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Learning and teaching mathematics K-7: Book 3

Jack Bana

Brian Farrell

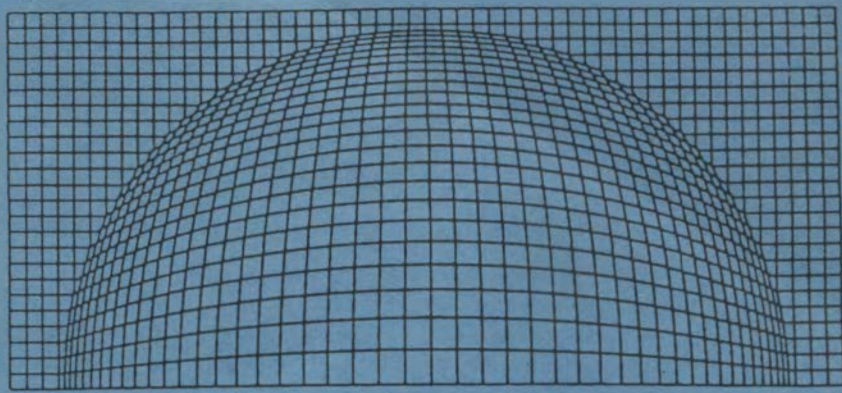
Ron Gleeson

Kevin Jones

Alistair McIntosh

See next page for additional authors

Authors
Jack Bana, Brian Farrell, Ron Gleeson, Kevin Jones, Alistair McIntosh, and Paul Swan



Teaching and Learning Mathematics K-7

Jack Bana
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Paul Swan

Book 3

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Table of Contents

Introduction

This book is designed as a text for the ECU unit MPE3108 and is the third one in a series. The first two books are as follows:

Swan, P. & Sparrow, L. (1996). *Teaching and Learning Mathematics K-7, Book 1.* Second edition. Perth: MASTEC, Edith Cowan University; and

Gleeson, R. & Farrell, B. (1997). *Teaching and Learning Mathematics K-7, Book 2.* Perth: MASTEC, Edith Cowan University.

In Book 2 there was an emphasis on preparing you for the classroom. There was a major focus on numeration and number computation in all its forms. Additional topics included further studies in space, measurement, data and graphical representation.

This text has a major emphasis on planning and assessment with particular attention to the EDWA Student Outcome Statements and associated pupil worksamples. Also included is further work on the major strands of Working Mathematically, Number, Measurement, Space and Chance and Data.

Chapter 1: Student Outcome Statements

As a result of studying this chapter and completing the set tasks you will:

- obtain an initial understanding of the WA documents related to the Mathematics Learning Area;
- have an understanding of the relationships among these documents, and between them and earlier EDWA curriculum documents including the K-7 Mathematics Syllabus;
- be aware of the meaning and implications of OBE (Outcomes Based Education); and
- begin to come to grips with implications of these documents for planning, teaching and assessing primary students in mathematics.

The Change

You will already be aware of a fundamental change affecting the curriculum for all schools in Western Australia. The heart of this change is a move from prescribing what is to be taught to prescribing what is to be learned. OBE (Outcomes Based Education), as it is often called, is quite new to Western Australia, and the move to full implementation in all learning areas is planned to take place from the beginning of 1999 to the end of 2003.

Everyone is feeling their way in this new situation and you must expect it to take you a great deal of time and study before you feel comfortable and knowledgeable within the system. Remember that everyone else from teachers in schools to university staff is in the same situation. You cannot expect to find ready-made solutions based on long experience to be always at hand.

This chapter is intended to do no more than start you on the road by ensuring that you are aware of, and have initial familiarity with, the various documents which have been introduced in their final form since the middle of 1998.

Relevant Documents

You will need access to the following documents — at least one between two. Locate them and distinguish between them.

- *Curriculum Framework*
- *Outcomes and Standards Framework: Student Outcome Statements —Overview* (not essential)
- *Outcomes and Standards Framework: Student Outcome Statements —Mathematics* (SOS)
- *Outcomes and Standards Framework: Work Samples —Mathematics*
- *Learning Mathematics: K-7 Mathematics Syllabus* (not essential)

Note the Internet locations below for the essential documents:

The WA Curriculum Council Curriculum Framework document is available at
http://www.curriculum.wa.edu.au/

The EDWA Outcomes and Standards Framework mathematics SOS and worksamples documents are at *http://www.eddept.wa.edu.au/centoff/outcomes/maths/mathmenu.htm*

Activities

Spend a few minutes looking through the documents. Try to establish the relationship between them. Mathematics is one of the eight learning areas. Name all eight before referring to the documents

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We will now spend some time looking at each of the main documents.

The Curriculum Framework

1. Read the purpose of the Curriculum Framework on page 6.

2. The Curriculum Framework consists (p. 15) of an Overarching Statement and eight Learning Area Statements (including one for Mathematics). The Overarching Statement (p.11) outlines seven key principles which underpin the Curriculum Framework. Find and list them:

____

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____

____

The Overarching Statement also describes the Overarching Learning Outcomes to which all learning areas contribute. There are thirteen of them and they are listed below:

1. Students use language to understand, develop and communicate ideas and information and interact with others.

2. Students select, integrate and apply numerical and spatial concepts and techniques.

3. Students recognise when and what information is needed, locate and obtain it from a range of sources and evaluate, use and share it with others.

4. Students select, use and adapt technologies.

5. Students describe and reason about patterns, structures and relationships in order to understand, interpret, justify and make predictions.

6. Students visualise consequences, think laterally, recognise opportunity and potential and are prepared to test options.

7. Students understand and appreciate the physical, biological and technological world and have the knowledge and skills to make decisions in relation to it.

8. Students understand their cultural, geographic and historical contexts and have the knowledge, skills and values necessary for active participation in life in Australia.

9. Students interact with people and cultures other than their own and are equipped to contribute to the global community.

10. Students participate in creative activity of their own and understand and engage with the artistic, cultural and intellectual work of others.

11. Students value and implement practices that promote personal growth and well-being.

12. Students are self-motivated and confident in their approach to learning and are able to work individually and collaboratively.

13. Student recognise that everyone has the right to feel valued and be safe, and, in this regard, understand their rights and obligations and behave responsibly.

(Curriculum Council, 1998, pp. 18-19)

3. (a) List below the numbers of the Overarching Learning Outcomes to which you think the mathematics learning area can significantly contribute and say why:

(b) List below the numbers of the Overarching Learning Outcomes to which you think the mathematics learning area can make some, though not significant, contribution and say why:

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4. The Mathematics Learning Area Statement begins on page 177. Locate the following sections of the Mathematics Learning Area Statement, which are common to all the learning area statements:

- Definition and Rationale
- Learning Area Outcomes (Read all of these)
- Scope of the Curriculum
- Learning, Teaching and Assessment
- Links Across the Curriculum
    - to the Overarching Statement
    - to other learning areas

5. Consider the relationship between the Overarching Learning Outcomes (pp. 18-19) and the Mathematics Learning Area Outcomes (pp. 180-181).

The Overarching Learning Outcomes are meant to be achieved through the eight learning areas, including of course mathematics. Look at each of the first five Overarching Learning Outcomes and decide which of the Mathematics Learning Area Outcomes link directly with each. List them below. One example is given:

OLO	MLAO
1	3, 4, 5
2	
3	
4	
5	

6. Now compare your entries with those on pages 42-45 of the Curriculum Framework.

7. Note the section on assessment (pp. 210-212). We will consider this section in more detail later.

It should be noted that the Curriculum Council's Curriculum Framework to be phased in over the 1999-2003 five-year period is mandatory for all schools and school systems in Western Australia. However, the Outcomes and Standards Framework and the associated Student Outcome Statements are and initiative of EDWA and will apply to EDWA schools. Non-EDWA schools can decide whether or not to use these.

EDWA Student Outcome Statements — Mathematics

1. The Student Outcome Statements in Mathematics are clustered under six strands. Name them.

2. Note that these strands correspond exactly to six of the seven clusters of the Mathematics Learning Area Outcomes in the Curriculum Framework (pp. 180-181). Find which is missing and find a reason given for its omission.

3. Look at the table on pages 6-7 of the Student Outcome Statements — Mathematics. Determine what each of the three columns represents (and where else in the documents each appears).

   (a) What is the significance of the numbers in bold type in the centre column?

(b) What is the significance of the arrows between Columns 2 and 3?

The descriptions of sub-strands, levels and pointers are set out in the same way for each of the strands. As an example we will look at the structure of the Working Mathematically Strand which appears on pages 11-50.

First there is a three-page fold-out on pages 12-13 which describes outcomes for each of the four sub-strands of the Working Mathematically Strand at each of eight levels. Before Level 1 is a row labelled FOS. Find what this means.

The right hand column (the part folded out) gives the strand outcome statements for each level of Working Mathematically. They summarise the outcomes of all four substrands at each level. It is this column which provides the focus for assessment of children. When the teacher assesses a child as being at Level 3 in the Working Mathematically strand, it means that the child has achieved the outcome described in the right hand column of the row labelled Level 3.

On pages 14-22, each of those strand outcome statements for each level of Working Mathematically is described more fully.

4. Locate the strand outcome statement for Level 3 of Working Mathematically on the fold-out page and then compare it with the fuller description given on page 17.

Pages 24-47 now take each of the level descriptors for each substrand of Working Mathematically, and provide pointers, or examples of actions which would indicate achievement at that level.

5. Find the pointers for Level 3 of each of the four substrands and compare them with the description you read on page 17. Do you find that the two complement each other, or do you find discrepancies between them? Discuss.

EDWA Student Outcome Statements — Work Samples

This book provides some annotated work samples showing children performing at levels indicated in each strand. You should be able to find your way around it and recognise its links to the Student Outcome Statements — Mathematics.

1. Find the four pages of examples of work of children performing at Level 3 of the Working Mathematically strand. Consider them carefully and see whether you can see the relation of the children's work and the annotated comments to the descriptions and pointers which you read earlier.

2. At what Year levels would you normally expect students to perform at Level 3 on these tasks? Explain.

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Learning Mathematics: K-7 Mathematics Syllabus

The 1989 K-7 syllabus gives considerable detail on the mathematical content for all those levels. For example, an entry in the number strand for Stage 7, corresponding approximately to Year 7, states:

> Record and interpret data using
>
> (a) standard tally;
>
> (b) standard tally, with grouped data.

Other information is then provided for the teacher under the headings of related syllabus entries, background, focus ideas and skills, focus questions, suggested materials, suggested activities, resources, and extensions. This format is typical of all entries at all levels.

1. Discuss how the syllabus differs from the current frameworks structure.

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2. Is the 'old' syllabus material still useful? Why?

Note that the emphasis in all school syllabuses has been on *content.* while the new emphasis is on performance. Thus, materials such as syllabus documents, textbooks, multimedia and so on are simply resources as means to achieving the stated outcomes for students.

References

Curriculum Branch, Ministry of Education. (1989). *Learning mathematics: Pre-primary to Stage 7 mathematics syllabus.* Perth: Author

Curriculum Council. (1998). *Curriculum framework for Kindergarten to Year 12 in Western Australia.* Perth: Author.

Education Department of Western Australia. (1998a). *Outcomes and standards framework: Student outcome statements — Mathematics.* Perth: Author.

Education Department of Western Australia. (1998b). *Outcomes and standards framework: Student outcome statements — Overview.* Perth: Author.

Education Department of Western Australia. (1998c). *Outcomes and standards framework: Worksamples — Mathematics.* Perth: Author.

Websites:

http://www.curriculum.wa.edu.au/

http://www.eddept.wa.edu.au/centoff/outcomes/maths/mathmenu.htm

Chapter 2: The Measurement Strand and Student Outcome Statements

As a result of studying this chapter and completing the set tasks you will:

- have a greater understanding of the Student Outcome Statements with respect to measurement;
- reflect on the various substrands in the measurement section;
- be able to compare and contrast the Student Outcome Statements with the WA K-7 Mathematics Syllabus; and
- successfully conduct some measurement activities.

The Measurement Strand

A brief summary of the nature and content of each strand in the Mathematics Learning Area is provided in the Student Outcomes Statements book. The description of the Measurement strand is as follows:

> The Measurement strand focuses on the choice and use of suitable units, tools and formulas for measurement, and the direct estimation and measurement of physical attributes and time. As a result of their learning, students are expected to make better and more skilful use of direct and indirect measurement and estimation to describe, compare, evaluate, plan and construct. Progression through the levels indicates that the student increasingly:
>
> - decides what needs to be measured and carries out measurements of length, capacity/volume, mass, area, time and angle to needed levels of accuracy;
> - selects, interprets and combines measurements, measurement relationships and formulas to determine other measures indirectly; and
> - makes sensible direct and in direct estimates of quantities and is alert to the reasonableness of measurements and results.

(EDWA, 1998a, p. 4)

Some Tasks

All of the following tasks should be carried out in groups of two or three.

1. Select two different containers. Find a way to determine 'Which one holds more?'. Draw a picture below to illustrate your result.

2. By using simple illustrations, sequence six activities you carried out in a day. Write the time for each activity beneath each drawing.

1.	2.	3.
4.	5.	6.

3. Estimate the length and width of this room.

Estimates: length _____ width _______.

Explain how you arrived at your estimates.

4. Find the mass of a grain of rice. Use the materials provided. Illustrate your method below, then explain it briefly in writing. Mass of grain of rice is: _______

5. How could you measure the surface area of an orange (or other spherical object)? Illustrate and describe the process.

6. Investigate shapes with an area of 36 square centimetres. Use the one-cm grid below to help you if necessary. Explain your processes and result below.

Student Work Samples

Obtain a copy of the Mathematics Work Samples publication. On pages 64, 70, 73, 74, 80, and 83 are student work samples of tasks that are identical or similar to those that you have completed above.

1. With a colleague, study and discuss the -
   - background,
   - child's work,
   - relevant outcome, and
   - summary comment, as provided for each of the above tasks.

2. For each example of a student's work, several comments are provided. Working with a colleague, see if you can justify these comments for the mass of a grain of rice (p. 80) by using the pointers from the Student Outcome Statements.

3. Examine your group's attempts and comment on them in comparison to the work samples. Provide assessment comments for your own work.

   Task 1:

   Task 2:

   Task 3:

Task 4: ______________________________

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Task 5: ______________________________

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Task 1: ______________________________

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The Substrands

The Measurement strand of the Student Outcome Statements consists of four substrands as follows.

- **Understanding Units** Students select what attributes to measure and what units to use.
- **Direct Measure** Students measure length, capacity/volume, mass, area, time and angle to needed levels of accuracy.
- **Estimate** Students make sensible direct and indirect estimates of quantities and are alert to the reasonableness of measurements and results.
- **Indirect Measure** Students select, interpret and combine measurements, measurement relationships and formulas to determine other measures indirectly.

(EDWA, 1998a, pp. 92-93)

1. For the substrand *Understanding Units* provide an activity which could be used to assess whether or not students are performing at Level 2.

____

____

____

2. For the substrand *Direct Measure* provide an activity which could be used to assess whether or not students are performing at Level 3.

3. For the substrand *Estimate* provide an activity which could be used to assess whether or not students are performing at Level 4.

4. For the substrand *Indirect Measure* provide an activity which could be used to assess whether or not students are performing at Level 5.

The K-7 Mathematics Syllabus

Many teachers would be familiar with the WA K-7 Mathematics Syllabus (Curriculum Branch, 1989). This is the document used by primary schools for the past 10 years to guide their mathematics curriculum. In the past the curricular emphasis has been on the mathematical content to be taught across the various stages or year levels. The focus now, via the SOS, is shifting to the outcomes. Thus the syllabus is now one of the resources which can be used to attain the outcomes.

1. Examine the measurement section of the syllabus and compare this with the Measurement Strand in the Student Outcome Statements.

   (a) Write comments on the different substrands.

(b) Write comments on the different emphases and layout.

____

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(c) Suggest reasons for the different organisation of the measurement strand in the Student Outcome Statements.

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(d) How could you use the WA K-7 Mathematics Syllabus in your planning of measurement activities?

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More Tasks

Below is another set of tasks suitable for various primary school levels. Many of these can be compared with similar examples in the Student Work Samples. Complete the tasks as you would expect a primary student to do so. Then, for each one, provide assessment comments on your efforts.

1. Compare your height to someone else in your group. The tallest person is ________________.
   Draw a picture to illustrate your finding.

2. Use 'handspans' to measure -
   the length of the cupboard about _______ handspans
   the length of the table about _______ handspans
   the width of the door about _______ handspans
   the width of a blackboard about _______ handspans
   Which object is the longest? ________________________

3. Compare your 'stride' length with three other people.
   If you all took 20 paces who would walk the furthest?
   ______________ would walk the furthest.

   Explain your conclusion.

____

____

4. Choose four or five objects and estimate their order of mass.

1________________2________________3________________4________________5
(lightest) (heaviest)

By balancing them against each other on a balance beam put them in order and compare the result with your estimation.

1________________2________________3________________4________________5
(lightest) (heaviest)

5. List some objects which have a mass of about -
   - a kilogram ________________________________
   - less than a kilogram ________________________________
   - more than a kilogram. ________________________________

6. Find two objects of the same volume but of different mass.

____

7. Investigate shapes with a perimeter of 48 centimetres and explain your findings.

____

____

8. Investigate the surface area of prisms made from 64 unit cubes and explain your findings.

____

____

References

Curriculum Branch, Ministry of Education. (1989). *Learning mathematics: Pre-primary to Stage 7 mathematics syllabus.* Perth: Author.

Education Department of Western Australia. (1998a). *Outcomes and standards framework: Student outcome statements — Mathematics.* Perth: Author.

Education Department of Western Australia. (1998b). *Outcomes and standards framework: Worksamples — Mathematics.* Perth: Author.

Chapter 3: Assessment and the Student Outcome Statements

As a result of studying this chapter and completing the set tasks you will:

- be aware of guidelines regarding assessment contained in the Curriculum Framework;
- be aware of a range of strategies for assessing children's levels and progress in mathematics; and
- be able to use the Student Outcome Statements to assess students' performance levels in mathematics.

Assessment and the Curriculum Framework

1. Read the section on assessment (pp. 210-212) of the Curriculum Framework, carefully analysing what is said about the relevance of each of the assessment items given in the margins of the pages.

   (a) Which assessment item (from [a] to [j]) do you think is particularly good? Say why.

   ____

   ____

   ____

   ____

   (b) Detail some doubts or query you have about one of the assessment items.

   ____

   ____

   ____

   ____

   ____

Assessment and the Student Outcome Statements

Assessing students' outcome levels of achievement and planning appropriate experiences based on these assessments is a complex process. At this stage we will confine ourselves to two goals — first, to understand the structure of the Student Outcome Statements (Mathematics) and what it is that we are assessing; and second, to have some practice in aligning examples of children performing practical tasks and also examples of children's work to levels of achievement or performance described in the Student Outcome Statements.

Assessment Requirements

All pupils must be assessed to determine the level they have achieved in each of the six Strands of the Student Outcome Statements (Mathematics). They are not necessarily assessed for each substrand. For primary children this means that each child is assessed in each of five strands: Working Mathematically, Space, Measurement, Chance and Data, and Number.

1. Which is the sixth strand, on which primary children are generally not assessed? Explain why not (Page 212 of the SOS may help).

