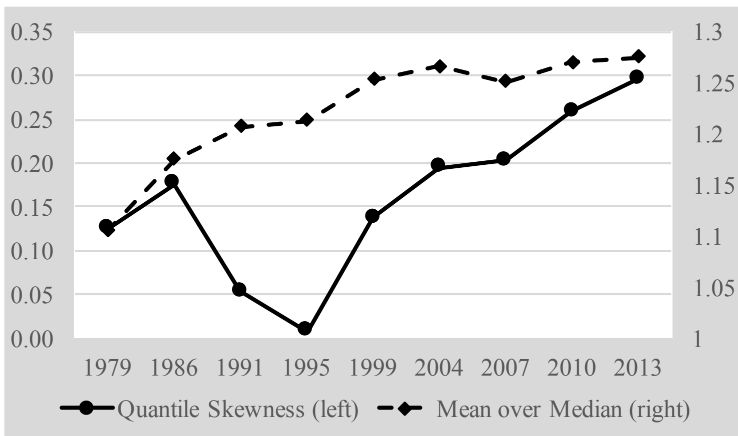
(b) Quantile Skewness

Notes. The top panel depicts gross and net income density over gross income support. The horizontal axis is truncated after the 95% gross income percentile. The vertical lines, from left to right, are gross income quantiles at $\bar{p}$, 0.5 (median income), and $1 - \bar{p}$, respectively. The bottom panel depicts quantile measures of skewness, $\lambda_p(G)$ and $\lambda_p(F)$ as functions of p. The vertical line represents $\bar{p}$.

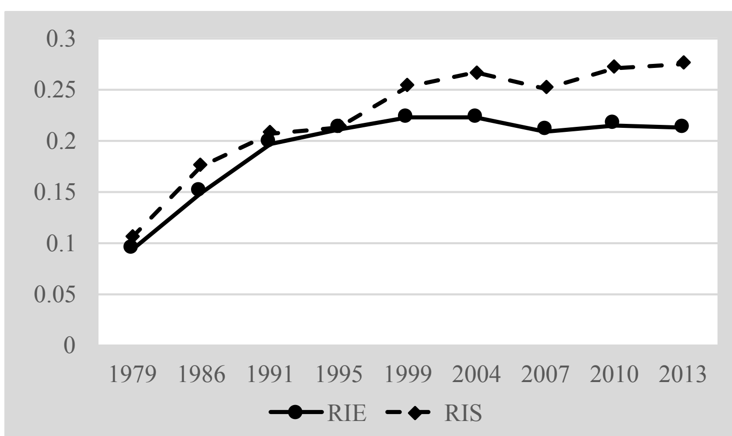
Figure C17. : Equivalized data, UK'13



(a) Quantile skewness of gross and net income distributions



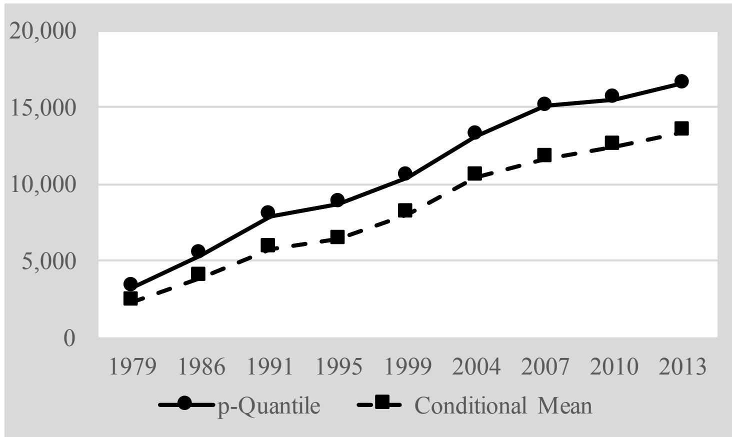
(b) Left y axis: quantile skewness of gross income. Right y axis: gross mean to median ratio