____

____

2. Would there ever be a need to assess this strand for primary students? Explain.

____

____

3. What is the strand that has no assessable levels? Explain the reason.

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____

Achievement Levels

There are eight achievement levels covering kindergarten to Year 12 in the SOS. A rough unofficial but helpful guide is that most primary children will achieve up to Level 4. Some implications of this are that:

(a) children can be at the same level for eighteen months or so on average,
(b) children in Year 1 may well not have achieved any level, and
(c) you can expect that the children in any classroom may cover, for any strand, at least three levels.

Two things are important to remember here: first, levels have nothing to do with Year level; second, a child is not 'at' Level 2. Either the child has achieved Level 2, and is therefore working towards Level 3, or he/she is working towards Level 2, having achieved Level 1. A child will also very likely have achieved differing levels in different strands.

Structure of the Student Outcome Statements

We will take the Working Mathematically strand to illustrate the structure of each strand in the Student Outcome Statements (Mathematics).

1. There are three aspects to consider. Locate each of these in the Student Outcome Statements (Mathematics).

(a) Pages 12-13 (and the foldout) contain an overview of each of the four substrands (pages 12-13) and of the strand as a whole (foldout). This foldout contains the level statements against which the student is assessed.

(b) Pages 14-22 contain an elaboration of the strand outcome statement for each level. You will notice that each level (except Level 1) is described in four paragraphs.

(c) Pages 24-47 contain pointers for each level of each substrand. These pointers give detail actions on the part of children which would indicate that the child had achieved that level. These pointers are intended as examples only, and there is intentionally room left for schools and teachers to add further pointers.

2. Complete the table below, giving the page numbers for the elements for each of the strands.

Strand	Overview	Strand Outcome Statement	Pointers (Levels 1-2)	Pointers (Levels 3-5)
Working Mathematically	12-13	14-22	24-31	32-39
Space				
Measurement				
Chance and Data				
Number				
(Algebra)				

3. Why was a column for pointers for Levels 6- 8 not included in this table?

Student Work Samples

A separate book contains annotated Work Samples which provide examples of children's work with indications of the level shown by the piece of work. The following activities will ask you to use your observation and judgment to align examples of children's performance on particular tasks to achievement levels on particular sub-strands of the SOS (M). While you should find these activities valuable in familiarising yourself with structure of the SOS (M) and with the process of assessing against levels, it is important to remember two things:

(a) You should never in practice make formal assessments based on only one piece of work. Teachers will make judgments about levels based on experience and observation of children's performance on a wide variety of tasks over a prolonged period of time.

(b) Levels of achievement are assigned based on performance over all substrands of a strand, not on performance on an individual strand.

1. Turn to page 69 of the book of Work Samples (Mathematics). Look carefully at the various parts of the page and write answers to the following:

(a) Write down the four sub-strands of the Measurement strand and tick those which you think might be relevant to this activity:

____________________ ____________________

____________________ ____________________

(b) Two relevant outcomes for the activity are given at the top right hand side of the page. What does each symbol in the code M2.3 mean?

The M means ____________________

The 2 means ____________________

The 3 means ____________________

(c) On which page of the SOS (M) does each of these outcomes appear?

M 2.2 is on page _____. M 2.3 is on page _____

(d) For each of these two outcomes, decide which pointer(s) is(are) relevant to the activity and write these below:

M2.2 ____

____

M2.3 ____

____

(e) For each of these two outcomes, look at the description for the level immediately above and below that given. Write your reasons for either agreeing or disagreeing with the suggested level for each outcome.

M2.2 ____

____

____

M2.3 ____

____

____

2. The following task is modified from the book of Work Samples (Mathematics), but there is no information about it. Consider the task and result and write below what you judge to be the missing elements, including your own assessment of the level of achievement.

> Task: Working in pairs make and test a spinner that you think is most likely to stop on red, least likely to stop on green and have the same chance of stopping on yellow and blue. In your report, say what you did and what you found.

We made a spinner with 8 equal parts.
So we had 3 parts red
2 " yellow
2 " blue
1 " green
We spun the spinner 30 times and got these results.

R	Y	B	G
卌 卌 |	卌 ||	卌 ||	卌

Yellow and blue got the same results. Green had the least and as expected Red had the most.
We then made a spinner with unequal parts. But still having the rules as in the question.

We then spun the spinner 30 times. Here are our results.

R	Y	B	G
卌 卌 卌	卌 |	卌 ||	||

Level: ___________

Relevant Outcomes (2)

Summary Comment

Assessing Student Performance

Watch each of the following video clips of children being interviewed as they perform a range of practical tasks. Relevant pages of the SOS (M) are given from each. After you have watched each video clip, look at the references to the SOS (M) and write your assessment of the levels of some of the children's performances, giving your reasons.

<table>
<tr><th colspan="3">Video Clip 1</th></tr>
<tr><td>Year 3</td><td colspan="2">Strand: Space</td></tr>
<tr><td>Incident: Plans of classroom
Timing: 15:30 - 26:00
Length: 10:30</td><td colspan="2">Substrands:
1. Represent Location
2. Represent Space
3. Represent Transformation
4. Reason Geometrically</td></tr>
<tr><td>Strand Outcome Statements:</td><td colspan="2">pages 55-57</td></tr>
<tr><td>Student Outcome Statements:</td><td>S1.1, S2.1, S3.1
S1.2, S2.2, S3.2, S4.2</td><td>(pp. 65, 72)
(pp. 67, 74-75)</td></tr>
<tr><td>also</td><td>M3.4b</td><td>(p. 118)</td></tr>
</table>

Your comments and assessments of Video Clip 1:

Video Clip 2		
Year 3	Strand: Measurement	
Incident: Weight of Frogs Timing: 1:50 - 10:10 Length: 8:20	Substrands: 1. Understand Units 2. Direct Measure 3. Estimate 4. Indirect Measure	
Strand Outcome Statements:	pages 95-98	
Student Outcome Statements:	M1.1, M2.1, M3.1, M4.1 M1.2, M2.2, M3.2, M4.2	(pp. 104-105, 112-113) (pp. 106-107, 114-115)

Your comments and assessments of Video Clip 2:

Video Clip 3		
Year 3	Strand: Number	
Incident: "9 take 8" Timing: 6:45 - 11:02 Length: 4:17	Substrands 1. Understand Number 2. Understand Operations 3. Calculate 4. Reason About Number Patterns	
Strand Outcome Statements:	pages 174-178	
Student Outcome Statements:	N1.2, N2.2, N3.2, N4.2 N1.3, N2.3, N3.3, N4.3	(pp. 187, 192-193) (pp. 189, 194-195)

Your comments and assessments of Video Clip 3:

Video Clip 4	
Year 5	Strand: Space
Incident: Properties of Prisms Timing: 12:40 - 15:00 Length: 2:20	Substrands: 1. Represent Location 2. Represent Space 3. Represent Transformation 4. Reason Geometrically
Strand Outcome Statements: pages 55-58	
Student Outcome Statements: S1.4, S2.4, S3.4 (pp. 70-71, 78-79)	

Your comments and assessments of Video Clip 4:

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Video Clip 5	
Year 5	Strand: Measurement
Incident: Comparing Two Glasses Timing: 19:25 - 21:18 Length: 1:53	Substrands: 1. Understand Units 2. Direct Measure 3. Estimate 4. Indirect Measure
Strand Outcome Statements: pages 95-98	
Student Outcome Statements: M1.1, M2.1, M3.1, M4.1 (pp. 104-105, 112-113) M1.2, M2.2, M3.2, M4.2 (pp. 106-107, 114-115)	

Your comments and assessments of Video Clip 5:

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Video Clip 6		
Year 5	Strand: Number	
Incident: Plans of classroom Timing: 15:30 - 26:00 Length: 10:30	Substrands 1. Understand Number 2. Understand Operations 3. Calculate 4. Reason About Number Patterns	
Strand Outcome Statements:	pages 174-178	
Student Outcome Statements:	N1.2, N2.2, N3.2, N4.2 N1.3, N2.3, N3.3, N4.3	(pp. 187, 192-193) (pp. 189, 194-195)

Your comments and assessments of Video Clip 6:

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References

Curriculum Branch, Ministry of Education. (1989). *Learning mathematics: Pre-primary to Stage 7 mathematics syllabus.* Perth: Author

Curriculum Council. (1998). *Curriculum framework for Kindergarten to Year 12 in Western Australia.* Perth: Author.

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Notes

Chapter 4: Assessment in the Classroom

As a result of studying this chapter and completing the set tasks you will:

- identify different reasons for and purposes of assessment and evaluation;
- identify and evaluate a variety of assessment strategies for the primary classroom; and
- appreciate the role of assessment in the learning process.

Assessment and Evaluation: Is there a Difference?

Webb and Welsch (1993) draw the following distinction between the two.

> Evaluation is using standardised tests, surveys, demographic analyses, and other means to determine the value of a program. Assessment is using classroom tests, observations, questioning, projects, portfolios, and other means to determine what a student knows, can do, feels, and believes.
>
> (Webb and Welsch, 1993, p. 299)

In simple terms, assessment is focussed more on the individual child rather than to the broader goals of evaluation. In this chapter we will mainly be concerned with assessment strategies as they apply to the classroom.

Purposes of Assessment

Before detailing some of the many different assessment strategies it is important that we consider the major reasons for assessing student progress in the classroom.

1. Discuss this issue with a colleague. List some of the purposes of assessment below, then share your ideas with your group.

Webb and Welsch (1993) suggest the following as some of the reasons for evaluation and assessment:-

(i) To direct student learning.

Teachers use a variety of formal and informal techniques such as questioning, testing, observing, marking work etc to determine children's understanding of different aspects of mathematics. This is the most important purpose of assessment. The Student Outcome Statements are designed to give direction to student learning as well as guiding assessment.

(ii) To place students into groups or programs.

Rightly or wrongly students can be placed into groups or classes on the basis of results of tests and/or teacher observations. This may be to select students for a remedial program, for an enrichment program, for the formation of classes with more than one year group, or for the formation of groups within a class.

(iii) To grade students.

Students are given grades that will be used in reporting to parents and later employers and institutions. More significant grades are based on assessments that are either norm-referenced (e.g., grades for your core units of study) or criterion-referenced (e.g. grades for your teaching practice). As indicated in Chapter 3, the Student Outcome Statements are designed to validate and make more uniform the grades given to students.

(iv) To motivate students.

Some children will be extrinsically motivated by tests and assignments and the desire to do well. Webb and Welsch however, suggest that "informal assessment that explores student knowledge and that creates interesting and perplexing situations can build on intrinsic motivation that will lead to deeper understanding of mathematics". (Webb and Welsch, 1993. p. 302)

(v) To identify student attitudes.

A number of assessment strategies can be employed to give an indication of children's feelings and perceptions of mathematics. These include attitude tests using techniques such as a Likert scale, and student journal writing.

Assessment Strategies

In their CD-ROM package Herrington et al (1997) list the following strategies:-

Observing	Checklists
	Anecdotal records

Questioning	Higher order questioning
	Factual questioning
	Open-ended questioning
Interviewing	Structured interviews
	Open interviews
	Parent interviews
Testing	Diagnosis
	Performance-based
	Pencil and paper
	Multiple choice tests
	Problem solving
	Attitude
Reporting	Oral report
	Written report
	Portfolio
	Investigation
	Modelling
Self-Assessment	Journal writing
	Reflective prompts
	Self-questioning
	Peer assessment

1. Consider the list of strategies above and check those you would assume most teachers would use. Place a question mark next to those that you are unfamiliar with. Compare your results with others in the group or class. Peruse several of the unfamiliar ones on the CD-ROM. Write a sentence or two to indicate what you found out.

2. The list of assessment strategies exemplified by Herrington et al (1997) is not meant to be an exhaustive one. Can you think of any others? Discuss with colleagues and list them below.

In reference to the Question 2 above, did you include concept maps, for example, as an assessment strategy? We will now consider some of the strategies listed above in a little more detail, so that you may explore them further.

The Structured Interview

In Chapter 3 the structured interviews examined in with the videoclips were used as a strategy for assessing children's understanding in space, measurement and number. In the classroom the teacher should take opportunities to question and discuss with children both formally and informally regarding their understanding of mathematics. This is a very fruitful strategy for determining the individual student's thinking.

1. What are the advantages/disadvantages of this assessment strategy?

2. What demands does it place on the teacher?

Paper and Pencil Tests

The paper and pencil or written test is an assessment strategy used by most teachers whether it be written work collected at the end of a lesson, a teacher-constructed test or a standardised test from a testing agency like ACER. The most important function of a test is not so much to give children a mark or grade but to gain some insight into children's understanding of that area of mathematics. The Curriculum Framework emphasises the importance of the educative value of assessment: "Assessment should make a positive contribution to students learning"

(Curriculum Council, 1998, p. 211)

The written test often suffers from two problems, as follows:

(i) Work is often marked right or wrong resulting in children gaining no reward for the knowledge demonstrated through the processes used.

(ii) There is little follow-up of test items with individual children; though this is of less concern if the purpose of the test is the evaluate a program or a group of students as a whole, as many of the standardised tests do.

1. Mark the sample "test paper" provided, assigning marks for each question in a way that better reflects the "student's" performance. The total mark allocation for the test is to be 25. To a colleague, explain your marking criteria, the major deficiencies noted re concepts and skills, and the order in which you would try to remediate these.

Student Test Sample

1. (a) Write in digits, *seventy five thousand and thirty three*. 7533

   (b) Now write the number that is *one thousand* more. 7633

2. Write in ascending order: 28, 15, 8, 6, 3, 2. 28, 15, 8, 6, 3, 2

3. Add the numbers 16, 378, and 4273

$$\begin{array}{r} 16 \\ 378 \\ +\ 4273 \\ \hline 4667 \end{array}$$

4. Find the difference between 3.142 and 6.

$$\begin{array}{r} 6 \\ -\ 3\cdot142 \\ \hline 3\cdot142 \end{array}$$

5. What is the area of a tunnel's roadway that is 36.5 m long and 4.3 m wide?

$$\begin{array}{r} 36\cdot5 \\ \times\ 4\cdot3 \\ \hline 1095 \\ +\ 14600 \\ \hline 15695 \end{array}$$

6. A cricketer scored these runs in five innings: 16, 15, 20, 37, 12. What was her average score?

$$\frac{16+15+20+37+12}{5} = \frac{90}{5} = 18$$

7. There are 32 children in a class. One quarter of them are boys. How many are girls?

$$\frac{1}{\cancel{4}_1} \times \frac{\cancel{32}^8}{1} = 8$$

8. Ron has $^1/_4$ metre of string and halves it. How long is each piece?

9. What time is shown on the clock faces?

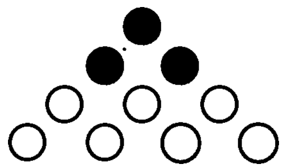

8:00


8:25

10. A 12 metre piece of string is used to mark out a rectangular garden plot which is twice as long as it is wide. What is its area? $\boxed{8 \quad 4}$ A = L x B = 8 cm x 4 cm = 32 cm

2. Indicate how would you follow up each test item below with individual children in a way that enables the item to be the medium for a learning experience.

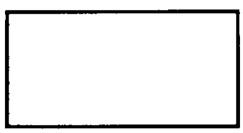
(i) Write the decimal that shows the part that has been shaded.



Answer: 3.7

(ii) Calculate the area of the rectangle below.

60 cm



30 cm

Answer: 1800 cm

(iii) Place either >, < or = between the fractions below to make a true statement.

Answer: $^{1}/_{3} < {}^{1}/_{10}$

Writing as an Assessment Strategy

The use of children's writing as both an assessment tool and a learning experience is very well documented in the NCTM *Standards*:-

> Middle school students should have many opportunities to use language to communicate their mathematical ideas . . . Opportunities to explain, conjecture, and defend one's ideas orally and in writing can stimulate deeper understandings of concepts and principles . . . Writing and talking about their thinking clarifies students' ideas and gives the teacher valuable information from which to make instructional decisions.

(NCTM, 1989, pp. 78-79)

The students' writing tasks can take many different forms, such as -

- observations about a graph,
- an explanation of how they solved a problem,
- a story problem for a simple number operation, and
- explaining their understanding of a concept (e.g., symmetry, capacity, area).

1. Write an explanation of what you understand by one of the following:
   area, capacity, symmetry.
   Then share your explanation with your group.

Explanation of area, capacity or symmetry:

Another useful activity is to select a concept, such as multiplication, and ask students to list all the concepts, ideas, activities, applications associated with the concept.

2. Read the following solutions/explanations for the given problem. What does it tell you about each student's level of understanding? Use the Student Outcomes publication to guide you.

A man has three bags containing a mixture of black and white balls to choose from, wearing a blindfold. To win a $5 000 prize he has to draw a black ball from one of the bags. Which bag should he choose to draw from to give himself the best chance of winning the $5 000? Justify your choice mathematically.

Bag A	contains:-	4 black balls	2 white balls
Bag B	contains:-	5 black balls	3 white balls
Bag C	contains:-	6 black balls	4 white balls

Student 1:

By drawing a diagram of 3 bags and placing the black & white balls inside we can easily see that Bag C has the most black balls inside. By choosing Bag C the man would have the greatest chance of choosing a black ball. A simplier way would be to look at the question and we can see that Bag C contains 6 black balls which is more than A & B.

Comments:

Student 2:

A 6 balls, 4B, 2W = $\frac{1}{3}$
B 8 balls 5B, 3W = $\frac{5}{8}$
C 10 balls 6B, 4W = $\frac{3}{5}$

A = $0.33\bar{3}$
B = 0.625 *
C = 0.6
Answer = Bag B

Comments: ____

____

____

Student 3:

all the bags give him an equal chance to draw out a black ball. Each bag contains 2 more black balls than white balls - the balls are only increasing in numbers but still have an even chance of winning the man a $5000 prize.

Comments: ____

____

____

Affective Issues

Assessment is too often concerned with cognitive learning outcomes ignoring the affective outcomes of a lesson or program. Affective outcomes relate to attitudes, interests, feelings, values, and beliefs. Many children (and adults) have a negative attitude towards and often a fear of mathematics. Some common complaints are *maths is boring, I'm no good at maths, I don't understand maths, maths is meaningless,* and so on. As teachers we have to be aware of the importance of creating positive attitudes towards mathematics. Factors such as confidence, persistence, perceived usefulness and motivation to learn are important in learning.

> What students believe about mathematics, what they believe about themselves doing mathematics, how they get excited about doing mathematics, and how they value mathematics all play a central role in mathematics learning and instruction and should be assessed. The NCTM *Curriculum and Evaluation Standards* provides seven areas to consider when assessing disposition toward mathematics: (a) confidence in using mathematics; (b) flexibility in exploring mathematical ideas; (c) willingness to persevere in mathematical tasks; (d) interest, curiosity and inventiveness in doing mathematics ; (e) inclination to monitor and reflect on their own thinking and performance; (f) valuing of the application of mathematics; and (g) appreciation of the role of mathematics in our culture . . . Systematically observing students, talking with students, and recording information using checklists based on the above seven areas are means for assessing mathematical disposition.
>
> (Webb and Welsch, 1993, pp. 306-307)

You may recall an attitude scale that you completed in the early phase of the MPE1108 unit.

Journal Writing

A less common assessment strategy is journal writing.