(c) Income excess/shortfall relative to gross median

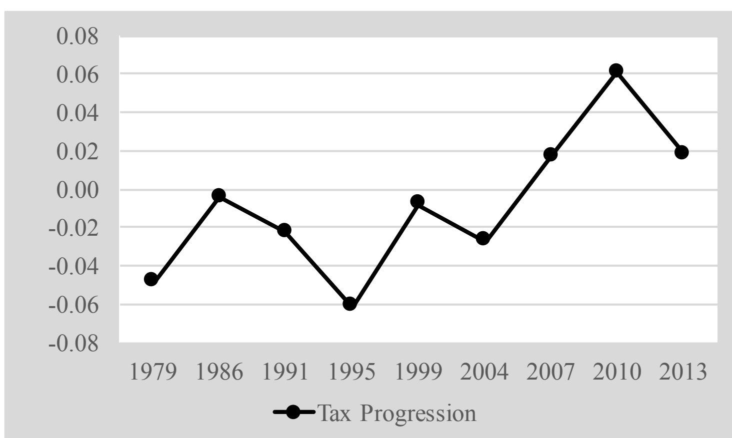
Notes. All panels use equivalized data. Top panel: quantile measure of skewness over time for gross and net incomes. Second panel: quantile skewness evaluated at $\bar{p}$ (left y-axis) and the mean over the median ratio (right y-axis). Bottom panel: Relative income excess and relative income shortfall over time.

Figure C18. : Quantile Skewness and Demand for Redistribution



Notes. Based on equivalized data. $\bar{p}$-quantile (solid line) versus conditional mean (dashed line) over time.

Figure C19. : Condition (7) for POUM hypothesis holds in equivalized data



Notes. Based on equivalized data. Difference between the quantile skewness measures of gross and net incomes evaluated at $\bar{p}$.

Figure C20. : Tax progression over time

Table C4—: Gross income distributions over time, equivalized data.

	Gross Income (G)				
Year	Skewness Parameter	Mean	Median	$\bar{p}$-Quantile	Conditional Mean
	$\bar{p}$	$\mu^G_{1-\bar{p}}$	μ^G	$\mu^G_{\bar{p}}$	
1979	0.431	3,964	3,588	3,253	2,295
1986	0.403	7,438	6,332	5,392	3,916
1991	0.391	11,920	9,875	7,935	5,759
1995	0.392	13,434	11,079	8,743	6,359
1999	0.372	16,785	13,400	10,425	8,024
2004	0.362	21,374	16,890	13,135	10,467
2007	0.372	23,820	19,044	15,074	11,625
2010	0.361	25,094	19,752	15,507	12,425
2013	0.362	26,761	20,990	16,536	13,364

Table C5—: Net income distributions over time, equivalized data.

	Net Income (F)				
Year	Skewness Parameter	Mean	$1-\bar{p}$-Quantile	Median	$\bar{p}$-Quantile
	$\bar{p}$		$\mu^F_{1-\bar{p}}$	μ^F	$\mu^F_{\bar{p}}$
1979	0.431	3,177	3,157	2,887	2,658
1986	0.403	5,812	5,844	5,073	4,420
1991	0.391	9,125	9,088	7,717	6,444
1995	0.392	10,503	10,447	8,784	7,229
1999	0.372	13,319	13,297	10,875	8,760
2004	0.362	17,038	16,855	13,674	11,071
2007	0.372	19,645	19,666	16,241	13,353
2010	0.361	20,680	20,639	16,955	13,880
2013	0.362	22,344	22,449	18,260	14,982

Table C6—: Gross and net income skewness over time, equivalized data.

	Skewness Parameter	Quantile Skewness		Mean over Median	Relative Income Excess	Relative Income Shortfall
		Gross Income (G)	Net Income (F)			
	$\bar{p}$	$\lambda_{\bar{p}}(G)$	$\lambda_{\bar{p}}(F)$	m^G	$1-\mu_{\bar{p}}^G/\mu^G$	$\mu_{1-\bar{p}}^G/\mu^G-1$
1979	0.431	0.125	0.174	1.105	0.093	0.105
1986	0.403	0.176	0.181	1.175	0.149	0.175
1991	0.391	0.054	0.077	1.207	0.196	0.207
1995	0.392	0.008	0.069	1.213	0.211	0.213
1999	0.372	0.138	0.146	1.253	0.222	0.253
2004	0.362	0.194	0.222	1.266	0.222	0.266
2007	0.372	0.203	0.186	1.251	0.208	0.251
2010	0.361	0.258	0.198	1.270	0.215	0.270
2013	0.362	0.296	0.278	1.275	0.212	0.275