> Asking children to keep a journal or a diary of their mathematical experiences can provide teachers with an opportunity to directly assess students' achievements, as well as enabling students to make judgements about their own capabilities. Journals can also give insights into the way students feel, about their learning, their beliefs and the difficulties they are experiencing. Journals can be used regularly after each lesson or less frequently to reflect upon a topic of work.
>
> (Herrington et al, 1993, p. 35)

Burns (1995, p.52) suggests the following structure for journal writing:

- Write about what you did.
- Write about what you learned.
- Write about what you're not sure about or wondering about.

Teachers can structure the writing to more specifically reflect the content of the lesson. The success or otherwise of journal writing depends on whether teachers have the time and inclination to monitor and respond to children's writings.

Self-directed learning

View the video on assessment by Clarke (Herrington & Herrington, 1998) and answer the questions below.

1. Explain in your own words what Clarke means by the following purposes of assessment: to model, to monitor, to inform.

2. Clarke suggests that many assessment items are focussed on what the teacher wants to know rather than encouraging children to show what they know. He uses a question based on a hexagon to explain this point. Give another example of a typical teacher-centred question and transform it to a learner-centred question.

3. Make notes on the "tool" analogy he used for mathematical knowledge.

4. Clarke lists five strategies that teachers can add to their class assessment strategies which he claims will give optimum results for teacher effort. What are the stated advantages of these methods?

5. Explore the Herrington et al (1997) CD-ROM and write notes on the following assessment strategies highlighted in the Clarke video

   (a) Anecdotal notes.

   (b) Peer assessment

   (c) Portfolios

   (d) Performance or practical tasks

   (e) Self-assessment

6. What does the CD ROM say about -

   (a) Journal writing? and

   (b) Attitude tests?

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Chapter 5: Planning the Mathematics Program

As a result of studying this chapter and completing the set tasks you will:

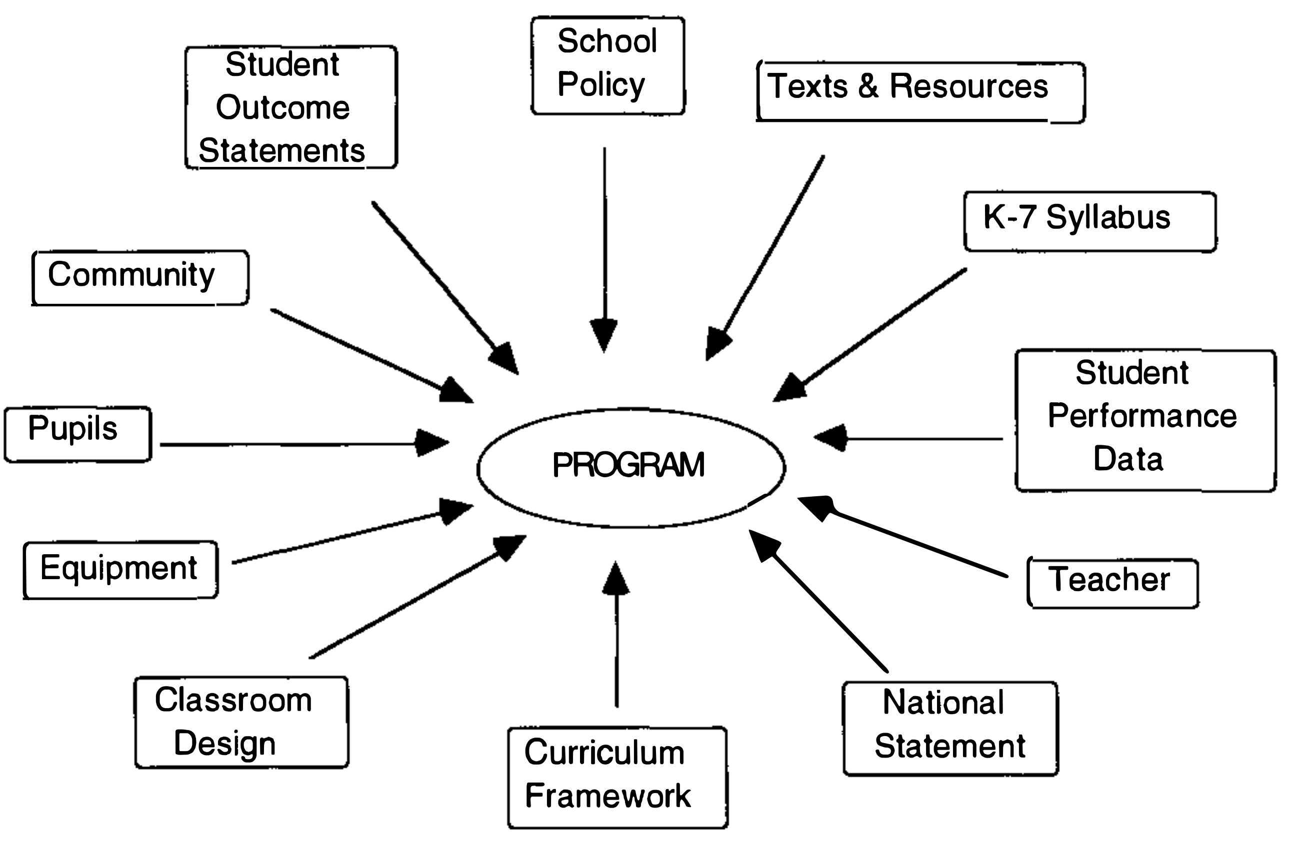
- identify the major planning principles and develop planning rationales;
- identify and evaluate a variety of planning strategies for the primary mathematics classroom; and
- be aware of the significant resource documents to be used in the planning process.

Programming is a source of concern for most teachers on ATP and in the early years of their teaching. In recent times, the programming demands on experienced teachers has diminished, often only being a brief summary or outline. The need for programs is often at the discretion of the school principal. However, detailed programs are usually expected of and are desirable for beginning teachers Planning in the past was based on the content of the WA Syllabus and many teachers used syllabus-related text books and worksheets as the basis of their teaching. With the introduction of Student Outcome Statements, the syllabus will be used more as a resource. Teachers will select from the syllabus objectives and activities that suit the outcomes they wish their children to work towards. No doubt the text book writers will now model their materials more on the outcomes.

Planning Principles

Study the planning model in Figure 1 below, then complete the activities that follow:

Figure 1: Factors influencing planning for mathematics teaching and learning



1. Add any further factors that you feel might impinge on the planning process and give one or two reasons for each.

2. For each of the factors in Figure 1, discuss with a colleague the rationale for its inclusion and write some explanatory notes below.

School Policy:

Student Outcome Statements:

Community:

Pupils:

Equipment:

Classroom Design:

Curriculum Framework:

National Statement: ____________________

____

____

Teacher: ____________________

____

____

Student Performance Data: K-7 Syllabus: ____________________

____

____

Texts & Resources: ____________________

____

____

3. Review the list above (from Figure 1), together with other factors you have added, and categorise and label their importance in the planning process as very important (V), of medium importance (M), or not so important (N).

4. Which factor do you think is the most important? Give reasons.

____

____

____

Planning Approaches

Stone et al (1995, p.86) present a continuum to indicate teachers' rationales for planning or programming in mathematics. Their model is represented below in Figure 2.

Figure 2: Planning Continuum

Begin with rich mathematical activities, then identify learning objectives	<---------------->	Begin with learning objectives, then find suitable rich mathematical activities

1. Discuss with your colleague and indicate at least one advantage and one disadvantage of each extreme of the continuum in Figure 2.

____

____

____

2. Mark the continuum in Figure 2 to show your personal position. Give reasons for your stance in a sentence or two.

Replacement Units

So-called replacement units are based on a set of problem solving or investigative tasks which are packaged and marketed by the Curriculum Corporation — a national body based in Melbourne (and not to be confused with the Western Australian Curriculum Council). You may like to visit their website at *http://www.srl.rmit.edu.au/mar/PSTC/cc/index.html.* There are several interesting places to visit at this site, and in addition you can also obtain more information about replacement units at *http://www.srl.rmit.edu.au/mar/PSTC/index.html.*

It should be noted that the replacement unit structure is fairly new and somewhat radical so student teachers and new teachers may find this approach a little difficult to manage. Teachers wishing to adapt the replacement unit idea could easily do so by grouping together problem solving tasks of a similar nature. You should already have a bank of useful tasks as a result of having completed the MPE1108 and MPE2107 units. You will become acquainted with further tasks during this unit.

Replacement units have a three-week structure with the assumption that teachers have arranged a Week Zero to familiarise themselves with the material and plan how they will adapt activities to local conditions. The structure suggested is detailed below.

Week 1 — Introduction:
Exploration of a collection of short investigations. Pupils choose the tasks they tackle and the order in which they tackle them from the menu. Their menu is signed off on satisfactory completion of the task. Informal assessment by observation, to inform Week 2.

Week 2 — Formalisation:
A series of formal lessons identifying key concepts, formal language and rules which, during the previous week, the teacher had noted as needing review, including

- teaching the Working Mathematically process,
- teaching report writing,
- teaching content, and
- making use of other resources.

Here assessment is more formal, perhaps with topic tests depending on the lessons that teachers plan.

Week 3 — Investigation

At least one in-depth investigation selected by the pupil from the menu. Assessment is by a portfolio:

- Pupils write initial project reports as drafts,
- Drafts are discussed with the teacher and other pupils and checked against expected performance standards, and
- Reports are re-drafted and the final product goes into the student's portfolio for semester assessment.

A Topic Approach

Another approach to planning is based on a 'topic approach'. Teachers adopting this approach would choose a topic such as 'time' from the Measurement strand and program around that topic. A two-page sample from a topic program on mental mathematics is included in the appendices of this chapter (pp. 58-59). It should be noted, however, that some ongoing practise of basic number facts and revision of previously taught concepts should also be included in the plan if adopting this approach. Below is another example of a topic approach as shown in Figure 3. A template for this format is also included in the appendices to this chapter (p. 55).

Figure 3: Example of a topic approach to planning

Possible Sequence of Learning Activities	**Other Learning Contexts**
<u>Initial Open Task</u> This task should relate to the topic being studied so that children become focussed on the topic. Children of all abilities should be able to complete the task to some point. By looking at the children's work, you should be able to gain some insight into the their ability and identify areas that require more work. The data gathered from the children should help the teacher to determine children's levels of performance. <u>Early Tasks / Activities</u> Activities pitched at the appropriate level and may be sourced from the syllabus, teacher texts, commercial student texts, etc. <u>Continuing Activities</u> Reference may be made to specific books and page numbers. <u>Extension Activities</u> Teachers should plan to challenge more able students in the group.	<u>Mathematics</u> Relate to other mathematics topics. <u>Other Learning Areas</u> Look for ways to integrate with other learning areas.

The above approach is based on gathering information about what level the children are at, and hence relies on the choice of an appropriate open-ended task which will allow children to demonstrate their understanding at various levels. Beginning teachers will need to take considerable care in choosing good, open-ended tasks.

3. What are some of the advantages and disadvantages of a topic approach?

4. With a colleague, select a topic, then design an initial open assessment task, suitable for level of your choice, which will cater for the demonstration of student performance at more than one level. Justify your choice, and give a follow-up task.

A Thematic Approach

As discussed earlier in this text, there are eight specified learning areas. This categorisation is one of convenience rather than absolute necessity. Ideally all learning should take place in an integrated way, but this proves to be rather difficult. Nevertheless, thematic approaches are quite popular and can be very successful at the primary school level. Such approaches are ideal for integrating learning in meaningful contexts. Examples could be the zoo, aeroplanes, or other such topics.

5. Working in twos or threes, select the school year level of your recent practice (or one close to that) and choose a suitable theme which could occupy the class for the major part of each school day for at least two weeks. Briefly indicate the following:

   (a) Content and processes:

(b) The mathematics to be covered:

(c) Work in the other learning areas:

6. List some of the major advantages and disadvantages of a thematic approach to teaching and learning mathematics.

Post Programming

This method of programming, as the name implies, is done after a period of teaching. This approach would not be suitable for beginning teachers. However, it does suit the new trend towards the use of student outcome statements. You may see an example of this technique in Hammond, Vincent, and Williams (1995).

7. Discuss and list some of the major advantages and disadvantages of a post-programming method.

Planning Rationales

There are many factors that need to be taken into account before planning a program. At the beginning of any program, schools may require teachers to prepare a rationale that summarises the philosophy and planning behind their program. The rationale should indicate the aspects of mathematics and the learning of mathematics that the teacher sees as being important. It may include some of the following:

- nature and importance of mathematics;
- general aims or broad teaching emphases; e.g., problem solving, attitudes, real-life relevance, use of calculators, emphasis on mental mathematics, etc;
- teaching and learning strategies; e.g., activity based, investigation orientated, use of materials, etc.
- main resources; e.g., text book, work cards, etc;
- organisation, e.g., seating, time allocation; and
- evaluation.

Examine the extract of a mathematics program rationale below.

> Mathematics is a science of space and number. A body of knowledge and skills that provides a powerful, precise and concise means of communication. Further, it is a creative activity that triggers a search for patterns and relationships within the environment.
>
> Mathematical learning is facilitated in situations where children are able to explore and manipulate their environment. That is, understandings are gained when children are actively, rather than passively, involved in their learning environment. Therefore, this program is based on a hands-on material centred approach so that children will learn by doing.
>
> It is intended that the teacher will maximise the understandings gained by acting as a guide to the children's discoveries; questioning, directing and focussing their attention on relevant detail as they manipulate the materials.
>
> Language is a tool that allows children to make meaning from their experiences and to communicate the extent of these understandings to others. As a result, this program aims to provide the children with numerous opportunities to discuss and describe what they have discovered. The teacher will provide a model of suitably formal language that children can adopt as their need and ability allow.
>
> A major reason for the inclusion of mathematics in the curriculum is to equip children skills to solve real life problems. This program aims to develop problem solving skills by including word problems that involve new concepts in real life situations.
>
> A positive attitude towards mathematics and a confidence in his/her ability to succeed are important factors which influence the effectiveness of a mathematics program. This

- monitoring children's work,
- records,
- clarifying expectations,
- opportunities for feedback,
- opportunities for sharing, and
- assessment

(Linke, 1990, pp. 91-95)

While Linke's article has a literacy focus and is not based on an outcomes approach to planning, it does clearly outline the process the author goes through when programming for mathematics and thus provides a useful summary.

9. Examine the *Curriculum Framework* document (Curriculum Council, 1998, pp. 178-182) and also *A National Statement on Mathematics for Australian Schools*. (Australian Education Council, 1991, pp. 1-24). Then write your own rationale (about a page).

Planning Periods and Formats

In this section we will consider issues such as the length of the planning period, the planning format, and the selection of content.

The Planning Period

The learning period to be covered by a plan will usually range from as little as one lesson to as much as a one-year program. The *K-7 Syllabus* (EDWA, 1989) is a useful source of activities. A sample page from that document is included in the appendices to this chapter (p. 54). Several short-term approaches to planning have already been considered, and again, you are referred to the appendices. The planning period will depend on many factors. For example if it is dependent on a theme it may only last, say, four weeks. Planning for a school term of 10 weeks is very common. Also included in the appendices is a one-year long-term planner sample. The length of the planning period will significantly affect the amount of detail in the document.

Content Selection

For the ATP most student teachers will (or should be) guided by their classroom teacher in content selection. Students are usually given some topics to cover and occasionally are free to select some areas of their own. You may be given some broad task such as "problem solving" to cover. Or, you may be directed to follow a text book which will dictate your content selection. As already mentioned, the syllabus is a good source of material. Other sources include various texts, computer software and other multimedia material, and many useful Internet sites.

What is MEOW?

The authors and other mathematics educators are preparing a set of mathematical tasks for K-12 and these are being placed on a website at *http://www.cowan.edu.au/ses/education/meow/meow.html.* What is MEOW? Mathematics Education On the Web.

Sequencing Content

Some beginning teachers find it difficult to decide what order in which to teach various topics. This is not an easy task. However, in mathematics, there are some commonsense principles regarding the sequencing of topics. Firstly, however, it is essential to always follow the well-established teaching/learning sequence of Concrete ---> Pictorial ---> Abstract.

1. For each of the substrands listed, discuss with a colleague and indicate some of the sequences of development that need to be observed.

   (a) Number: ______________________

   (b) Measurement: ______________________

   (c) Chance & Data: ______________________

   (d) Space: ______________________

Format of Plan

There is *no one way* of setting out your program. This is *your choice*, in consultation with the school, and it needs to be a format that you are comfortable with and can use effectively. One technique is to use the format of the sample program already provided. Several formats may be found in the appendices. Here is another example:

Outcomes	Syllabus	Learning	Resources	Evidence	Reference	Activities

The Outcomes will be drawn from the Student Outcome Statements.
The Learning Activities will include each significant task associated with the objectives.
Resources will include the teaching aids to be used as well as text and other references.
The Evidence column will include any specific forms of assessment associated with the activities.

This format is usually structured according to syllabus entries and gives a broad overview of what has to be taught. When segments are completed they are checked off and comments can be included on areas that need re-teaching, etc. Each week the teacher needs to select what he or she will cover. If this format is used then it is useful to have a brief outline of what will be covered week by week.

A common method of "programming" is the week-by-week lesson-by-lesson format. This can almost become a daily work pad. Many student teachers find it easier to plan like this. Headings similar to the above can be used. The main difficulties with this type of program are the amount of work needed initially and the fact that lessons may need to be repeated, as interruptions will occur from time to time.

Amount of Detail

This is the 64 thousand dollar question. Many practising, experienced teachers have contracted their programming to brief outlines. This is made possible by the use of Syllabus references or text book pages. This is again a question of choice, tempered by the school requirements.

2. Discuss, then list some of the factors you consider will affect the extent of detail in a mathematics planning document.

____

____

____

____

3. Examine the appendices and design a planning format that you could use for the six-week period during ATP.

4. Working in twos or threes, select the school year level of your recent practice (or one close to that) and write a six-week mathematics program (as for ATP).

   The following documents will be provided to assist you:

   Curriculum Framework

   Student Outcomes Statements

   Student Work Samples

   Learning Mathematics K-7 Syllabus

   National Statement

   Calculators and Computers Policy Document

Self Directed Learning

The following websites may be of use when developing a mathematics program.

1. Take time to look at these sites and search for others, as it may save you time in the long term.

   Some innovative assessment tasks -
   http://www.educ.msu.edu/MARS/ or a mirror site *http://www.nottingham.ac.uk/education/MARS*

   A good starting point when looking for resources -
   http://www.anglia.co.uk/education/mathsnet/intro.html

   http://explorer.scrtec.org/explorer/

   http://www.col-ed/cur/math.html#math1

   http://www./uc.edu/schools/education/csimath/csitext.htm

   http://www.learner/org/K12/sami/

   http://www.ex.ac.uk/cmit/

2. Add at least two useful URLs to the list above by searching the WWW.

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Websites:

http://explorer.scrtec.org/explorer/

http://www./uc.edu/schools/education/csimath/csitext.htm

http://www.anglia.co.uk/education/mathsnet/intro.html

http://www.col-ed/cur/math.html#math1

http://www.cowan.edu.au/ses/education/meow/meow.html

http://www.curriculum.wa.edu.au/

http://www.eddept.wa.edu.au/centoff/outcomes/maths/mathmenu.htm

http://www.educ.msu.edu/MARS/

http://www.ex.ac.uk/cmit/

http://www.learner/org/K12/sami/

http://www.nottingham.ac.uk/education/MARS

http://www.srl.rmit.edu.au/mar/PSTC/cc/index.html

Appendices

The appendices listed below are included in the following pages of this chapter.

- EDWA *K-7 Syllabus* — Overview of Stage 5;
- Sample page from EDWA *K-7 Syllabus;*
- Sample short-term mathematics outcomes planner
- Sample long-term yearly planner;
- A sample program framework; and
- Initial section of a topic program (Mental Mathematics).

This recording sheet may be useful for teachers when planning mathematics

Bold italic print indicates objectives being introduced at this stage

C Using Concrete Materials
M Mental Computation
W Written Computation

SPACE

PART ONE: Movement and Position in the Environment

1. Carry out activities based on movement and position:
   (a) ***finding positions from directions, including 45°, 90°, 180°, 360° turns, formulated by teacher and children;***
   (b) ***locating points on a grid.***
2. Investigate position and layout in the environment.
3. Carry out activities to investigate regions, paths, routes and networks.
4. Make scale models and drawings of familiar structures and areas.

PART TWO: Exploring Three-dimensional Shape

1. Carry out activities in which attributes of shapes are investigated:
   (a) vertices, faces and edges;
   (b) symmetry;
   (c) dissections and sections.
2. Carry out activities involving the classification of shapes.
3. Carry out activities to investigate congruence of shapes.
4. Make models and drawings of a variety of three-dimensional shapes including sphere, cylinder, cone, triangular prism, pyramid, cube, rectangular prism, tetrahedron.
5. Carry out activities in which the features and functions of objects and structures in the environment are investigated.

PART THREE: Exploring Two-dimensional Shape

1. Carry out activities in which attributes of the shape are investigated and used:
   (a) sides, angles and diagonals in polygons;
   (b) symmetry;
   (c) dissections.
2. Carry out activities involving the classification of shapes.
3. Carry out activities to investigate congruence of shapes.
4. Make and draw a variety of shapes including triangle, rectangle, square and circle.
5. Investigate the features and functions of shape in the environment.

PART FOUR: Transforming Shapes

1. Carry out activities to investigate changes in position.
2. ***Carry out activities to investigate reflectional and rotational symmetry.***
3. Carry out activities to investigate tessellations and other patterns.
4. Reduce and enlarge:
   (a) three-dimensional shapes;
   (b) two-dimensional shapes ***using grids and projective light.***
5. Investigate change of shape by distortion.

MEASUREMENT

PART ONE: Length

1. Measure in metres and centimetres, including approximation to the nearest centimetre. Select the appropriate unit of measure.
2. ***Measure small lengths in millimetres (where appropriate).***
3. ***Carry out activities which give practical experience of one kilometre. Recognise and recall: 1 000 metres = 1 kilometre.***
4. Carry out activities involving direct measurement of:
   (a) perimeters of polygons;
   (b) perimeters of other shapes;
   (c) perimeters of circles.
5. Relate measurement of length to other measures.

PART TWO: Area

1. Find the area of regular and irregular shapes by counting squares (arbitrary units) ***making allowances for parts of squares near boundaries. Make shapes with a given number of square units.***
2. Manipulate two-dimensional shapes:
   (a) to solve puzzles;
   (b) to construct patterns.
3. ***Investigate surface areas of three-dimensional shapes.***
4. Relate measurement of area to other measures.

PART THREE: Volume and Capacity

1. Carry out practical activities measuring in litres ***and millilitres*** to:
   (a) ***ascertain the capacity of containers;***
   (b) ***compare capacities;***
   (c) ***order containers by capacity.***
   Approximate to the nearest unit while measuring. Select the appropriate unit of measure.
2. ***Use standard measures to compare and order the volume of solids by displacement of liquids.***
3. Construct three-dimensional shapes using cubes as arbitrary units. Investigate and compare the volumes of shapes ***and of containers*** by counting the cubes.
4. Relate measurement of volume and capacity to other measures.

PART FOUR: Mass

1. Compare and order sets of objects and quantities of material by hefting (lifting), ***suspending*** and balancing. ***Select the appropriate unit to measure mass; either grams or kilograms.***
2. ***Carry out practical activities to measure, compare and order objects or quantities by mass using kilograms and grams.***
3. Relate measurement of mass to other measures.

PART FIVE: Time

1. ***Solve problems based on the calendar. Recognise and recall: 365 days = 1 year; 366 days = 1 leap year; 52 weeks = 1 year.***
2. Participate in activities requiring rhythmic response:
   (a) musical experience;
   (b) speech;
   (c) various dance activities.
3. Estimate, measure, compare and order intervals of time:
   (a) with arbitrary units including the use of mechanical devices;
   (b) ***in minutes;***
   (c) ***in seconds.***
   Recognise and recall: 60 seconds = 1 minute.
4. Read clocks (twelve ***and twenty-four*** hour) to:
   (a) the nearest five minutes;
   (b) ***the nearest minute.***
   (include reading from digital clocks.
5. ***Read simple timetables relating times to the clock face.***

NUMBER

PART ONE: Addition and Subtraction

ADDING

1. M ***Add a whole number less than 100 to one less than 1 000.***
2. M Analyse, discuss and practise methods which make mental calculations easier by:
   (a) reordering terms;
   (b) regrouping terms;
   (c) ***expressing a term as the sum or difference of simpler terms;***
   (d) ***using equal additions.***

SUBTRACTING

3. M Subtract a whole number less than 100 from:
   (a) a number less than 1 000 ***with no regrouping;***
   (b) ***multiples of 10 or 100 (less than 1 000) with regrouping.***

ADDING AND SUBTRACTING

4. M Recall the basic sums to 18 and differences with minuends to 18.
5. M Estimate sums and differences.
   (a) where only an approximation is required.
   (b) to compare with answers in written examples
6. W Add and subtract whole numbers (to a level limited by ability and practical needs).
7. M ***Add and subtract in sequence, as in a given pattern.***

DECIMALS

8. C Add and subtract numbers ***to three decimal places with regrouping:***
   (a) ***with like numbers of decimal places;***
   (b) ***with unlike numbers of decimal places.***
9. M Add and subtract decimals to ***three decimal places.***
10. W Add and subtract numbers ***to three decimal places (including money and measures).***

FRACTIONS

11. C ***Add and subtract fractions with like denominators (denominators to 10, and of 100 and 1 000). Sum or minuend should not exceed 1.***
12. M ***Add and subtract fractions with like denominators to 10 only. Sum or minuend should not exceed 1.***
13. W ***Add and subtract fractions with like denominators to 10, and of 100 and 1 000. Sum or minuend should not exceed 1.***

PART TWO: Relations and the Number System

PLACE VALUE

1. C Carry out activities which give experience with large numbers or objects, using ***cardinal numbers up to 1 000 000. Extend beyond 1 000 000 within reasonable limits.***
2. C Carry out activities with small numbers ***to three decimal places.***
3. C Carry out place value games and activities.
4. C Study the relationship between places in the base ten system of numeration.
5. C Identify place value and meaning of each digit in numerals to ***three decimal places.***
6. C Round:
   (a) whole numbers to the nearest ten, hundred or thousand;
   (b) ***decimals to the nearest whole number.***
7. C Investigate the Roman system of numeration.
8. C Continue to use ordinal numbers in real-life situations.
9. C Continue to show order relations between:
   (a) ***whole numbers to 1 000 000;***
   (b) ***numbers to three decimal places.***
10. C Carry out pattern-making and pattern-searching activities.
11. C Carry out activities with particular sets of numbers.
    (a) odd, even, prime and composite and square numbers;
    (b) investigate numbers for divisibility by 2, 3, 5 and 10.
12. C Carry out practical activities which involve investigation of
    (a) equality;
    (b) inequality.

FRACTION CONCEPTS

13. C Carry out activities which demonstrate:
    (a) the part/whole nature of fractions;
    (b) the subset/set nature of fractions. Experience with denominators to 10 ***and of 100 and 1 000.***
14. C Carry out practical activities involving:
    (a) equivalence of fractions;
    (b) ordering of fractions with like denominators (Limit: see entry 13).

LOGIC

15. C ***Construct and use codes.***
16. C Carry out games and activities involving problem solving, including classification and the application of logic.

PART THREE: Multiplication and Division

MULTIPLYING

1. M ***Apply basic multiplication facts to multiples of 10 (to 90) and multiples of 100 (to 900).***
2. M Carry out activities requiring grouping of objects in equivalent sets, up to, and beyond, the basic facts.
3. M Multiply a number up to ***1 000 by a number up to 10, with regrouping.*** Include money and measures.
4. W Multiply any two numbers, each less than 100, with regrouping.
5. W ***Multiply decimals (with up to two decimal places) by whole numbers less than 100.***
6. M Multiply a number less than 100 by a number to 10.
7. M ***Multiply three or more numbers together.*** (Limit numbers: 0, 1, 2 ... 10).
8. C ***Analyse, discuss and practise methods which make mental calculations easier by:***
   (a) ***reordering factors;***
   (b) ***regrouping factors;***
   (c) ***expressing a factor as the sum of simpler terms.***
9. M Estimate products:
   (a) where only an approximate answer is required.
   (b) to compare answers with written examples.
10. W Divide whole numbers, money and decimals by whole numbers to 10, with no more than four digits in the dividend:
    (a) with no regrouping and no remainder;
    (b) with regrouping but no remainder;
    (c) with remainder (whole numbers only).
11. M ***Apply basic division facts to multiples of 10 and 100.***

MULTIPLYING AND DIVIDING

12. C ***Multiply and divide whole numbers and decimals (with no more than three decimal places) by a whole number to 10.***
13. M Recall the basic multiplication and division facts. Apply to simple examples involving decimals.

PART FOUR: Statistics

1. Construct and read graphs, and interpret graphical information:
   (a) represent data by, and interpret bar graphs;
   (b) ***represent data by line graphs.***
2. Record and interpret data using
   (a) standard tally;
   (b) ***standard tally with grouped data.***
3. Record data in and interpret:
   (a) organised lists;
   (b) tables.
4. ***Carry out activities to develop an understanding of:***
   (a) ***spread of scores;***
   (b) ***middle, and most common scores.***
5. Construct and interpret:
   (a) Venn diagrams;
   (b) Carroll diagrams;
   (c) arrow diagrams;
   (d) tree diagrams.
6. Select appropriate forms of pictorial or graphic representation of data.

PART FIVE: Chance Processes

1. Investigate everyday events/games and activities that have a chance outcome:
   (a) predict, and rank in order of likelihood all possible outcomes;
   (b) choose the situation in which an event is most likely to occur.
2. Carry out activities in which a numerical probability is assigned to an event.

EDWA K-7 Syllabus Entry

[Stage 6 Space Strand (EDWA, 1989, p. 17)]

ENTRY 5

Carry out activities in which the features and functions of objects and structures in the environment are investigated.

Related Entries

S6:P1:2 S6:P1:4 S6:P2:1 S6:P2:2 S6:P2:3 S6:P2:4
S6:P3:5

Background

An understanding of how, why and where shapes are used in the environment gives children an enhanced appreciation and awareness of the role of shape in their surroundings.

Focus Ideas and Skills

Properties of three-dimensional shapes Relationship of shapes to lines, points and planes of symmetry
Discussing Investigating Constructing

Focus Questions

- What is the purpose of ball-bearings inside wheels?
- How is a building designed to minimise damage from cyclones?

Suggested Materials

Three-dimensional shapes, familiar objects which are symmetrical. Straws and pins.

Suggested Activities

- Discuss objects in the environment which have planes of symmetry (buildings, tents, people ...)
- Make models of local features and familiar places.
- Build houses from cardboard (or use cardboard boxes) and construct a roof using straws and pins. Investigate the shapes that are strongest. Introduce the concept of interior diagonals if required.
- Investigate the operation of cranes, pulleys, syphons, etc. (Integrate with Science program.)
- Investigate the changing shape of racing cars, rockets, yachts, safety helmets...

Resources

Language, Recording and Representation

Discuss as appropriate.

Mathematics Outcomes Planner

Outcome Level A: ____
A = The level chn have reached

Outcome Level B: ____
B = The level chn working towards

Outcome Level C: _ __
C = The next level, which a few chn may reach

Time period: Enter number of teaching sessions covered by the plan.

Key Teaching Points:
Specify the focus of the activities and the key concepts to be developed

Sequence of Learning Activities

Introductory Activity:
A rich mathematical task should be used to motivate the children and to gather evidence of 'where the children are at'; e.g., at Level 'A'

Initial Tasks:
Either give a brief description of the task(s) or refer to the syllabus, text or other resource. When choosing tasks, try to choose those that will cater for the wide range of abilities in your class.

Extension Tasks:
Provide these for chn who have succeeded at the tasks above. remember that some chn will be moving towards Level 'C'

Other Learning Contexts:
Integration of two or more strands

Other Learning Areas:
Integration across any of these

Assessment Procedures:
Indicate the strategies of assessment to be used

Evidence:
Results of Assessments to show SOS attainment

Sample Yearly Planner

Week	Term 1	Term 2	Term 3	Term 4
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Program Framework

Content	Processes	Attitudes	Assessment
• movement and position in the environment • spatial concepts • transformation of two- and 3D shapes • measurement concepts of length, area, volume and capacity, and mass and time • relationships among the measures • accurate use of measuring instruments • the relationship of the metric system of measurement to the number system • number concepts and relationships in the number system • probability concepts • competence in the fundamental mathematical operations • appropriate use of aids such as calculators and computers • application of mathematical skills in practical situations	• comprehension of mathematical information given in oral and/or written forms • selection of appropriate strategies • purposeful use of materials • selection of appropriate operations to solve problems • reflection on actions to formulate ideas • expression of mathematical ideas in words, pictures and symbols • construction of lists, tables and graphs • estimation in number and measurement activities • identification of patterns and relationships • classification, ordering and preparing • analysis and interpretation of information • formulation of hypotheses • justification of conclusions and inferences	• enjoyment of mathematics • satisfaction and fulfilment in mathematical achievement • confidence to experiment and solve problems • willingness to express ideas and hypotheses • appreciation of the relevance of mathematics in life	• Chance meeting • Teacher-initiated conversions • Student self-assessment • Investigations • Questionnaires • Observations • Schedules • Projects / assignments • Published attitude tests • Classroom tests • Checklists • Practical work • Published diagnostic tests • Examinations • Standardised tests

Class Organisation	Teaching Approaches	Resources	Time Allocation	Program Evaluation

Mental Mathematics Program

Rationale	Goals
Mental computation is a practical life skill used in at least 75% of all computations. Mental computation is a very efficient means of calculation and is necessary for effective estimation and use of calculators. Mental computation is a creative process in which students may use different methods to solve problems. It requires understanding of number properties and mathematical processes.	• To develop students' knowledge of number properties and relationships useful for calculating purposes. • To develop students' network of connections between number facts and relationships. • To develop students' range of calculating strategies. • To develop students' confidence in mathematics
Implementation	**Evaluation**
• Mental maths is timetabled daily for 15 minutes per day. • Emphasis is on learning and sharing ideas and strategies, not 'testing'. • Problems will be presented in oral and written form, but answers will be mostly oral. • Calculators, dice, playing cards and other materials will be used wherever appropriate.	Students' progress will be monitored by: • observations • questioning • student self assessment

Objectives	Learning Activities	Resources	Evaluation
1. Recall the basic sums to 18. (K-7 Syllabus, N5:P1:4, N4:P1:7) 2. Add a number less than 10 to any number (N4:P1:1)	Card Activities: "Tally Ho!" 1. Organise pairs 2. Demonstrate game 3. Distribute cards 4. Play game 5. Pack away	Ref: *Think Mathematically,* p. 52 • Each student will need A, 2, 3, 4, 5, 6, 7, 8, 9, 10 from a pack of cards. • Recording sheet for each student	Use observation of selected students to determine: 1. Could students recall basic sums to 18? 2. Could students add any number less than 10 to any number? Record on checklist
Identify the relationship between place value and value in the base ten system for whole numbers to 1000 (Syllabus, N4/5:P2:4)	Dice Activities: "Dice Digits"	Ref: *Dice Dilemmas*, pp. 12-14. • ten-sided dice	Use observation of selected students to determine: Could student identify the relationship between place value and value in the base ten system for whole numbers to 1000? Record on checklist.
1. State that simple number problems are best solved mentally. 2. State that more complex problems are better solved using a calculator. (Process : selection of appropriate strategies)	Calculator Activities: "Beat the calculator" 1. Divide class into two groups 2. Explain game procedure 3. Play game 4. Discuss which method is "best" and under what circumstances this method is useful	Ref: *Kids Calculators and Classrooms* pp. 17-18	Use questioning of selected students to determine: 1. Did student state that simple number problems are best solved mentally? 2. Did student state that more complex problems are better solved using a calculator? Record on mathematical processes checklist (selection of appropriate strategies)

Objectives	Learning Activities	Resources	Evaluation
Select appropriate operation to solve problems. (Process)	"Today's Number is..." - 12 1. Write number on board, ask students to make up calculation for that number. Write them up as suggested, using columns for each type of calculation. After 8 or 10 responses, focus on a particular type of response.	Ref: *Think Mathematically*, p. 23 • whiteboard/blackboard	Use observation and questioning of <u>selected students</u> to determine: Was student able to select appropriate operation to solve problems? (Process) Record on mathematical processes sheet.
Select appropriate strategies to solve mental calculations. (Process)	"How did you do it? " Focus: 15 + 11. Introductory lesson for this strategy emphasising that there are many different ways to do mental arithmetic	Ref: *Think Mathematically*, p. 27. • whiteboard/blackboard	Use questioning of selected students to determine: Could student select appropriate strategies to solve mental calculations? (Process) Record on mathematical processes sheet.
Recall basic facts (N4:P3:12; N4:P1:7; N5:P1:4, N5:P3:13)	<u>Card Activities:</u> "Discard" 1. Distribute cards 2. Explain game 3. Play game 4. Pack away	• One set of "Discard!" cards per pair of students.	Use observation of selected students to determine: Could student recall basic facts to successfully play the game? Record on checklist
1. Recall the basic sums to 18 (N5:P1:4, N4:P1:7) 2. Add a number less than 10 to any number (N4:P1:1) 3. Show a willingness to express ideas and hypotheses	<u>Dice Activities:</u> "Pig" 1. Demonstrate game 2. Distribute dice 3. Students play game in pairs 4. Discuss strategies used	Ref: *Think Mathematically* p. 66 • 2 dice per pair of students • pencil and paper to maintain score Also see *Dice Dilemmas, Go for Broke*, p. 21	Use observation of selected students to determine: 1. Could student recall basic sums to 18? 2. Could student add any number less than 10 to any number? 3. Was student willing to express ideas and hypotheses? Record on Checklist
Carry out place value game (N4&5:P2:3)	<u>Calculator Activities:</u> "Wipe-out" 1. Demonstrate game using OHP 2. Distribute calculators 3. Give first number and talk students through steps 4. Student practice	Ref: *Kids Calculators and Classrooms*, p. 19 • one calculator per pair of students • one overhead calculator • overhead projector	Use observation to determine: Could student play game which involved place value? Record on checklist

Notes:

Chapter 6: Working Mathematically and Problem Solving

As a result of studying this chapter and completing the set tasks you will be able to:

- explain the meaning and role of 'working mathematically' in the mathematics curriculum;
- detail the heuristics and strategies of problem solving; and
- demonstrate greater confidence in personally solving mathematical problems

The Working Mathematically Strand

The Working Mathematically strand is the most important strand of all, since it pervades all the other strands. It is not a content-based strand such as Space or Chance and Data, but is one that is generic in nature and applies to all mathematics teaching and learning, irrespective of the content being covered. It is concerned with the process or approach rather than with the content of the mathematics curriculum.

You may recall that the first chapter in the text for the unit MPE1108 was devoted to this strand so as to "set the scene" for the significant change being promoted for mathematics classrooms. Check pages 12-13 in the Student Outcomes book (EDWA, 1998) for the WM strand and read the Level statements for each substrand.

1. Write the four substrand titles for WM below.

   (i) ____

   (ii) ____

   (iii) ____

   (iv) ____

2. How does the topic of problem solving fit into the WM strand? Discuss with a colleague and list some points below.

   ____

   ____

   ____

   ____

'School Maths' and 'Real Maths'

The mathematics of the classroom is often very different from that of the real world. Thus the terms "school maths" and "real maths". Classroom mathematics can too often consist of a diet of contrived problems based on artificial contexts, together with standard written algorithms. Lesh and Zawojeski (1992) make some interesting comments on this issue. A comparison of school and "real world" or everyday mathematical problem solving reveals a number of self-evident contrasts:

School mathematics and problem solving	**Real world mathematics and problem solving**
formal (algorithmic) approach used	informal, ad-hoc techniques used
mathematics is decontextualised	mathematics is contextualised
data is often artificially clear	data is "noisy" with many distractors

Lesh and Zawojeski (1992) summarise these and other differentials in a little more detail as follows:

> The characteristics of problems in the classroom should be similar to the characteristics of real-world situations in which students will be required to use mathematics in their everyday and work worlds . . . the "informal mathematics" used in everyday life captures the flavour of the types of problem-solving experiences we should be providing for students. In everyday situations mathematics is embedded in a task rather than abstract. The mathematical processes needed in everyday situations are not explicitly apparent . . . The data in realistic situations is often ill defined and "noisy" rather than well defined and presented neatly. The level of accuracy required in real-world problems is defined by the situation rather than given or assumed, and the language of problems in a real-world context is imprecise and responsive to the setting rather than carefully worded to elicit precisely the correct response. The more experiences we provide for students that have characteristics similar to those encountered in the outside world, the more students will be able to readily use their school-learned mathematics on substantive problems both inside and outside the classroom environment.

(p. 72)

Maier (1991) discussed extensive contrasts between school mathematics and what he called "folk mathematics". He too saw a corresponding dichotomy in terms of the problem solving/posing children experienced inside and outside school.

> In school almost all problems are presented to students pre-formulated and accompanied by the requisite data. For folk outside school, problems are seldom clearly defined to begin with and the information necessary for solving them must be actively sought from a variety of sources.

(p. 65)

1. Should most of the mathematics undertaken in the classroom be "real maths"? Why/Why not? Give examples. Discuss with a colleague and report to the class.

____

____

____

2. Devise two different real-world contexts for each of the following computations:

(a) 12 - 5

(i)

(ii)

(b) $77 \div 5$

(i)

(ii)

(c) 50 - (3 x 4.76)

(i)

(ii)

3. What benefits do you see in children determining a variety of 'real world' contexts for calculations as in the above exercise?

4. Note the following two problems, then answer the questions below:

Problem 1:

I am thinking of a number. It is between 10 and 30. It is not 29. It is odd. It is less than 24. It is prime. It is not 17. It is greater than 15. Its digits add up to an even number. What is my number?

Problem 2

I had $25 dollars in my purse when I went out. I only bought a pizza and some apples. The apples were $4 and the pizza was $9.50. Do I have the right amount left in my purse?

(a) How do these two problems relate to the 'real maths' and 'school maths' labels described above?

(b) Discuss the significant aspects of each problem with a colleague, and jot these down below.

Problem 1:

Problem 2:

5. Find or create three real-world problems that would be useful for primary school classrooms as indicated below, and that would occupy the pupils for approximately 20 minutes.

Years 1-3:

Years 3-5:

Years 5-7:

The daily newspaper is a very rich source for "real world" mathematical problem posing and solving. The content is relevant and topical, and therefore generally very motivating for students. The newspaper is an ideal vehicle for *contextualising mathematics* — one of the substrands of the Working Mathematically strand. Moreover, any one daily newspaper can provide countless activities in the number, and in the chance and data strands; and to a slightly lesser extent in the space and measurement strands. Below are two extracts from *The West Australian* (February 13, 1999) which can provide examples of the richness of such material for real-life problem posing and problem solving.

6. Study the extract below from the stock market report then answer the questions that follow.

58 THE WEST AUSTRALIAN SATURDAY FEBRUARY 13 1999

Industrial

BUY c	SELL c		1900 STOCK CODE	LAST SALE c	UP/ DWN c	VOL 100s	12 MONTH HIGH c	12 MONTH LOW c	YLD %	P/E
47½	48	Macmahon	MAH	47½	-2	2841	82	38	7.37	8.3
96	97	Macmahon pref	MAHPA	96		156	103	78	10.42	
1752	1780	Macq Bank $1	MBL	1752	+27	485	1805	1201	3.42	18.0
1280	1290	Macq Bank ca7cw	MBLWCE	822			976	750		
330	338	Macq Bank cw00	MBLWAB	333	+18	100	382	95		
78	83	Macq Bank cw00	MBLWAC	92			285	92		
8.1	14	MacroCorp 20c	MAC	8			17	6		
33	35	Magna Pac 25c	MPH	33	+3	300	46	6		
12	13	Magna Pac opt01	MPHO	10½			18½	1.3		
875	890	Maryborough 20c	MSF	900			1020	800.7	5.62	10.6
110	115	[illegible] 20c	MAX	110	+2	87	115	80	3.64	10.0
36½	37	Maxitrans 20c	MXO	37½	-½	1864	38	22	6.00	10.4
36		Maxitrans ncprf	MXOPA	37			40	36		
579	582	Mayne N	MAY	582	-13	11260	821.2	578	4.29	52.9
215	240	Mayne N [illegible]	MAYWME	243			450	225½		
17	20	Mayne N ca/99	MAYWAA	19	-5	700	165	19		
76	81	Mayne N pw/99	MAYWAP	66			122	60		
276	290	McCDowell	MDC	290			385	270	2.76	8.5
275	280	McGuigan	MGW	275	+3	145	280	160	2.55	18.0
95	100	McPherson 20c	MCP	95		66	161	85	5.84	13.6
17	20	Med Quest 20c	MQL	17	-3	2856	32	10		
3		MedCorp 20c	MOD	6¼			13.4	5		
62	82	Media Ent 20c	MED	61		1220	80	23½		9.7
38	38½	Media Ent opt99	MEDO	38	+1	1211	54	6.6		
30	31	Medic Inv 20c	MIN	30		424	40	16		
15	15½	MenzCourt 25c	MCT	15½	+1	1075	18	7		
3	3½	MenzCourt opt01	MCTO	3	-1	2486	5	3		
.7	1.2	MenzCourt opt99	MCTOA	1.8			4½	.4		
10	10½	Merchant 1c	MHI	10			11½	9.2	10.00	40.0
83	84	Meridian units $1	MRD	83		764	120	75	7.23	4.8
26½	27	Metabolic	MBP	27		440	33	14		
12½	13	Metabolic opt03	MBPO	13	-½	1500	22	5		
133	140	MetalCorp	MTL	132	-11	[illegible]	283	110	3.03	28.9
268	270	Metal Man	MMF	270		853	285	132	20.37	4.1
8½	9.4	Metroland 20c	MTD	8½		25	20	5½		
1.2	1.8	Metroland opt01	MTDO	1.8			5	.4		
20½	21	Micromed 25c	MMD	20½	+1½	1047	38	12½		
8	10	Micromed opt00	MMDO	10	+½	2	11	2		
118	120	Mid E Min 25c	MEM	116			140	105		3.1
21½	38	Midwest R 20c	MDR	23			27	16		
21½	23	MikohCorp 20c	MIK	21½	-1½	880	28½	10		
7.1	8	MikohCorp opt00	MIKO	8		500	10½	3		
151	155	Millers FI 1c	MFL	155	+5	430	165	100		4.4
925	930	Milton $1	MLT	925	+25	130	950	740	4.11	12.1
11	12	Minproc 20c	MNE	11½		20879	27	10½	8.70	2.5
177	178	Mirvac	MRV	178		1456	240	140	5.82	9.1
29	31½	Mirvac opt01	MRVO	31½	+½	500	42	27		
148	153	Mirvac PT units 1.50	MPT	148	+3	1894	158	137	7.73	9.8
1180	1190	MobileCorp	MBC	1180	+40	93	1205	420		
270	285	Monadelphous 25c	MND	270			330	250	4.81	9.5
20		Moon Star 20c	MSH	20			24½	15½	10.00	8.5
19	20	[illegible] 25c	MSG	20			31	19	7.50	11.4
9.8	14	[illegible] 20c	MSI	12		100	25	7		
27	30	NSL Health 20c	NSL	27			90.2	22		
77	79	NamoiCot 92 1c	NAM	77	-1	318	85	68	5.19	5.5
2620	2635	NAB $1	NAB	2635.3	+65.9	45386	2707	1880	3.87	18.8

(a) Note that the previous 12-month 'high' and 'low' prices are given for each stock listed. Hindsight is a wonderful thing, but suppose you were able to predict these highs and lows 12 months ago, which would have been the most profitable share to buy and sell in that period? Which would have been the next best? Use estimation first, then check with a calculator.

(i) Estimate — Best: ____________________ Next Best: ____________________

(ii) Calculation — Best: ____________________ Next Best: ____________________

Does the solution above depend on the amount of money you had to invest? Give reasons.

____

____

(b) Pose two different problems related to the extract above.

(i) ____

____

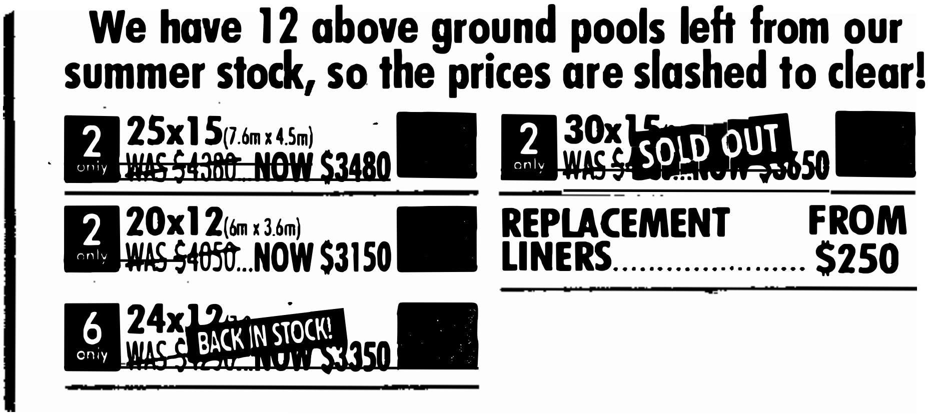
____

(ii) ____

____

____

7. Use the advertisement below to generate three different problems.



(i) ____

____

____

(ii) ____

____

____

(iii) ____

____

____

Now, share these examples with the group.

Problem Solving Steps and Strategies

The classic book by Polya (1957) on problem solving — *How to Solve it* — was originally published in 1945. The well-known heuristics of problem solving are embodied in Polya's four-step approach:

1. Understand the problem,
2. Devise a plan,
3. Carry out the plan, and
4. Look back.

These steps may be expressed in simpler terms for primary school students as:

Look
Plan
Do
Check

Following are some brief explanations/examples of each strategy.

Understand the problem

- Explain the problem in your own words
- Relate to similar problems
- Ask questions
- Highlight important parts
- Draw a diagram

Devise a plan

- Consider various strategies
- Compare to similar problems
- Determine subgoals

Carry out the plan

- Make estimates
- Make adjustments as necessary
- Write the solution

Look Back

- Check that all important information has been utilised
- Check each step
- Check computations
- Consider the reasonableness of the solution
- Look for alternative solutions

1. For each of the four steps, indicate at least one more act that would be helpful.

   Look: ______________________________

   Plan: ______________________________

   Do: ______________________________

   Check: ______________________________

2. There is considerable evidence that problem solvers — both children and adults— tend not to follow these essential steps. What is the typical behaviour?

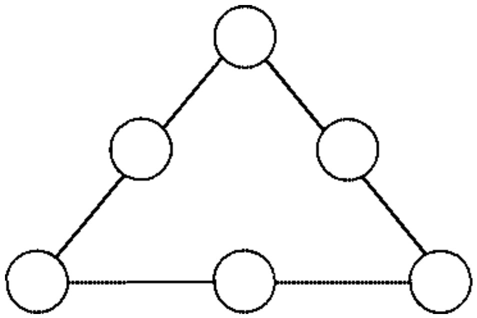
   ____

Following is an example of the four steps in operation, using a 'guess and check' (or trial and error) strategy.

Problem Example

Problem

Place the digits 1, 2, 3, 4, 5, 6 in the circles in the diagram so that the sum of the three numbers on each side of the triangle is 12.



Understand the Problem

- one circle for each digit
- use each digit once
- each line of three digits adds to 12

This indicates that the problem is understood.

Devise a plan

- Try guess and test

This particular strategy can be used in three different ways ;

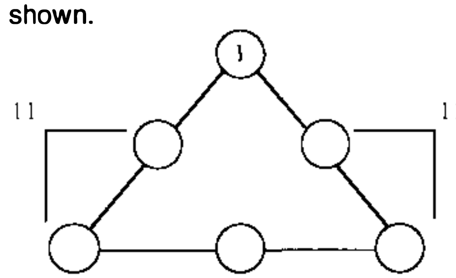
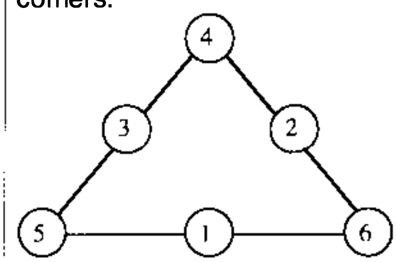
- random,
- systematic, and
- inferential.

Use six counters numbered 1-6.

Carry out the plan

The planning and "doing" is summarised in the tables below for the three different "guess and test" techniques.

	Devise a plan	*Carry out the plan*
Random	Arrange counters randomly checking each new combination until one works.	Carry out the plan as described. Unless random tests are recorded in some way you will probably inadvertently duplicate some guesses a number of times.
Systematic	Guess corner numbers are 1, 2, 3; if that does not work try 2, 4, 6; and so on.	With 1, 2, 3 in the corners, the side sums are too small. Try larger corner numbers until a solution is found. This technique reduces the number of combinations we need to try.

	Devise a plan	*Carry out the plan*
Inferential	Start by assuming 1 is in a comer and explore the consequences.	If 1 is placed in a corner we must find two pairs from the remaining digits (2, 3, 4, 5, 6) whose sum is 11 as shown.  Can't be done. Only 6+5=11 If 2 is tried in a corner we need two pairs totalling 10 - this can not be done. If 3 is tried in a comer we need two pairs totalling 9 - this also can not be done. Thus if there is a solution, 4, 5 and 6 must be in the corners:  Placing 1, 2, 3 as shown, we have a solution.

Look back

The solution satisfies all the requirements. Some comments on these three techniques may be useful:

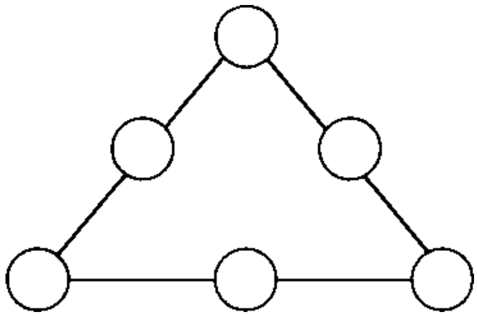
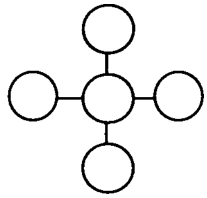
Random Guess and Test — very useful for getting started but you can lose track of your various trials if a large number of options are available.

Systematic Guess and Test — this ensures all possibilities are canvassed, but is inefficient if a large number of options exist.

Inferential Guess and Test — this is generally superior to both of the previous techniques, as it saves time and provides information which was not given.

Solve the two problems below by following the steps in the above example. The diagrams are on the following page.

1. Place the digits 1, 2, 3, 4, 5, 6 in the circles in the diagram in the previous problem so that the sum of the three numbers on each side of the triangle is 10.
2. Using numbered counters place the digits 1 to 5 into the 5 circles so that any three numbers in a line through the centre will give the same sum. Find all three solutions.

(1)  (2) 

Problem Solving Strategies

The heuristics of problem solving outlined above need to be implemented in conjunction with appropriate strategies. One such strategy — Guess and Test — has already been exemplified. Below is a list of useful strategies (not necessarily exhaustive) which can be used by primary school students. You would have met and used most of these before.

Think of a similar problem
Guess and Test
Simplify the problem
Construct a table
Look for a pattern
Act out the problem
Construct a Model
Draw a diagram
Investigate all possibilities
Identify the subgoals
Work backwards

We will now illustrate these strategies with several problem examples. One problem that you would already have met is as follows.

Problem

There are 30 people in this class. If everyone in the class shakes hands with everyone else, how many handshakes will there be? Consider what strategies could be used.

Think of a Similar Problem

If you can think of a similar problem and recall the method of solution then this would provide the starting point here.

Guess and Test

This has been illustrated above, but is not appropriate for this problem.

Simplify the Problem

It is often very useful to take the simplest case where possible — in this instance, two people, which results in one handshake, which provides the basis for starting a table.

Construct a Table

A table for this situation is shown below:

People	Handshakes	Increase	Pattern
2	1	1	1
3	3	2	1+2
4	6	3	1+2+3
5	10	4	1+2+3+4
..	..	..	
..	..	..	
29	..	28	1+2+3+ . . . +28
30	..	29	1+2+3+ . . . +29

Look for a Pattern

It is clear from the table that there is a pattern. Note that this had to be derived in order to avoid listing all the instances. It also allows us to generalise for any number of people.

Act out the Problem

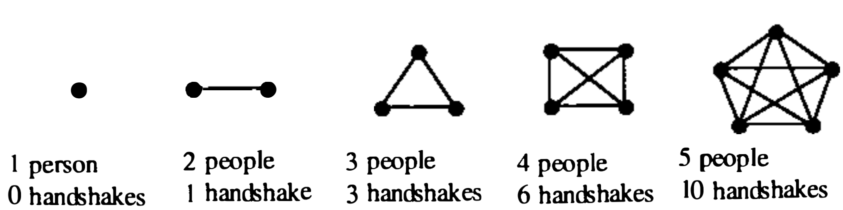
The problem could be solved by everyone in the class actually shaking hands with everyone else, which could take considerable time. However, acting out the first few simple cases with, say, two, three, and four people would provide the path to a solution.

Construct a Model

This strategy is inappropriate for this problem.

Draw a Diagram

This is of some help here, but such a strategy should always be used whenever possible in order to provide a visual image.



Investigate all Possibilities

The table shown above generated a pattern which rendered this strategy unnecessary. Nevertheless, it would have been possible to list the numbers of handshakes for all cases from 2-30.

Identify the Subgoals

This could be to determine what happens for a small number of people (in this instance, a simpler case of the same problem)

Work Backwards

Not appropriate here.

The handshakes problem above lends itself to the use of many of the standard strategies. Try the problems below using the strategies indicated.

1. How many diagonals in a centagon? [Use strategies as for the handshakes problem] __________

2. Construct a 3 x 3 magic square by using the digits 1 to 9 so that each row, column and diagonal add to the same total. [Identify subgoals]

3. Two barrels, A and B, contain unspecified amounts of liquid, A more than B. From A pour into B as much liquid as B already contains. Now pour from B into A as much liquid as A now contains. Finally pour from A to B as much liquid as B now contains. At the end of this, each barrel finally has 64 litres. How many litres did each barrel contain at the start?

More Problems

Solve each of the following problems and indicate the strategies used. Also state other strategies that could have been used in each case.

1. The Green Tomatoes rock group has four members: Bob, Denny, Gil and Pete. They play bass, drums, guitar and piano. No one plays an instrument that begins with the same letter as his first name. Bob hates the piano player. Pete and the drummer are best friends. Gill plays the bass, which he borrowed from Bob. Which instrument does each person play?

	Bob	Denny	Gil	Pete
Bass	No		Yes	
Drums		No		
Guitar				
Piano				

Strategies: ____

____

2. By zig-zagging vertically and horizontally through the arrangement how many "triangles" can you see? ____

TRIANGLE
RIANGLE
IANGLE
ANGLE
NGLE
GLE
LE
E

Strategies: ____

____

3. Make a 3 x 3 square array of 9 blocks or tiles. Can you keep the perimeter constant by removing 1 tile?___ 2 tiles?___ 3 tiles?___ 4 tiles?___ 5 tiles?___ 6 tiles?___ 7 tiles?___

Strategies: ____

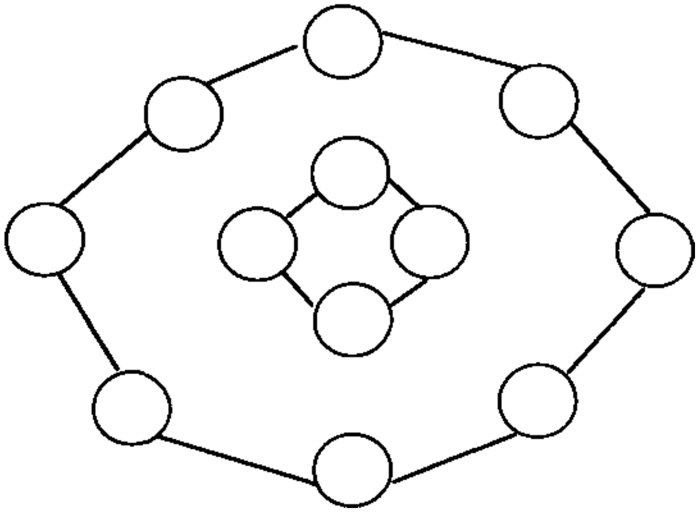
____

4. The farmer's will said, "My wife is to have the house in the corner of my square shaped farm together with a quarter of the land in the shape of a square which includes the house. Each of my 4 sons must share the remainder equally so that each of their 4 farms are the same size and shape". Draw this.

Strategies: ____

____

5. Place the numbers from 1 to 12 in the circles below in such a way that the sum in the outer ring is twice the sum in the inner ring.



Strategies: ____

____

6. Mr E Nigma looked at the photograph of a man and stated: Brothers and sisters I have none but the man's father is my father's son. Whose photo was it? ______________________

Strategies: ____

____

7. Three friends went to a restaurant for a meal and were charged \$10 each. They paid this to the waitress, but she realised that she had overcharged them a total of \$5 so she brought that amount back to the table. They gave her \$2 as a tip and kept \$1 each, resulting in a cost of \$9 each. But 3 x \$9 = \$27 and \$27 + \$2 = \$29. What happened to the other \$1? ______________

Strategies: ____

____

8. How many squares in a 10 x 10 array of square tiles? _____ What is the rule for solving this type of problem?____

Strategies: ____

____

9. A man wants to plant five shrubs along each side of his triangular garden. What is the minimum number of shrubs he will need to purchase?_______

Strategies: ____

____

10. In the example below, find a digit to replace each letter. ______. Is there is more than one possible answer? ______

$$\begin{array}{r} FIVE \\ -\ FOUR \\ \hline ONE \\ \hline \end{array}$$

Planning for Problem Solving/Posing

Lesson planning formats are dependent on the particular problem solving/posing pedagogy being employed, the respective teacher/student roles, and the scope of the problem. For this reason it is difficult to nominate a single format suitable for all situations. One rather common misconception about mathematical problems is that they should generally be able to be solved very quickly. This is not the case. A simple classification of problems in terms of their scope could be as stated below.

1. Discuss with a colleague and give an example of each.

(i) Short Problems — Can be solved in about five minutes

Example: ____

____

____

____

(ii) Medium Problems — Can take a class maths period to solve

Example: ____

____

____

____

(iii) Long Problems — Can take a week or more to solve

Example: ____

____

____

____

The following structure described by Kroll and Miller (1993) identifies teaching actions and purposes before, during, and after problem solving, and provides valuable assistance for teachers.

Teaching Action	Purpose
Before	
1. Read the problem to the class or have a student read the problem. Discuss words or phrases students may not understand.	Illustrate the importance of reading problems carefully and focus on words that have special interpretations in mathematics.
2. Use a whole-class discussion about understanding the problem.	Focus attention on important data in the problem and clarify parts of the problem.
3. (Optional) Use a whole-class discussion about possible solution strategies.	Elicit ideas for possible ways to solve the problem.
During	
4. Observe and question students to determine where they are in the problem-solving process.	Diagnose students' strengths and weaknesses related to problem solving.
5. Provide hints as needed.	Help students past blockages in solving a problem.
6. Provide problem extensions as needed.	Challenge the early finishers to generalise their solution strategy to a similar problem.
7. Require students who obtain a solution to "answer the question."	Require students to look over their work and make sure it makes sense.
After	
8. Show and discuss solutions.	Show and name different strategies used successfully to find a solution.
9. Relate the problem to previously-solved problems and discuss or have students solve extensions of the problem.	Demonstrate that problem-solving strategies are not problem-specific and that they help students recognise different kinds of situations in which particular strategies may be useful.
10. Discuss special features of the problem, such as a picture accompanying the problem statement.	Show how the special features of a problem may influence how one thinks about a problem.

(Kroll & Miller, 1993, p. 69)

Depending on teacher experience the above sequence might need to be preceded by intended outcomes, materials possibly needed, and organisational options. Reference to intended evaluation specifics (outcomes and recording formats) might also be necessary. Missing from the teaching action list is any reference to motivation. When the problem is provided by the teacher, this aspect needs to be explicitly addressed. Unless the problem has contextual relevance, intrinsic appeal, or amusement value, simply presenting it to the class/group is unlikely to serve this purpose. Student problem-posing on the other hand can be expected to impact more effectively in this respect.

2. With a colleague, discuss the use of the above "teaching action" sequence to plan a lesson based on each of the following problems, then share with others.

(a).If you ride exactly one kilometre on your bicycle, how many times do the wheels go round? How many times do you have to push the pedals round?

(b).In a family the girls have twice as many sisters as brothers and the boys have five time as many sisters as brothers. How many boys and girls are there in the family?

Assessing the Problem Solving/Posing

Suitable long-term assessment strategies should be selected to monitor student progress in the above areas. Herrington, Sparrow, Herrington and Oliver (1997) identified many strategies appropriate for these particular applications; with some of the most relevant being portfolios, written and oral reports on investigations and problem solving, checklists, journal writing, modelling, and interviews; as well as conventional paper and pencil tests. It is essential that all of these strategies be employed to profile both individual and class achievements, and the success or otherwise of the problem solving/posing program.

Additionally:

> Rather than relying on approaches that provide assessment solely for the purposes of grading, ranking, and credentialling, assessment practices are needed that integrate with learning activities, that support students' construction of knowledge, and that reflect the diversity found in the curriculum and in the learners themselves.

(Herrington et al, 1997, p. 2)

1. Select three of the assessment strategies from the CD-ROM *Investigating assessment strategies in the classroom* (Herrington et al, 1997) which would be particularly appropriate in assessing problem-solving activities. Describe briefly why you have chosen each.

(i) ______________________________

____

____

(ii) ______________________________

____

____

(iii)______________________________

____

____

Attitude Inventory Items

Pretend your class has been given some math story problems to solve. Mark true or false depending on how the statement describes you.

1. I will put down any answer just to finish problem ______
2. It is no fun to try to solve problems ______
3. I will try almost any problem ______
4. When I do not get the right answer right away I give up ______
5. I like to try hard problems ______
6. My ideas about how to solve problems are not as good as other students' ideas ______
7. I can only do problems everyone else can do ______
8. I will not stop working on a problem until I get an answer ______
9. I am sure I can solve most problems ______
10. I will work a long time on a problem ______
11. I am better than many students at solving problems ______
12. I need someone to help me work on problems ______
13. I can solve most hard problems ______
14. There are some problems I will just not try ______
15. I do not like to try problems that are hard to understand ______
16. I will keep working on a problem until I get it right ______
17. I like to try to solve problems ______
18. I give up on problems right away ______
19. Most problems are too hard for me to solve ______
20. I am a good problem solver ______

Scoring is as follows:

Willingness to engage in problem solving (2,3,5,14,15,17)
Perseverance during problem solving process (1,4,8,10,16,18)
Self-confidence with respect to problem solving (6,7,9,11,12,13,19,20)
Positive (3,5,8-11,13,15,17,20) — 0 for false, 1 for true
Negative (1,2,4,6,7,12,14,15,18,19) — 0 for true, 1 for false

Analytic Scoring Scale

Cruikshank and Sheffield (1992) describe an analytic scoring scale developed by Charles, Lester, and O'Daffer (1987) where *all* written evidence produced by a student in solving a problem is considered. The focus of holistic scoring is the process of solving (or attempting to solve) a problem — not just the solution. Analytic scoring requires the teacher to assign points to selected aspects of the problem solving process, and apply specified criteria to assess the student's work. Charles, Lester, and O'Daffer (1987) (cited in Cruikshank & Sheffield, 1992, p. 49) provided the following example:

Analytic Scoring Scale		
Understanding the problem	0:	Complete misunderstanding of the problem
	1:	Part of the problem misunderstood or misinterpreted
	2:	Complete understanding of the problem
Planning a Solution	0:	No attempt, or totally inappropriate plan
	1:	Partially correct plan based on part of the problem being interpreted correctly
	2:	Plan could have led to a correct solution if implemented properly
Getting an Answer	0:	No answer, or wrong answer based on an inappropriate plan
	1:	Copying error; computational error; partial answer for a problem with multiple answer
	2:	Correct answer and correct label for the answer

The correspondence with Polya's stages is evident, and the criteria specify what the teacher should look for. While the marks can be added to give a single score, the intention is that the part-by-part analysis indicates student developmental priorities.

2. One omission from the Analytic Scoring Scale given above is that it does not include criteria for judging the ability of the student to communicate. In the box below describe criteria for assessing the ability to communicate.

Communicating	0:	
	1:	
	2:	

Investigations

A mathematical investigation is usually a more extensive type of problem solving exercise. However, it is usually more open-ended and can be interpreted or extended by the learner in many different ways. A major use of the investigation process is to enable students to explore important mathematical concepts and relationships in their own way. Students are thus able to make important mathematical discoveries. In most instance such discoveries will already have been made before them, but in some cases new discoveries are made. Investigations provide a useful medium for the student to work mathematically.

The main stages of investigations are:

- data gathering
- analysis of data for findings, conclusions
- explanation and justification
- extension

Being open ended, most mathematical investigations lend themselves to extended inquiry over long periods of time if the student so desires.

1. One useful source of material for mathematical investigations is provided by Bastow, Hughes, Kissane and Mortlock (c1983). Undertake the following investigations from their collection.

   (a) The country of Funnymoney can mint only two denominations of coin and no paper money. Investigate the amounts of money that can be made. Investigate.

   (b) Investigate the number of rectangles in rectangles drawn on grid paper.

   (c) On what day of the week would your 100th birthday fall? Investigate this and other days of interest.

2. Investigations are not only for middle and upper primary years. There are possibilities at the junior primary levels as well. For instance, the "Hundreds Chart" is a rich source for investigation of patterns and relationships for all primary year levels.

   (a) Give an example of a stimulus you could give to encourage investigation of the "Hundreds Chart" (see next page) at each of the levels indicated.

   Years 1-3: ____

   ____

   Years 3-5: ____

   ____

   Years 5-7: ____

   ____

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

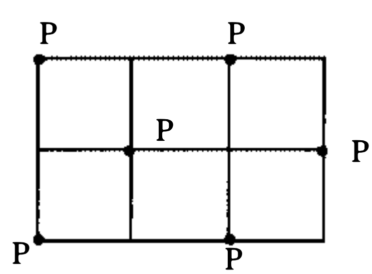
(b) Investigate what happens if you use a different Hundreds Chart.

3. (a) A large cube is constructed of smaller cubes and is painted blue. When the paint is dry, the cube is dismantled into the original smaller cubes. With a colleague, investigate the number of blue-painted faces on the smaller cubes. Write a brief report.

   (b) Check pp. 26-27 of the Student Work Sample and rate your performance.

4. In a Square town, street blocks are square. Patrolling policemen can effectively see for a distance of one length of a block in any direction. Investigate the number of policemen needed to effectively patrol city areas of various sizes.

e.g.



6 police needed to patrol this city area.

Self Directed Activity

1. Find four resources — such as books, journals, CD-ROMs, Internet sites — that you would find useful as sources of a variety of problems for the primary school classroom. Reference each resource correctly below, and add a brief annotation of its contents and relevance:

Resource 1: ______________________________

Comment: ______________________________

Resource 2: ______________________________

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Resource 3: ___

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References

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Chapter 7: Space — Scale and Similarity

As a result of studying this chapter and completing the set tasks you will:

- understand the concept of similarity in the context of geometry;
- identify real-world situations where scaled enlargements and reductions are relevant;
- determine the relationships between similar shapes in terms of length, area and volume;
- be able to use a variety of strategies to make scaled enlargements and reductions of two- and three-dimensional shapes; and
- appreciate the importance of scale and similarity in developing and reinforcing a variety of mathematical processes and content.

Syllabus and SOS References

Mathematically if one shape is a scaled enlargement or reduction of another, we say that they are geometrically *similar*. Note that in everyday use the word "similar" does not have this precise meaning. This is common problem associated with language in mathematics. The study of similarity and scale is a valuable part of the mathematics program because it gives scope for the treatment of most of the areas of Working Mathematically. Children will certainly get many opportunities to *contextualise mathematics* because scale enlargements and reductions are very much part of a child's world. This is well typified by examples such as —

- model cars and planes;
- a dolls house;
- enlargements of photographs;
- maps in an atlas; and
- viewing through a magnifying glass.

Part Four of the Space Strand of the WA K-7 Syllabus (Ministry of Education, 1989) is entitled Transforming Shapes. One aspect of this section of the Syllabus is involved with the scaled enlargement or reduction of shapes. It is part of a child's experience from Stage 1 onwards, becoming more formal from Stage 4 onwards. Remember that the so-called stages in the syllabus correspond approximately to school year levels.

The Student Outcome Statements (EDWA, 1998) indicate students should attain this concept in the Space strand at Level 5. The entries are given below.

Substrand: Represents Transformations

Students visualise and show the effect of transformations on shapes and arrangements.

Level S 5.3

Visualises and sketches the effect of straightforward translations, reflections, rotations and enlargements of figures and objects using suitable grids.

(EDWA, 1998, p. 53)

Pointers

Consult the SOS book (p. 77) to check the pointers for Level S 5.3.

Activities

As you perform the tasks below keep in mind the ways you may be Working Mathematically by -

- Contextualising Mathematics;
- Using Mathematical Strategies;
- Reasoning Mathematically; and
- Applying and Verifying.

1. List three other examples of scale enlargements.

2. Are the following examples of scale enlargements/reductions? Explain.

   (a) A 300 mL, a 500 mL, and a litre coke bottle.

   (b) A Barbie Doll.

3. A toy truck is made on a scale of 1: 80; i.e., its linear dimensions are one eightieth of the original.

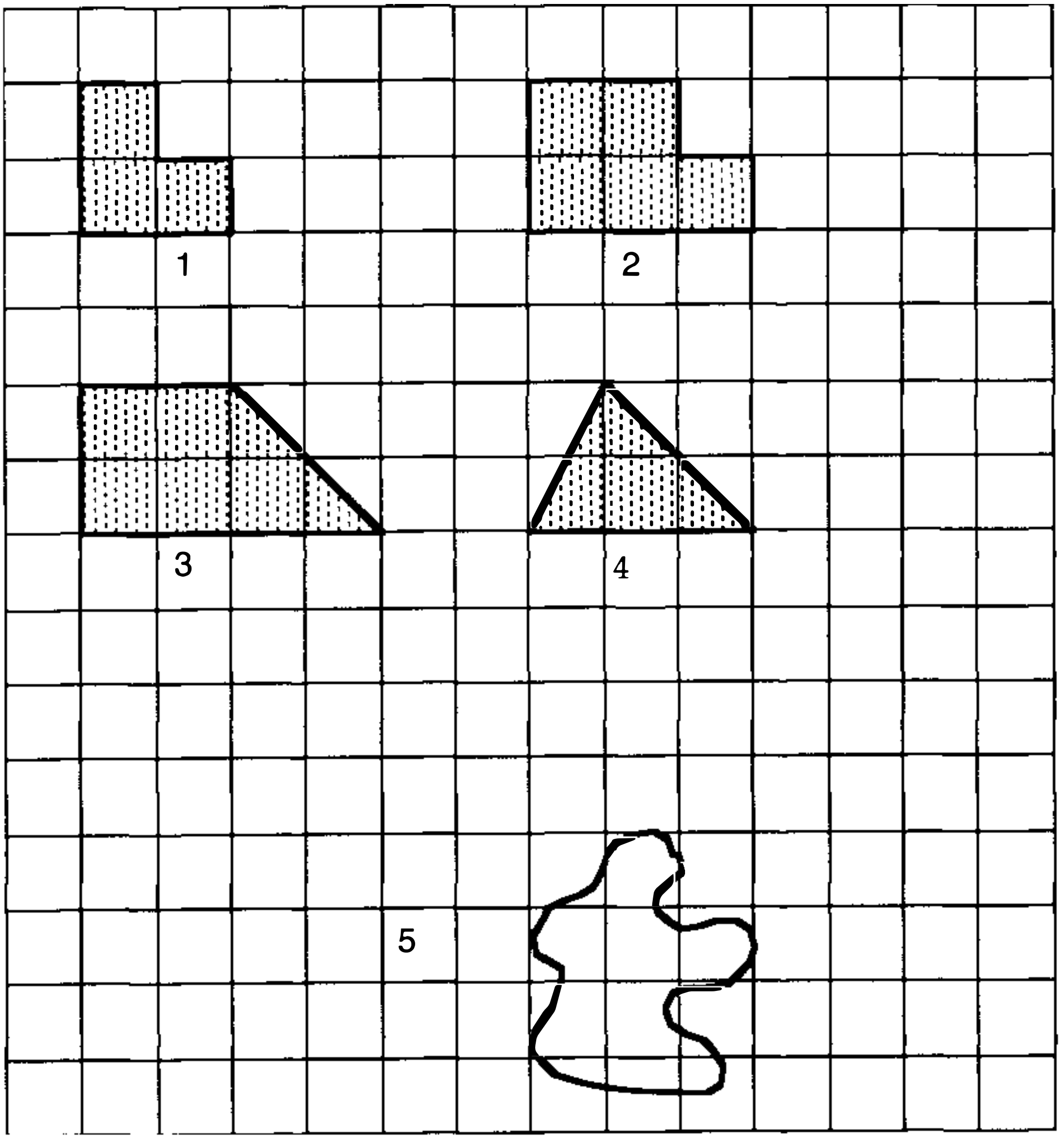
   (a) In what way is the body of a model car or truck not an accurate scaled reduction?

   (b) If the toy truck's mass is 55 grams, determine the mass if the scale model is enlarged to the original size. At the end of this chapter you should be able to answer this question.

Integration of Content

In the activities that follow, try to identify the different content areas of space, measurement and number that are incorporated in the tasks.

4. On the grid paper supplied, construct a double scale model of each shape below.



(a) Complete the table below.

Shape	Perimeter original	Perimeter double scale	Area original	Area double scale
1				
2				
3				
4				
5				

(b) How do the perimeters of the originals compare with those of the double scale? Explain why.

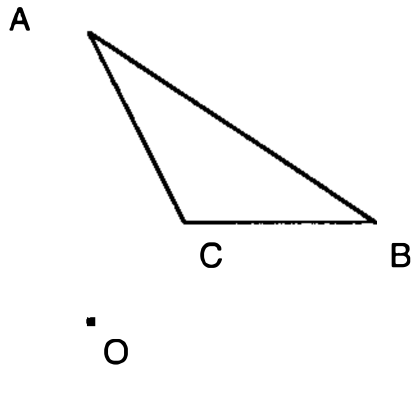
(c) How do the areas of the originals compare with those of the double scale? Explain why.

(d) With shapes 1 to 4 try to fill the double-scale shape with copies of the original shape. Explain your findings.

(e) Suggest a more "user friendly" method of enlarging Shape 5.

(f) Predict the area and perimeter of Shape 2 if enlarged by a scale factor of three. Verify your result. What if the scale factor was 100:1?

5. Another method of enlarging (or reducing) shapes is the centre-of-enlargement technique. Copy this diagram onto a sheet of paper in the lower left-hand corner, then follow the directions below.



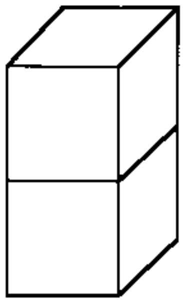
From O, lines are drawn through A, B, and C.

For a double scale enlargement, we extend OA to A' so that OA' is twice OA,
OB to B' so that OB' is twice OB, and
OC to C' so that OC' is twice OC.

Triangle A'B'C' will be a double-scale enlargement of triangle ABC.

Note that the centre of enlargement can be inside, outside or on the shape. Verify that the resulting shape is a double scale enlargement of the original. Investigate with different places for the centre and scale factors other than 2:1.

6. Using two of the cubes provided, construct the following model.



(a) In turn, construct a double, triple, and quadruple scale model of this shape and complete the first four rows of the table below. Without constructing the 5:1 and 6:1 scale models, complete the table by the identification of number patterns.

Scale factor	Height (units)	Surface Area (squares)	Volume (cubes)	$\frac{\text{surface area}}{\text{volume}}$
1	2	10	2	$10 \div 2 = 5$
2				
3				
4				
5				
6				

(b) What happens to the surface area in relation to the volume as the shape gets larger?

____

____

(c) Suggest a "real life" application of this relationship.

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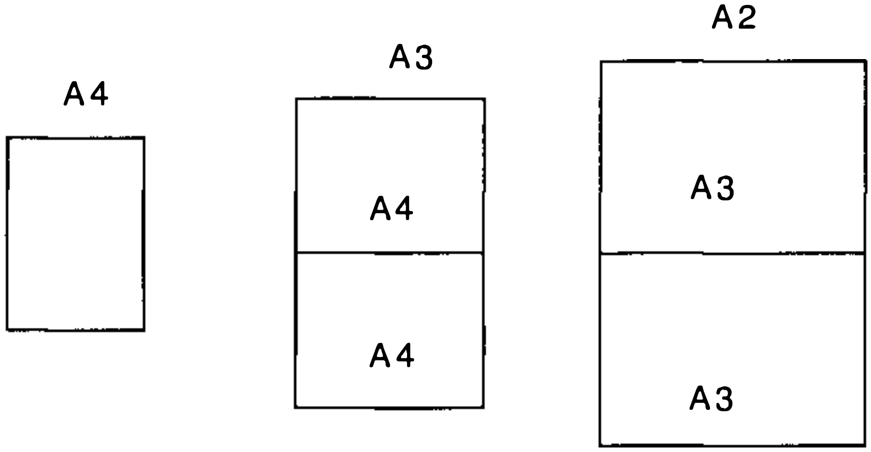
7. This page is an A4 size. When two A4 sheets are put together they make an A3 sheet. When two A3 sheets are put together they make an A2 sheet, and so on as illustrated below (not drawn to scale).



(a) Complete the table below, starting with the dimensions of an A4 sheet.

Paper	length	width	area	l/w
A6				
A5				
A4	29.7 cm	21.0 cm	623.7 cm^2	1.41
A3				
A2				
A1				
A0				

(b) What information on the table indicates that the sheets are geometrically similar?

____

(c) What is the significance of the area of the A0 sheet?______________________________

(d) What is the inverse of 1.41 and how are these two numbers important on a photocopier?

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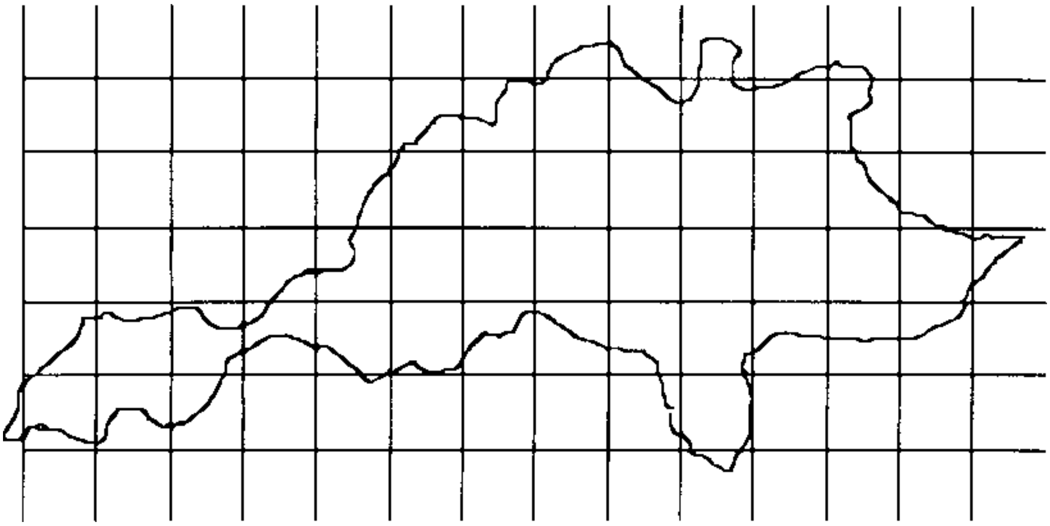
8. Below is a map of Rottnest Island on a one-centimetre grid. The scale of the map is 1 cm : 0.8 km. Determine the approximate area of the map of the island in square centimetres. Use the scale factor to determine an estimate of the area of Rottnest in square kilometres.



9. In groups of four, construct a 3 : 1 scale model of a matchbox and match. Each person constructs one of the following:

- the outer section, the label, the inside container, a match.

10. Select an object in the classroom and construct a half-scale or quarter-scale model of that object using cardex.

Reflection

After completing some or all of these activities make a list of

- the mathematical content in the activities;
- the mathematical processes used; and
- ways in which you consider you were working mathematically.

References

Curriculum Branch, Ministry of Education. (1989). *Learning mathematics: Pre-primary to Stage 7 mathematics syllabus.* Perth: Author.

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Notes

Chapter 8: Space — Tessellations

On completing this chapter you will be able to:

- describe a tessellation;
- identify common shapes that will tessellate;
- explain why some shapes will tessellate and others will not;
- identify examples of tessellations in the real world;
- create tessellating patterns using a variety of methods; and
- locate Internet websites related to tessellations.

Background Information

Below is a dictionary definition of *tessellation* from the *Illustrated Maths Dictionary for Australian Schools*

> A tessellation is a complete covering of a surface by one or more figures in a repeating pattern with no overlapping of or gaps between the figures. Certain shapes will cover a surface completely; for example squares, rectangle and triangles. These are said to tessellate. A circle is not a tessellating shape.

Tessellating shapes can be a useful source of enrichment in the primary mathematics program. As well as leading to a simple but effective art form, they offer us a vehicle for the investigation of shape and angle properties that can be adapted to a wide range of mathematical abilities. Studies in this area can directly relate to Curriculum Framework Overarching Learning Outcomes numbers 2, 4, 10 and indirectly to several others. This will also involve most of the sub-strands of the Working Mathematically Student Outcome Statements. In the Space Strand the sub-strand Represent Transformations embraces this topic from Level 2 onwards. The Level 2 Space SOS is as follows:

> **S 2.3** The student uses multiple copies of shapes to construct repetitive patterns and follows and describes simple movement rules for generating such patterns.

(EDWA, 1998, p. 69)

As with most teaching areas we should try to begin with the child's experiences in the real world.

1. What are some of the situations where a child will see tessellating patterns ?

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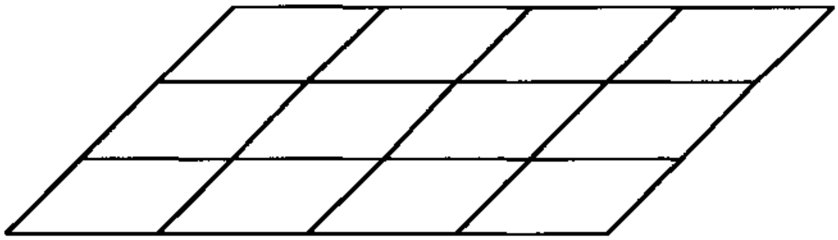
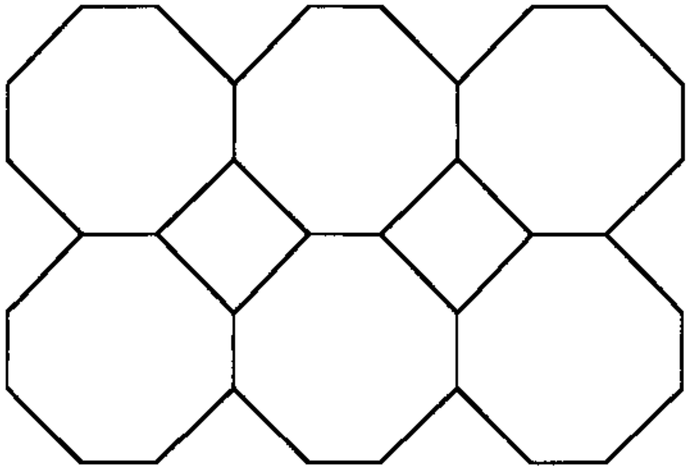
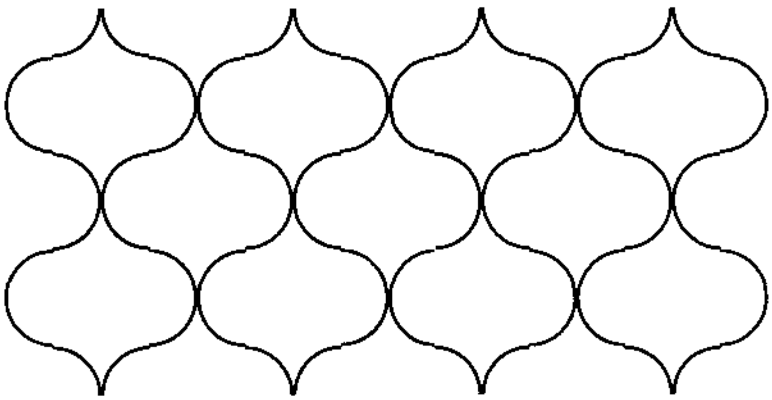
Some tessellation patterns use just one shape, as shown below.



Others use two shapes to form the tessellating pattern.



There are many curved shapes that tessellate, and these are often features of designs on wall paper and floor coverings.



Children's first experiences should be with concrete materials that they can handle and explore.

Activities

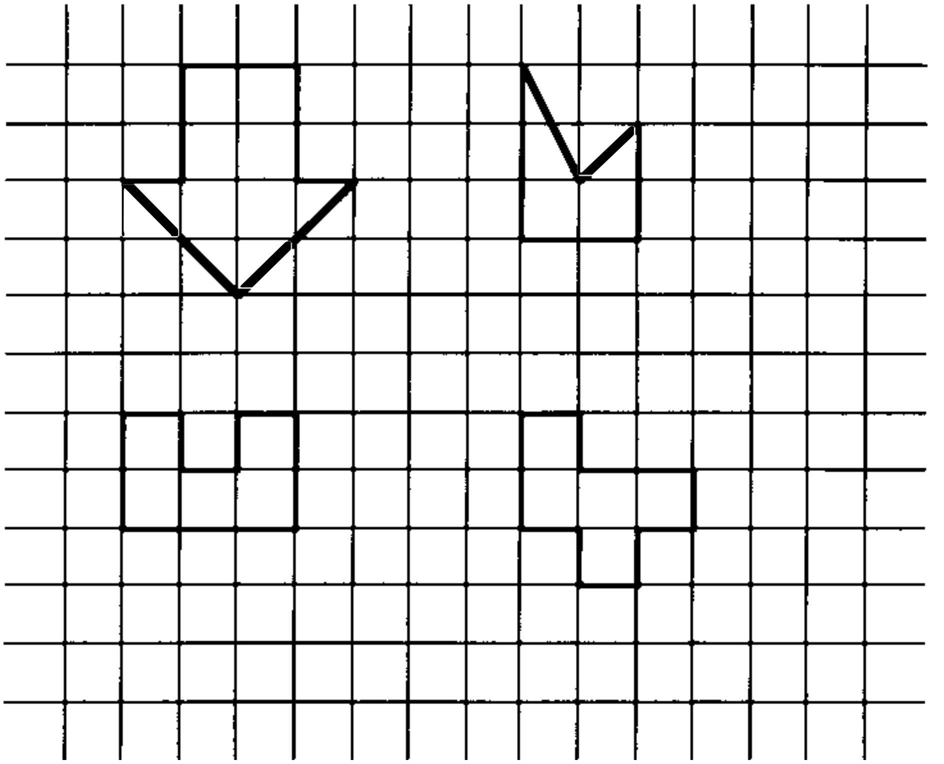
1. Make patterns with the tessellation kit supplied.

2. Using the pattern blocks make a list of shapes that will tessellate.

In later activities children can be given some experiences tessellating shapes on grid paper; i.e., they progress to pictorial representations.

3. On the grid paper supplied draw three different patterns for brick paving, using 2 by 1 bricks.

4. Using grid paper show that each of the following shapes tessellates.



5. You are to set up a factory to make ceramic floor and wall tiles. What different shapes could you make so that the tiles will tessellate?

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____

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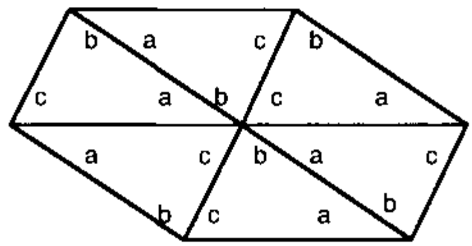
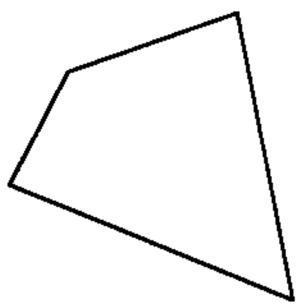
After exploring both concrete and pictorial examples of tessellations, children in the upper grades can explore the mathematics behind the concept of tessellation. It is obvious that shapes such as rectangles and parallelograms will tessellate as the property is easy to visualise with those shapes. You can verify intuitively that triangles and trapeziums will tessellate because in pairs they will form parallelograms. To explore mathematically whether or not a shape will tessellate requires an investigation of the angle properties of the shape. Clearly for shapes to tessellate, it is necessary for them to make complete joins (no gaps) where they meet; i.e., it must be possible to use the angles of the shape to make up 360° joins.

Triangles will tessellate because the angle sum is 180°.



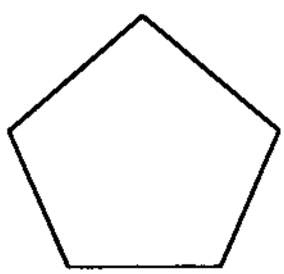
Squares and rectangles will tessellate because their 90° angles will also form 360° joins. The following activity looks at quadrilaterals in general.

6. Verify that the sum of the angles of a quadrilateral is 360°, and show that this shape tessellates by folding a sheet of paper three times and cutting out 8 copies of the quadrilateral.

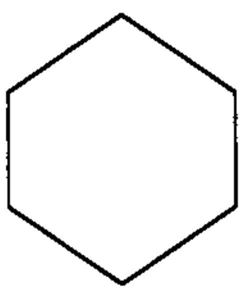


7. Investigate the angle sizes of regular polygons to determine which of the shapes below will tessellate. Use the strategy developed in MPE1108. Note that the diagrams are not accurate.

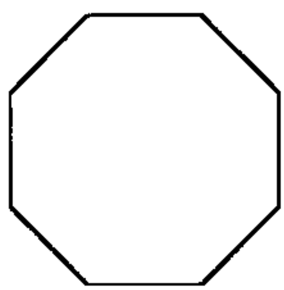
Regular Pentagon



Regular Hexagon



Regular Octagon

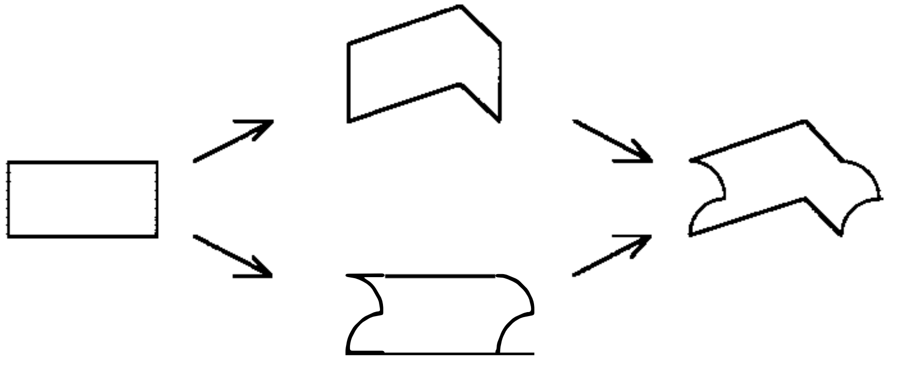


What can be concluded from the angle sizes?

Having explored real-world, concrete, pictorial and abstract aspects of tessellations we now return to the real world for applications of tessellation in the area of art and design. Mathematical concepts often begin and end with the real world representations.

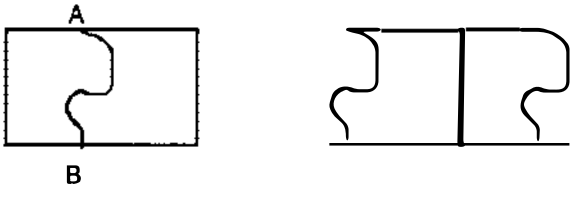
Escher-Type Drawings

Dutch artist Maurits Escher extended the mathematics of tessellations to an intricate art form. His simple technique was to begin with a basic shape that tessellates and to transform it into a more interesting design that preserves the tessellating property of the shape. Identical changes are made to the opposite pairs of sides, the combination producing a new shape.



You can this method to create your own tessellating pattern.

8. A more concrete approach to creating a tessellating pattern is as follows. Start with a small square or rectangular piece of cardboard. Make a curved cut from a point A on one side to the point B below, then rearrange the shape by sliding one piece horizontally as below. The join should then be taped. This process can be continued by cutting from one point on the curved side to a point horizontally opposite, then rearranging by sliding one piece vertically. The new shape can be tessellated on a piece of newspaper using a texta to trace around it. Make such a "tile".



Self-Directed Learning

1. Investigate the computer software package *Tesselmania*. This enables children in the upper primary year levels to create their own colourful tessellations.

2. Explore the Internet to see what you can find out about tessellations. There are a number of good web-sites that can be accessed by using the word *tessellation* using one of the more common search engines. Also, investigate the web-site, Tantalising *Tessellations* at

   http://mathcentral.uregina.ca/RR/database/RR.09.96/archamb1.html

References

Education Department of Western Australia. (1998). *Outcomes and standards framework: Student outcome statements — Mathematics*. Perth: Author.

Illustrated Maths Dictionary for Australian Schools.

Website:

http://mathcentral.uregina.ca/RR/database/RR.09.96/archamb1.html

Chapter 9: Number Sense

As a result of studying this chapter and completing the set tasks you will be able to:

- explain the meaning of number sense and its relation to the mathematics curriculum;
- describe some components of number sense;
- discuss some research into the number sense of school students; and
- evaluate activities intended to develop the number sense of primary school students.

Relationship to Previous Study

In your two previous mathematics education units you have met a number of ideas which can be grouped under the general heading of 'number sense'.

In your first year unit (MPE1108) one of the topics you studied was *Rational Number: Fractions.* In this you learned the following about fraction concepts:

- that fractions represent parts of wholes and subsets of a set, and later ratios,
- that different fractions can represent the same amount (for example $^3/_4$ and $^6/_8$),
- that fractions, decimals, percentages and ratios are all different ways of representing the same number (for example $^1/_2$, 0.5, 50%, 1:2), and
- that children often have misconceptions about fractions as a result of wrongly applying knowledge of whole numbers (for example $^1/_3$ is bigger than $^1/_2$ because 3 is bigger than 2).

In your second year unit (MPE2107) you studied *Early Number Concepts* and *Concepts and Properties of the Four Operations.* You also studied *Mental Computation, Pencil and Paper Computation* and *Calculator Computation.* One chapter, *Computation*, considered the relationship among these three computational methods and the importance of estimation.

In this chapter we will

- draw together the threads of number sense so that we can consider it as a whole,
- look at some recent research into the number sense of primary children, and
- consider a range of activities designed to help children develop number sense.

Revision Activities

Before we start, here are a number of quick challenges to remind you of earlier work. You may want to discuss them in pairs, or you may prefer to work on them on your own.

1. Explain why $^1/_3$ is smaller than $^1/_2$.

2. Represent, with reasons, $2/5$ as a decimal, a percentage and a ratio.

3. Suggest some ways of classifying numbers.

4. Why might a child say that 0.3 is smaller than 0.21?

5. Using place value ideas I can re-write 314 as 200, 11 tens, and 4 units. Give two other ways of re-writing 314.

6. Subtraction can arise from three quite different situations: 'take away', 'difference between', 'complementary addition'. Give an example of each and explain why they are all different.

7. Mentally calculate 25 + 89. Describe your strategy to a friend and compare it with theirs. How many different strategies can you think of for mentally calculating 25 + 89?

8. What is meant by a 'formal written algorithm'? What are some of the reasons why it is suggested that less attention is paid to the development of formal written algorithms in school?

9. What are some of the reasons why calculator use is encouraged in primary schools?

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10. Estimate the answer to $^{12}/_{13} + {}^{7}/_{8}$. Which of the answers 1, 2, 19, 21 do you think is the best estimate? Why?

____

____

11. Why do you think being able to estimate the answer to this fraction addition might be considered by some a better indicator of number sense than being able to calculate the answer exactly?

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What Number Sense is and What it isn't

To take the second part first:, Number Sense isn't the same as skill in written computation. Here are two examples which illustrate the point. Discuss each in turn. Explain why each might be considered by some to make the point that Number Sense isn't the same as skill in written computation, and then state your own opinion with reasons.

1. The Cost of Two Diaries (A True Story)

One of the authors went into a newsagent shortly after Christmas one year to buy two identical diaries. Each was marked "$2.50 but above the pile was the sign "Diaries half marked price". He took them to the counter and handed them to the assistant. "How much?" he asked. The assistant took one diary, wrote the marked price on a piece of paper, performed a written calculation and obtained the answer '$1.25' She then picked up the other (identical) diary, wrote down the marked price, performed a second written calculation and again obtained the answer '$1.25'. She then wrote these two answers as an addition sum and obtained the answer. With no trace of recognition she handed the diaries to him with the remark, "that will be two dollars fifty".

Comments:______________________________________

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2. Calculating versus Estimating (extract from account of research from Taiwan)

A twenty-item Written Computation Test (WCT) was constructed by the researcher for Year 7 and Year 9. All of the test items were displayed in the native language of Taiwanese students. The WCT items were open-ended and consistent with the Taiwanese national mathematics curriculum. A forty-item Number Sense Test (NST) was also constructed by the researcher. The same test was used for both year levels. The first 20 items of the NST were developed to parallel the WCT items. That is, the same numbers were involved but the format of the items was different. Some examples show a sharp difference between their written computation performance and number sense ability. For example, Table 1 shows the percentage of the same cohorts of students giving each response on two parallel items from the WCT and the NST respectively.

Table 1: Percentages of Taiwanese student respondents for NST & WCT items (N = 115 Yr 7, 119 Yr 9)

NST Item: Without calculating an exact answer, circle the best estimate for $^{12}/_{13} + {}^{7}/_{8}$			WCT Item: Calculate $^{12}/_{13} + {}^{7}/_{8}$		
	Yr 7	Yr 9		Yr 7	Yr 9
A. 1	10	20	Correct	61	63
B. 2 *	25	38	Incorrect	39	37
C. 19	36	14			
D. 21	16	12			
E. I don't know	10	16			
F. No response	3	0			

Reys, Reys, McIntosh, Bana & Farrell (1997, p. 49)

Comments:____

What then is number sense? The following is a recent description:

> Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations. It is characterised by a desire to make sense of numerical situations, by looking for links between new information and previously acquired knowledge, and by an innate drive within the learner to make the forming of these connections a priority. Those with well-developed number sense have strongly connected knowledge and in learning they appreciate the importance of making links between new and previously acquired knowledge. They approach problems involving number by seeking to make sense of the whole situation and by using the multiple representations and connected knowledge that they have...Various "indicators" of number

sense have been hypothesised. These include: well-understood number meanings, existence of and reliance on multiple numerical relationships, recognition of relative magnitude of numbers, awareness of the relative effect of operating on numbers; and use of referents for measures of common objects and situations in their environments offers additional indicators: using the decimal structure of the number system to decompose and recompose numbers to simplify calculations, tending to want to "make sense" of situations involving number and quantity, and using "benchmark" information/data to derive new information (such as using a known number fact to calculate an unknown fact).

(Adapted from Reys, Reys, McIntosh, Bana, & Farrell, 1997)

3. Do you find any parts of this description difficult to follow? If so, discuss these and decide what each means.

   Comments:____

4. Write in your own words a short description which you think conveys the essential meaning of *number sense*.

   Comments:____

Six Strands of Number Sense

On the following pages we consider six components or strands of number sense. For each strand a sample assessment item is given, together with some data on the results of giving that item to school students in Western Australia (Reys, Reys, McIntosh, Bana, & Farrell; 1997). In each case:

(a) Write what the results show and write a comment on these findings; and

(b) Write another question which you think would assess some aspect of the same number sense strand.

Understanding the Meaning and Size of Numbers (Number Concepts)

Description

Understanding of the base 10 number system (whole numbers, fractions, and decimals), including patterns and place value which provide clues to the meaning/size of a number (e.g., $^5/_6$ is a fraction less than one, it is close to one because of the relationship between the numerator and denominator;

OR 1000 is a large number if you are referring to the population of a school but a small number if you are referring to the population of a town). Could involve relating and/or comparing numbers to standard or personal benchmarks. Includes comparing the relative size of numbers within a single representational form.

Sample Item (Years 7, 9)

How many different <u>fractions</u> are there between $^2/_5$ and $^2/_5$? Circle your answer and then fill in the blanks.	A None. Why? ____________________ B One. What is it ? _____ C A few. Give two:________ and ______ D Lots. Give two. ________ and ________

Results

Response %	Aus Yr 7	Aus Yr 9
A	57	29
B	29	21
C	4	7
D & examples*	7	40
Other	3	3

* Correct response

Summary Comments: ____

____

____

Another Item: ____

____

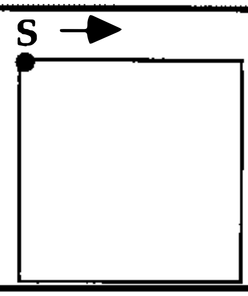
Understanding and Using Equivalent Forms and Representations of Numbers (Multiple Representations)

Description

Recognition that numbers take many different numerical and representational forms (e.g., a fraction as a decimal, a whole number in expanded form, or a decimal on a number line) and can be thought about and manipulated in many ways to benefit a particular purpose. Ability to identify and/or reformulate numbers to produce an equivalent form. Use of decomposition and recomposition to reformulate numbers for ease of processing. Relating and/or comparing sizes of numbers to a physical referent (e.g. collection of items, shaded region, or position on a number line). Includes crossing among various representational forms.

Sample Item (Years 5, 7)

You are going to walk once around a square-shaped field. You start at the corner marked **S** and move in the direction shown by the arrow. Mark with an **X** where you will be after 1/3 of your walk.



Results

Response %	Aus Yr 5	Aus Yr 7
In range*	25	51
Right edge	19	19
Other	56	30

Summary Comments: ______________________________

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____

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Another Item: ______________________________

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Understanding the Meaning and Effect of Operations (Effect of Operations)

Description

Understanding the meaning and effect of an operation either generally or as it relates to a certain set of numbers (e.g., division means breaking a number into a specified number of equivalent sub-groups; OR multiplying by a number less than one produces a product less than the other factor). Includes judging the reasonableness of a result based on understanding the numbers and operations being employed.

Sample Item (Years 5, 7)

When a 3-digit number is added to a 3-digit number the result is:

A always a 3-digit number

B always a 4-digit number

C always a 5-digit number

D either a 3, 4 or 5-digit number

E either a 3 or 4 digit number

Results

Response %	Aus Yr 5	Aus Yr 7
A	16	10
B	3	5
C	3	4
D	43	29
E *	32	52
No Response	3	0

Summary Comments: ____

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____

____

Another Item: ____

____

Understanding and Use of Equivalent Expressions (Equivalent Expressions)

Description

Translation of expressions to equivalent forms. Generally used to re-evaluate and/or more efficiently process computation. Includes understanding and use of arithmetic properties (commutativity, associativity, distributivity) to simplify expressions and develop solution strategies (e.g., use of distributive property to multiply 7 x 52).

Sample Item (Years 7, 9)

0.5 x 840 is the same as:	A $840 \div 2$ B 5 x 840 C 5 x 8400 D $840 \div 5$ E 0.50 x 84

Results

Response %	Aus Yr 7	Aus Yr 9
A *	32	65
B	18	2
C	14	19
D	7	6
E	30	9

Summary Comments: ____________________

____

____

____

Another Item: ____________________

____

Computing and Counting Strategies

Description

Applying various number sense components previously described in the formulation and implementation of a solution process to a counting or computational (estimation, mental computation, paper/pencil, calculator) situation (e.g., Is 29 x 38 more or less than 400? OR How many birds are in the sky?)

Sample Item (Years 3, 5, 7)

About how many days have you lived? (Circle the nearest answer.)	A 300 B 3000 C 30 000 D 300 000

Results

Response %	Aus Yr 3	Aus Yr 5	Aus Yr 7
A	31	7	1
B *	34	35	38
C	24	37	48
D	11	21	13

Summary Comments: ____________________

____

____

____

Another Item: ____________________

____

Measurement Benchmarks

Description

Applying various number sense components previously described in the formulation and implementation of a solution process to a measuring situation. Requires an understanding and use of standard, non-standard and/or personal benchmark units of measure (e.g., a textbook weighs about a kilogram or 5 oranges make a pound, or the angle is slightly less than a right angle so it must be about 85 degrees). Involves measuring attributes such as mass, length, area, capacity, volume, time, angles.

Sample Item (Year 5, 6)

What is the weight of a man who is 180 cm tall? Ring the best answer.	A. 10 kg B. 40 kg C. 80 kg D. 180 kg

Results (Not available)

Another Item: ____

Activities to Develop Number Sense

The following pages provide activities which can be used to focus on some aspect(s) of number sense. When using them with children it is important to allow time for discussion and for individual children to explain their thinking. These activities are designed for use by a teacher with a whole class, and the activity can be put on an overhead projector or each child can be given a copy.

For each activity answer the following questions:

1. What is the purpose or focus of the activity? How might you use the stimulus given?
2. For what age group(s) is the activity suitable? Why?
2. To which of the six number sense strands does the activity relate? Why?
3. Suggest some possible responses of children to the activity.
4. Suggest some possible follow-up whole-class activity.
5. Suggest an extension activity for individual children.
6. Suggest a form of assessment for the activity.

References

Reys, B, Reys, R, McIntosh, A., Bana, J., & Farrell, B. (1997).*Number sense in school mathematics: Student performance in four countries.* Perth: MASTEC, Edith Cowan University.

McIntosh, A.J., Reys, B.J., Reys, R.E., & Hope, J. (1996a). *Number sense: Grades 1-2.* Palo Alto: Dale Seymour Publications.

McIntosh, A.J., Reys, B.J., Reys, R.E., & Hope, J. (1996b). *Number sense: Grades 3-4.* Palo Alto: Dale Seymour Publications.

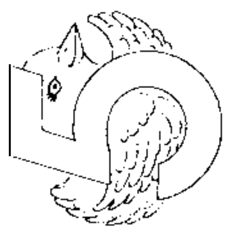
McIntosh, A.J., Reys, B.J., Reys, R.E., & Hope, J. (1996c). *Number sense: Grades 4 -6.* Palo Alto: Dale Seymour Publications.

Appendix

The appendix which follows includes copies of several pages from McIntosh, Reys, Reys, & Hope (1996a, 1996b, 1996c).

EXPERIENCE 13 / *Activity 1*

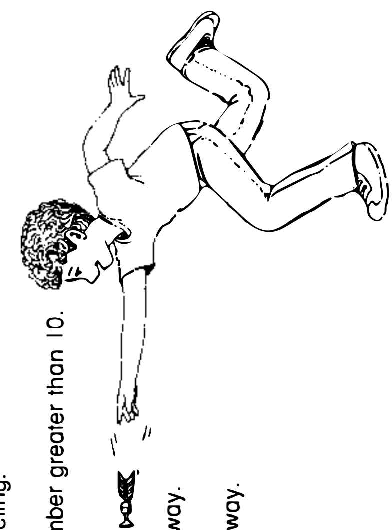
Today's Target



Today's target is **5**

Try to make the target by

1. adding two numbers.
2. subtracting two numbers.
3. adding three numbers.
4. adding and subtracting.
5. starting with a number greater than 10.
6. starting with 5.
7. using the easiest way.
8. using the hardest way.



66 Number SENSE / Grades 1-2

EXPERIENCE 14 / *Activity 3*

Adding Compatibles

Cover pairs of numbers that add to 10.

Which number is left over?

3 5 9

6 2

8 4 1

5 7 3 5

1 6 2 9

7 4 8

74 Number SENSE / Grades 1-2

Find a Connection

Choose any three numbers, and show how they are related.

35 **7** **21**

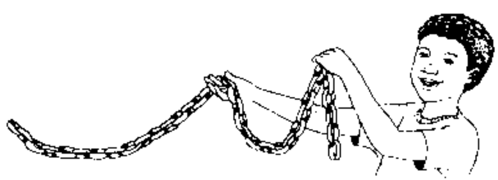
9

4 **15** **8**

12

32 **3**

5 **11**



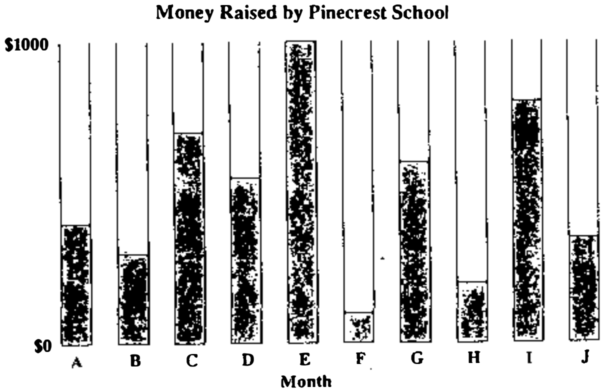
Name a Fraction

Working in your team, make up one answer to fit each statement.

	Possible Answers
1. A fraction between $\frac{1}{4}$ and $\frac{1}{2}$	________
2. A fraction greater than $\frac{3}{4}$	________
3. A fraction less than $\frac{1}{3}$	________
4. A fraction that is a little more than 1	________
5. A fraction between 2 and 3	________
6. A fraction near 0	________
7. A fraction that is almost 1	________
8. A fraction that is almost $\frac{1}{2}$	________
9. A fraction between $\frac{1}{3}$ and $\frac{1}{4}$	________
10. A fraction that is nearly 3	________

How Much?

Money Raised by Pinecrest School

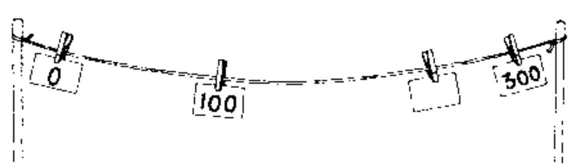


Pinecrest School is raising money to buy new computers. They raised $1000 in 10 months. The bars show how much money they had at the end of each month.

Put the bars in order. About how much money did Pinecrest School have at the end of each month?

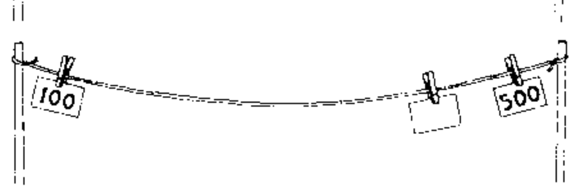
Numbers on a Line

1.

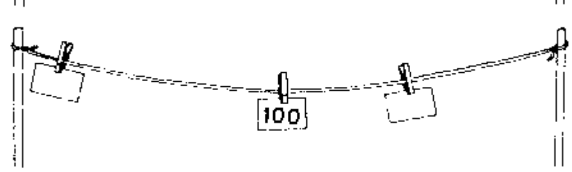


2.

3.



4.



Estimate the value of each blank number card.

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Mathematics, Science and Technology Education Centre
Edith Cowan University, Perth, Western Australia